Habit formation and Interest-Rate Smoothing

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Abstract

In this paper we show that a habit formation model, suitably specified, can explain why monetary authorities smooth interest rates. To do this we follow Kozicki and Tinsley (2002) and adopt a geometric form for the way in which the stock of habit accumulates from past consumption. This matters because the use of an additive habit stock otherwise violates reasonable postulates of a utility function. As Wendner (2002) has shown a multiplicative form of the habit term in the utility function, recently employed by Carroll (2000) and Fuhrer (2002) has some undesirable properties if the habit function is itself still additive. In addition the particular kind of habit formation (geometric) that we have adopted provides a significant improvement to the dynamic structure of the New Keynesian model. Because the welfare function of the policymaker is that of the representative agent, and consumers dislike large changes in consumption relative to the level of consumption to which they aspire, the optimal (one-period) rule penalises changes in income and also responds sluggishly to shocks. This goes some way towards accounting for the common observation that the responses of output and inflation to shocks are drawn out, and the interest rate used for policy is persistent, even when account has been taken of the persistence in output and inflation.

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1 Introduction

Recently, the idea of habit formation has forced itself back onto the agenda of modern macroeconomics. As Carroll (2001) has pointed out, the view that past consumption patterns of both the individual and others can affect the utility of current consumption is as old as economics itself. Its revival, in part, has been driven by its ability to account for a number of anomalies in first generation stochastic general equilibrium models, such as the equity premium puzzle, identified by Mahra and Prescott (1985), Abel (1990) and Campbell and Cochrane (1999). It has also been invoked to account for the excess smoothness of consumption (Muellbauer, 1988, Deaton, 1992).

In this paper we put habit formation to another use. In particular we show that a habit formation model, suitably specified, can explain why monetary authorities smooth interest rates. In order to do this we follow Kozicki and Tinsley (2002) and adopt a geometric form for the way in which the stock of habit accumulates from past consumption. This matters because the use of an additive habit stock otherwise violates reasonable postulates of a utility function (Wendner, 2002). Wendner (2002) has shown that the multiplicative form of the habit term in the utility function, recently employed by Carroll (2000) and Fuhrer (2002) has some undesirable properties if the habit function is itself still additive. This problem does not arise if the subtractive (linear) form of the habit term in the utility function that was originally used by Muellbauer (1988) is used in combination with an additive habit formation function.

Sack and Wieland (2000) have argued that interest rate smoothing by the monetary authorities may be the optimal response when stabilising output and inflation. They point to three widely cited explanations for interest rate smoothing. First, asset markets are typically forward looking (Woodford, 1999). Hence, history-dependent central-bank behaviour, when anticipated by private agents, can serve stabilisation objectives even when the reduction in the magnitude of interest rate movements is not a social objective *per se*. The persistence in interest rates allows the monetary authority to manipulate long-term rates and hence aggregate demand with relatively modest movements in the short-term rate. Goodfriend (1991) and Roberts (1992) focus on the stabilising role that smooth interest rate responses have on capital markets: sharp interest rate reversals can expose firms and financial intermediaries to interest rate risks. Another view extends Brainard's (1967) work on the impact that uncertainty has on monetary policy. According to this interpretation since central banks have limited knowledge about the economy, they prefer to move cautiously and smooth interest rates.

We generalise the habit formation model of consumption to allow for both a multiplicative utility function and a habit\aspiration function which is a geometrically weighted average of past consumption. The geometric form of the aspiration function addresses the recent concerns of Wendner (2002). The geometric form allows us to derive an optimising model of the IS-PC form in which there is a greater degree of inertia in both inflation and output compared to the additive form of habit formation. The geometric form also allows us to derive a rule for the interest rate in which there is equivalent inertia in the setting of interest rates by the monetary authority.

Because the welfare function of the policymaker is that of the representative agent, and consumers dislike large changes in consumption relative to the level of consumption to which they aspire, the optimal (one-period) rule penalises changes in income and also responds sluggishly to shocks. This goes some way towards accounting for the common observation that the responses of output and inflation to shocks are drawn out, and the interest rate used for policy is persistent, even when account has been taken of the persistence in output and inflation.

We establish the case for interest rate smoothing within the standard New Keynesian paradigm. We combine habit persistence in consumption with sluggishness in price setting that arises in this model from wage indexation. There are a number of other forms of price stickiness - Calvo contracts for example are particularly common - that are likely to generate similar results. But the central point is that the particular kind of habit formation that we have adopted provides an improvement to the dynamic structure of the New Keynesian model. When we set up the policymaker's problem, the implied feedback rule for the interest rate which minimises the policymaker's loss function includes current, future and lagged terms in inflation and output and the interest rate. In section 2 we discuss the form in which habit appears in the utility function of the representative household. In section 3 we integrate the habit function into a standard version of the New Keynesian model and derive the optimal feedback rule from the welfare function of the government. In section 4 we report some simulations of a linearised version of the model and demonstrate how important the particular form of habit formation is for the properties of the model.

2 Properties of a generalised Habit function

Empirical findings on consumption have stimulated intensive research on habit formation (Fuhrer, 2000 and Carrol, Overland and Weil, 2000). Fuhrer(2000) introduces a model with an endogenous additive linear habit with a multiplicative utility function (which becomes non-separable) and shows that optimising behaviour leads to an augmented version of the IS equation where the output gap depends on the exante real interest rate and on past and expected output. More recently Tinsley and Kozicki (2002) have critically reviewed traditional output models and have introduced an aspiration level whose log is approximated by a weighted average of past log consumption. This formulation produces linearised FOCs with higher order, self-reciprocating polynomials in lag and lead operators. We analyse the properties of both habit specifications in a multiplicative utility function.

The representative household is infinitely lived and is assumed to maximise its expected utility, U:

$$U = E_t \left\{ \sum_{j=0}^{\infty} \beta^j U_{t+j}(.) \right\} \tag{1}$$

$U(.)$ is the instantaneous utility function, $\beta = 1/(1+\theta)$ measures a household's impatience to consume and θ is the subjective rate of time preference. The utility function now takes the form common in the literature[1]:

$$U_t = \frac{\left(C_t H_t^{-v}\right)^{1-\alpha}}{1-\alpha} \tag{2}$$

C_t is consumption at time t, α is the inverse of the intertemporal elasticity of substitution and H is the stock of habits. The parameter v indexes the importance of the habit stock. If $v = 0$, only the absolute level of consumption matters, while if $v = 1$, then consumption relative to the stock of habit is all that matters[2]. This specification of the utility function is referred to as multiplicative, in contrast to the subtractive formulation originally introduced by Deaton and Muellbauer (1980). The stock of habit, or reference level of consumption, can be expressed as:

$$H_t = \lambda \digamma(H_{t-1}) + (1-\lambda) \digamma(C_{t-1}) \tag{3}$$

The function $\digamma$ is a general specification of the habit function which can be either linear in its additive formulation (Fuhrer, 2000) or logarithmic in its geometric specification (Tinsley and Kozicki, 2002).

By assuming that $\lim_{n\to\infty} \lambda^n H_{t-n} = 0$ habit formation in its additive form can be expressed as:

$$H_t^a = (1-\lambda) \Sigma_{i=1}^{n} \lambda^{i-1} C_{t-i} \tag{4}$$

where the superscript a indicates that aspiration levels are additive in past levels of consumption.

Alternatively, the habit stock is a geometrically weighted average of past consumption:

$$H_t^m = \Pi_{i=1}^{n} C_{t-i}^{(1-\lambda)\lambda^{i-1}} \tag{5}$$

which corresponds to the multiplicative habit specification proposed by Tinsley and Kozicki (2002) where $\digamma$ is logarithmic. For $\lambda = 0$ we get $H_t = C_{t-1}$ which is the specification adopted by Fuhrer (2000). The parameter λ measures the strength with which previous levels of consumption matter for current aspiration levels.

As Wendner (2002) shows with habit persistence there are some desirable properties that a utility function should satisfy:

[1] For a separable problem this utility function is the constant relative risk aversion utility function with the coefficient of relative risk aversion equal to α and independent of C. For the problem we have this does not hold.

[2] In the literature, the standard of comparison against which current consumption is measured can be internal to the household, so it is only what levels of consumption were in the past that matters, or it can be external so it is consumption relative to what other households consume. In this paper we confine ourselves to the internal form of comparison. Carroll et al (2000) also refer to this as inward and outward looking.

$$\partial U(.)/\partial v < 0 \tag{P1}$$

$$\partial U(.)/\partial C_t > 0 \tag{P2}$$

$$\partial U(.)/\partial H_t \leq 0 \tag{P3}$$

$$\partial MRS_{C_t,C_{t+1}}/\partial v \leq 0 \tag{P4}$$

Property P1 requires that an increase in the strength of habits, v, with no change in current or past consumption, reduces utility. This happens because the larger is v the less is the utility generated from current consumption. Hence, habit forming consumers will postpone consumption since households benefit not only from consumption levels but also from consumption growth (Deaton, 1992). Property P2 requires that an increase in current consumption, with no change in past consumption, and therefore with no change in the habit stock, increases utility. Property P3 requires that an increase in the habit stock with no change in current consumption reduces utility. This happens because when a household gets used to a given habit stock, he will derive less utility from a given amount of current consumption. Property P4 requires that an increase in the importance of a given habit stock in period t, as measured by v, requires a lower marginal rate of substitution C_t for C_{t+1}. Higher consumption today adds to the future habit stock which then lowers future effective consumption.

With respect to property P1 we can show that:

$$\frac{\partial U}{\partial v} = -\Sigma_{j=1}^{\infty}\beta^j U_{t+j}\ln(H_{t+j}) \tag{6}$$

where the log of the habit stock is given by:

$$\ln(H^a_{t+j}) = \ln(1-\lambda)\,\Sigma_{i=1}^{\infty}\lambda^{i-1}C_{t+j-i} \lesseqgtr 0 \tag{7}$$

$$\ln(H^m_{t+j}) = (1-\lambda)\,\Sigma_{i=1}^{\infty}\lambda^{i-1}\ln C_{t+j-i} < 0 \tag{8}$$

As (7) shows, in the additive case the argument of the additive specification can be either greater or less than one (hence positive or negative) so we cannot sign the derivative, whereas in the multiplicative case an increase in the strength of habits, v, always decreases utility since, by assumption, the aggregate consumption level, $C_{t+j-i} > 1$.

We now examine the properties of the intertemporal utility function with respect to consumption in order to verify whether properties P2 and P3 hold for both specifications.

With respect to property P2[3]:

$$\frac{\partial U^a}{\partial C_t} = \frac{\widehat{C}_t^{1-\alpha}}{C_t} - v\left(1-\lambda\right)\Sigma_{j=1}^{\infty}\beta^j\lambda^{j-1}\frac{\widehat{C}_{t+j}^{1-\alpha}}{H_{t+j}^a} \tag{9}$$

$$\frac{\partial U^m}{\partial C_t} = \frac{\widehat{C}_t^{1-\alpha}}{C_t} - v\left(1-\lambda\right)\Sigma_{j=1}^{\infty}\beta^j\lambda^{(j-1)}\frac{\widehat{C}_{t+j}^{1-\alpha}}{C_t} \tag{10}$$

Where $\widehat{C}_t = C_t H_t^{-v}$. A change in consumption today will in general increase the instantaneous utility at time t. However, this will also raise the consumer's aspirations so that when consumption falls back to its previous level, there will be a measure of dissatisfaction. This will continue until the aspiration level falls back to its previous level[4]. To ensure that this dissatisfaction does not outweigh the increase in instantaneous utility from higher consumption and that the utility function remains concave[5] we assume that λ is small[6].

We now analyse whether property P3 holds. We can show that:

$$\frac{\partial U^a}{\partial H_t} = -v\Sigma_{j=1}^{\infty}\beta^j\lambda(j)\frac{\widehat{C}_{t+j}^{1-\alpha}}{H_{t+j}^a} \leq 0 \tag{11}$$

[3] The derivative of $U = U_t + \beta U_{t+1} + ...$ with respect to C_t is:

$$\frac{\partial U}{\partial C_t} = \frac{\partial U_t}{\partial C_t} + \beta\frac{\partial U_{t+1}}{\partial H_{t+1}}\frac{\partial H_{t+1}}{\partial C_t} + ...$$

where $\frac{\partial U_t}{\partial C_t} = \frac{\widehat{C}_t^{1-\alpha}}{C_t}$; $\frac{\partial U_{t+j}}{\partial H_{t+j}} = -\frac{v\widehat{C}_{t+j}^{1-\alpha}}{H_{t+j}}$; $\frac{\partial H_{t+j}^a}{\partial C_t} = \left(1-\lambda\right)\lambda^{j-1}$; $\frac{\partial H_{t+j}}{\partial C_t} = \left(1-\lambda\right)\lambda^{j-1}\frac{H_{t+j}}{C_t}$.

[4] In this model the household is richer for one day. However, this gives the household a taste for the higher level of consumption and it suffers disutility in the future as actual consumption returns to its former level. Eventually the household's aspirations return to their former levels. We assume that this windfall gain cannot be taken in the form of an increase in wealth so that its benefits cannot we spread across time as in the standard life cycle model.

[5] The curvature of the utility function in both specifications is given by:

$$\begin{aligned}\frac{\partial^2 U^a}{\partial C_t^2} &= -\alpha\frac{\widehat{C}_t^{1-\alpha}}{C_t^2} + v\Sigma_{j=1}^{n}\frac{\widehat{C}_{t+j}^{1-\alpha}}{(H_{t+j}^a)^2}\beta^j\lambda^2(j)\ \left[v\left(1-\alpha\right)\right) + 1\right] \\ \frac{\partial^2 U^m}{\partial C_t^2} &= -\alpha\frac{\widehat{C}_t^{1-\alpha}}{C_t^2} + v\Sigma_{j=1}^{\infty}\frac{\widehat{C}_{t+j}^{1-\alpha}}{C_t^2}\beta^j\lambda(j)\ \left[\lambda(j)v\left(1-\alpha\right) + 1\right]\end{aligned}$$

where $\lambda(j) = (1-\lambda)\lambda^{(j-1)}$. Again, both specifications will be concave if we assume either that the autoregressive habit parameter, λ, is small.

[6] The assumption concerning a low value for λ is confirmed by some empirical studies (see in particular Fuhrer, 2000).

$$\frac{\partial U^m}{\partial H_t} = -v\Sigma_{j=1}^{\infty}\beta^j\lambda(j)\frac{\widehat{C}_{t+j}^{1-\alpha}}{H_t} \leq 0 \tag{12}$$

hence habit-forming consumers dislike large and rapid changes in consumption in both specifications. As a household builds a stock of habits, he gets used to a given consumption level. The higher the habit stock the less is the utility derived from a given amount of consumption.

To demonstrate P4 we first derive the marginal rate of substitution for both forms of habit specification:

$$MRS^a_{C_tC_{t+1}} = \frac{\frac{\widehat{C}_t^{1-\alpha}}{C_t} - v\left(1-\lambda\right)\sum_{j=1}^{T}\beta^j\lambda^{j-1}\frac{\widehat{C}_{t+j}^{1-\alpha}}{H^a_{t+j}}}{\beta\frac{\widehat{C}_{t+1}^{1-\alpha}}{C_{t+1}} - v\left(1-\lambda\right)\sum_{j=2}^{T}\beta^j\lambda^{i-2}\frac{\widehat{C}_{t+j}^{1-\alpha}}{H^a_{t+i}}} \tag{13}$$

$$MRS^m_{C_tC_{t+1}} = \frac{\frac{\widehat{C}_t^{1-\alpha}}{C_t} - v\left(1-\lambda\right)\sum_{j=1}^{T}\beta^j\lambda^{j-1}\frac{\widehat{C}_{t+j}^{1-\alpha}}{C_t}}{\beta\frac{\widehat{C}_{t+1}^{1-\alpha}}{C_{t+1}} - v\left(1-\lambda\right)\sum_{j=2}^{T}\beta^j\lambda^{j-2}\frac{\widehat{C}_{t+j}^{1-\alpha}}{C_{t+1}}} \tag{14}$$

Appendix A shows that if we assume that consumption grows at a constant positive rate $\sigma \equiv \frac{C_{t+1}}{C_t}$ and denoting $A = \beta\lambda\sigma^{-(\alpha+v(1-\alpha))}$ then the MRS can be rewritten as:

$$MRS^a_{C_tC_{t+1}} = \frac{1 - v\frac{(\sigma-\lambda)}{\lambda}\sum_{j=1}^{T}A^j}{A - v\frac{(\sigma-\lambda)}{\lambda}\sum_{j=2}^{T}A^j} \tag{15}$$

$$MRS^m_{C_tC_{t+1}} = \frac{\sigma\left(1 - v\frac{(1-\lambda)}{\lambda}\sum_{j=1}^{T}(\sigma A)^j\right)}{\sigma A - v\frac{(1-\lambda)}{\lambda}\sum_{j=2}^{T}(\sigma A)^j} \tag{16}$$

Appendix A also shows that as $T \to \infty$:

$$\partial MRS^a_{C_tC_{t+1}}/\partial v = \partial MRS^m_{C_tC_{t+1}}/\partial v = \frac{(1-\alpha)}{A}\ln\sigma \tag{17}$$

When consumption growth is positive, such that $\sigma > 1$ the consistency with property P4 requires in both specifications $\alpha > 1$. In this case an increase in the strength of habits reduces the marginal rate of substitution between C_t and C_{t+1} at a given consumption path. Consumption today adds to the future habit stock thus reducing future *effective* consumption. Therefore, a lower amount of C_{t+1} is needed to compensate the household for a marginal reduction in C_t as v rises.

3 Habit Formation in the New Keynesian Model

3.1 The optimising IS schedule

Here we integrate the general form of habit formation of the previous section into a more or less standard New Keynesian Model. We start from a representative household who is infinitely lived and maximises expected utility given by (57). The one period utility, U_t, comprises a multiplicative endogenous habit as discussed above and the disutility of labour:

$$U_t = \frac{\left(C_t H_t^{-v}\right)^{1-\alpha}}{1-\alpha} - \frac{L_t^{1+\gamma}}{1+\gamma} \tag{18}$$

L is the supply of labour, γ the elasticity of labour supply. Utility is increasing at a decreasing rate in consumption and disutility is increasing at an increasing rate in labour supply In each period the representative household is subject to the budget constraint:

$$C_t + B_t = \frac{W_t}{P_t} L_t + R_t \ B_{t-1} \tag{19}$$

B_t denotes bonds, W_t denotes the nominal wage rate for employed hours and R_t is the real, one-period gross return to bonds. P_t is the price level.

The first order conditions for optimal consumption, bond allocations and hours worked are:

$$\frac{\partial U}{B_t} = E_t \left\{-\Lambda_t + \beta R_{t+1} \Lambda_{t+1}\right\} = 0 \tag{20}$$

$$\frac{\partial U}{\partial C_t} = E_t \left\{(1-\alpha)\left(\frac{U_t}{C_t} - \frac{v}{C_t} \Sigma_{j=1}^{\infty} \beta^j (1-\lambda) \lambda^{j-1} U_{t+j} - \Lambda_t\right)\right\} = 0 \tag{21}$$

$$\frac{\partial \Upsilon}{\partial L_t} = E_t \left\{-L_t^{\gamma} + \Lambda_t \frac{W_t}{P_t}\right\} = 0 \tag{22}$$

where Λ_t is the Lagrangian multiplier on the t period flow constraint, with $\Lambda_t \geq 0$.

We can easily see (Tinsley, 2001) that the first relationship in its log-linear form reproduces the term structure of interest rates. Log-linearising (20):

$$x_t = E_t \left\{x_{t+1} + \log(\beta) + \rho_{t+1}\right\} = 0 \tag{23}$$

Where we define the short-term interest rate as $\rho_{t+1} = \log R_{t+1} = r_{t+1} - \pi_{t+2}$ and then solving forward we get:

$$x_t = (\rho_{n,t} - \overline{\rho}_n) = \sum_{i=0}^{\infty}(\rho_{t+1+i} - \overline{\rho}). \tag{24}$$

the n period real interest rate is the sum of the real short term interest rates.

Replacing 24 in 21 Appendix B shows that a log deviation formulation of 21 is:

$$E_t \left\{ -g_0 y_t + \sum_{j=1}^{\infty} g_j (y_{t-j} + \beta^j y_{t+j}) - (\rho_{n,t} - \overline{\rho}_n) \right\} = 0 \tag{25}$$

where we have normalised on y_t by assuming market clearing $c_t = y_t$. Let us denote $w(\lambda) = \frac{(1-\lambda)}{1-\beta\lambda}$ and $z(\lambda) = \frac{(1-\lambda)}{1-\beta\lambda^2}$. The coefficients are defined as:

$$\begin{aligned}
g_0 &= \frac{\alpha - \beta v \left[v(1-\alpha)(1-\lambda)z(\lambda) + w(\lambda)\right]}{1 - \beta v w(\lambda)} \\
g_1 &= (1-\lambda)\frac{(1-\alpha)\, v\, (-1 + \lambda\beta v z(\lambda))}{1 - \beta v w(\lambda)} \\
&. \\
&. \\
g_j &= \lambda^{j-1}(1-\lambda)\frac{(1-\alpha)\, v\, (-1 + \lambda\beta v z(\lambda))}{1 - \beta v w(\lambda)}
\end{aligned}$$

The relationship in (25) reduces to a higher-order Euler equation which has the desirable feature, as stressed by Tinsley and Kozicki (2001) that it improves consistency with what we observe in the data while preserving the simplicity of the minimalist model.

We can easily verify that the augmented IS aggregate demand relationship in (25) nests Fuhrer's specification. For $\lambda = 0$, we have $z(\lambda) = w(\lambda) = 1$,so the coefficients become:

$$g_0^f = \frac{\alpha - \beta v\,(v(1-\alpha) + 1)}{(1 - \beta v)} < g_0 \tag{26}$$

$$g_1^f = -\frac{(1-\alpha)\, v}{(1 - \beta v)} < g_1 \tag{27}$$

Note that the coefficients on current, lagged and future consumption are higher in the multiplicative form.

For $n \rightarrow \infty$ the infinite backward and forward sum can be expressed as:

$$\sum_{j=1}^{n} \lambda^j \left(y_{t-j} + \beta^j y_{t+j}\right) = y_t \left(\frac{1}{(1-\lambda L)} + \frac{1}{(1-\lambda\beta F)} - 2 \right) \tag{28}$$

$$E_t\left\{-gy_t-\lambda\frac{g_1(y_{t-1}+\beta y_{t+1})}{1+\beta\lambda^2-\lambda\left(L+\beta F\right)}-(\rho_{n,t}-\overline{\rho}_n)\right\}=0 \tag{29}$$

L is the lag operator, so $L^i x_t = x_{t-i}$ and F is the lead operator, so $F^i x_t = x_{t+i}$. So now the IS equation can be written as a function of the long-term real interest rate $\rho_{n,t}$:

$$y_t = b_0 + b_1(y_{t-1}+\beta y_{t+1}) - b_2\rho_{n,t} + b_3\left(\rho_{n,t-1}+\beta\rho_{n,t+1}\right) \tag{30}$$

where:

$$\begin{aligned}
b_1 &= \frac{1}{(1+\beta\lambda^2)}\left(\lambda-\frac{g_1}{g_0}\right)\\
b_2 &= \frac{1}{g_0}\\
b_3 &= \frac{\lambda}{(1+\beta\lambda^2)}\frac{1}{g_0}
\end{aligned}$$

In contrast with the standard optimising IS equation, the relationship for output in (30) contains extra leads and lags. The additional lead and lag terms lead to smoother output responses to movements of real interest rates (and shocks) compared to a model which is only forward looking.

3.2 The Supply Side

A revenue maximizing monopolistic firm indexed by z produces a single differentiated nontraded good, also indexed by z, employing a continuum of differentiated labor inputs indexed by k. The representative firm takes wages as given and chooses prices and labour inputs to maximize profits:

$$\max_{P(z),\ L(k)} P_t(z)Y_t(z) - W_t L_t \tag{31}$$

subject to the production function:

$$Y_t(z) = L_t \tag{32}$$

and to the demand function for a particular good z:

$$Y_t^d(z) = \left[\frac{P_t(z)}{P_t}\right]^{-\psi} Y_t \tag{33}$$

where ψ is the price elasticity of demand faced by each monopolist and P_t is the general price level. The production function (32) of the monopolistic producer exhibits decreasing returns with respect to labor L_t which is defined as:

$$L_t = \left[\int_0^1 L_t(k)^{\frac{\phi-1}{\phi}} dk \right]^{\frac{\phi}{\phi-1}} \qquad \phi > 1 \tag{34}$$

with $L_t(k)$ denoting differentiated labor inputs, where $k \in [0, 1]$. The parameter ϕ is the (constant) elasticity of substitution between labor inputs.

Let $W_t(k)$ denote the nominal wage of worker k. Then, the aggregate nominal wage W_t is equal to:

$$W_t = \left[\int_0^1 W_t(k)^{1-\phi} dk \right]^{\frac{1}{1-\phi}} \qquad \phi > 1 \tag{35}$$

Hence, the firm's maximization problem implies a demand for labor of type k, $L_t^{k^d}$, equal to:

$$L_t^{k^d} = \left(\frac{P_t(z)}{W_t^k} \right)^{\phi} Y_t(z) \tag{36}$$

We assume that nominal wages are indexed to price changes between time t and time $t-1$:

$$W_t^k = \left(\frac{P_t}{P_{t-1}} \right)^{\mu_k} \tilde{W}_t^k \tag{37}$$

where $\tilde{W}$ denotes the fully flexible nominal wage, and $\frac{P_t}{P_{t-1}}$ is one plus the rate of inflation at time t. The parameter μ captures the extent to which the i wage is indexed.

For the sake of simplicity, and in common with much of the literature, we impose symmetry across agents so that $\mu_k = \mu$ and $\tilde{W}_t^k = \tilde{W}_t$, so (37) reduces to:

$$W_t = \left(\frac{P_t}{P_{t-1}} \right)^{\mu} \tilde{W}_t \tag{38}$$

and substituting (38) into (36):

$$L_t^d = \left(\frac{P_t(z)}{\tilde{W}_t} \right)^{\phi} \left(\frac{P_t}{P_{t-1}} \right)^{-\phi\mu} Y_t(z) \tag{39}$$

where we have dropped the index z Thus in a symmetric environment, output $Y_t(z)$ and all prices $P_t(z)$ are equal in equilibrium across firms, hence $Y_t(z) = Y_t$ and $P_t(z) = P_t$.

Given the consumer' demand schedule (33) and the labor demand (39), a profit maximizing firm will set the optimal price, P_t, according to the following markup rule:

$$\frac{\tilde{W}_t}{P_t} = \frac{\psi - 1}{\psi} \tag{40}$$

Relationship (40) is the standard pricing rule with fully flexible real wages followed by monopolistic firms which face a constant elasticity of demand. By replacing (40) in (38) the real wage can be expressed as :

$$\frac{W_t}{P_t} = \left(\frac{P_t}{P_{t-1}}\right)^{\mu} \frac{\psi - 1}{\psi} \tag{41}$$

Replacing (41) and (39) in the first order condition for the household given by equation (22)[7]:

$$\left(\frac{\psi}{\psi - 1}\right)^{1+\gamma\phi} \left(\frac{P_t}{P_t - 1}\right)^{-\mu(1+\gamma\phi)} Y_t^{\gamma}(z) = \Lambda_t \tag{42}$$

where we have replaced L_t with (39) and the real wage with (41). By replacing in Λ_t the FOC (22) assuming $c = y$ and log-linearizing we get:

$$\mu(1 + \gamma\phi)\pi_t = y_t(\gamma + g) - \sum_{j=1}^{n} g_j (y_{t-j} + \beta^j y_{t+j}) \tag{43}$$

where $\pi_t = \log(P_t) - \log(P_{t-1})$. We replace again the infinite backward and forward sum using 28:

$$\pi_t = a_1 \left(\pi_{t-1} + \beta\pi_{t+1}\right) + a_2 y_t - a_3 \left(y_{t-1} + \beta y_{t+1}\right) \tag{44}$$

where

$$\begin{aligned} a_1 &= \tfrac{\lambda}{(1+\beta\lambda^2)} \\ a_2 &= \tfrac{(\gamma+g_0)}{\mu(1+\gamma\phi)} \\ a_3 &= \tfrac{\lambda}{\mu(1+\gamma\phi)(1+\beta\lambda^2)} \left(\gamma + g_0 - g_1\right) \end{aligned}$$

Note that as $\lambda \to 0$ 44 reduces to the standard Phillips specification where inflation, π_t, depends on current output gap, y_t.

[7] $-\left(\frac{\psi}{\psi-1}\right)^{\gamma\phi} \left(\frac{P_t}{P_{t-1}}\right)^{-\gamma\phi\mu} Y_t^{\gamma}(z) + \Lambda_t \frac{\psi-1}{\psi} \left(\frac{P_t}{P_{t-1}}\right)^{\mu} = 0$

3.3 Maximising Welfare

We next need to derive a loss function for the monetary authority. Assume the monetary authority shares household preferences over consumption and labour:

$$W_t = U_t(C_t, H_t) - V(L_t) \tag{45}$$

We can approximate 45 using a Taylor's series expansion:

$$U_t(C_t, H_t) = U + U_{C_t}\widetilde{C}_t + U_{H_t}\widetilde{H}_t + \frac{U_{C_t,C_t}\widetilde{C}_t^2}{2} + \tag{46}$$

$$\frac{U_{H_t,H_t}\widetilde{H}_t^2}{2} + \frac{U_{C_t,H_t}\widetilde{C}_t\widetilde{H}_t}{2} + \frac{U_{H_t,C_t}\widetilde{H}_t\widetilde{C}_t}{2} \tag{47}$$

$$V_t(L_t) = V + V_L\widetilde{L}_t + V_{LL}\frac{\widetilde{L}_t^2}{2} \tag{48}$$

where all the derivatives are evaluated at the steady-state levels C and H and L. Variables with a tilde denote deviation of consumption, habit and labour (in levels) from their steady-state. Appendix C shows that a utility-based welfare function for the policymaker is given by:

$$W_t = U(1-\alpha)\left(\begin{array}{c} \frac{1}{2}\sigma_y\left(-(\alpha+\gamma)\left(1-2\lambda+\lambda^2\right)+(1-\alpha v)(1-\lambda)^2\right) \\ -\frac{1}{2}\sigma_\pi(1+\gamma)\phi^2\mu^2\left(1-2\lambda+\lambda^2\right)+I+O\left(\parallel\xi\parallel^3\right) \end{array}\right)$$

where I denotes terms which are not affected by monetary policy and $O\left(\parallel\xi\parallel^3\right)$ are higher order terms which can be neglected. So the policy objective function can be rewritten as:

$$W_t = -U(1-\alpha)\left(\frac{1}{2}L_1\sigma_y + \frac{1}{2}L_2\sigma_\pi - I - O\left(\parallel\xi\parallel^3\right)\right) \tag{49}$$

with

$$\begin{aligned} L_1 &= (\alpha+\gamma)\left(1-2\lambda+\lambda^2\right) - (1-\alpha v)(1-\lambda)^2 \\ L_2 &= (1+\gamma)\phi^2\mu^2 \end{aligned}$$

where we assume that authorities can control the volatility of current output and inflation but they cannot affect past realisations of inflation and output. Also we assume that the correlation between output and inflation is time invariant. The resulting objective function is similar to the utility based loss derived by Gali and Monacelli (2002). Note that if $v = \lambda = 0$ then $L_1 = (\alpha+\gamma)$; hence in a utility-based framework if habits play no role, the monetary authority will place a larger weight on the volatility of inflation relative to the volatility of output.

3.4 The Optimal Reaction Function

Standard literature originating from Taylor (1979) postulates that the objective for monetary policy is to minimize some combination of the variance of inflation and output around their equilibrium level. When the utility function is time separable the maximisation of household welfare is equivalent to minimising the volatility of output and inflation (Woodford,1997 and Woodford, 1999). However, when habit formation is allowed for, the implied time non-separability of the utility function changes the authority's objective in addition to enriching the dynamics of the output and inflation specification.

From (49) it follows that the welfare based loss function can be expressed as:

$$\min_{\rho_t} W_t = U(1-\alpha)\left(\frac{1}{2}L_1 y_t^2 - \frac{1}{2}L_2\pi_t^2 + C + O\left(\| \xi \|^3\right)\right) \tag{50}$$

Given the time non-separability of the utility function, standard dynamic programming cannot be applied. However we could focus on the one-period loss function and by minimising (50) with respect to y_t, we get the reaction function:

$$y_t = -\frac{L_2}{L_1}\frac{(\gamma+g_0)}{\mu(1+\gamma\phi)}\pi_t = -\frac{L_2}{L_1}a_2\pi_t \tag{51}$$

By replacing (30) in (51) and rewriting everything in terms of ρ_t we derive the optimal feedback rule:

$$\rho_t = k_1\Delta\pi_t + k_2(\Delta\pi_{t-1} + \beta\Delta\pi_{t+1}) + k_3\left(\rho_{t-1} + \beta\rho_{t+1}\right) \tag{52}$$

where

$$k_1 = g_0\tfrac{L_2}{L_1}a_2, k_2 = \tfrac{1}{(1+\beta\lambda^2)}\tfrac{L_2}{L_1}a_2\left(\tfrac{g_1}{g_0} - \lambda\right), \text{ and } k_3 = \tfrac{\lambda}{(1+\beta\lambda^2)}$$

If $\lambda = 0$ (Fuhrer, 2000) then habits just depend on past consumption $H_t = C_{t-1}$ and the coefficients of the feedback rule become:

$$\begin{aligned} k_1^f &= g_0^f\tfrac{L_1^f}{L_2^f}a_2^f \\ k_2^f &= \tfrac{1}{(1+\beta\lambda^2)}\tfrac{L_1^f}{L_2^f}a_2^f\tfrac{v(1-\alpha)}{1-v\beta} \\ k_3^f &= 0 \end{aligned}$$

where as shown in section 3.1 $g_0^f < g_0$, $\frac{L_1^f}{L_2^f} = \frac{\alpha+\gamma-(1-\alpha v)}{(1+\gamma)\phi^2\mu^2}$, and $a_2^f = \frac{(\gamma+g_0^f)}{\mu(1+\gamma\phi)}$. In this specification the short term interest rate responds to past output, current inflation, one period lagged and leading change of output but it does not respond to the lagged and leading short-term interest rate.

Note when $v = 0$, $\lim_{v \to 0} g_0 = \alpha$, $\frac{L_1^t}{L_2^t} = \frac{\alpha+\gamma-1}{(1+\gamma)\phi^2\mu^2}$, and $a_2^t = \frac{(\gamma+\alpha)}{\mu(1+\gamma\phi)}$, so the short-term interest rate responds only to inflation:

$$k_1^t = \alpha \frac{L_1^t}{L_2^t} a_2^t, k_2^t = 0 \text{ and } k_3^t = 0$$

The response of the short-term interest rate to the output gap and to current and expected inflation will be higher in a world where λ, in the habit function, is positive and habits are multiplicative. In this case there will also be interest rate smoothing so interest rate also respond to past and expected interest rate. This is a strong result and implies that interest rates respond more smoothly than in Fuhrer's case where just past consumption enters the utility function. We show in the next section how this modification affects the properties of a calibrated model.

4 Simulation Exercise

In the following sections we calibrate the log-linearised new keynesian macro-model derived in section (3) under the assumption of multiplicative habits and show the implications for output and inflation.

Our stylised, linearised model takes the form:

$$y_t = b_0 + b_1(y_{t-1} + \beta y_{t+1}) - b_2\rho_{n,t} + b_3\left(\rho_{n,t-1} + \beta\rho_{n,t+1}\right) \tag{53}$$

$$\pi_t = a_1\left(\pi_{t-1} + \beta\pi_{t+1}\right) + a_2 y_t - a_3\left(y_{t-1} + \beta y_{t+1}\right) \tag{54}$$

$$\rho_t = k_1\Delta\pi_t + k_2(\Delta\pi_{t-1} + \beta\Delta\pi_{t+1}) + k_3\left(\rho_{t-1} + \beta\rho_{t+1}\right) \tag{55}$$

$$r_t - \pi_{t+1} = \rho_{n,t} + n(\rho_{n,t+1} - \rho_{n,t}) \tag{56}$$

where (53) is the IS relationship with multiplicative habits, (54) is the hybrid Phillip's equation, (55) is the utility-based loss function and (56) is an equilibrium relationship which links the short term real interest rate to its long term real interest rate[8].

[8] Under the assumption of efficient markets and rational expectations, the expected return of an n period bond will equal the real return from investing in one-period bills for n periods hence $\rho_{n,t} = \frac{1}{n}\sum_{i=0}^{n-1} E_t(\rho_{t+i})$. The real interest rate is recovered from the Fisher equation $E_t\left\{\rho_{t+i}\right\} = E_t\left\{r_{t+i} - \pi_{t+i+1}\right\}$. Therefore:

5 AIM

We simulate the model using the Anderson-Moore (AIM) algorithm. AIM is a rational expectations algorithm for computing the vector autoregressive reduced-form of a forward-looking linear structural model;

$$\sum_{i=-\tau}^{\theta} H_i \underset{n\times 1}{x_{t+i}} = \varepsilon_t \tag{57}$$

where H_i, with $i = -\theta, ..., \tau$ represent the structural coefficients of the model.

We start rewriting the model in a more compact form:

$$HZ = \varepsilon_t \tag{58}$$

where H is the vector of structural coefficients for different orders of lead and lags:

$$\underset{(n\times n)(\tau+\theta+1)}{H} \equiv [H_{-\tau}........., H_0, ..., H_\theta] \tag{59}$$

$$Z_t \equiv [x_{t-\tau}......x_t........x_{t+\theta}]' \tag{60}$$

and Z collects for various leads and lags the vector of variables $x = \left[y_t, \pi_t, r_t, \rho_t, \rho_{n,t}\right]$. Finally $\varepsilon = [\varepsilon_{y_t}, \varepsilon_{\pi_t}, \varepsilon_{r_t}, 0, 0]$ is the vector of shocks.

5.1 Conditioning equations

To derive the restricted Vector Autoregressive Representation of the structural model AIM constructs a matrix P which is a $\theta n \times (\theta + \tau)n$ matrix which comprises a set of auxiliary conditions (Q) which are imposed to ensure the non-singularity of the leading matrix H_θ plus a set of stability condition on the forward-looking part of the system which provide the additional equations that close the system under saddle-path stability.

$$\rho_{n,t} = \frac{1}{n} E_t \left[(r_t - \pi_{t+1}) + ...(r_{t+n-1} - \pi_{t+n})\right]$$

We can derive an iterative method to calculate the ex-ante $n-$period long term real interest rate as:

$$r_t - \pi_{t+1} = n\rho_{n,t+1} - (n-1)\rho_{n-1,t} = \rho_{n,t} + n(\rho_{n,t+1} - \rho_{n,t})$$

which will be used in our simulations and where to get the result we have assumed that $(n-1)\rho_{n-1,t} \simeq (n-1)\rho_{n,t}$.

$$\overbrace{\begin{bmatrix} Q \\ V_u' \end{bmatrix}_{\theta n \times (\theta+\tau)n}}^{P} \begin{bmatrix} x_{t-\tau} \\ .... \\ x_{t+\theta-1} \end{bmatrix} = 0 \tag{61}$$

The matrix P can then be partitioned into left and a square right blocks so that $P = (P_1 \mid P_2)$:

$$P \equiv \begin{bmatrix} Q \\ V_u' \end{bmatrix} = \begin{bmatrix} \underset{\theta n \times \tau n}{P_1} & \underset{\theta n \times \theta n}{P_2} \end{bmatrix} \tag{62}$$

The conditioning equation can be rewritten as:

$$P_1 \begin{bmatrix} x_{t-\tau} \\ .... \\ x_{t-1} \end{bmatrix} + P_2 \begin{bmatrix} x_t \\ .... \\ x_{t+\theta-1} \end{bmatrix} = 0 \tag{63}$$

where x_t now depends on its past values through the coefficient matrix P_1 and on its future through the coefficient matrix P_2.

5.2 Vector Autoregressive Representation

Given $\det(P_2^{-1}) \neq 0$ and premultiplying by $-P_2^{-1}$ yields the auto-regressive representation:

$$x_t = \sum_{i=1}^{\tau} B_i x_{t-i} \tag{64}$$

where B is given by the first n rows of $-P_2^{-1}P_1$ and

$$B \equiv [B_\tau \quad B_{\tau-1}.......B_1] \tag{65}$$

Replacing the implied forecast formula of (64) in (57) yields the observable structure:

$$S_0 x_t = \sum_{i=1}^{\tau} S_i x_{t-i} + \varepsilon_t \tag{66}$$

Therefore for S_0 non-singular, the restricted VAR representation is:

$$x_t = B_{t-i} x_{t-i} + u_t \tag{67}$$

where $B_{t-i} \equiv S_0^{-1} S_i$ and $u_t \equiv S_0^{-1} \varepsilon_t$. Therefore a VAR can be written in the standard MA(∞) form as:

$$x_t = \sum_{i=0}^{\infty} B_{t-i}^i u_{t-i} = \sum_{i=0}^{\infty} B_{t-i}^i S_0^{-1} \varepsilon_{t-i} = \sum_{i=0}^{\infty} C_{t-i} \varepsilon_{t-i}$$

Therefore the responses of x_t are determined by the rows of B_{t-i}^i.These are the responses of x to standard shocks in ε_t.

$$z_t = C_t \varepsilon_t + C_{t-1} \varepsilon_{t-1} + C_{t-2} \varepsilon_{t-2} ... \tag{68}$$

Therefore the matrix C_k has the interpretation:

$$\frac{\partial z_t}{\partial \varepsilon_{t-k}} = C_{t-k} \tag{69}$$

which will be used in our impulse response analysis

6 Calibration and Policy Experiment

Table 1 reports the values assigned to the parameters in our simulation exercise. The value of the power utility coefficients, α, is set to 6.11 as in the FIML estimates of Fuhrer (2000). Since we simulate the model on a quarterly basis we set the fractional discount factor β at 0.99 which corresponds to an annual rate of 3.96% (Woodford and Rotemberg (2000)). As in Ravenna and Natalucci (2002) we set γ to 2, implying an elasticity of labour supply equal to $\frac{1}{2}$.

Moving to the parameter in the wage and price equation we set the elasticity of demand, ψ equal to 7.88 as in Woodford and Rotemberg (2000). We do not have a specific estimate for the elasticity of labour inputs, ϕ, so we set it equal to 3 as in the calibration exercise of Sarajevs (2000). As in Alogoskoufis-Manning (1998) the estimate of the wage indexation index μ is set to 0.96 to reflect the almost total indexation experience of most OECD countries.

We then move to our simulation exercise where we set v to 0.2[9]. We then compare the response of inflation, output and interest rates to a standardised unitary shock to inflation ,output and the interest rate in two cases. One which corresponds to Fuhrer's specification for habits and the other to the geometric form used in this paper. This corresponds to λ equal to zero in Fuhrer's case and λ equal to 0.4 in our case.

The results are graphed in Figures 1 to 3. A number of features of these results stand out. The immediate jump in all state variables is identical for the two cases of additive and multiplicative habit formation. However, the subsequent adjustment

[9] The habit strength parameter is set in order to ensure that the properties of the utility function illustrated in section 2 are satisfied.

Table 1: Parameter Values

Parameter Values		
Utility Function		
α	6.11	Fuhrer (2000)
β	0.99	Woodford and Rotemberg (2000)
γ	2	Ravenna (2002)
Wages and Prices		
ϕ	3	Sarajevs (2000)
ψ	7.8	Woodford and Rotemberg (2000)
μ	0.96	Alogoskoufis-Manning (1998)
Parameters		
ν		0.2
λ		0 & 0.4

is quite different. For the multiplicative habit case adjustment in output, inflation and interest rates is much more sustained. A quite modest re-specification has a very large impact on the properties of the New Keynesian model. For the inflation shock the response of output (and the long term interest rate) in the multiplicative case is hump shaped.

7 Conclusions

We have shown that a relatively modest re-formulation of the habit specification of the consumption model of Carroll (2000) and Fuhrer (2002) to make the stock of habit a geometric average of past levels of consumption achieves two things. Firstly, it addresses some recent concerns of Wendner (2002) that the approach used by Carroll and Fuhrer violates some reasonable postulates of the utility function. Secondly, when this habit function is incorporated into an otherwise standard New Keynesian model, we are able to generate a much smoother response to shocks in output and inflation. Interest rate adjustment is also much smoother. Nevertheless, what we have done does not fully generate other features of the data. For example, the initial response of inflation to an innovation in output is still immediate, when there is considerable evidence to suggest that inflation responds with a delay. Recent developments in the literature on optimal price adjustment (Mankiw and Reis, 2003) with sticky information suggest further ways of modifying the model to improve the response of the model to shocks.

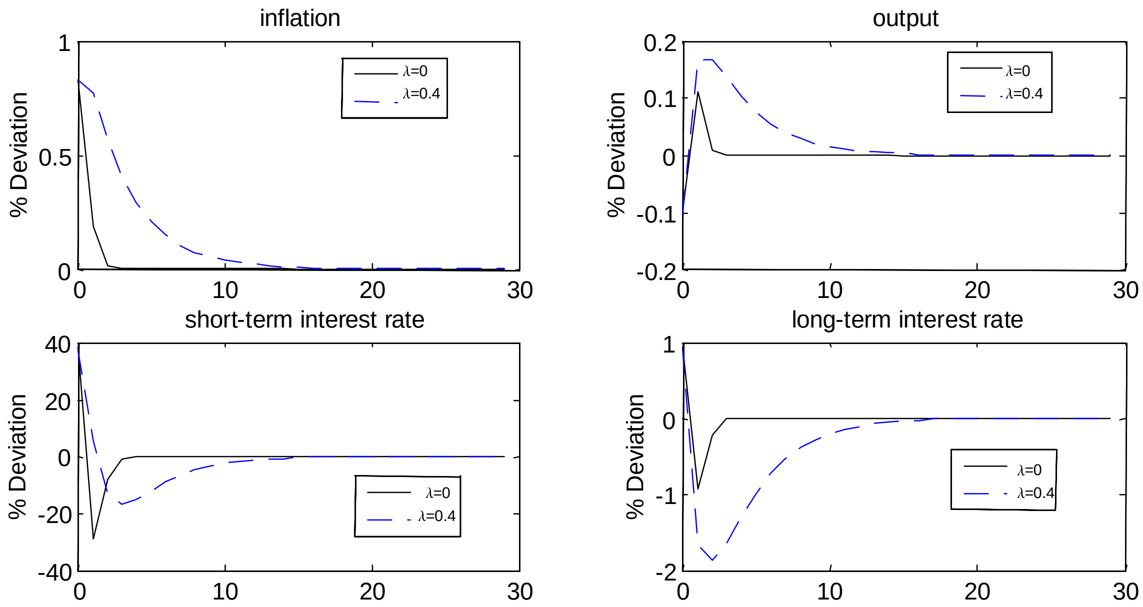

Figure 1: Shock to inflation

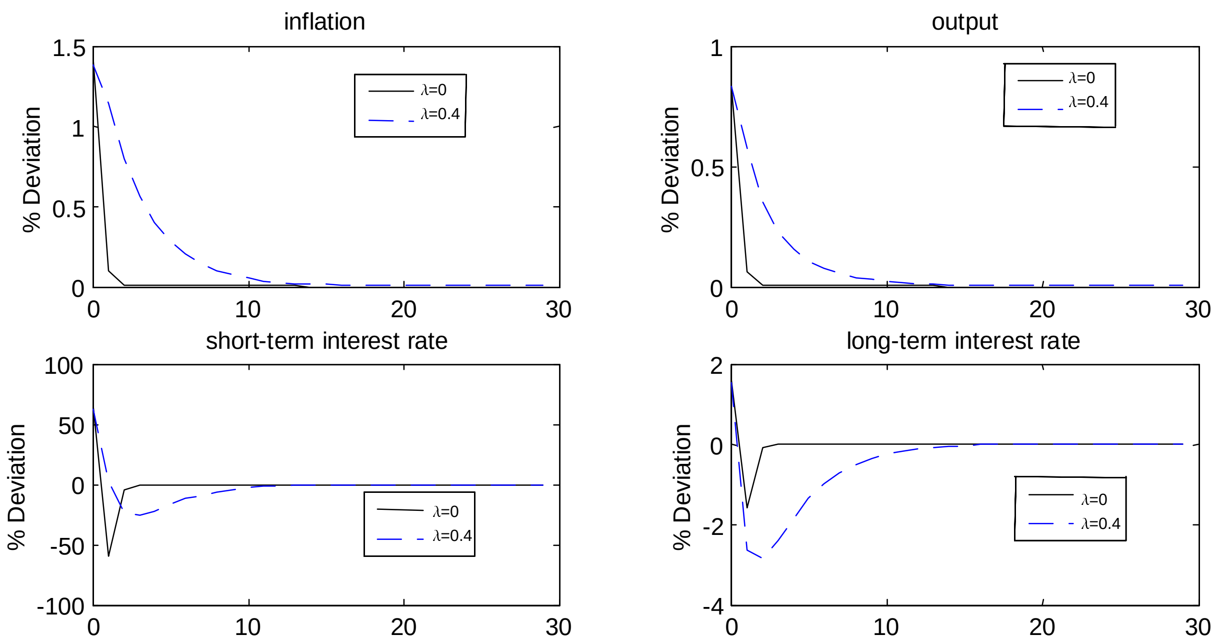

Figure 2: Shock to output

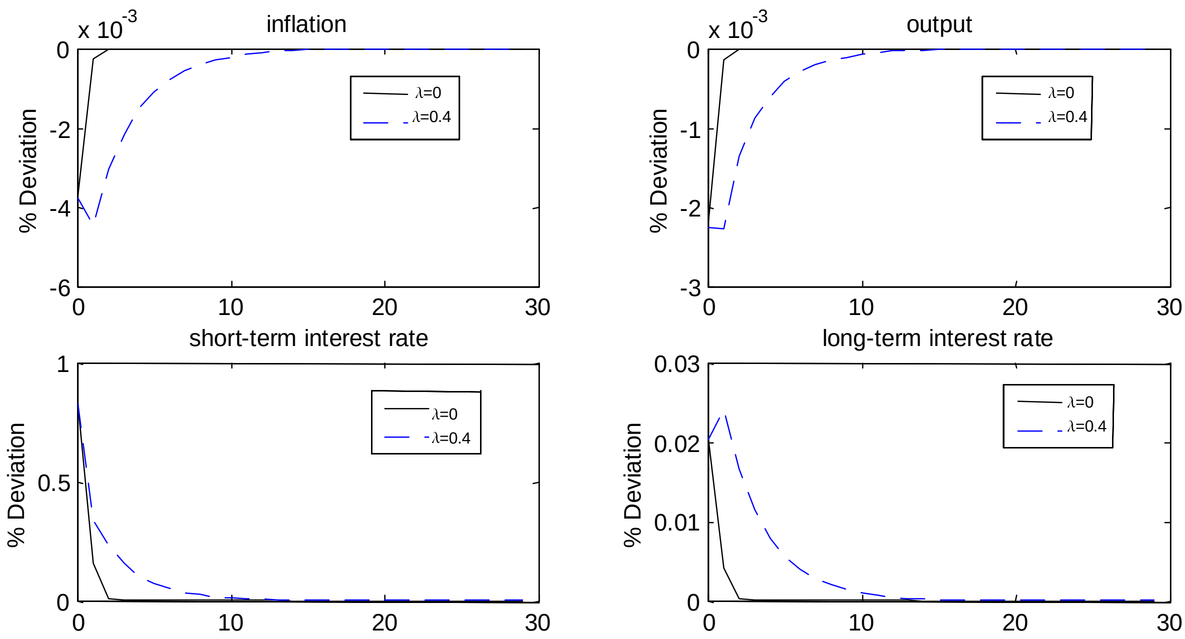

Figure 3: Shock to the interest rate

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A The Marginal Rate of Substitution

We start from the additive habit specification. To prove property P4 we first rewrite 13 as:

$$MRS^a_{C_tC_{t+1}} = \frac{1 - v\,(1-\lambda)\sum_{j=1}\beta^j\lambda^{j-1}\frac{C_{t+j}^{(1-\alpha)}H_{t+j}^{-v(1-\alpha)-1}}{C_t^{-\alpha}H_t^{-v(1-\alpha)}}}{\beta\left(\frac{H_{t+1}}{H_t}\right)^{-v(1-\alpha)}\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha} - v\,(1-\lambda)\sum_{j=2}\beta^j\lambda^{j-2}\frac{C_{t+j}^{(1-\alpha)}H_{t+j}^{-v(1-\alpha)-1}}{C_t^{-\alpha}H_t^{-v(1-\alpha)}}} \tag{70}$$

Note that if consumption grows at a constant rate σ, implying $\frac{C_{t+1}}{C_t} = \sigma$ then $H_t^a = \frac{(1-\lambda)}{(\sigma-\lambda)}C_t$[10] and we can rewrite $\frac{C_{t+j}^{(1-\alpha)}H_{t+j}^{-v(1-\alpha)-1}}{C_t^{-\alpha}H_t^{-v(1-\alpha)}} = \frac{\left(\sigma^iC_t\right)^{(1-\alpha)}\left(\sigma^iC_t\frac{(1-\lambda)}{(\sigma-\lambda)}\right)^{-v(1-\alpha)-1}}{C_t^{-\alpha}\left(C_t\frac{(1-\lambda)}{(\sigma-\lambda)}\right)^{-v(1-\alpha)}} = \sigma^{-i[\alpha+v(1-\alpha)]}\frac{(\sigma-\lambda)}{(1-\lambda)}$. If we denote $A = \beta\lambda\sigma^{-\alpha-v(1-\alpha)}$:

$$MRS^a_{C_tC_{t+1}} = \frac{1 - v\frac{(\sigma-\lambda)}{\lambda}\sum_{j=1}A^j}{A - v\frac{(\sigma-\lambda)}{\lambda}\sum_{j=2}A^j} = \frac{1 - v\frac{(\sigma-\lambda)}{\lambda}\left(\frac{1-A^T}{1-A} - 1\right)}{A - v\frac{(\sigma-\lambda)}{\lambda}\left(\frac{1-A^T}{1-A} - 1 - A\right)} \tag{71}$$

Dividing both the numerator and the denominator by A we can rewrite 71 as:

$$MRS^a_{C_tC_{t+1}} = \frac{(1-A) - v\frac{(\sigma-\lambda)}{\lambda}\left(A - A^T\right)}{A(1-A) - v\frac{(\sigma-\lambda)}{\lambda}\left(A^2 - A^T\right)} \tag{72}$$

If $\sigma > 1$ as $T \to \infty$ then $\lim_{T\to\infty} A^T = 0$ and the MRS becomes:

$$MRS^a_{C_tC_{t+1}} = \frac{1}{A} \tag{73}$$

hence

$$\partial MRS^a_{C_tC_{t+1}}/\partial v = (1-\alpha)\frac{\ln\sigma}{A} \tag{74}$$

In the multiplicative case we start from rewriting 14 as:

[10] In the additive case we can rewrite the habit function as:

$$H_t = \lambda^T H_{t-T} + (1-\lambda)\sigma C_t\sum_{i=0}^{T-1}\left(\frac{\lambda}{\sigma}\right)^i = C_t\frac{1-\lambda}{\sigma-\lambda} \text{ as } T \to \infty$$

since we assume that $\lim_{T\to\infty}\lambda^T H_{t-T} = 0$

$$MRS^m_{C_tC_{t+1}} = \frac{1 - v(1-\lambda)\sum_{j=1}\beta\lambda^{j-1}\left[\left(\frac{H^m_{t+j}}{H^m_t}\right)^{-v}\frac{C_{t+j}}{C_t}\right]^{(1-\alpha)}}{\beta\left[\left(\frac{H_{t+1}}{H_t}\right)^{-v}\frac{C_{t+1}}{C_t}\right]^{(1-\alpha)} - v(1-\lambda)\sum_{j=2}\beta\lambda^{i-2}\left[\left(\frac{H^m_{t+j}}{H^m_t}\right)^{-v}\frac{C_{t+j}}{C_t}\right]^{(1-\alpha)}} \tag{75}$$

Since consumption grows at a rate σ then: $\left(\frac{H^m_{t+j}}{H^m_t}\right)^{-v(1-\alpha)}\left(\frac{C_{t+j}}{C_t}\right)^{(1-\alpha)} = \sigma^{j(1-\alpha)(1-v)}$, $\left[\left(\frac{H_{t+1}}{H_t}\right)^{-v}\frac{C_{t+1}}{C_t}\right]^{(1-\alpha)} = \sigma^{(1-\alpha)(1-v)} = \sigma A$ we can rewrite 75 as:

$$MRS^m_{C_tC_{t+1}} = \frac{\sigma\left(1 - v\frac{(1-\lambda)}{\lambda}\sum_{j=1}(\sigma A)^j\right)}{\sigma A - v\frac{(1-\lambda)}{\lambda}\sum_{j=2}(\sigma A)^j} \tag{76}$$

and following the same procedure we rewrite 76 as:

$$MRS^m_{C_tC_{t+1}} = \frac{\sigma\left((1-\sigma A) - v\frac{(1-\lambda)}{\lambda}\left(\sigma A - \sigma^T A^T\right)\right)}{\sigma A(1-\sigma A) - v\frac{(1-\lambda)}{\lambda}\left((\sigma A)^2 - \sigma^T A^T\right)} \tag{77}$$

Note that $\lim_{T\to\infty}\sigma^T A^T = \sigma^{T(1-\alpha)(1-v)} = 0$ if $\alpha > 1$. In this case the MRS becomes:

$$MRS^m_{C_tC_{t+1}} = \frac{1}{A} \tag{78}$$

$$\partial MRS^m_{C_tC_{t+1}}/\partial v = (1-\alpha)\frac{\ln\sigma}{A} \tag{79}$$

B The optimising IS schedule

$$\frac{\partial U}{\partial C_t} = E_t\left\{\Pi^n_{i=1}C^{-v(1-\lambda)\lambda^{i-1}}_{t-i} - (1-\lambda)vC_t^{-1}\sum_{j=1}^{\infty}\beta^j\lambda^{j-1}C_{t+j}\Pi^n_{i=1}C^{v(1-\lambda)\lambda^{i-1}}_{t+j-i} - \Lambda_t\right\} \tag{80}$$

A log-deviation formulation of 80 is:

$$
\begin{aligned}
\frac{\partial U}{\partial C_t} &= \left\{ \frac{-\alpha+\beta v(1-\lambda)\left[v(1-\lambda)\left(1-\alpha\right)\sum_{i=0}^{n}\left(\beta\lambda^2\right)^i+\sum_{i=0}^{n}v\left(\beta\lambda\right)^i\right]}{1-\beta v(1-\lambda)\sum_{i=0}^{n}\left(\beta\lambda\right)^i} \right\} c_t \\
& +\lambda^0 \frac{(1-\alpha)\,v(1-\lambda)\left[-1+\beta v\lambda(1-\lambda)\sum_{i=0}^{n-2}\left(\beta\lambda^2\right)^i\right]}{1-\beta v(1-\lambda)\sum_{i=0}^{n}\left(\beta\lambda\right)^i}\left(c_{t-1}+\beta c_{t+1}\right) \\
& \ldots \\
& +\lambda^{j-1}\frac{(1-\alpha)\,v(1-\lambda)\left[-1+\beta v\lambda(1-\lambda)\right]}{1-\beta v(1-\lambda)\sum_{i=0}^{n}\left(\beta\lambda\right)^i}\left(c_{t-j}+\beta c_{t+j}\right)
\end{aligned}
$$

Hence

$$
\frac{\partial U}{\partial C_t} = g_0 c_t + \sum_{j=0}^{n} g_j \left(c_{t-j}+\beta c_{t+j}\right) \tag{81}
$$

$$
g_0 = \frac{-\alpha+\beta v(1-\lambda)\left[v(1-\lambda)\left(1-\alpha\right)\sum_{i=0}^{n}\beta^i\lambda^{2i}+\sum_{i=0}^{n}\beta^i\lambda^i\right]}{1-\beta v(1-\lambda)\sum_{i=0}^{n}\beta^i\lambda^i}
$$

$$
g_j = \frac{\lambda^{j-1}\left(1-\alpha\right)v(1-\lambda)\left[-1+\beta v\lambda(1-\lambda)\sum_{i=0}^{n-j-1}\left(\beta\lambda^2\right)^i\right]}{1-\beta v(1-\lambda)\sum_{i=0}^{n}\left(\beta\lambda\right)^i} \tag{82}
$$

So even if the number of addends changes with j we can always approximate the summation in the squared brackets of 82, for a given value of j, with $lim_{n\to\infty}\sum_{i=0}^{n-j-1}\left(\beta\lambda^2\right)^i = \frac{1}{1-\beta\lambda^2}$ In fact, the last elements of the summation will drop as j increases but as they are very small they won't affect the size of the coefficients.

So we can now approximate the infinite sum in g and g_j and obtain the coefficients:

$$
\begin{aligned}
g_0 &= \frac{\alpha-\beta v(1-\lambda)\left(\frac{v(1-\lambda)(1-\alpha)}{1-\beta\lambda^2}+\frac{1}{1-\beta\lambda}\right)}{(1-\frac{\beta v(1-\lambda)}{(1-\beta\lambda)})} \\
g_1 &= \lambda^0\frac{(1-\alpha)\,v\frac{(1-\lambda)}{\lambda}\left(-1+\beta v\frac{\lambda(1-\lambda)}{1-\beta\lambda^2}\right)}{(1-\frac{\beta v(1-\lambda)}{(1-\beta\lambda)})} \\
& . \\
& . \\
g_j &= \lambda^{j-1}\frac{(1-\alpha)\,v\frac{(1-\lambda)}{\lambda}\left(-1+\beta v\frac{\lambda(1-\lambda)}{1-\beta\lambda^2}\right)}{(1-\frac{\beta v(1-\lambda)}{(1-\beta\lambda)})} = \lambda^{j-1}g_1
\end{aligned}
$$

C Maximising Welfare

Assume the monetary authority shares households' preferences over consumption and labour:

$$W_t = U_t(C_t, H_t) - V(L_t) \tag{83}$$

We can approximate using a Taylor's series expansion

$$\begin{aligned} U_t(C_t, H_t) &= U + U_{C_t}\widetilde{C}_t + U_{H_t}\widetilde{H}_t + \frac{U_{C_t,C_t}\widetilde{C}_t^2}{2} + \\ &\quad \frac{U_{H_t,H_t}\widetilde{H}_t^2}{2} + \frac{U_{C_t,H_t}\widetilde{C}_t\widetilde{H}_t}{2} + \frac{U_{H_t,C_t}\widetilde{H}_t\widetilde{C}_t}{2} \end{aligned} \tag{84}$$

where all the derivatives are evaluated at the steady-state levels C and H and the variable with a $\sim$ denote deviation of consumption and habit (in levels) from their steady-state. Defining $c_t = \log(C_t/C)$ and $h_t = \log(H_t/H) = (1-\lambda)\sum_{i=1}^{\infty}\lambda^{i-1}c_{t-i}$ allows us to approximate $\widetilde{C}_t$ and $\widetilde{H}_t$ as:

$$\widetilde{C}_t = C\left(c_t + \frac{1}{2}c_t^2 + O\left(\| \xi \|^3\right)\right) \tag{85}$$

$$\widetilde{H}_t = H\left(h_t + \frac{1}{2}h_t^2 + O\left(\| \xi \|^3\right)\right) \tag{86}$$

Plugging (85) and (86) in (84)[11] :

$$\begin{aligned} U_t(C_t, H_t) &= U + U_{C_t}C\left(c_t + \frac{1}{2}c_t^2\right) + U_{H_t}H\left(h_t + \frac{1}{2}h_t^2\right) \\ &\quad + \frac{U_{C_t,C_t}C^2c_t^2}{2} + \frac{U_{H_t,H_t}H^2h_t^2}{2} + U_{C_t,H_t}CHc_th_t \end{aligned} \tag{87}$$

Substituting the derivatives of U[12] in (87) we get:

$$U_t(C_t, H_t) = U + U(1-\alpha)\left[\frac{1}{2}(1-\alpha)c_t^2 + \frac{1}{2}(1-\alpha v)h_t^2 + (c_t - vh_t) - v(1-\alpha)c_th_t\right] \tag{88}$$

We then consider the log-linearisation of the disutility of labour as in Woodford and Monacelli (2002):

[11] The result in (87) uses the symmetry in the cross-derivatives $U_{C_t,H_t}\widetilde{C}_t\widetilde{H}_t$ and $U_{H_t,C_t}\widetilde{H}_t\widetilde{C}_t$

[12] The partial derivatives of U_t are:$U_{C_t} = (1-\alpha)\frac{U}{C}$; $U_{H_t} = -(1-\alpha)v\frac{U}{H}$; $U_{C_t,C_t} = -\alpha(1-\alpha)\frac{U}{C^2}$; $U_{H_t,H_t} = (1-\alpha)\left(v(1-\alpha)+1\right)\frac{U}{H^2}$; $U_{H_t,C_t} = U_{C_t,H_t} = -\frac{v(1-\alpha)^2}{C}\frac{U}{H}$.

$$V_t(L_t) = V + V_L \widetilde{L}_t + V_{LL} \frac{\widetilde{L}_t^2}{2} \tag{89}$$

where again we can log-linearise L_t around the steady state L :

$$\widetilde{L}_t = L\left(l_t + \frac{1}{2} l_t^2 + O\left(\| \xi \|^3\right)\right) \tag{90}$$

given the function $V_t(L_t) = \frac{L_t^{1+\gamma}}{1+\gamma}$ in steady-state $V_{LL}^2 L^2 = \gamma V_L L$ so:

$$V_t(L_t) = V + V_L L\left(l_t + \frac{1}{2}(1+\gamma) l_t^2\right) \tag{91}$$

The log-linearisation of $L = \left(\frac{\psi}{\psi-1}\right)^{\phi}\left(\frac{P_t}{P_t-1}\right)^{-\phi\mu} Y_t$ gives:

$$l_t = y_t - \phi\mu\pi_t \tag{92}$$

which substituted in (91):

$$V_t(L_t) = V + V_L L\left(y_t - \phi\mu\pi_t + \frac{1}{2}(1+\gamma)\left(y_t - \phi\mu\pi_t\right)^2\right) + O\left(\| \xi \|^3\right) \tag{93}$$

Subtracting (93) from (88) assuming that in equilibrium $c_t = y_t$ and considering that in steady-state $V_L L = U(1-\alpha)$[13] gives:

$$W_t = U(1-\alpha)\left(\frac{1}{2}\left[(1-\alpha)y_t^2 + \frac{1}{2}(1-\alpha v)h_t^2 - (1+\gamma)\left(y_t - \phi\mu\pi_t\right)^2\right] + C + O\left(\| \xi \|^3\right)\right) \tag{94}$$

where $C = -vh_t(1-\alpha c_t) - \phi\mu\pi_t$. Since $h_t = (1-\lambda)\sum_{i=1}^{\infty}\lambda^{i-1}y_{t-i} = \frac{(1-\lambda)}{\lambda}\left(\frac{1}{1-\lambda L} - 1\right) = \frac{(1-\lambda)L}{1-\lambda L} y_t$ we can express the above expression as:

$$W_t = U(1-\alpha)\left(\begin{array}{c} \frac{1}{2}\sigma_y\left\{-(\alpha+\gamma)\left(1-2\lambda+\lambda^2\right) + (1-\alpha v)(1-\lambda)^2\right\} \\ -\frac{1}{2}\sigma_\pi(1+\gamma)\phi^2\mu^2\left(1-2\lambda+\lambda^2\right) + I + O\left(\| \xi \|^3\right) \end{array}\right) \tag{95}$$

where

$$\begin{aligned} I \;=\; & -v(1-\lambda)\left(y_{t-1} - \lambda y_{t-2}\right) - v(1-\alpha)(1-\lambda)\left(y_t y_{t-1} - \lambda y_{t-1} y_{t-2}\right) \\ & + (1+\gamma)\phi\mu\left(\pi_t y_t - 2\lambda\pi_{t-1}y_{t-1} + \lambda^2\pi_{t-2}y_{t-2}\right) \end{aligned} \tag{96}$$

---

[13] We know that in steady-state $U_c = (1-\alpha)\frac{U}{C}$; $R = \frac{1}{\beta} \cong 1$; $V_L = U_c \frac{W}{P}$ and that in equilibrium the budget constraint is:

$$C + B = \frac{W}{P} L + RB$$

then $\frac{C}{L} = \frac{W}{P}$ and $V_L L = U_c = (1-\alpha)U$