Business Cycle Implications of Habit Formation

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Abstract

The inability of a wide array of dynamic stochastic general equilibrium (DSGE) models to generate fluctuations that resemble actual business cycles has lead to the use of habit formation in consumption. For example, habit formation has been shown to help explain the negative response of labour input to a positive, permanent technology shock, several asset pricing puzzles, and the impact of monetary shocks on real variables. Investigating four different DSGE models with the Bayesian calibration approach, this paper observes that, especially in a new Keynesian monetary business cycle model with both staggered price and wage, habit formation fails to mimic the shape of the output growth in the frequency domain: it counterfactually emphasizes low frequency fluctuations in the output growth, compared to the U.S. data. On the other hand, habit formation has no clear implications on other business cycle aspects including impulse responses and forecast error variance decompositions of output to permanent and transitory shocks. These observations cast doubt on habit formation as an important ingredient of the DSGE model with a rich set of internal propagation mechanisms.

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1. Introduction

It is 'folk-theorem' of macroeconomics that, "All models are false." Given a sufficiently rich collection of stylized facts, any dynamic stochastic general equilibrium (DSGE) model of the business cycle will be rejected by the data. One response to this problem is to find the most powerful sample moments for model evaluation in the econometric sense, a line of attack begun by Hansen (1982). Another approach to the evaluation of DSGE models is to focus on the sample moments most relevant for students of the business cycle.

This paper follows the latter tack to study a slew of DSGE business cycle models that have consumption habits in common. Beginning with Constantinides (1990), habit formation has been at the center of stories that unravel quandaries about asset prices and returns. In general, past consumption restricts current and future consumption for a habit-forming consumer. A consumer on a binge in the recent past tends to consume more in the current period. Therefore, habit formation creates a smoother consumption process. Habit formation resolves the equity premium puzzle and the risk-free rate puzzle because smoother consumption implies a larger marginal rate of intertemporal substitution on average. In turn, the risk-free rate is smaller given just a moderate degree of a risk aversion. Jermann (1988) and Boldrin, Christiano and Fisher (2001) exploit consumption habits to match asset pricing moments in a one-sector real business cycle (RBC) model and a two-sector RBC (TSRBC) model, respectively. Consumption habits have been also proposed as propagation mechanism to explain business cycle properties of the actual data. Francis and Ramey (2002), Fuhrer (2000), Estrella and Fuhrer (2002), Christiano, Eichenbaum and Evans (2003), Smets and Wouters (2003) adapt consumption habits to replicate the negative response of hours worked to permanent technology shocks attributed to Galí (1999), and explain the effects of monetary policy shocks on real activity, respectively.

Questions linger about the precise role habit formation plays in the propagation of business cycles, in spite of this success. Lettau and Uhlig (2000) show that habit formation in consumption produces excessive smoothness in consumption compared to the actual U.S. data. Similar results are obtained by Otrok, Ravikumar, and Whiteman (2002). They show that the power of habit formation to generate a small risk-free rate and a large equity premium relies on short-run, high-frequency consumption dynamics not usually thought to be important for asset pricing. Furthermore, in the context of a new Keyensian monetary business cycle (NKMBC) model with a sticky price, Bouakez, Cardia, and Ruge-Murcia (2003) observe that implausibly strong habit formation and adjustment costs of investment must be accompanied together to generate the hump-shaped response of output to monetary policy shocks.

This paper adds to the evidence that habit formation in consumption can solve business cycle and asset pricing puzzles, but at a price. We evaluate the impact of consumption habits on business cycle fluctuations in the frequency domain, based on four DSGE models with consumption habits: a one-sector RBC model with costly adjustment of capital, a two-sector RBC (TSRBC) model with limited intersectoral factor mobility, a monetary business cycle (MBC) model with the money-in-utility function (MIUF) and flexible prices, and an NKMBC model with both sticky price setting and staggered nominal wage contracts. In the recent business cycle literature, a series of studies by Watson (1993), Cogley and Nason (1995b), Ellison and Scott (2000), Christiano and Vigfusson (2003), and Jung (2004) evaluates the empirical fit of DSGE models in the frequency domain. Among them, Jung (2004) claims that, regarding the spectral density functions (SDFs), habit formation improves the matching performance of several NKMBC models with sticky prices. We check the robustness of this claim by examining a broad set of DSGE models, which includes an NKMBC model with both staggered price and wage, as in Christiano, et al. (2003) and Smets and Wouters (2003).

This paper addresses the fit of the four DSGE models in the frequency domain within the context of the Bayesian calibration approach developed by DeJong, Ingram, and Whiteman (1996). In this approach, we assess the fit of a DSGE model by comparing the theoretical distributions of the statistical properties implied by the model with the empirical posterior distributions of those statistical properties numerically generated by vector autoregressions (VARs) under prior distributions of the parameters of the VARs.

We observe that, especially in the NKMBC model, habit formation fails to mimic the empirical posterior distributions of the SDFs of the growth rates of output: the model counterfactually emphasizes low and business cycle frequency fluctuations in the output growth, compared to the U.S. data. On the other hand, we find that, within the NKMBC model, habit formation has no clear implications on other business cycle facts such as the impulse response functions (IRFs) and the forecast error variance decompositions (FEVDs) of output. This observation, together with the failure of habit formation with respect to the SDFs, casts doubt on habit formation as an important ingredient of the DSGE model endowed with a rich set of internal propagation mechanisms.

2. DSGE Models and Habit Formation

This section presents several closed-economy DSGE models with habit forming preferences.[1] The recent literature that claims habit formation is important in understanding business cycle and asset pricing puzzles motivates our choice of DSGE models. Francis and Ramey (2002) argue a one-sector RBC model with adjustment costs of investment and habit formation is able to replicate the negative correlation between labor and the permanent component of productivity. Boldrin, et al. (2001) claim a TSRBC model with habit formation and limited intersectoral factor mobility resolves many outstanding asset pricing puzzles. In their one-sector MBC models, Fuhrer (2000) and Estrella and Fuhrer (2002) contend that habit formation captures the short-run dynamics of real variables and inflation and solves the counterfactual jump behavior of real variables to monetary shocks implied by a broad set of forward-looking, rational expectation models. Edge (2000), Christiano, et al. (2003), and Bouakez, et al. (2003) champion habit-forming MBC models, but with the new Keynesian features of staggered nominal contracts. Smets and Wouters (2003) and Amato and Laubach (2004) embed a Taylor-type monetary policy rule into NKMBC models with habits. In this section, we construct four DSGE models with habit-forming preferences: a one-sector RBC model with adjustment costs of investment, a two-sector RBC model with limited intersectoral factor mobility, a one-sector MBC model with the MIUF, and an NKMBC model with

[1]Several recent papers also study the implications of habit formation in the small open economy-DSGE models; see Bouakez (2003), Kano (2003), Karayalçin (2003), and Letendre (2003).

sticky price and nominal wage. The next subsections introduce the four DSGE models this paper studies.

2.1 A one-sector RBC model with adjustment costs of investment

Our version of the standard one-sector RBC model with habit formation assumes period utility of the representative household is linear in the disutility of labor and adopts the "internal habit" specification. The specification of internal habits assumes the lagged household consumption enters period utility rather than aggregate past consumption as in the "external habit" or "catching-up-with-the-Joneses" specification of Abel (1990). The internal habit formation has been adopted by Boldrin, et al. (2001), Fuhrer (2000), Francis and Ramey (2002), Edge (2000), Christiano, et al. (2003), Bouakez, et al. (2003), and Amato and Laubach (2004).[2]

The period utility of the representative household is characterized with internal habits and linear disutility of labor

$$(1) \qquad U(c_t, c_{t-1}, n_t) = \ln(c_t - hc_{t-1}) - \gamma_1 n_t, \quad 0 < c_t - hc_{t-1}, \forall t,$$

where c_t and n_t denote household consumption and labour supply at period t, and h is the habit parameter. If $0 < h < 1$, the representative household faces habits in her consumption.[3] In this case, the household wants to smooth the growth as well as the level of consumption across time. This fact makes the optimal path of consumption more sluggish than in the case without habits. In this paper, the period utility function is log with respect to consumption,

[2] The specification of habit formation is also distinguished with respect to its functional form. For example, Fuhrer (2000), Bouakez, et al. (2003), and Amato and Laubach (2004) specify consumption habits so that period utility is a function of the ratio of current consumption to the habit stock, while, in the models of Francis and Ramey (2002), Edge (2000), Boldrin, et al. (2001), Christiano, et al. (2003), and Smets and Wouters (2003), habit formation is specified so that period utility is a function of the difference between current consumption and the habit stock. Campbell, et al. (1997, chapter 8) discuss the difference between these two specifications in the asset-pricing context.

[3] When h is strictly negative, local substitutability in consumption arises in period utility. Heaton (1993) contains a complete discussion of relationship between habit formation and durability in consumption.

given $h = 0$. The utility function (1) also reflects the assumption that labour supply is indivisible. As discussed by Hansen(1985), Rogerson (1988), and Christiano and Eichenbaum (1992), given a constant $\gamma_1 > 0$ the non-convexity in individual labour choice exists when households buy lotteries over employment.

The expected lifetime utility function of the representative household is

$$(2) \qquad \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i U(c_{t+i},\, c_{t+i-1},\, n_{t+i}), \quad 0 < \beta < 1,$$

where $\mathbf{E}_t$ is the conditional expectation operator on the information set at period t and β is the subjective discount factor. The budget constraint of the household is

$$(3) \qquad w_t n_t + r_{K,t} k_t = c_t + x_t + \tau_t,$$

where w_t, $r_{K,t}$, k_t, x_t, and τ_t represent the real wage, the rental rate of capital, and investment, and government tax, respectively. Investment is implemented with adjustment costs. Jermann (1998), Francis and Ramey (2002), Christiano, et al. (2003), Bouakez, et al. (2003), and Smets and Wouters (2003) all combine habit formation with costly adjustment of capital to improve the fit of their models to the data. We follow Christiano, et al. (2003) and Smets and Wouters (2003) to specify the law of motion of capital with adjustment costs of investment

$$(4) \qquad k_{t+1} = (1 - \delta) k_t + \left[1 + S\left(\frac{x_t}{x_{t-1}}\right)\right] x_t,$$

where $0 < \delta < 1$ is the depreciation rate of capital. The function S is strictly convex and characterized with $S(1) = S'(1) = 0$ and $S''(1) = \kappa > 0$.[4]

The representative firm combines capital and (efficiency units of) labor to produce output in a constant returns to scale (CRS) technology. Production of the single consumption-investment goods employs

[4] As discussed by Christiano, et al (2003) and Smets and Wouters (2003), these assumptions on the function S imply that the deterministic steady state of the economy is independent of adjustment costs.

$$Y_t \;=\; K_t^{\psi}(A_t N_t)^{1-\psi} \quad 0 < \psi < 1, \tag{5}$$

where Y_t, K_t, N_t, and A_t denote aggregate output, capital, labor, and labor-augmenting technical change, respectively. We assume the log of A_t evolves as a random walk with drift α

$$A_t \;=\; A_{t-1}\exp(\alpha + \varepsilon_t), \quad \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2). \tag{6}$$

The firm rents the household capital and purchases labor services from the household in perfectly competitive markets.[5] Throughout this paper, the government budget is balanced at each period. In this model, government spending G_t is financed with lump-sum tax τ_t. Therefore, the government budget constraint is given as $G_t = \tau_t$.

The aggregate resource constraint of the economy is

$$Y_t \;=\; C_t + I_t + G_t, \tag{7}$$

where C_t is aggregate consumption and I_t is aggregate investment. The stochastic process of government spending G_t is as the transitory component g_t equal to the ratio of G_t to aggregate output Y_t follows an AR(1) process

$$g_t \;=\; g^{*(1-\rho_g)} g_{t-1}^{\rho_g}\exp(\eta_t), \quad 0 < \rho_g < 1, \quad \eta_t \sim \mathcal{N}(0, \sigma_\eta^2). \tag{8}$$

This introduces an aggregate income shock to the RBC model in the spirit of Christiano and Eichenbaum (1992).

Optimal allocations arise from the solution of the household's and firm's optimization problems. The household maximizes (2) subject to (3) and (4), given (1) and the initial

[5] The firm is owned by households through equity holdings. We push the equity market into the background without loss of generality.

conditions k_0, and $c_{-1} \geq 0$. The firm maximizes its profit function equal to the production function (5) net of factor costs, $w_t n_t + r_{K,t} k_t$. The resulting optimality conditions, along with the aggregate resource constraint (7) conditional on the exogenous shock processes (6) and (8), provide necessary conditions a potential equilibrium path must satisfy. Equilibrium in this decentralized RBC economy requires it to clear its goods and labor markets. At the market clearing wage rate, w_t, and rental rate of capital, $r_{K,t}$, $N_t = n_t$ and $K_t = k_t$, $C_t = c_t$, and $I_t = x_t$, which define the equilibrium of this economy. Together with the transversality conditions for the state variables K_t and C_{t-1}, the optimality conditions evaluated at market clearing characterize a unique equilibrium path for the economy.

2.2 A Two-Sector RBC Model

Boldrin, et al. (2001) examine the business cycle and asset pricing implications of habit formation in a two-sector RBC (TSRBC) model with limited intersectoral factor mobility. They show that their habit model is successful in replicating the U.S. sample moments related to asset pricing (e.g., the mean risk free rate and equity premium, and the Sharp ratio). However, there is no clear evidence that, in their TSRBC model, habits improve the model's ability to replicate the U.S. sample moments related to business cycles.[6] We investigate the role of habits in this TSRBC model in explaining the U.S. statistical properties in the frequency domain.

In their model, consumption goods and investment goods are produced with different technologies:

$$Y_{c,t} = K_{c,t}^{\psi}(A_t N_{c,t})^{1-\psi} = C_t + G_t, \tag{9}$$

$$Y_{i,t} = K_{i,t}^{\psi}(A_t N_{i,t})^{1-\psi} = K_{c,t+1} + K_{i,t+1} - (1-\delta)(K_{c,t} + K_{i,t}), \tag{10}$$

where $K_{c,t}$ and $K_{i,t}$ denote capital stocks in the consumption and investment sectors, respectively. Similarly, $N_{c,t}$ and $N_{i,t}$ represent hours worked in the consumption and investment

[6] One exception is that the TSRBC model with habits can replicate the sample estimate of the coefficient of the relative risk aversion in Campell and Mankiw's (1989) regression fairly well.

sectors, respectively, and are restricted with $N_t = N_{c,t} + N_{i,t}$. A_t and G_t follow the processes (6) and (8), respectively.

In this model, N_t, $N_{c,t}$, and $N_{i,t}$ are determined prior to the realization of ε_t and η_t: it is difficult to adjust quickly aggregate employment and its sectoral allocation in response to shocks. Moreover, once installed in one sector, capital cannot be shifted to the other sector: $K_{c,t+1}$ and $K_{i,t+1}$ are determined prior to the realization of ε_{t+1} and η_{t+1}.

Notice that the technologies in the two sectors are symmetric. This implies that in the deterministic steady state, the relative price of investment goods to consumption goods should be one. Boldrin, et al. (2001) measure aggregate output in the base year price, i.e., the unit relative price at the deterministic steady state. In this case, the aggregate output Y_t and aggregate capital K_t are simply constructed by adding up the two sector's output and capital: $Y_t = Y_{c,t} + Y_{i,t}$ and $K_t = K_{c,t} + K_{i,t}$. The law of motion of capital is without adjustment costs of investment: $K_{t+1} = (1-\delta)K_t + I_t$. The aggregate resource constraint (7) and the impulse structure (6) and (8) are still applicable for this model.

2.4 An MBC model with the MIUF

The standard MBC model with the MIUF, which has the seminal works of Sidrauski (1967) and Brock (1974) as its predecessors, is the basis of recent NKMBC models with nominal and real rigidities. In an MBC model with a cash-in-advance (CIA) constraint, Nason and Cogley (1994) show that the simple MBC model lacks real propagation mechanisms enough to replicate the IRFs of output and hours worked to monetary shocks observed in the U.S. sample. Fuhrer (2000) claims that habits give strong business cycle propagation for matching the U.S. real data in his one-sector MBC model. We scrutinize the business cycle implication of habits in the frequency domain within the context of the MBC model with the MIUF.

Let M_t and P_t denote the nominal money stock and the aggregate price level at period t. In this model, the utility function contains the real balance M_t/P_t as its argument because holding money reduces real transaction costs:

$$(11) \qquad \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i U(c_{t+i},\, c_{t+i-1},\, n_{t+i},\, M_{t+i}/P_{t+i}), \quad 0 < \beta < 1,$$

where the period utility is specified as an additive-separable form

$$(12) \qquad U(c_t,\, c_{t-1},\, n_t,\, M_t/P_t) \;=\; \ln(c_t - hc_{t-1}) - \frac{n_t^{1+\frac{1}{\gamma_2}}}{1+\frac{1}{\gamma_2}} \;+\; \ln(M_t/P_t), \quad 0 < \gamma_2.$$

The household maximizes the lifetime utility function (11) subject to the budget constraint

$$(13) \qquad M_{t+1}/P_t \;+\; c_t \;+\; x_t \;\; + \; \tau_t = \; r_{K,t}k_t \;+\; w_t n_t \;+\; M_t/P_t,$$

and the law of motion of capital with adjustment costs of investment (4). As in the standard one-sector RBC model, the representative firm maximizes its profit equal to the production function (5) net of factor costs $w_t n_t + r_{K,t}k_t$.

In this model, the government finances its spending G_t by correcting lump-sum tax τ_t and printing new money (i.e. seigniorage revenue) $(M_{t+1} - M_t)/P_t$. Therefore the government's budget constraint at period t is

$$(14) \qquad G_t = \tau_t + (M_{t+1} - M_t)/P_t,$$

where the stochastic process of the government spending G_t is given as $G_t = gY_t$ with the government spending-output ratio g constant. In the standard MBC model, the monetary policy is characterized with the following exogenous process of the growth rate of the monetary base

$$(15) \qquad \Delta \ln M_{t+1} \;=\; (1-\rho_M)m^* \;+\; \rho_M \Delta \ln M_t \;+\; \mu_t, \quad 0 < \rho_M < 1, \quad \mu_t \sim \mathcal{N}(0, \sigma_\mu^2)$$

where m^* is the steady state level of the money growth rate. Note that $\Delta \log M_{t+1}$ is in the information set of the household at period t: the household's decision at period t is taken place after the realization of the monetary policy shock μ_t. Note that, in this model, the underlying shocks of this model are composed of the permanent technology shock and the money growth rate shock.

2.5 An NKMBC model

The NKMBC model of this paper is a simplified version of Christiano, et al. (2003): it is composed of (i) the Calvo (1983)-type staggered price-setting behavior of monopolistic final goods firms, as in Yun (1996), (ii) the Calvo-type staggered wage setting behavior of households as monopolistic suppliers of heterogenous labour, as in Erceg, Henderson, and Levin (2000), (iii) variable capital utilization, (iv) adjustment costs of investment, and (v) habit formation in consumption.[7] Mainly due to staggered wage contracts, this model can yield strong, hump-shaped responses of output, consumption, and investment to money growth shocks.[8]

In this model, the households consume a Dixit-Stiglitz type consumption index, c_t, that consists of final goods produced by monopolistically competitive firm:

$$c_t = \left[\int_0^1 y_{D,t}(j)^{\frac{\xi-1}{\xi}} dj \right]^{\frac{\xi}{\xi-1}}, \tag{16}$$

where $y_{D,t}(j)$ is the demand for the final goods produced by a typical firm j under the price $p_t(j)$. This consumption index implies the downward demand function with the price elasticity ξ: $y_{D,t}(j) = [P_t/p_t(j)]^{\xi} Y_{D,t}$, where $Y_{D,t}$ is the aggregate demand and P_t is the consumption price index $P_t = [\int_0^1 p_t(j)^{1-\xi}]^{\frac{1}{1-\xi}}$. All final goods firms, which are endowed with an identical Cobb-Douglas technology $y_t(j) = K_t(j)^{\psi}[A_t N_t(j)]^{1-\psi}$ and facing the downward demand functions for their own goods, maximize their real profits by setting their prices $p_t(j)$ optimally. However, in each period, a firm can set its price to the desired level $p_{c,t}$ only

[7] The complete specification of the Christiano, et al. (2003) model further requires limited participation restriction of financial market.

[8] An NKMBC model with only staggered price setting behavior such as Chari, et al. (2000) is difficult to generate the hump-shaped response of real variables to monetary shocks under plausible values of structural parameters.

with probability $1-\mu_p$. With probability μ_p, the firm should set its price to the one-period past level multiplied by the steady state inflation rate, $\pi^* P_{t-1}$. Then, the aggregate price at period t is shown as

$$P_t = [(1-\mu) P_{c,t}^{1-\xi} + \mu \left(\pi^* P_{t-1}\right)^{1-\xi}]^{1/(1-\xi)} \tag{17}$$

where $\pi^* = \exp(m^* - \alpha)$. The optimal price of a firm at period t is

$$p_{c,t} = \left(\frac{\xi}{\xi - 1}\right) \frac{\mathbf{E}_t \sum_{i=0}^{\infty} \left(\beta \mu \pi^{*-\xi}\right)^i \Gamma_{t+i} \phi_{t+i} Y_{D,t+i} P_{t+i}^{\xi}}{\mathbf{E}_t \sum_{i=0}^{\infty} \left(\beta \mu \pi^{*1-\xi}\right)^i \Gamma_{t+i} Y_{D,t+i} P_{t+i}^{\xi-1}}, \tag{18}$$

where $\beta^i \Gamma_{t+i}$ is the stochastic discount factor. ϕ_t is the real marginal cost of producing final goods, which is related to the real wage W_t/P_t and the real rate of return of capital $R_{K,t}/P_t$ through

$$\frac{W_t}{P_t} = (1-\psi)\phi_t \frac{Y_{D,t}}{N_t}, \qquad \frac{R_{K,t}}{P_t} = \psi \phi_t \frac{Y_{D,t}}{K_t}. \tag{19}$$

As shown in Erceg, et al. (2000), the households in this model are monopolistically competitive suppliers of their heterogenous labour service, $n_t(j)$, which can set their own nominal wage $W_t(j)$ optimally. The final goods firms, which are competitive in the labour market, need all differenciated labour services to produce their final goods through the aggregate labour input function

$$N_t = \left[\int_0^1 n_t(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} \tag{20}$$

where θ is the coefficient of the wage elasticity of labour demand. This labour input function then implies downward labour demand functions for differenciated labour services: $n_t(j) = [W_t/W_t(j)]^{\theta} N_t$, where W_t is the aggregate nominal wage index $W_t = [\int_0^1 W_t(j)^{1-\theta} dj]^{\frac{1}{1-\theta}}$. This model assumes complete financial markets in which the households can buy or sell state-contingent claims to diversify away their idiosyncratic risks. Hence, in equilibrium, all the households are identical with respect to consumption and asset holdings. Furthermore, we assume that the utilization rate of capital u_t is variable and the households have to pay costs in terms of consumption goods to set the utilization rate to the level $u_t \in [0,1]$. The budget constraint of household j is given as

$$(21)\quad M_{t+1}/P_t + c_t + x_t + a(u_t)k_t + + \tau_t = r_{K,t}u_t k_t + w_t(j)n_t(j) + M_t/P_t + D_t/P_t,$$

where D_t is the nominal profits of the final goods firms, u_t is the utilization rate of the physical capital, and $a(u_t)$ is the costs of setting the utilization rate to u_t.[9]

Subject to the downward demand functions for their labour services, the budget constraint (21), and the law of motion of capital with capital adjustment costs (4), the households maximize the lifetime utility (11) with the period utility function (12) by setting their nominal wage $W_t(j)$ optimally. However, similar to the price setting behaviour of the final goods firms, the households can set their nominal wages to the desired level $W_{c,t}$ only with probability $1-\mu_\omega$. With probability μ_ω, the households should set their nominal wages to the one-period past level multiplied by a constant $\pi_W^* > 1$, $\pi_W^* W_{t-1}$.[10] Then, the aggregate nominal wage at period t is shown as

$$(22)\qquad W_t = [(1-\mu_\omega)W_{c,t}^{1-\theta} + \mu_\omega\left(\pi_W^* W_{t-1}\right)^{1-\theta}]^{1/(1-\theta)}.$$

The desired nominal wage of the household at period t is

$$(23)\qquad W_{c,t} = \left(\frac{\theta}{\theta-1}\right)\frac{\mathbf{E}_t\sum_{i=0}^{\infty}(\beta\mu_\omega)^i n_{t+i}(j)^{1+1/\gamma_2}}{\mathbf{E}_t\sum_{i=0}^{\infty}\left(\beta\mu_\omega\pi_W^*\right)^i\lambda_{t+i}P_{t+i}^{-1}n_{t+i}(j)}$$

where λ_t is the shadow price of consumption goods or the marginal utility of consumption such that $\lambda_t = (c_t - hc_{t-1})^{-1} - \beta h\mathbf{E}_t(c_{t+1} - hc_t)^{-1}$. Finally, we assume that the underlying

[9] As in Christiano, et al.(2003), we specify the increasing, convex function $a(u_t)$ with $a(1) = 0$ and $a''(1)/a'(1) = 0.01$.

[10] It can be shown that, to derive the balanced growth equilibrium path, we need to set π_W^* to $exp(m^*)$ i.e., the steady state growth of money. We impose this restriction through the analysis.

impulse structure of this model consists of the random walk technology (6) and the exogenous AR(1) money growth rate (15).

3. Model Evaluation Strategy, Calibration, and Solution Method

This section discusses our model evaluation strategy and describes our calibration and solution methods.

3.1 Model Evaluation Strategy — Bayesian Calibration Approach

In this paper, we assess the fit of the DSGE models by exploiting the Bayesian calibration approach developed by DeJong, et al. (1996). Using prior probability distributions, we illustrate the calibrator's uncertainty concerning structural parameters of theoretical DSGE models in this approach. The calibrator then constructs the probability distributions of statistical properties — i.e. the SDFs, the IRFs, and the FEVDs — of artificial data generated by the DSGE models. These theoretical distributions are then compared with the empirical probability distributions of the statistical properties of the actual U.S. data. We employ vector autoregressions (VARs) as statistical models, from which we can yield the posterior distributions of the statistical properties of the actual U.S. data, given prior probability distributions defined over the parameters of VARs. The fit of the DSGE models is evaluated by observing how well the theoretical distributions overlap the empirical posterior distributions.

As statistical models of the U.S. data, we use three bivariate VARs with the data spanning the period 1954Q1 to 2002Q4. These VARs are different in their information sets: *VAR1* corresponds to $[\Delta \ln Y_t \ \ \ln N_t]'$, *VAR2* to $[\Delta \ln Y_t \ \ \ln(C_t/Y_t)]'$, and *VAR3* to $[\Delta \ln Y_t \ \ \ln(I_t/Y_t)]'$.[11] For each VAR, we generate an empirical joint posterior distribution of

[11]The optimal lag for each VAR is chosen with the general to specific likelihood ratio tests, Starting 8 lags as the maximum lag, the LR tests pick 3 lags for *VAR1*, 2 lags for *VAR2*, and 4 lags for *VAR3*, respectively. We have also investigated four other information sets, $[\Delta \ln C_t \ \ \ln N_t]'$, $[\Delta \ln C_t \ \ \ln(C_t/Y_t)]'$, $[\Delta \ln I_t \ \ \ln N_t]'$, and $[\Delta \ln I_t \ \ \ln(I_t/Y_t)]'$. The results based on these additional information sets are provided in the appendix.

the unrestricted VAR coefficients.[12] Using 5000 posterior draws of the VAR coefficients, we then construct the posterior distributions of the SDFs, the IRFs, and the FEVDs, which we discuss in the following subsections. More detailed account of the data is provided in the appendix.

Our Bayesian Monte Carlo experiments are based on the four DSGE models discussed in the last sections. For each DSGE model, we create two versions: the habit version and the non-habit version. This is done by allowing the habit parameter h to take a positive value with uncertainty for the former version and, for the latter version, setting the habit parameter h to zero. The two versions of the DSGE models generate their own theoretical distributions of the SDFs, the IRFs, and the FEVDs. We evaluate the two versions of each DSGE model by measuring the degree of overlap between the theoretical and empirical distributions. For example, if the theoretical distribution of the SDF of output growth implied by the non-habit version of the NKMBC model overlaps the corresponding empirical posterior distribution to a greater extent than that implied by the habit version, we conclude that habits are not important when generating the SDF of output growth within the context of the NKMBC model.

To measure the proximity of the theoretical distributions to the empirical distributions, we construct the confidence interval criterion (CIC) introduced by DeJong, et al. (1996). In general, when we have the theoretical distribution $P(s)$ and the empirical posterior distribution $D(s)$ with respect to a function s to be used to evaluate the model's fit, the CIC statistics is defined as the integral of $P(s)$ over the inter-q quantile range of $D(s)$, normalized by $1/(1-q)$: $CIC = (1-q)^{-1} \int_a^b P(s)ds$, where a and b equal the $q/2$ and $1-q/2$ quantile of $D(s)$. The measure CIC takes a value between 0 and $1/(1-q)$, and the value close to zero signifies a poor fit. The closer the CIC is to one, the more successful the model is in replicating the empirical posterior distributions with respect to the statistical property of interest s. Throughout this paper, we set $q = 0.25$, and hence we investigate whether the inter-75 quantile range of the theoretical distribution can match that of the empirical

[12] We use John Geweke's BCAA program (http://www2.cirano.qc.ac/ $\sim$ bacc/) to generate the posterior distributions of the VAR coefficients.

counterpart. We consider a CIC over 0.3 indicates a good fit of the underlying DSGE model, as discussed by DeJong, et al. (1996) in evaluating the fit of their RBC models. In particular, when the CIC of the non-habit version with respect to a statistical property is greater than 0.3, and that of the habit version is less than 0.3, we claim that habits deteriorate the matching performance of the underlying DSGE model in the statistical dimension.

3.2 Calibration and Solution Methods

To construct the theoretical distributions of the statistical properties, we solve and calibrate the DSGE models. This paper log-linearly approximates the equilibrium path around the deterministic steady state for each model. The resulting linear rational expectation model is solved by Sims's (2001) method to derive the state-space representation of the equilibrium path, which in turn is used to conduct Monte Carlo simulations to generate artificial data of aggregate variables[13].

The models are calibrated with prior distributions of their structural parameters. Our prior assumes that all structural parameters are independently distributed with truncated normal distributions with means and standard deviations summarized in Table 1. For instance, $\beta = 0.992$ is an uncontroversial value for the subjective discount factor in the DSGE literature, which includes Christiano and Eichenbaum (1992), Cogley and Nason (1995b), and Christiano, et al.(2003). Therefore, we set a very small value of $1e-7$ as the standard deviation of β to reflect our small uncertainty in this parameter. The means and standard deviations of the deterministic growth rate α, the capital share ψ, the depreciation rate δ, and the indivisible labour coefficient γ_1, are based on the GMM estimates of Christiano and Eichenbaum (1992). We use their sample point estimates as our prior means and their standard errors as our prior standard deviations. The Bayesian estimation of Smets and Wouters (2003) and Chang, Gomes, and Shorfheide (2002) reports the posterior means and standard deviations of adjustment costs of investment χ and the elasticity of labour supply γ_2, respectively, which we use as our prior means and standard deviations for the two

[13]We use the Gauss version of Sim's Matlab code, which is written by Frank Schorfheide (http://www.econ.upenn.edu// shorf/computing.html) accompanied with Paul Söderlind's code of the generalized Schur decomposition.

parameters.

The parameters, ξ, η, μ_p, and μ_w, are related to the NKMBC model. Yun(1996) calibrates the elasticity of intermediate goods, ξ, to 6, which matches the estimate of Christiano, et al.(2003) in their limited information-minimum distance estimation. Bouakez, et al. (2003) calibrate this parameter to 10. To include these two values in the existing studies into the 95 per cent confidence interval of our prior distribution, we set our prior mean and standard deviation of ξ to 8 and 1.1, respectively. Christiano, et al.(2003) and Smets and Wouters (2003) calibrate the elasticity of labour demand, η, to 21. We also use this number as our prior mean of ξ with a small prior standard deviation $1e-7$. Smets and Wouters (2003) estimates the posterior mode of the probability of no price change μ_p to be 0.908. Our prior distribution of μ_p has this number as the mean, and 0.035 as the standard deviation to include the ML estimate of 0.847 reported by Bouakez, et al. (2003) within the 95 per cent confidence interval. Similarly, as the prior mean of the probability of no wage change, μ_w, we use Smets and Wouters's (2003) estimate of the posterior mode of 0.737. As the prior standard deviation of μ_w, we set 0.03 to include the estimate of Christiano, et al. (2003) of 0.64 within the 95 per cent confidence interval.[14]

We use our sample data to estimate g^*, ρ_g, and σ_η of the government spending-output ratio process (8), and m, ρ_m, and σ_m of the monetary growth rate process (15) as well. The OLS estimates (standard errors) of g^*, ρ^g, and σ_η are 0.253 (0.009), 0.958 (0.0203), and 0.012, while the OLS estimates of m^*, ρ_m, and σ_μ are 0.0053 (0.0008), 0.649 (0.0579), and 0.0038, respectively[15]. The OLS estimates are exploited to construct our prior distributions of these parameters. The prior mean of the standard deviation of permanent technology shocks, σ_η, is calibrated by matching the mean growth rate of output from each of the DSGE models to

[14]For the calibration of capital utilization, see footnote 9.

[15]The OLS estimates of the government spending-output ratio process are obtained by regressing $\ln g_t$ on $\ln g_{t-1}$, constant, and deterministic trend. To estimate the monetary growth rate process, we use the M1 stock series distributed by the Federal Reserve Bank of St. Louis. The series are monthly and span the periods 1959:1-2003:2. We convert the monthly series to quarterly series by taking 3 month average of them and estimate the monetary growth process by using the quarterly series spanning the periods Q2:1959-Q4:2002.

the U.S. sample average of the output growth rate.

For each DSGE model, we construct the non-habit version by setting both the prior mean and standard deviation of the habit parameter h to zero. On the other hand, to construct the habit version, we set the prior mean and standard deviation of the habit parameter h to 0.65 and 0.150, respectively. The two standard deviation interval implied by this prior distribution is [0.35 0.95], and it includes almost all of the estimates of the habit parameter h in the recent literature (e.g., Boldrin, et al. 2001, Christiano, et al. 2003, and Smets and Wouters 2003).

4. Results

This section reports the results of our calibration exercises with respect to the SDFs, the IRFs, and the FEVDs.

4.1 The SDFs

The theoretical and empirical means of the SDFs are plotted in Figures 1(a)-(d), which correspond to the four DSGE models, the RBC, TSRBC, MBC, and NKMBC models, respectively. Each figure is composed of four small windows, and each of the windows plots the empirical posterior mean constructed with the VAR as the solid line, and the theoretical means of the SDFs implied by the non-habit and habit versions of the underlying DSGE model as the large dashed and small dotted lines, respectively.[16] The first window corresponds to output growth, $\Delta \ln Y$; the second to the log of hours worked, $\ln N$; the third to the log of the consumption-output ratio, $\ln C/Y$; and the fourth to the log of the investment-output ratio, $\ln I/Y$.

The most left window in Figure 1(a) shows that, even with the adjustment costs of investment, the one-sector RBC model cannot replicate the well known property of the U.S. data that the relatively large portion of the variations in output growth is attributed

[16] We observe that the three different information sets, *VAR1*, *VAR2*, and *VAR3*, predict the closely similar shape of the empirical posterior mean of output growth. Therefore, we report only the empirical means of the SDFs of output growth predicted with *VAR1* in the most left window of each figure.

to business cycle frequencies. While the non-habit version of the RBC model can explain the empirical mean of the SDF of $\Delta \ln Y_t$ at zero frequency fairly well, the theoretical mean predicted by this version monotonically declines over business cycle frequencies and deviates away from the empirical counterpart. On the other hand, the habit version overstates the mean of the SDF of output growth at zero frequency. However, at business cycle and higher frequencies, this version can mimic the empirical mean of the SDF of output growth better than the non-habit version. In the rest of the three windows in Figure 1(a), we cannot observe the significant differences in the theoretical means of the SDFs of $\ln N_t$, $\ln C_t/Y_t$, and $\ln I_t/Y_t$ between the two versions of the RBC model. While the habit version predicts the SDF of $\ln N_t$ slightly better than the non-habit version, both versions yield the almost same shapes of the SDFs of $\ln C_t/Y_t$ and $\ln I_t/Y_t$ through all frequencies. In particular, the most right window implies that the one-sector RBC model fails to generate the spectral shape of $\ln I_t/Y_t$ in the U.S. data.

An astonishing result observed in Figure 1(b) is that the habit and non-habit versions of the TSRBC model have the almost same implications on the spectral shapes of the four variables, $\Delta \ln Y_t$, $\ln N_t$, $\ln C_t/Y_t$, and $\ln I_t/Y_t$, on average. In all windows, the theoretical means of the SDFs predicted by the two versions of the model trace each other fairly closely. This means that we cannot find any important role of habit formation in the frequency domain in this model.

The results from the two monetary business cycle models, the MBC and NKMBC models, are illustrated in Figures 1(c) and (d), respectively. Similarly to the case of the one-sector RBC model, the simple MBC model without habits fails to mimic the posterior means of the SDFs of $\Delta \ln Y_t$ over business cycle frequencies, while the habit version of the model puts too much emphasis on long-run variations in output growth: it overstates the SDF of output growth at zero frequency relatively to the empirical counterpart. This DSGE model also has poor matching performance with respect to the mean SDFs of $\ln N_t$ and $\ln I_t/Y_t$. However, from the results of the MBC model, we cannot draw a strong inference on whether habits help explain the business cycle fluctuations in frequency domain.

Contrary to the case of the MBC model, we can clearly distinguish the non-habit and habit versions of the NKMBC model with respect to the spectral shape of $\Delta \ln Y_t$. As shown in the most left window of Figure 1(d), the non-habit version can track the empirical means of the SDFs of $\Delta \ln Y_t$ fairly closely at all frequencies. In fact, this version yields a flat portion of the means of the SDFs at some business cycle frequencies, which Cogley and Nason (1995b) point out as a stylized fact of the U.S. business cycle. On the other hand, the habit version of the NKMBC model predicts extremely large power spectra of $\Delta \ln Y_t$ from zero frequency throughout business cycle frequencies, which deviate away from the empirical counterparts. In summary, we can infer that, within the context of the NKMBC model, habit formation generates too much fluctuations of output growth at low and business cycle frequencies to mimic the empirical posterior means of the output growth spectra observed in the U.S. data.

The formal CIC statistics support the above inference from the "eye ball" comparison. Table 2 reports the CICs calculated for the SDFs of $\Delta \ln Y_t$, $\ln N_t$, $\ln C_t/Y_t$, and $\ln I_t/Y_t$ implied by the two versions of the four DSGE models at the zero frequency (i.e. infinite years per cycle), 8 years, 4 years, 2 years and a year per cycle, respectively. The most striking difference in the CIC between the non-habit and habit versions is observed in the SDF of $\Delta \ln Y_t$ predicted by the NKMBC model, which is shown in the last small table. At the zero frequency, the non-habit version yields 0.81 of the CIC, while the habit version generates only 0.07 of this statistic. We can observe this significant difference in the CIC statistic between the two versions even at 8 years and 4 years per cycle, although the CIC of the habit version becomes greater than that of the non-habit version at 2 years per cycle. The average of the CICs over the low and business cycle frequencies is 0.64 for the non-habit version and 0.24 for the habit version. Therefore, the CIC statistics also provide evidence that habits deteriorate the matching performance of the NKMBC model with respect to the SDF of $\Delta \ln Y_t$.

The above analysis comparing the theoretical means of the SDFs with the empirical counterparts is based only on piesewise information of the whole shapes of the empirical and theoretical distributions. To reflect the whole information of the spectral shape into our evaluation of the role of habit formation in the U.S. business cycle, we construct the quasi Kolmogorov-Smirnov (QKS) and quasi Cramer-von Mises (QCVM) statistics. These statistics are Bayesian versions of the classical Kolmogorov-Smirnov and Cramer-von Mises statistics, which Cogley and Nason (1995a) first applied to the business cycle literature.[17] Let $f_n^{VAR}(\omega)$, $f_n^{NH}(\omega)$, and $f_n^{H}(\omega)$ denote the nth repetition within 5000 Monte Carlo simulations of an SDF at frequency ω based on the VAR and the non-habit and habit versions of the DSGE models, respectively. Moreover, we define variables $R_n^{VAR}(\omega)$, $R_n^{NH}(\omega)$, and $R_n^{H}(\omega)$ as the ratios of the posterior mean of $f_n^{VAR}(\omega)$, $Ef_n^{VAR}(\omega)$, to $f_n^{VAR}(\omega)$, $f_n^{NH}(\omega)$, and $f_n^{H}(\omega)$, respectively:

$$(24) \qquad R_n^{VAR}(\omega) = \frac{Ef_n^{VAR}(\omega)}{f_n^{VAR}(\omega)}, \quad R_n^{NH}(\omega) = \frac{Ef_n^{VAR}(\omega)}{f_n^{NH}(\omega)}, \quad \text{and} \quad R_n^{H}(\omega) = \frac{Ef_n^{VAR}(\omega)}{f_n^{H}(\omega)}.$$

When we compute a partial sum of $R_n^m(\omega)$ as $U_n^m(2\pi j/T) = (2\pi/T)\sum_{i=1}^{j} R_n^m(2\pi i/T)$ for $m = \{VAR, NH, H\}$, we can then construct the nth repetition of the QKS and QCVM statistics, QKS_n^m and $QCVM_n^m$:

$$(25) \qquad QKS_n^m = \max|B_n^m(\tau)|, \quad \text{and} \quad QCVM_n^m = \int_0^1 B_n^m(\tau)^2 f_n^m(\pi\tau)d\tau,$$

where $B_n^m(\tau) = (\sqrt{2T}/2\pi)\,[U_n^m(\pi\tau) - \tau U_n^m(\pi)]$. We repeat the same process 5000 times, and obtain the empirical posterior distribution of QKS_n^m and $QCVM_n^m$. Notice that, by construction, if $f_n^H(\omega)$ and $f_n^{NH}(\omega)$ are distributed closely to $f_n^{VAR}(\omega)$, the distributions of QKS_n^H and QKS_n^{NH} should resemble that of QKS_n^{VAR} in their shapes. The same is

[17] Cogley and Nason show that the Hodrick-Prescott (1980) filter can generate spurious business cycle dynamics in the frequency domain by using the Kolgomorov-Smirnov (KS) and Cramer-von Mises (CVM) statistics. These statistics are constructed as follows. Let $I_T(\omega)$ and $f(\omega)$ denote the spectral density of the sample and that implied by a DSGE model at frequency ω, respectively. Furthermore, let $R_T(\omega)$ denote the ratio of $I_T(\omega)$ to $f(\omega)$: $R_T(\omega) = I_T(\omega)/f(\omega)$. In this case, a partial sum of $R_T(\omega)$ over the frequency domain, $U_T(2\pi j/T) = (2\pi/T)\sum_{i=1}^{j} R_T(2\pi i/T)$, converges to a uniform distribution function under the hypothesis that $I_T(\omega)$ is drawn from a population governed by $f(\omega)$. The KS statistic is given as $KS = \max|B_T(\tau)|$, where $B_T(\tau) = (\sqrt{2T}/2\pi)\,[U_T(\pi\tau) - \tau U_T(\pi)]$. On the other hand, the CVM statistic is defined as $CVM = \int_0^1 B_T(\tau)^2 d\tau$. The null hypothesis can be tested with their limiting distributions.

true for the QCVM statistic. To evaluate the non-habit and habit versions of each DSGE model, we compare QKS_n^H and QKS_n^{NH} with QKS_n^{VAR}, and $QCVM_n^H$ and $QCVM_n^{NH}$ with $QCVM_n^{VAR}$, by plotting their nonparametric density estimates and reporting the CIC statistics.[18]

Figures 2(a)-(d) plot the estimated density functions of the QKS and QCVM statistics for the RBC, TSRBC, MBC, and NKMBC models, respectively. Each figure contains two rows: the first row corresponds to the QKS statistics, and the second row to the QCVM statistics. The kernel-smoothed densities of the QKS statistics for $\Delta \ln Y_t$, $\ln N_t$, $\ln C_t/Y_t$, and $\ln I_t/Y_t$ are shown in the four small windows in the first row, while those of the corresponding to the QCVM statistics are plotted in the four small windows in the second row. Each window plots three kernel-smoothed densities: the first from the VAR as the solid line, the second from the non-habit version as the dashed line, and the third from the habit version as the dotted line. Besides, we report the CIC statistics corresponding to the non-habit and habit versions inside each window.

In Figure 2(a), we observe the significant difference in both the QKS and QCVM statistics for the SDF of $\Delta \ln Y_t$ in favour of the habit version of the RBC model. This result is consistent with the observation in Figure 1(a): the habit version of the RBC model can track the empirical mean of the SDF of $\Delta \ln Y_t$ at business cycle frequencies fairly well. The same observation is applicable to the case of the MBC model, as shown in Figure 2(c). As illustrated in Figure 2(b), the QKS and QCVM statistics also support evidence that we cannot observe any significant differences in the SDFs of the four aggregate variables between the non-habit and habit versions of the TSRBC model. Similarly to the RBC model, the QKS and QCVM statistics show that the TSRBC model poorly explains the SDF of $\ln N_t$, regardless of habit formation.

Finally, in Figure 2(d), the QKS and QCVM statistics reveal the superior of the non-habit version of the NKMBC model to the habit version with respect to the SDFs of $\Delta \ln Y_t$ and $\ln C_t/Y_t$. In particular, the non-habit version yield the kernel-smoothed density of the

[18]Throughout this paper, we non-parametrically estimate probability density functions with the normal density kernel $N(x) = \exp(-0.5x^2)/\sqrt{2\pi}$.

QKS statistic for the SDF of $\Delta \ln Y_t$ overlapping the empirical counterpart to the greater extent than the habit version. In fact, the estimated density corresponding to the habit version is too diffused to explain the shape and position of the empirical density precisely both in the cases of the QKS and QCVM statistics.

In summary, we obtain the following results from our analysis on the SDF.

- Habits improve the fits of the RBC and MBC models with respect to the power spectra of the output growth at business cycle frequencies.

- Habits are neutral for the fit of the TSRBC model regarding output growth, the log of hours worked, and the logs of the two golden ratios.

- Habits deteriorate the fit of the NKMBC model with respect to the power spectra of output growth and the consumption-output ratio.

These results lead us to the following inference: habits are helpful to explain business cycle fluctuations only if a DSGE model lacks a strong internal propagation mechanism, as in the RBC and MBC models. On the other hand, if a DSGE model is already endowed with a rich set of internal propagation mechanisms, as in the NKMBC model with nominal and real rigidities of Christiano, et al. (2003), habits yield too much persistence of output growth at low and business cycle frequencies. This might be the cost a researcher has to pay to explain the other dimension of the data — e.g. the IRFs and FEVDs to monetary shocks — by exploiting habit formation.

4.2 The IRFs and FEVDs

Christiano, et al. (2003) observe that habit formation in consumption is less important in explaining the hump-shaped IRFs of output to monetary policy shocks in the U.S. data than staggered nominal wage contracts and capital utilization costs. Based on different identification with different information sets from Christiano, et al. (2003), we find that habits are crucial for explaining none of the IRFs of output, hours worked, and the golden ratios in the NKMBC model: its non-habit version yields almost the same shapes of the IRFs as the habit version.

We identify the IRFs of the aggregate variables to both permanent and temporary shocks by applying Blanchard and Quah's (1989) long-run decomposition to bivariate VARs.[19] We evaluate the fit of the two versions of the NKMBC model with respect to the IRFs in two ways. First, we compare the theoretical means of the IRFs with the corresponding empirical posterior means. This comparison uses only piesewise information of the IRF at each forecast horizon. To reflect joint information of the IRFs at several forecast horizons in the comparison between the two versions, we construct a Bayesian version of the quasi-LM (QLM) statistic of Cogley and Nason (1995b). Let $IRF_n^{VAR}(j)$, $IRF_n^{NH}(j)$, and $IRF_n^{H}(j)$ denote the $j \times 1$ column vector containing the nth repetitions within 5000 Monte Carlo simulations of IRFs for j periods after the impact based on the VAR, the non-habit, and habit versions of the NKMBC model, respectively. When we define $\mathbf{E}IRF_n^{VAR}(j)$ as the posterior mean of $IRF_n^{VAR}(j)$, we construct the QLM statistics, $QLM_n^m(j)$, as

$$QLM_n^m(j) = [IRF_n^m(j) - \mathbf{E}IRF_n^{VAR}(j)]'[IRF_n^m(j) - \mathbf{E}IRF_n^{VAR}(j)], \tag{26}$$

where $m = \{VAR, NH, H\}$. Notice that, by construction, if the distributions of $IRF_n^{NH}(j)$ and $IRF_n^{H}(j)$ are close to that of $IRF_n^{VAR}(j)$, the distributions of $QLM_n^{NH}(j)$ and $QLM_n^{H}(j)$ are also close to that of $QLM_n^{VAR}(j)$. We calculate the CIC statistics to evaluate how well the theoretical QLMs overlap the empirical QLM.

Figure 3 plots empirical and theoretical means of the IRFs of $\ln Y_t$, $\ln N_t$, $\ln C_t/Y_t$, and $\ln I_t/Y_t$ to permanent and temporary shocks.[20] Regardless of the non-habit and habit versions, the NKMBC model generally does a good job in replicating the empirical means of the IRFs of the four variables to the permanent shock.[21] More importantly, we cannot find any clear differences between the habit and non-habit versions of this model with respect to

[19] Since the model has only two structural shocks, we can interpret the permanent and temporary shocks identified with the long-run restriction as the technology and monetary policy shocks, respectively. The IRFs and FEVDs of output and hours worked are based on $VAR1$, and those of the consumption-output ratio on $VAR2$, and those of the investment-output ratio on $VAR3$, respectively.

[20] The results of the IRFs of $\ln Y_t$ reported in Figure 3 are based on the information set $VAR1$. Even with the information sets $VAR2$ and $VAR3$, we obtain the almost same inferences on the IRFs of $\ln Y_t$ as those shown in Figure 3.

[21] An exception is observed in the IRF of $\ln N_t$ to the permanent shock. Regardless of the habit and

the IRFs to the permanent shocks.

On the other hand, the theoretical means of the IRFs to the temporary shocks are far from their empirical counterparts. In particular, "eyeball" comparison detects differences between the two versions regarding the IRFs of $\ln Y_t$ and $\ln N_t$ to the temporary shocks in the short run: the habit version seems to generate these IRFs closer to the empirical means than the non-habit version. To check whether these differences are significant, we generate the QLM statistics with eight and twelve forecast horizons, $QLM_n^m(8)$ and $QLM_n^m(12)$ for $m = \{VAR, NH, H\}$, for the IRFs of $\ln Y_t$ and $\ln N_t$ to the temporary shocks, and compute the corresponding CIC statistics, as reported in Table 3. The non-habit and habit versions yield the almost same value of the CICs calculated for the QLM statistics with eight and twelve forecast horizons regarding the IRFs of $\ln Y_t$ to the temporary shocks. Similarly, the two versions generate the same value of the CICs for the QLM statistics with eight and twelve forecast horizons regarding the IRFs of $\ln N_t$ to the temporary shocks. From these results, we have no clear evidence that habits help significantly improve the matching performance of the NKMBC model in the dimension of the IRFs.

Figure 4 illustrates the theoretical and empirical means of the FEVDs of $\ln Y_t$, $\ln N_t$, $\ln C_t/Y_t$, and $\ln I_t/Y_t$ in the case of the NKMBC model. Three results are worth noting. First, the two versions of the NKMBC model predict the almost same FEVDs of $\ln Y_t$, although their predictions are far from the empirical posterior means. Combined with our inference on the IRFs of $\ln Y_t$, this result supports our claim that habit formation costs business cycle researchers bizarre shapes of the power spectra of output growth at low and business cycle frequencies when they exploit the NKMBC model to describe output dynamics.

Second, the non-habit version does a better job in matching the empirical means of the FEVD of $\ln N_t$ at impact than the habit version. Finally, as the third result, the habit version implies the FEVDs of the two golden ratios much closer to the empirical means than the non-habit versions. Notice that the FEVDs of $\ln I_t/Y_t$ predicted by the habit version is far from the empirical means yet. Hence, the FEVD of $\ln C_t/Y_t$ at short forecast horizons

non-habit versions, the NKMBC yields an extremely large negative IRF of hours worked to the permanent technology shock, relative to the empirical counterpart.

is the only statistical dimension in which we can observe the improvement of the matching performance of the NKMBC model when introducing habit formation into the model.

5. Conclusion

[Material forthcoming.]

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Appendices

Appendix 1: Data Description and Construction

This appendix describes the source and construction of the data. All the time series data are distributed by FRED II maintained by the Federal Reserve Bank of St.Louis (mnemonics follow in parentheses)[22]. The NIPA data are quarterly, real at chained 1996 billion dollars, and seasonally adjusted at annual rates. The consumption series are constructed by *Real Personal Consumption Expenditures on Nondurables* (PCNDGC96) plus *Real Personal Consumption Expenditures on Services* (PCESVC96). The investment series are constructed by *Real Personal Consumption Expenditures on Durables*(PCDGCC96) plus *Real Gross Private Domestic Investment* (GPDIC1) plus *Real National Defense Gross Investment* (DGIC96) plus *Real Federal Nondefense Gross Investment* (NDGIC96). The government spending series are constructed by *Real Government Consumption Expenditures and Gross Investment* (GCEC1) minus *Real National Defense Gross Investment* minus *Real Federal Nondefense Gross Investment.* The output series are simply constructed by summing up the consumption, investment and government spending series. All the series are divided by *Civilian Labor Force* (CLF16OV) to convert them to the per capita series. The employment rate series are obtained by diving *Civilian Employment* (CE16OV) by *Civilian Labor Force.* Since the series of *Civilian Labor Force* and *Civilian Employment* are monthly, this paper takes three month average of each series to construct the quarterly series. Finally, the monetary growth rate series are constructed from nominal, seasonally adjusted, *M1 Money Stock*(M1SL). This is monthly data. Hence, this paper takes three month average of the data to construct the quarterly series.

[22]The webpage is *http://research.stlouisfed.org/fred2/*

Table 1: Calibrated Structural Parameters of the DSGE Models

		Mean	S.D.	Source
β	Subjective discount rate	0.992	1e-7	CE, CN, CEE
α	Deterministic growth rate	0.004	0.0015	CE, CN, BCF
ψ	Capital share	0.344	0.010	CE, CN, CEE
δ	Depreciation rate	0.021	0.002	CE, CN, CEE, SW
χ	Adjustment costs of investment	6.771	1.026	SW
γ_1	Indivisible labour coefficient	0.0037	0.0005	CE, CN
γ_2	Elasticity of labour supply	1.3088	0.3196	CGS
ξ	Elasticity of intermediate goods demand	8	1.1	TK, CEE, BCR
η	Elasticity of labour demand	21	1e-7	CEE, SW
μ_p	Probability of no price change	0.908	0.035	SW, BCR
μ_w	Probability of no wage change	0.737	0.03	SW, CEE
g^*	Mean of g_t	0.253	0.009	U.S. data
m^*	Mean of $\Delta \ln M_t$	0.0053	0.0008	U.S. data
ρ_g	AR coefficient of g_t	0.9603	0.0203	U.S. data
ρ_m	AR coefficient of $\Delta \ln M_t$	0.649	0.0579	U.S. data
σ_ε	S.D. of technology shock	0.010	1e-7	Simulation
σ_η	S.D. of government expenditure shock	0.0116	1e-7	U.S. data
σ_m	S.D. of money growth rate shock	0.0038	1e-7	U.S. data
h	Habit parameter	0.650	0.150	CEE, BCF, SW

Note 1: All parameters are drawn from the normal density with the corresponding mean and standard deviation.

Note 2: CE denotes Christiano and Eichenbaum (1992); CN, Cogley and Nason (1995b); CEE, Christiano, et al. (2003); BCF, Boldrin et al. (2001); SW, Smets and Wouters (2003); CGS, Chang, et al. (2002), TK, Yun (1996); BCR, Bouakez, et al. (2003).

Note 3: "U.S. data" means that calibration is implemented with the U.S. data. The standard deviation of technology shock, σ_ϵ, is calibrated so that the mean of the output growth rate implied by the DSGE models matches its U.S. sample counterpart.

Table 2: The CIC Statistics of the SDFs

RBC

	$\Delta \ln Y_t$		$\ln N_t$		$\ln C_t/Y_t$		$\ln I_t/Y_t$	
Years per Cycle	NHabit	Habit	NHabit	Habit	NHabit	Habit	NHabit	Habit
∞	0.94	0.47	0.42	0.80	0.72	0.57	0.67	0.64
8	0.72	0.76	0.47	0.79	0.76	0.95	0.50	0.52
4	0.02	0.69	0.29	0.45	1.13	0.92	0.55	0.52
2	0.29	0.58	0.69	1.02	1.01	1.12	0.64	0.57
1	0.46	0.89	0.53	0.06	0.80	0.86	0.47	0.43
Ave.	0.54	0.67	0.47	0.65	0.90	0.86	0.56	0.54

TSRBC

	$\Delta \ln Y_t$		$\ln N_t$		$\ln C_t/Y_t$		$\ln I_t/Y_t$	
Years per Cycle	NHabit	Habit	NHabit	Habit	NHabit	Habit	NHabit	Habit
∞	0.96	0.96	0.89	0.88	1.13	0.69	1.19	0.84
8	0.75	0.77	0.92	0.91	1.16	0.99	0.75	0.75
4	0.42	0.46	0.56	0.56	1.15	1.13	0.77	0.55
2	1.02	0.99	1.12	1.12	0.91	1.17	1.06	0.78
1	0.18	0.16	0.05	0.06	1.00	1.16	0.76	0.54
Ave.	0.69	0.70	0.74	0.74	1.12	1.02	0.93	0.72

MBC

	$\Delta \ln Y_t$		$\ln N_t$		$\ln C_t/Y_t$		$\ln I_t/Y_t$	
Years per Cycle	NHabit	Habit	NHabit	Habit	NHabit	Habit	NHabit	Habit
∞	0.83	0.24	0.26	0.38	0.63	0.53	0.77	0.55
8	0.37	0.33	0.28	0.17	0.69	0.61	0.59	0.45
4	0.01	0.82	0.24	0.43	1.02	0.80	0.73	0.43
2	0.33	0.99	0.47	0.32	0.95	0.74	0.65	0.49
1	0.33	0.74	0.85	0.32	0.87	0.81	0.46	0.43
Ave.	0.40	0.57	0.39	0.26	0.85	0.72	0.65	0.47

NKMBC

	$\Delta \ln Y_t$		$\ln N_t$		$\ln C_t/Y_t$		$\ln I_t/Y_t$	
Years per Cycle	NHabit	Habit	NHabit	Habit	NHabit	Habit	NHabit	Habit
∞	0.81	0.07	0.83	0.93	0.63	0.51	0.66	0.55
8	0.69	0.06	0.76	0.91	0.55	0.60	0.49	0.50
4	0.47	0.19	1.11	1.01	1.04	0.84	0.52	0.44
2	0.45	0.77	0.71	0.62	0.75	0.80	0.52	0.58
1	0.65	0.31	0.21	0.47	0.87	0.80	0.47	0.43
Ave.	0.64	0.24	0.73	0.81	0.77	0.71	0.53	0.50

Table 3: The CICs for the QLM Statistics of the IRFs to Temporary Shocks

$\ln Y_t$ to T shock			
	CIC		CIC
$QLM^{NH}(8)$	0.321	$QLM^{NH}(12)$	0.344
$QLM^{H}(8)$	0.393	$QLM^{H}(12)$	0.385

$\ln N_t$ to T shock			
	CIC		CIC
$QLM^{NH}(8)$	0.000	$QLM^{NH}(12)$	0.000
$QLM^{H}(8)$	0.000	$QLM^{H}(12)$	0.000

Figure 1(a): Empirical and Theoretical Means of SDFs: RBC

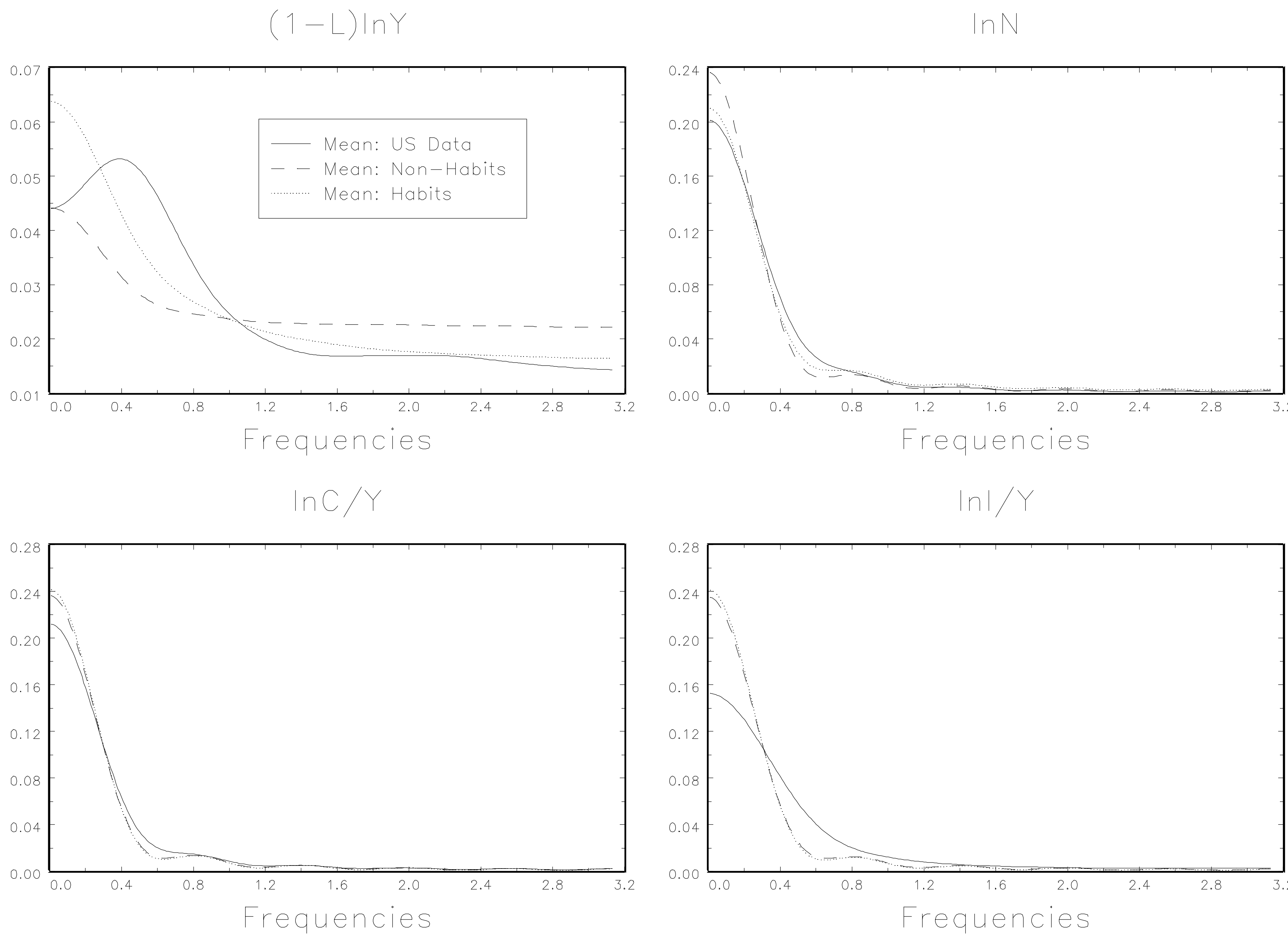

Figure 1(b): Empirical and Theoretical Means of SDFs: TSRBC

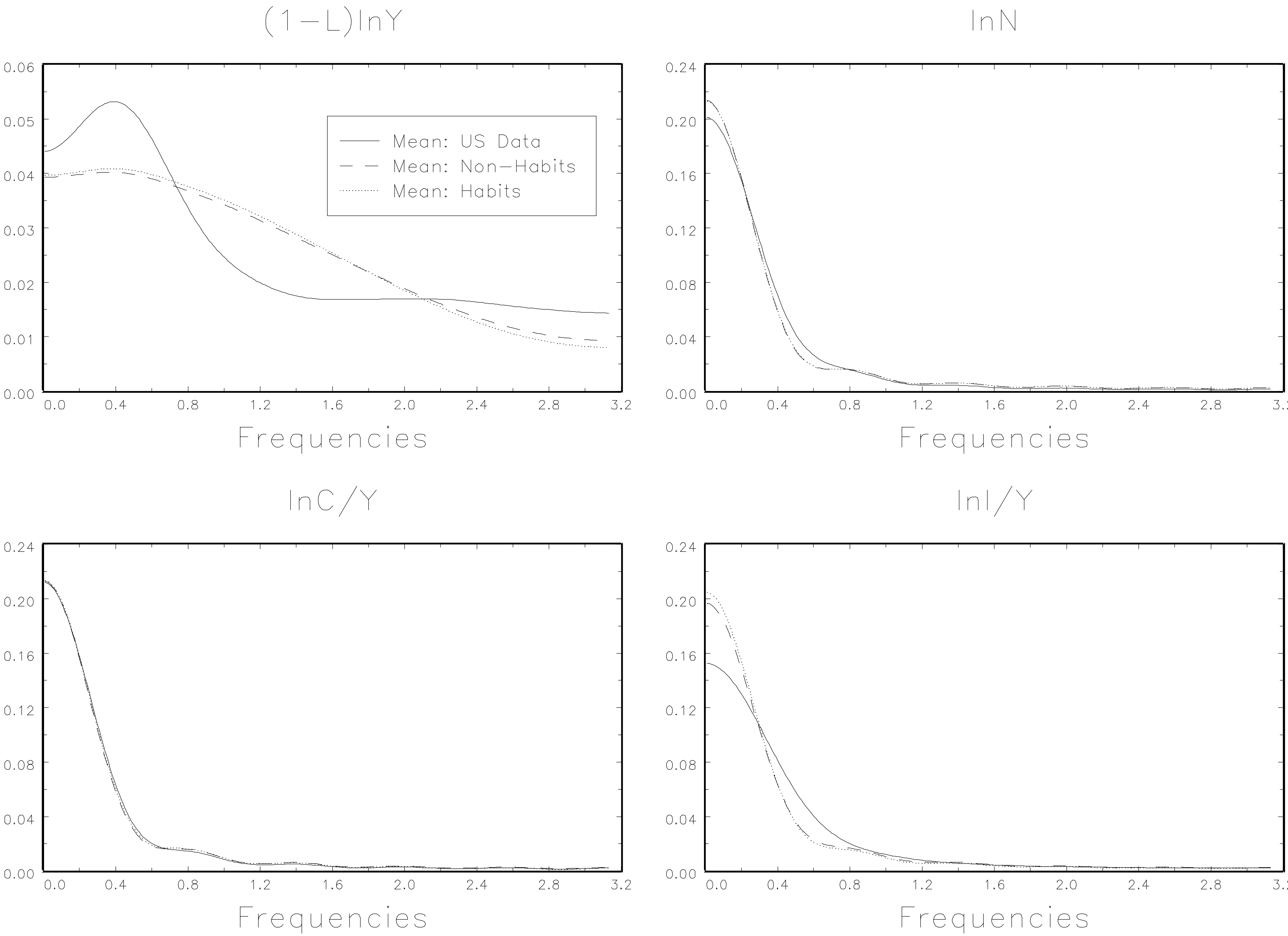

Figure 1(c): Empirical and Theoretical Means of SDFs: MBC

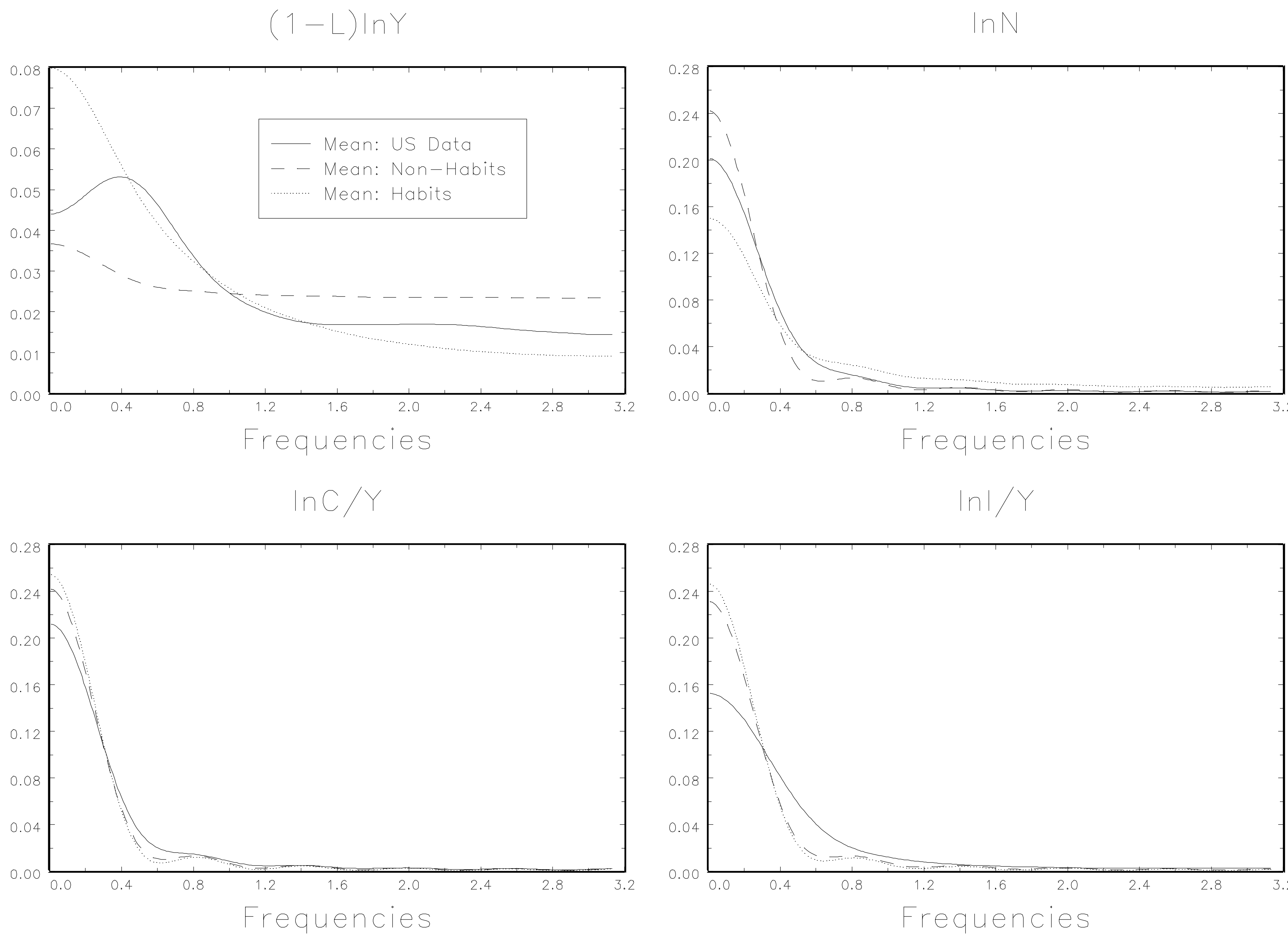

Figure 1(d): Empirical and Theoretical Means of SDFs: NKMBC

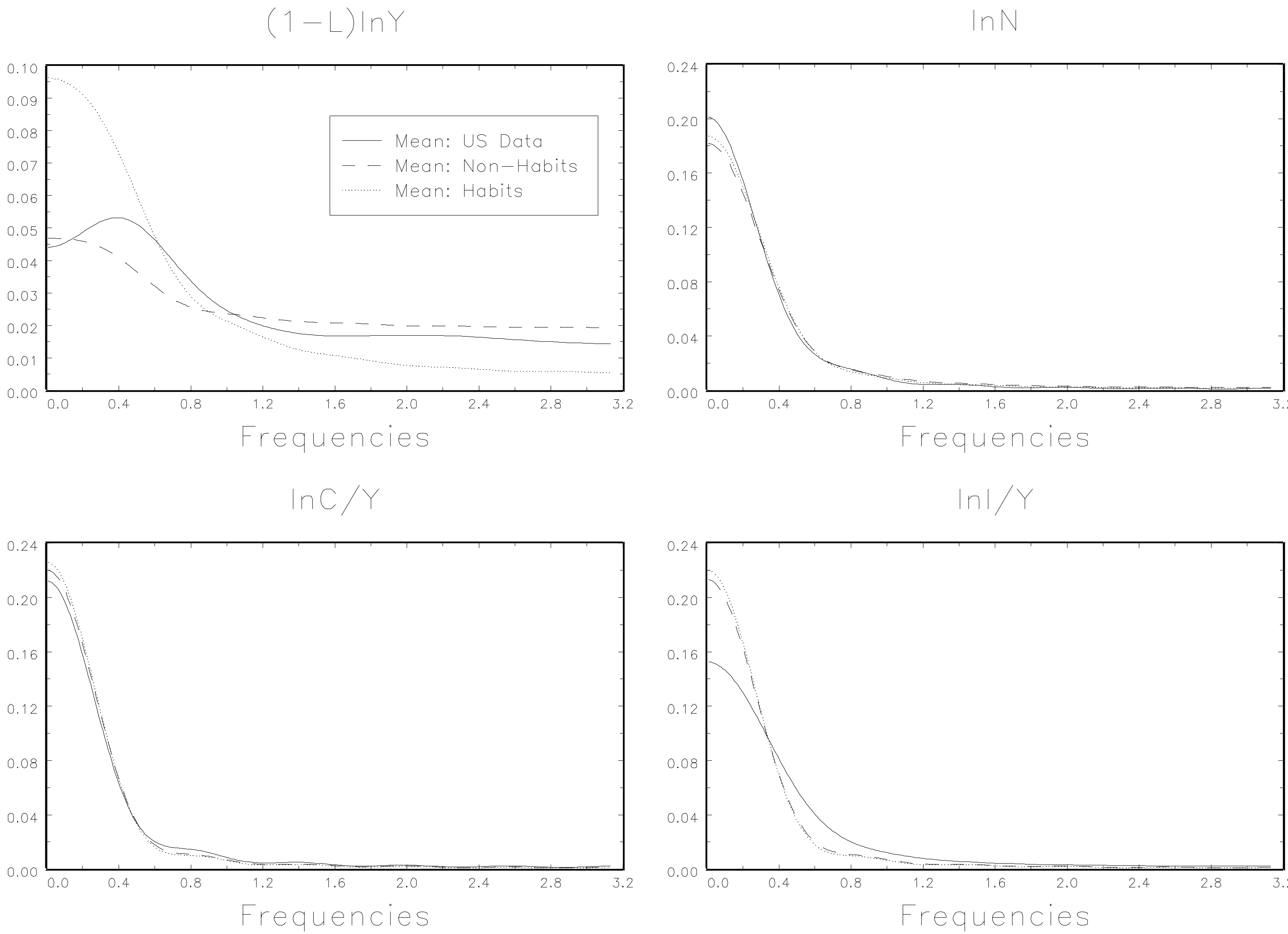

Figure 2(a): Empirical and Theoretical Distributions of the QKS and QCVM statistics: RBC

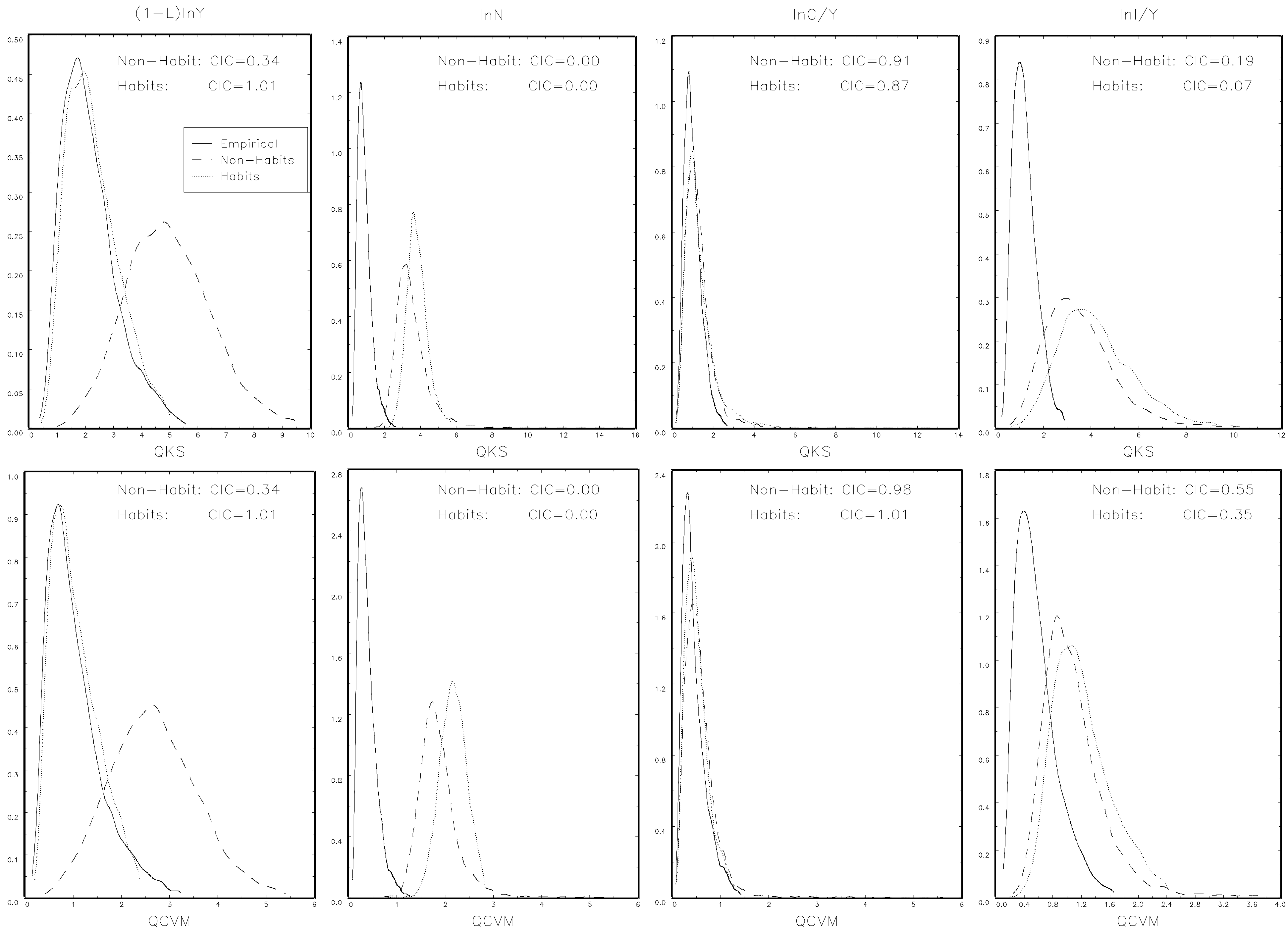

Figure 2(b): Empirical and Theoretical Distributions of the QKS and QCVM statistics: TSRBC

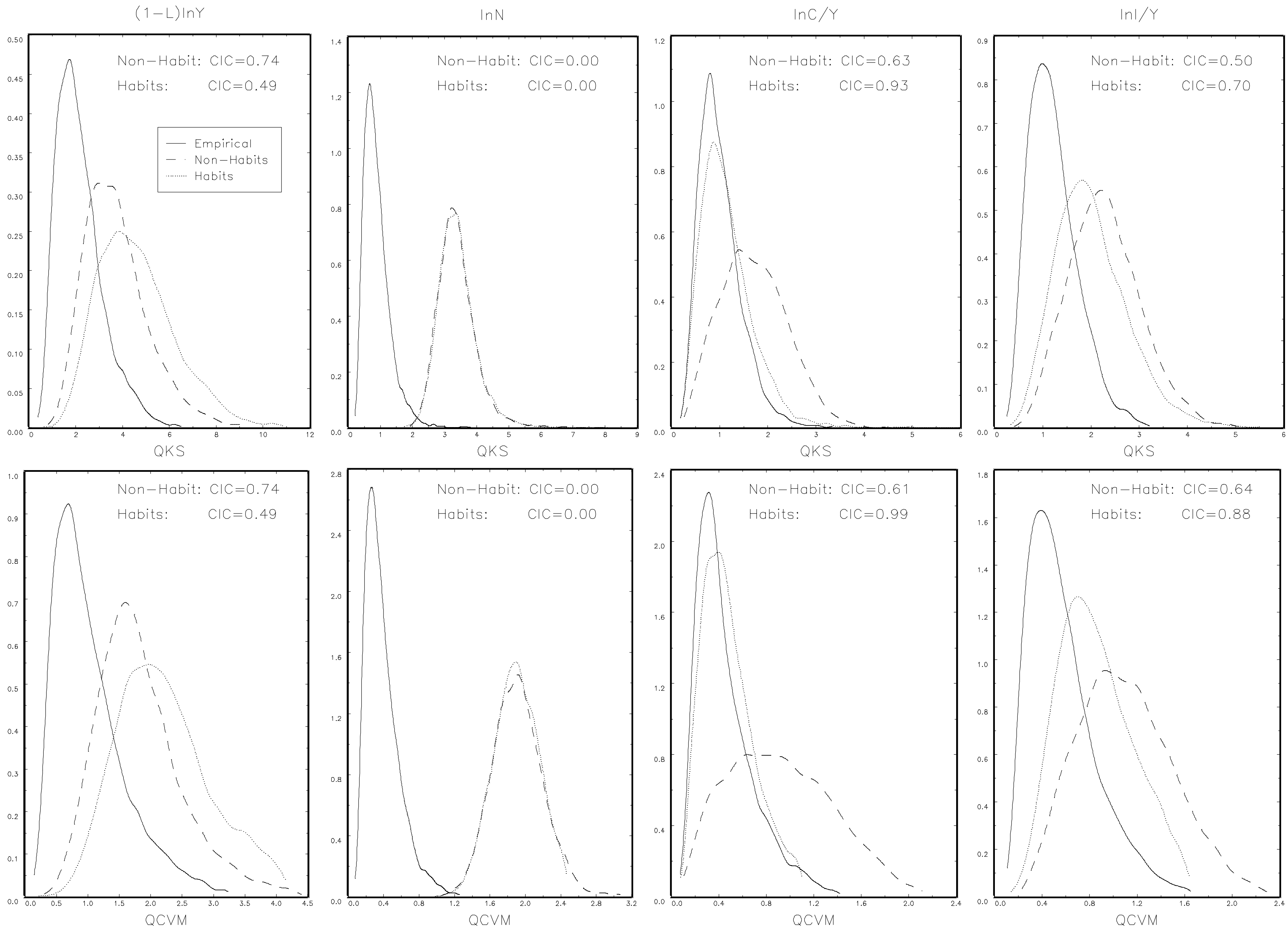

Figure 2(c): Empirical and Theoretical Distributions of the QKS and QCVM statistics: MBC

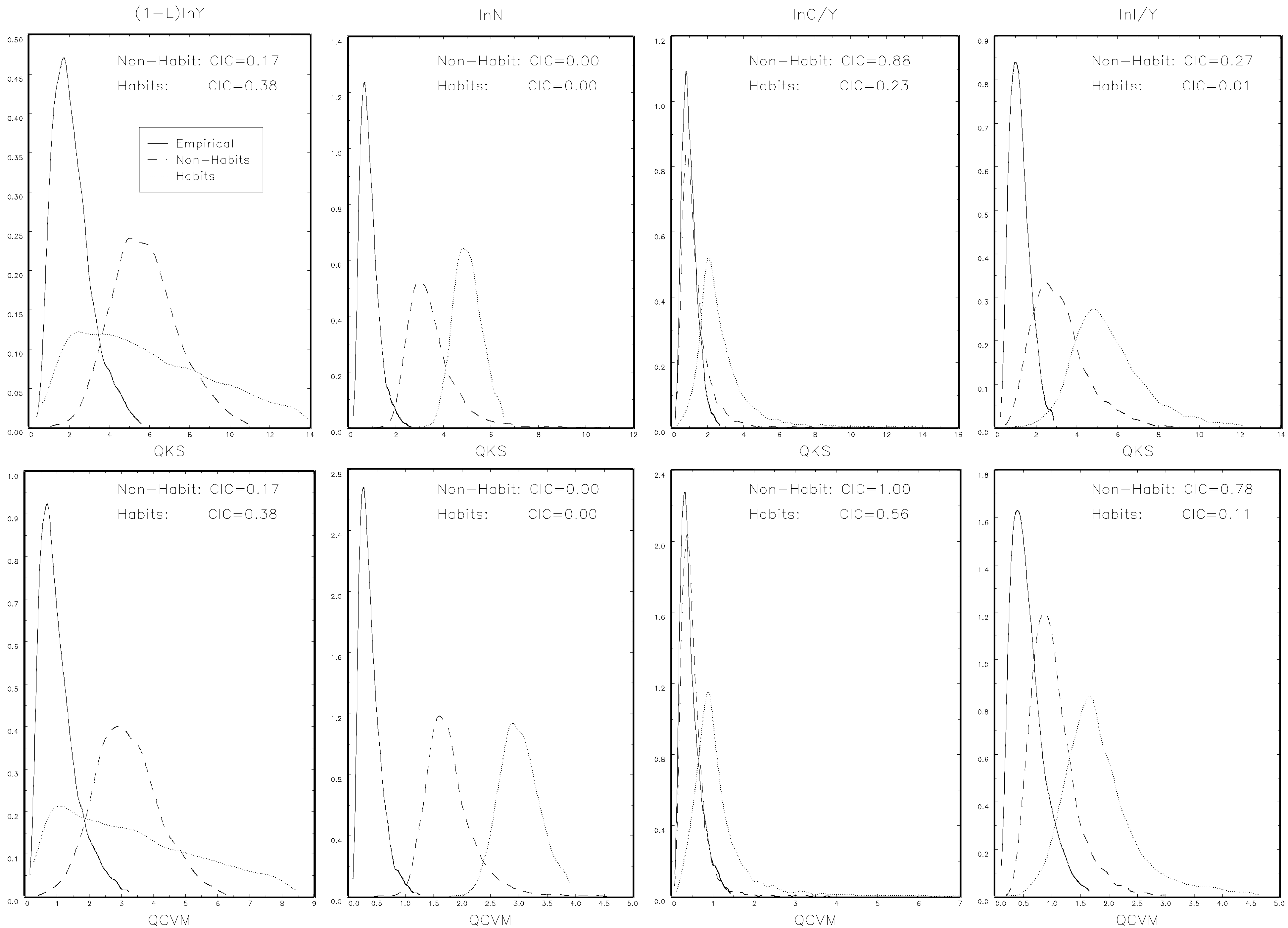

Figure 2(d): Empirical and Theoretical Distributions of the QKS and QCVM statistics: NKMBC

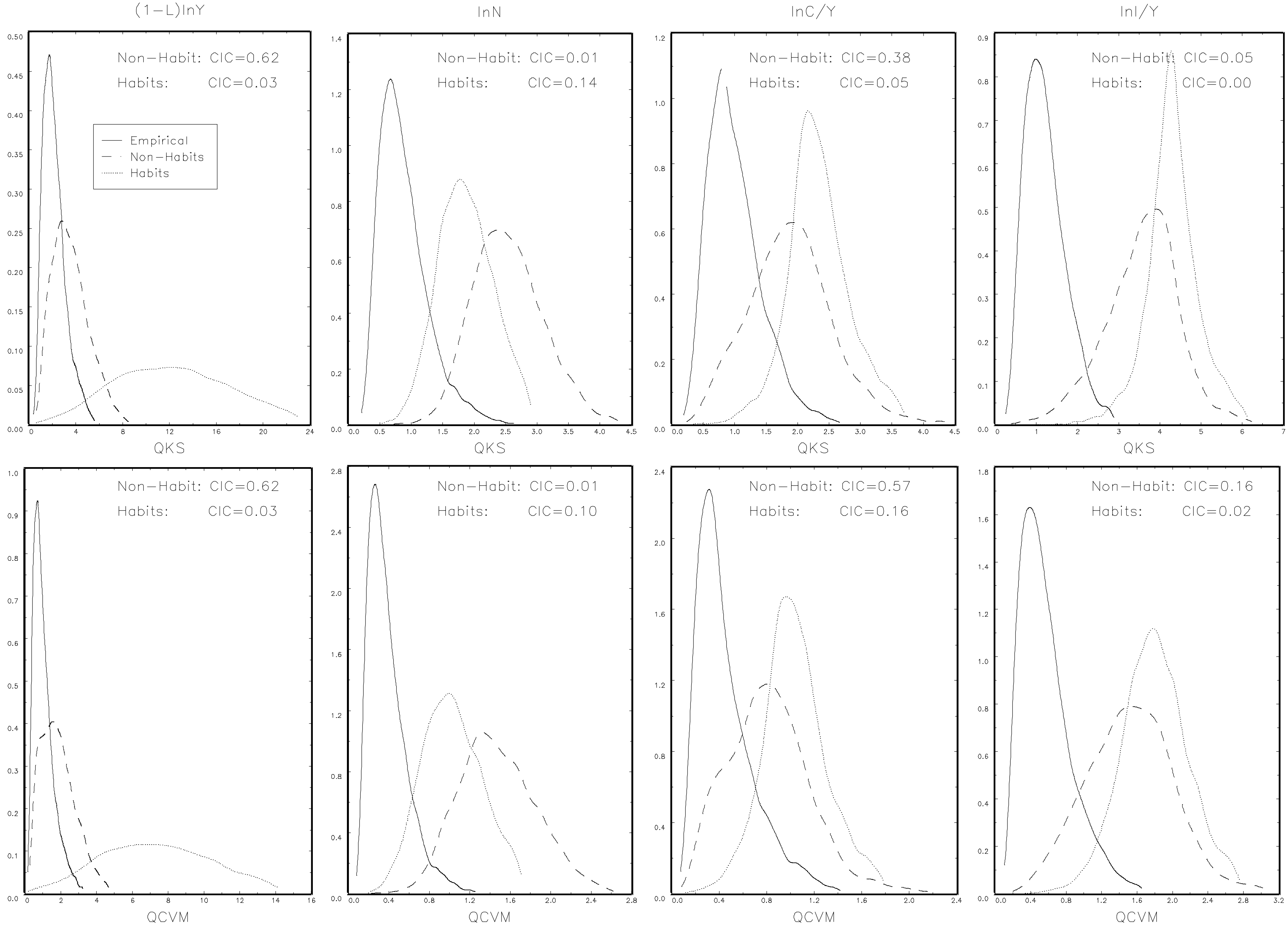

Figure 3 Empirical and Theoretical Means of IRFs: NKMBC

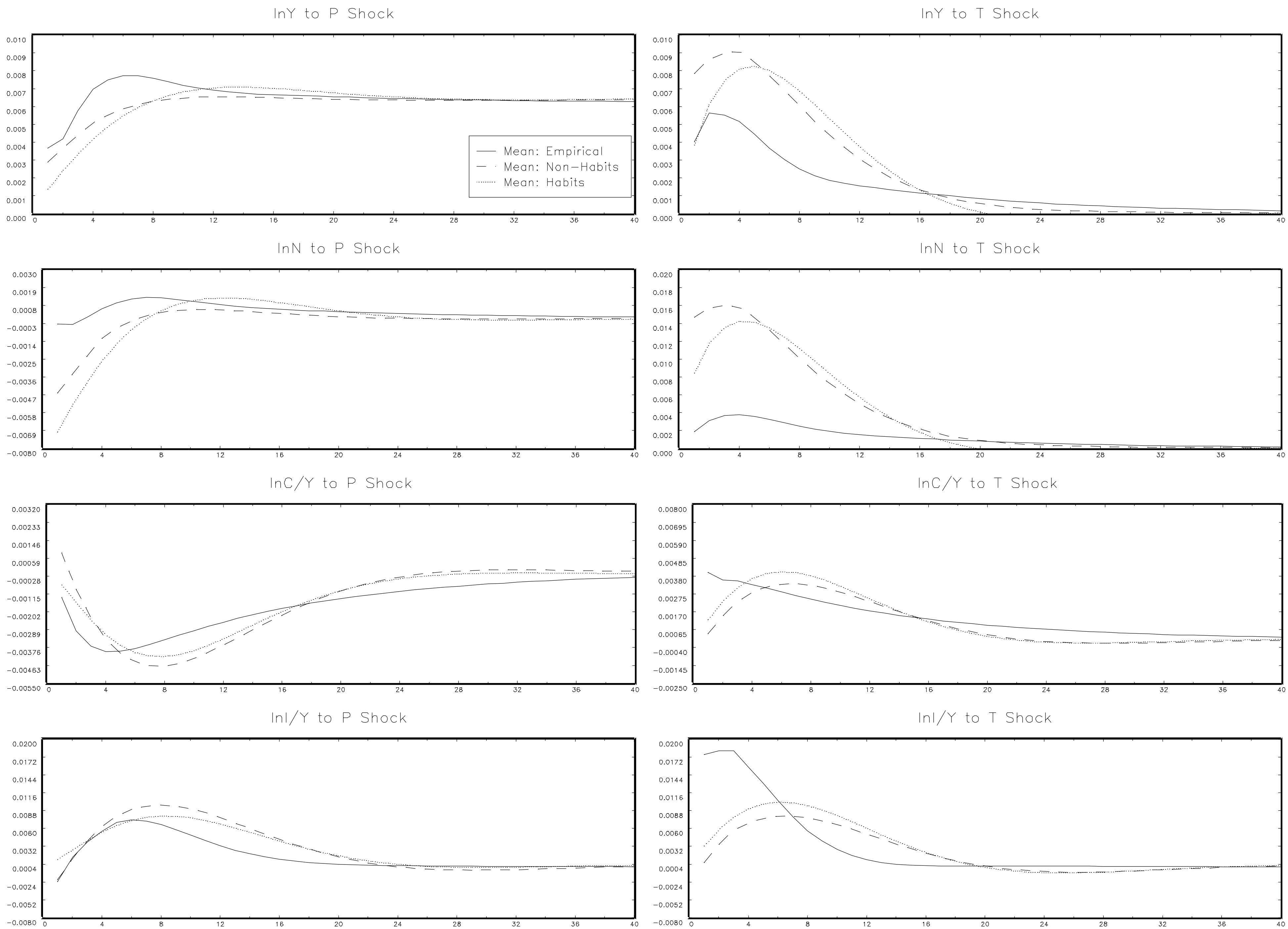

Figure 4 Empirical and Theoretical Means of FEVDs: NKMBC