Minority Games, Local Interactions, and Endogenous Networks

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First Draft

Abstract

In this paper we study a local version of the Minority Game where agents are placed on the nodes of a directed graph. Agents care about being in the minority of the group of agents they are currently linked to and employ myopic best-reply rules to choose their next-period state. We show that, in this benchmark case, the smaller the size of local networks, the larger long-run population-average payoffs. We then explore the collective behavior of the system when agents can: (i) assign weights to each link they hold and modify them over time in response to payoff signals; (ii) delete badly-performing links (i.e. opponents) and replace them with randomly chosen ones. Simulations suggest that, when agents are allowed to weight links but cannot delete/replace them, the system self-organizes into networked clusters which attain very high payoff values. These clustered configurations are not stable and can be easily disrupted, generating huge subsequent payoff drops. If however agents can (and are sufficiently willing to) discard badly performing connections, the system quickly converges to stable states where all agents get the highest payoff, independently of the size of the networks initially in place.

Keywords: Minority Games, Local Interactions, Endogenous Networks, Adaptive Agents.

JEL Classification: C72, C73.

1 Introduction

In the last years, both physicists and economists have become increasingly interested in investigating the collective properties of dispersed dynamical systems composed of many boundedly-rational agents who directly interact over time (Kirman, 1997).

A well-known instance of such a system is the Minority Game (MG), firstly introduced as a model of inductive rationality in the famous "El-Farol Bar Problem" by W. Brian Arthur (Arthur, 1994) and then explored in details by Challet and Zhang (1997).

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In a nutshell, the standard formulation of the MG envisages a population of N (odd) players who have to repeatedly choose a binary state (-1 or $+1$). In each time period $t = 1, 2, ...$, the state chosen by the minority wins. Agents who are in the minority get a point, the others get zero. Agents are only allowed to observe the last $m \geq 1$ winning sides (i.e. history is the only common information). To choose their next-period state, players use one of their $k \geq 2$ strategies, each strategy being a lookup table assigning an output (i.e. the state to be chosen in the next period) to any of the 2^m possible states configurations. Agents always select their best-performing strategy to make their choice. The performance of any given strategy evolves through time. Agents are initially endowed with a random *repertoire* of strategies, drawn at random from the pool of all 2^{2^m} conceivable ones. If an agent is successful in a given time period, it assigns a point to all strategies that would have predicted the winning state (no matter if they were actually used or not) and zero otherwise.

The standard MG has become a paradigm for studying systems where adaptive agents compete for scarce resources and has been recently employed to study the dynamics of stock markets and market-entry games[1]. From a theoretical perspective, the standard MG model has been extensively studied both numerically and analytically (Challet and Zhang, 1998) and a huge number of contributions have been exploring a large spectrum of possible extensions[2]. In particular, a recent stream of research has investigated the consequences of relaxing two key assumptions of the basic framework, namely global interactions (i.e. each agent cares about being in the minority of the whole population) and common information (i.e. all players have access to the same information, i.e. the globally winning side).

The underlying idea is that each agent playing the MG could instead have access only to a local source of information, e.g. the state played in the past by those agents which are the "closest ones" in some underlying socio-economic space[3]. For example, Paczuski, Bassler, and Cooral (2000) model a MG where agents are placed in a Kauffman NK-network (Kauffman, 1993). Agent can only observe the state of the agents they are currently linked with and hold only one strategy. The latter maps the past state of one's neighbors into the state to be chosen in the next period. Similarly, Kalinowski, Schulz, and Briese (2000) and Slanina (2000) place their agents over one-dimensional, regular, boundary-less lattices (i.e. circles) and allow them to observe the states held in the recent past by their nearest-neighbors. Notice, however, that in these models agents interact *locally* but play

[1]See *inter alia* Challet, Marsili, and Zhang (2000), Marsili and Challet (2001), Bottazzi, Devetag, and Dosi (2003) and Ochs (1990, 1995).

[2]See the Minority Game's Web Page (http://www.unifr.ch/econophysics/minority) in the Econophysics Forum Internet Site for an exhaustive annotated bibliography.

[3]For a similar perspective in the context of "spatial evolutionary games", cf. the survey in Fagiolo (1998).

globally the MG: players are indeed rewarded only if they choose the *globally* winning side. Conversely, Moelbert and De Los Rios (2001) study a local MG where the population is spatially distributed over a regular lattice with nearest-neighbor interactions. Agents only observe time $t-1$ states played by their neighbors and care about being in the minority of their *local* group.

Although the study of "local" versions of the standard setup has been shedding some light on the ways in which heterogeneous information affects collective MG dynamics, very little attention has been paid to the role played by network structures and local interactions in shaping long-run aggregate outcomes. For example, the existing literature has extensively addressed the study of MGs played over very regular network structures (e.g. homogeneous lattices), without exploring the consequences of assuming more asymmetric structures (e.g. generic graphs with directed links). Furthermore, local MGs have almost exclusively focused on "frozen" or "static" interaction structures. This means that agents always interact with the same opponents and do not have the faculty to endogenously modify the structure of their network[4].

In this paper, we begin instead to explore in more details the role played by networks (and the evolution thereof) in local MGs.

We consider a population of N agents living in a discrete-time economy. Each agent is initially connected through unilateral links (directed edges) to a fraction of other players, whose last-period states is the only information they are allowed to observe. Agents are rewarded with one point if they are successful, i.e. if they play as the (strict) minority of their local group does (and zero otherwise).

In any time period, only a fraction of agents are allowed to consider whether to change their state. Agents decide their next-period state by evaluating the state of their local network.

We implement two network evaluation setups. In the first one, we assume *non-weighted links*: each opponent always counts as one, no matter what it did in the past. Therefore, agents simply choose the state played by the (strict) minority of their peers. In the second setup, we introduce *weighted links*. Each agent separately evaluates every link (i.e. every opponent) and assigns to it a weight that increases only if the linked agent has allowed the evaluating agent to take the winning decision in the past. State decisions will then depend upon a comparison between the sum of weights associated to agents playing $+1$ and -1 in the local group. We call this second setup a "weighted" minority game (WMG).

As far as network dynamics is concerned, we simulate the behavior of the system in

[4]As we argue at more length elsewhere (Fagiolo, 2004; Fagiolo, Marengo, and Valente, 2004), allowing for endogenous network formation might crucially help in better understanding the properties of collective dynamics in spatial games.

two scenarios. The first scenario assumes *frozen networks.* The network initially in place cannot be modified and only their weights might possibly evolve. In the second scenario (*endogenous networks*), agents can decide whether to remove a badly performing link, i.e. a link whose weight becomes lower than a given threshold. Since agents do not have a good knowledge of the region of the economy outside their local network, we assume that the new opponent is chosen at random from the remaining set of agents.

We explore system behavior when the initial density of the network changes in alternative network evaluation setups and network dynamics scenarios. Preliminary simulation exercises show that, in the non-weighted local MG with *frozen networks*, the smaller the density of the network, the larger average payoffs. If however we assume a WMG with *frozen* networks, the population tends to build a network of small clusters composed of highly coordinated agents choosing the same state. Agents of one cluster keep assigning more and more importance to agents of other clusters. These global configurations are not in general robust and can be easily disrupted by subsequent network reassessments. Fluctuating patterns for average payoffs are likely to emerge. Finally, we study what happens when agents play the WMG over *endogenous networks.* In this case, the population splits in two subgroups playing opposite states. Agents in a group are prevalently linked with agents of the other group. Average payoffs converge to one. Thus, the population learns to "globally win" the WMG by selecting the most convenient set of opponents in the game.

The rest of the paper is organized as follows. Section 2 formally describes the model. In Section 3 we present preliminary simulation results. Finally, Section 4 concludes and discusses future developments.

2 The Model

We study a population of boundedly-rational agents $i \in I = \{1, ..., N\}$ playing a minority game in discrete time periods $t = 0, 1, 2, ...$ Each agent i is characterized by its binary state $s_i^t \in \{-1, +1\}$ in the game and by the set $V_i^t \subseteq I - \{i\}$ of other agents is currently linked to (which we call *interaction group*). Interaction groups govern the way in which agents gather information in the model: at the beginning of each period $[t, t+1)$, any agent i can only observe the state of the agents it is currently connected with, i.e. $\{s_j^{t-1}, j \in V_i^t\}$.

We denote by ij_t the directed edge linking agent i to j at time t. Since links are directed, the fact that $j \in V_i^t$ does not necessarily imply that $i \in V_j^t$. The collection $\{V_i^t, i \in I\}$ thus induces at every t a directed graph over I. The set of directed edges (links) held by the agents may be interpreted as their "window over the world". Contrary to what happens in the standard MG, agents cannot observe the whole system (i.e. cannot get information about the globally winning side) and must adapt to a small, local, set of signals[5].

In each period, agents get a positive (unit) payoff if they play the same as the minority of their interaction group V_i^t does. More formally:

$$\pi_i^t = \pi_i^t(s \mid n_i^{t-1}(s), n_i^{t-1}(-s)) = \begin{cases} 1, & \text{if} \quad n_i^{t-1}(s) < n_i^{t-1}(-s) \\ 0, & \text{otherwise} \end{cases}, \tag{1}$$

where $s\{-1,+1\}$ and $n_i^{t-1}(s) = \#\{j \in V_i^{t-1} : s_j^{t-1} = s\}$ is the number of agents playing s in V_i^{t-1}.

We assume that, in each time period, any agent is allowed to revise its current state with a probability $\theta \in (0,1]$ and that agents earn the payoff (1) even if they must stay put (or if they do not actually change) their current state.

We consider two alternative setups as to how updating agents evaluate the information coming from their local network and use that information in deciding their next-period state.

In the first setup, every linked agent (i.e. every edge ij_{t-1}) always counts as one, independently of the past behavior of the opponent j. Therefore, agent i chooses its (new) action using a standard, deterministic, best-reply rule[6]:

$$s_i^t = \begin{cases} +1, & \text{if} \quad n_i^{t-1}(+1) < n_i^{t-1}(-1) \\ -1, & \text{if} \quad n_i^{t-1}(+1) > n_i^{t-1}(-1) \end{cases}. \tag{2}$$

In the second setup, we implement a weighted version of the MG (WMG). We suppose that each agent attaches an indicator of importance (*weight*) η_{ij}^t to each link ij_t it maintains. Weights are updated over time. In each period, each link is assigned a *score* e_{ij}^t equal to 1 or 0, depending on whether the "suggestion" coming from j was correct (i.e. if it indicated the state played by the minority of the linked agents) or not. More formally:

$$e_{ij}^t = \begin{cases} 1, & \text{if} \quad \pi_i^t = 1 \quad \text{and} \quad s_i^{t-1} \neq s_j^{t-1} \\ 1, & \text{if} \quad \pi_i^t = 0 \quad \text{and} \quad s_i^{t-1} = s_j^{t-1} \\ 0, & \text{if} \quad \pi_i^t = 0 \quad \text{and} \quad s_i^{t-1} \neq s_j^{t-1} \\ 0, & \text{if} \quad \pi_i^t = 1 \quad \text{and} \quad s_i^{t-1} = s_j^{t-1} \end{cases}. \tag{3}$$

[5] We did not assume undirected edges (i.e. undirected graphs) because of two considerations. First, there is no obvious reason why if i observes j, the opposite should hold as well. Second, if bilateral links are assumed, both the payoff structure and link removal rules (see below) should be supplemented by some additional coalition-level rule that might overly complicate (and possibly bias) our analysis.

[6] We have implemented different tie-breaking rules (TBR) for the case $n_i^{t-1}(-1) = n_i^{t-1}(+1)$. However, our results are not dramatically altered if one assumes stochastic TBRs (e.g. agents choose at random when a tie occurs) or state-dependent ones (e.g. agents stick/switch to their current choice). Therefore, we have decided to avoid any tie-breaking complications simply by assuming that all interaction groups always contain an odd number of players (see below).

Agents then use the score to update the weight of each link as follows[7]:

$$\eta_{ij}^t = \alpha\eta_{ij}^{t-1} + (1-\alpha)e_{ij}^t. \tag{4}$$

Notice that by applying the weight updating rule in (4), agents trade-off their memory about the "past contributions" from agent j with its "current contribution" (e_{ij}^t). The parameter α tunes the memory effect. If $\alpha \simeq 0$, weights track very closely current scores (no memory), whereas, if $\alpha \simeq 1$, memory becomes very important. The η_{ij}^t series are smoother and are quite robust to new scores. See Figure 1 for an example of weight series generated by applying rule (4) to the same series of e_{ij}^t for alternative values of α.

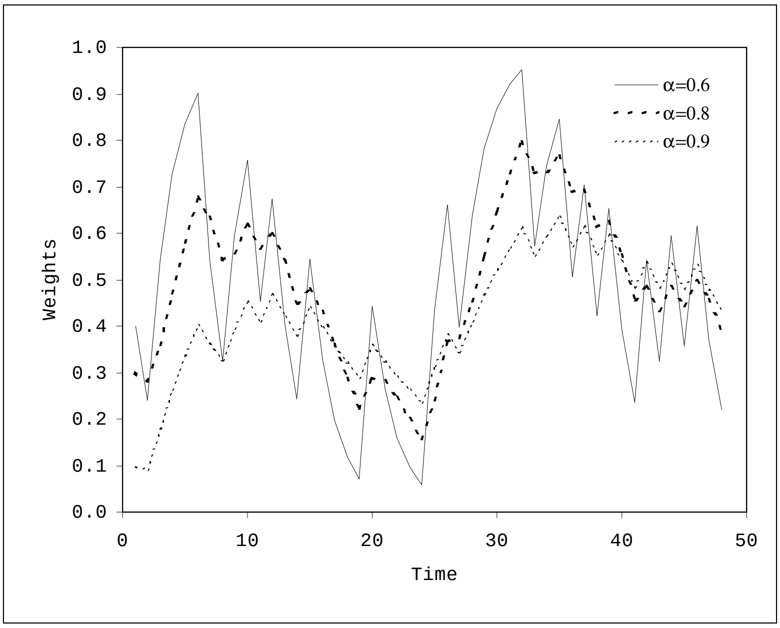

Figure 1: An example of the dynamics of individual weights. Equation (4) is applied over the same time-series of scores e_{ij}^t for three values of the memory parameter α =0.6, 0.8, 0.9.

Given current weights η_{ij}^t attached to agents $j \in V_i^{t-1}$, agent i will then choose its state by employing a weighted-minority rule as follows:

$$s_i^t = \begin{cases} +1, & \text{if} \quad \sum_{j \in V_i^{t-1}(-1)} \eta_{ij}^t < \sum_{j \in V_i^{t-1}(+1)} \eta_{ij}^t \\ -1, & \text{if} \quad \sum_{j \in V_i^{t-1}(-1)} \eta_{ij}^t > \sum_{j \in V_i^{t-1}(+1)} \eta_{ij}^t \end{cases}, \tag{5}$$

where we denote by $V_i^{t-1}(s) = \{j \in V_i^{t-1} : s_j^{t-1} = s\}$ and $s \in \{-1, +1\}$. In other words,

[7]See Weisbuch, Kirman, and Herreiner (2000) and Kirman and Vriend (2001) for a similar approach.

agents now use a weighted best-reply rule, where opponents do not all count as one and their relative importance is given by their performance in the past[8].

Weights provide an indication of how useful a link has been in the past in helping to take the correct decision, no matter if the player actually did choose a new state or was forced to stick to its current one. In a sense, η_{ij}^t is a measure of the trust that i develops through experience in evaluating the information coming from j.

Notice that if an agent turns out to hold very similar weights, the latter will not be very useful as the opponents are all conveying both wrong and right messages. Conversely, an agent having very different values for its η_{ij}^t finds some linked agents consistently useful (and others consistently wrong). In other terms, an agent with very differentiated η_{ij}^t can be interpreted as an agent with "strong opinions", while an agent attaching very similar weights to its opponents is an "uncertain" decision maker. Therefore, the dispersion (Herfindahl) index

$$H_i^t = \frac{\sum_{j \in V_i^{t-1}} \left(\eta_{ij}^t\right)^2}{|V_i^{t-1}|} \tag{6}$$

provides some information about "opinion convergence" for agent i: the larger H_i^t, the wider the dispersion of weights η_{ij}^t.

Finally, let us describe how network evolve through time. At time $t = 0$, each agent is endowed with the same (odd) number of links $L_i^0 = L^0 = \lfloor \delta N \rfloor$, where $\delta \in (0, 1)$ is a proxy for the "density" of the initial network and $L^0 \in \{1, 3, 5, ..., N-1\}$ (i.e. each agent is randomly linked L^0 other agents)[9].

We experiment with two alternative scenarios as far as network dynamics is concerned.

In the first one, *frozen networks* are assumed. Links cannot change in the entire process and, under the WMG setup, weights only can be updated. This allows one to disentangle the roles played by coordination, local interactions, and, possibly, weight updating in shaping collective behavior.

In the second setup, we study an *endogenous networks* system. After state updating, any agent is allowed with some small probability $\beta > 0$ to discard badly-performing links[10]. More formally, we introduce a threshold $\mu \in (0, 1)$ and we suppose that agent i deletes all links such that:

$$\eta_{ij}^t < \mu \frac{\sum_{h \in V_i^{t-1}} \eta_{ih}^t}{|V_i^{t-1}|} = \mu \bar{\eta}_i^t. \tag{7}$$

In line with the local nature of information diffusion in our economy, we assume that

[8]Notice that, if $\alpha = 0$, the weighted rule (5) reduces to (2).

[9]We assume here N even. Notice that this choice allows us to avoid tie breaks (see also above).

[10]If $\beta = 0$ the *frozen networks* setup is recovered.

agents replace any discarded link with a new one (randomly chosen from the set of all currently unconnected agents) and that any new link is initially assigned a weight equal to the average of the weights of all undiscarded links. Notice that all agents always keep L^0 open at any t.

3 Simulation Results

In this Section, we present some preliminary simulation Montecarlo exercises and we discuss the most important properties displayed by the collective behavior of the system.

We begin with a benchmark parametrization[11] where: (i) the frequency of state updating is $\theta = 0.20$; (ii) the population size is $N = 100$. Our main goal is to explore how networks influence collective dynamics. Therefore, we start by studying what happens to the distribution of individual payoffs and to the interaction structure (i.e. links and weights) when the "density" of the initial network (δ or equivalently L^0) changes in each of the following three setups:

1. Agents play a *non-weighted* MG and networks are *frozen*.
2. Agents play a *weighted* MG and networks are *frozen*.
3. Agents play a *weighted* MG and networks are *endogenous*.

In all our exercises, we average our observations over $M = 10$ independent Montecarlo simulations in order to wash away across-simulation variability. Moreover, we observe the system dynamics until the economy has reached a sufficiently stable behavior, which typically happens for $2000 \leq T \leq 4000$[12].

3.1 Non-Weighted Minority Games over Frozen Networks

In the first set of simulations, we study a system where agents play a non-weighted MG over exogenously fixed networks. This means that the initial interaction structure $\{V_i^0, i \in I\}$ is not allowed to change. Agents always observe the same local network (i.e. their "observation window") and are rewarded only if they play as the minority of their opponents. Furthermore, each opponent always count as one, irrespective of its past behavior (i.e. weights are always equal to one for each link).

[11] A thorough Montecarlo analysis exploring the robustness of our results to the parameters kept fixed in our preliminary simulation is the next point in our agenda, see also Section 4.

[12] Our results are fairly robust to alternative (larger) Montecarlo sample sizes (M), population sizes (N) and time horizons (T).

In such a setting, an interesting question concerns whether collective coordination can be affected by the size of the observation window held by the agents. In Figure 2 we plot the evolution over time of population-average payoffs

$$\overline{\pi}^t = \frac{1}{N} \sum_{i=1}^{N} \pi_i^t \qquad (8)$$

as the number of links which we endow each agent with (L^0) changes.

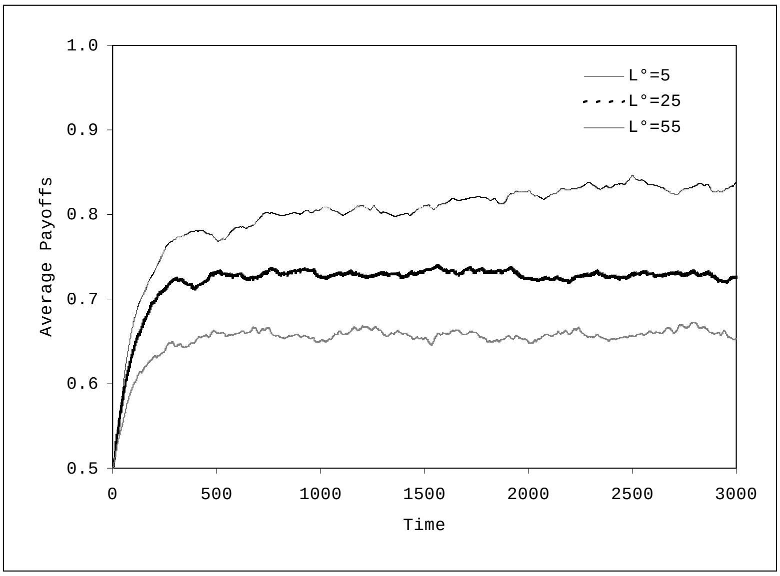

Figure 2: Montecarlo mean of population-average payoffs when agents play a *non-weighted* MG over *frozen* networks and the initial number of links changes (L^0=5, 25, 55). Montecarlo mean performed over $M = 10$ runs. Setup: $N = 100$, $\alpha = 0.99$, $\theta = 0.20$.

Populations with a smaller number of links provide, on average, higher payoff levels. A better coordination is then achieved by playing with a smaller number of opponents. In fact, if agents are only able to observe the world through a smaller window, it is more likely that these windows do not overlap (that is, $i \in V_j^0$ but $j \notin V_i^0$). This allows agents to better coordinate and the system to reach a quite good average performance.

Notice also that the high instability often displayed in standard MG (e.g. frustration) is avoided[13]. Our results suggests that local interactions can in part surrogate individual memory: a stable collective behavior characterized by a better-than-average coordination can be reached even if players cannot observe the last global winning sides and cannot learn in the strategy space (cf. also Bottazzi, Devetag, and Dosi (2003)).

[13]For similar findings in the context of nearest-neighbors interactions on lattices, cf. Burgos, Ceva, and Perazzo (2003), Kalinowski, Schulz, and Briese (2000) and Moelbert and De Los Rios (2001).

3.2 Weighted Minority Games over Frozen Networks

Let us turn now to the case where networks are still frozen but agents can weight the importance of the links they maintain. Notice that in this setup agents cannot discard poorly performing links and add other ones, but they can attach different beliefs as to whether the information coming to a particular opponent will be useful in deciding their next-period state.

Each opponent j in a given V_i^0 will then contribute to the decision at time t in proportion to its link-weight η_{ij}^t. In what follows, we suppose that the importance of memory in weights dynamics is very high (i.e. $\alpha = 0.99$). This means that the score e_{ij}^t currently earned by the opponent j has a small impact in changing the assessments of agent i about its beliefs about j.

When agents are allowed to "select which details to observe from their window", the collective behavior of the system sensibly changes as compared to the non-weighted case. Indeed, populations holding a large number of links are now able to focus on smaller subsets of trusted connections. The latter are the ones that in the past have guaranteed high scores (see eq. 3). As a consequence, the population tends to break down into groups of agents sharing the same state, which, however maintain very weak (if no) connections among them. Agents belonging to, say, a $+1$ group will instead build connections with players belonging to a -1 group.

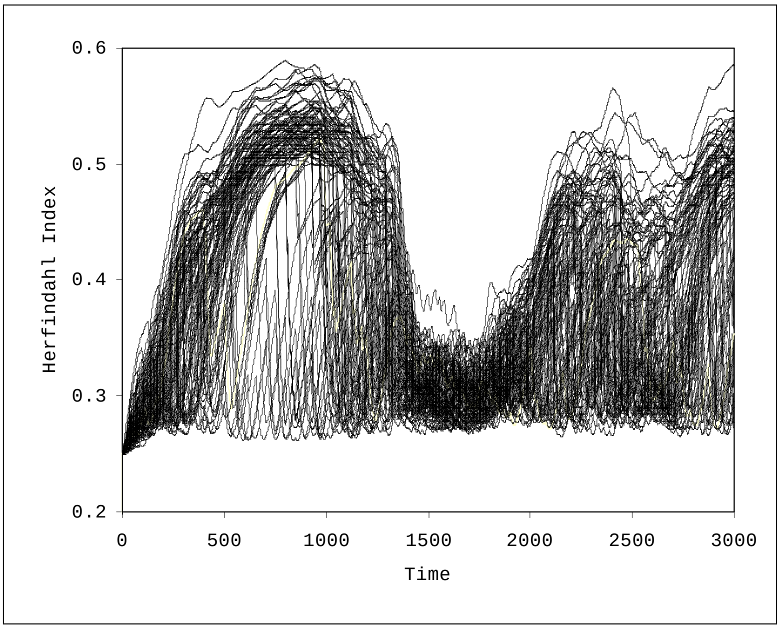

Figure 3: Time-series of individual Herfindahl index H_i^t, $i = 1, ..., N$, when agents play a *weighted* MG over *frozen* networks. Values of H_i^t refer to a benchmark run. Setup: $N = 100$, $\alpha = 0.99$, $\theta = 0.20$, $L^0 = 55$.

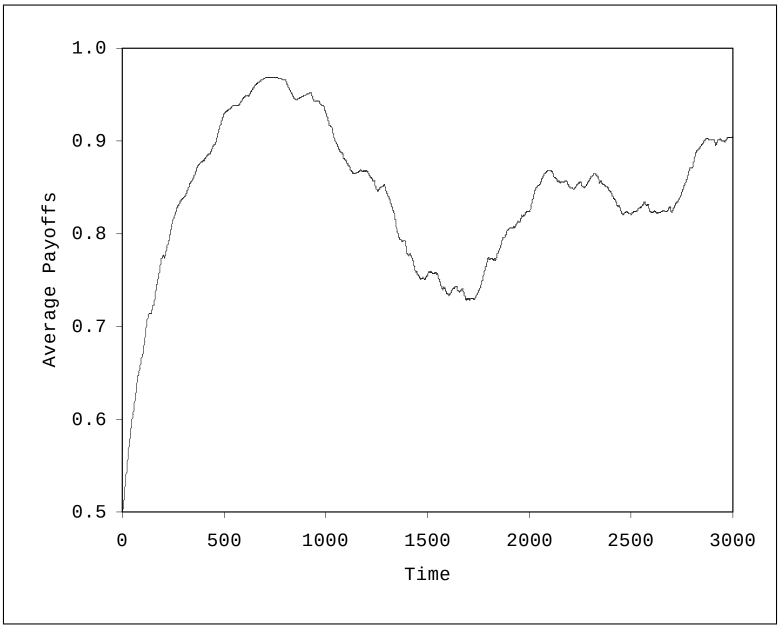

Figure 4: Time-series of population average payoffs when agents play a *weighted* MG over *frozen* networks. Payoff values refer to the same benchmark run of Figure 3. Setup: $N = 100$, $\alpha = 0.99$, $\theta = 0.20$, $L^0 = 55$.

This clustering process entails a high degree of coordination among agents: it is therefore very difficult to achieve and very sensible to small fluctuations. To see this, we report in Figure 3 all Herfindahl individual indices H_i^t (see eq. 6) for a typical simulation run. Notice that all dispersion indices increase when a feasible coordination pattern is under formation. However, after such a clustered network has been built, any small change in the global state-weight configuration is able to disrupt the coordination pattern that has just emerged[14]. Population-average payoff accordingly increases during the construction of the clustered network and then falls after the latter has been destroyed, see Figure 4. The initial wave (with Herfindahl indices all increasing) is due to a larger and larger number of agents strengthening their links to agents belonging to an opposite-state cluster. While more and more agents joint the cluster, the network falls apart. This causes a huge drop in Herfindahl values.

Interestingly enough, populations with an initially low number of links get now *lower* average payoffs than their connected counterparts (see Figure 5). This is because weight assessment over a small number of agents does not provide a big enough pool from which selecting a robust subset of opponents behaving consistently over time.

Note, however, that all three populations reach now (on average) higher payoff levels than in the previous case. This suggests that allowing for some endogeneity in network for-

[14]This dynamic pattern closely resembles that of open-ended economies self-organizing "on the edge of chaos" and displaying *punctuated equilibria*, cf. Lindgren (1991).

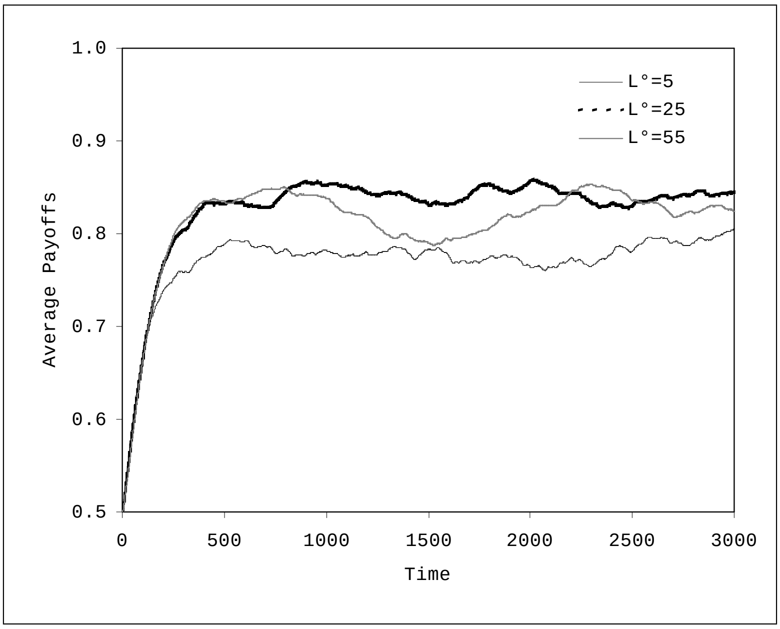

Figure 5: Montecarlo mean of population-average payoffs when agents play a *weighted* MG over *frozen* networks and the initial number of links changes (L^0=5, 25, 55). Montecarlo mean performed over $M = 10$ runs. Setup: $N = 100$, $\alpha = 0.99$, $\theta = 0.20$.

mation (e.g. a payoff-dependent weight dynamics) implies a better collective performance. To further explore this intuition, we move now to the *endogenous networks* setup.

3.3 Weighted Minority Games over Endogenous Networks

Let us now suppose that agents play a WMG and can endogenously delete badly-performing links. We set the cutoff value to $\mu = 0.50$ and the network updating frequency to $\beta = 0.10$. This means that an agent holding a link ij_t such that $\eta_{ij}^t < \frac{1}{2}\overline{\eta}_i^t$ will delete it and replace it by another ij_t', where $j\prime$ is drawn at random from the pool of all currently non-linked players.

In this setup, the coordination process becomes quite efficient and very rapid. Agents typically split into two (almost) equally-sized groups: players in the first one persistently choose the state +1, those in the other one always select the state −1. Agents in one group maintain a large majority of links with agents in the other group. This allows them to effortlessly get a positive reward and rapidly converge to an overall system configuration characterized by average payoffs $\overline{\pi}^t$ very close (if not equal) to one.

Therefore, despite the local nature of interaction patterns, the population is able to “globally win” the MG by endogenously selecting the right set of opponents.

What is more, convergence to a stable interaction pattern allowing for an efficient collective behavior does not generally depend on the initial degree of connectivity. Figure

6 shows the population-average Herfindahl index:

$$\overline{H}^t = \frac{1}{N}\sum_{i=1}^{N} H_i^t, \tag{9}$$

for three populations characterized by $L^0 = 5, 25, 55$. The two *initially* highly connected populations (i.e. $L^0 = 25, 55$) quickly converge to a steady-state with very high values[15]. Average payoffs $\overline{\pi}^t$ for these two populations quickly converge to 1.

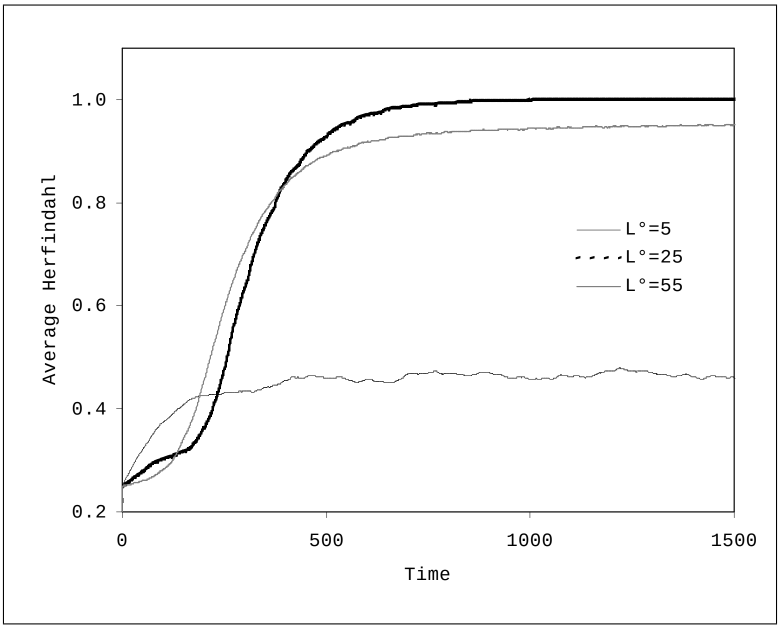

Figure 6: Montecarlo mean of population average Herfindahl index $\overline{H}^t$ when agents play a *weighted* MG over *endogenous* networks and the initial number of links changes (L^0=5, 25, 55). The cutoff value for all three populations is $\mu = 0.50$. Montecarlo means performed over $M = 10$ runs. Setup: $N = 100$, $\alpha = 0.99$, $\theta = 0.20$, $\beta = 0.10$.

Incidentally, notice that the average dispersion index follows a *s*-shaped pattern, i.e. the signature of diffusion processes (Dosi, 1991). Indeed, when the two groups start forming, an agent which is not yet part of any group can simply join one of them by selecting a linked agents in the other group, similarly to what happens in an epidemic process.

Only the $L^0 = 5$ population fails to stabilize. The H_i^t values are rather low, as well as the population-average of weights. This prevents the worst performing links to fall below the deletion threshold. Therefore, agents cannot effectively employ network adaptation to

[15]Notice that $\overline{H}^t \to 1$ when $L^0 = 55$, while $\overline{H}^t$ stabilizes around values very close to one when $L^0 = 25$. Note that the $\overline{H}^t$ value for the $L^0 = 55$ population settles to a stable level which is strictly below 1.0. This happens because agents in any of the two groups must necessarily keep 5 links open to agents playing their same state (i.e. in their own group). The weights for these links will always be very small (and decreasing in time), implying a smaller-than-one average dispersion index.

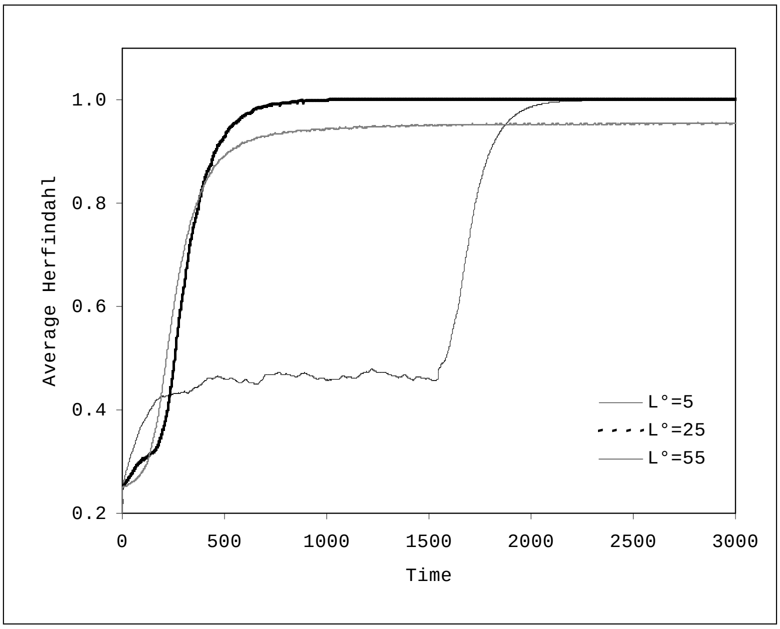

Figure 7: Continuation of the simulation runs presented in Figure 6 after the cutoff value for the $L^0 = 5$ population has been increased to $\mu = 0.80$ at $t = 1550$. Montecarlo mean of population average Herfindahl index performed over $M = 10$ runs. Setup: $N = 100$, $\alpha = 0.99$, $\theta = 0.20$, $\beta = 0.10$.

reach coordination.

However, if the cutoff value increases, agents find it easier to get rid of badly-performing links. To see whether this is sufficient to trigger convergence to the efficient state, we raised on-the-fly (around $t = 1550$) the cutoff values of agents belonging to the $L^0 = 5$ population to $\mu = 0.80$. Figure 7 shows the continuation of the simulation presented in Figure 6: also the initially weakly-connected population is now able to converge to the efficient stable state.

4 Concluding Remarks

In this paper, we have studied a local version of the MG where agents initially hold directed edges connecting them with other players in the population. Agents can only observe the state chosen by their opponents in the last-period and care about being in the minority of their own interaction group. To choose their next-period state, agents employ simple, best-reply, rules.

We have explored two network evaluation setups. In the first one, links are non-weighted and agents simply count the number of their opponents in either state to decide which side to take in the next period. In the second one, links (that is, opponents) are attached a *weight*. A weight path-dependently evolves through time: its value increases only if the information provided by the linked agent has been helpful in the past.

We started from a scenario where networks are *frozen* (i.e. links cannot be deleted/added) and we then moved to a system where *endogenous* networks are assumed (i.e. agents can discard badly-performing links and replace them with other ones chosen at random).

Our results indicate that the very possibility of locally playing the MG and endogenously acting over the network structure might strongly affect the efficiency of collective behavior. For example, even when agents cannot path-dependently discriminate among connections and must stick to an exogenously given network, efficiency can be increased if agents hold small, local, interaction groups. In a sense, individual memory can be surrogated by the information locally gathered by the players in a myopic way.

Furthermore, simulations show that the efficiency of the system can be greatly enhanced by allowing players to act upon the structure of the network. Indeed, our results suggest that populations playing a WMG over frozen networks are able to self-organize and build transient clustered networks which attain high payoff levels. These self-organized configurations are very sensible to subsequent network reassessments: their disruption may generate abrupt fluctuations in average payoffs.

Such huge fluctuations, as well as sub-optimal payoff levels, may be however avoided if agents are able to delete badly performing links and replace them with random ones. In that case, the population is able to "globally win" the WMG and consistently reach a stable state where all agents get a positive payoff. Provided that agents are sufficiently willing to discard links, this conclusion holds independently on the initial number of connections assigned to each player.

These quite promising results should be more carefully scrutinized *vis-à-vis* an extensive Montecarlo analysis over the entire parameter space. For instance, preliminary exercises generate no clear indications about whether efficiency is affected by the frequency of state updating (θ). Nevertheless, a more thorough exploration of how the long-run properties of the system jointly depend on $(\delta, \theta/\beta, \alpha, \mu)$ seems to be needed.

Finally, the relationships between network structure and collective behavior should be studied more deeply. For example, it might be interesting to assess how the fine properties of the network structure initially in place affect the coupled network-state dynamics in the system (as well as the properties of the emergent network structure). More generally, the robustness of our results might be tested against alternative network formation processes allowing e.g. the size of interaction groups to endogenously change as well.

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