

The Optimality of the US and Euro Area Taylor Rule

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Abstract

The purpose of this paper is to examine the optimality of the monetary authorities reaction function in the two-area medium size model MARCOS (US and euro areas). The parameters and the horizons of output gap and inflation expectations of the Taylor rule are computed in order to minimise a loss function of the monetary authorities. However, investigating the optimality of the Taylor rule in the context of a large scale macroeconomic model raises several difficulties: the model is non-linear and all the state variables potentially enter the optimal monetary policy rule. Furthermore, the optimality of the Taylor rule is assessed by the minimisation of the loss function under the constraint of a large forward-looking model. To overcome these problems, Black, Macklem and Rose [1998] propose a stochastic simulation based method which has been applied to single-country macroeconomic models. To study the optimality of the Taylor rule in the case of a two-area model, we suppose that the economy is stochastically hit by numerous shocks (supply, demand, monetary, exchange rate and world demand) in each area and simulate MARCOS stochastically.

Résumé

L'objectif de ce travail est d'examiner l'optimalité de la fonction de réaction des autorités monétaires dans le modèle à deux zones (USA et zone euro) de taille moyenne MARCOS. Les paramètres et l'horizon des anticipations de l'*output gap* et de l'inflation sont déterminés de façon à minimiser une fonction de perte des autorités monétaires. Cependant, l'examen de l'optimalité de la fonction de réaction des autorités monétaires dans le contexte d'un modèle de grande taille soulève plusieurs difficultés : le modèle est non linéaire et toutes les variables d'état entrent potentiellement dans la règle de politique monétaire. En outre, l'optimalité de la règle de Taylor est établie par la minimisation d'une fonction de perte sous la contrainte d'un modèle de grande taille avec des anticipations rationnelles. Pour résoudre ces difficultés, Black, Macklem and Rose [1998] proposent une méthode fondée sur des simulations stochastiques qui a été appliquée sur un modèle à un seul pays. Pour étudier l'optimalité de la règle de Taylor dans le cadre d'un modèle à deux zones, on suppose que chaque zone subit divers chocs stochastiques (offre, demande, monétaire, taux de change, demande mondiale) et on simule MARCOS de façon stochastique.

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1- Introduction

The optimality of the monetary reaction function has been intensively investigated in many respects. First, the properties of a simple monetary rule, generally a Taylor rule, are compared to a more sophisticated one deduced from an optimisation framework (Rudebusch and Svensson [1999]). Second, the parameters and the horizon of inflation expectation of the Taylor rule are computed in order to minimise a loss function of the monetary authorities (Batini and Haldane [1999], Batini and Nelson [2000], Jondeau and Le Bihan [2000]). We retain the second approach to examine the optimality of the monetary reaction function.

For tractability purpose, the optimality of the monetary reaction function is usually studied in a simplify framework where the economy is described as a VAR or a small structural model. These models are generally composed of two or three equations: an IS curve, a Phillips curve and an UIP relation. However, in those cases the description of monetary policy channels is rather poor. The impacts of interest rate are considered only through a reduced form equation (the IS curve) entangling contradictory effects such as the substitution and wealth effects on consumption, capital cost on investment and competitiveness through real exchange rate on external trade.

Black, Macklem and Rose [1998], Drew and Hunt [1999], Yuong [2000] investigate different monetary policy rules or the uncertainty about the monetary transmission delay in the context of a large scale macroeconomic model (QPM and FPS). However, computing the efficiency frontier to examine the optimal monetary policy in the context of a large scale model raises several difficulties. Because the model is non linear and all the state variables enter the optimal monetary policy rule, its computation becomes intractable for large scale models. Furthermore, the optimality of the Taylor rule is assessed by the minimisation of a loss function under the constraint of the model. In the context of a large scale model, especially if it is calibrated, the task is rather tricky. To overcome this problem, Black, Macklem and Rose [1998] propose a stochastic simulation based method which has been applied to single-country macroeconomic models (Black, Macklem and Rose [1998] for QPM and Drew and Hunt [1999], Yuong [2000] for FPS).

The aim of this paper is to examine the optimality of the Taylor rule in the case of a calibrated two-area model, MARCOS (Jacquinot and Mihoubi [2003a], [2003b]). The optimal Taylor rule will be the one with the parameter set minimising the criterion composed of the variances of output, inflation and interest rate. We suppose that the economy is stochastically hit by numerous shocks (supply, demand, monetary, exchange rate and world demand). For this purpose, MARCOS is stochastically simulated. The optimality of the Taylor rule is examined with respect to either the parameters or the horizon related to inflation and output gap using Black, Macklem and Rose [1998] methodology.

The first part of the paper is devoted to MARCOS presentation whereas the optimality of the Taylor rule is discussed in the second part. The last part deals with the results of stochastic simulations.

2- MARCOS at a glance

MARCOS (*Modèle à Anticipations Rationnelles de la CONjoncture Simulée*, Jacquinot and Mihoubi [2000]) is a yearly model designed for economic policy evaluation. It is calibrated and composed of two area-blocks: the euro and US areas. The goal of MARCOS is to get a comprehensive and understandable tool to analyze economic policies. MARCOS is a medium-size model (around 100 equations for each area) with a coherent accounting framework and rational expectations.

The overall coherence of the model is ensured by a top down strategy (from theoretical structure to equations). A balanced growth path exists and explicitly comes from the short-term dynamics of the model. Parameters in equations are structural and invariant to economic policy shocks. They are directly derived from different agents optimising framework (households, unions, firms). The MARCOS's supply side homogeneity is thus guaranteed and the wage-setting follows a bargaining process. Forward looking expectations are model-path consistent. They appear in the real sphere: consumption, investment, fiscal-authority reaction function; as well as in the nominal sphere: Phillips curve, monetary-authority reaction function, Fisher equation, uncovered interest rate parity.

Recent works implementing this approach include Laffargue [1995], MARMOTTE (Cadiou, Stéphane, Guichard, Kadareja, Laffargue, Rzepkowski [2001]), QPM (Black, Laxton, Rose, and Tetlow [1994], Coletti, Hunt, Rose and Tetlow [1996]), QUEST II (Roeger and in't Veld [1997]), FPS (Black, Cassino, Drew, Hansen, Hunt, Rose, and Scott [1997]), and MULTIMOD Mark III (Laxton, Isard, Faruqee, Prasad, and Turtelboom [1998]). MARCOS slightly differs from these models by its more general theoretical framework: we simultaneously assume monopolistic competition, wage bargaining and life cycle hypothesis. Five agents are retained in MARCOS: households, firms, public administration, rest of the world and unions.

2.1- MARCOS agents

Households

Consumption is split between workers and retired in a pay-as-you-go retirement scheme. Following Gertler [1997], at each date working age households face a constant probability to become retire and retired households face a constant probability to die. In addition, two kinds of households are distinguished whether they can access or not to the financial markets. The neo-classical households, that are non-constrained, hold treasury bonds, capitalized value of social security benefits and firms and determine their consumption by maximising their inter-temporal utility function. Income is determined by real wages under the assumption of a life cycle bell-shaped (Faruqee, Laxton and Symansky [1997]).

Firms and Unions

The firms operate in a monopolistic competition framework with a CES technology. Their decisions obey to the following sequence: first, they determine investment and capital; second they bargain wages with unions and then fix unilaterally employment. The profit maximisation program including capital adjustment costs gives investment thus related to the

Tobin's q .³ Employment, subject to adjustment cost, is determined by the labour demand given the wage bargained. Wages are deduced from a right to manage model. Furthermore, modelling the wage-bargaining process allows computing an equilibrium unemployment rate consistent with both workers and firms' objectives. In a profit optimizing framework, the labour demand equation cannot be distinguished from the value added price equation. The value added price is thus the implicit GDP price.

Government

The government raises direct and indirect taxes. The personal income tax rate is endogenous and adjusted by the government in order to reach a public debt target. The employer social contribution rate is endogenously determined in order to guarantee the long-term social budget equilibrium. In the short run the employer social contribution rate is exogenous and the government guarantees the equilibrium of social account.

Rest of the world

The foreign trade equations are rather traditional with exports and imports respectively depending upon world demand, domestic demand and price competitiveness.

The nominal rigidities

The nominal block is composed of seven prices: demand price, value added price, consumption price, investment price, public expenditures price, import and export deflators. In order to take account of nominal rigidities and pressures on the price setting, the demand price is modelled by a hybrid Phillips curve with model consistent expectations. The Phillips curve describes the relationship between the rate of inflation (π_t) and the output gap where the potential output is equal to its steady state level given by the model $\left(\frac{Y_t}{Y_t^*} - 1\right)$. Due to nominal rigidity, an inflation-unemployment dilemma is quite possible in the short term.

$$\pi_t = \zeta\pi_{t-1} + (1-\zeta)\pi_t^a + \psi\left(\frac{Y_{t-1}}{Y_{t-1}^*} - 1\right)$$

With $\psi > 0$. Current inflation depends upon the past inflation (π_{t-1}) and the expected inflation for the next period ($\pi_t^a = E_t(\pi_{t+1})$). Furthermore, expectations completely forward looking (*i.e.* when $\zeta = 0$) are excluded. Otherwise, the monetary policy will be limited to the announcement of an inflation target that will be immediately verified by all agents' expectations.

Interest rates

Four interest rates are included in MARCOS: the one-year interest rate, the ten-year interest rate, the composite interest rate associated to the public debt (mix of the two previous ones) and the foreign short-term interest rate. Short term and long term interest rates are related by a yield curve with a constant term premium. Monetary authorities fix the short-term interest rate according to an inflation and output-gap targeting. These interest rates have three types of effects on the real sphere. First, they directly influence the neo-classical households'

³The stock market is supposed in perfect information situation.

consumption *via* wealth and saving-consumption substitution effects as well as investment *via* the optimal capital stock which equates the long-term capital productivity and the real interest rate. Second, they directly determine levels of public and external debts and thus the households' wealth. Third, they determine the exchange rate by the uncovered interest rate parity relationship and then modify the price competitiveness and the trade balance.

The reaction function

The reaction function is a Taylor rule type (Taylor [1993]): monetary authorities control the nominal short-term interest rate (r_t), reacting to shocks on inflation or deviations of output from its potential level:

$$r_t = r_{t-1} + \mu(\pi_{t+1} - \pi_{t+1}^*) + \tau \left(\frac{Y_t}{Y_t^*} - 1 \right)$$

with π_t^* as the inflation target, $\left(\frac{Y_t}{Y_t^*} - 1 \right)$ represents the output gap. However, this version of

the Taylor rule differs from the original version by the fact that it is not the level of the interest rate but its changes that enter the monetary rule. Thus, to bring back the interest rate to its based line value, it is not sufficient that the initial shock vanishes, but this shock has to be compensated with an opposite shock of the same magnitude. In others words, this version of the Taylor rule displays some hysteresis effect. The direct consequence is that in case of inflation pressure the monetary authorities behave more aggressively than in the case of an usual Taylor rule (in case of a permanent shock the interest rate does not increase once for all but increase continuously). A second implication of this version of the Taylor rule is the invalidity of the usual indeterminacy condition ($\mu < 1$): small values of μ could be considered. Furthermore, this specification captures the high value of the optimal parameter associated to the lagged interest rate in several models. As pointed out by Levin, Vieland and Wiliams [1999], this reaction function form performs much better than the traditional Taylor rule, provides efficiency frontiers comparable to those given by more complicated rules (including additional lags or variables), and is less sensitive to model uncertainty. In addition, this specification takes into account the strong persistency of the interest rate.⁴

Area blocks linkage

The linkage variables between the areas are interest rates, exchange rates, foreign demand and foreign prices. Hence, exchange rates (euro/US-dollar, euro/foreign currencies and US-dollar/foreign currencies) are deduced from an uncovered interest rate parity. The foreign demand for each area is the sum of the US, the euro area and the rest-of-the-world imports weighted by their respective shares in the area imports. For each area, the foreign price depends upon exports prices of the other areas.

In the long run, the real interest rates are identical in the three areas. To balance the model, the rest of the world guarantees the trade balance equilibrium.

⁴ Note however that examining the optimal Taylor rule letting the past value of interest rate unknown is time consuming. So, we have not used such a specification in order to limit the number of parameters over which we have to optimize in the Taylor rule. Furthermore, empirical evidences indicate that this coefficient is rather close to one.

2.2- Calibration

The specifications of both euro area and US are identical, obviously calibrations are different. Jacquinot and Mihoubi [2003b] report the coefficients values and the main features of the steady state for the euro area and the US. The euro area and US model are calibrated using respectively annual data provided by Eurostat and the ECB and OECD Main Economic Indicator database.

Methodology and common features

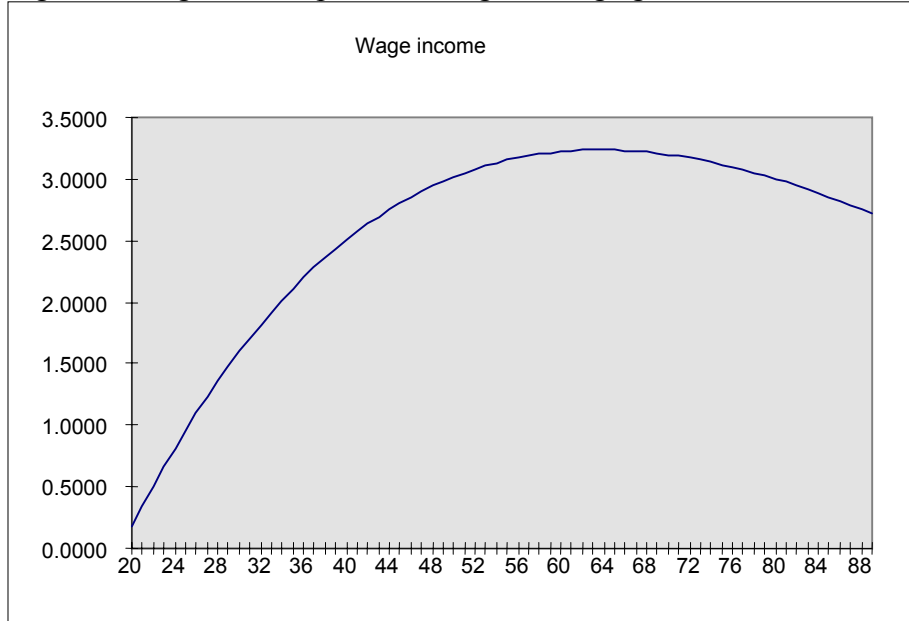
The calibration relies on the assumption that the economies are on average at their steady state during the period 1985-1997. Thus variables describing the steady state are put to their 1985-1997 sample mean values. During the calibration, variables in level (GDP and employment for example) are set to their 1997 values. Ratios and rates (shares of the different components of the demand in the GDP, ratios of the different debts to the GNP and taxes rates,...) are supposed to be equal to their mean on the 1985-1997 sample. For unobserved parameters two cases could be considered: parameters considered as endogenous during the calibration – the model is inverted – and parameters set to realistic values.

The share of Keynesian household consumption is endogenous during the calibration. Its value (44%) is deduced from the simulation of the overall model taking into account constraints on the household wealth and on their consumption. The retire probability is 2.5% implying an expected working time of 40 years and the death probability is equal to 5% corresponding to an expected adult life time of 60 years. The coefficients determining the path of the wage income during the adult life time are set such that the labour income has the usual life cycle pattern (Figure 1).

The capital depreciation rate is set to 4.5% in order to be in line with the investment rate at the steady state. We get a capital life time of 22 years. The bargaining power of the union is set to 0.5 leading to a gain, coming from the matching of a vacant job with an unemployed worker, equally shared between the employer and the employee. In order to get a mark-up rate about 10%, the price elasticity of the good demand is equal to 11. The adjustment cost is set to 3 on capital and to 2 on labour.

Parameters of the monetary policy reaction function are those proposed by Taylor [1993]: the parameter related to inflation μ is equal to 1.5 and the parameter which measure the sensibility to the output gap is set to 0.5. Thus the central bank is more aggressive on inflation than on activity.

Figure 1: Wage income profile during working age



Main differences

Main differences between the two areas rely on the choice of retirement ratio for pay-as-you-go system (the ratio of public pension per capita over wages per capita is 40% for euro area and 5% for US). The technology differs also from the euro area: the elasticity of substitution between labour and capital is close to 1 for euro area but smaller for the US with a value of 0.6. At the steady state, the unemployment rate (which measures only the compensated unemployment) is equal to its estimated equilibrium value of 8.6% for the euro area and 6.13% for the US. Phillips curve is less sensitive to output gap in the for euro area (0.045 against 0.12 for the US). The equations for external trade have been estimated. Our finding is that price elasticity of imports is much higher in the US (1.92 versus 0.8 in the euro area).

3- Optimal Taylor rule

Optimal coefficients

The optimality of the monetary policy rule is defined as the suitable calibration of the Taylor rule. We mean by suitable, the values of the coefficients of the Taylor rule that minimise a weighted sum of variances of output, inflation and interest rate conditional on the model.

Formally the program is:

$$\left\{ \begin{array}{l} \text{Min}_{\mu, \tau} \sum_{j=0}^T \beta^j \left[\lambda_y \cdot V(y_{t+j} - y_{t+j}^*) + \lambda_\pi \cdot V(\pi_{t+j} - \pi_{t+j}^*) + \lambda_r \cdot V(\Delta r_{t+j}) \right] \quad (1) \\ r_t = r_{t-1} + \mu(\pi_{t+1} - \pi_{t+1}^*) + \tau(y_t - y_t^*) + \varepsilon_t^r \quad (2) \\ F(Z_t, X_t) = \varepsilon_t^Z \quad (3) \end{array} \right.$$

Where y_t is the logarithm of output, y_t^* its potential value, π_t the inflation, π_t^* the inflation target, and r_t the nominal interest rate. Equation (2) is the usual Taylor rule whereas equation (3) corresponds to the overall model (the monetary policy rule excepted) with Z_t and X_t the endogenous (determining y_t , π_t and y_t^*) and exogenous (π_t^*) variables respectively. ε_t^r and ε_t^Z are the innovations of the Taylor rule and of the rest of the model. The coefficient β is the discount factor.

The model (3) is usually (Ball [1997], Jondeau Le Bihan [2000]) composed of an IS curve and a Phillips curve. The problem could be rewritten as:

$$\left\{ \begin{array}{l} \text{Min}_{\mu, \tau} \sum_{j=0}^T \beta^j \left[\lambda_y \cdot V(y_{t+j} - y_{t+j}^*) + \lambda_\pi \cdot V(\pi_{t+j} - \pi_{t+j}^*) + \lambda_r \cdot V(\Delta r_{t+j}) \right] \\ r_t = r_{t-1} + \mu(\pi_{t+1} - \pi_{t+1}^*) + \tau(y_t - y_t^*) + \varepsilon_t^r \\ y_t = \rho y_{t-1} - \alpha_1(r_t - \pi_t) + \alpha_2 + \varepsilon_t^y \\ \pi_t = A(L)\pi_{t-1} + \beta(y_{t-1} - y_{t-1}^*) + \varepsilon_t^\pi \end{array} \right.$$

with $A(1) = 1$ to verify the long-run verticality of the Phillips curve. Due to the linearity of the model, the analytical solution is then straightforward. It can be shown (Svensson [1998]) that the model admits the following AR(1) form:

$$\tilde{Z}_t = B\tilde{Z}_{t-1} + \varepsilon_t$$

with $\tilde{Z}_t = (r_t, Z_t)'$.

With B a matrix depending upon μ and τ

The whole system could then be rewritten as:

$$\begin{pmatrix} V(\Delta r_t) \\ V(y_t - y_t^*) \\ V(\pi_t - \pi_t^*) \end{pmatrix} = [I - (B \otimes B)]^{-1} \text{vec}[V(\varepsilon_t)]$$

the optimal coefficients of the Taylor rule are deduced from the minimisation of

$$\sum_{j=0}^T \beta^j \begin{pmatrix} \lambda_y & \lambda_\pi & \lambda_r \end{pmatrix} \begin{pmatrix} V(\Delta r_{t+j}) \\ V(y_{t+j} - y_{t+j}^*) \\ V(\pi_{t+j} - \pi_{t+j}^*) \end{pmatrix} = \frac{1}{1-\beta} \begin{pmatrix} \lambda_y & \lambda_\pi & \lambda_r \end{pmatrix} [I - D]^{-1} \text{vec}[\Omega]$$

with $V(\varepsilon_t) = \Omega$ and $D = B \otimes B$ a quadratic form of the model coefficients including μ and τ .

Optimal horizon

The monetary policy affects the economy with some delay. The monetary authority has to take into account the transmission delay to conduct its policy. And in presence of transmission lag, it will be sub-optimal to target the current inflation rate rather than its future value. The issue here is: what is the optimal horizon of monetary policy? In other words, what are the

leads in the Taylor rule for inflation rate target as well as for output gap that minimise the loss criterion? This question of the optimal horizon could be related to Batini and Nelson [2000] OFH definition: “the optimal feedback horizon (‘OFH’) [is] the best point in the future for which the authorities should form the inflation forecast that enter their policy rule”. Batini and Nelson [2000] consider only inflation targeting. We extend the optimal horizon to the output-gap target.

The transmission delay depends upon the openness of the area. For an open economy the exchange rate channel operates faster than the output-gap channel. Nevertheless, even in open countries the transmission delay is still significant. In MARCOS, both areas present a weak degree of openness.

Concretely, to compute the optimal horizon we minimise the loss function (1) respect to k_μ and k_τ

$$r_t = r_{t-1} + \mu(\pi_{t+k_\mu} - \pi_{t+k_\mu}^*) + \tau \left(\frac{Y_{t+k_\tau}}{Y_{t+k_\tau}^*} - 1 \right)$$

Implementation with MARCOS

Applying Ball [1997] or Svensson [1998] methods to MARCOS raises several difficulties. First, MARCOS presents strong non-linearity. Its linearization around the steady state is a cumbersome task and leads to an approximated rather than an exact solution. The other solution consisting in simulating stochastically the model for a set of coefficients rather than solving it analytically has to be considered despite its difficult implementation. In this case, each shock in MARCOS has to be uncorrelated with the contemporaneous endogenous variables. Due to the forward-lookingness of MARCOS, at each period the model is solved taking into account the path formed by all the future periods. If we simulate stochastically at once a full path, the residuals of future periods will be correlated with the current endogenous. To avoid such a problem, we have to simulate the model as follow. At each date we introduce a shock at the current period and we set it to zero for all the following periods and we simulate the entire path. This procedure could be repeated date after date to get a complete path of stochastic shocks. So to simulate a complete path of T periods stochastic shocks we have to run T forward-looking simulations of the model.

Second, MARCOS is calibrated around its equilibrium steady state rather than estimated. It means that we do not dispose of a residuals covariance matrix. Thus, the matrix Ω in the minimisation problem is unknown and furthermore applying stochastic simulations is impossible since we do not know the residual distribution.

4- Stochastic simulations

The stochastic simulation strategy

The main issue here is how to run stochastic simulations in MARCOS whereas it is mainly calibrated. For estimated models, the exercise is quite easy: shocks are simply drawn from the distribution of estimated residuals. In our case, we suppose (as Black et al. [1997] for QPM and Drew and Hunt [1998] for FPS) that the economy could be approximated by a reduced form core model and the estimation of the core model will give the distribution of shocks

required by the stochastic simulation. The VAR methodology is the most appropriated to get such a core model. According to the VAR literature, the economy is hit by independent innovations and impulse responses are run in order to identify them. Each residual of the VAR model could finally be expressed as combination of these innovations. In order to proceed to stochastic simulations, residual terms are added to some behavioural equations of MARCOS and defined such that the model could mimic the impulse responses given by the VAR.

Two VAR are estimated: one for each area. This strategy of estimating two different VAR for a two-country model can be first justified by the weakness of the links between areas (in the model). Furthermore, each VAR should be large enough to capture the main shocks the US and euro zones are supposed to face. But estimating a too large VAR (with annual data) could raise some degree of freedom problems.⁵ For both zones, the VAR is composed of the five following variables: world demand, world demand deflator, consumption, demand deflator and yield curve. Volumes and the yield curve are in level, prices are computed in growth rate. The order of appearance of the variables gives the causal ordering of the VAR and consequently its identification scheme. The priority of foreign variables with respect to domestic variables indicates the top position of the foreign sector in the causal hierarchy of the model. The last position of interest rates signifies that the monetary authority reacts to all the previous information. The interpretation of innovations associated to each equation is quite standard. The first two shocks are respectively the world demand shock and the terms of trade shock whereas the shock to the consumption can be viewed as a demand shock and the shock to the demand price as a supply shock on the Phillips curve. The shock to the yield curve is interpreted as a monetary shock.

The VAR is estimated over the 1970-2000 period for the euro area and the 1974-1997 period for the US. The number of lags, equal to one, has been determined by AICC and Schwarz criteria. Impulses are responses to a one-standard-deviation shock on each innovation.

Tables 1 and 2 show the instantaneous impulse responses of the VAR models, computed from a Cholesky decomposition of residuals variance-covariance matrix. Results are rather standard. For a world demand innovation, we have a positive impact on consumption and thus on prices implying a short-term interest rate increase (a decrease in the yield curve). The foreign inflation shock induces a rise in domestic inflation, a consumption contraction and a short-run interest rate increase. The positive shock on consumption involves a weak deflationary effect whereas the impact on the yield curve is positive in the euro area and negative but negligible in the US. This difference suggests that ECB is less tolerant to inflation than the fed. Finally the positive shock on domestic inflation involves an increase in the short-run interest rate.

⁵ A VAR for both areas will contain 10 variables. With two lags, the number of parameters to be estimated for each equation is equal to 20! That is particularly expensive for annual data.

Table 1: Instantaneous response of the VAR model variable for the euro area

IRF / Innovation of	World demand	World demand deflator growth rate	Consumption	Consumption price index growth rate	Yield curve
World demand	2685	0	0	0	0
World demand deflator growth rate	-0.002	0.038	0	0	0
Consumption	14829	-5693	6895	0	0
Consumption price index growth rate	0.002	0.01	-0.001	0.008	0
Yield curve	-0.0002	-0.002	0.001	-0.005	0.007

Table 2: Instantaneous response of the VAR model variable for the United-States

IRF / Innovation of	World demand	World demand deflator growth rate	Consumption	Consumption price index growth rate	Yield curve
World demand	9677	0	0	0	0
World demand deflator growth rate	0.015	0.055	0	0	0
Consumption	22200	-15036	50046	0	0
Consumption price index growth rate	0.008	0.004	-0.003	0.01	0
Yield curve	-0.004	-0.003	-0.0002	-0.003	0.006

Introduction of stochastic shocks in MARCOS

The VAR gives an estimate of the response for the five variables to each innovation and the problem is how to design MARCOS to exactly replicate the impulse responses function (IRF) of the VAR over the first period (one year), i.e. before any effect of economic policy. The aim is here to catch the purely exogenous shocks hitting the economy and for this reason the period should be free of any policy effect. On the one hand, the VAR identifies the innovations, their standard deviations, and also produces a precise picture of the dynamics of the economy. On the other hand, and by construction, MARCOS has no residuals. The strategy will be to use the information given by the VAR to introduce the appropriate residual terms in behavioural equations of MARCOS. These terms are added to the level of behavioural equations of MARCOS whose economic definition is the nearest to the one of the VAR. Their role is to give the deviation from the steady state that will permit the replication of the IRF. These residuals will be a combination of the innovations. Once weights determined, the stochastic simulations can finally be implemented from a normal distribution $N(0,1)$.

Contributions of innovations to MARCOS residuals are computed as follow. Let us take the case of a world demand shock in the euro zone. First the IRF of the VAR to a world demand shock (responses of the five variables to this shock) are retrieved. Second, add-factors are introduced in MARCOS not only in the corresponding five equations of the euro area but also in the same five equations of the US area. They are introduced in both areas in order to take account of all inter-relationships. Third, the model is simulated over 50 periods (the time to be sure that all variables reach their steady state) with add-factors as endogenous and behavioural variables as exogenous. The monetary reaction function is switched off to assure the

independence of computed residual terms from the structural form of the reaction function. The weights of the first innovation to MARCOS residuals are then retrieved. The contributions of the world demand innovation to the one-period residuals are obtained. Fourth, the procedure is applied for all IRF and fifth, re-iterated for the next period. MARCOS could now be stochastically simulated.

As previously noticed, the forward-lookingness induces a stochastic simulation run date by date to get a complete path. These simulations have to be repeated for the number of replications. The simulation protocol retained is of 30 replications over 50 periods. As pointed out for FPS by Drew and Hunt [1998], for less than 30 replications standard deviation of output display instability.⁶

Formally, given the estimated reduced form:

$$X_t = A(L) X_{t-1} + v_t$$

with $V(v_t) = \Sigma$ and where X_t is the vector of the five dependent variables, $A(L)$ the lag polynomial matrix, L the lag operator. The associated structural VAR is:

$$X_t = A(L) X_{t-1} + B\varepsilon_t$$

The shocks ε_t are iid $N(0, I)$ with I the identity matrix. B is a matrix such that $B'B = \Sigma$. ε_t has five independent components ε_t^j where a single-period unitary shock on ε_t^j produces the IRF j . Since $v_t = B\varepsilon_t$ we can write:

$$v_t = \sum_{j=1}^5 B t^j \varepsilon_t^j$$

where t^j is a selector vector of zeros excepted the j th row equal to one. ε_t^j are $N(0, 1)$ random numbers.

Now let us construct the residual terms that MARCOS needs to replicate the IRF. To the equation i is associated the residual term u_t^i (with $i = 1, \dots, 10$ in order to catch direct as well as indirect effects, 5 for each area). As noticed before, the random number ε_t^j represents the innovation associated to the variable j of the VAR. Each simulation gives the numerical value $\alpha_{i,t}^j$ for the effect of shock j on variable i at date t . $t = 1$ since the IRF are replicated for the first period only. Finally,

$$u_1^i = \sum_{j=1}^{10} \alpha_{i,1}^j \varepsilon_1^j$$

whereas $u_t^i = 0$ for $t \neq 1$.

It is worth noting, that in addition of cross correlation, Drew and Hunt [1998] allow for serial correlation among the structural model residuals (u_t^i). In this case residuals of the model will be:

⁶ For more than 30 replications the marginal change of standard deviation is less than 1%.

$$u_t^i = \sum_{k=0}^K \sum_{j=1}^{10} \alpha_{i,1+k}^j \varepsilon_{t-k}^j$$

with K the order of autocorrelation correction. For MARCOS $K=0$.

In order to get comparable results, our different stochastic simulations innovations have to be identical for all experiments. For this reason we maintained the same seed to the random generator. We also have skip the 20 first observations of each stochastic simulations in order to have results unaffected by initial conditions (here deterministic).

5- Results

Optimal Taylor rule coefficients

Theses simulations are carried out for each area (euro and US) on a mesh composed of 30 nodes ($\mu = 0.4, 0.8, 1.2, 1.6, 2$ and $\tau = 0, 0.2, 0.4, 0.6, 0.8, 1$). For each shock, we compute the Taylor rule coefficients minimising the overall criterion or specifically the variance of the output gap, the variance of the inflation gap or the variance of interest rate changes. For the global criterion the weighting parameters $\lambda_y, \lambda_\pi, \lambda_r, \beta$ have been set equal respectively to 1, 1, 0.5 and 1/1.04. For one stochastic simulation we report the coefficients which minimise the different criteria for each area (for the other area the Taylor rule coefficients are: $\mu = 1.5$ and $\tau = 0.5$).

It is worth noting that Blanchard and Kahn [1980] conditions depend on μ and τ values. For some range of values these conditions could not be met. For example, $\mu = 0$ leads to indeterminacy (the number of non redundant lead variables exceeds the number of eigenvalues greater than one) and this case has been ruled out.⁷

Results reported in Tables 3 exhibit analogies between the two areas. Hence, the optimal coefficients are nearly identical except for the overall criteria. For an output objective, the optimal Taylor rule is obtained by putting all the weight on the output-gap parameter and the smallest value to the inflation-gap parameter. In the same manner, when the monetary authorities have an inflation objective, their optimal strategy is to react strongly to inflation. It is worth noting that even if the monetary authorities have a unique objective of inflation, they still put a weak but strictly positive weight on the output gap: 0.2 for the euro area and 0.4 for the US. This difference could be related to the larger Phillips effect in the US. To get an inflation-gap reduction the fed can also target the output gap because of the magnitude of the Phillips effect. The higher the output-gap coefficient in the Phillips curve the stronger is the improvement in the inflation reduction. So for the US the monetary authorities take advantage of this highest Phillips effect. Concerning the interest rate change criteria, results are quite obvious: the variance of interest rates change is minimised if the central bank does not react, i.e. μ and τ are equal to 0.

As far as the global criterion is retained by central banks, the US presents a reaction function less aggressive to inflation than the euro area (less than 0.4 for the US against more than 2 for the euro area). But according to our findings, the Fed seems to pay more attention to the

⁷ See Taylor rule presentation in section 2.1 for the remark about first difference Taylor rule properties and indeterminacy conditions.

output gap than the ECB. The difference between the two areas could be related to their different volatility features for output gap and inflation. The US seems to be characterised by a higher variance on output gap whereas the inflation appears to be more volatile in the euro area. Because the objective of the central bank is a mixture of the three variances, its optimal attitude is to put more weight on the more volatile component.

It is noticeable that output-gap coefficient is, whatever the objective of the central bank, always strictly positive (contrary to the Batini and Nelson [2000] assumption). The coefficient τ is always greater than 0.2 (at the exception of the particular interest-rate variance criterion) implying a reaction function containing the output gap.

Table 3: Optimal coefficients in the Taylor rule

	EA		US	
	μ	τ	μ	τ
$V(Y)$	≤ 0.4	≥ 1	≤ 0.4	≥ 1
$V(\pi)$	≥ 2	0.2	≥ 2	0.4
$V(dr)$	≤ 0.4	0	≤ 0.4	0
V(criteria)	≥ 2	0.4	≤ 0.4	≥ 1

Optimal Taylor rule horizon

To compute the optimal horizon, we proceed as previously by searching on a grid formed of 30 nodes ($k_\mu = (0, 1, 2, 3, 4)$ and $k_\tau = (0, 1, 2, 3, 4, 5)$).

When an objective of output-gap stability is tracked, the optimal behaviour of the monetary authorities is to react to immediate deviations of output and inflation from their target (Table 4). As noted previously, if the stability of the interest rate is targeted the central bank should avoid large movement in the interest rate and hence retain the longer horizon as possible. These results could be compared to those of Batini and Nelson [2000] despite their different specification of the reaction function (only inflation and an auto-regressive term enter the reaction function). With a small forward-looking model they found an optimal horizon inferior to one year (2 quarters) and a coefficient related to inflation in the Taylor rule equal to 1.2. In our case, with the same criterion ($\lambda_y, \lambda_\pi, \lambda_r$ and β set equal to 1, 1, 0.5 and 1/1.04), we found an optimal lag of 0 year on both inflation and output gap.

Table 4: Optimal leads in the Taylor rule

	EA		US	
	$k\mu$	$k\tau$	$k\mu$	$k\tau$
$V(Y)$	0	0	0	0
$V(\pi)$	1	3	0	1
$V(dr)$	1	5	4	5
V(criteria)	0	0	0	0

Efficient frontiers

Efficient frontiers for the euro area and the US are exhibited in Figures 2 and 3. We found the usual trade-off between inflation and output gap. As expected, the frontier is convex to the origin. Its top left value corresponds to a unique objective of inflation stabilisation whilst its bottom right value corresponds to a pure objective of output-gap variance minimisation.

The main difference relies on the larger variance of output gap in the US than in the euro area, at the opposite of what could be observed for inflation. Two mechanisms could be invoked to explain this large dispersion of the output gap: first, the high volatility of the innovations associated to the output gap; second, the strength of the persistency of the propagation in the model. Clearly, the propagation mechanism in MARCOS are more persistent for the euro area than for the US (the Phillips effect and the sensitivity of imports to the real exchange rate are both higher for the US). Furthermore, the variance of the innovations is systematically larger in the US. As a consequence, the larger variance of the output gap could be originated from a greater dispersion of the shock governing output gap in the US.

Figure 2: Efficient frontier for the EA with $\lambda_r = 0.5$

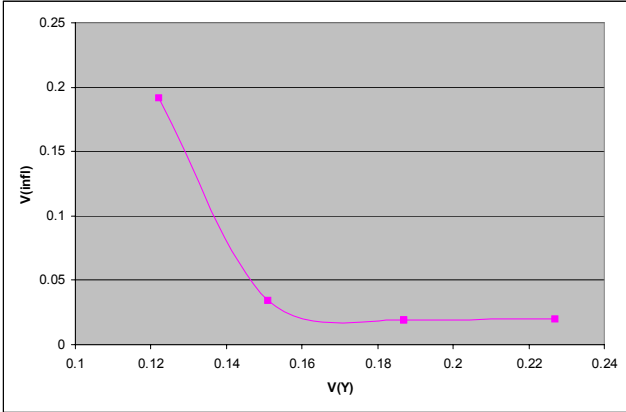
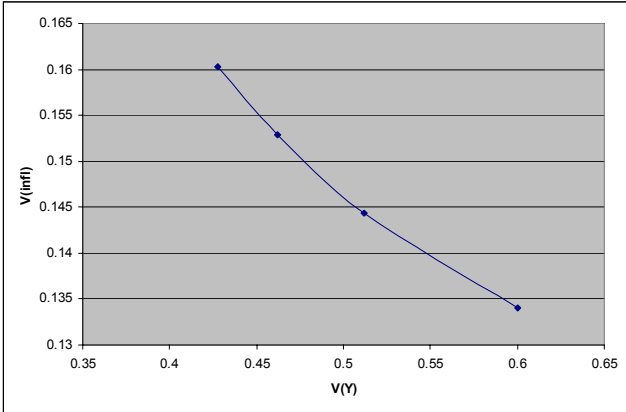


Figure 3: Efficient frontier for the US with $\lambda_r = 0.5$



Concluding Remarks

This paper investigates the optimality of the Taylor rule using the two-area model MARCOS with respect to the parameters as well as horizon. The results suggest that if the criterion considered by the central bank combines variances of output gap, inflation gap and interest rate changes, the Fed seems to be more sensitive to output gap whereas ECB seems to be more aggressive with respect to inflation. Considering the same criterion, the optimal horizon for both central banks corresponds to an infra-annual targeting on inflation and output gap as well.

Three aspects could be improved. So far, we have used a constrained form of the Taylor rule with the changes in the interest rate depending on output gap and inflation gap. In other words, we have not examined the degree of interest-rate smoothing in the reaction function. This parameter provides additional information on the timing of the optimal monetary policy rule. In a same perspective, stochastic simulations could also be extended to minimise the different criteria with respect to parameters and leads of the Taylor rule. However, as noted above, it is an expensive task in CPU time. In a multi-area model perspective, we could also investigate the optimal monetary policy of one area considering the optimality of the reaction function of the other area.

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