Community structure and labour market segmentation in a stochastic model of human capital accumulation

Davide Fiaschi - Matteo Marsili*

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Abstract

We analyse an economy where heterogeneous agents are partitioned in communities and individual human capital accumulation, the source of growth, is the joint result of private investment in education, public expenditure and externalities within a community. We characterize the long-run growth rate and the distribution of human capital under alternative specifications regarding community structure, the method to finance public expenditure and labour market. The maximum growth rate is reached for a full integrated economy (just one community). In a stratified economy public expenditure financed by government is preferred to locally financed education. The segmentation of labour market has a negative effect on aggregate growth by decreasing the resources devoted to education.

Keywords: social interactions, human capital, neighbourhood effects, segmented labour market, taxation.

*Respectively, Dipartimento di Scienze Economiche, University of Pisa, Via Ridolfi 10, 56100 Pisa Italy, e-mail: dfiaschi@ec.unipi.it and The Abdus Salam International Centre for Theoretical Physics, Strada Costiera 11, 34014 Trieste Italy, email: marsili@ictp.trieste.it

1 Introduction

In this paper we consider an economy where agents with different endowments are organized in local communities. They invest in education to increase their human capital. For each agent the effect of individual investment in education depends on the level of public expenditure and on the human capital of the agents who are in the same community. This framework aims at capturing the relevance in the accumulation of human of three aspects generally highlighted in the literature: (i) the investment in human capital can be limited by individual resources in presence of credit market imperfection (see Galor e Zeira (1993)), (ii) the benefit of participating to rich community, which can provide higher public services to education (see Bènabou (1996)) and (iii) the positive externality of staying in a community where aggregate human capital stock is higher and more equally distributed (see Durlauf (2003)).

We assume that there exists a complementarity among individual human capital stocks, so that the level of aggregate output depends on the distribution of human capital. This should reflect the intuitive fact that between two economies with the same aggregate level of stock of human capital the most productive will be the one where human capital stock is more evenly distributed (see Bènabou (1996) for more detailed discussion on this point).

The interactions among individual decisions crucially affect the dynamics of accumulation. Agents in more endowed community show a greater accumulation of human capital both for the higher level of public expenditure, when locally financed, and for the stronger positive externalities. Moreover, the human capital stocks of all members of a community tend to equalize, so improving the overall efficiency (because of complementarity in production). On the contrary, a stratified economy can hurt this equalizing process, leading to a lower long-run growth rate. Moreover, the presence of a segmented labour market can enhance the negative effects of such stratification by decreasing the return to human capital of agents in poor communities and providing less resources to devote to education.

We characterized the properties of long-run dynamics under different specifications of social organization.[1] A full integrated economy shows the highest long-run growth rate and the lowest income inequality. The method to finance public expenditure is relevant when economy is not integrated; in this case government should directly finance public expenditure because this increases the aggregate growth rate and reduces income inequality. A

[1] It is out of scope of this paper to endogenize the community structure as in Durlauf (1996).

segmented labour market, which is a likely event when economy is stratified, is a further channel by which inequality negatively affects the long-run growth rate. This is particularly relevant when investment in education is constrained by the individual income due to capital market imperfections and public investment is locally financed. The model's predictions find some corroboration in some econometric studies, as reported by Perotti (1996), Manski (2000) and Durlauf (2003).

Our model is close to Bènabou (1996). The main difference is that we consider the accumulation of human capital as the result of expenditure in education, while Bènabou (1996) as the result of an allocation of time. Thus in our setting more talented agents can be constrained in their investment in education. Moreover, we propose a new channel by which inequality negatively affects growth rate, the segmentation of labour market. Finally, we characterize the dynamics for a general community structure, while Bènabou (1996)'s analysis is limited to full integrated or totally stratified economy. We are also close in spirit to Durlauf (1996), but his main focus is to endogenize community structure. Manski (2000) provides a very stimulating discussion of models of social interactions. Durlauf (2003) provides both a theoretical and empirical survey of neighbourhood effects.

The paper is organized as following. Section 2 presents the model, Section 3 analyzes the dynamics and Section 4 concludes. Appendix gathers proofs.

2 The model

Consider an economy populated by $|I|$ agents partitioned in K communities P_k, $k = 1, \ldots, K$ and $I = \cup_k P_k$ is the set of all agents.[2] We shall label agents with $i \in I$ and denote the generic community by $P \subseteq I$. We shall generally use lower case letters for agents and upper case for communities.

At period t agent i is endowed with a stock of human capital $h_i(t)$. She lives one period and has to decide how to divide her income $y_i(t)$, derived by employing her human capital in production, between her consumption $c_i(t)$ and the expenditure in education for her son $e_i(t)$. The level $h_i(t+1)$ of human capital stock of her son depends on

- the private expenditure in education $e_i(t)$,

[2]This framework can represent various economic situations, both geographic organization (e.g. cities and suburbs) and social stratification (e.g. private schools only for rich agents). We take as given this partition. The endogenous determination of the latter is the subject of various contributions (see e.g. Durlauf (1996)).

- the public expenditure in education $E_G(t)$ which is financed by taxes collected within a community G, where G can be different from P, in particular $i \in G \supseteq P$. Here the cases of interest are: (i) community G includes all agents, i.e. $G = I$, which means that public education is financed by government and (ii) $G = P$, that is public education is locally financed.

- The positive externality of human capital arising from other members of her community P, represented by an index $H_P(t)$.

- A random component $a_i(t)$ representing idiosyncratic effects, as individual talents.

In particular we assume that:

$$h_i(t+1) = \mu a_i(t)\left[e_i(t)\right]^{\alpha}\left[E_G(t)\right]^{\beta}\left[H_P(t)\right]^{\gamma}, \quad i \in P \subseteq G \tag{1}$$

where μ is a scale factor; we assume that $\log a_i(t)$ is independently normally distributed with mean $-s^2/2$ and variance s^2.[3] We assume that $\alpha \in (0,1]$, $\beta \in (0,1]$, and $\gamma \in (0,1]$.

The level of public expenditure on education is independently decided and financed by each community G.[4] We assume that this expenditure is run on balanced budget and it is financed by a flat tax on income $\tau_G(t)$ imposed on all members of community G, so that:

$$E_G(t) = \frac{\tau_G}{|G|^{\sigma}} \sum_{i \in G} y_i(t), \equiv \tau_G Y_G(t) \tag{2}$$

where σ measures the congestion in the furniture of public education. In particular, the higher is σ the higher is this congestion. For $\sigma = 1$ the benefit for each individual is equal to the per-capita public expenditure of community G, so that public expenditure appears to be rival as well as a private good. Therefore empirical plausible value would be in the range $(0,1)$.[5] In Eq. (2) $y_i(t)$ is the income of agent i which is proportional to her

[3] We assume that $E[\log a_i] = -s^2/2$ since it implies that $E[a_i] = 1$. Thus we avoid a possible positive relationship between growth rate of human capital and variance of distribution of human capital induced by the assumption on the shape of random shocks (see Bènabou (1996)).

[4] In this expenditure we can include any factor which increases the productivity of investment in human capital, as building, equipment, etc..

[5] The case $\sigma > 1$ would be justified on the consideration that provision of public good in very large communities can be affected by a structurally inefficient due to asymmetric information. In this case σ could be a non linear function of G. Our analysis is easily extendable to such a case.

level of human capital

$$y_i = w_i h_i \tag{3}$$

where w_i is the wage, to be discussed below.

We assume that agents have log-utility, so that the problem of agent i which is in community P and pays taxes to government G is given by:[6]

$$\max_{c_i(t), e_i(t)} U_i = \log c_i(t) + \delta \log h_i(t+1)$$

$$s.t. \begin{cases} [1-\tau_G(t)]\, y_i(t) = c_i(t) + e_i(t); \\ y_i(t) = w_i(t)\, h_i(t); \\ h_i(t+1) = \mu a_i(t)\, [e_i(t)]^{\alpha} [E_G(t)]^{\beta} [H_P(t)]^{\gamma}; \\ E_G(t) = \tau_G(t)\, Y_G(t); \\ \log a_i(t) \sim N(-s^2/2, s^2). \end{cases}$$

By simple algebraic manipulation we get the indirect utility of agent i

$$V_i = \log\{[1-\tau_G(t)]\, w_i(t)\, h_i(t) - e_i(t)\} \tag{4}$$

$$+\, \delta \log\left\{\mu a_i(t)\, [e_i(t)]^{\alpha} [\tau_G(t)\, Y_G(t)]^{\beta} [H_P(t)]^{\gamma}\right\}; \tag{5}$$

Indirect utility V_i is maximized with respect to e_i for

$$e_i^*(t) = \left(\frac{\alpha\delta}{1+\alpha\delta}\right) [1-\tau_G(t)]\, w_i(t)\, h_i(t). \tag{6}$$

To close the model we have to specify how the wage w_i is determined, how fiscal policy of community is decided and the type of externality generated within a community.

2.1 Labour market

In order to distinguish the case of a single labour market for the whole economy from that of a labour market which is segmented, let us introduce the set L and assume that all agents $i \in L$ belong to the same labour market. We take $P \subseteq L \subseteq I$.

Following Bènabou (1996) we assume that individual human capital are complement in production. In particular, we assume that in a labour market

[6] In our formulation we implicitly assume that agents cannot borrow in the capital market to finance investment in education as it is standard in literature on human capital accumulation.

L the final output is given by:[7]

$$Z_L = \left[\sum_{i \in L} h_i^{\rho}\right]^{\frac{1}{\rho}}, \tag{7}$$

where ρ is the elasticity of substitution among inputs. In particular, $\rho = 1$ means perfectly substitutability among inputs (i.e. $Z_L = \sum_{i \in L} h_i$), $\rho = 0$ the Cobb-Douglas production function, and $\rho \to -\infty$ the Leontief production function (i.e. $Z_L = \min_{i \in L} \{h_i\}$). For $\rho \leq 1$ the isoquants are convex, that is, given the total stock of factors, higher equality in the individual stocks implies a higher output. On the contrary $\rho > 1$ implies concave isoquants, which implies that more unequal distributions show a higher output. In the rest of paper we follow the standard assumption in literature, i.e. $\rho \leq 1$, but the particular properties of human capital can justify also $\rho > 1$ (see Bènabou (1996)).

We assume that human capital is paid to its marginal productivity (constant returns to scale guarantees that all output is distributed):

$$w_i(t) = \frac{\partial Z_L(t)}{\partial h_i(t)} = Z_L(t)^{1-\rho} h_i(t)^{\rho-1} \;\; \forall i \in L. \tag{8}$$

The assumption on the size of labour market is crucial for our results. In fact, in the case of a common labour market, i.e. $L = I$, the latter is a channel by which the accumulation of human capital of all agents positively affects the level of education of any single agent. In particular, an increase in the aggregate income benefits all agents independent of the community structure by increasing the wage of all individuals and therefore their level of education. Notice that this can be also a consequence of an unique competitive market in the production of good which leads to equalizing the factors' prices.

However, the presence of different communities could also suggest that labour market is segmented, i.e. $L \subset I$.[8] For instance, if the pattern of consumption of poor agents is different from the one of rich agents and some goods are locally produced, e.g. personal services, then the same good can be sold at different prices and therefore the wage rates can be different between poor and rich communities, hence $L = P \subset I$.

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[7] Here we follow Bènabou (1996), in which the final output is produced by competitive firms with the technology:

$$Z = \left[\sum_{s} x_s^{\rho}\right]^{\frac{1}{\rho}},$$

where x_s is an intermediate input and every agents must specialized in a single output.

[8] For a theory on segmented labour market see Akerlof and Yellen (1990) and Kremer and Maskin (1996).

2.2 Fiscal policy

To determine the fiscal policy consider the preference of agent $i \in G$ with regard to $\tau_G(t)$. It is straightforward to show that indirect utility (5) is maximized for

$$\tau_G(t) = \tau^* \equiv \frac{\beta\delta}{1+\delta(\alpha+\beta)} \; \forall G \subseteq I, \forall t, \tag{9}$$

which is independent of the individual endowments. This implies that under any possible political rule Eq. (9) gives us the adopted fiscal policy in G.

The public expenditure in community G will depend on the level of wages of all agents which are in G; these wages, in turn, depend on the condition of job market (see (2) and (8)):

$$E_G(t) = \tau^* \frac{\sum_{i\in G} Z_L(t)^{1-\rho} h_i(t)^\rho}{|G|^\sigma} \tag{10}$$

$$= \tau^* \frac{Z_L(t)^{1-\rho} Z_G(t)^\rho}{|G|^\sigma} \; \forall G \subseteq L \subseteq I, \tag{11}$$

where we assumed that all agents in the same community G have to belong to the same labour market.

Therefore, if $P = G$, i.e. the public expenditure is locally financed, the richer communities will have a higher expenditure in education independent of labour market structure. When labour market is not segmented, i.e. $L = I$, the public expenditure for each community is positively related to the aggregate income and this plays a crucial role in the aggregate dynamics.

We notice that when labour market is segmented in K different markets, such that $L_k \subset G$, Eq. (11) still holds true as long as the size of each market is the same, i.e. $|L_k| = |I|/K$, and the latter is large enough. Indeed $Z_{L_k}(t)$ can be easily seen to depend on structural parameters (s^2, α, β, ...), which are assumed to be constant across the different markets, and on the size of the labour market. If $|L_k| = |I|/K \; \forall k$ and $|L_k|$ is large enough, then $Z_{L_k}(t)^{1-\rho}$ can be factored out of Eq. (10), so that Eq. (11), and the results following from it, again hold true even when $L_k \subset G$.

2.3 Externality within a community

Finally, we have to specify the index of human capital $H_P(t)$, which reflects the positive externality in the accumulation of human capital within a community. There is a large literature on the neighbourhood effects in education (see Durlauf (2003)); the main conclusions are that the higher is the average human capital stock and the more homogeneous is the distribution of individual human capital within a community, the more favourable is the environment to the accumulation of human capital. The following index satisfies these properties:

$$H_P = \frac{1}{|P|^{\nu}} \left[\sum_{i \in P} h_i^{\theta} \right]^{\frac{1}{\theta}}, \tag{12}$$

where $\theta \leq 1$; here larger θ means an higher substitutability among individual human capital stocks (more equal communities show higher positive externalities) and ν measures the degree of diffusion of the positive externality of human capital stocks among the members of the community P. In particular, the higher ν the lower is this diffusion. Heuristically parameter ν measure the scale effect of participating to a large community. Notice that H_P should be a not decreasing function of the size $|P|$ of the community, which implies $\nu\theta < 1$. In the following we assume that the latter condition is always satisfied.

3 Dynamics

3.1 The accumulation equation

Substituting the optimal expenditure in education (6), given the level of wage (8) and the level of tax rate (9), and the level of public expenditure (11), in the accumulation Eq. (1) we get:

$$h_i\,(t+1) = \frac{\tilde{\mu}}{|G|^{\beta\sigma}} a_i(t) Z_L^{(\alpha+\beta)(1-\rho)}(t) Z_G^{\beta\rho}(t) H_P^{\gamma} h_i^{\alpha\rho}(t), \forall i \in P \subseteq G \subseteq L \subseteq I, \tag{13}$$

where

$$\tilde{\mu} = \mu \left[\frac{\alpha\delta}{1+\phi\delta} \right]^{\alpha} \left[\frac{\beta\delta}{1+\delta\,(\alpha+\beta)} \right]^{\beta}$$

(notice that $\tilde{\mu} < \mu$).

Let (P, G, L) be denoted as the community structure of agent i (labour market is included in the definition of community structure for sake of simplicity).

In order to disentangle the dependence on the community structure it is convenient to introduce the notation

$$\langle F[h(t)] \rangle_X \equiv \frac{1}{|X|} \sum_{i \in X} F[h_i(t)]$$

for any set $X \subseteq I$. The idea is that in the limit of a very large community $|X| \to \infty$ averages over X satisfy laws of large numbers and can then be estimated easily. Then we can write

$$Z_X = |X|^{1/\rho} \left[\langle h^\rho(t)\rangle_X\right]^{1/\rho}, \qquad H_P = |P|^{1/\theta-\nu} \left[\langle h^\theta(t)\rangle_P\right]^{1/\theta}$$

and hence from (13) we get:

$$h_i(t+1) = \tilde{\mu} a_i(t)|G|^{\beta(1-\sigma)}|L|^{\frac{(\alpha+\beta)(1-\rho)}{\rho}}|P|^{\frac{\gamma}{\theta}-\gamma\nu}\langle h^\rho(t)\rangle_G^\beta \langle h^\rho(t)\rangle_L^{\frac{(\alpha+\beta)(1-\rho)}{\rho}}\langle h^\theta(t)\rangle_P^{\frac{\gamma}{\theta}} h_i^{\alpha\rho}(t), \tag{14}$$

which holds $\forall i,\ P,\ G,\ L$ such that $i \in P \subseteq G, L$.

The analysis is greatly simplified by setting:

$$m_i(t) = E[\log h_i(t)], \qquad x_i(t) = \log h_i(t) - m_i(t), \tag{15}$$

where $x_i(t)$ is the log of human capital stock of agent i normalized with respect to its expected level (i.e. $x_i(t)$ is the stochastic component of $\log h_i(t)$). Then, taking the logarithm of Eq. (14) we find

$$m_i(t+1) = v_{CS} + v_{IT} + (\alpha+\beta+\gamma)m_i(t), \tag{16}$$

where

$$v_{CS} = \log\tilde{\mu} + \beta(1-\sigma)\log|G| + \frac{(\alpha+\beta)(1-\rho)}{\rho}\log|L| + \gamma(1/\theta-\nu)\log|P| \tag{17}$$

and

$$v_{IT} = -\frac{s^2}{2} + \beta E[\log\langle e^{\rho x}\rangle_G] + \frac{(\alpha+\beta)(1-\rho)}{\rho}E[\log\langle e^{\rho x}\rangle_L] + \frac{\gamma}{\theta}E[\log\langle e^{\theta x}\rangle_P], \tag{18}$$

where $i \in P \subseteq G, L$.

We are now in a position to analyze the long run properties of the model. We notice that v_{CS} depends on the community structure (P, G, L) ,where $i \in P \subseteq G, L$, while all stochastic elements are included in v_{IT}.

3.2 Preliminary results

Firstly we state the conditions under which the expected growth rate of human capital of agent i grows at a constant positive growth rate:

Proposition 1 *Suppose that* $\alpha+\beta+\gamma=1$. *Then the long-run growth rate of human capital* v *for each agent* $i \in P \subseteq G, L$ *is given by:*

$$v \equiv E\left[\log\frac{h_i(t+1)}{h_i(t)}\right] = v_{CS} + v_{IT}.$$

For $\alpha+\beta+\gamma \neq 1$ *the long-run growth rate is either zero or infinity.*

Proof. This is evident from Eq. (16). In particular $m_i(t) = m_i(0) + vt$. ■

We shall concentrate henceforth on the case $\alpha+\beta+\gamma=1$.

The next Proposition states that the asymptotic distribution of the detrended human capital stocks $x_i(t)$ is normal:

Proposition 2 *The accumulation Eq. (14) with* $\alpha+\beta+\gamma=1$ *and* $\alpha\rho<1$ *leads to normal distribution of the logarithm of detrended human capital* $x_i(t)$ *for every initial distribution of individual human capital stocks. The variance of* $x_i(t)$, *in the long run, is*

$$V[x_i(t)] = \frac{s^2}{1-\alpha^2\rho^2}.$$

Proof. The evolution equation of $x_i(t)$ is

$$x_i(t+1) = \alpha\rho x_i(t) + \xi_i(t)$$

where $\xi_i(t) = \log(a_i(t)) + s^2/2$ is a Gaussian variable with mean zero and variance s^2. Then,

$$x_i(t) = \sum_{q=0}^{t-1}(\alpha\rho)^q\xi_i(t-q) + (\alpha\rho)^t x_i(0)$$

which is Gaussian, because it is a sum of Gaussian variables, and it is independent of $x_i(0)$ when $t\to\infty$ as the last term vanishes in this limit because $\alpha\rho<1$. The variance of $x_i(t)$ is easily computed from the above formula. ■

As a consequence (see Marsili et al. (1998)),

Corollary 3 *For very large community sizes,* $|X| \gg 1$, *the quantity* $\langle e^{\eta x(t)}\rangle_X$ *satisfies the Central Limit Theorem, and is well approximated by*

$$\langle e^{\eta x(t)}\rangle_X \cong e^{\frac{\eta^2 s^2}{2(1-\alpha^2\rho^2)}}\left[1+\sqrt{\frac{e^{\frac{\eta^2 s^2}{1-\alpha^2\rho^2}}-1}{|X|}}\zeta(t)\right] \tag{19}$$

where $\zeta(t)$ *is a Gaussian variable with zero mean and unit variance.*

Taking the logarithm and the expected value[9] we find that, asymptotically as $|X| \to \infty$,

$$E\left[\log\langle e^{\eta x(t)}\rangle_X\right] \cong \frac{\eta^2}{2}\frac{s^2}{1-\alpha^2\rho^2} - \frac{e^{\frac{\eta^2 s^2}{1-\alpha^2\rho^2}} - 1}{2|X|} \tag{20}$$

where terms which vanish faster than $1/|X|$ have been disregarded.

We shall focus on the case where community sizes are very large henceforth. Therefore we shall use the leading term in the asymptotic limit of infinite community sizes $|G|, |L|, |P| \to \infty$, with the understanding that the finite size of the communities only affects the results by corrections which are of the order of one over the community size.

Eqs. (16), (17) and (18) suggest to separate the analysis of the effect of community structure and of financing of public expenditure from the effect of the random component in the human capital accumulation, representing individual talents. In fact, we notice that when $|G|, |L|, |P| \to \infty$:

$$v_{IT} = -\frac{s^2}{2}\left[\frac{1-\rho(\alpha+\beta)-\theta\gamma}{1-\alpha^2\rho^2}\right], \tag{21}$$

which means v_{IT} is independent of the community structure, that is

Remark 4 *The effects on the expected growth rate of agent i's human capital v of community structure and random component are, respectively, given by v_{CS} and v_{IT}.*

Following Remark 4 first we analyze the effect of community structure on the growth rate v.

3.3 Community structure

The following Proposition states the relationships between the expected growth rate of agent i's human capital and the community structure:

Proposition 5 *The growth rate v is an increasing function of the size $|P|$ of the community, of the size $|L|$ of labour market and of the size $|G|$ for $\sigma < 1$. Hence the highest possible expected growth rate of agent i's human capital attains when $P = G = L = I$, i.e. the labour market is not segmented and when education is financed by the government.*

[9] Taking the logarithm is possible only if Eq. (19) is positive. This requires a truncation of the support of the distribution of $\zeta(t)$ so that the probability that $|\zeta(t)| > K|X|^\epsilon$ is zero, for some $K > 0$ and $\epsilon > 0$. This is not problematic, as the Central Limit Theorem only concerns the central part of the distribution of $\zeta(t)$. For our purposes if $\epsilon < 1/2$, for any K there is a $|X|$ large enough that the argument of the logarithm never becomes negative.

Proof. Growth rate v is defined by Proposition 1, where v_{CS} is given by (17) and v_{IT} by (21). Then $\partial v/\partial P$, $\partial v/\partial G$ and $\partial v/\partial L$ are always greater than zero for $\nu\theta$, ρ and $\sigma < 1$. ■

The dimension of community structure of agent i, i.e. the number of agents with whom agent i is interacting with, crucially affects the growth rate of her human capital. In the literature this phenomenon is called *scale effect*. We could eliminate the latter effect by setting $\frac{1}{\theta} = \nu$ (this eliminates the scale effect due to the externality of human capital within a community), $\sigma = 1$ (this eliminates the scale effect due to the public expenditure in education) and $\rho = 1$ (this eliminates the scale effect due to the labour market). It is worth to remark that in this model these scale effects are an endogenous result of the economic and social interactions among the agents.

In the case of full integrated economy, i.e. $P = I$, then $P = G = L$ and growth rate of human capital is equal for each agent; therefore v is also the growth rate of aggregate economy. Proposition 5 states that this setting corresponds to the case of maximum aggregate growth rate.

Bènabou (1996) assumes that the effect of public education is equally shared among members of communities, i.e. $\sigma = 1$. In such a case in our model the effect of different financing system of public expenditure disappears. Moreover, he sets $\nu = 1$, so that his result that the most efficient community structure is given by $P = I$ is not surprising.

The size of labour market $|L|$ as determinant of growth rate of human capital is a novelty with respect to the existing literature. This phenomenon is the sum of two single effects: (i) the direct effect, measured by α, of a lower wage in smaller (less productive) labour market, which provides less resources for the investment in education of sons, and (ii) the lower aggregate efficient, measured by β, which provides less resource for the public investment in education. Of course, both effects disappears if composition of work force is not relevant, i.e. $\rho = 1$ (perfect substitutability between different individual human capitals).

3.4 Individual talents

The random component can be interpreted as the individual talent of each agent. A greater inequality (variance) of such variable among agents means a lower growth rate of human capital of agent i:

Proposition 6 *The growth rate v is a decreasing function of the variance s^2 of the random component in the accumulation equation.*

Proof. Firstly notice s^2 only enters in v_{IT}. The latter is an increasing function of θ (see (21)). For $\theta = 1$, v_{IT} is a decreasing function of s (remember that $\gamma = 1 - \alpha - \beta$). The same must be true for $\theta < 1$. ■

Given the linear relationship between s^2 and $V[x_i]$ (see Proposition 2), we should expect a negative empirical relationship between growth rate and income inequality. This fact is found in many empirical works (see for example Perotti (1996)). It is interesting to remark that the higher is the substitutability between individual human capital (i.e. the higher ρ) the higher is the variance of the observed human capital distribution.

The economic intuition is straightforward: the technology and the shape of externality favour an economy with similar agents and penalize economy (community) where agent are very heterogeneous. The remarkable result is that inequality determines a *growth effect* and not only a *level effect*. The dependence of v_{IT} on ρ and θ confirms this intuition; indeed, in Eq. (21) setting $\rho = \theta = 1$, i.e. by eliminating the negative effects of inequality on production and on externality in the human capital accumulation, we get that $v_{IT} = 0$. This result shows that α, β and γ only magnifies the effect of ρ and θ. As it will be shown below, the effect of greater inequality on overall growth rate of economy is fostered in a stratified economy, since most talented agents can be rationed in their investment in education.

3.5 Stratified economy

A stratified economy is an economy where there is more than one community, education is locally financed and where labour market is segmented, e.g. when $P = G = L \subset I$. We can apply to each communities the results of the previous sections. Therefore community k will have a growth rate $v^{(k)} = v_{CS}^{(k)} + v_{IT}^{(k)}$ depending on its community structure ($P^{(k)}$, $G^{(k)}$ and $L^{(k)}$) and on the volatility $s^{(k)}$. In principle, one could also envisage a dependence of the parameters $\alpha, \beta, \theta, \nu$ on dynamics of community k. The following Proposition states the aggregate long-run properties of a stratified economy:

Proposition 7 *Let $v^{(k)}$ be the average long-run growth rate of human capital in community k. The growth rate of the economy in the long run v^{AE} equals that of the fastest growing community, that is*

$$v^{AE} = \max_k v^{(k)} \tag{22}$$

Proof. See Appendix A. ■

Given that $v^{(k)}$ is an increasing function of P, G and L and $P \subseteq G, L$, while $v_{IT}^{(k)} = v_{IT}\ \forall k$ we have that:

Corollary 8 *The growth rate of the whole economy v^{AE} equals that of the largest community structure, that is $v^{AE} = v_{IT} + \max_k v_{CS}^{(k)}$.*

In the following we analyze two particularly interesting issues related to stratified economies: (i) the method to finance public education and (ii) the segmentation of labour market.

3.5.1 Locally vs centrally financed public education

Smaller communities show a lower growth rate, so that we observe a ever-increasing gap between per-capital income of communities with different community structure. In this case education financed by the government, i.e. $G = I$, can decrease such inequality among communities. Moreover, since v_{CS} is increasing in G, this also increases the aggregate growth rate (see Corollary 8). Therefore, locally financing of education can lead both to a lower growth rate and to an increase in the inequality among communities.[10]

3.5.2 Segmented labour market

The segmentation of labour market introduces a source of inefficiency in the economy. This is particularly relevant for stratified economy, because the lack of capital market reduces the resources that agents in small community structure can invest in the education of their sons. To focus on the labour market structure assume that public education is financed by government, i.e. $G = I$. We discussed that Eq. (11) still holds also for $L \subset G$ if there exists K different communities with different labour markets but of equal size, i.e. $|L_k| = |I|/K$. Therefore we can apply Corollary 8 and conclude that in a stratified economy aggregate growth rate v^{AE} is lower than in an economy with a common labour market. In the case public expenditure were locally financed v^{AE} will be even lower.

3.6 Numerical simulations

In this section we simulate the effects of different community structures on long-run growth rate of economy in order to test our theoretical results. In the numerical simulations we use the following parameters' values. The results in Borjas (1995) suggest to set $\alpha = 0.24$ and $\gamma = 0.20$, from which $\beta = 0.56$. The other parameters are set to the following values: $\sigma = 0.9$, $\rho = 0.8$, $\theta = 0.9$, $\nu = 1$, $\tilde{\mu} = 0.03$ and $s = 0.085$.[11] Finally, we assume that there is a new generation every 25 years.

[10] Bènabou (1996) considers only the first effect and neglects the effect of public expenditure on aggregate income inequality.

[11] The latter parameter is calculated using the italian income distribution, while the other are set in order to get a plausible long-run growth rate. Codes for simulations are available on the author's website (http://www-dse.ec.unipi.it/fiaschi).

First of all we test if a finite sample introduces a significative bias in our theoretical results. Figures 1 and 2 report the comparison between theoretical annual growth rates (solid circle) calculated by Proposition 1 and the simulated annual growth rates (circle) for simulations of 10 periods (in these simulations we use $\tilde{\mu} = 0.3$).

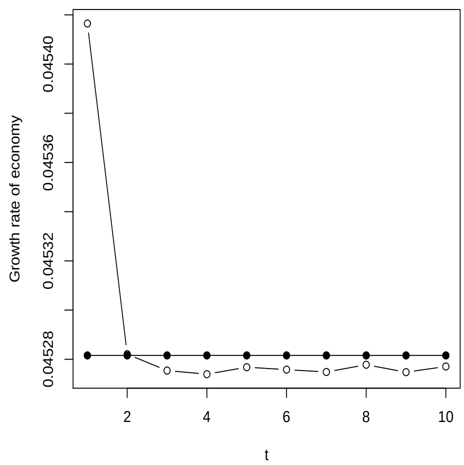

Figure 1: Theoretical growth rates (solid circle) vs simulated growth rates (circle) for centrally financed education

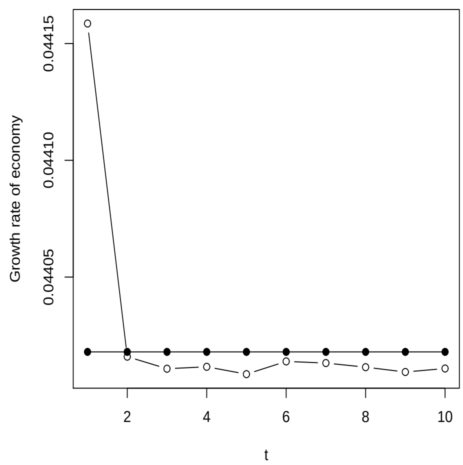

Figure 2: Theoretical growth rates (solid circle) vs simulated growth rates (circle) for locally financed education

We considered an economy composed by 560.000 individuals partitioned into 5 regions and 350 communities of the same size. Labour markets always coincides with the regions. Figure 1 reports the results for an economy where public expenditure is financed by Government, while Figure 2 for an economy where public expenditure is financed by each regions. Both figures show that the bias is very low and negative, as we expected (see Eq. (20)).[12]

To test the magnitude of the effect on growth rate of different community structures we consider an economy with 56 millions of persons and 20 regions of the same size. If not specified, labour markets coincide with the regions. Finally, we suppose that there are several communities P, whose cardinality is equal to 20000 (a small town or a city quarter). In such setting from Proposition 1 together with Eqs. (17) and (21) we find that a centrally financed public education determines an annual growth rate of 2.01%. If public education is financed by the single regions, annual growth rate decreases

[12] It is worth remarking that numerical standard deviations of detrended distribution of log of human capital stocks show a very low bias with respect to the theoretical ones of Proposition 2.

to 1.67%. This means that to decentralize the public education would imply a loss of about 0.4% of the annual growth rate. To test the significance of this finding we consider alternative values of σ (the crucial parameter measuring the congestion of public education) and report the results in Figure 3. Locally financed public education (solid circles in the Figure 3) always shows lower growth rates.

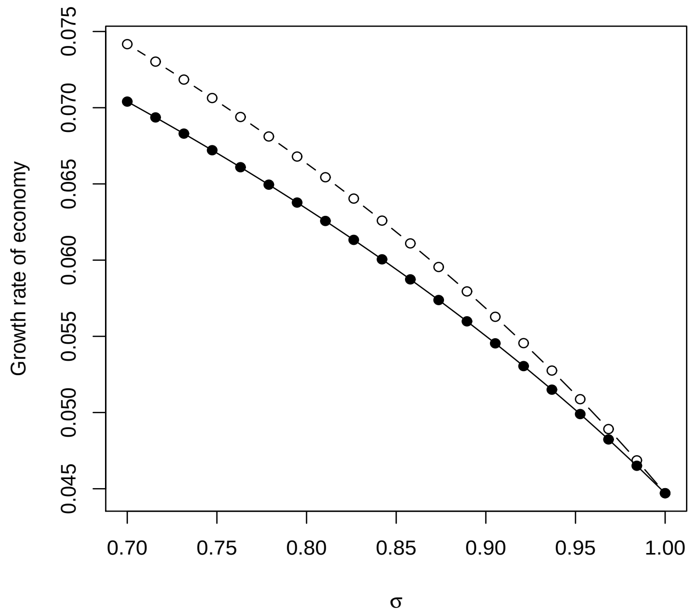

Figure 3: The effect on long-run growth rate of a different degree of congestion of public education (solid circle is related to locally financed public education)

We find a confirmation that a decrease in σ (a decrease in the congestion of public expenditure) increases the negative effect of a locally financed public education.

Another interesting issue is the effect of segmentation of labour market on the long-run growth rate. Figure 4 reports the results for the cases both of locally and of centrally financed public education. Centrally financed public education (solid circles in the Figure 4) always shows higher growth rates.

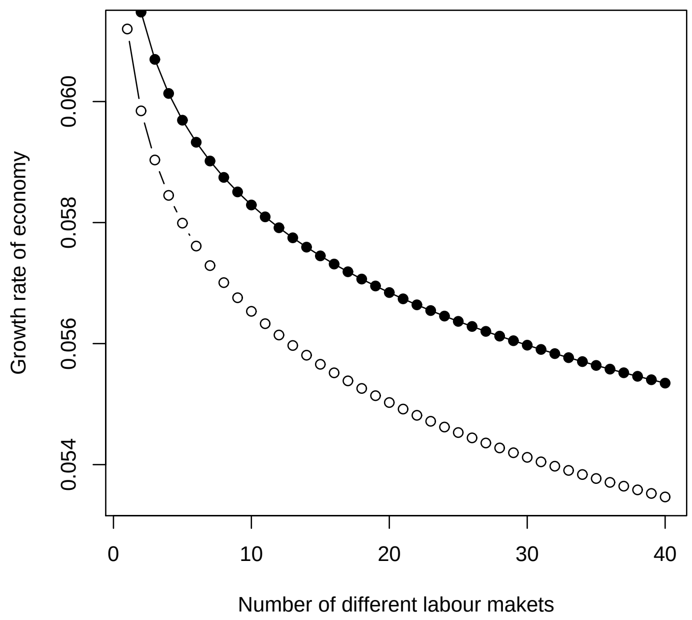

Figure 4: The effect on long-run growth rate of a different segmentation of labour market (solid circle is related to centrally financed public education)

Segmentation of labour market appears to have a strong impact on the long-run growth rate: for centrally financed public education the difference between one labour market and 40 different labour markets is 1.6%. This impact is magnified by a public education financed by the regions: the difference between one labour market and 40 different labour markets becomes 1.75%.

4 Conclusions

We have explored the properties of an economy populated by agents with heterogeneous endowments of human capital under different settings. The main results are that the institutional settings, as the segmentation of labour market and the method to finance public expenditure, have effects on the long-run growth rate. In particular, a stratified economy has always a lower growth rate than a fully integrated economy. In a stratified economy public expenditure financed by the government can both increase the long run growth rate of economy and decrease the income inequality of agents of different communities. Finally, the segmentation of labour market can decrease the growth rate of economy because of both less efficient allocation of resources and lower resources that agents can devote to the education of their sons. By simple numerical simulations we have shown that these effects can have a relevant impact on long-run growth rate of a country.

References

Akerlof, G. and J. Yellen (1990), The Fair Wage-Effort Hypothesis and Unemployment, *Quarterly Journal of Economics*, CV, 255-283.

Bènabou, R. (1996), Heterogeneity, Stratification, and Growth: Macroeconomic Implications of Community Structure and School Finance, *American Economic Review*, 86, 584-609.

Bènabou R. (1996), Equity and Efficiency Human Capital Investment: The Local Connection, *Review of Economic Studies*, 63, 237-264.

Borjas, G. (1995), Etchnicity, Neighborhoods, and Human Capital Externalities, Amercian Economic Review, 85, 365-390.

Durlauf, S. (1996), A Theory of Persistent Inequality, *Journal of Economic Growth*, 1, 75-93.

Durlauf, S. (2003), Neighbourhood Effects, forthcoming in *Handbook of Regional and Urban Economics*, vol. 4, Economics, J. V. Henderson and J. F. Thisse, eds.

Galor O. e Zeira J. (1993), "Income Distribution and Macroeconomics", *Review of Economic Studies,* vol. 60, pp. 35-52.

Lucas R.E. (1988), "On the Mechanics of Economic Development", *Journal of Monetary Economics*, vol. 22, pp. 3-42.

Kremer M. and E. Maskin (1996), working paper NBER, n.5718.

Manski, C. (2000), Economic Analysis of Social Interactions, *Journal of Economic Perspectives*, 14, 115-136.

Marsili, M., S. Maslov and Y. Zhang (1998), Dynamical Optimization Theory of a Diversified Portfolio, *Physica A253* (1-4): 403-418.

Perotti, R., 1996. Growth, income distribution and democracy: what the data say. Journal of Economics Growth 1, 149-187.

A Proof of Proposition 7

Here we prove that in a stratified economy the long-run growth rate is equal to the growth rate of the most performing community.

Consider a set of independent log-normal processes $X_i(t)$ with different growth rates r_i and variances σ_i^2

$$X_i(t) = e^{r_i t + \sigma_i W_i(t)}$$

where $W_i(t)$ are iid Gaussian variables with zero average and variance t.

Then, in the long run, the process

$$X(t) = \sum_i X_i(t)$$

has the growth rate of the fastest process $X_i(t)$

$$\lim_{t\to\infty} E[\log X(t)]/t = r \equiv \max_i r_i$$

Proof: Let $r_0 > r_i$, for all i be the fastest process. Then

$$E[\log X(t)]/t = r_0 + E\left[\log\left(1 + \sum_{i\neq 0} e^{X_i(t)-X_0(t)}\right)\right]$$

Now call

$$Q = \sum_{i\neq 0} e^{X_i(t)-X_0(t)}$$

It is easy to show by direct calculation that, for any $\epsilon > 0$, $P(Q > \epsilon) \to 0$ as $t \to \infty$. Likewise $P(Q \in [q, q+dq)) = p(q)dq$ vanishes as $t \to \infty$ for all $q > \epsilon$. Then

$$E\left[\log\left(1+Q\right)\right] = \int_0^\infty \log(1+Q)p(Q)dQ =$$

$$= \int_0^\epsilon \log(1+Q)p(Q)dQ + \int_\epsilon^\infty \log(1+Q)p(Q)dQ \leq \log(1+\epsilon)[1 - P(Q > \epsilon)]$$

$$+ \int_\epsilon^\infty \log(1+Q)p(Q)dQ \to \log(1+\epsilon)$$

which vanishes as $\epsilon \to 0$.

The intuition is that

$$e^{X_i(t)-X_0(t)} = e^{-(r_0-r_i)t+\sigma_i W_i - \sigma_0 W_0}$$

which is dominated by the factor $e^{-(r_0-r_i)t}$ for $t \to \infty$.