

Nonlinear Growth and the Productivity Slowdown

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Abstract

In this paper we study the productivity slowdown taking as a starting point the nonlinear shape of the growth path. We relate the slowdown to the evolution of the world income distribution and show that i) in the periods before and after the oil shocks growth is nonlinear; ii) the productivity slowdown consists in a downward shift of the nonlinear growth path; iii) in both periods we observe a medium run tendency to polarization, but the long run distribution features convergence in the first period and polarization in the second. We provide theoretical and empirical arguments to suggest that the process of international transfer of technology in the context of nonlinear growth can provide useful insights to understand the productivity slowdown.

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1 Introduction

During the 70s and 80s many countries have been affected by a phenomenon commonly indicated as productivity slowdown, consisting in a decrease in productivity that had a permanent effect on the growth rate. It is a common wisdom that the oil shock in 1973 was the source of this slowdown, but the causes of its long-run effects on the growth rate are still debated (see Fischer (1988) and the papers in the same issue). Early contributions on endogenous growth theory appear motivated not only from the observed lack of convergence across countries, but also by an attempt at explaining the slowdown (e.g. see Romer (1990)).

The aim of this paper is to offer a new perspective on the productivity slowdown, which takes as a central point the nonlinear shape of the growth process and the evolution of world income distribution. We argue that this is crucial to understand how the productivity slowdown affected the cross-country growth dynamics, an issue that is not receiving attention so far. Moreover, this perspective can allow to gain insights on the causes of the slowdown.

We compare two periods: 1950 – 1973 and 1974 – 1997. Our findings are: i) in both periods we find support to the hypothesis that growth is nonlinear (see Fiaschi and Lavezzi (2003)); ii) in the second period the productivity slowdown consists in a downward shift of the nonlinear growth path; iii) in both periods we observe a medium-run dynamics indicating a tendency to polarization, but in the first period the long-run distribution features convergence at high income levels, while in the second period the long-run tendency is for polarization (which contrasts with Lucas (2000)).

We suggest that these results provide evidence in favor of an endogenous growth model, in which countries' growth rate is the result of both an internal process of accumulation, and of the relative GDP with respect to other countries, as the latter affects the the international transfers of technology (see Basu and Weil (1998)). A Solovian growth model with exogenous technological progress freely available for all in fact does not find support when the dynamics of the two periods are compared.

We do not find support for the explanations of the slowdown emphasizing a reduction in the rate of accumulation of capital (see e.g. Hamilton and Montegudo (1998)). We argue that the causes of the slowdown, and its effects on the world income distribution, can be summarized in the following way: the oil shocks caused a reduction in productivity due to a sudden increase of the rate of depreciation of capital (see Baily and Schultze (1990)); this caused a downward shift of the growth path which caused the appearance of a development trap for poor countries. A tendency for global convergence

was substituted by a tendency for polarization (see Quah (1997)). A higher spread of the world income distribution made more difficult, or interrupted, the international flow of technologies, as suggested by the theory of appropriate technology. This had perverse effects on the productivity of the lower tail of the distribution, where countries, having to rely solely on internal factors, were not able to escape the poverty trap.¹ Productivity at the higher tail of the distribution was not able to recover due to a strong decrease in the investment rate, and to increased competition from follower countries with respect to the technological leaders (see Goodfriend and McDermott (1998)).

This paper adopts the distribution dynamics approach for the empirical analysis of the productivity slowdown, and is therefore related to works like Quah (1997) on convergence. We argue that this approach is particularly appropriate given its focus on the relative economic distances among countries. As noted, we adopt the view the the international transfer of technology is a relevant aspect to consider for the issue at hand (Bernard and Jones (1996)), specifically when the relative levels of capital/labor ratios or per capita income are crucial to understand the process of technological spillovers. This point is made in the literature on appropriate technology, proposed by Atkinson and Stiglitz (1969) and more recently by Basu and Weil (1998). We will contrast our results with the empirical implications of other models of growth and international technological spillovers, namely that of Lucas (2000) and Basu and Weil (1998).

The paper is organized as follows: Section 2 presents an empirical analysis of growth dynamics before and after the oil shock, adopting the distribution dynamics approach, and evaluates the relation between investment and growth; Section 3 assesses the capacity of alternative theoretical frameworks to be supported by the empirical evidence, and advances a theory based on a nonlinear growth model with international technological transfer; Section 4 contains some concluding remarks.

2 Empirical Analysis

In this section we provide an extended empirical analysis for a large sample of countries. In particular in Section 2.1 we analyze the distribution dynamics of a sample of 122 countries in two subperiods: 1950 – 1973 and 1974 – 1997.² In particular, we first consider GDP in absolute terms, and then GDP in

¹ Atkinson and Stiglitz (1969), p. 577, emphasize the dependence of technological progress on history.

²Pooled data are from Maddison (2001). See Appendix A for the country list. Figures are in 1990 constant dollars

relative terms, that is expressed with respect to the sample average.

The use of absolute values helps to evaluate the possible presence of poverty traps in absolute terms, and abstracts from the assumption of a world technological trend. However, it has one drawback: care should be used when formulating long-run predictions, given that the behavior identified in some GDP classes (see below) may not assumed to be the same in distant periods. The use of relative values avoids this problem, but has the drawback of being based on a normalization which may not be completely appropriate. As an alternative found in the literature (see, e.g. Jones (1997)), we report in Appendix B.3 estimates also for GDP normalized with respect to US GDP.

2.1 Distribution Dynamics

2.1.1 Absolute Values

In Figures 1 and 2 we report a nonparametric regression of growth rates against the absolute value of per capita GDP.³

³The nonparametric estimate is obtained with the statistical package included in Bowman and Azzalini (1997). We used the standard settings suggested by the authors (e.g. optimal normal bandwidth and weights on observations according to their density). To test the robustness of this estimate, we ran an alternative nonparametric regression by using the plug-in method to calculate the kernel bandwidth, and obtained a similar picture. We refer to Bowman and Azzalini (1997) for more details. Data sets and codes used in the empirical analysis are available on the authors' websites (<http://www-dse.ec.unipi.it/fiaschi> and <http://www-dse.ec.unipi.it/lavezzi>).

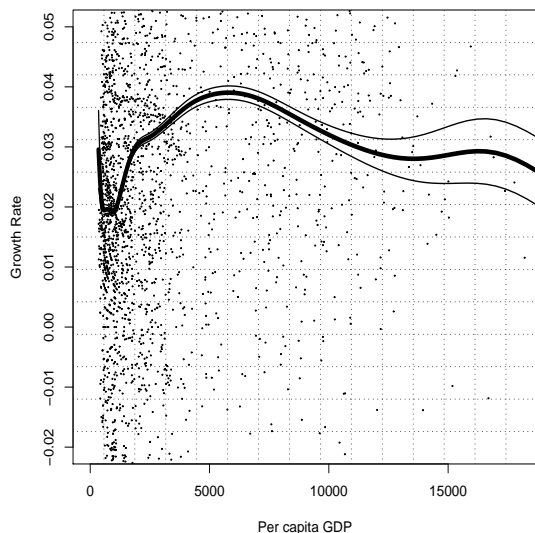


Figure 1: Growth rates vs per-capita GDP : 1950-1973

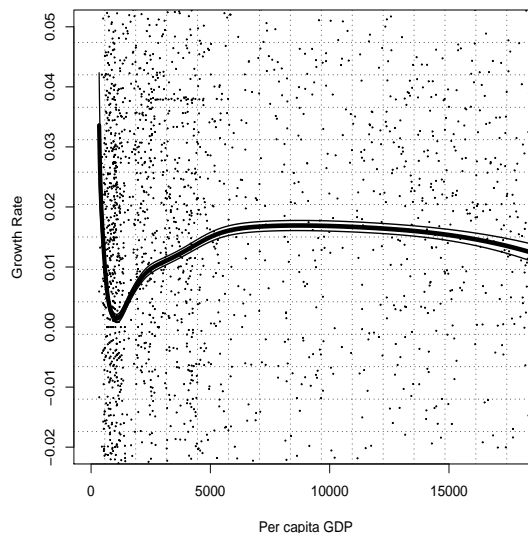


Figure 2: Growth rates vs per-capita GDP : 1974-1997

In both cases the estimated growth path is nonlinear, with the turning points broadly coincident. However, in the second subperiod we observe a downward shift of the entire path, which can be interpreted as a slowdown in the overall trend. Therefore, the productivity slowdown seems to have affected the world growth path, *without substantial changes in the shape of the growth process*. This evidence can be compared with the insights provided by Galor (1996), Fig. 3, where a change in a technology parameter modifies a nonlinear growth path. In that case the change is responsible for the elimination of convergence clubs; in our case it seems that the opposite has taken place (as we explain in Section 3). These hypotheses are tested by the estimation of Markov transition matrices.

Following the method proposed in Fiaschi and Lavezzi (2003) we define the state space of the Markov process by inspection of Figures 1 and 2. In particular we first define the growth rate classes on the basis of the average growth rate of the samples. In the first period the average growth rate is equal to 2.6%, while in the second period it is equal to 0.9%. The growth rate classes are defined by adding and subtracting one percentage point. The GDP classes are defined with respect to the turning points of the growth path (for details see Fiaschi and Lavezzi (2003) and refer to Figure 9 below for the exact placement of the GDP classes' boundaries). This leads to the definition of two state spaces in Table 1.

GDP\Growth rate 1950-73	< 1.6%	1.6%, 3.6%	> 3.6%
GDP\Growth rate 1974-97	< -0.1%	-0.1%1.9%	> 1.9%
0 – 1250	I-	I+	I++
1250 – 4000	II-	II+	II++
4000 – 9000	III-	II+	III++
> 9000	IV-	IV+	IV++

Table 1: state space definition

In Tables 2 and 3 we represent the distribution dynamics in the two periods.⁴ We consider the distribution in the four GDP classes in the initial and final year, along with the ergodic distribution.⁵

GDP	<i>I</i>	<i>II</i>	<i>III</i>	<i>IV</i>
1950	0.44	0.40	0.11	0.04
1973	0.28	0.36	0.17	0.19
Ergodic	0	0	0	1.00

Table 2: Distribution dynamics 1950-1973: absolute GDP

GDP	<i>I</i>	<i>II</i>	<i>III</i>	<i>IV</i>
1973	0.28	0.36	0.17	0.19
1997	0.29	0.25	0.20	0.27
Ergodic	0.15	0.15	0.19	0.51

Table 3: Distribution dynamics 1974-1997: absolute GDP

In Table 2 we show that in the first period GDP class *IV* is an absorbing state. Differently, in the second period (see Table 3) 49% of the countries are expected to fail to catch up with the richest in the long-run, and to cross the value of 9000 constant dollars of per capita GDP.

The nonlinearity in the two periods can be appreciated by looking at the transition matrix in Appendix B.1, and to the “*normalized*” ergodic distribution in Table 4.⁶ Given that in the first period there is an absorbing state, we represent it only for the second period.⁷

⁴The distribution of observations is not symmetric, given our criterium for the choice of the GDP classes. In particular, the distribution of observations is: 0.38, 0.39, 0.15, 0.08 (first period); 0.28, 0.30, 0.20, 0.22 (second period).

⁵We considered 3-year transitions in order to circumvent the possible presence of autocorrelation of growth rates due to measurement errors. The transition matrix are presented in Appendix B.1.

⁶In Fiaschi and Lavezzi (2003) we discuss in detail how to detect nonlinearities from the values of a transition matrix. Briefly, we take as evidence of nonlinear growth: i) a relatively high tendency to have persistently low/decreasing growth in GDP class *I*; ii) a relatively high tendency to have high/accelerating growth in GDP class *III*; and iii) a relatively high tendency to have medium/high growth in GDP class *IV*.

⁷The mass in GDP class *IV* in the ergodic distribution of the first period is distributed as: 0.38, 0.24, 0.38. The ergodic distribution exists even if there exists an absorbing state, as it is unique.

	-	+	++
I	0.40	0.23	0.37
II	0.34	0.22	0.43
III	0.35	0.16	0.49
IV	0.20	0.26	0.54

Table 4: ergodic distribution normalized for each GDP class

We take as evidence for the nonlinearity the following facts: the probability to have a low growth rate is relatively higher in GDP class *I*, decreases in GDP class *II* and *III* (although the values in the latter classes are very close), and decreases further in GDP class *IV*. The relative probability of having a medium growth rate is highest in GDP class *IV*. The probability of having a very high growth rate increases from classes *I* to *IV*. Notice that there is no decrease in the fourth column in GDP class *IV*, as in Fiaschi and Lavezzi (2003). This can be justified as in the long run there is positive growth as predicted by the family of *AK* models, and not a monotonic decrease in the growth rate as predicted by Solovian models.

2.1.2 Relative Values

In this subsection we repeat the analysis using data normalized with respect to the world average, as generally done in the distribution dynamics literature. This approach has the advantage of taking into account the possible presence of a world trend.

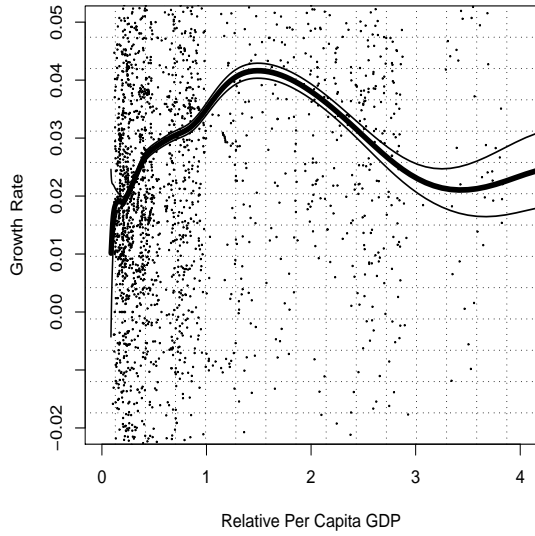


Figure 3: Growth rates vs per-capita *GDP*: 1950-1973

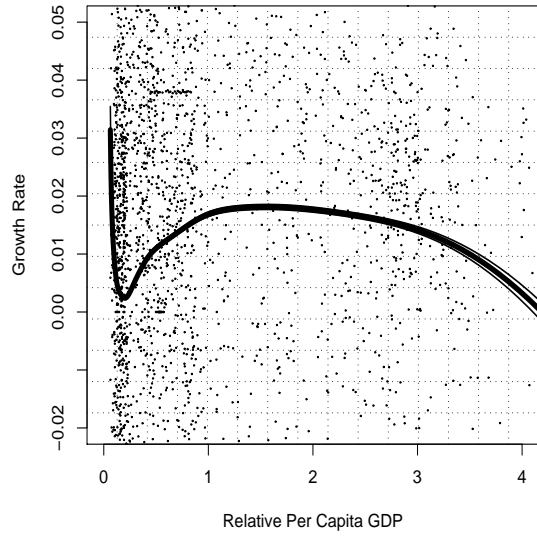


Figure 4: Growth rates vs per-capita *GDP*: 1974-1997

Figures 3 and 4 confirm the presence of a nonlinear growth path (notice that the estimate for high and low relative GDP values is not very precise, as shown by the large variability band⁸). The transition matrices in Appendix B.2 confirm the result.

The state space in this case is reported in Table 5.⁹

\Growth rate 1950-73	< 1.6%	1.6%, 3.6%	> 3.6%
Income\Growth rate 1974-97	< -0.1%	-0.1%1.9%	> 1.9%
0 – 0.25	I-	I+	I++
0.25 – 0.9	II-	II+	II++
0.9 – 2.1	III-	II+	III++
> 2.1	IV-	IV+	IV++

Table 5: state space definition

In Tables 6 and 7 we report the distribution dynamics.¹⁰

⁸For details on variability bands see Bowman and Azzalini (1997).

⁹The distribution of observations in the four classes is the following: 0.22, 0.40, 0.17, 0.13 (first period); 0.30, 0.36, 0.16, 0.18 (second period).

¹⁰The transition matrix and the ergodic distribution for all states are reported in Appendix B.2

GDP	<i>I</i>	<i>II</i>	<i>III</i>	<i>IV</i>
1950	0.21	0.53	0.15	0.11
1973	0.28	0.42	0.12	0.18
Ergodic	0.05	0.06	0.04	0.85

Table 6: Distribution dynamics 1950-1973: relative GDP

GDP	<i>I</i>	<i>II</i>	<i>III</i>	<i>IV</i>
1973	0.28	0.42	0.12	0.18
1997	0.34	0.31	0.16	0.19
Ergodic	0.36	0.24	0.17	0.23

Table 7: Distribution dynamics 1974-1997: relative GDP

Observe that in the first period, the initial distribution has a peak in GDP class *II*. However, in 1973, the extreme classes show a substantial higher mass at the expenses of the central classes (as a matter of fact, it can be noted that there are two peaks, in GDP class *II* and *IV*). We consider this as evidence of a medium-run tendency to polarization. In particular, in GDP class *II* all countries are growing faster than countries in GDP class *I*. In addition, within GDP class *II*, richer countries are growing faster. This implies that those who are growing more slowly transit to GDP class *I*, as the world average is increasing over time. In the long run there is clearly a tendency to converge to GDP class *IV*.¹¹

In the second period the two peaks in the long run are at the extreme classes. If we compare the initial distribution and the ergodic, we notice that in 1973 the peaks are in GDP classes *II* and *IV*, while in the ergodic they are in GDP classes *I* and *IV*.

In Tables 8 and 9 we report the normalized ergodic distributions for the two periods. Both broadly confirm the presence of nonlinearities along the lines suggested in the previous section.

	-	+	++
I	0.50	0.25	0.25
II	0.40	0.24	0.36
III	0.22	0.18	0.60
IV	0.34	0.25	0.41

Table 8: ergodic distribution normalized for each GDP class. 1950-73

	-	+	++
I	0.40	0.22	0.38
II	0.33	0.22	0.45
III	0.31	0.14	0.55
IV	0.19	0.29	0.52

Table 9: ergodic distribution normalized for each GDP class. 1974-97

With relative GDP, the cross-country dynamics in the first period is compatible with the Lucas (2000)'s model, in which growth in the aggregate is

¹¹It is not completely meaningful that almost all countries converge to a GDP level which is more than twice the average of the sample. This is an example of the limits of using this type of normalization highlighted by Kremer et al. (2001). If we normalize GDP with respect to US GDP the tendencies are broadly confirmed (see Appendix B.3).

nonlinear, and countries polarize in the medium term but all converge to the same income levels in the long run.¹² This type of dynamics is generated by a process of technological spillovers across countries: countries starting the growth process late can benefit from the technology developed by the leading countries, and therefore have an initial strong increase in growth, which is stronger the further a country is from the technological leader, that subsequently slows down.

However, Lucas (2000)'s and all models where relative backwardness is an advantage fail to explain the dynamics of the second period. In the first period 5 out of 122 countries had negative average annual growth rate; in the second period the number raised to 35: almost all are African countries, some South American countries and some oil producers. It seems that countries affected more severely by the slowdown are many poor countries, which moved from positive to negative growth rates (this is particularly clear when we use absolute GDP). The Lucas' model should not be affected by the slowdown in the long run, as its mechanism of growth should not prevent all countries to converge to the highest GDP levels.

In general, this fact is not coherent with the idea that productivity slowdown regards in particular countries on the technological frontier, while lagging countries should be affected only marginally given that they are far away from the frontier. In Section 3 we discuss some theoretical models which can help to explain these facts.

2.2 Investment and Growth

Is the productivity slowdown related to investment rates? The standard Solow theory suggests that a reduction in the saving rate causes a reduction of the growth rate in the short run, while a simple AK model predicts that the reduction should extend also to the long run. As Fischer (1988), p. 4, puts it a decline in the rate of investment is a "*suspect*" in the search for the causes of productivity growth slowdown. We consider a restricted sample for which we can obtain data on investment rates (measured by the ratio of investment to GDP) from the Penn World Tables 6.1. The restricted sample includes 91 countries for the period 1961 – 1997.¹³

¹²Taken literally, in Lucas' model countries are completely stagnant until they begin to grow exploiting the international spillovers of technology. The beginning of growth is marked by a jump to a high growth rate, which eventually converges to the growth rate of technological leaders. In the first period we observe countries with a positive but low growth rate. This can reflect the presence of internal sources of growth, which allow a country to grow while it is "*waiting*" for the spillovers.

¹³See Appendix A for the country list.

In Figure 5 we plot a nonparametric estimation of the relation between investment rates and the growth rate of GDP for the two periods.¹⁴

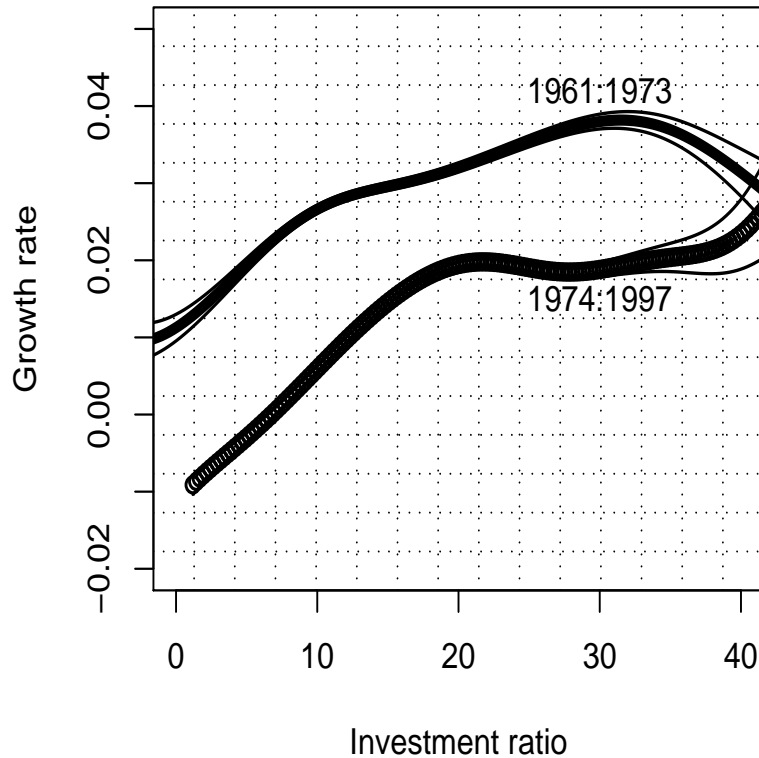


Figure 5: Growth rates vs investment ratio

We can observe that in both periods the relation is positive, although characterized by some nonlinearities (the estimates for high values of the investment ratio are not very precise). What is striking is that the estimate for the first period lies above the one of the second period and the gap is nearly constant. Hence, we argue that an exhaustive explanation for the slowdown in growth rates cannot be found in changes in rates of accumulation.

In fact, the growth rate increases with the levels of investment rates in both periods but, in the second period, the same level of investment share is invariably associated to a remarkably lower growth rate of per capita GDP. This means that *something else* must have been responsible for the reduction in the growth rate in the second period rather than a reduction in investment rates. This result contrasts with Hamilton and Montenegro (1998).

¹⁴The criterion used for these estimates is the same of the figures in Section 2. Here we used 4-year averages to reduce the influence of cyclical factors.

They study the slowdown in the neoclassical framework and find a positive coefficient of variation of investment on variation of growth rate across two periods before and after the oil shocks. Their finding is however explainable by a downward on the growth-investment relation, as shown in Figure 5, and not by a movement on the same curve, as implied by their analysis.

This is further demonstrated in Table 10. We separate the 91 countries in two groups: those who decreased the average investment rate s over the two periods: 1961 – 1973 and 1974 – 1997, and those who increased it.

	GDP 1961	GDP 1974	s_1	s_2	g_1	g_2
$\Delta s < 0$ (48 countries)	1.31 (4067)	1.30 (6321)	22.94%	18.15%	3.00%	0.9%
$\Delta s > 0$ (43 countries)	0.67 (2110)	0.66 (3221)	11.12%	14.60%	2.97%	1.7%

Table 10: Investment ratios and growth rates: relative and absolute (in parenthesis) per capita GDP

In Table 10 we report for both groups of countries: the average GDP in the first year of both periods (respectively, 1961 and 1974); the average investment share over the first and second period (s_1 and s_2); the average yearly annual growth rate over the two periods (g_1 and g_2).

Both groups of countries saw a remarkable decrease in the growth rate in the second period. This, despite the fact that a group of countries considerably increased the investment share. Those who experienced a decrease in the rate of investment were on average richer than those who increased it.

We can represent this piece of evidence in a more detailed fashion, by partitioning the countries according to our GDP state space. In particular, we partition the observations in 1973, and relate them to the average investment rates in the first and in the second period.

GDP (relative)	s_1	s_2	Δs	g_1	g_2	Δg
<i>I</i> (24 countries)	9.17 (6.17)	10.28 (5.35)	1.11	1.5%	0.7%	-0.8%
<i>II</i> (39 countries)	15.53 (7.76)	15.45 (6.85)	-0.08	2.9%	1.2%	-1.7%
<i>III</i> (14 countries)	21.81 (8.66)	19.59 (5.43)	-2.2	4.4%	1.5%	-2.9%
<i>IV</i> (17 countries)	29.12 (5.03)	24.89 (3.95)	-4.23	3.5%	1.6%	-1.9%
All countries	17.36 (9.84)	16.49 (7.62)	-0.87	2.92%	1.20%	-1.71%

Table 11: Investment shares and growth rates: countries partitioned in (relative) GDP classes

GDP (absolute)	s_1	s_2	Δs	g_1	g_2	Δg
<i>I</i> (24 countries)	9.17 (6.17)	10.28 (5.35)	1.11	1.5%	0.7%	-0.8%
<i>II</i> (31 countries)	14.72 (7.51)	15.28 (7.22)	0.56	2.8%	1.2%	-1.6%
<i>III</i> (18 countries)	20.70 (7.8)	18.25 (4.77)	-2.45	4.3%	1.4%	-2.9%
<i>IV</i> (18 countries)	29.48 (5.11)	25.10 (3.93)	-4.38	3.6%	1.6%	-2.0%
All countries	17.36 (9.84)	16.49 (7.62)	-0.87	2.92%	1.20%	-1.71%

Table 12: Investment shares and growth rates: countries partitioned in (absolute) GDP classes

Tables 11 and 12 show that the decrease in the investment rate affected the countries differently according to their GDP level (standard deviations are in parentheses). Results are comparable with relative and absolute GDP observations. In both cases, countries in GDP classes *III* and *IV* strongly decreased the investment share, while countries in GDP class *I* did not decrease it. Data for countries in GDP class *II* show little change. From Tables 11 and 12 we also see that there is a positive relation between the level of GDP and the level of investment rate.¹⁵

Overall, we argue that the growth slowdown cannot be entirely reconducted to a generalized decrease in the rate of accumulation although, as we discuss below, the strong decrease in investment in rich countries may be played a role.

3 A Discussion of Alternative Theoretical Models

In this section we relate our results to different theoretical frameworks. In particular we discuss the exogenous and endogenous growth models in order to assess their capacity to explain the empirical evidence provided in the previous section. Finally, we analyze models of international transfer of technology as an alternative framework to gain further insights on the causes of slowdown.

¹⁵This does not depend on the discretization of data on GDP into classes. A non parametric regression of investment rates against GDP level confirms this result (also for the two periods).

3.1 Solovian Framework (Exogenous Growth)

In the standard Solovian model the capital stock is governed by the following rule:¹⁶

$$\frac{\dot{\kappa}}{\kappa} = s \frac{f(\kappa)}{\kappa} - (\delta + n + \gamma), \quad (1)$$

where s is the saving rate, $\kappa = k/A$ is the capital stock in efficiency units ($k = K/L$ is the per capita capital), A is the level of exogenous technological progress, δ the rate of depreciation of capital, n the growth rate of population and γ the exogenous growth rate of technological progress.

Standard assumptions on technology are that $f' > 0$ and $f'' < 0$ plus the Inada conditions: $\lim_{k \rightarrow 0} f' = \infty$ and $\lim_{k \rightarrow \infty} f' = 0$. In the long run equilibrium all per capita variables grow at the constant rate γ .

Countries with different parameters should nonetheless grow in the long run at the same rate γ . In a model with exogenous technological progress the average growth rate of per capita GDP of the sample over the period can be considered as a (rough) estimate of the exogenous growth rate of technological progress γ .

Even taking into account cross-country heterogeneity and the implied tendency for conditional convergence, the standard Solow growth model represented by Eq. (1) is not appropriate to explain our empirical results for the following reasons:

1. the nonlinearity of the growth path excludes that countries are characterized by concave production functions with different parameters.¹⁷
2. In the first period, data in relative terms shows a tendency for basically all countries to converge to the same GDP class. If the cross-country heterogeneity were present, it would be in this case sufficiently weak not to impede this type of dynamics. In the second period, on the contrary, convergence is expected to occur in different GDP classes. Then two cases are possible: i) convergence to different classes is not a problem of heterogeneity but of nonlinearity (see point 1. above); ii) heterogeneity characterizes the period after 1973, and hence the shock affected asymmetrically the countries in the sample. In this case we need a theory explaining why countries appeared as homogeneous before the shock and heterogeneous after the shock.

¹⁶In this and in the following sections we assume that s is constant. However, as we showed, the investment rate is increasing in GDP. Therefore, a more precise specification should consider $s = s(k)$, with $s' > 0$. For simplicity we abstract from this point, which, in any case, would strengthen the nonlinearity of the growth path that we introduce below.

¹⁷In Fiaschi and Lavezzi (2003) we control for cross-country heterogeneity in a similar sample, and reject the hypothesis of conditional convergence.

3. In the second period there appears a poverty trap in absolute terms, i.e. a relevant fraction of countries is not expected to reach the highest GDP class in the long run. This is at odds with the crucial assumption of the conditional convergence hypothesis, according to which all countries in the long run are expected to grow at the same long run growth rate, determined by the exogenous technological progress γ .

The next step consists in considering a nonlinear Solovian growth model with exogenous technological progress. With respect to Eq. (1), we assume that $f(\kappa)/\kappa$ is nonlinear in κ . In particular we assume that $f(\kappa)/\kappa$ is first decreasing, then increasing and finally decreasing in κ , with $\lim_{\kappa \rightarrow 0} f' = \infty$ and $\lim_{\kappa \rightarrow \infty} f' = 0$.¹⁸ Define the capital level where the function reaches a local minimum, and changes concavity, as κ_{min} .

The nonlinear path can be due to the existence of an intermediate level of capital characterized by increasing returns to scale. This assumption is justified by a large literature which goes from classical contributions of Lewis (1956) and Rostow (1960) to Murphy et al. (1989) and Peretto (1999), which highlight structural change in the growth process as a cause for this pattern. Structural change consists in a transformation of the economy such that the weight of traditional sectors (e.g. agriculture) declines, and the weight of industrial sectors increases.

In order to obtain the dynamics of the first period, we should have a path showing only one stable equilibrium. Figures 6 and 7 illustrate two alternatives.

¹⁸A longer discussion on this type of function is in Barro and Sala-i-Martin (1999), p. 53.

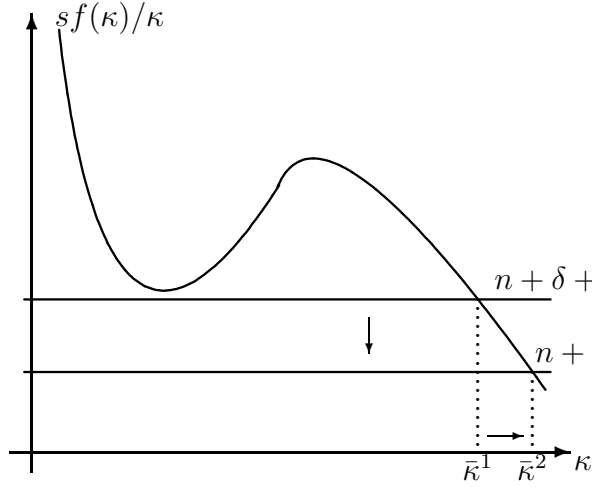


Figure 6: Nonlinear solovian model:
case A

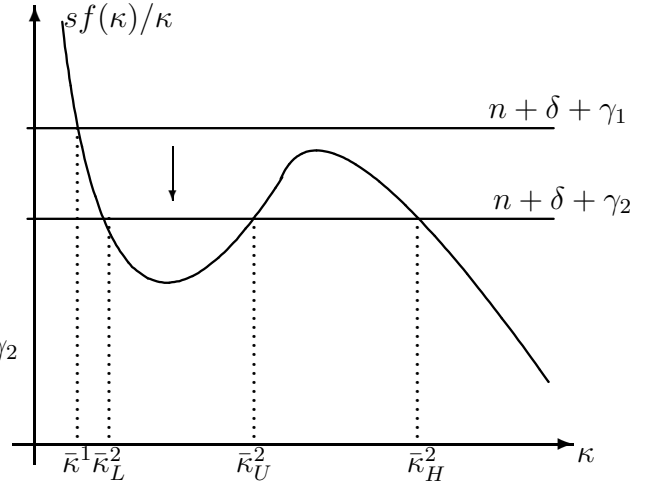


Figure 7: Nonlinear solovian model:
case B

In Figures 6 and 7 we represent two possibilities to have a nonlinear path and one stable equilibrium in a Solovian model with a nonlinear production function and exogenous technological progress. In the first case (Figure 6) all countries converge to $\bar{\kappa}^1$. In steady state all countries grow (in per capita terms) at the common rate γ_1 . If we proxy the long-run growth rate by a simple average of the sample over a period of time, then we conclude that it is decreased in the second period (as noted, from 2.6% to 0.9%). Hence, the horizontal line in Figure 6 should shift down. This would produce a tendency to convergence to the same capital level (in efficiency units) $\bar{\kappa}^2$. All countries would still be expected to grow in the long run (in per capita terms) at the common growth rate $\gamma_2 < \gamma_1$. This implication does not find support in the data, as in the second period we observe polarization in relative terms: with polarization, countries may well be described as growing at the same rate in the long run, but maintaining a distance in GDP levels.

In Figure 7 we represent another case. Here all countries are still expected to converge to the same capital level (in efficiency units) $\bar{\kappa}^1$, and to grow in the long run at the exogenous growth rate γ_1 . The decrease in γ in this case is compatible with the evidence in the second period, where countries polarize at different relative GDP levels.

We see in Figure 7 that a decrease in γ from γ_1 to γ_2 generates three equilibria: a stable “low” equilibrium $\bar{\kappa}_L^2$, a stable “high” $\bar{\kappa}_H^2$, and an unstable equilibrium, $\bar{\kappa}_U^2$. However, this picture is not compatible with the evidence for

the first period. There, we observed in relative terms a medium-run tendency to polarization, with increasing fractions of countries in GDP classes *I* and *IV* and decreasing fractions in *III* and *IV*. In Figure 7, poor countries, that is those starting from a GDP level lower than $\bar{\kappa}^1$, have the highest growth rates in the transition. In particular their growth rate is higher than the long run growth rate γ_1 . Countries starting from per capita GDP levels higher than $\bar{\kappa}^1$ should instead show lower growth rates in per capita terms, in particular growth rates lower than γ_1 (these growth rates may even be negative). These countries should display a tendency to converge to equilibrium by jumping from the higher GDP classes to the lower. Moreover, therefore we should not observe polarization in GDP class *I* in the medium run.

3.2 Nonlinear Endogenous Growth Model

Given that nonlinearities are a salient feature of aggregate picture, consider the following nonlinear endogenous growth model:

$$\frac{\dot{k}}{k} = s \frac{f(k, B)}{k} - (\delta + n), \quad (2)$$

where $f(k, B)/k$ is nonlinear in k ; in particular we assume that $f(k, B)/k$ is first decreasing, then increasing and finally decreasing in k , with $\lim_{k \rightarrow 0} f' = \infty$ and $\lim_{k \rightarrow \infty} f' = B$. A necessary condition to have growth in the long run is that $B > (\delta + n)/s$; in this case in the long run $g = sB - (\delta + n)$. B parametrizes the level of average productivity for a given level of k , i.e. $\partial f(k, B)/\partial B > 0$ (see Galor (1996), p. 1067). This is a convenient way of representing in this framework multifactor productivity, another variable often quoted as responsible for the slowdown.¹⁹ Productivity slowdown can therefore be represented as a reduction in B . In this model the capital stock should be interpreted as a composite index of physical and human capital.

Eq. (2) shows a *AK* dynamics, but only for a high level of capital.²⁰ In general, a downward shift of the growth path generated by Eq. (2), and

¹⁹See e.g. Baily and Schultze (1990) on the United States. In growth accounting exercises, the growth of output over a specified period is expressed as a weighted sum of growth rates of capital and labor and of multifactor productivity, taken as a proxy of exogenous technological progress in the Solovian framework. An estimate of multifactor productivity is provided by the residual after deducting the weighted growth rates of capital and labor from the growth rate of output. Weights are the factor shares in national GDP, under the hypothesis of perfect competition.

²⁰The *AK* model has been criticized on the grounds of results on conditional convergence. See, e.g. Barro and Sala-i-Martin (1999), p. 167. However, if the growth path is nonlinear in the transition, the evidence on conditional convergence may not be sufficient to reject the *AK* model. In fact, with a nonlinear path, countries in the sample may

a decrease in the long run growth rate, may be caused by four factors: (i) a decrease in s ; (ii) an increase in the growth rate of population n ; (iii) a decrease in the average product of capital $f(k, B)/k$ and (iv) an increase in the rate of depreciation of capital δ .²¹

In Section 2.2 we did not find that variations in s have a significant explanatory power for the decrease in growth rates after 1973. Moreover, we find that the growth rate of population appears almost constant in the two periods (2.3% vs 2.0% in our restricted sample for respectively the first and the second period).²² This is not surprising because, with the exception of very unlikely events, the growth rate of population follows a smooth path (in addition the oil shock is difficult to be reconciled with a change in population dynamics). This excludes another potential explanatory factor of the decreasing of the growth rate, which is generally found significantly inversely related to growth.²³ In the rest of this section we concentrate on the effect of a change in productivity measured by B and on δ (points iii) and iv)).

3.2.1 On a Decrease in Factor Productivity

A decrease in total factor productivity strongly characterizes the productivity slowdown after 1973, as remarked among others by Baily and Schultze (1990) and Fischer (1988). In this framework without exogenous technological progress but with embodied technological change this phenomenon is represented by a decrease in B .²⁴

Figure 8 reports a graphical representation of the possible dynamics in the two periods.

display a period of convergence, in which poor countries grow faster than rich countries. The problem in this case would consist in discerning between a nonlinear model with a model which is asymptotically AK , but satisfies the neoclassical assumptions otherwise (Barro and Sala-i-Martin (1999), p. 161). To the best of our knowledge, an empirical test of a nonlinear AK model has not been provided yet.

²¹Clearly, the variation in s , n and δ affect in the same manner the growth path generate by Eq. (1).

²²Data on population are from PWT 6.1.

²³Romer (1990) finds a negative long-run relation between the growth rate of labor force and the rate of growth of productivity.

²⁴A related literature emphasizes that the 60s witnessed a reduction in inventive activity, which may have been reflected in the slowdown of productivity in the 70s (see Fischer (1988) and the references therein.).

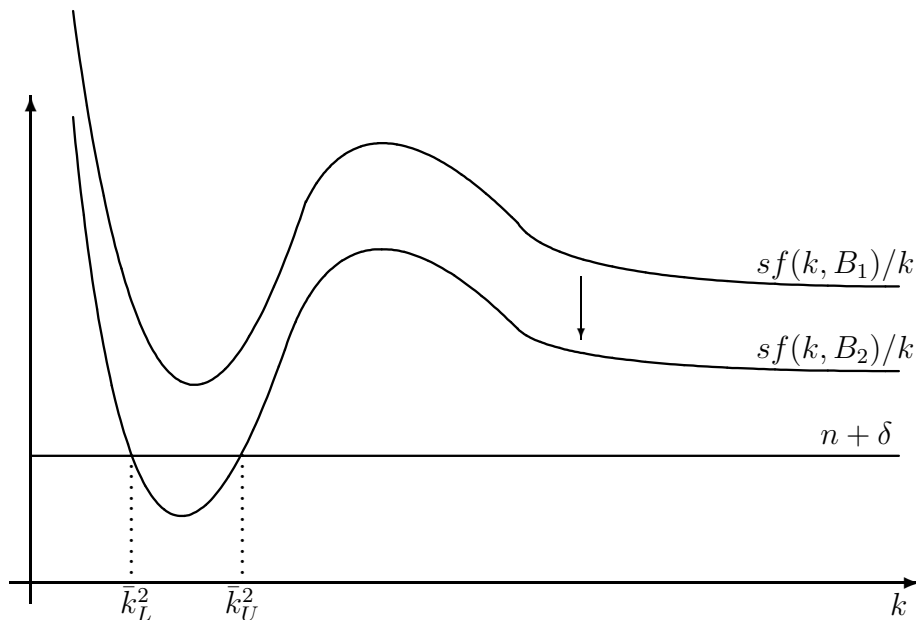


Figure 8: Nonlinear endogenous growth model: the effect of a decrease in the productivity parameter B

In the figure \dot{k}/k is measured by the vertical distance between the curve $sf(k, B)/k$ and the straight line representing $n + \delta$. In the first period productivity is sufficiently high that $sf(k, B_1)/k$ is always above $n + \delta$: all countries independent of their initial levels of capital grow at a positive rate in the long period. The resulting growth pattern is compatible with our empirical result: in particular with the estimates reported in Figures 1 and 2. Notice that GDP and growth rate classes allow to test the empirical implications of the model in Figure 8.

First we observe that, in the transition, rich countries can grow faster than poor ones. This dynamics is compatible with the medium-run polarization in the relative data observed in the first period (compare the GDP distributions for 1950 and 1973 in Table 6), and with the tendency of all countries to converge to the same absolute and relative GDP class in the long run (see the ergodic distributions).

A productivity slowdown caused by a decrease in B from B_1 to B_2 determines a downward shift of the $sf(k, B)/k$ curve and a lower growth rate for any level of k . A sufficiently high reduction of B leads to the appearance of multiple equilibria. In particular in Figure 8 we have two equilibria: k_L^2 is a

stable equilibrium, while k_U^2 is unstable. All countries whose capital is lower than k_U^2 will see their capital converge to k_L^2 , while countries with a initial capital higher than k_U^2 will have a positive growth in the long-run (but lower than the previous period).

This is compatible with the empirical evidence in absolute values (see Table 3): a relevant fraction of countries appears unable to reach the highest GDP class, neither in the medium run (in 1997), nor in the long-run limit (the ergodic distribution). Notice that in 1973 28% of the sample was in GDP class *I*, and that in 1997 this share is 29%. This is compatible with appearance of a stable equilibrium in 1973. In the ergodic limit 15% of countries are still in the same class. This is also compatible with the evidence in relative terms (see Table 7): in the second period in fact we observe a marked tendency to polarization both in the medium and in the long-run. In particular, notice that the mass in GDP class *II* has been reducing clearly since 1973. This agrees with the appearance of an unstable equilibrium in GDP class *II*. In the same period, the mass in GDP class *I* has increased remarkably both in the medium run and in the ergodic limit; this confirms the emergence of a stable equilibrium in GDP class *I*.

3.2.2 An increase in the rate of depreciation

Baily and Schultze (1990), p. 397, argue that the rate of obsolescence of capital δ may have increased suddenly as a consequence of the oil shock. A change in the relative price of factors like the price of energy can make installed capital economically obsolete, and the capital services may be reduced. Then, even if the investment rate is not changing, the growth rate of GDP per capital and productivity decrease because part of the new capital must be devoted to restoring the efficient allocation of capital among sectors, when installed capital is largely specific.

In Figure 8 the effect of higher δ can be represented by an upward shift of the straight line. The empirical implications are the same of a decrease in B . However, it is not clear that δ has remained permanently to its higher post-shock level (see Baily and Schultze (1990)), and therefore it should be explained why a transitory change in δ has a permanent effect on growth and on the world income distribution. In the next section we address this issue.

3.3 A Nonlinear Endogenous Growth Model with Technological Transfers

In this section we argue that further insights on the productivity slowdown can be gained with a growth model with technology transfers across countries.

We have already discussed the drawback of Lucas (2000)'s model.

Here, we suggest to utilize a different perspective on technological transfers, namely the one advanced by Atkinson and Stiglitz (1969) and more recently by Basu and Weil (1998) centered on the concept of appropriate technology. According to this view, technological progress is "local", in the sense that it affects only some combinations of capital/labor and not the entire production function.

In the perspective of cross-country technological transfers, this implies that technology can be transferred across "similar" countries, in the sense that they must have similar capital/labor ratios. Moreover, two countries may reciprocally benefit from their proximity in terms of capital/labor ratio.²⁵ The relevance of economic proximity suggests that the distribution dynamics approach, explicitly focused on economic distance across countries, in particular in terms of per capita GDP, can be appropriate to understand the effects and the causes productivity slowdown. The presence of polarization in the Lucas' model has no effects in the long run, in the sense that polarization does not preclude poor countries to grow and catch up. On the contrary, if the transfer of technologies occurs in the way described by Basu and Weil (1998), polarization in the medium run may have an effect in the long run, as countries lagging behind become unable to exploit the technologies developed in advanced countries.

However, in Basu and Weil (1998) the production technology is basically AK , even if the growth rate of a country also depends on the interactions with other countries. This implies that poor countries that increase their saving rate when richer countries decrease it, should increase their growth rate in the steady state.²⁶ We have observed in Tables 11 and 12, in a comparison between the two periods, that only countries in GDP class *I* significantly increased their saving rate, countries in GDP class *II* show a moderate increase or decrease according to the type of data we consider, while countries in GDP classes *III* and *IV* show a remarkable decrease. Nonetheless, growth rate for poor countries has decreased in the second period and it is the lowest with respect to the growth rate for the other GDP classes.

Differently from Basu and Weil (1998), in a nonlinear growth model (with international technological transfers) it is possible that poor countries increase their saving rate but their long run growth rate remains fixed at zero.

²⁵If the nonlinearity in the equation for the capital stock holds also for the dynamics of per capita GDP, capital per capita and GDP per capita may be used interchangeably.

²⁶There exists a case in which poor countries that increase the saving rate do not increase the growth rate: the case in which richer countries increase their saving rate faster. This implies that poor countries stop benefiting from the technological spillovers from richer countries.

This happens when the increase in the saving rate is not sufficient to allow a country to escape the poverty trap. As noted, in this model the growth rate depends on the interplay of internal sources and technological transfers. This allows to understand why temporary shocks may have permanent effects. In particular, if a process of technological transfers was at work before the oil shock, the shock could have temporarily increased the distance between countries, causing an interruption in the flow of technologies from rich countries to poor ones. The marked decrease in the growth rates of rich countries can be only partially explained by the decrease of their saving rate; in our view it remains to understand completely why rich countries have permanently halved their growth rate after the oil shock.

Figures 9 and 11 compare the estimates of the growth path for four sub-periods: 1950 : 1961, 1962 : 1973, 1974 : 1985, 1986 : 1997.²⁷ Figures 10 and 12 compare the density estimations of the sample income distribution in 1950, and in the final year of each subsample: 1961, 1973, 1985 and 1997. Figures 9, 10, 11 and 12 report the boundaries of the relative and absolute GDP classes we used in the previous sections.

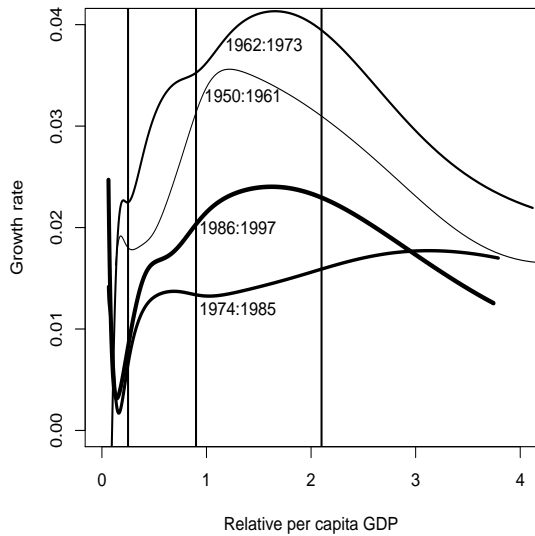


Figure 9: Estimate of growth path for four subperiods (relative per capita GDP)

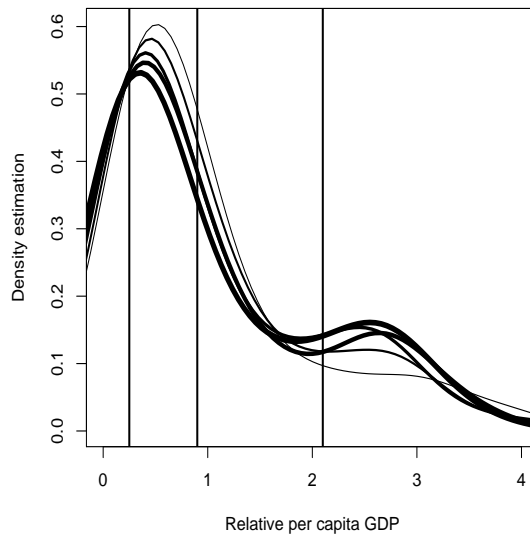


Figure 10: Estimate of density for relative per capita GDP in five selected years

²⁷We consider the restricted sample of 91 countries in order to make reference to the data on investment rates. Hence, the growth path and the density estimation are very similar but not necessarily identical, to those estimated from the full sample in Section 2.

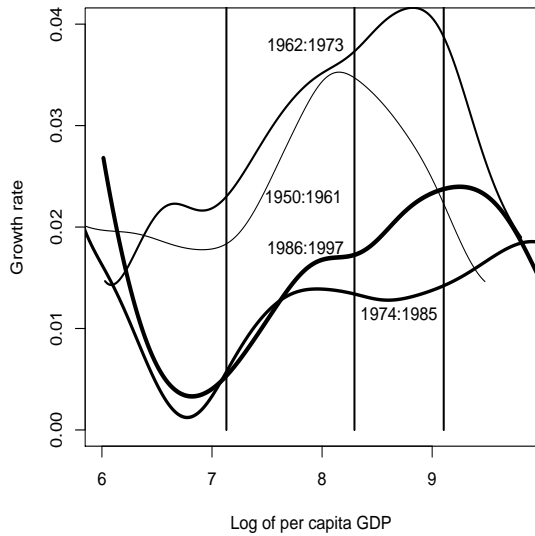


Figure 11: Estimate of growth path for four subperiods (log of per capita GDP)

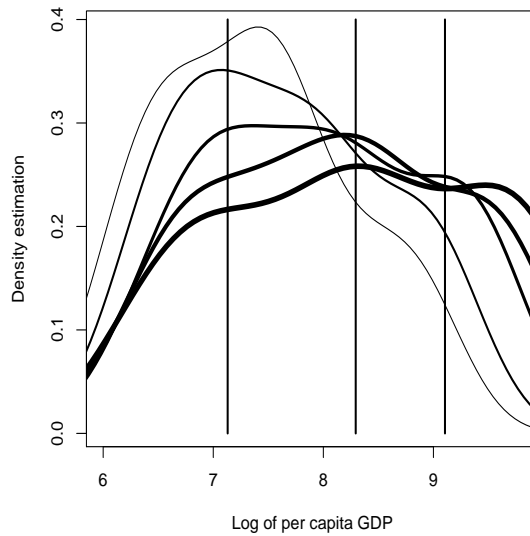


Figure 12: Estimate of density for log of per capita GDP in five selected years

We observe that all the growth paths display an increasing part which ends with a peak in GDP class *III* for relative GDP (see Figure 9); this is less evident for the log of GDP, in which the peak is GDP class *II* in the first subperiod, near to the limit of GDP class *III*, and then moves into GDP class *III* (see Figure 11). A partial exception is the growth path in the period 1974 : 1985, which becomes rather flat after reaching a peak in GDP class *II*. It is remarkable that, in the last subperiod, the path shifts upwards, and becomes more similar to the paths of the first two subperiods. We take this as indicating that some of the causes of the slowdown had been removed in 1986 : 1997.

The evolution of the world income distribution displays the following features:²⁸ in relative terms, the distribution becomes twin peaked over time (see Figure 10). Notice that in 1973 there appears a marked increase in the peak in GDP class *IV*, which subsequently stabilizes. The distribution of the log of GDP shows a sharp decrease in the masses of GDP classes *I* and *II* in the first two periods (1950 and 1961), coupled with an increase in the mass of GDP classes *III* and *IV*. After 1973 this reduction of the mass in GDP class *I* slows down, and is more evident in GDP class *II*. At the same time, in 1973 there is a more visible increase of the mass in GDP class *IV*, which subsequently increases slowly.

²⁸We did not label the estimates in Figure 10 for clarity of exposition. The thinner line refer to 1950, the thicker to 1997.

In the following section we propose a theoretical model that can account for the evidence presented in Figures 9, 10, 11 and 12.

3.3.1 A Theoretical Model

The model in Eq. (2) can be extended to provide an intuition of the effect of technological transfers among countries with limited appropriability of technology. Consider Eq. (3):

$$\frac{\dot{k}_i}{k_i} = s \frac{f(k_i, B_i)}{k_i} - (\delta + n), \quad (3)$$

where B_i represents an index of knowledge available to country i . We suppose that B_i depends on the technological transfers which country i can benefit from other countries, i.e.:

$$B_i = \mu \left(\vec{A}_i \right),$$

where $\vec{A}_i = (a_i^1, \dots, a_i^{i-1}, 1, a_i^{i+1}, \dots, a_i^N)$ is a ranked vector, i.e. $a_i^{j-1} < a_i^j$, representing the relative level of knowledge of each country with respect to the knowledge of country i ($N > 1$ is the total number of countries).

In the endogenous growth literature it is common to consider per capita capital as a proxy of the level of knowledge accumulated in a country (see Barro and Sala-i-Martin (1999)), so that the vector of relative knowledge for country i becomes:

$$\vec{A}_i = \left(\frac{k_1}{k_i}, \dots, \frac{k_{i-1}}{k_i}, 1, \frac{k_{i+1}}{k_i}, \dots, \frac{k_N}{k_i} \right).$$

Function μ should reflect two crucial properties of models with technological transfers and limited appropriability of technology: (i) a follower gains from technological leader more than the other way around and, (ii) following Basu and Weil (1998), technological spillovers are effective only for similar economies, i.e. with similar k . Accordingly, we assume that:

$$B_i = \mu \left[\frac{\sum_{j \in P} \left(\frac{k_j}{k_i} \right)^\beta}{|P_i|} \right], \quad (4)$$

where $\mu > 0$, $\beta > 1$ and P_i represents the set of countries with which country i interacts; that is:

$$P_i = \left\{ j : \frac{k_j}{k_i} \in (1 - \bar{a}^L, 1 + \bar{a}^H) \right\}. \quad (5)$$

In Eq. (5) we are assuming that both follower and leader countries are reciprocally affected by technological transfers. This means that $1 + \bar{a}^H$ defines the range of the relative capital of the more advanced countries which country i benefits from. On the contrary $1 - \bar{a}^L$ measures the negative effect that less advanced countries have on technological leader's economy (notice that each term $\frac{k_j}{k_i} < 1$ decreases B_i with respect to the case in which country i is not affected by technological transfers). Here we follow the intuition of Goodfriend and McDermott (1998). In their model productivity increases by two sources: (i) learning by doing generated by the local production of intermediate goods and (ii) technological transfers by import of intermediate goods. Imitative activity of laggards, which allows them to produce intermediate goods previously imported from leader countries, reduces the accumulation of knowledge in leader countries and therefore their growth rate of productivity. Parameter $\beta > 1$ measures the advantage of the follower in benefiting from the flow of knowledge of more advanced countries; higher β means higher benefit, but it also measure the disadvantages of the leader form the presence of the follower. This hypothesis differs from Basu and Weil (1998), where a country benefits from both leaders and followers if their capital/labor ration falls within a certain range.²⁹

Finally, μ is now a parameter representing a scale factor: when $P_i = \{i\}$ then $B_i = \mu$. Technological transfers have a positive impact on the level of productivity of a country only if the latter can benefit from a more advanced countries. In the case all countries share the same level of capital it is straightforward to see from (5) that $B_i = \mu$.³⁰ The most favourable case for country i is when all countries N are in P_i and they have a level of k equal to $k_i (1 + \bar{a}^H)$, that is:

$$\vec{A}_i = (1, 1 + \bar{a}^H, \dots, 1 + \bar{a}^H).$$

Then:

$$B_i = \mu \frac{(N-1)(1 + \bar{a}^H)^\beta + 1}{N},$$

which is increasing in N and converges to $\mu (1 + \bar{a}^H)^\beta$ when N increases. The worst case for country i is when it is the leader country and all countries are on the limit of lower range, that is:

$$\vec{A}_i = (1 - \bar{a}^L, \dots, 1 - \bar{a}^L, 1).$$

²⁹In this case the function form of $\mu(\cdot)$ should be U -shaped with a minimum in 1.

³⁰This is another difference with respect to Basu and Weil (1998), where the cardinality of P_i matters. That is, if all countries in P_i are identical, an increase in $|P_i|$ increases productivity of country i . This effect can be easily obtained by raising $|P_i|$ in Eq (4) to an exponent lower than 1.

Then:

$$B_i = \mu \frac{(N-1)(1-\bar{a}^L)^\beta + 1}{N},$$

which is decreasing in N and converges to $\mu(1-\bar{a}^L)^\beta$ when N increases.

In order to match the empirical results we impose the following two conditions on the parameters' values:

1. Possibility of positive long-run growth:

$$s\mu(1-\bar{a}^L)^\beta > \delta + n; \quad (6)$$

2. Possible existence of a poverty trap:

$$\exists \bar{k}_S, \bar{k}_U : s \frac{f(k, \mu)}{k} = \delta + n \text{ for } k = \bar{k}_S, \bar{k}_U; \quad (7)$$

Condition (6) says that the leader country grows at a positive rate even if all other countries are followers (remember that $\lim_{k \rightarrow \infty} \dot{k}_i/k_i = sB_i - (\delta + n)$); Condition (7) says that growth path of a country which does not benefit from technological transfers can display a poverty trap. In this respect let $B^{\min} > \mu$ be the minimum level of productivity for which a poverty trap exists, that is:

$$\frac{f(k, B^{\min})}{k} = \delta + n. \quad (8)$$

We notice that B^{\min} positively depends on δ and n .

Now we suggest an interpretation of the dynamics observed in the period 1950 : 1997 on the basis of our model. For sake of simplicity we assume that in an economy there exist three main sets of countries: the poor, belonging mainly to GDP class *I*, the followers belonging to GDP classes *II* and *III* and the leaders, belonging to GDP class *IV*.

In the first subperiod 1950 : 1961 all countries belonging to these three sets grow at positive but different rates. In particular, the productivity of poor countries B_P is sufficiently high to avoid the appearance of a poverty trap, i.e. $B_P > B^{\min}$, thanks to the technological transfers of follower countries, but their growth rate is low because they are still in the range of low and decreasing productivity of capital. Follower countries show a higher growth rate than the other two sets of countries because they benefit from both technological spillover of leader countries and of the increasing returns of capital. Finally, leader countries are in a phase of decreasing growth rate, but still sustained. Figure 13 illustrates this situation; we remark that the shape of $sf(k, B)/k$ curve is affected by the interaction (proximity) of three

sets of countries (k_P , k_F and k_L indicates possible average values of capital for these three country sets).

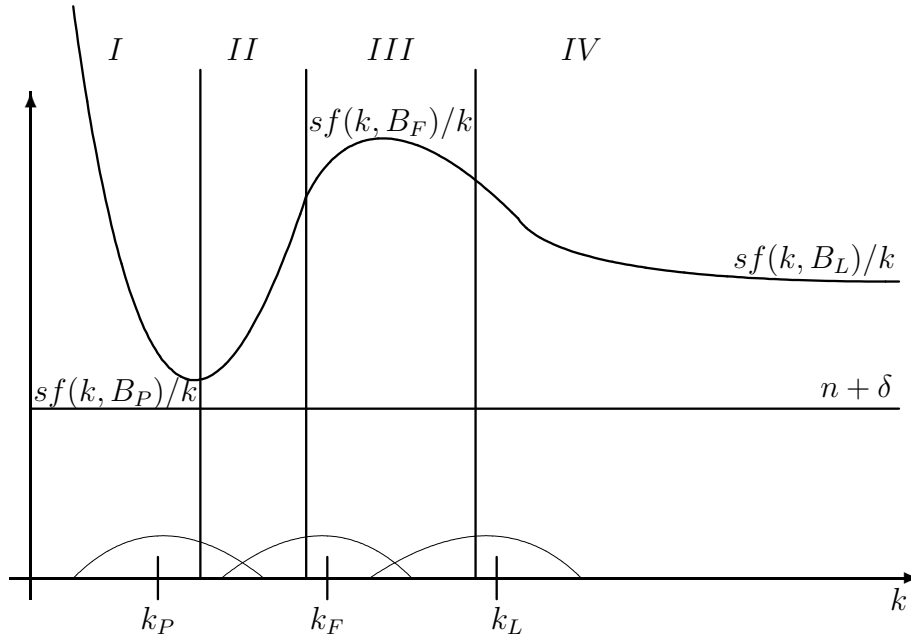


Figure 13: Nonlinear endogenous growth model with technological transfers

The resulting dynamics is compatible with the empirical evidence previously reported: in particular in absolute terms the mass in the first class tends to decrease in absolute terms, while in relative terms it slightly increases due to the higher pace of follower countries (the poorest among follower countries fall in GDP class *I*). Follower countries tend to reduce the gap with leader countries and the mass in GDP classes *II* and *III* tends to decrease in relative terms. In absolute terms, we observe that the densities of 1950 and 1961 cross in GDP class *II*. The mass in 1961 is lower before the intersection and higher after, indicating a global process of positive growth. Figures 10 and 12 broadly confirm this dynamics.

In the second subperiod 1962 : 1973 the dynamics of the first subperiod is consolidated. In absolute terms we have a further decrease in the mass of GDP classes *I* and *II* and an increase of the mass in GDP class *III* (in relative terms this regard only the last part of the range of GDP class *III*, and the first part of the range of GDP class *IV*), caused by an increasing

growth of follower countries. The latter is reflected in an upward shift of growth path in Figures 9 and 11 for all GDP classes, but with particular evidence for GDP class *III*. In terms of our model this means that B_F is increased for the higher number of follower countries which benefit from technological spillover of leader countries or that, alternatively, for a country in GDP classes *II* and *III* the number of countries with a higher k within P has increased (in lower scale this phenomenon also happens between poor and follower countries). Given the positive growth of all countries, in this period the mass in GDP class *IV* increases in absolute terms, and also in relative terms, giving raise to what we have defined as medium-run polarization.

The third subperiod 1974 : 1985 starts with the first oil shock (the second is in 1979). We have already argued that this could have caused a sudden increase in the rate of depreciation of capital δ . This increase could have been so strong that $B_P < B^{\min}$ (see Eq. (8)), that is poor countries can be trapped in a low income equilibrium.

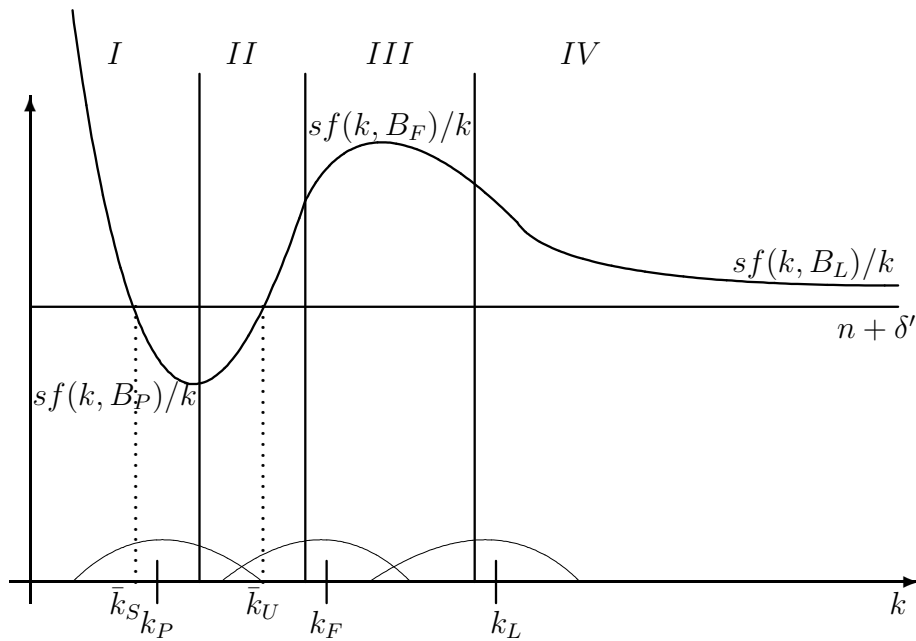


Figure 14: Nonlinear endogenous growth model with technological transfers: the effect of oil shocks

Figure 14 shows the dynamics resulting from an increase in δ . The growth path of all countries shifts down, but this is particularly disastrous for poor

countries because for some of them, i.e. the ones with capital in the range (\bar{k}_S, \bar{k}_U) , this means a decrease in their GDP per capita (they tend to the low equilibrium \bar{k}_S in Figure 14). We conjecture that the negative effects of oil shocks were especially strong for follower countries, whose economies were heavily based on energy intensive technology (see the decrease in growth path in Figures 9 and 11). The distributions of relative and absolute GDP reflect this dynamics showing an increasing polarization. GDP class *I* shows a substantially constant mass in absolute terms and an increasing mass in relative terms. This holds also for GDP class *IV*.³¹ The mass of central GDP classes *II* and *III* shows a reduction in relative terms. In absolute terms the mass in GDP class *II* decreases and the mass in GDP *III* increases, but these changes are less pronounced than in previous years, indicating that the process of growth was slowed, but not checked. We conjecture that the increase in polarization caused a reduction in B_P , that is of the rate of technological transfers to poor countries.

The fourth subperiod 1986 : 1997 witnesses a decrease in the rate of depreciation δ according to Baily and Schultze (1990). However, the decrease in B_p was so strong that B_p is still below B^{\min} (even if B^{\min} has decreased with δ , and therefore poor countries remain trapped in the low income equilibrium \bar{k}_S). We find evidence of this fact in the behavior of the growth path in Figures 9 and 11: the path is basically invariant for GDP class *I* for both relative and absolute GDP, and for absolute GDP this also hold for the first part of the path in GDP class *II*. On the contrary, the growth path is strongly shifting upwards for absolute and relative GDP class *III*. Finally, leader countries in GDP class *IV* show a lower increases than follower countries in their growth path, maybe due to the negative effect of new follower countries, as argued by Goodfriend and McDermott (1998). This, coupled with a possible negative effect on productivity due to the strong decreasing in investment rate, may explain why productivity in rich countries as not completely recovered.

4 Conclusions

The pieces of evidence presented in this paper provide some support to the following hypothesis: before 1973 a tendency for convergence characterized the world income distribution, with all countries following a nonlinear path toward the highest GDP class. Polarization in this case was a temporary

³¹Notice that, in absolute terms, the mass in GDP classes *I* and *II* does not change much from 1973 to 1986, when compared with the changes occurred between previous years.

phenomenon. In particular, the internal process of accumulation was the key factor for the nonlinear shape of the path, while international spillovers allowed productivity to be sufficiently high to avoid poverty traps. In 1973 a shock reduced productivity in the aggregate, in particular shifting downwards the growth path. This may have been caused by a sudden increase in the rate of obsolescence of capital. The post-1973 years have been characterized by a generalized decrease in the parameter B , even if the reasons behind a reduction in B may differ across countries, in particular because of changes in the pattern of international transfer of technologies.

Given the nonlinear shape, the temporary shock on δ has produced permanent effects as, for low-income countries, this has meant the appearance of a low development trap. Countries started to fall apart and, as suggested by the literature on appropriate technology, the international flow of technologies was interrupted, or at least reduced. The evolution in the subsequent period witnessed a tendency for richer countries to isolate from the rest. At this point the internal factors of accumulation become more important, in the sense that laggards could not benefit from the technologies developed in the richest countries.

The absence of recovery in terms of productivity may find an explanation in the following facts: richest countries reduced their investment rates which may have decreased the rate of innovations at the technological frontier. In fact, a reduction in inventive activity in the US is quoted as one possible source of the slowdown (see Fischer (1988)). A higher spread in the world income distribution may be responsible for a difficult transmission of technologies across countries. Notice that the growth path in 1986-1997 becomes steeper, indicating that it is possible that a cause of the slowdown has been removed, like the increase in the depreciation rate. Baily and Schultze (1990), p. 397, notice that Tobin's q in the US fell well below unity after the first shock, indicating a low evaluation of installed capital (and therefore an increase in the rate of obsolescence), while in 1987 it reached a value of 0.91, indicating the reversal of the previous tendency. However, the path did not return to previous levels, in particular leaving room for a poverty trap and signalling that some extra forces were still at work.

Future research should demonstrate the robustness of these results, which in some cases appear more as speculations from the proposed perspective on the productivity slowdown, than facts grounded in empirical evidence. For instance we could adopt, a more disaggregated view of the countries in the sample like OECD countries, for which it should be possible to analyze in more details the process transfer of technology.

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A Country lists

Table 13 contains the sample of countries from Maddison (2001).

Country			
Algeria	Sierra Leone	Paraguay	Burma
Angola	Somalia	Puerto Rico	Hong Kong
Benin	South Africa	Trinidad Tobago	Malaysia
Botswana	Sudan	Australia	Nepal
Cameroon	Swaziland	New Zealand	Pakistan
Cape Verde	Tanzania	Canada	Singapore
Central African Republic	Togo	United States	Sri Lanka
Chad	Tunisia	Bahrain	Afghanistan
Comoros	Uganda	Iran	Cambodia
Congo	Zambia	Iraq	Laos
Côte d'Ivoire	Zimbabwe	Israel	Mongolia
Djibouti	Argentina	Jordan	North Korea
Egypt	Brazil	Kuwait	Vietnam
Gabon	Chile	Lebanon	Austria
Gambia	Colombia	Oman	Belgium
Ghana	Mexico	Qatar	Denmark
Kenya	Peru	Saudi Arabia	Finland
Liberia	Uruguay	Syria	France
Madagascar	Venezuela	Turkey	Germany
Mali	Bolivia	UAE	Italy
Mauritania	Costa Rica	Yemen	Netherlands
Mauritius	Cuba	West Bank Gaza	Norway
Morocco	Dominican Republic	China	Sweden
Mozambique	Ecuador	India	Switzerland
Namibia	El Salvador	Indonesia	UK
Niger	Guatemala	Japan	Ireland
Nigeria	Haiti	Philippines	Greece
Reunion	Honduras	South Korea	Portugal
Rwanda	Jamaica	Thailand	Spain
Senegal	Nicaragua	Taiwan	
Seychelles	Panama	Bangladesh	

Table 13: Sample from Maddison (2001): 122 countries

Table 14 contains the restricted sample of 91 countries.

Country			
Benin	Senegal	Jamaica	Malaysia
Botswana	Seychelles	Nicaragua	Nepal
Cameroon	South Africa	Panama	Pakistan
Cape Verde	Tanzania	Paraguay	Sri Lanka
Central African Republic	Togo	Trinidad Tobago	Austria
Chad	Tunisia	Australia	Belgium
Comoros	Uganda	New Zealand	Denmark
Congo	Zambia	Canada	Finland
Côte d'Ivoire	Zimbabwe	United States	France
Egypt	Argentina	Iran	Italy
Gabon	Brazil	Israel	Netherlands
Gambia	Chile	Jordan	Norway
Ghana	Colombia	Syria	Sweden
Kenya	Mexico	Turkey	Switzerland
Madagascar	Peru	China	UK
Mali	Uruguay	India	Ireland
Mauritania	Venezuela	Indonesia	Greece
Mauritius	Bolivia	Japan	Portugal
Morocco	Costa Rica	Philippines	Spain
Mozambique	Dominican Republic	South Korea	
Namibia	Ecuador	Thailand	
Niger	El Salvador	Taiwan	
Nigeria	Guatemala	Bangladesh	
Rwanda	Honduras	Hong Kong	

Table 14: Sample from Maddison (2001) and PWT 6.1: 91 countries

B Transition Matrices: full sample

In this appendix we report the transition matrices computed for the full sample from Maddison (2001), in both absolute and relative values.

B.1 Absolute values

We present first the results for the first period (1950-1973) and then for the second (1974-1997).

	Obs	I-	I+	I++	II-	II+	II++	III-	III+	III++	IV-	IV+	IV++
I-	444	0.59	0.15	0.23	0.01	0.01	0.01	0	0	0	0	0	0
I+	264	0.37	0.39	0.20	0.02	0.02	0.01	0	0	0	0	0	0
I++	250	0.34	0.20	0.32	0.02	0.02	0.11	0	0	0	0	0	0
II-	377	0.01	0	0	0.45	0.23	0.29	0.01	0	0.02	0	0	0
II+	256	0	0	0	0.36	0.32	0.27	0.01	0.03	0.02	0	0	0
II++	375	0	0	0	0.23	0.18	0.47	0.01	0.01	0.10	0	0	0
III-	87	0	0	0	0	0	0.01	0.32	0.20	0.40	0.02	0.02	0.02
III+	89	0	0	0	0	0	0	0.18	0.25	0.39	0.03	0.02	0.12
III++	198	0	0	0	0	0	0	0.17	0.17	0.51	0.03	0.03	0.10
IV-	79	0	0	0	0	0	0	0	0	0	0.52	0.14	0.34
IV+	52	0	0	0	0	0	0	0	0	0	0.38	0.29	0.33
IV++	85	0	0	0	0	0	0	0	0	0	0.24	0.32	0.45

Table 15: transition matrix 1950-1973, 122 countries, absolute values

	I-	I+	I++	II-	II+	II++	III-	III+	III++	IV-	IV+	IV++
ergodic	0	0	0	0	0	0	0	0	0	0.38	0.24	0.38

Table 16: ergodic distribution 1950-1973, 122 countries, absolute values

	Obs	I-	I+	I++	II-	II+	II++	III-	III+	III++	IV-	IV+	IV++
I-	258	0.36	0.33	0.29	0	0.01	0	0	0	0	0	0	0
I+	210	0.33	0.30	0.35	0	0	0.01	0	0	0	0	0	0
I++	281	0.22	0.25	0.41	0.03	0.02	0.07	0	0	0	0	0	0
II-	227	0.07	0.05	0.04	0.33	0.28	0.21	0	0	0	0	0	0
II+	207	0.01	0	0	0.24	0.35	0.34	0.01	0.02	0.02	0	0	0
II++	372	0	0	0	0.17	0.17	0.52	0.04	0.04	0.06	0	0	0
III-	137	0	0	0	0.06	0.07	0.05	0.20	0.21	0.40	0	0	0.01
III+	125	0	0	0	0.02	0.01	0.02	0.22	0.23	0.45	0.01	0.02	0.03
III++	264	0	0	0	0	0	0	0.21	0.18	0.48	0.03	0.02	0.08
IV-	107	0	0	0	0	0	0	0.09	0.06	0.04	0.24	0.21	0.36
IV+	189	0	0	0	0	0	0	0	0	0	0.11	0.38	0.51
IV++	307	0	0	0	0	0	0	0	0	0	0.14	0.29	0.56

Table 17: transition matrix 1974-1997, 122 countries, absolute values

	I-	I+	I++	II-	II+	II++	III-	III+	III++	IV-	IV+	IV++
ergodic	0.05	0.05	0.06	0.04	0.04	0.07	0.05	0.05	0.10	0.08	0.15	0.27

Table 18: ergodic distribution 1974-1997, 122 countries, absolute values

B.2 Relative values

We present first the results for the first period (1950-1973) and then for the second (1974-1997).

	Obs	I-	I+	I++	II-	II+	II++	III-	III+	III++	IV-	IV+	IV++
I-	261	0.58	0.18	0.22	0.01	0	0.01	0	0	0	0	0	0
I+	168	0.36	0.43	0.18	0.01	0.01	0.01	0	0	0	0	0	0
I++	141	0.37	0.18	0.32	0.05	0.02	0.06	0	0	0	0	0	0
II-	505	0.04	0.01	0.02	0.48	0.20	0.25	0	0	0	0	0	0
II+	309	0.02	0	0.01	0.37	0.31	0.26	0.01	0.01	0.01	0	0	0
II++	424	0	0	0	0.24	0.18	0.48	0.02	0.01	0.05	0	0	0
III-	108	0	0	0	0.08	0.03	0.05	0.33	0.13	0.37	0	0	0.01
III+	97	0	0	0	0.01	0.03	0	0.23	0.34	0.36	0	0.01	0.02
III++	221	0	0	0	0.02	0	0	0.15	0.14	0.60	0.01	0.03	0.05
IV-	113	0	0	0	0	0	0	0	0	0	0.44	0.19	0.37
IV+	87	0	0	0	0	0	0	0	0	0.01	0.33	0.24	0.41
IV++	128	0	0	0	0	0	0	0	0	0	0.25	0.31	0.44

Table 19: transition matrix 1950-1973, 122 countries, relative values

	I-	I+	I++	II-	II+	II++	III-	III+	III++	IV-	IV+	IV++
ergodic	0.02	0.01	0.01	0.02	0.01	0.02	0.01	0.01	0.02	0.29	0.22	0.35

Table 20: ergodic distribution 1950-1973, 122 countries, relative values

	Obs	I-	I+	I++	II-	II+	II++	III-	III+	III++	IV-	IV+	IV++
I-	286	0.36	0.34	0.29	0.01	0	0	0	0	0	0	0	0
I+	220	0.33	0.30	0.36	0	0	0	0	0	0	0	0	0
I++	299	0.23	0.25	0.45	0.01	0.01	0.05	0	0	0	0	0	0
II-	254	0.10	0.02	0.02	0.27	0.30	0.28	0	0	0	0	0	0
II+	257	0.01	0.01	0.01	0.25	0.35	0.35	0.01	0	0.02	0	0	0
III+	451	0.01	0	0	0.20	0.20	0.50	0.02	0.02	0.05	0	0	0
III-	106	0	0	0	0.07	0.04	0.08	0.28	0.20	0.34	0	0	0
III+	98	0	0	0	0.05	0	0.03	0.12	0.26	0.49	0	0.03	0.02
III++	235	0	0	0	0	0	0.01	0.18	0.16	0.56	0.02	0.02	0.05
IV-	83	0	0	0	0	0	0	0.11	0.04	0.05	0.19	0.23	0.39
IV+	156	0	0	0	0	0	0	0.01	0	0.01	0.11	0.37	0.49
IV++	239	0	0	0	0	0	0	0.01	0	0.01	0.13	0.32	0.53

Table 21: transition matrix 1974-1997, 122 countries, relative values

	I-	I+	I++	II-	II+	II++	III-	III+	III++	IV-	IV+	IV++
ergodic	0.12	0.11	0.14	0.06	0.07	0.11	0.04	0.03	0.09	0.03	0.08	0.12

Table 22: ergodic distribution 1974-1997, 122 countries, relative values

B.3 Relative values to US

\Growth rate 1950-73	< 1.6%	1.6%, 3.6%	> 3.6%
GDP\Growth rate 1974-97	< -0.1%	-0.1%, 1.9%	> 1.9%
0 - 0.05	I-	I+	I++
0.05 - 0.18	II-	II+	II++
0.18 - 0.65	III-	II+	III++
> 0.65	IV-	IV+	IV++

Table 23: state space definition for relative US GDP

FIRST PERIOD: 1950:1973

	I-	I+	I++	II-	II+	II++	III-	III+	III++	IV-	IV+	IV++
ergodic	0.02	0.01	0.01	0.09	0.05	0.06	0.07	0.06	0.13	0.18	0.12	0.20

Table 24: ergodic distribution 1950-1973, relative to US GDP

	-	+	++
I	0.50	0.26	0.24
II	0.45	0.24	0.31
III	0.29	0.21	0.50
IV	0.35	0.24	0.40

Table 25: ergodic distribution normalized for each GDP class, 1950-1973, relative to US GDP

	<i>I</i>	<i>II</i>	<i>III</i>	<i>IV</i>
1950	0.07	0.49	0.34	0.10
1973	0.11	0.42	0.31	0.16
ergodic	0.05	0.20	0.26	0.50

Table 26: distribution dynamics, 1950-1973, relative to US GDP

SECOND PERIOD: 1974:1997

	I-	I+	I++	II-	II+	II++	III-	III+	III++	IV-	IV+	IV++
ergodic	0.23	0.16	0.22	0.08	0.05	0.11	0.03	0.02	0.06	0.01	0.01	0.02

Table 27: ergodic distribution 1974-1997, relative to US GDP

	-	+	++
I	0.38	0.26	0.36
II	0.33	0.21	0.46
III	0.31	0.15	0.54
IV	0.17	0.30	0.53

Table 28: ergodic distribution normalized for each GDP class, 1974-97, relative to US GDP

	<i>I</i>	<i>II</i>	<i>III</i>	<i>IV</i>
1973	0.11	0.42	0.31	0.16
1997	0.30	0.30	0.26	0.15
ergodic	0.61	0.25	0.10	0.04

Table 29: distribution dynamics, 1974-1997, relative to US GDP

C Data on per worker GDP

In this appendix we present the results on labor productivity, using data from the Penn World Tables 6.1 for a restricted sample of 91 countries for the period 1961 – 1997. Growth and GDP classes are calculated applying the same procedure used for relative and absolute GDP.

C.1 Absolute per worker GDP

\Growth rate 1961-73	< 2.1%	2.1%, 4.1%	> 4.1%
GDP\Growth rate 1974-97	< 0.1%	0.1%, 2.1%	> 2.1%
0 – 3330	I-	I+	I++
3300 – 6500	II-	II+	II++
6500 – 22000	III-	II+	III++
> 22000	IV-	IV+	IV++

Table 30: state space definition for absolute per worker GDP

FIRST PERIOD: 1961:1973

	Obs	I-	I+	I++	II-	II+	II++	III-	III+	III++	IV-	IV+	IV++
I-	106	0.49	0.11	0.32	0.03	0.00	0.05	0.00	0.00	0.00	0.00	0.00	0.00
I+	40	0.42	0.13	0.35	0.07	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
I++	95	0.35	0.14	0.29	0.15	0.02	0.05	0.00	0.00	0.00	0.00	0.00	0.00
II-	79	0.01	0.03	0.05	0.51	0.09	0.18	0.05	0.03	0.06	0.00	0.00	0.00
II+	18	0.00	0.00	0.00	0.50	0.06	0.17	0.00	0.17	0.11	0.00	0.00	0.00
II++	60	0.02	0.00	0.00	0.23	0.07	0.35	0.12	0.02	0.20	0.00	0.00	0.00
III-	122	0.00	0.00	0.00	0.01	0.00	0.00	0.39	0.24	0.31	0.02	0.02	0.02
III+	88	0.00	0.00	0.00	0.01	0.00	0.01	0.32	0.26	0.33	0.01	0.01	0.05
III++	163	0.00	0.00	0.00	0.00	0.00	0.00	0.26	0.20	0.44	0.02	0.03	0.05
IV-	44	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.41	0.34	0.25
IV+	49	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.43	0.24
IV++	46	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.46	0.22

Table 31: transition matrix 1961-1973, 91 countries, per worker GDP

The associated ergodic distribution is:

	I-	I+	I++	II-	II+	II++	III-	III+	III++	IV-	IV+	IV++
ergodic	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.36	0.40	0.24

Table 32: ergodic distribution 1961-1973, 91 countries, per worker GDP

	<i>I</i>	<i>II</i>	<i>III</i>	<i>IV</i>
1961	0.31	0.21	0.38	0.10
1973	0.20	0.16	0.42	0.22
ergodic	0	0	0	1.00

Table 33: distribution dynamics 1961-1973, 91 countries, per worker GDP

SECOND PERIOD: 1974:1997

	Obs	I-	I+	I++	II-	II+	II++	III-	III+	III++	IV-	IV+	IV++
I-	207	0.54	0.15	0.29	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
I+	55	0.53	0.20	0.27	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
I++	141	0.32	0.16	0.40	0.04	0.00	0.08	0.00	0.00	0.00	0.00	0.00	0.00
II-	106	0.12	0.02	0.06	0.44	0.09	0.23	0.02	0.01	0.01	0.00	0.00	0.00
II+	36	0.03	0.00	0.00	0.22	0.19	0.42	0.03	0.06	0.06	0.00	0.00	0.00
II++	125	0.00	0.00	0.01	0.18	0.13	0.49	0.03	0.02	0.14	0.00	0.00	0.00
III-	269	0.00	0.00	0.00	0.03	0.00	0.01	0.38	0.21	0.36	0.00	0.00	0.01
III+	159	0.00	0.00	0.00	0.01	0.01	0.00	0.37	0.24	0.31	0.02	0.00	0.04
III++	361	0.00	0.00	0.00	0.01	0.00	0.00	0.29	0.16	0.46	0.01	0.01	0.07
IV-	175	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.01	0.02	0.31	0.23	0.39
IV+	129	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.26	0.27	0.46
IV++	239	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.27	0.28	0.44

Table 34: transition matrix 1974-1997, PWT, 91 countries, per worker GDP

	I-	I+	I++	II-	II+	II++	III-	III+	III++	IV-	IV+	IV++
ergodic	0.05	0.02	0.03	0.02	0.01	0.02	0.11	0.06	0.12	0.16	0.15	0.26

Table 35: ergodic distribution 1974-1997, PWT, 91 countries, per worker GDP

	-	+	++
I	0.49	0.16	0.34
II	0.44	0.13	0.43
III	0.38	0.21	0.42
IV	0.28	0.27	0.45

Table 36: ergodic distribution normalized for each GDP class, PWT, 91 countries 1974-97, per worker GDP

	<i>I</i>	<i>II</i>	<i>III</i>	<i>IV</i>
1973	0.20	0.16	0.42	0.22
1997	0.21	0.10	0.35	0.34
ergodic	0.09	0.05	0.29	0.57

Table 37: distribution dynamics, PWT, 91 countries, 1974-1997, per worker GDP

C.2 Relative per worker GDP

\Growth rate 1961-73	< 2.1%	2.1%, 4.1%	> 4.1%
GDP\Growth rate 1974-97	< 0.1%	0.1%, 2.1%	> 2.1%
0 – 0.25	I-	I+	I++
0.25 – 0.6	II-	II+	II++
0.6 – 1.8	III-	III+	III++
> 1.8	IV-	IV+	IV++

Table 38: state space definition for relative per worker GDP

FIRST PERIOD: 1961:1973

	I-	I+	I++	II-	II+	II++	III-	III+	III++	IV-	IV+	IV++
ergodic	0.08	0.03	0.06	0.06	0.01	0.05	0.10	0.07	0.12	0.14	0.16	0.11

Table 39: ergodic distribution 1961-1973, PWT, 91 countries, relative per worker GDP

	-	+	++
I	0.49	0.16	0.36
II	0.48	0.11	0.41
III	0.36	0.24	0.41
IV	0.34	0.39	0.27

Table 40: ergodic distribution normalized for each GDP class, PWT, 91 countries 1961-73, per worker GDP

	<i>I</i>	<i>II</i>	<i>III</i>	<i>IV</i>
1961	0.18	0.27	0.36	0.19
1973	0.22	0.19	0.37	0.22
ergodic	0.17	0.12	0.29	0.42

Table 41: distribution dynamics, PWT, 91 countries, 1974-1997, per worker GDP

SECOND PERIOD: 1974:1997

	I-	I+	I++	II-	II+	II++	III-	III+	III++	IV-	IV+	IV++
ergodic	0.13	0.04	0.10	0.07	0.03	0.08	0.10	0.06	0.14	0.07	0.07	0.11

Table 42: ergodic distribution 1974-1997, PWT, 91 countries, relative per worker GDP

	-	+	++
I	0.49	0.15	0.35
II	0.39	0.18	0.43
III	0.34	0.20	0.46
IV	0.28	0.28	0.44

Table 43: ergodic distribution normalized for each GDP class, PWT, 91 countries 1974-97, relative per worker GDP

	<i>I</i>	<i>II</i>	<i>III</i>	<i>IV</i>
1973	0.22	0.19	0.37	0.22
1997	0.24	0.18	0.34	0.24
ergodic	0.27	0.18	0.30	0.25

Table 44: distribution dynamics, PWT, 91 countries, 1974-1997, relative per worker GDP