The Political Economy and the Interaction between Endogenous Fertility and Inequality

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Abstract

We simulate a two period olg-model with heterogeneous agents. Parents receive utility from quantity and quality of their offspring. Generating a trade-off between the former and the latter, an increasing wage rate leads to higher opportunity costs, lower fertility, and higher quality of the children. All this leads to an intergenerational persistence in fertility decisions and wages. We show that growth increases inequality and fertility differentials controlled for the initial distribution of wealth. Furthermore, we endogenize redistribution by implementing a median voter-system. Due to fertility differentials the median-voter moves from upper to lower income percentiles.
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1 Introduction

Existing literature analyzes the decline of fertility as an endogenous phenomenon based on Becker (1960). In this setting children show up as durable goods in the utility function of their parents. Surely, pioneering works in this field are Barro/Becker (1988,1989), Becker/Murphy/Tamura (1990).

A unified framework about the transition from a Malthusian trap to a modern growth path with increasing per capita income and declining fertility is offered by Galor/Weil (2000). As soon as children's quality is considered the problem of the well known quality quantity trade-off (Becker/Lewis (1973)) appears. As far as fertility decisions are concerned we consider the opportunity costs of rearing a child as the economic driving force (for empirical evidence see Kögel (2004)), which lead to the decline of fertility in recent decades (similar Galor/Weil (1996)). If fertility is negatively related to the wage rate and a high wage rate in turn is associated with a high quality per child, it is very reasonable to analyze the latter in a macroeconomic setting with income heterogeneities. Recent research suggests that there is a positive correlation between inequality and fertility differentials which is meassured by the Total Fertility Rate by women's years of education. Kremer and Chen (2000) find that changing from a relatively equal country as Indonesia (Gini=0.32) to a country with higher inequality like Brazil (Gini=0.545) would lead to an increase in the fertility differential by 0.020. This result can be interpreted as follows. Comparing two women with ten and zero years of scholling means that the relation of the expected number of children between both women is about 1.22 children higher in Brazil than in Indonesia.

De la Croix/Doepke (2003) close the link between inequality, fertility differentials and growth. They find empirical evidence for a growth harming effect of fertility differentials based on educational differentials. There, as well as in Dahan/Tsiddon (1999) and Morand (1999) inequality is introduced by the initial distribution of human capital. The steady state however is characterized by an equal distribution of human capital. Hence, there is no scope for redistributional policies.

We in turn argue, building on Schäfer (2003,2002), similar as Galor/Zaira (1993), Galor/Zang (1997), Persson/Tabellini (1994), Benabou (1996) that the distribution of wealth in an economy is essential for the overall growth performance. Furthermore, given the findings by Kremer/Chen (2000) and De la Croix/Doepke (2003), we find that inequality is not only growth harming, but also leading to increasing redistributional pressure during the transition to the steady state. Hence, due to differential fertility the median voter moves from upper to lower income percentiles and votes for more redistribution given his/her preferences for equality in the economy.

As it seems that recent findings provide little support for the Kuznets-curve stating that growth determines inequality (Anhand/ Kanbur (1993) and Deininger/Squire (1998)) the question is how the reversed causality relates inequality to growth. [1] Forbes (2000) finds a positve relationship (within a country) between inequality and growth, at least in short and medium term, while the coefficient gets insignifcant for long-run considerations. This result, however, does not apply for very poor countries as they were excluded from the data set. Furthermore, her finding does not contradict any finding that there exists a long-run negative relationship between inequality and growth. In fact, most of the theoretical channels suggesting a negative relationship would have an effect only over longer period of time (Galor/Zeira (1993) - capital market imperfection, Alesina/Rodrik, Persson/Tabellini (1994), Perotti (1996) and Rodrik (1997) - political economy and social conflicts). As we considere a two-period olg-framework with endogenous fertility and therefore a period-length between 30 and 40 years, we derive definetly rather implications for the long-run performance of an economy.

Given that labor supply is endogenous, in a stochastic growth setting, we are forced to apply numerical solution methods. More in detail, we analyze a two period OLG-model. Parents receive utility from quality and quantity (measured

[1] Barro (2000) finds evidence for the Kusnetz-Curve, but most likely driven by transition countries (Ferreira (1999)).

in a wide definition of wealth) of their offspring. We show that the amount of wealth which includes human and physical capital bequested to the next generation leads to an intergenerational persistence in wage rates and fertility decisions. We find, that growth leads to higher inequality and (possibly) higher fertility differentials. Higher initial inequality slows down growth and increases transitory inequality leading to redistributional pressure. When a redistributional tax scheme is introduced, we show that redistribution lowers inequality and increases average growth. Fertility differentials are closing. The effect of long-run fertility differentials, however is non-monotonous
In a last step we close the missing link between growth, inequality, fertility and the political economy by implementing a median voter system. Given that relatively poorer people have a higher fertility, redistrbiutional pressure must be the consequence. The amount of redistribution depends on the median's preferences for equality. If he/she prefers a relatively low tax-rate, redistribution of wealth in terms of future opportunities is low and the subsequent growth rate is also low, as inequality is high.

2 The Model

We consider an economy populated by a continuum of households which are assumed to live for exactly two periods, childhood and adulthood. During childhood each child consumes a constant fraction z out of its parents' time budget

$$zn_t^{t-1} + l_t^{t-1} = 1. \tag{1}$$

Consequently, the decision between the number of children and labor supply l_t^{t-1} is done simultaneously. Children are priced by the foregone wage income during child-rearing time $w_t z n_t^{t-1}$. When adult each individual receives bequests x_{t-1}^{t-1} from his/her parents and utility from the number of children n_t^{t-1}, consumption c_t^{t-1} and x_t^t, whereas x_t^t represents the amount of wealth per child left as bequest. For further interpretations it is important to consider x as a wide defined capital stock. It includes physical and human capital or in other words education as well. Therefore, the amount of bequest x has to be interpreted as the sum of efforts undertaken by parents in order to equip their offspring with quality in terms of future opportunities.

The production technology is common knowledge, such that each adult has access to the following production technology

$$y_t^{t-1} = B {x_{t-1}^{t-1}}^{\alpha} {l_t^{t-1}}^{1-\alpha}. \tag{2}$$

In addition, each household produces an exogenously given amount of home produced goods h.

The optimization problem of household i is given by

$$\max u_t^{t-1,i} = \gamma \ln n_t^{t-1,i} + \beta \ln x_t^{t,i} + \delta \ln c_t^{t-1,i}, \tag{3}$$

subject to: [2]

$$\underbrace{y_t^{t-1,i} + h}_{I_t^{t-1,i}} = w_t^{t-1,i} z n_t^{t-1,i} + c_t^{t-1,i} + n_t^{t-1,i} x_t^{t,i}, \tag{4}$$

$$\text{and } z n_t^{t-1,i} + l_t^{t-1,i} = 1,$$

$$\text{where } y_t^{t-1,i} = \frac{\partial y_t^{t-1,i}}{\partial l_t^{t-1,i}} l_t^{t-1,i} + \frac{\partial y_t^{t-1,i}}{\partial x_{t-1}} x_{t-1}.$$

Solving the problem leads to optimal decisions

$$c_t^{t-1,i} = \frac{\delta}{\gamma + \delta} I_t^{t-1,i} \tag{5}$$

$$n_t^{t-1,i} = \frac{\gamma - \beta}{\delta + \gamma} \frac{I_t^{t-1,i}}{z w_t^{t-1,i}} \tag{6}$$

$$x_t^{t,i} = \frac{\beta}{\gamma - \beta} z w_t^{t-1,i}, \quad \text{with } \gamma > \beta. \tag{7}$$

It becomes apparent from Eq.(6) and (7) that the model is characterized by a quality quantity trade-off between the number of children and the amount of wealth. Parents with a high amount of wealth receive a high wage income, have high opportunity costs of child rearing, hence a low fertility and leave a high amount of wealth as bequests for their offspring. All this leads to an intergenerational persistence in fertility decisions and wealth.

$$n_t^{t-1,i} = \varphi(x_{t-1}^{t-1,i}), \quad \text{with } \frac{\partial \varphi}{\partial x_{t-1}^{t-1,i}} < 0, \tag{8}$$

and

$$w_t^{t-1,i} \quad = \quad w_t(x_{t-1}), \quad \text{with } \frac{\partial w_t}{\partial x_{t-1}^{t-1,i}} > 0. \tag{9}$$

Consequently, the economy converges as a whole with declining fertility and increasing wages towards its steady state. Whereas the steady state is characterized by zero growth in per capita terms and constant fertility

$$n_t^{t-1,i} = n^{i^*} \quad \text{and} \quad x_t^{t,i} = x^{i^*} \qquad \forall \quad t, i. \tag{10}$$

If this is true it is reasonable to examine how heterogeneities in wealth and abilities affect the outcome of the model.

3 Heterogeneities in Terms of Wealth and Abilities

Heterogeneities are assumed to work through two distinct channels. First, the initial distribution of wealth in a society, for example the constitution of a society or norms concerning equity in living standards which distribute landownership, access to education or any other accumulateable asset, important for production. For reasons of tractability and coinciding with empirical findings we follow the common assumption that wealth is log-normal distributed. The advantage of this specification is that the moments of the distribution are described by μ and σ. Furthermore, for any log-normal distributed random variable, it must be true that the logarithm of this variable is normal distributed. The initial distribution of wealth is therefore given by

$$x_{t=0}^i \quad \sim \quad LN(\mu_{t=0}^x, \sigma_{t=0}^x), \tag{11}$$

or equivalently

$$\ln x_{t=0}^i \quad \sim \quad N(\mu_{t=0}^x, \sigma_{t=0}^x). \tag{12}$$

Consequently, the distribution and its moments are completely described by μ^x and σ^x, such that

$$
\begin{aligned}
E[x^i_{t=0}] &= \exp(\mu^x_{t=0} + \frac{\sigma^{x^2}_{t=0}}{2}), && (13) \\
Var[x^i_{t=0}] &= \exp(2\mu^x_{t=0} + \sigma^{x^2}_{t=0})(\exp(\sigma^{x^2}_{t=0}) - 1). && (14)
\end{aligned}
$$

Second, we allow for some social mobility (even in the steady state) by assuming that intellectual ability ϵ^i is log-normal distributed as well (Loury (1981)). Contrary to the distribution of wealth, the distribution of abilities does not change over time

$$\epsilon^i_t \sim LN(\mu^\epsilon, \sigma^\epsilon), \qquad \text{with } \mu^\epsilon = 0 \text{ and } \sigma^\epsilon = \text{const.} \ \forall \ t. \tag{15}$$

The production technology is common knowledge, such that each adult i born in $t-1$ has access to the following production technology

$$y^{t-1,i}_t = \epsilon^i_t B x^{t-1,i^\alpha}_{t-1} l^{t-1,i^{1-\alpha}}_t. \tag{16}$$

Individual wealth develops according to (7) as

$$x^{t,i}_t = \frac{\beta}{\gamma - \beta} z w^{t-1,i}_t. \tag{17}$$

The received bequests in period t are a function of the parental wage rate, which in turn is a positive function of their received bequests and intellectual ability. Substituting for the wage rate and taking logarithms, leads to

$$\ln x^i_t = \underbrace{\ln[\frac{\beta}{\gamma - \beta}] + \ln z + \ln B + \ln(1-\alpha)}_{C} + \ln \epsilon^i_t + \alpha \ln x^i_{t-1} - \alpha \ln l^i_t. \tag{18}$$

Obviously, the amount of wealth per child is governed by the parental ability term and their endowment of wealth. As the ability shock translates into endogenous variables of the model the assumption of uncorrelated abilities over time is not as limiting as it might appear. Parents with a low ability will leave lower bequests. Therefore, their children will do worse, independent from their own ability.

Developing the expectation value and the variance over Eq.(18) leads to two difference equations governing the development of the moments of the wealth distribution

$$E[\ln x_t^i] = \mu_t^x = \ln C + \alpha\mu_{t-1}^x - \alpha\mu_t^l, \tag{19}$$
$$Var[\ln x_t^i] = \sigma_t^{x^2} = \sigma^{\epsilon^2} + \alpha^2(\sigma_{t-1}^{x^2} + \sigma_t^{l^2}), \tag{20}$$

with

$$E[\ln l_t^i] = \mu_t^l = g(\mu_{t-1}^x) + \frac{1}{2}\sigma_t^{x^2} g''(\mu_{t-1}^x), \tag{21}$$

and

$$Var[\ln l_t^i] = \sigma_t^{l^2} = g'(\mu_{t-1}^x)^2\sigma_{t-1}^{x^2}. \tag{22}$$

Proof

Define

$$\ln l_t = ln(1 - zn_t(x_{t-1})) \equiv g(x_{t-1}) \tag{23}$$

leads after developing a Taylor-series approximation over $\ln n_t$ to:

$$E[\ln l_t] = E[g(\overline{x}) + (x - \overline{x})g'(\overline{x}) + \frac{1}{2}(x - \overline{x})^2 g''(\overline{x})]. \tag{24}$$

Setting, further

$$\overline{x} = \mu_{t-1}^x \quad \text{yields} \tag{25}$$
$$E[\ln l_t] = \mu_t^l = g(\mu_{t-1}^x) + \frac{1}{2}\sigma_{t-1}^{x^2} g''(\mu_{t-1}^x) \tag{26}$$

and

$$Var[\ln l_t] = \sigma_t^{l^2} = g'(\mu_{t-1}^x)^2\sigma_{t-1}^{x^2} \quad \bullet \tag{27}$$

As the steady state is characterized by zero growth in per-capita terms, such that

$$E[x^*_{t-1}] \;=\; E[x^*_t] = E[x^*] \tag{28}$$

which implies

$$E[w^*_{t-1}] \;=\; E[w^*_t] = E[w^*], \tag{29}$$

$$E[n^*_{t-1}] \;=\; E[n^*_t] = E[n^*], \tag{30}$$

any steady state is characterized by a stationary distribution of wealth, such that $\mu^x_t = \mu^x_{t=1} = \mu^{x^*}$ and $\sigma^{x^2}_t = \sigma^{x^2}_{t-1} = \sigma^{x^{2^*}}$

$$E[\ln x^{i^*}] \;=\; \mu^{x^*} = \ln C + \alpha\mu^{x^*} - \alpha\mu^{l^*}, \tag{31}$$

$$Var[\ln x^{i^*}] \;=\; \sigma^{x^{2^*}} = \sigma^{\epsilon^2} + \alpha^2(\sigma^{x^{2^*}} + \sigma^{l^{2^*}}), \tag{32}$$

with

$$E[\ln l^{i^*}] \;=\; \mu^{l^*} = g(\mu^{x^*}) + \frac{1}{2}\sigma^{x^{2^*}} g''(\mu^{x^*}), \tag{33}$$

and

$$Var[\ln l^{i^*}] \;=\; \sigma^{l^{2^*}} = g'(\mu^{x^*})^2\sigma^{x^{2^*}} \qquad \forall \quad t. \tag{34}$$

The transition towards the steady state is entirely driven by the moments of the wealth distribution and the structural parameters of the model.
Given the properties of the production function, especially positive, but decreasing marginal returns to each production factor and taking into account that $l < 1$ we can state the following relationships

$$g(\mu^x_{t-1}) \;<\; 0, \tag{35}$$

$$g'(\mu^x_{t-1}) \;>\; 0, \tag{36}$$

$$g''(\mu^x_{t-1}) \;<\; 0. \tag{37}$$

An increase of $\sigma_t^{x^2}$ leads directly to an increase of the inequality in wealth, accelerated by the increase in labor supply and fertility differentials. [3]

In addition, there works an indirect effect. A higher value of μ_{t-1}^x implies a higher value of $g(\mu_{t-1}^x)$ and $g''(\mu_{t-1}^x)$ leading to a higher μ_{t-1}^x. On the other hand $\sigma_t^{l^2}$ is lowered by the decrease of $g'(\mu_{t-1}^x)$.

The overall effects are examined in the subsequent section.

4 Development of the Wealth Distribution

In this section the development of the wealth distribution is analyzed. In order to run numerical experiments, the set of parameters $\{\alpha, \beta, \gamma, \phi, z, B, h, \sigma^\epsilon\}$ has to be specified. Since, we consider agents who differ in their innate abilities and in their factor endowments, the first period of the economy ($t = 0$) is characterized by the initial parameters of the wealth distribution $\mu_{t=0}^x$ and $\sigma_{t=0}^x$ determining the amount of inequality in the economy.

After specifying the parameter set, we first explore 'empirically' - by generating an artificial sample of households -, if the log-normality of the wealth distribution during the transition process is maintained. In a second step, we simulate the development of the theoretic wealth distribution as it can be expected by the considerations undertaken above. Here and hence force we assume without further lost of generality the following parameter constellation:

$$\begin{aligned} \alpha &= 0.4; \quad \beta = 0.3; \quad \gamma = 0.5; \quad \delta = 0.4; \quad z = 0.5; \\ A &= 6; \quad h = 1; \quad \sigma^\epsilon = 0.4. \end{aligned} \tag{38}$$

Given the set of parameters thousand artificial households are generated by drawing a stochastic vector $\vec{\epsilon}_{t=0} = [\epsilon_{t=0}^{i=1}, ..., \epsilon_{t=0}^{i=1000}]'$ out of the log-normal distribution,

[3] Labor supply and fertility are negatively correlated.

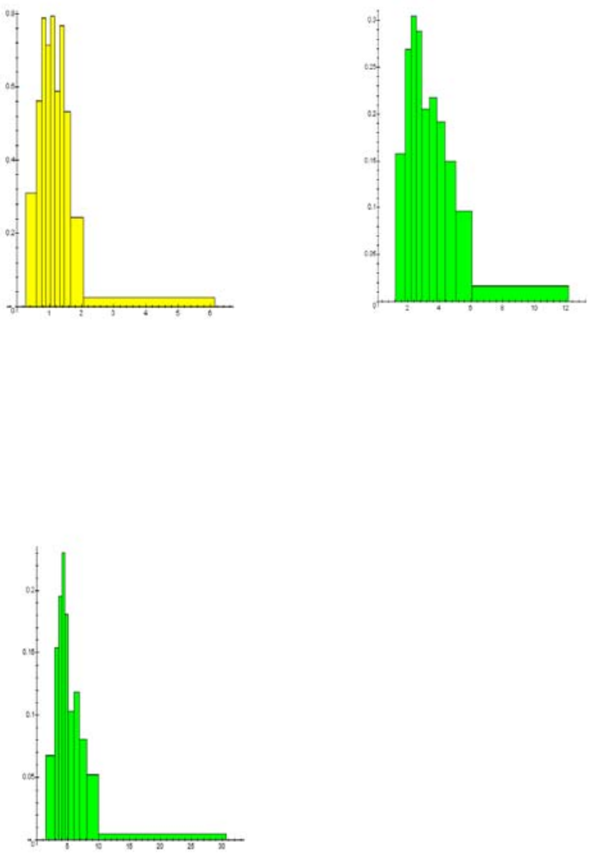


Figure 1: *Dynamics of the wealth distribution within an artificial sample (N=1000).*

satisfying Eq. (15). The endowment vector of the intial period is generated similary , such that $\vec{x}_{t=0} = [x^{i=1}_{t=0}, ..., x^{i=1000}_{t=0}]'$, with $\mu_{t=0} = 1.5$ and $\sigma_{t=0} = 0.5$ given. Each of the thousand agents takes its decisions according to the underlying optimization problem which is known and the same for all of them. Solving that problem leads to the set of optimal decisions of the sample, especially fertility decisions $n^i_{t=0}$ and optimal choices for bequests $x^i_{t=0}$. The latter ones constitute the endowment vector $\vec{x}_{t=1}$.
In order to take into account for the different fertility decisions, each element i of $\vec{x}_{t=1}$ is weighted by the relative fertility of the household i with respect to the mean of the economy. At the end of period one, the working generation dies and the children enter the labor market, equipped with some $x^i_{t=1} \in \vec{x}_{t=1}$. Given the endogenously determined vector $\vec{x}_{t=1}$ the amount of inequality for period one is given, as well, such that the loop starts again by generating a new stochastic ability vector $\vec{\epsilon}_{t=1}$. The procedure of this loop continues until the distribution of wealth within this artificial sample becomes stationary.
The histograms are shown in Figure 1. Apparently, the characteristics of a log-normal distribution are preserved during the transition to the steady state, whereas the steady state by itself is characterized a stationary distribution.
In order to examine the interaction between inequality, fertility and growth we explore now the theoretic derived dynamics given by the set of difference equations (19) and (20).

1. $x^i_{t=0} \sim LN(\mu^x_{t=0} = 1.5;\ \sigma^x_{t=0} = 0.4)$,

2. $x^i_{t=0} \sim LN(\mu^x_{t=0} = 1.5;\ \sigma^x_{t=0} = 0.8)$.

The simulation results are shown in Table 1, 2 and Figure 2.

Obviously the initial distribution of wealth has 'only' transitory effects, or the other way around the initial wealth distribution does not affect the long-run performance of the economy but the transition process. The latter becomes important, if the economy is still far away from its steady state and/or the transitory

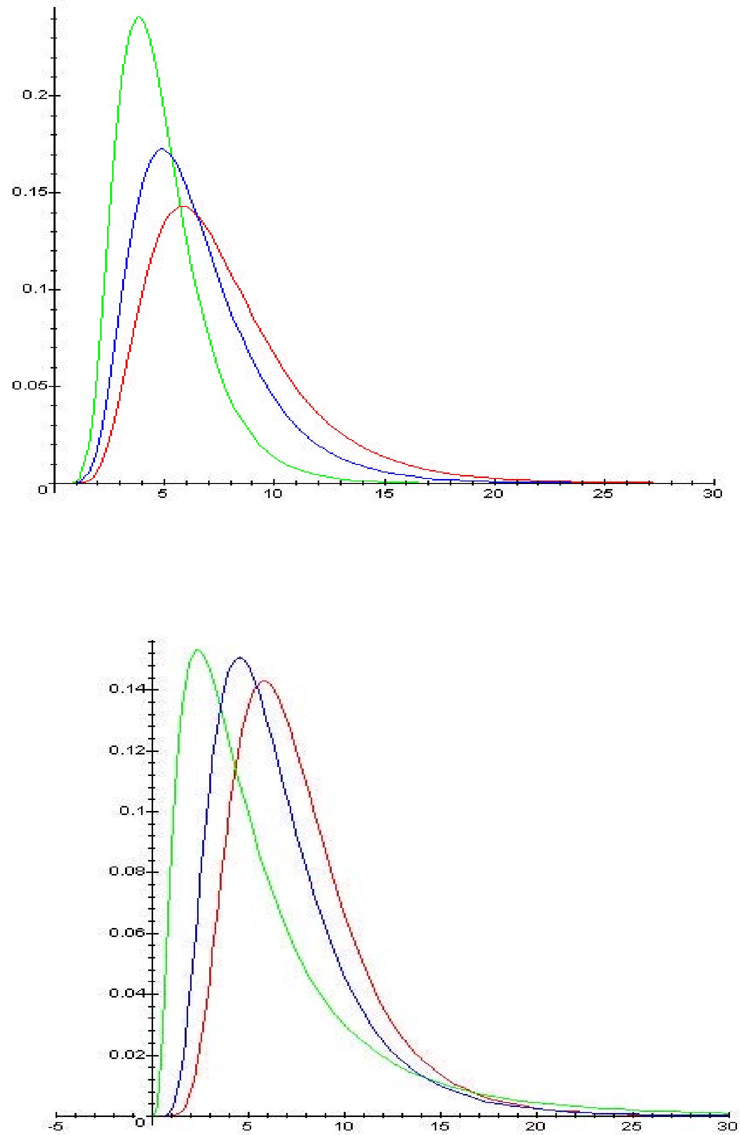


Figure 2: *Dynamics of the wealth distribtuion* ($\mu^x_{t=0} = 1.5, \sigma^x_{t=0} = 0.4$, *and* $\mu^x_{t=0} = 1.5, \sigma^x_{t=0} = 0.8$)

Parameter: $\mu^x_{t=0} = 1.5$ and $\sigma^x_{t=0} = 0.4$

t	$E[x_t]$	$Var[x_t]$	$E[l_t]$	$Var[l_t]$	$\frac{\mu^x_t}{\mu^x_{t-1}}$	μ^x_t	σ^x_t	μ^l_t	$\sigma^{l^2}_t$
1	4.8549	4.0897	0.63846	$0.1582\ 10^{-4}$	-	1.5	0.4	-0.4487	$0.3881\ 10^{-4}$
2	6.4594	8.5095	0.6410	$0.1158\ 10^{-4}$	1.1818	1.7727	0.4308	-0.4446	$0.2818\ 10^{-4}$
3	7.2071	10.8501	0.6419	$0.1005\ 10^{-4}$	1.0606	1.8802	0.4355	-0.4433	$0.2440\ 10^{-4}$
4	7.5220	11.8639	0.6422	$0.0948\ 10^{-4}$	1.0225	1.9226	0.4362	-0.4427	$0.2298\ 10^{-4}$
..	.....	......	.....	......	......	.....	.....	.....	.....
14	7.7340	12.5511	0.6424	$0.0921\ 10^{-4}$	1	1.9503	0.4364	-0.4424	$0.2213\ 10^{-4}$
15	7.7340	12.5511	0.6424	$0.0921\ 10^{-4}$	1	1.9503	0.4364	-0.4424	$0.2213\ 10^{-4}$

Table 1: *Daynamics of average wealth $E[x_t]$ and labor supply $E[l_t]$, the respective variances, the moments of the respective distributions, and the break-even wealth $\tilde{x}_t$ for $\mu^x_{t=0} = 1.5$ and $\sigma^x_{t=0} = 0.4$*

Parameter: $\mu^x_{t=0} = 1.5$ and $\sigma^x_{t=0} = 0.8$

t	$E[x_t]$	$Var[x_t]$	$E[l_t]$	$Var[l_t]$	$\frac{\mu^x_t}{\mu^x_{t-1}}$	μ^x_t	σ^x_t	μ^l_t	$\sigma^{l^2}_t$
1	6.1718	34.1486	0.6362	$0.6285\ 10^{-4}$	-	1.5	0.8	-0.4522	$1.552\ 10^{-4}$
2	6.7217	13.5581	0.6408	$0.1631\ 10^{-4}$	1.1827	1.7741	0.5122	-0.4450	$0.3973\ 10^{-4}$
3	7.2566	11.7871	0.6418	$0.1068\ 10^{-4}$	1.0601	1.8809	0.4494	-0.4433	$0.2597\ 10^{-4}$
4	7.5316	12.0295	0.6422	$0.0956\ 10^{-4}$	1.0223	1.9929	0.4385	-0.4427	$0.2318\ 10^{-4}$
..	.....	......	.....	......	......	......	.....	.....	.....
16	7.7340	12.5511	0.6424	$0.0921\ 10^{-4}$	1	1.9503	0.4364	-0.4424	$0.2213\ 10^{-4}$
17	7.7340	12.5511	0.6424	$0.0921\ 10^{-4}$	1	1.9503	0.4364	-0.4424	$0.2213\ 10^{-4}$

Table 2: *Dynamics of average wealth $E[x_t]$ and labor supply $E[l_t]$, the respective variances, the moments of the respective distributions for $\mu^x_{t=0} = 1.5$ and $\sigma^x_{t=0} =$ 0.8*

outcomes lead to political or social unstable situations and force the economy to another path. In addition, it has to be taken into account that we are arguing in an OLG-framework which means that individuals care only about themselves and not about future generations and the long-run performance of the economy. We can state the following results.

1. ***Growth leads to a higher expected value of wealth, but also to a higher variance, hence inequality.***
2. ***Higher initial inequality raises during the transition both expected value and variance of x.***
3. ***Higher initial inequality lowers during the transition the expected value of labor supply and increases its variance, hence fertility differentials.***
4. ***The more equal the initial distribution the higher is the expected growth rate.***
5. ***Fertility differentials and labor-supply differentials can be declining or increasing during the transition, depending for example on the time costs z.***[4]

It follows that the inequality rising (transitory) effect of growth is weakened in its levels by a more equal initial distribution of wealth. Growth leads to higher labor supply, declining fertility and decreasing fertility differentials. This means that in earlier stages of economic development lower income percentiles have a higher fertility than the upper ones controlled for the initial inequality. As growth increases opportunity costs for child-rearing, labor market participation increases

[4]Setting for example $z = 0.2$ leads contrary to the scenario presented above to inreasing fertility differentials (from $0.1580\,10^{-4}$ to $0.4520\,10^{-3}$).

and fertility differentials decline.[5]

Faster growth caused by a more equal initial distribution leads to a higher growth rate of labor market participation and a faster decline of fertility. The growth rate of inequality is accelerated. The latter might lead to re-distributional pressure, if individuals care not only about levels but also about relative changes in the economic environment.

These results correspond very much to the increasing arm of the Kuznets-Curve, as higher growth leads to more inequality and higher fertility differentials, controlled for the initial wealth distribution. If that is the case, higher inequality and higher fertility for lower income percentiles should translate into redistributional pressure. Therefore, it seems to be very unlikely that an economy converges as described to its steady state. One should expect that growth with increasing social pressure leads to redistribution of wide defined capital and opportunities. A norm of justice could be that everybody with the same innate abilities can engage into the same venture, independent from its social background.[6] The effects of wealth redistribution are examined in the following section.

5 Exogenous Redistribution

In this section we seek to analyze the effects of wealth redistribution. Here and henceforth, it is very important to note that redistribution takes place before entering the labor force and not after production. An after-production tax would reverse the outcomes. Such a redistribution scheme would increase the non-labor market income for transfer receiving households and lower opportunity costs of child-rearing time for the contributing households. Fertility would increase economy wide and growth slows down because of a lower wealth or quality per

[5] In the second scenario inequality is declining during the transition to the steady state because the initial level of inequality is above that of the limiting distribution.

[6] See for a similar argumentation Galor and Zeira (1993). We abstract so far from the existence of a capital market and consider the non-existence of a capital market as a limiting case of capital market discrimination.

adult.

Before entering the labor force, wealth is redistributed according to the following tax scheme (Benabou(1996))

$$\hat{x}_t^{t-1,i} = x_t^{t-1,i^{1-\tau}} \tilde{x}_t^{\tau}, \tag{39}$$

whereas $\hat{x}_t^{t-1,i}$ represents the after-tax wealth, $\tilde{x}_t^{t-1,i}$ the break-even wealth and $0 < \tau < 1$ the tax rate. Obviously, the tax scheme is progressive. Households with wealth lower than $\tilde{x}$ have an higher after-tax wealth and the opposite is true for the richer ones

$$\hat{x}_t^{t-1,i} \quad > \quad x_t^{t-1,i} \quad \forall \quad x_t^{t-1,i} | x_t^{t-1,i} < \tilde{x}_t, \tag{40}$$

$$\hat{x}_t^{t-1,i} \quad < \quad x_t^{t-1,i} \quad \forall \quad x_t^{t-1,i} | x_t^{t-1,i} > \tilde{x}_t. \tag{41}$$

Given, that $d = x_t^{t-1,i} - \hat{x}_t^{t-1,i}$ represents the net-transfer received or contributed according to Eq.(39), it has to be taken into account that the sum of net-transfers sum up to zero, in each period

$$\int_0^{N_t} x_t^{t-1,i} - \hat{x}_t^{t-1,i} \quad dx^i = 0. \tag{42}$$

Equation (39) and (42) are determining the budget constraint which is binding as long as it is abstracted from intertemporal debt policies. It follows that the expectation value of the after-tax wealth equals the expectation value of the pre-tax wealth, such that the mean is equally well-off after redistribution

$$\int_0^{N_t} x_t^{t-1,i} f(x_t^{t-1,i}) \quad dx^i \quad = \quad \tilde{x}_t^{\tau} \int_0^{N_t} x^{i^{1-\tau}} f(x^{i^{1-\tau}}) dx^i, \tag{43}$$

$$\text{and} \quad x^{i^{1-\tau}} \quad \sim \quad LN((1-\tau)\mu_{t-1}^x, (1-\tau)^2 \sigma_{t-1}^{x^2}). \tag{44}$$

Therefore, it must also be true that

$$\ln\left[\exp\left(\mu^x_{t-1}+\frac{\sigma^{x^2}_{t-1}}{2}\right)\right]$$
$$=\ln\tilde{x}_t+\ln\left[\exp\left((1-\tau)\mu^x_{t-1}+\frac{(1-\tau)^2\sigma^{x^2}_{t-1}}{2}\right)\right] \tag{45}$$

Hence, the break-even wealth which satisfies the budget constraint is given by the following relation

$$\ln\tilde{x}_t = \mu^x_{t-1}+\frac{1}{\tau}[1-(1-\tau)^2]\frac{\sigma^{x^2}_{t-1}}{2}. \tag{46}$$

A higher tax rate leads everything else constant to a lower break-even wealth, hence more redistribution concentrating on the lower income percentiles. The break-even wealth has to decline because the after-tax wealth $\hat{x}^{t-1,i}_t$ is increasing in the tax rate[7], whereas the pre-tax income remains constant. Consequently, Eq.(43) requires $\tilde{x}_t$ to decline.

Equation (7) modifies to

$$x^{t,i}_t = \frac{\beta}{\gamma-\beta}z\epsilon^i_t(1-\alpha)Bx^{t-1,i^{\alpha(1-\tau)}}_{t-1}\tilde{x}^{\alpha\tau}l^{t-1,i^{-\alpha}}_t, \tag{47}$$

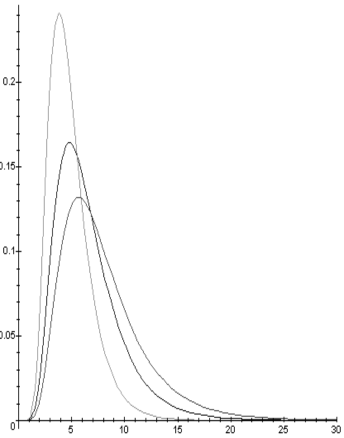
taking logarithms and using (46) yields

$$E[\ln x^i_t] = \mu^x_t=\ln C+\alpha\mu^x_{t-1}+\alpha[1-(1-\tau)^2]\frac{\sigma^{x^2}_{t-1}}{2} -\alpha\mu^l_t, \tag{48}$$

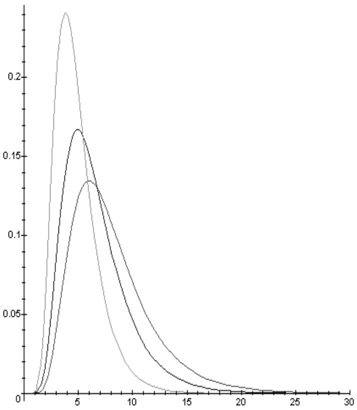
and

$$Var[\ln x^i_t] = \sigma^{x^2}_t=\sigma^{\epsilon^2}+\alpha^2((1-\tau)^2\sigma^{x^2}_{t-1}+\sigma^{l^2_\tau}_t), \tag{49}$$

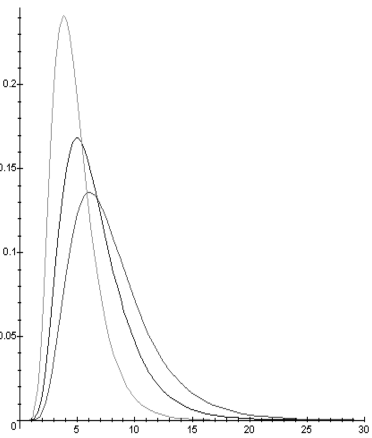
[7] $\frac{\partial\hat{x}^{t-1,i}_t}{\partial\tau}=(1-\tau)+\tau\frac{x^{t-1,i}_t}{\tilde{x}_t}>0$



(a)



(b)



(c)

Figure 3: *Dynamics of the wealth distribution with* $\mu^x_{t=0} = 1.5, \sigma^x_{t=0} = 0.4$*, and* $\tau = 0.2; \tau = 0.5; \tau = 0.8$ *for tax rates (a)* $\tau = 0.1$*; (b)* $\tau = 0.5$*; (c)* $\tau = 0.8$*.*

Parameter: $\mu^x_{t=0} = 1.5$, $\sigma^x_{t=0} = 0.4$, and $\tau = 0.1$

t	$E[x_t]$	$Var[x_t]$	$E[l_t]$	$Var[l_t]$	$\frac{\mu^x_t}{\mu^x_{t-1}}$	μ^x_t	σ^x_t	μ^l_t	$\sigma^{l^2}_t$	$\tilde{x}_t$
1	4.8549	4.0897	0.6937	0.0836	0	1.5	0.4	-0.4456	0.1601	5.2174
2	6.5586	9.8592	0.7129	0.1165	1.1850	1.7776	0.4542	-0.4415	0.2064	7.1968
3	7.3790	13.3841	0.7187	0.1270	1.0625	1.8887	0.4688	-0.4401	0.2198	8.1461
4	7.7290	15.6171	0.7205	0.1301	1.0235	1.9331	0.4729	-0.4395	0.2237	8.5474
..	.....	......	.....	......	......	......	.....	.....	.....	.....
16	7.9656	16.0297	0.7213	0.1315	1	1.9625	0.4745	-0.4392	0.2253	8.8153
17	7.9656	16.0297	0.7213	0.1315	1	1.9625	0.4745	-0.4392	0.2253	8.8153

Table 3: *Dynamics of average wealth $E[x_t]$ and labor supply $E[l_t]$, the respective variances, the moments of the respective distributions, and the break-even wealth $\tilde{x}_t$ for $\mu^x_{t=0} = 1.5$, $\sigma^x_{t=0} = 0.4$ and $\tau = 0.1$.*

where μ^l_t and σ^l_t transform to

$$E[\ln l^i_t] = \mu^l_t = g(\mu^x_{t-1}, \ln\tilde{x}, \tau) + \frac{1}{2}\sigma^{x^2}_t g''(\mu^x_{t-1}, \ln\tilde{x}, \tau), \quad (50)$$

$$Var[\ln l^i_t] = \sigma^{l-\tau^2}_t = [g'(\mu^x_{t-1}, \ln\tilde{x}, \tau)]^2 \sigma^{x^2}_{t-1}. \quad (51)$$

As far as we seek to analyze the effects of redistribution on growth, fertility differentials and inequality controlled for the initial wealth distribution, we examine the following scenarios

1. $\mu^x_{t=0} = 1.5$, $\sigma^x_{t=0} = 0.4$, and $\tau = 0.1$,

2. $\mu^x_{t=0} = 1.5$, $\sigma^x_{t=0} = 0.4$, and $\tau = 0.5$,

3. $\mu^x_{t=0} = 1.5$, $\sigma^x_{t=0} = 0.4$, and $\tau = 0.8$.

The results are shown in Tables 3, 4, 5 and Figure 3.
First of all, it is important to state that redistribution does not effect the long-run growth performance, in the sense that long-run per capita growth ceases.

Parameter: $\mu^x_{t=0} = 1.5$, $\sigma^x_{t=0} = 0.4$, and $\tau = 0.5$

t	$E[x_t]$	$Var[x_t]$	$E[l_t]$	$Var[l_t]$	$\frac{\mu^x_t}{\mu^x_{t-1}}$	μ^x_t	σ^x_t	μ^l_t	$\sigma^{l^2}_t$	$\tilde{x}_t$
1	4.8549	4.0897	0.7009	0.0852	0	1.5	0.4	-0.4353	0.1600	5.0530
2	6.6021	9.2266	0.7153	0.1083	1.1942	1.7913	0.4381	-0.4309	0.1920	6.9267
3	7.4648	12.2287	0.7187	0.1133	1.0667	1.9109	0.4454	-0.4294	0.1984	7.8444
4	7.8382	13.5788	0.7196	0.1144	1.0252	1.9591	0.4468	-0.4287	0.1996	8.2394
..	.....	......	.....	......	......	......	.....	.....	.....	.....
16	8.0932	14.5022	0.7200	0.1148	1	1.9910	0.4472	-0.4284	0.2000	8.5081
17	8.0932	14.5022	0.7200	0.1148	1	1.9910	0.4472	-0.4284	0.2000	8.5081

Table 4: *Dynamics of average wealth $E[x_t]$ and labor supply $E[l_t]$, the respective variances, the moments of the respective distributions, and the break-even wealth $\tilde{x}_t$ for $\mu^x_{t=0} = 1.5$, $\sigma^x_{t=0} = 0.4$ and $\tau = 0.5$.*

Parameter: $\mu^x_{t=0} = 1.5$, $\sigma^x_{t=0} = 0.4$, and $\tau = 0.8$

t	$E[x_t]$	$Var[x_t]$	$E[l_t]$	$Var[l_t]$	$\frac{\mu^x_t}{\mu^x_{t-1}}$	μ^x_t	σ^x_t	μ^l_t	$\sigma^{l^2}_t$	$\tilde{x}_t$
1	4.8549	4.8549	0.7052	0.0863	0	1.5	0.4	-0.4291	0.1600	4.9332
2	6.6125	8.9713	0.7177	0.1057	1.1970	1.7956	0.4320	-0.4249	0.1866	6.7370
3	7.4844	11.9731	0.7205	0.1092	1.0677	1.9173	0.4370	-0.4233	0.1910	7.6288
4	7.8622	13.0689	0.7212	0.1099	1.0254	1.9661	0.4379	-0.4227	0.1917	8.0145
..	.....	......	.....	......	......	......	.....	.....	.....	.....
16	8.1202	13.9523	0.7215	0.1101	1	1.9983	0.4381	-0.4223	0.1919	8.2776
17	8.1202	13.9523	0.7215	0.1101	1	1.9983	0.4381	-0.4223	0.1919	8.2776

Table 5: *Dynamics of average wealth $E[x_t]$ and labor supply $E[l_t]$, the respective variances, the moments of the respective distributions, and the break-even wealth $\tilde{x}_t$ for $\mu^x_{t=0} = 1.5$, $\sigma^x_{t=0} = 0.4$ and $\tau = 0.8$.*

This is because the properties of the production function remain the same. But taxation changes the long-run distribution of wealth and not only the transitory distributions.
We can establish the following results

1. ***Redistribution lowers inequality and increases growth, but with decreasing rates.***

2. ***The effect on the long-run average labor supply (fertility) is ambigous, as the long-run level of labor supply (fertility) is an (inverted) u-shaped function in the tax rate.***

3. ***Long-run and transitory fertility and labor supply differentials are declining with the amount of redistribution.***

4. ***The break-even wealth is declining in the tax rate.***

5. ***Compared to the scenario without redistribution, fertility and labor supply differentials are increasing.***

A higher level of redistribution in a more equal society has therefore little effect on the average growth and fertility rate. This is due to the properties of the underlying production function. If the amount of inequality is very low and the average is relatively rich, the dispersion in marginal returns to wealth is very low, as well.
That labor supply and fertility differentials are increasing compared to the scenario without redistribution is no contradiction. Responsible for that is the distribution of abilities. Individuals with a very low ability are still, though richer in terms of wealth, not willing to increase there labor supply. Therefore, there fertility remains compared to the rest relatively high and their labor supply relatively low. The question is however, if redistribution is an appropriate measure to close fertility and labor supply differentials. If the reason for differential fertility lies in the innate abilities and not in the distribution of economic factors, further

redistribution would disadvantage individuals with high abilities. Consequently, a further compression of fertility differentials should be achieved by a subsidy of labor supply for low-income percentiles. This would increase their opportunity costs, lower their fertility and increase the quality per child.

A further question which arising out of the persistence of fertility differentials is the above mentioned emergence of redistributional pressure. If low-income percentiles exhibit a higher fertility this should translate into redistributional pressure. In order to analyze these effects, we endogenize the tax rate, hence the amount of redistribution by means of a median-voter system.

6 Endogenous Redistribution

In the last section the effects of redistribution were analyzed, whereas the tax rate was exogenously fixed. In this section, we endogenize the amount of redistribution by a majority-voting system.

Each household formulates a voting function, given the optimal choices for fertility $n_t^{t-1,i}$, quality $x_t^{t-1,i}$, consumption $c_t^{t-1,i}$, and the break-even wealth $\tilde{x}_t$

$$V_t^i = \gamma \ln n_t^{t-1,i} + \beta \ln x_t^{t,i} + \delta \ln c_t^{t-1,i} + \Upsilon \ln \tilde{x}_t. \tag{52}$$

The last term in utility function captures therefore the individual preferences for redistribution, hence the tolerated amount of inequality. For the break-even wealth the following relationships have to be true

$$\ln \tilde{x}_t = \mu_{t-1}^x + \frac{1}{\tau_t}[1-(1-\tau_t)^2]\frac{\sigma_{t-1}^{x^2}}{2} \tag{53}$$

and

$$\ln \tilde{x}_t = \frac{1}{\tau_t}[\ln \hat{x}_t^{t-1,i} - (1-\tau_t)\ln x_t^{t-1,i}]. \tag{54}$$

As stated above, the break-even wealth is declining in the tax rate and increasing in the amount of inequality. On the other hand, the second equation states a rather egoistic argument, in the sense that each individual tries to maximize the distance between pre and after-tax wealth, such that each individual seeks to set a tax rate increasing his or her after-tax wealth. Therefore, a low weight Υ of $\tilde{x}_t$ reflects a preference for a low distance between $\hat{x}_t^{t-1,i}$ and $x_t^{t-1,i}$, hence less selfishness and a relatively high preference for a lower σ_{t-1}^x associated with a higher tax rate. Alternatively, Υ can be interpreted as the voting-power. A high Υ would represent a system which is more biased to the rich.

In a majority-voting system, we assume that the median voter is decisive for the tax-rate. Although this is the common assumption in the literature of income distribution, one should take into consideration that this is only true, if political power is independent from economic power. Despite the fact that rich as poor people have only one vote, rich people have more resources available to organize and articulate their preferences. To the contrary, the voting participation of poor people might be lower, due to lower education, and frustration. Therefore, any majority-voting system might be more biased to the rich, rather than to the poor. However, the interval around the median can definitively serve as a point of reference, as it represents more than fifty percent of the population.

The moments of the wealth distribution are known, such that the pre-tax wealth of the median is known, too

$$x_t^{t-1,med} = \exp(\mu_{t-1}^x). \tag{55}$$

Consequently, the after-tax wealth of the median is given by

$$\hat{x}_t^{t-1,med} = (\exp(\mu_{t-1}^x))^{1-\tau_t}(\tilde{x}_t)^{\tau_t} \tag{56}$$

implying a median after-tax income

$$\begin{aligned} y_t^{t-1,med} &= (1-\alpha)(\hat{x}_{t-1}^{t-1,med})^{\alpha}(l_t^{t-1,med})^{-\alpha} \\ &\quad +\alpha(\hat{x}_{t-1}^{t-1,med})^{\alpha-1}(l_t^{t-1,med})^{1-\alpha}. \end{aligned} \tag{57}$$

Parameter: $\mu^x_{t=0} = 1.5$, $\sigma^x_{t=0} = 0.4$, $\Upsilon = 0.35$

t	$E[x_t]$	$Var[x_t]$	$E[l_t]$	$Var[l_t]$	$\frac{\mu^x_t}{\mu^x_{t-1}}$	μ^x_t	σ^x_t	τ^{med}_t	$E[x_t]-x^{med}_t$	$\tilde{x}_t$
1	4.8549	4.0897	0.6976	0.0844	0	1.5	0.4	0.3023	0.3732	5.1336
2	6.5847	9.4978	0.7142	0.1117	1.1904	1.7857	0.4450	0.3106	0.6208	7.0500
3	7.4330	12.6903	0.7185	0.1186	1.0654	1.9025	0.4547	0.3137	0.7300	7.9796
4	7.7995	14.1156	0.7196	0.1201	1.0247	1.9497	0.4568	0.3149	0.7727	8.3775
..	.....	......	.....	......	......	......	.....	.....	.....	.....
16	8.0500	15.0743	0.7201	0.1206	1	1.9811	0.4573	0.3156	0.7992	8.6472
17	8.0500	15.0743	0.7201	0.1206	1	1.9811	0.4573	0.3156	0.7992	8.6472

Table 6: *Scenario I(a): Dynamics of average wealth $E[x_t]$ and the variance of wealth $Var[x_t]$, the moments of the wealth distribution, the tax rate, the difference between mean and median $E[x_t]-x^{med}_t$ and the break-even wealth $\tilde{x}_t$ for $\mu^x_{t=0} = 1.5$, $\sigma^x_{t=0} = 0.4$, $\Upsilon = 0.35$.*

Plugging (57) together with the set of optimal decisions (Eqs. (5)-(7)) into Eq.(52), we yield the voting function of the median voter $V^{med}_t(\tau^{med}_t)$. Forming the first-order condition $\frac{\partial V^{med}_t(\tau^{med}_t)}{\partial \tau^{med}_t} = 0$ (and $\frac{\partial^2 V^{med}_t(\tau^{med}_t)}{\partial \tau^{med^2}_t} < 0$) leads together with Eq (6) to a system of equations with τ^{med}_t and $n^{t-1,med}_t$ as unknowns. Solving this system leads to the preferred tax rate, in period t

$$\tau_t = \tau^{med}_t. \tag{58}$$

With the tax rate known, the break even wealth $\tilde{x}_t$ is - given the moments of the wealth distribution - also known. Consequently the economy develops according to

$$\begin{aligned} E[\ln x^i_t] &= \mu^x_t = \ln C + \alpha\mu^x_{t-1} + \alpha[1-(1-\tau^{med}_t)^2]\frac{\sigma^{x^2}_{t-1}}{2} \\ &\quad -\alpha\mu^l_t, \end{aligned} \tag{59}$$

and

$$Var[\ln x^i_t] = \sigma^{x^2}_t = \sigma^{\epsilon^2} + \alpha^2((1-\tau^{med}_t)^2\sigma^{x^2}_{t-1} + \sigma^{l^2_\tau}_t), \tag{60}$$

In order to explore for the effects of inequality within a median-voter system, we simulate two scenarios with different preferences for equality and different

Parameter: $\mu^x_{t=0} = 1.5,\ \sigma^x_{t=0} = 0.55,\ \Upsilon = 0.35$

t	$E[x_t]$	$Var[x_t]$	$E[l_t]$	$Var[l_t]$	$\frac{\mu^x_t}{\mu^x_{t-1}}$	μ^x_t	σ^x_t	τ^{med}_t	$E[x_t] - x^{med}_t$	$\tilde{x}_t$
1	5.2134	9.6011	0.7493	0.1983	0	1.5	0.55	0.3034	0.7318	4.4816
2	6.7958	12.0532	0.7266	0.1378	1.2002	1.8003	0.4815	0.3112	0.7439	6.0518
3	7.5326	13.5896	0.7214	0.1246	1.0619	1.9118	0.4633	0.3140	0.7667	6.7658
4	7.8422	14.4115	0.7203	0.1216	1.0221	1.9542	0.4588	0.3150	0.7835	7.0587
..	.....	......	.....	......	......	......	.....	.....	.....	.....
16	8.0500	15.0743	0.7201	0.1206	1	1.9811	0.4573	0.3156	0.7992	8.6472
17	8.0500	15.0743	0.7201	0.1206	1	1.9811	0.4573	0.3156	0.7992	8.6472

Table 7: *Scenario I(b): Dynamics of average wealth $E[x_t]$ and the variance of wealth $Var[x_t]$, the moments of the wealth distribution, the tax rate, the difference between mean and median $E[x_t]-x^{med}_t$ and the break-even wealth $\tilde{x}_t$ for $\mu^x_{t=0} = 1.5$, $\sigma^x_{t=0} = 0.55,\ \Upsilon = 0.35$.*

Parameter: $\mu^x_{t=0} = 1.5,\ \sigma^x_{t=0} = 0.4,\ \Upsilon = 0.505$

t	$E[x_t]$	$Var[x_t]$	$E[l_t]$	$Var[l_t]$	$\frac{\mu^x_t}{\mu^x_{t-1}}$	μ^x_t	σ^x_t	τ^{med}_t	$E[x_t] - x^{med}_t$	$\tilde{x}_t$
1	4.0897	4.8549	0.6915	0.0831	0	1.5	0.4	-0.008408	0.3732	5.2628
2	6.5413	10.0871	0.7126	0.1198	1.1815	1.7723	0.4600	0.002893	0.6569	7.2693
3	7.3454	13.7877	0.7193	0.1323	1.0609	1.8803	0.4770	0.007038	0.7899	8.2240
4	7.6865	15.4521	0.72142	0.1362	1.0208	1.9233	0.4820	0.008637	0.8429	8.6246
..	.....	......	.....	......	......	......	.....	.....	.....	.....
16	7.9162	16.5421	0.7224	0.1378	1	1.9517	0.4834	0.009678	0.8749	8.8898
17	7.9162	16.5421	0.7224	0.1378	1	1.9517	0.4834	0.009678	0.8749	8.8898

Table 8: *Scenario II(c): Dynamics of average wealth $E[x_t]$ and the variance of wealth $Var[x_t]$, the moments of the wealth distribution, the tax rate, the difference between mean and median $E[x_t]-x^{med}_t$ and the break-even wealth $\tilde{x}_t$ for $\mu^x_{t=0} = 1.5$, $\sigma^x_{t=0} = 0.4,\ \Upsilon = 0.505$.*

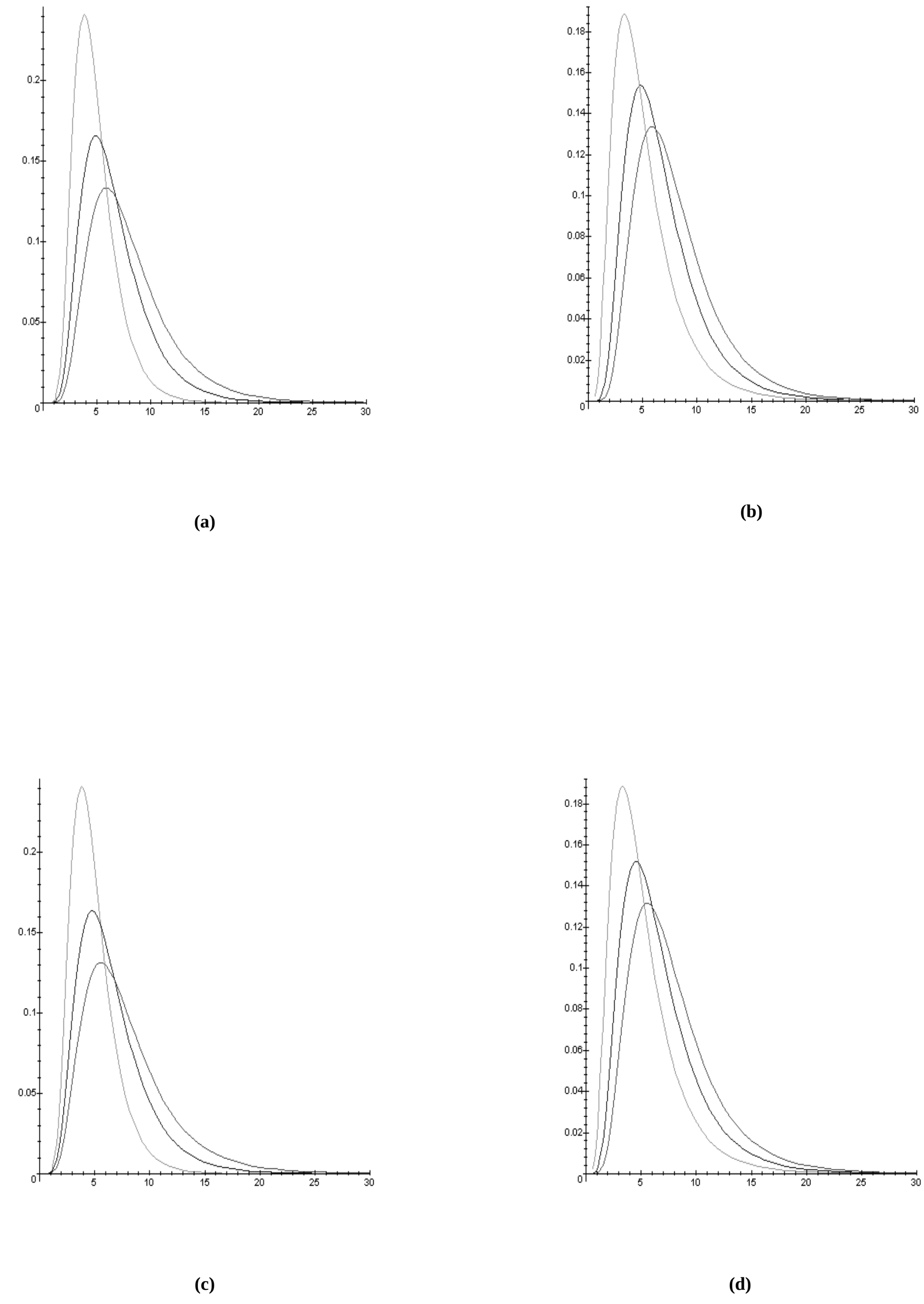


Figure 4: *Dynamics of the wealth distribution with an endogenous tax rate* $\tau_t = \tau_t^{med}$ *and different preferences for equality* Υ. *Scenario I:* $\Upsilon = 0.35$, $\mu^x_{t=0} = 1.5$ *and (a)* $\sigma^x_{t=0} = 0.4$; *(b)* $\sigma^x_{t=0} = 0.55$. *Scenario II:* $\Upsilon = 0.505$, $\mu^x_{t=0} = 1.5$ *and (c)* $\sigma^x_{t=0} = 0.4$; *(d)* $\sigma^x_{t=0} = 0.55$.

Parameter: $\mu^x_{t=0} = 1.5$, $\sigma^x_{t=0} = 0.55$, $\Upsilon = 0.505$

t	$E[x_t]$	$Var[x_t]$	$E[l_t]$	$Var[l_t]$	$\frac{\mu^x_t}{\mu^x_{t-1}}$	μ^x_t	σ^x_t	τ^{med}_t	$E[x_t] - x^{med}_t$	$\tilde{x}_t$
1	5.2134	9.6011	0.7417	0.1945	0	1.5	0.55	-0.008461	0.7318	6.0725
2	6.6931	13.1661	0.7290	0.1563	1.1814	1.7772	0.5076	0.002896	0.8090	7.6105
3	7.4004	15.0113	0.7246	0.1440	1.0600	1.8804	0.4921	0.007046	0.8441	8.3461
4	7.7053	15.8778	0.7231	0.1399	1.0208	1.9234	0.4868	0.008642	0.8610	8.6657
..	.....	......	.....	......	......	......	.....	.....	.....	.....
16	7.9162	16.5421	0.7224	0.1378	1	1.9517	0.4834	0.009678	0.8749	8.8898
17	7.9162	16.5421	0.7224	0.1378	1	1.9517	0.4834	0.009678	0.8749	8.8898

Table 9: *Scenario II(d): Dynamics of average wealth $E[x_t]$ and the variance of wealth $Var[x_t]$, the moments of the wealth distribution, the tax rate, the difference between mean and median $E[x_t]-x^{med}_t$ and the break-even wealth $\tilde{x}_t$ for $\mu^x_{t=0} = 1.5$, $\sigma^x_{t=0} = 0.55$, $\Upsilon = 0.505$.*

amounts of inequality within that scenario. Scenario I is characterized by a relatively high preference for equality $\Upsilon = 0.35$ and Scenario II by a relatively weak preferences for equality $\Upsilon = 0.505$. The results are shown in Tables 6 - 9 and Figure 4.

As can be seen clearly from the simulations performed in Tables 6 - 9, an increase in inequality leads, everything else constant, to a higher variance in wealth. The mean is richer and therefore the labor supply of the mean is higher and its fertility lower, but the distance between mean and median has increased. Despite the fact that the average of wealth has increased and the fertility of the person with average wealth is lower the variances for both, wealth and labor supply are higher. As the person with the average wealth does not coincide with the average in the population a higher distance between the median and the mean leads to a higher tax rate and a lower break-even wealth. As the initial inequality is above its long-run value, fertility differentials are declining in Scenario I (b) and increasing in Scenario I (a). The effect of redistribution in a more unequal society on the growth rate is due to the diminishing returns to wealth bigger than in a more equal society. Therefore, the expected growth rate is initially higher in Scenario

(b) higher than in Scenario (a). This effect vanishes, however, during the course of the transition, as it is dominated by a level of inequality which is overshooting its long-run level.[8] With the same parameter constellation, the long-run characteristics of Scenario I (a) and (b) are entirely the same.

The effects of an increase in inequality in Scenario II which is characterized by a lower preference for equality, are entirely the same as described above, however compared to Scenario I the transitory and long-run characteristics changed essentially. With a lower tax rate (which is even regressive at the beginning) the mean gets poorer and the distance between median and mean increases. The variance of wealth and labor supply has increased both during the transition and in the steady state. Surprisingly, the average labor supply has increased and therefore the average level of fertility declined. This is no contradiction to the argumentation undertaken so far. It rather emphasizes the importance of looking at the moments of the redistribution and not only at the mean. The increase in average labor supply is caused by an increase in σ_t^x which translates into the variance of wealth and labor supply as much as into the expectation value of labor (see for example Eqs. (50)-(51)). It is therefore caused by an increase in inequality which is accompanied by a higher dispersion arroud the mean. The higher expectation value of labor supply is in this sense no result of a more prosperous development in the economy.

7 Conclusions

According to empirical findings we have shown that fertility is declining during the process of economic development and interacting with the distribution of wealth.

The growth process leads by itself to more inequality and (eventually) rising fertility differentials; the more, the more unequal the initial distribution. All this gives rise to redistributional pressure during the process of economic development.

[8] For an initial value of $\sigma_{t=0}^x = 0.3$ we receive $\frac{\mu_t^x}{\mu_{t-1}^x} = \{1.1857, 1.0671, 1.0260, ..., 1\}$

Once a redistributional policy is applied, the (average) growth rate increases if the initial inequality is below its long-run level, otherwise it slow down.
In a median-voter system the preferred tax-rate and the transitory as well as the limiting distribution of wealth depend on the preference for redistribution, hence equality in the society. During the transition the median-voter gets due to increasing fertility differentials relatively poorer and poorer. As a consequence the tax rate is increasing during the transition, the more the more unequal the wealth distribution and the higher the preference for equality. Contrary to existing literature, redistributional pressure does not cease once the steady state is reached and is interacting with demographic variables.

References

Alesina, A. and Rodrik, B. (1994), Distributive Politics and Economic Growth, *The American Economic Review*, Vol. 40(6), 1203-1228.

Anhand, S. and Kanbur, S.M. (1993) , inequality and Development: A Critique. *Journal of Development Economics*, VOl. 41(1), pp.19-43.

Barro, R. (2000), Inequality and Growth in a Panel of Countries, *Journal of Economic Growth*, Vol. 5, 5-32.

Barro, R. and Sala-i-Martin, X. (1995), Economic Growth, *McGraw-Hill*, New York.

Becker, G. S. (1960), An Economic Analysis of Fertility, in Ansley J. Coale, ed., *Demographic and Economic Change in Developed Countries, Princeton University Press*, Princeton, NJ, 209-240.

Becker, G. S. and Lewis, H. G. (1973), On the Interaction Between Quantity and Quality of Children, *Journal of Political Economiy*, Vol. 82, 279-288.

Becker, G. S. and Barro, R. J. (1988), A Reformulation of the Economic Theory of Fertility, *The Quarterly Journal of Economics*, Vol. 103(2), 1-26.

Becker, G. S. and Barro, R. J. (1989), Fertility Choice in a Model of Economic Growth, *Econometrica*, Vol. 57(2), 481-501.

Becker, G. S., Barro, R. J. and Tamura, R. (1990), Human Capital, Fertility, and Economic Growth, *Journal of Political Economy*, Vol. 98(5), 12-37.

Benabou, R. (1996), Inequality and Growth, *NBER*, Working Paper 5658.

Croix de la, D. and Doepke, M (2003), Inequality and Growth: Why Differential Fertility Matters, *American Economic Review*, Vol. 93(4), 1091-1113.

Dahan, M. and Tsiddon, D. (1998), Demographic Trasition, Income Distribution and Economic Growth, *Journal of Economic Growth*, Vol. 3, 29-52.

Deininger, K. and Squire, L. (1998), New Way of Looking at Old Issues, *Journal of Development Economics,* Vol. 57, pp. 259-287.

Diamond, P. A. (1965), National Debt in a Neoclassical Growth Model, *American Economic Review*, Vol. 55, 1126-1150.

Forbes, K. (2000), A Reassessment of the Relationship Between Inequality and Growth, *American Economic Review*, Vol. 90(4), pp. 869-887.

Galor, O. and Zeira, J. (1993), Income Distribution and Macroeconomics, *Review of Economic Studies*, Vol. 60, 35-52.

Galor, O. and Weil, D. N. (1996), The Gender Gap, Fertility, and Growth, *The American Economic Review*, Vol. 86(3), 374-387.

Galor, O. and Weil, D. N. (2000), Population, Technology and Growth: From Malthusian Stagnation to Growth and Beyond, *The American Economic Review*, Vol. 90, 806-828.

Galor, O. and Zang, H. (1997), Fertility, Income Distribution, and Economic Growth: Theory and Cross-Country Evidence, *Japan and the World Economy*, Vol. 9, 197-229.

Kögel, T. (2004) Did the Association between Fertility and Femal Employment within OECD countries really change its sign?, *Journal of Population Economics*, Vol. 17(1), pp. 45-65.

Kremer, M. and Chen, D. (2000), Income Distribution Dynamics with Endogenous Fertility, *NBER*, Working Paper 7530.

Loury, G. C. (1981), Intergenerational Transfers and the Distribution of Earnings, *Econometrica*, Vol. 49, pp. 843-867.

Lucas Jr., R. E. (1988), On the Mechanics of Economic Development, *Journal of Monetary Economics*, Vol.22, pp. 3-42.

Morand, O. F. (1999), Endogenous Fertility, Income Distribution, and Growth, *Journal of Economic Growth*, Vol. 4, 331-349.

Owen, A. and Weil, D.N. (1998), Intergenerational Earnings Mobility, Inequality and Growth, *Journal of Monetary Economics*, Vol. 41, 71-104.

Persson, T. and Tabellini, G. (1994), Is Inequality Harmful for Growth? Theory and Evidence, *The American Economic Review*, Vol. 84(3), 600-621.

Schäfer, A. (2003), On the Interaction between Inequality and Fertility in Hausen, Resinek, Schümann, Stierle, eds., *Determinants of Growth and Business Cycles* INFER-Research Ed. Vol.9 , Berlin, 61-78.

Schäfer, A. (2002), Endogenous Growth with Endogenous Fertility and Social Discrimination in Education *Thuenen-Series of Applied Economic Theory* Working Paper No. 35 , University of Rostock.