Lumpy Investment, Sectoral Propagation, and Business Cycles

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Abstract

This paper proposes a model of endogenous fluctuations in investment. A monopolistic producer has an incentive to invest when the aggregate demand is high. This causes a propagation of investment across sectors. When the investment follows an (S,s) policy, the propagation size can exhibit a significant fluctuation. We characterize the probability distribution of the propagation size, and show that its variance can be large enough to match the observed investment fluctuations. We then implement this mechanism in a dynamic general equilibrium model to explore an investment-driven business cycle. By calibrating the model with the SIC 4-digit level industry data, we numerically show that the model replicates the basic structure of the business cycles.

1 Introduction

This paper concerns a propagation mechanism in investment across sectors. The large fluctuation in investment is often considered as a driving force of business cycles. Also the investment fluctuation is characterized by the synchronized oscillation across sectors. We propose a model of investment propagation which quantitatively explains this phenomenon and identifies the parameters at work.

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The sectors are linked each other by derived factor demand when each sector uses other sectors' product as intermediate inputs. Their interaction forms a positive feedback in capital adjustments in the network of input-output relations. Suppose that a capital adjustment takes a form of discrete decision. Then there is a chance of a chain-reaction of investment in which an investment in one sector triggers an investment in another sector, and so on. This chain-reaction turns out to be represented by a branching process in an equilibrium model. We then find that the total size of the chain-reaction can exhibit a very large variance in some parameter range.

Quantitatively, we ask the following question: given the magnitude of sectoral oscillations in the U.S. economy, how do the sectoral fluctuations add up to the aggregate fluctuations? It is immediately clear that just summing up the independent series of the sectoral oscillations do not amount to the aggregate fluctuations observed in the U.S. production. There must be some sectoral comovements. Our model provides a model of sectoral comovements which is able to simulate the magnitude of aggregate fluctuations observed in the U.S. when the responses of real wage and real intrest rate to aggregate production are modest.

This paper casts a new perspective on the much discussed issue of investment fluctuations. Traditional macroeconomics as well as the benchmark real business cycle theory supposes the aggregate shocks, such as money supply, aggregate productivity, or animal spirits of investers, as the fundamental shock. Without apparent evidence of such aggregate shocks as the consistence cause of the business cycles,[1] however, the literature needs some mechanism that propagate and amplify the shocks on disaggregated parts of economies. The disaggregated model of the aggregate fluctuations turns out to face the law of large numbers: the tendency that disaggregated shocks cancel out each other. In many models the tendency is so strong that a realistic magnitude of an individual shock does not generate aggregate fluctuations large enough to match the data. For example, Long and Plosser (1983) show that a general equilibrium model can in principle generate comovement across sectors when sectors bear idiosyncratic productivity shocks. In a successive research, however, Dupor (1999) establishes that their model cannot generate the aggregate fluctuations unless the individual shock is of order the size of the number of individuals in the economy.

This paper shows that the law of large numbers can be overcome. We show that the propagation distribution in our model has a heavier tail than the normal distribution which characterizes a large class of aggregative models. The propagation size also exhibits critical fluctuations in which the propagation size does not have mean and variance in the limiting case at which wage and interest rate are determined independently

[1]See Cochrane (1994) for a careful discussion.

from the product market. This proposition assures that any magnitude of aggregate fluctuation can be obtained in the model when the price response to aggregate product is sufficiently slow.

Another line of research on investment fluctuations focused on the endogenous fluctuations which result from non-linearity of economic dynamics. The models of multiple equilibria, chaos, or self-fulfilling expectation show the possibility that the aggregate fluctuations occur in a deterministic environment of economic fundamentals if the non-linearity is sufficiently strong. This paper explores a new approach along this line, in which an interaction of many small non-linear behaviors causes a deterministic fluctuation. We suppose that individual sectors follow a deterministic pattern of capital oscillations with occasional large adjustments and periods of inertial depreciation. The sectors monopolistically compete each other, so an increase in production in a sector induces other sectors to increase their production (and cut prices). Thus the timing of occasional capital adjustments may be endogenously synchronized. This interrelation makes the product markets a multi-dimensional non-linear dynamical system which in principle is capable of generating an endogenous complex fluctuation. The result obtained here can be seen as a generalization of the critical fluctuations demonstrated by Bak, Chen, Scheinkman, and Woodford (1993) in particular. They show a power-law distribution of production propagation in a network of locally interacting producers. We implement a similar propagation mechanism in an equilibrium model of globally interacting sectors. We find that a power-law distribution appears at a limiting case, and near the limit any magnitude of fluctuation is observed for a system of a large number of individuals.

This paper is related to the question of whether a micro discrete choice, in particular an (S,s) behavior, is relevant in aggregate fluctuations. It has been noted that an establishment level capital is adjusted only occasionally but by a jump in size. A series of research, among others Cooper, Haltiwanger, and Power (1999), has stressed the role of the lumpy adjustments played in business cycles. Theoretical studies on aggregation of (S,s) behaviors, for example Caplin and Spulber (1987) and Caballero and Engel (1991), have largely found that such an individual lumpiness does not contribute to aggregate fluctuations. Again, the law of large number is the logic: the individual lumpiness tends to cancel out each other. To the contrary, this paper shows that the (S,s) behavior can generate a considerable magnitude of aggregate fluctuations. In fact, the fluctuation is scale free, in the sense that the variance does not depend on the number of agents, at the limiting case when the wage and interest rate are determined independently from the product markets. The propagation size exhibits a power-law distribution whose mean and variance diverge. This implies that, if there are numerous establishments in an economy, their lumpy investments generate stochastic synchronization which results in a considerable aggregate fluctuation.

After establishing the analytical results on the propagation distribution in a partial equilibrium setup, we simulate a general equilibrium calibrated by a finely disaggregated sectoral data to examine under what conditions the model generates the right magnitude of fluctuations. There have been successful attempts in reproducing the production fluctuations by simulating coupled oscillators (see Selover, Jensen, and Kroll (2003) for example). Here we embed the coupled oscillators in a general equilibrium framework which incorporates a representative household's response to prices, construct the coupling parameters by the fundamentals such as technology and preference, and examine the structure of fluctuations among aggregate variables. We also show that the autocorrelation and correlation structure of the production and demand components matches the usual business cycle patterns.

The rest of the paper is organized as follows. The next section presents the model of investment propagation and the analytical propositions in a partial equilibrium setup. Section 3 numerically examines the quantitative properties of the propagation and the business cycle fluctuations by explicitly incorporating the consumer's behavior. Section 4 concludes the paper.

2 Model of Investment Propagation

In this section we focus on the inter-industrial equilibrium relations in the product market provided with the other prices, namely wage and interest rate. The product market consists of N monopolists and a representative household. Each monopolist j produces a differentiated good Y_j, using capital K_j and labor h_j.

Let us specify the production technology by a constant returns to scale Cobb-Douglas function: $Y_{j,t} = K_{j,t}^{\alpha}(A_t h_{j,t})^{1-\alpha}$, where A_t is a labor-augmenting technology parameter which grows at rate g. We consider a balanced growth path where $Y_{j,t}$, $K_{j,t}$, and consumption C_t grows at rate g and h_t stays constant. Let us normalize the variables by a growth factor A_t as $y_{j,t} \equiv Y_{j,t}/A_t$, $k_t \equiv K_{j,t}/A_t$, $c_t \equiv C_t/A_t$, $i_{j,t} \equiv I_{j,t}/A_t$, etc. Then the production function is written in the normalized terms as:

$$y_{j,t} = k_{j,t}^{\alpha} h_{j,t}^{1-\alpha}. \tag{1}$$

The capital is accumulated over time as:

$$g k_{j,t+1} = (1 - \delta_j) k_{j,t} + i_{j,t} \tag{2}$$

where δ_j is an industry specific depreciation rate. Investment $i_{j,t}$ is a composite good produced by combining all the goods symmetrically as:

$$i_{j,t} = N^{1/(1-\xi)}(\sum_{l=1}^{N}(z_{l,j,t}^{I})^{(1-\xi)/\xi})^{\xi/(1-\xi)} \tag{3}$$

where $\xi > 1$ is the elasticity of substitution between inputs in the production of investment good.

We assume that the investment rate is chosen from a discrete set. Specifically, we assume that:

$$i_{j,t}/k_{j,t} \in \{0, (1-\delta_j)(\lambda_j^{\pm 1} - 1), (1-\delta_j)(\lambda_j^{\pm 2} - 1), \ldots\} \tag{4}$$

for $\lambda_j > 1$. This implies that the next period capital $k_{j,t+1}$ has to be either the naturally depreciated level $k_{j,t}(1-\delta_j)/g$ or its multiplication or division of λ_j. By this assumption, the producer is forced to invest in a lumpy manner. Thus this constraint is a shortcut for the lumpy behavior which typically occurs when a fixed cost incurs in investment. This is the only modification from the usual model of monopolistic economies. The main objective of this paper is to examine the aggregate consequence of a non-linear behavior of producers induced by the discreteness constraint.

Let $p_{j,t}$ denote the price of good j at t. Define a price index $p_t \equiv (\sum_{j=1}^{N} p_{j,t}^{1-\xi}/N)^{1/(1-\xi)}$ and normalize it to one. Let w_t denote a real wage for an efficiency unit of labor. Then the monopolist's profit (normalized by A_t) at t is written as:

$$\pi_{j,t} \equiv p_{j,t}y_{j,t} - w_t h_{j,t} - \sum_{l=1}^{N} p_{l,t} z_{l,j,t}^{I} \tag{5}$$

The demand function for good j is derived by usual procedure as in Dixit and Stiglitz (1977). Let us suppose that the representative household has a preference over the sequence of consumption and labor:

$$\sum_{t=0}^{\infty} \beta^t U(C_t, h_t) \tag{6}$$

where $C_t = A_t c_t$ is a composite consumption good produced identically as the investment good:

$$c_t = N^{1/(1-\xi)}(\sum_{l=1}^{N}(z_{l,t}^{C})^{(1-\xi)/\xi})^{\xi/(1-\xi)}. \tag{7}$$

The representative household maximizes the utility function subject to the sequences of budget constraints:

$$\sum_{j=1}^{N} p_{j,t} z_{j,t}^{C} = w_t h_t + \sum_{j=1}^{N}((\pi_{j,t} + q_{j,t})v_{j,t} - q_{j,t}v_{j,t+1}). \tag{8}$$

where $v_{j,t}$ is the stock holding for firm j and $q_{j,t}$ is its price.

The cost minimization of the consumer given the level of consumption C_t implies $z^C_{j,t} = (p_{j,t}/p_t)^{-\xi} c_t/N$. Similarly, the derived demand for good j by the monopolist l given the level of investment $i_{l,t}$ is obtained as $z^I_{j,l,t} = (p_{j,t}/p_t)^{-\xi} i_{l,t}/N$. With the equilibrium condition for good j, $y_{j,t} = z^C_{j,t} + \sum_{l=1}^N z^I_{j,l,t}$, these yield the demand function for good j as: $y_{j,t} = (p_{j,t}/p_t)^{-\xi}(c_t + i_t)/N$ where $i_t \equiv \sum_{j=1}^N i_{j,t}$. Define a production index $y_t \equiv N^{1/(1-\xi)}(\sum_{j=1}^N y_{j,t}^{(\xi-1)/\xi})^{\xi/(\xi-1)}$. Then we have relations $\sum_{j=1}^N p_{j,t} y_{j,t} = p_t y_t$, $\sum_{j=1}^N p_{j,t} z^C_{j,t} = p_t c_t$, and $\sum_{j=1}^N p_{j,t} z^I_{j,l,t} = p_t i_{l,t}$. Combining with the consumer's budget constraint (8) and the equilibrium condition for labor, $h_t = \sum_j h_{j,t}$, we obtain the demand function:

$$y_{j,t} = (p_{j,t}/p_t)^{-\xi} y_t/N \tag{9}$$

The monopolist maximizes its discounted future profits as instructed by the representative household. The discount rate, r_t^{-1}, is the intertemporal ratio of marginal utility of consumption. Then the monopolist's problem is defined as follows.

$$\max_{\{y_{j,t}, k_{j,t+1}, h_{j,t}, i_{j,t}, z^I_{l,j,t}\}} \sum_{t=0}^{\infty} (r_1 \cdots r_t)^{-1} A_t \pi_{j,t} = A_0 \sum_{t=0}^{\infty} (r_1 \cdots r_t)^{-1} g^t (p_{j,t} y_{j,t} - w_t h_{j,t} - \sum_{l=1}^N p_{l,t} z^I_{l,j,t}) \tag{10}$$

subject to the production function (1,3), the capital accumulation (2), the discreteness of investment rate (4), and the demand function (9).

Let us define the aggregate capital index k_t as follows.

$$k_t \equiv (\sum_{j=1}^N k_{j,t}^{\alpha(\xi-1)/(\xi\alpha+1-\alpha)}/N)^{(\xi\alpha+1-\alpha)/(\alpha(\xi-1))} \tag{11}$$

By using the optimality condition for $h_{j,t}$, the profit at t is reduced to a function of $(k_{j,t}, k_{j,t+1})$ as:

$$\pi_{j,t} = D_0 w_t^{(\alpha-1)/\alpha} k_t^{1/(\xi\alpha+1-\alpha)} k_{j,t}^{\alpha(\xi-1)/(\xi\alpha+1-\alpha)} - g k_{j,t+1} + (1-\delta_j) k_{j,t} \tag{12}$$

where $D_0 \equiv (1 - (1-1/\xi)(1-\alpha))((1-1/\xi)(1-\alpha))^{(1-\alpha)/\alpha}$. The discounted sum of the profit sequence is concave in $k_{j,t}$. Thus the optimal policy is characterized by an inaction region in $k_{j,t}$ with a lower bound $k^*_{j,t}$ and an upper bound $\lambda_j k^*_{j,t}$. Consider two sequences of $k_{j,s}$ which are identical except for $k_{j,t}$. Such sequences can be constructed by assigning a positive investment at $t-1$ and zero investment at t in one sequence and zero investment at $t-1$ and a positive investment at t in the other. Then the lower bound of the inaction region is derived by solving for $k^*_{j,t}$ at which the two sequences yield the same discounted profit. If $k_{j,t}$ is strictly less than $k^*_{j,t}$, the producer is better off by adjusting it upward rather than waiting. The sequence has $k_{j,t} = k^*_{j,t}$ if zero investment is assigned at $t-1$. Then $k_{j,t-1} = (g/(1-\delta_j))k^*_{j,t}$. The other sequence has $k_{j,t} = \lambda_j k^*_{j,t}$. Note that by this construction the both sequences have the same capital at $t+1$: $(\lambda_j(1-\delta_j)/g)k^*_{j,t}$. Solving for $k^*_{j,t}$ which equates the discounted profits of the two sequences, we obtain:

$$k^*_{j,t} = D_j(w_t^{(1-\alpha)/\alpha}(r_t - 1 + \delta_j))^{-(\xi\alpha+1-\alpha)}k_t \tag{13}$$

where $D_j \equiv ((\lambda_j^{\alpha(\xi-1)/(\xi\alpha+1-\alpha)} - 1)D_0/(\lambda_j - 1))^{\xi\alpha+1-\alpha}$.

Equation (13) expresses the feedback relation from the mean capital level k_t to the threshold for an individual capital level $k_{j,t}$. Note that the feedback effect on $k_{j,t}$ is non-linear because of the threshold behavior. The mean capital level k_t affects the threshold of the inaction region, but it may or may not induce the adjustment of $k_{j,t}$. Also note that the effect on the inaction region is linear. This implies that, in the situation when an individual capital adjustment occurs continuously ($\lambda_j \to 1$), the feedback effect from the mean capital to an individual capital is linear. The linear feedback means that the individual capital moves proportionally to the mean capital level. These two observations are summarized as local inertia and global strategic complementarity of the individual behavior. The individual capital is insensitive to a small perturbation in the mean capital level, while it synchronizes with the mean capital if the perturbation is large.

The global strategic complementarity is perfect in a sense that the percentage changes coincide in an individual and mean capital. We will show shortly that this perfect complementarity induces a large fluctuation in propagation of capital adjustments. The perfect complementarity results from the constant returns to scale of the technology. This point is shown as follows. Consider an identical economy as above except for that the production incurs only capital as in $Y_j = A_j^{1-\theta}K_j^{\theta}$. Then the lower bound of the inaction region of capital is shown to be proportional to $k_t^{1/(1+\xi(1/\theta-1))}$. The lower bound is linear in k_t only when the returns to scale is constant ($\theta = 1$). The strategic complementarity is less than or more than proportional depending on whether the returns to scale is diminishing or increasing.

For simplicity, let us for a while focus on this feedback network of producers in the product markets while abstracting the rest of the economy by assuming that the equilibrium wage and interest rate only depends on the mean capital level. We come back to the equilibrium price functionals in Section 4. Suppose that the equilibrium wage and interest are approximated by a constantly elastic function of the mean capital k_t in the vicinity of the steady state $\tilde{k}$. Namely, using a tilde to designate a steady state value, I assume that:

$$(r_t - 1 + \delta_j)/(\tilde{r} - 1 + \delta_j) = (k_t/\tilde{k})^{\theta_r} \quad (14)$$

$$w_t/\tilde{w} = (k_t/\tilde{k})^{\theta_w} \quad (15)$$

Then the threshold (13) is written simply:

$$k^*_{j,t}/\tilde{k}^*_j = (k_t/\tilde{k})^{\phi} \quad (16)$$

where $\tilde{k}^*_j$ is a threshold corresponding to the steady state $\tilde{r}, \tilde{w}, \tilde{k}$ and ϕ is the strategic complementarity between the individual and mean capital:

$$\phi = 1 - (\alpha\theta_r + (1-\alpha)\theta_w)(\xi - 1 + 1/\alpha) \quad (17)$$

Note that ϕ is less than one. This implies that the strategic complementarity between producers is decreased from the perfect complementarity due to the equilibrium response of the wage and interest rate. The wage and interest rise in our approximation when the production is higher than the steady state level. This price response works as a dampening factor in the investment propagation.

The equilibrium of the product markets is given by a capital profile which satisfies $k_{j,t} \in [k^*_{j,t}, \lambda_j k^*_{j,t}]$. This condition allows multiple equilibria in general. Here we employ best response dynamics as an equilibrium selection algorithm. Suppose that a predetermined capital $k_{j,t}$ resides in the inaction region. The next period capital $k_{j,t+1}$ only decreases by depreciation and technology progress unless adjusted. In the first step of the best response dynamics, the producers adjust capital by λ_j if their capital level goes below $k^*_{j,t}$ given k_t. Note that assuming $\delta_j + g < \lambda_j$, the adjustment never exceeds λ_j. In the second step, k_t is calculated by a new capital profile, and the producers adjust their capital responding to the revised k_t. We repeat this procedure until the capital profile converges. The adjustments after the second step can be upward or downward, depending on whether the first step upward adjustments by some producers weigh more or less than the inertial depreciation of overall capital. Let us formally define the best response dynamics as follows. Set the initial point of the dynamics as $k^0_{j,t} = k_{j,t}(1-\delta_j)/g$ and $k^0_t = k_t$. Succeeding mean capital k^u_t is defined by the profile $k^u_{j,t}$. Then $k^u_{j,t}$, $u = 0, 1, \ldots$, evolves according to the (S,s) rule:

$$k^{u+1}_{j,t} = \begin{cases} \lambda_j k^u_{j,t} & \text{if } k^u_{j,t} < k^{*u}_{j,t} \\ k^u_{j,t}/\lambda_j & \text{if } k^u_{j,t} > \lambda_j k^{*u}_{j,t} \\ k^u_{j,t} & \text{otherwise} \end{cases} \quad (18)$$

We can show that this dynamics converges at a finite stopping time T with probability one when $N \to \infty$. Thus the best response dynamics is a valid equilibrium selection algorithm. Then we define the converged point as an equilibrium capital profile at $t+1$, namely, $k_{j,t+1} = k^T_{j,t}$.

The best response dynamics is a realistic equilibrium selection mechanism in a situation where many agents interact each other, as Vives (1990) argues. All information needed for an agent to make decision is the prices and the mean capital level. This selection mechanism precludes big jumps that occur due to the informational coordination among agents. In this sense, the best response dynamics selects an equilibrium path that is least volatile among possible equilibrium paths.

The aggregate investment fluctuates along the equilibrium path depending on the evolution of configuration of the capital profile in the inaction region. To evaluate the magnitude of fluctuations analytically, we regard the capital configuration as being a random variable that takes values within the inaction region. Specifically, we assume that the position of an individual capital relative to the lower bound of its inaction region (in log-scale) follows a uniform distribution independent across sectors.[2] The uniformity assumption has an analytical ground. It is known that a variable which grows linearly and is controlled by an (S,s) policy converges to a uniform distribution in the (S,s) band when the initial value is random. See Engel (1992) for the mathematical reference and also Nirei (2003) for a rigorous treatment in our specific economic model.

Define a producer's position in an inaction region as $s_{j,t} = (\log k_{j,t} - \log k^*_{j,t})/\log \lambda_j$. We assume for a while that λ_j and δ_j are common across j. Define $m_0 = N(\log k^1 - \log k_t)/\log \lambda$ where k^1 is the mean capital at the first step of the best response dynamics. At the first step, all capital is depreciated by $(1-\delta)/g$ and some producers increased capital due to the direct effect of the depreciation. Thus m_0 indicates the deviation of mean capital growth from the steady state level in the unit of the number of producers at the first step of the adjustment process. Also define $W = N(\log k_{t+1} - \log k_t)/\log \lambda$. W indicates the deviation of mean capital growth from the steady state level in the unit of the number of producers in the entire best response dynamics. Define $\mu = |\log((1-\delta)/g)|/\log \lambda$. Here we place our main analytical proposition.

Proposition 1 *Suppose that λ_j and δ_j are common across j. Suppose that $s_{j,t}$ is a random variable which follows a uniform distribution independently across j. Then $m_0/\sqrt{N}$ asymptotically follows a normal distribution with mean zero and variance $\mu(1-\mu)$. Let m be a positive integer. $|W|$ conditional to $|m_0| = m$ follows a distribution function asymptotically as we take $\xi \to 1$ first and then $N \to \infty$:*

$$\Pr(|W| = w \mid |m_0| = m) = (m/w)e^{-\phi w}(\phi w)^{w-m}/(w-m)! \tag{19}$$

[2] See Nirei (2003) for the case in which the distribution is not uniform.

for $w = m, m+1, \ldots$. The tail of the probability function is approximated by:

$$\Pr(|W| = w \mid |m_0| = m) \approx (m(\phi e)^{-m}/\sqrt{2\pi})(\phi e^{1-\phi})^w w^{-1.5} \tag{20}$$

The unconditional distribution of W is symmetric.

Proof is deferred to Appendix A.1. The key to the proof is to embed the best response dynamics in a branching process so that the recursivity of the branching process becomes available. Let $G(s)$ be the generating function of the total adjustment W given the initial deviation from steady state, $m_0 = 1$. Let x be the number of sectors that adjust capital due to m_0, and $F(s)$ be its generating function. Each adjustment of x then has a chance to propagate in the next step just like the initial adjustment m_0. Thus the total number of offsprings which are originated from each of x follows $G(s)$. Hence we obtain a functional equation $G(s) = sF(G(s))$, from which we derive the distribution of W. A similar functional equation obtains for a large class of models with features such as heterogeneous λ_i and δ_i, non-uniformly distributed $s^0_{j,t}$, or non-constant returns to scale technology, as shown in Nirei (2003). The functional equation characterizes the propagation distribution completely, because all the moments can be derived from it.

Proposition 1 implies that the capital growth $\log k_{t+1} - \log k_t$ conditional to m_0 is approximated by a power distribution $w^{-1.5}$ truncated by an exponential distribution that declines at rate $1-\phi$. We can calculate moments when $\phi < 1$. The capital growth conditional to $m_0 = 1$ has an asymptotic mean $\log\lambda/(N(1-\phi))$ and variance $(\log\lambda/N)^2(2-\phi)/(1-\phi)^3$. The variance of an unconditional capital growth rate is calculated as $((\log\lambda)^2/N)(\mu(1-\mu)(1-2/\pi)/(1-\phi)^2 + \sqrt{2\mu(1-\mu)/(\pi N)}/(1-\phi)^3)$ by approximating m_0 by an integer random variable. This is a natural result as obtained in usual models: a fraction μ of sectors are induced to adjust by the deterministic trend in mean. The variance of the capital growth rate declines linearly in N, hence the law of large numbers obtains. One notable difference is that the variance has a $1/(1-\phi)^3$ term, which can be quite large when ϕ is close to one. In a continuously adjusting model, the variance is of order $1/(1-\phi)^2$. We can regard the extra $1/(1-\phi)$ as the contribution of the discrete propagation to the fluctuations.[3].

The fluctuation of the capital growth exhibits quite a different behavior, however, when $\phi = 1$. The distribution of W becomes a power law distribution. With the exponent 0.5 (in a cumulative distribution), it is known that the distribution does not have either mean or variance. That is, the sample moments diverge as the sample size increases.

In fact, the variance of the capital growth rate ceases to depend on N when $\phi = 1$.

[3]See Nirei (2003) for details.

Proposition 2 *When $\phi = 1$, the variance of the aggregate capital growth rate converges to a non-zero constant as $N \to \infty$. The limit variance is approximated by:*

$$(\log \lambda)^2 \left(\mu(1-\mu) + \sqrt{2\mu(1-\mu)/(9\pi)} \right). \tag{21}$$

Proof: We concentrate on W/N which is the mean capital growth rate divided by $\log \lambda$. The unconditional variance $\mathrm{Var}(W/N)$ is decomposed as $E(\mathrm{Var}(W/N \mid m_0)) + \mathrm{Var}(E(W/N \mid m_0))$. Since W is symmetrically distributed and since $|W|$ conditional to m_0 follows the same distribution as the sum of m_0 number of $|W|$ conditional to $m_0 = 1$, we have $\mathrm{Var}(W/N \mid m_0) = |m_0| \mathrm{Var}(W/N \mid m_0 = 1)$ and $E(W/N \mid m_0) = m_0 E(W/N \mid m_0 = 1)$ for an integer m_0. We linearly interpolate this formula for any real number m_0. Then, using the asymptotic distribution of m_0, we obtain that $\mathrm{Var}(W/N) = \sqrt{(2/\pi)N\mu(1-\mu)} \mathrm{Var}(W/N \mid m_0 = 1) + N\mu(1-\mu)(E(W/N \mid m_0 = 1))^2$. Next we derive the moments of W/N conditional to $m_0 = 1$. At $\phi = 1$, the distribution of W becomes a pure power-law. Also by construction, W conditional to $m_0 = 1$ only takes integer values between 1 and N. Thus the distribution of W/N converges to a continuous distribution in $[0,1]$ with keeping the power-law exponent. For a large N, let us approximate the probability distribution of W conditional to $m_0 = 1$ by a density function $x^{-1.5}/(2(1-1/\sqrt{N}))$ for $x \in [1, N]$. Then the density function of W/N is given by $y^{-1.5}/(2(\sqrt{N}-1))$ for $y \in [1/N, 1]$. Note that the distribution converges to a delta function at zero only at the speed $1/\sqrt{N}$. Hence the mean and variance of W/N conditional to $m_0 = 1$ are of order $1/\sqrt{N}$. Combining with the previous result, we obtain that the unconditional variance of W/N is of order N^0. More precisely we obtain the following formula:

$$\mathrm{Var}(W/N) = \sqrt{2\mu(1-\mu)/\pi}(\sqrt{N}-1/N)/(3(\sqrt{N}-1)) + N\mu(1-\mu)(1-1/\sqrt{N})^2/(\sqrt{N}-1)^2 \tag{22}$$

By taking a limit of N, we obtain our result. $\square$

This result means that the growth rate fluctuation is scale-free, and that the law of large numbers is broken. No matter how large the aggregative system is, a non-linearity in an individual level can add up to an aggregate fluctuation. Consider the case of lumpy investment behaviors. Cooper, Haltiwanger, and Power (1999) documents that, in the Longitudinal Research Database, the investment episodes in which the investment-capital ratio exceed 20% constitutes 20% of the plants and account for 50% of gross investment. Considering that there are about 350,000 plans in U.S. manufacturing as they report, the aggregate fluctuation generated by the lumpy investment in individual level is negligible in a situation where the central limit theorem holds.

		lumpiness ($\log \lambda$)				
		0.02	0.05	0.1	0.2	0.4
periodicity	4	0.011	0.028	0.055	0.110	0.220
($1/\mu$)	6	0.010	0.024	0.049	0.098	0.195
	8	0.009	0.022	0.044	0.089	0.178

Figure 1: Scale-free standard deviation of capital growth g

To the contrary, our result establishes a possibility of the aggregate fluctuations via a stochastic propagation effect.

The formula (21) gives the standard deviation of growth rates as a function of λ and μ. λ is the lumpiness parameter, and $1/\mu$ is interpreted as the periodicity of capital oscillation in individual level. Some numerical examples are shown in Table 1. We observe that the magnitude of lumpiness observed in data is large enough to generate the fluctuations in aggregate production.

Our analytical results imply two things on the investment propagation. First, it challenges the conventional view that the sectoral propagation does not add up to a large aggregate fluctuation due to the law of large numbers effect. Our result shows that, when the price response is rigid enough so that ϕ is close to one, the sectoral propagation generates a significant fluctuation in aggregate level. Secondly, our result shows that the large, non-degenerate investment fluctuation can occur endogenously in a deterministic environment. This implies that an interdependence of a small non-linearity in a micro behavior may play a crucial role in aggregate investment fluctuations.

The propagation distribution derived here has an interesting link with other models of non-linear dynamics in a network, such as the self-organized criticality or a percolation in the Bethe lattice. These analytical connections are explored in Nirei (2003). In this paper, let us move on to the next question of how this propagation effect may explain the economic aggregate fluctuations quantitatively.

3 Business Cycle Simulation

In this section we examine quantitative properties of the equilibrium fluctuation by numerical simulations. We ask whether the sectoral oscillations of magnitude exhibited by the U.S. manufacturing sectors would add up in our model to the observed aggregate fluctuations and generate the business cycle patterns. The answer is affirmative when the intertemporal substitutions of consumption and leisure are close to perfect. If this is the case, the (S,s) policy at the individual level generates an endogeneous fluctuation of the aggregates.

In the previous section we demonstrate the possibility that an individual deterministic (S,s) policy generates aggregate fluctuations. The result is obtained by assuming a simple behavioral rule of consumer decisions and the stationarity of the cross-sectional distribution of producers' positions in the (S,s) band. We no longer impose these assumptions. The consumer's behavior is derived from a representative household's choice. By this we can analyze the impact of preference structure on the aggregate fluctuations. Moreover, the fluctuation is calculated by simulations without setting the cross-sectional distribution of producers' positions at the stationary distribution. Whereas simulations show that the distribution converges to a uniform distribution quickly, they can also exhibit interesting dynamics such as the echo-effect or mode-locking when a large deviation from the stationary state is present. We will study the dynamics which could not be examined in the setup of the previous section.

Our aim is to reproduce the second moment structure of business cycles. In particular, we attempt to explain the mechanism for the positive autocorrelation of the business cycle variables and the positive correlation between production and demand components. We do not focus on the amplification effect of the propagation, which was explored in the previous section.

Let us start from estimating the fluctuation magnitude of U.S. manufacturing sectors. We use the 4-digit SIC annual data compiled by Bartelsman and Gray (1996). We remove the trend by Hodrick-Prescott filter with smoothing parameter $\lambda = 100$. We estimate a second order autoregressive process of the detrended log sectoral capital as:

$$y_{j,t} = \phi_{1,j} y_{j,t-1} + \phi_{2,j} y_{j,t-2} + \epsilon_{j,t} \tag{23}$$

The regression shows that 434 sectors out of total 459 sectors exhibit a damped oscillation phase $\phi_{1,j}^2 + 4\phi_{2,j} < 0$. A second order autoregressive process with a damped oscillation displays a pseudo-periodic behavior. The pseudo-periodicity is calculated as $1/\mu_j \equiv 2\pi / \cos^{-1}(\phi_{1,j}/(2\sqrt{-\phi_{2,j}}))$, following the procedure similar to Yoshikawa and Ohtake (1987).

We emulate this oscillation by our lumpy behavior of sectoral investments. The presumption is that a sector has to commit to a sizable investment if it invests at all. If it does not invest, then the gap between the actual and desired level of capital increases as capital depreciation and technological progress takes effect. The lumpy adjustment generates a non-harmonic oscillation which is familiar in the (S,s) literature such as the Baumol-Tobin cash balance dynamics. It is more likely that the committed amount of investment is executed in several periods, if we consider the time to build.

By incorporating the time to build, the sectoral oscillation exhibits a more realistic harmonic oscillation, but the basic properties of aggregate behavior does not change by this modification.

We derive λ_j and δ_j from the observed oscillations μ_j and σ_j in the way that the periodicity and magnitude of oscillation the data shows are maintained. From the periodicity we have a relation $1/\mu_j = \log \lambda_j / |\log((1-\delta_j)/g)|$. Also, we numerically calculate the standard deviation of the model oscillation for $\log \lambda = 1$ and δ_j. Then $\log \lambda_j$ is derived by dividing σ_j by the calculated standard deviation. Thus we obtain λ_j and $(1-\delta_j)/g$.

Figure 2 shows the estimated periodicity in the first panel. The periodicity is distributed with mean 8.2 years and standard deviation 3.3. The second and third panels show the calibrated discreteness λ_j and the annual depreciation rate δ_j that match with the estimated parameters for oscillations. The mean of λ_j is 2.5 and standard deviation is 2, and the mean of δ_j is 0.09 and standard deviation 0.07. Let us notice the considerable heterogeneity shown in the periodicity. It casts a doubt on the view that the sectoral fluctuation is merely a reflection of aggregate fluctuations. It is worth exploring the possibility that a pseudo-random propagation effect across sectors causes the aggregate fluctuations.

We focus on the aggregate fluctuations by abstracting the mechanism for sectoral fluctuations in the individual level. Individual sectors may fluctuate for various reasons such as technological improvement or strategic complementarity among firms' behaviors within the industry. For convenience of analysis we assume the lumpy behavior of monopolists. The amplification effect of investment propagation is not an emphasis in this section either. The standard deviation of the aggregate growth rate amounts to about 1% even if we sum up the independent series of sectoral oscillations of magnitude we observe in data. Our emphasis in this section is thus on the structure of second moments of variables when the business cycle is driven by autonomous movements of investment.

We will show that our model of investment propagation is capable of reproducing the basic business cycle structure: the standard deviation of GDP around 1.7%, the positive correlations between production and demand components, and the strong autocorrelations of the production and demand components. To do so, we explicitly solve the representative household's choice between leisure and consumption. We discuss how the approximated parameters θ_w and θ_r in the previous section relate to the preference and technology parameters. Our model shares the basic quantitative characteristics of monopolistic models that have been studied by, for example, Galí (1994) or Rotemberg and Woodford (1995). In the following we concentrate on the investment fluctuation and its effect on production and consumption.

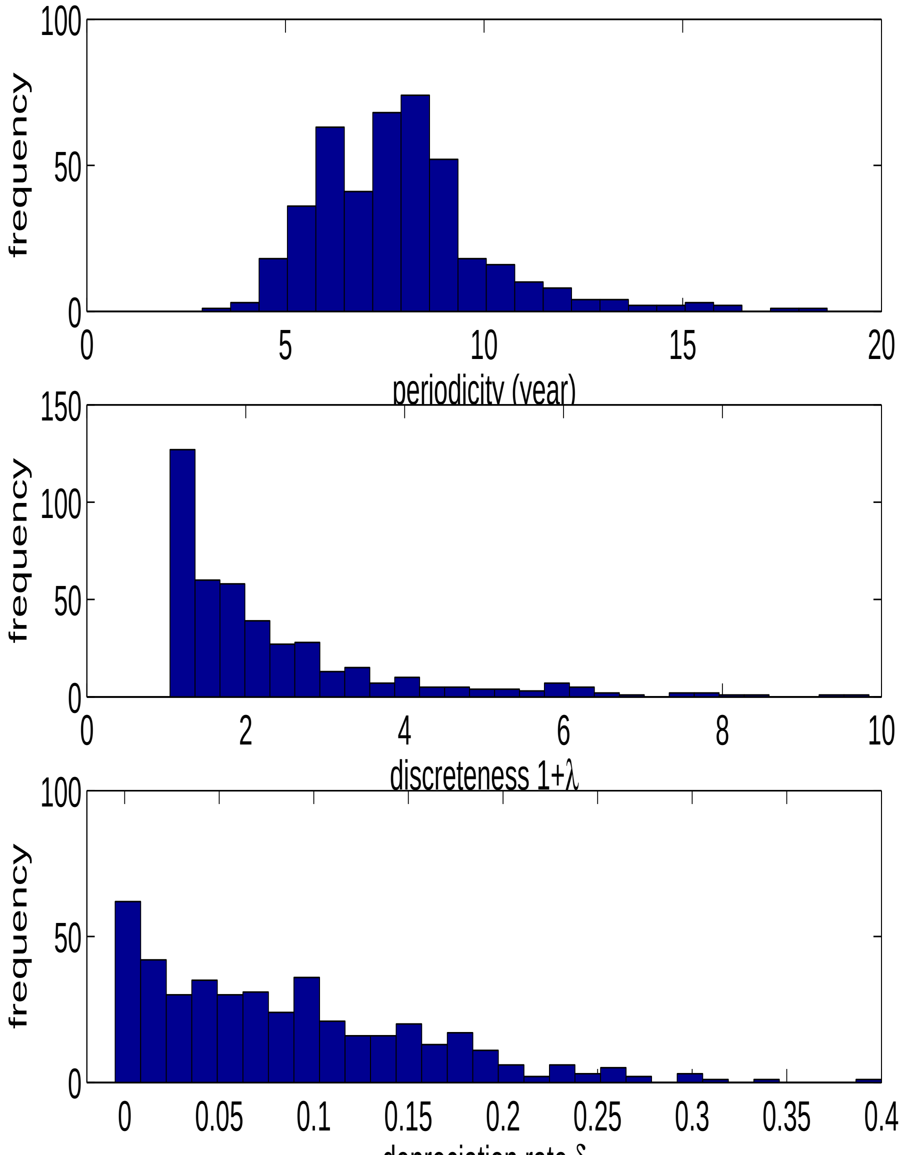

Figure 2: Properties of sectoral oscillations

We use the following utility specification:

$$U(c_t, h_t) = c_t^{1-\sigma}/(1-\sigma) - h_t^{1+\nu}/(1+\nu) \tag{24}$$

where $\sigma \geq 0$ and $\nu \geq 0$. This simple specification allows us to obtain some analytical insight as we see later, although the labor hour will not be stationary in the balanced growth path in this specification. We set the technological growth rate at $g = 1$ and inflate the depreciation rate δ_j by the observed productivity growth rate so that the simulated sectoral oscillations continue to match the oscillations in the data. From the utility specification we obtain the equilibrium price conditions immediately:

$$w_t = c_t^{\sigma} h_t^{\nu} \tag{25}$$
$$r_t = (c_{t+1}/c_t)^{\sigma}/\beta \tag{26}$$

A contemporaneous equilibrium (y_t, c_t, h_t, w_t) given k_t, i_t, r_t is determined by (25) and:

$$y_t = ((1-1/\xi)(1-\alpha)/w_t)^{(1-\alpha)/\alpha} N k_t \tag{27}$$
$$w_t h_t = (1-1/\xi)(1-\alpha) y_t \tag{28}$$
$$y_t = c_t + i_t \tag{29}$$

The first equation is derived by aggregating the optimal production level when the capital is given. The second equation is obtained by aggregating the optimal employment given capital. It shows that the labor share is equal to $(1-1/\xi)(1-\alpha)$. The third equation is a product market equilibrium condition. Given these equilibrium relations, the equilibrium path (k_t, i_t, r_t) is determined by the capital accumulation (2), the equilibrium interest rate (26), and the selection algorithm for i_t with the optimal threshold rule (13).

We resort to numerical simulations to solve the equilibrium path. In the simulation, we assume that the representative household and monopolists have a static expectation on future investment. Namely, the expected future investment is set at the steady state level $\sum_j \delta_j \tilde{k}_j$. Computational difficulty is the reason we do not solve for a perfect foresight equilibrium. Since the investment crucially depends on the details of the configuration of producers capital positions, solving the perfect foresight path requires prohibiting computational loads. Also, it is not realistic to suppose that the agents are able to form a perfect foresight. Besides the computational problem, the agents would have to have precise information about the capital configuration of the entire economy. When the economy has attained the stationary level, a noisy information would not contribute to the accuracy of prediction very much in our setting. We also tried another expectation formulation based on an AR(1) estimate of the past investment path. We

	GDP	Investment	Consumption	Capital	Hours	Wage
standard deviation (%)	1.83	11.63	1.80	1.88	1.82	0.02
	(0.52)	(1.07)	(0.50)	(0.54)	(0.52)	(0.01)
correlation with GDP	1	0.49	0.77	1.00	1.00	0.77
	–	(0.05)	(0.09)	(0.00)	(0.00)	(0.09)
autocorrelation	0.89	0.61	0.52	0.88	0.89	0.52
	(0.05)	(0.07)	(0.21)	(0.05)	(0.05)	(0.21)

Table 1: Simulated business cycle statistics

confirmed that the basic property of the fluctuations does not change, although we noted that the convergence to the rationally expected AR(1) parameters can be fragile depending on the fundamental parameters. Another issue in the simulation is the finiteness of the agents. The existence of equilibrium is shown in the previous section as an asymptotic property when the number of sectors N tends to infinity. When N is finite, with a positive probability the best response dynamics does not reach an equilibrium. We impose a rule that the dynamics stops either when all the sectors adjust upward or all the sectors which adjust at the initial step re-adjust downward. This case happens in the early periods of simulated paths. We did not observe this case once the equilibrium path is converged to a stationary state level.

Table 1 summarizes the simulation result on the second moments. The standard deviations of the estimated second moments in 500 runs are shown in parentheses. The parameter values are set as $\sigma = 0.01$, $\nu = 0$, labor share $(1 - 1/\xi)(1 - \alpha) = 0.58$, mark-up rate $1/(\xi - 1) = 1/3$, and annual discount rate $\beta = 0.96$. Although the correlation between production and investment is not strong enough, the simulation captures the basic feature of business cycles such as the magnitude of fluctuations in GDP, investment, and consumption, strong autocorrelations in GDP, positive correlations between production and demand components and input components, and small wage fluctuations.

Figure 3 shows typical paths of the simulated production and investment for the same parameter set. The variables are normalized by the stationary level GDP after convergence. The top left panel shows the entire paths of the GDP and the aggregate investment. The simulated path converges to a steady state level quickly and exhibits persistent fluctuations thereafter. The investment-production rate converges to a realistic 9.6%. The bottom left panel shows the capital paths of individual sectors. We observe an (S,s) behavior of the sectors. The right panels show the magnified plots of

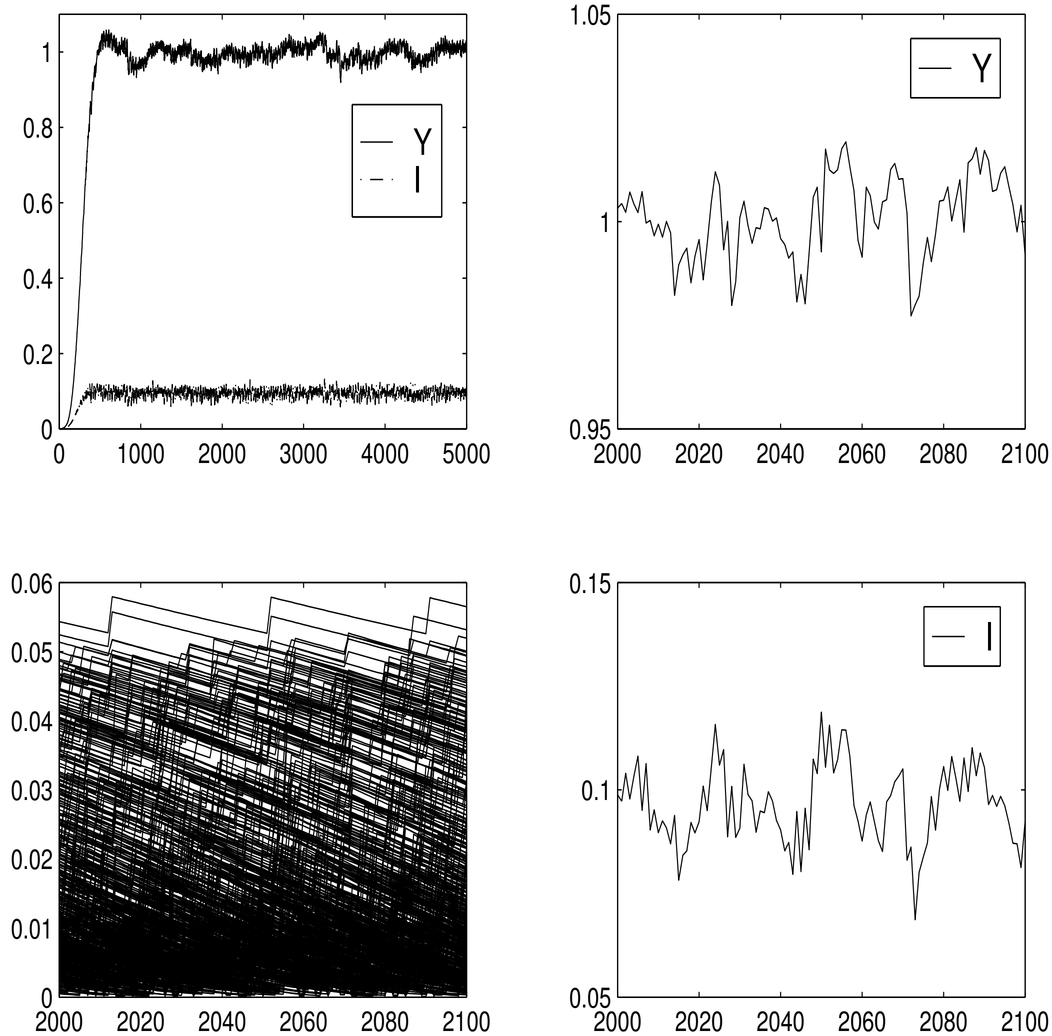

Figure 3: A simulation path of GDP and investment. X axis shows quarters. Y axis is scaled by the stationary level GDP.

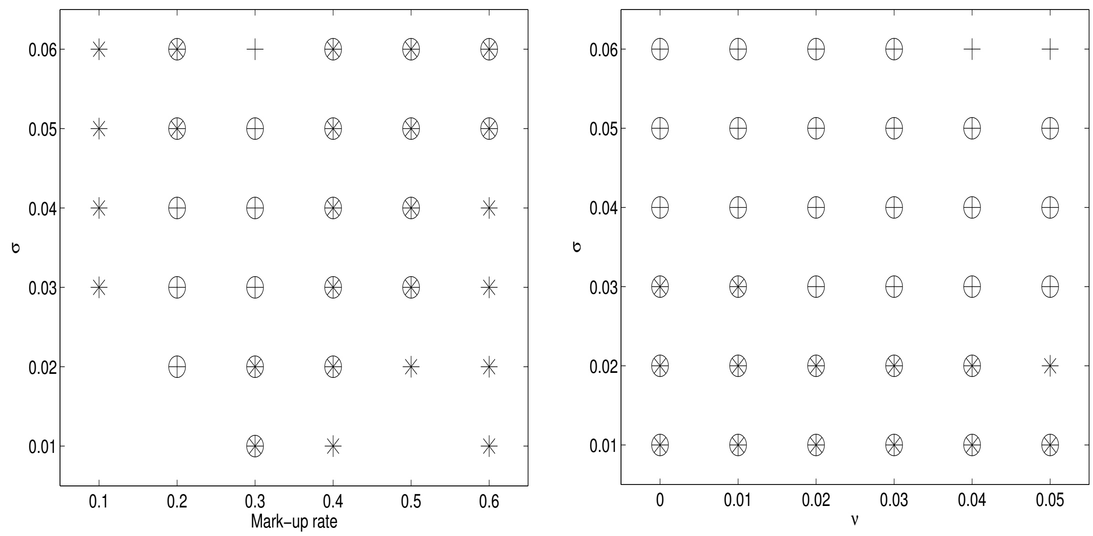

Figure 4: Admissible range of parameters

the same aggregate paths in a shorter time horizon. We observe a chaotic fluctuation (in the sense that the deterministic path appears random) with a certain degree of periodicity. Also we see a strong correlation between the production and investment.

The correlation structure shown in Table 1 is robust in parameters. Figure 4 shows the admissible range of parameters. For each parameter alignment, we take an average of estimates from 15 simulation runs. We plot a circle when the standard deviation of GDP is more than 1% and less than 3%, a cross when investment correlates with production, and a plus when consumption correlates with production. The plots show that our results depend on the preference specifications (σ and ν) sensitively but not on the markup rate ($1/(\xi-1)$). In the left panel, there exists an admissible range of σ for the markup rate larger than 30% (which corresponds to $\xi \leq 4$). The larger the markup rate is, the larger and the broader the admissible range of σ is. We observed that the business cycle patters obtain also for a smaller markup rate ($\xi \geq 10$). It is not certain, however, if this pattern is generated by the mechanism we analytically identified. For a small markup, the production goods are easily substitutable and the firms are competitive. Hence the price responds sensitively to the initial shock in the best response dynamics. The subsequent adjustment process occurs not in the direction to amplify the initial shocks but in the direction to mitigate the initial response. Hence our analysis does not apply to this case. It is nonetheless interesting that a competitive setting also generates an endogeneous fluctuation. The right panel

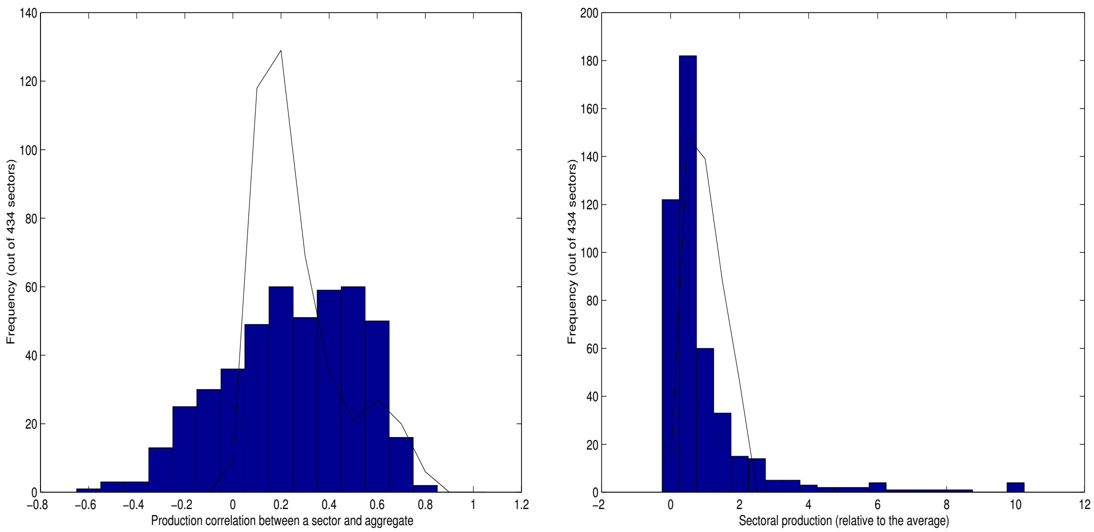

Figure 5: Correlation between sectoral and aggregate production (left) and cross sectional distribution of production relative to average (right). The bar and line respectively show the actual data and simulation.

shows that the admissible range for preference specifications (σ and ν). We obtain the aggregate fluctuations large enough when $\sigma + \nu$ is small enough. To obtain a meaningful stochastic propagation effect, the representative household needs to be sensitive enough to interest rate or wage. We also observe in the plot that σ needs to be small for the correlation between production and investment to obtain. When σ is larger, an investment by a sector increases the interest rate more, and dampens the propagation effect.

The simulation also replicates well the mean behavior of the pairwise correlation between sectoral production and GDP. The comovement of the sectoral production (and hence sectoral and aggregate production) is a defining characteristic of business cycles. However, the comovement is far from a perfect mode locking. The left panel of Figure 5 shows the histogram of the correlations in data (shown by a bar). The correlation between a sector and aggregate is only modest. This fact agrees with another fact we noted that the periodicity of sectoral oscillations varies much. These suggest contrary to the view that the business cycles are mainly driven by an aggregate factor and the sectoral movements are only a noise-ridden version of the same cycles. The modest correlation between the sectoral and aggregate production is captured by our simulation well. The histogram of the simulated correlations under our benchmark

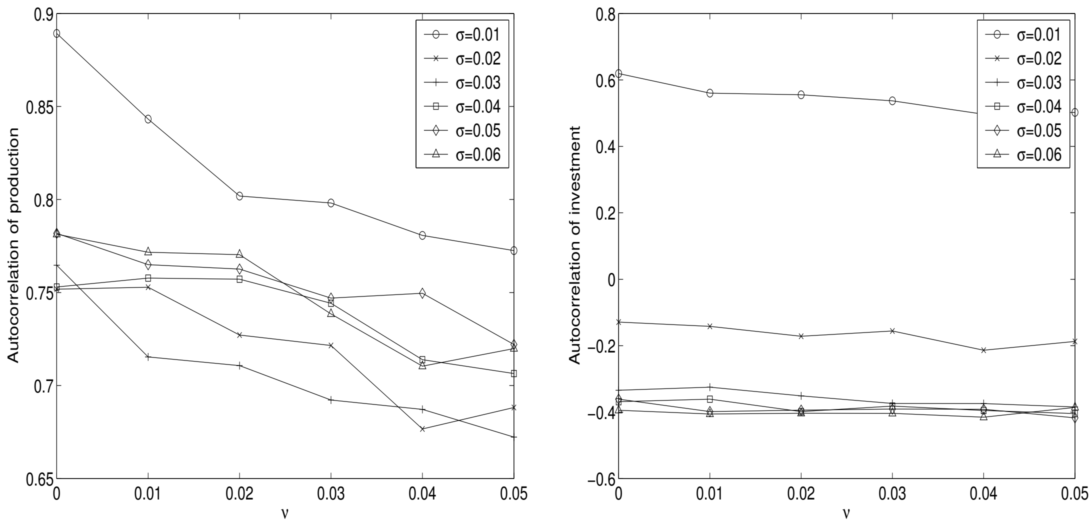

Figure 6: Autocorrelation of GDP (left) and investment (right)

parameter set (as for Table 1) is drawn by a real line. The simulated histogram is more centered than the real histogram, which is a natural consequence of our symmetric modeling of sectoral interactions. The real input-output matrix is far from symmetric, as Horvath (2000) emphasized, and the asymmetric input-output relation will generate more heterogeneity in the comovement structure across sectors. The mean of the correlation (0.24) is reproduced well by our simulation, however. This suggests that the symmetric modeling may be satisfactory insofar as the aggregate fluctuations is concerned. The right panel of Figure 5 shows the histograms of sector size in data (bar) and in simulation (line). The only source of heterogeneity in the model is depreciation rate (δ_j) and lumpiness (λ_j). The heterogeneity of the sector size is reproduced fairly well. This excludes the case in which the different variety in comovement stems from the different sector size distributions. Also this assures that the model fluctuation we observe does not result from an unrealistic distribution of sector size.

Figure 6 shows that the autocorrelations of production and investment depend on σ and ν. The autocorrelation is estimated by taking an average of 15 runs for each parameter set. The other parameters are set at the benchmark level. The left panel shows that the GDP autocorrelation is decreasing in ν. The right panel shows that the investment autocorrelation is not sensitive to the change in ν. This implies that the intertemporal substitution of labor affects the production autocorrelation not through the investment propagation but through the contemporaneous labor decisions.

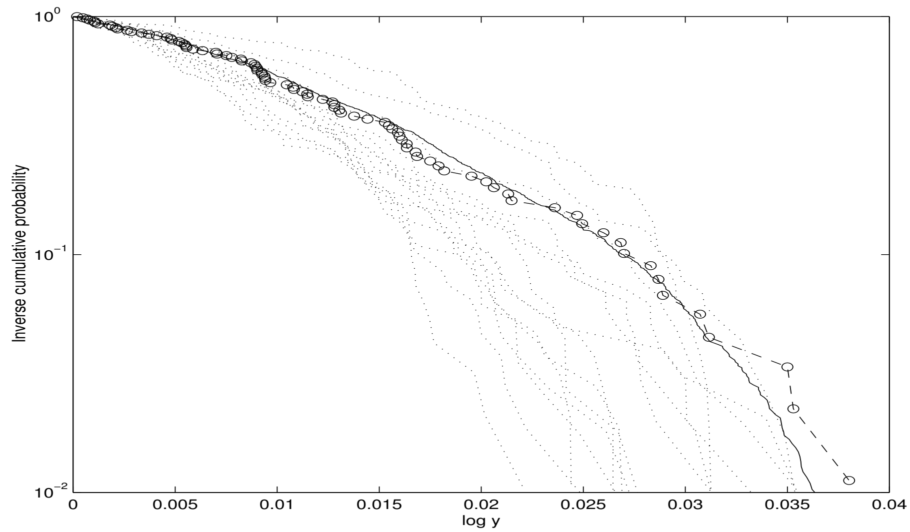

Figure 7: Distribution of GDP growth rates

In contrast, σ affects the autocorrelation of investment quite sensitively. This suggests that the large part of the decrease in autocorrelation of production from $\sigma = 0.01$ to the other values results from the decrease in autocorrelation in investment (and thus in capital). In the U.S. data, the autocorrelation of investment is about 0.12 for the post-war periods. To match this, σ has to be in between 0.01 and 0.02. It is a narrow range, but the other statistics for this level of σ are consistent with the data as seen in Table 1.

Finally, Figure 7 shows an inverse cumulative distribution of the growth rates in GDP. The probability shown in the vertical axis is cumulated from above. The plot is displayed in a semi-log scale, so a linear line would express an exponential distribution. We plot by the dashed circle the real distribution calculated by quarterly GDP from 1958 to 2002. The real line shows the simulated distribution. The dotted lines show several simulated distributions when the sample size is equal to that of the GDP data. Because of the small sample size, the distribution fluctuates across the simulation runs. However, the mean behavior of the distribution matches the data well. Yet, this should not be taken as a distinctive evidence for our distribution shown in Proposition 1. The distribution declines faster than an exponential distribution, hence a normal distribution would also fit well. Thus the distribution data by itself does not reject any aggregative model that results in a normal distribution by the central limit theorem. The plot only confirms that the propagation distribution exhibited in simulation is compatible with the data.

Let us now interpret the simulation results by our analytics. In the previous section we introduce the parameter ϕ to characterize the relation between the propagation fluctuations and the strategic complementarity across producers. The fluctuation exhibits an extreme variance when $\phi = 1$. In a static setup where the capital is replaced with intermediate input, we can derive when this critical fluctuation occurs under the same utility specification (Nirei (2003)). One case is $\sigma = \nu = 0$. In this case, the utility function is linear in consumption and labor, and thus both of the real wage and interest rate are fixed. Another case of criticality occurs when $\alpha = 1$ and the interest rate is fixed. Namely, the production is adjusted only by capital. In this extreme case of "production of commodities by means of commodities," there is no longer an aggregate resource constraint of labor. Thus the propagation lacks a dampening mechanism in which an increase in production is suppressed by a rising wage. In a general equilibrium, a rising interest rate still serves as a dampening factor. If we study the fluctuation of stationary level production, however, the interest rate is not a dampening factor since the stationary interest rate is given by fundamental parameters. Thus the fluctuation is still critical in a long run. This is because a rise in interest rate has to be followed by a decline to the steady state level eventually, which serves as an accelerator of the fluctuations.

It is not trivial in our model to have correlations between production and demand components. In the standard real business cycle model, the fluctuation in total factor productivity causes the procyclical movement of both consumption and investment. Instead, the investment fluctuates relatively independently from the economic environment in our model. This aspect gives the model a different mechanism for the procyclical movement of the consumption and investment. An increase in investment demand induces the monopolistic producers to produce more on one hand. On the other hand, since the capital level is predetermined, an increase in investment competes with the contemporaneous consumption given the production level. By using the equilibrium relations given k_t, we obtain $dy_t/di_t = 1/(1 + (\alpha + \nu)/(\sigma(1 - \alpha)(c_t/y_t)))$, which is always between 0 and 1. Hence, given the capital level, an investment has a positive effect on production, but the effect is no more than 1. Hence there is no multiplier effect of the investment demand on the production. The correlation between consumption and production rather stems from the fluctuations of accumulated capital. We also obtain $(dy_t/y_t)/(dk_t/k_t) = (1 + \nu)\alpha(c_t/y_t)/(\sigma(1 - \alpha) + (\alpha + \nu)(c_t/y_t))$ at equilibrium. This takes values between zero and one, and is close to one when σ and ν are close to zero, agreeing with our benchmark simulation. Since the investment is determined partly by an independent process of best response dynamics across producers which the representative household cannot predict deterministically, large production due to large capital can result in large consumption. The Keynesian multiplier effect would increase the correlations between production and demand components. This would be the case when the consumption function is more sensitive to income than our baseline model. If a significant number of consumers face liquidity constraint, for example, it would contribute to more synchronous movements between production and demands.

The autocorrelation of production is generated by the demand-smoothing effect of the real interest rate. In the previous section we saw that an increase in the interest rate sensitivity θ_r lowers ϕ and dampens the instantaneous investment propagation. In a dynamic setting, this dampening effect only postpones the investment propagation to the subsequent periods. Suppose that the interest rate is now above the steady state level due to a large concentration of sectors near the adjustment threshold. In the next period, the interest rate would decrease to the steady state level if the investment is at the steady state level. This decrease in interest rate increases the threshold for capital adjustment. Hence the investment in the next period tends to be larger than the steady state level. This is the mechanism for the autocorrelation in investment when $\sigma = 0.01$ in Figure 6. In this mechanism, the effect of delaying the investment is strong when the sensitivity parameter θ_r is large, and a large θ_r follows a small intertemporal substitution in consumption, $1/\sigma$. The autocorrelation in investment generates the autocorrelation in production in two routes: a contemporaneous effect on aggregate demand and subsequent effects on aggregate supply via capital accumulation.

It is helpful to examine our economy's smooth counterpart to understand the fundamental condition when the fluctuations occur. Suppose that there is no discreteness constraint (4); then any capital level can be chosen. The producers' optimal choice of capital yields an optimality condition which is linear in aggregate capital as in (13). By aggregating the optimality condition, we find that the aggregate capital level k_t is indeterminate in the product market. The capital level is thus solely determined by the consumer's choice between leisure and consumption. In our model, the steady state capital level (normalized by the total factor productivity) is also determined independently from technology. However, the investment is determined uniquely in the best response dynamics across producers. We saw that the propagation exhibits an extreme variance when the wage and interest rate are fixed. This corresponds to the indeterminacy of capital level in the smooth economy. When the wage and interest rate are not fixed, the aggregate capital does have a steady state level. However, the attraction power of the steady state in the dynamics of aggregate capital is vanishingly small as the wage and interest rate bacomes insensitive to production.

4 Conclusion

This paper explores a mechanism of investment propagation as a fundamental shock to the business cycle fluctuations. We consider industrial sectors which are characterized by constant returns to scale technology and monopolistic pricing. Demand for intermediate inputs forms a positive feedback of capital adjustment in the interindustrial relations. We suppose that the sectoral capital exhibits a intermittent adjustment where a large investment occurs occasionally. Under this environment, we derive the distribution function of the propagation size. The propagation size has a large variance when the real wage and real interest rate do not respond sensitively to the production level. In the limit case when the wage and interest rate are fixed, the variance of capital growth rates does not depend on the level of dissaggregation.

Simulations show that the investment propagation mechanism above explains the aggregate fluctuations of the U.S. economy quantitatively. We specify the representative household's utility as a separable function in leisure and consumption and solve for the equilibrium paths. The results show that the standard deviation, the correlations between production and investment and consumption, and the autocorrelation of production, investment, and consumption match the U.S. postwar business cycles well. Thus we show that, given the magnitude of oscillations that a manufacturing sector exhibits, the sectoral oscillations can add up to the aggregate fluctuations through the investment propagation mechanism with the correct second moments of the business cycle variables.

The paper leaves two points for further explorations. First, the simulation shows that the correlations between two demand components and production are not strong enough simultaneously. It stems from that the consumption responds weakly to income when capital level is fixed. The behavior of representative household needs to be modified in such a way that the income effect becomes strong, for example by incorporating the liquidity constraint. Secondly, the deterministic oscillation of sectoral capital is assumed. It is no doubt an over-simplification that a sectoral capital jumps in one period and depreciate capital over many years. Incorporating the time-to-build of capital would make the sectoral oscillations more realistic with keeping the results of the paper unaltered. Yet it is not obvious that a sectoral capital accumulation process incurs such degree of inflexibility. This leads to the question as to whether the business cycle patterns still obtain if we disaggregate the economy to the establishment level. Our analytics shows that the large aggregate fluctuations can occur in principle regardless of the number of agents, but a quantitative demonstration of the theoretical possibility is left open.

A Appendix

A.1 Proof of Proposition 1

The details of proof draw on Nirei (2003). Here we outline the proof. Let us rewrite the best response dynamics for the investment in t. We use u to denote the step in the dynamics and suppress t. Let k^u denote the mean capital defined by (11) with a profile k_j^u. Define k_j^{*u} by the threshold formula (16) with k^u except for $u = 0$ at which we define $k_j^{*0} = k_{j,t}^*$. Define $s_j^u = (\log k_j^u - \log k_j^{*u})/\log\lambda$. Then the dynamics of (k_j^u, s_j^u) is written as follows.

$$k_j^0 = k_{j,t}(1-\delta)/g \tag{30}$$

$$s_j^0 = s_{j,t} + (\log k_{j,t}^0 - \log k_{j,t})/\log\lambda \tag{31}$$

$$k_j^{u+1} = \begin{cases} k_j^u\lambda & \text{if } s_j^u < 0 \\ k_j^u/\lambda & \text{if } s_j^u > 1 \\ k_j^u & \text{otherwise} \end{cases} \tag{32}$$

$$s_j^{u+1} = s_j^u + (\log k_j^{u+1} - \log k_j^u - \log k^{u+1} + \log k^u)/\log\lambda \tag{33}$$

We consider for $u > 1$ the case $m_0 > 0$. The case $m_0 < 0$ is proved symmetrically by changing the sign of adjustments. $W = 0$ if $m_0 = 0$. Define H_u as the set of j such that $\log k_j^{u+1} - \log k_j^u = \log\lambda$. Define m_u as the size of H_u. First we derive a formula for $N(\log k^{u+1} - \log k^u)$. By definition, we have $\log k_j^{u+1} = \log k_j^u + \log\lambda$ for $u \in H_u$ and $\log k_j^{u+1} = \log k_j^u$ for $u \notin H_u$. Let us define $\varphi = \alpha(\xi-1)/(\xi\alpha+1-\alpha)$. Then the first term of a Taylor expansion is $\sum_{j\in H_u}(k_j^u/k^u)^\varphi \log\lambda$. All the terms after the second term either contain φ or are of order $1/N$. Also the series are absolutely convergent. We have $\varphi \to 0$ as $\xi \to 1$. Hence we obtain $N(\log k^{u+1} - \log k^u) \to \sum_{j\in H_u}\log\lambda = m_u\log\lambda$ as $\xi \to 1$ and $N \to \infty$.

Next we examine $m_0 = N(\log k^1 - \log k_t)/\log\lambda$. We break m_0 into two terms as $m_0 = N(\log k^1 - \log k^0)/\log\lambda + N(\log k^0 - \log k_t)/\log\lambda$. The first term represents the first step adjustments and the second term represents the depreciation. The second term is equal to $N\log((1-\delta)/g)/\log\lambda$. The first term converges to m_1 by the argument in the previous paragraph. Let us study m_1. By the assumption that $s_{j,t}$ follows a uniform distribution, we obtain $\Pr(s_j^0 < 0) = \mu$. Then the number of producers who adjust their capital at the first step, m_1, follows a binomial distribution $\mathrm{Bin}(N,\mu)$. By the central limit theorem, $m_1/\sqrt{N} - \sqrt{N}\mu$ asymptotically follows a normal distribution with mean zero and variance $\mu(1-\mu)$. Combining these results, we obtain that $m_0/\sqrt{N}$ asymptotically follows a normal distribution with mean zero and variance $\mu(1-\mu)$.

Next we examine m_u conditional to m_{u-1}. We have $\Pr(j \in H_u | j \notin \cup_{v=1,2,\ldots,u-1} H_v) = \phi(\log k^u - \log k^{u-1})/\log\lambda$. Thus m_u follows $\mathrm{Bin}(N - \sum_{v=1}^{u-1} m_v, \phi(\log k^u - \log k^{u-1})/\log\lambda)$. This defines the stochastic process m_u completely. As we let $\xi \to 1$ and $N \to \infty$, the binomial converges to a Poisson distribution with an asymptotic mean ϕm_{u-1}.

Since a Poisson distribution is infinitely divisible, the Poisson variable with mean ϕm_{u-1} is equivalent to a m_{u-1}-times convolution of a Poisson variable with mean ϕ. Thus the process m_u is a branching process with a step random variable being a Poisson with mean ϕ. Since $\phi \leq 1$, the process m_u reaches 0 by a finite stopping time with probability one. Thus the best response dynamics is a valid algorithm of equilibrium selection. Let T denote the stopping time. Using the asymptotic formula, we have $W \to \sum_{u=1}^{T} m_u$. By using the property of a Poisson branching process Kingman (1993), we obtain the infinitely divisible distribution for the accumulated sum $W = \sum_{u=1}^{T} m_u$ as in the proposition.

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