The US Phillips Curve and inflation expectations: A State Space Markov-Switching explanatory model[*]

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Abstract

This paper proposes a new empirical representation of US inflation expectations in a Stace-Space Markov-Switching framework in order to identify the expectations regimes which are associated with short and long term Phillips curves.

We explicitly identify the dynamic of inflation expectation errors using the expectations augmented Markov-switching Phillips curve as a measurement equation. In this paper we consider expected inflation as an underlying component of observed inflation. We thus use the same type of specification (occasionally integrated) to describe its dynamic.

We have found that dynamics of inflation expectation errors change across regimes. For the last 20 years we show the Phillips curve is vertical and associated with rational inflation expectations. Whereas for the period of economic instability (1973-1983) a negative Phillips curve is associated with adaptive expectations.

keywords: State-Space Markov-Switching model; Inflation expectation errors; Phillips curve; occasionally integrated process

JEL: E4, C32, C51

1 INTRODUCTION:

Inflation expectations play a central role in many different macroeconomic contents. Usually, strong hypotheses about them are formulate as they are not directly observed.

On the Phillips Curve, the "traditionnal" approach by Hibbs (1977) includes adaptive expectations. Whereas Sargent (1969) proposed a "rational" version of former models in which he incorporates rational expectations pioneered by Muth (1961).

As these expectations hypotheses have different implications for the theory, it seems important to know which ones are the more plausible. This leads to the following question: How to gauge inflation expectations? While there is a vast litterature on this topic no consencus has emerged among empiricists on how to measure these subjectives magnitudes. A first approach is to try to infer the expected inflation rate from prices of financial instruments (Bank of Canada, 1998; Mylonas and Schich, 1999). An alternative approach is to use quantitative information on inflation expectations from qualitative survey data (Carlson and Parakin (1977), Bakhsi and Yates (1998)).

In our article we are going to adopt a different approach. We will estimate explicitly (following Kim(1994) methodology) the inflation expectations process using a State-Space (SS) Markov-Switching (MS) model. Measurement equations which link observable with unobservable components are expectation errors and Phillips curve equations. The Phillips Curve is introduced in order to help to identify expectation errors. State equations (unobservable equations) are inflation expectations and natural rate unemployment specifications. We consider in this paper that, since inflation expectations is an underlying component of inflation rate, its behaviour should be the same.

How can we test expectations hypotheses in this framework? Our intuition is the following: if agents were rational, when a shift occurs in the economy, they should integrate it in their expectations, which should move the dynamics of inflation expectations. This imply that shift should leave expectation errors dynamic unchanged. Secondly rational expectations imply the process of expectation errors is white noise. Our Markov-Switching model in which the process of expectation errors is allowed to change across regimes will enable us to check these two points.

Moreover, in order to test which expectation regimes are associated with short and long run Phillips curves, we allow the slope of the Phillips Curve to change with expectation regimes.

The rest of this paper is organized as follows. In section 2 we show in a theoretical framework how inflation expectations play a role in the Phillips curve. In a third part, we will set out the econometric models: we first specify univariate representation for inflation rate, then we present the SS-MS model. Empirical results are presented in section 4. Section 6 concludes.

2 Inflation expectations in a regime-switching inverted Phillips curve:

Let us see now the role of expectation errors in the Phillips curve.

The Phillips curve in its modern form identifies three sources of inflation:

- inflation expectations: Π_t^e

- unemployment gap: $U_t - \bar{U}_t$. Where $\bar{U}_t$ is the natural rate of unemployement.

- supply shocks: v_t

These three factors are encompassed in the following equation:

$$\Pi_t = \Pi_t^e - \delta(U_t - \bar{U}_t) + v_t \tag{1}$$

with δ positive. For modeling convenience it will be preferable to rewrite the relation between unemployment and inflation in its reversed form as in the following :

$$U_t - \bar{U}_t = \lambda(\Pi_t - \Pi_t^e) + \eta_t \tag{2}$$

where $\lambda = -\frac{1}{\delta}$ is negative.

When agents have adaptive expectations, arbitrage between inflation and unemployment exists in the short run. For those who believes in rational expectations there isn't any arbitrage between inflation and unemployment even in the short run. And unemployment is at its natural rate. In order to take into account long run and short run Phillips curves we allow for a regime-switching λ. We rewrite (2) as:

$$U_t - \bar{U}_t = \lambda_{S_t}(\Pi_t - \Pi_t^e) + \eta_t \tag{3}$$

where S_t =(1,0) can be seen as the regime of inflation expectations at t. Let us assume that $S_t = 0$ is the regime of adaptive expectations. Then we expect $\lambda_0 < 0$. In the case of rational expectations ($S_t = 1$) we expect $\lambda_1 = 0$.

3 Econometric analysis:

3.1 Univariate representation of inflation :

Two unobservable components (inflation expectations and unemployment rate) are present in the Phillips curve. How could we specify them? In this article we consider inflation expectations as an underlying component of observed inflation. As a consequence we will use the same specification to describe its dynamic.

We consider the following representation for inflation rate:

$$\Delta\Pi_t = \mu_{S_t} + (\rho_{S_t} - 1)\Pi_{t-1} + \sum_{i=1}^{k} \phi_{iS_t}\Delta\Pi_{t-i} + \theta_{S_t}\eta_t \tag{4}$$

$$\mu_{S_t} = \mu_0(1 - S_t) + \mu_1 S_t$$

$$\rho_{S_t} = \rho_0(1 - S_t) + \rho_1 S_t$$

$$\phi_{iS_t} = \phi_{i0}(1 - S_t) + \phi_{i1} S_t$$

$$\theta_{S_t} = \theta_0(1 - S_t) + \theta_1 S_t$$

$$\eta_t \rightarrow N(0, 1)$$

$$S_t = 1, 0$$

$$p = \Pr(S_t = 1/S_{t-1} = 1)$$

$$q = \Pr(S_t = 0/S_{t-1} = 0)$$

Constant μ, the coefficient for persistence ρ, autoregressive parameters ϕ_i and volatility θ may change with the regime S_t which follows a first order Markov process. This model allows us to test later if an unit root is present within regimes ($\rho_{S_t} = 1$)[1].

3.2 State-Space Markov-Switching model of inflation expectations:

We will propose in this section to identify inflation expectations in a State-Space Markov-Switching framework. This framework enable us to take into account switching in the inflation expectations (which are unobservable) process.

[1] The same type of model was first proposed by Ang and Beckaert (1998) to describe the behaviour of interest rate

The model is the following:

Measurement equations

$$\Pi_t - \Pi_t^e = \alpha_{S_t} + \gamma_{S_t}(\Pi_{t-1} - \Pi_{t-1}^e) + \sigma^{\Pi}\varepsilon_t \quad (5)$$

$$U_t - \bar{U}_t = \lambda_{S_t}(\Pi_t - \Pi_t^e) + \sigma^U \eta_t \quad (6)$$

$$\varepsilon_t \rightarrow iidN(0,1) \quad (7)$$

$$\eta_t \rightarrow iidN(0,1) \quad (8)$$

Constant α and the autoregressive coefficient γ may be different accross regimes. In this way we allow for different type of inflation expectations. Let us assume $S_t = 0$ is the regime of adaptive expectations. Then we expect γ_0 significantly different from zero. For $S_t = 1$ (regime of rational expectations) we expect $\alpha_1 = \gamma_1 = 0$. These constraints will be tested.

The inverted Markov-Switching Phillips curve developped in section 1 is added in order to help identify expectation errors and to see if adaptive expectations are associated with a short run curve ($\lambda_0 < 0$) and if rational expectations are associated with a long run one ($\lambda_1 = 0$).

Π_t^e (inflation expectations) and $\bar{U}_t$ (natural rate of unemployement) are unobservable components. We specify them in the following state representation:

State equations

$$\Pi_t^e = \mu_{S_t}^{\Pi^e} + \rho_{S_t}^{\Pi^e}\Pi_{t-1}^e + \sigma_{S_t}^{\Pi^e}\varepsilon_t^{\Pi^e} \quad (9)$$

$$\bar{U}_t = \bar{U}_{t-1} + \sigma^{\bar{U}}\varepsilon_t^{\bar{U}} \quad (10)$$

As inflation expectations are considered in this paper as an underlying component of observed inflation rate we choose a similar specification. The specification of the unemployment rate and the calibration for $\sigma^{\bar{U}}$ are choosen as in Gordon (1997).

The State Space representation is the following:

Measurement equations

$$\begin{bmatrix} \Pi_t \\ U_t \end{bmatrix} = \begin{bmatrix} \alpha_{S_t} \\ \lambda_{S_t}\alpha_{S_t} \end{bmatrix} + \begin{bmatrix} \gamma_{S_t} \\ \lambda_{S_t}\gamma_{S_t} \end{bmatrix} \Pi_{t-1} \quad (11)$$
$$+ \begin{bmatrix} 1 & -\gamma_{S_t} & 0 \\ 0 & -\lambda_{S_t}\gamma_{S_t} & 1 \end{bmatrix} \begin{bmatrix} \Pi_t^e \\ \Pi_{t-1}^e \\ \bar{U}_t \end{bmatrix} + \begin{bmatrix} \sigma^{\Pi} & 0 \\ \lambda_{S_t}\sigma^{\Pi} & \sigma^{U} \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ \eta_t \end{bmatrix}$$

$$\begin{aligned} \varepsilon_t &\rightarrow iidN(0,1) && (12) \\ \eta_t &\rightarrow iidN(0,1) && (13) \end{aligned}$$

State equations

$$\begin{bmatrix} \Pi_t^e \\ \Pi_{t-1}^e \\ \bar{U}_t \end{bmatrix} = \begin{bmatrix} \mu_{S_t}^{\Pi^e} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \rho_{S_t}^{\Pi^e} & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Pi_{t-1}^e \\ \Pi_{t-2}^e \\ \bar{U}_{t-1} \end{bmatrix} + \begin{bmatrix} \sigma_{S_t}^{\Pi^e} \varepsilon_t^{\Pi^e} \\ 0 \\ \sigma^{\bar{U}} \varepsilon_t^{\bar{U}} \end{bmatrix}$$

$$\begin{aligned} \varepsilon_t^{\Pi^e} &\rightarrow iidN(0,1) && (14) \\ \varepsilon_t^{\bar{U}} &\rightarrow iidN(0,1) && (15) \end{aligned}$$

The regime ($S_t = 1, 0$) follows a first order Markov process and the transition probabilities are the following:

$$\begin{aligned} q &= \Pr(S_t = 0/S_{t-1} = 0) && (16) \\ p &= \Pr(S_t = 1/S_{t-1} = 1) && (17) \end{aligned}$$

4 Empirical results:

The analysis is driven on quarterly US data for the estimation period: 1973:02-2003:03. Data are US Consumer Price Index and unemployment rate.

4.1 Univariate representation of inflation rate:

For the following we pose : $\beta_{S_t} = \rho_{S_t} - 1$. Results of the univariate model (4) estimation are presented in Table 1. After seing the result ($\hat{\beta}_0 = -0.70$ with a t-stat=-2.95) we would like to check if the inflation process in regime 0 is stationary?

The critical values of the distribution of $(\frac{\hat{\beta}_0}{\hat{\sigma}(\hat{\beta}_0)})$ under the null $H_0 : \beta_0 = 0$ are computed by Monte Carlo as in presence of switching regimes the distribution is not known.

4.1.1 Simulation method:

The data generative process is G:

$$\Delta y_t = (\hat{\mu}_0 + \sum_{i=1}^{k} \hat{\phi}_{i0} \Delta y_{t-i} + \hat{\theta}_0 \eta_t)(1 - S_t) + (\hat{\mu}_1 + \sum_{i=1}^{k} \hat{\phi}_{i1} \Delta y_{t-i} + \hat{\theta}_1 \eta_t) S_t \tag{18}$$

$$\eta_t \to N(0,1)$$

$$\hat{p} = \Pr(S_t = 1/S_{t-1} = 1)$$

$$\hat{q} = \Pr(S_t = 0/S_{t-1} = 0)$$

where $\hat{\mu}_{S_t}, \hat{\phi}_{iS_t}, \hat{\theta}_{S_t}, \hat{p}, \hat{q}$ are estimated coefficients from model (6).

And we estimate the following model E:

$$\Delta y_t = (\mu_0 + \beta_0 y_{t-1} + \sum_{i=1}^{k} \phi_{i0} \Delta y_{t-i} + \theta_0 \eta_t)(1 - S_t) + (\mu_1 + \sum_{i=1}^{k} \phi_{i1} \Delta y_{t-i} + \theta_1 \eta_t) S_t \tag{19}$$

$$\eta_t \rightarrow N(0,1)$$

$$p = \Pr(S_t = 1/S_{t-1} = 1)$$

$$q = \Pr(S_t = 0/S_{t-1} = 0)$$

The t-stat of $\tilde{\beta}_0$ of the i-th replication under the null ($\beta_0 = 0$) is $(\frac{\tilde{\beta}_0}{\tilde{\sigma}(\tilde{\beta}_0)})$ where $\tilde{\sigma}(\tilde{\beta}_0)$ (the standard deviation of $\tilde{\beta}_0$) is calculated with a numerical procedure. Let us define the two following hypotheses:

H_0 : the process is integrated in both regimes ($\beta_0 = \beta_1 = 0$)

H_1 : the process is occasionnally integrated ($\beta_0 < 0, \beta_1 = 0$)

To test H_0 against H_1 we generate G and estimate E. Results of our simulation is reported in appendix in table 2. State 0 of inflation rate : H_0 is rejected for a risk level of 2.5% (t–stat=-2.95<-2.41). The process is occasionally integrated.

4.2 State-Space Markov-Switching representation:

As we found an occasionally process for inflation, one state for inflation expectations process is assumed to be integrated. We pose in (9):

$$\rho_1^{\Pi^e} = 1 \tag{20}$$

But for the other state we leave the persistence parameter free of variation. For the natural rate of unemployment we fix (as in Gordon(1997)) $\hat{\sigma}^{\bar{U}} = 0.2$.Table 3 in appendix shows the result of the estimation of the SS-MS model.

4.2.1 State classification:

State 0: In this state, agents seem to have rational expectations ($\hat{\alpha} \simeq \hat{\gamma} \simeq 0$). The Phillips curve is vertical ($\hat{\lambda} \simeq 0$). Figure 1 in appendix shows this state coincides with period of economic stability for the last 20 years (except in the early 1990s and 2000s). According to Figure 2 the unemployment rate is very closed to the natural rate for this period.

State 1: In this state, expectation errors are quite persistent ($\hat{\gamma} = 0.91$). There is a dilemma between inflation and unemployment ($\hat{\lambda} = -0.33$). This regime can be interpreted as a regime where adaptive expectations are associated with a short run negative Phillips curve. According to Figure 1 this state coincides with Volker and oil shocks periods.

4.3 Test of restrictions:

Rational expectations imply $\hat{\alpha} = \hat{\gamma} = 0$. That seems to be the case in state 0. Morever, as we want to check if rational expectations are associated with a vertical Phillips curve we will test $\hat{\alpha}_0 = \hat{\gamma}_0 = \hat{\lambda}_0 = 0$. In table 3 the Likelihood Ratio statistic is 4.76. For a 5% significant level the critical value of a $\chi^2(3)$ is 7.8. So we can not reject the restrictions for a standard significant level of 5 %.

5 CONCLUSION:

With the aim of identifying expectation regimes associated with short run and long run Phillips curves we develop a SS-MS empirical model. We use an inflation expectations augmented Markov-Switching Phillips curve as a measurement equation. Then we specify inflation expectations component as an underlying component of observed inflation which is occasionally integrated.

We have found the persistence of expectation errors is not the same across regimes. In one regime, the process of expectation errors is autoregressive whereas in the other one it is white noise. The regime with persistent expectation errors can be associated with the existence of an arbitrage between inflation and unemployment. In the rational expectations regime, the Phillips curve is vertical.

To summarize, we have identified a keynesian regime for the period (1973-1983). Whereas the period of relative economic stability of the 20 last years is consistent with a classical regime where agents have rational expectations and where the unemployment rate is closed to the natural rate.

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APPENDIX:

Table 1[2]: ML estimates of the univariate representation (1973:01-2003:03):

Inflation rate:

$$\Delta\Pi_t = (\mu_0 + \beta_0\Pi_{t-1} + \sum_{i=1}^{3}\phi_{i0}\Delta\Pi_{t-i} + \theta_0\eta_t)(1 - S_t) + (\mu_1 + \beta_1\Pi_{t-1} + \sum_{i=1}^{3}\phi_{i1}\Delta\Pi_{t-i} + \theta_1\eta_t)S_t$$ [3]

	$S_t = 0$	$S_t = 1$
$\hat{\mu}$	1.78 (2.68)	0.90 (1.69)
$\hat{\beta} = \hat{\rho} - 1$	-0.70 (-2.95)	-0.14 (-1.70)
$\hat{\phi}_1$	-0.18 (-0.88)	-0.41 (-3.61)
$\hat{\phi}_2$	-0.30 (-2.41)	-0.37 (-3.51)
$\hat{\theta}$	0.82 (5.48)	2.48 (11.47)
$\bar{\Pi}$[4]	2.66	5.9
$\hat{q}$	0.96 (2.82)	
$\hat{p}$	0.98 (3.22)	
duration	6.5 years	12.5 years
ln(L)	-253.9	

[2] T-stat are into parentheses

[3] The number of lags has been tested with the k-max method. We have found $k = 2$.

[4] $\bar{\Pi}$ is the empirical mean for either regimes. $\bar{\Pi}_0 = \frac{\sum_{t=1}^{T}\Pi_t P(S_t=0)}{\sum_{t=1}^{T}P(S_t=0)}$, $\bar{\Pi}_1 = \frac{\sum_{t=1}^{T}\Pi_t P(S_t=1)}{\sum_{t=1}^{T}P(S_t=1)}$

Table 2: Tables of critical values:

DGP=G. Estimated model=E

$H_0 : \beta_0 = \beta_1 = 0$

number of replications:5000

p-value	10%	5%	2.5%
critical values of $\frac{\hat{\beta}_0}{\hat{\sigma}(\beta_0)}$	-1.64	-2.06	-2.41

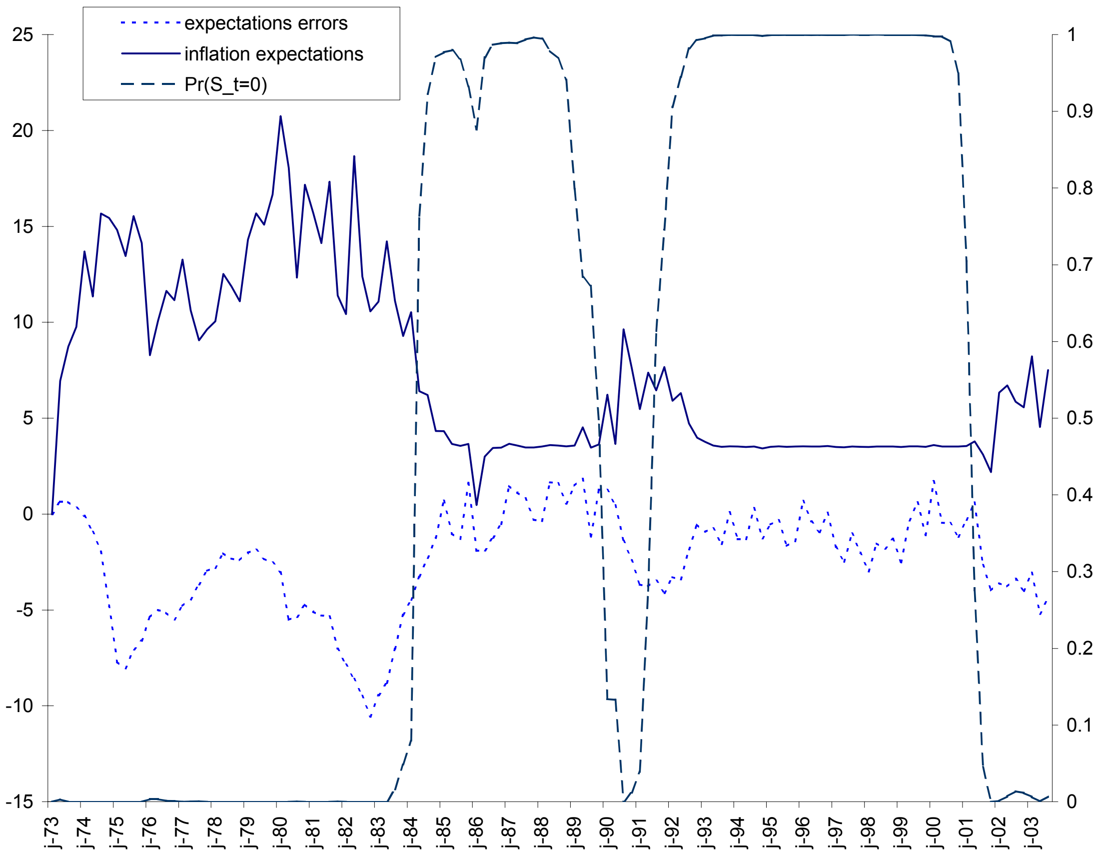
Table 3: Maximum Likelihood estimates of models (11) and (14)

$$\left\{ \begin{array}{c} \Pi_t - \Pi_t^e = \alpha_{S_t} + \gamma_{S_t}(\Pi_{t-1} - \Pi_{t-1}^e) + \sigma^{\Pi}\varepsilon_t \\ U_t - \bar{U}_t = \lambda_{S_t}(\Pi_t - \Pi_t^e) + \sigma^U \eta_t \end{array} \right\}$$

$$\Pi_t^e = \mu_{S_t}^{\Pi^e} + \rho_{S_t}^{\Pi^e}\Pi_{t-1}^e + \sigma_{S_t}^{\Pi^e}\varepsilon_t^{\Pi^e}$$

		$S_t = 0$		$S_t = 1$	
		param	T-stat	param	T-stat
	$\hat{\alpha}$	-0.44	-0.46	-0.35	-1.30
$\Pi_t - \Pi_t^e$	$\hat{\gamma}$	0.20	1.53	0.91	16.45
	$\hat{\sigma}^{\Pi}$	1.28	10.06	1.28	10.06
$U_t - \bar{U}_t$	$\hat{\lambda}$	-0.008	-0.49	-0.33	-7.10
	$\hat{\sigma}^U$	1e-6	3e-5	1e-6	3e-5
	$\hat{\mu}_{\Pi}$	2.73	2.17	0.13	0.32
Π_t^e	$\hat{\rho}_{\Pi}$	0.23	0.87	1	-
	$\hat{\sigma}_{\Pi}$	9e-5	3e-5	2.95	10.4
	q	0.97			
	p	0.96			
LR test[5]	4.76				

[5] We test $H_0 : \alpha_0 = \lambda_0 = \gamma_0 = 0$


expectations errors
inflation expectations
Pr(S_t=0)
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10
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0
-5
-10
-15
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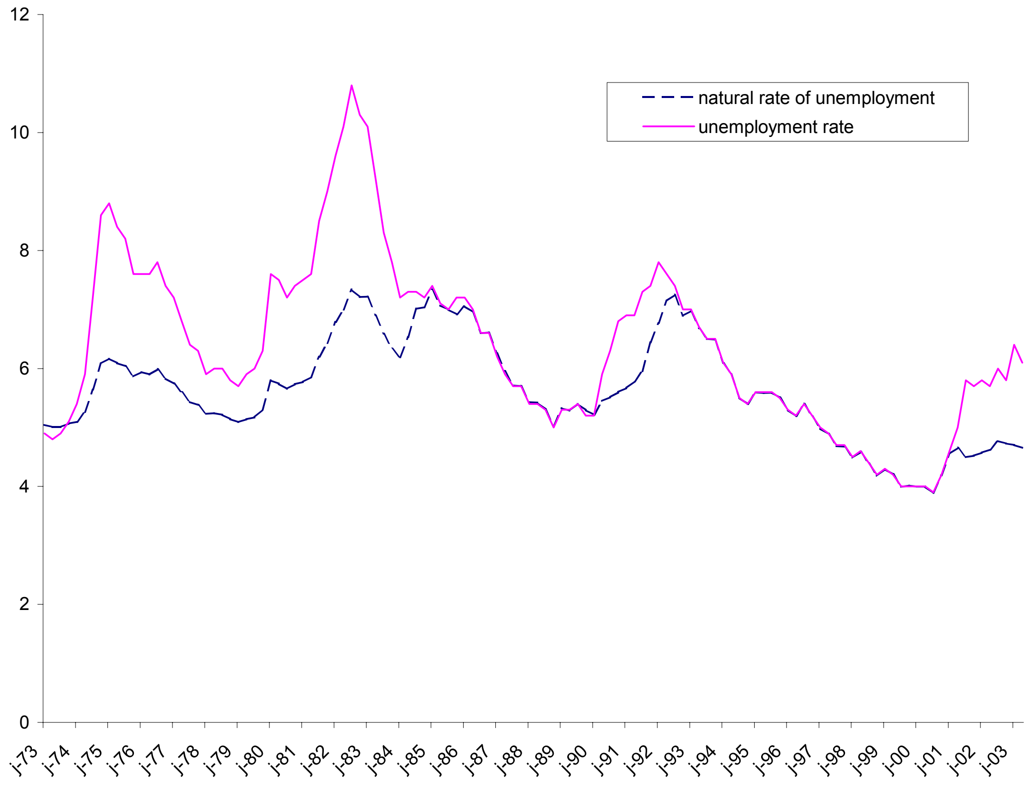
Figure 1: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natural rate of unemployment
unemployment rate
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Figure 2: