Learning About Which Measure of Inflation to Target*

Marco Airaudo† Luis-Felipe Zanna‡

This Draft: May 2005

Abstract

Using a closed economy model with a flexible-price good and a sticky-price good we study the conditions under which interest rate rules induce real determinacy and, more importantly, the MSV solution is learnable in the E-stability sense proposed by Evans and Honkapohja (2001). We show that these conditions depend not only on how aggressively the rule responds to inflation but also on the measure of inflation included in the rule and on whether the flexible-price good and the sticky-price good are Edgeworth complements, substitutes or utility separable. We consider three possible measures of inflation: the flexible-price inflation, the sticky price inflation and the core inflation; and we analyze three different types of rules: a forward-looking rule, a contemporaneous rule and a backward-looking rule. Our results suggest that in order to guarantee a unique equilibrium whose MSV representation is learnable, the government should implement a backward looking rule that responds exclusively to the sticky-price inflation. Forward-looking and contemporaneous rules that respond to either the flexible-price inflation or the core-inflation are more prone to induce multiple equilibria and E-instability of the MSV solution. More importantly backward-looking rules that react to either the flexible-price inflation or the core inflation may guarantee a unique equilibrium but in these cases the fundamental solution (MSV representation) is not learnable in the E-stability sense.

Keywords: Learning; Expectational Stability; Interest Rate Rules; Multiple Equilibria; Determinacy

JEL Classifications: E4, E5

*The authors thank the participants of the Federal Reserve Board IF workshop for their helpful comments. All errors remain ours. The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the view of the LLEE or the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System.

†LUISS Lab on European Economics, Rome, and Center for European Policy Studies, Place du Congrès 1 B-1000 Brussels, Belgium. Phone: +3222293939 E-mail: marco.airaudo@ceps.be.

‡Board of Governors of the Federal Reserve System, 20th Street and Constitution Avenue, NW, Washington, D.C., 20551. Phone: (202)452-2337. Fax: (202)736-5638. E-mail: Luis-Felipe.Zanna@frb.gov.

1 Introduction

In recent years there has been a revival of theoretical literature aimed at understanding the macroeconomic consequences of implementing interest rate rules in closed and open economies models.[1] This revival is partly explained by two reasons. First since the work by Taylor (1993) it has become common to think about monetary policy in terms of these rules whereby the central bank maneuvers the nominal interest rate in response to inflation and output. Second there is empirical evidence suggesting that some industrialized and developing economies have followed in the past, and are following in the present, forward-looking (and contemporaneous) interest rate rules as a manner to conduct their monetary policy.[2]

Recently, works by Benhabib, Schmitt-Grohé and Uribe (2001), Bernanke and Woodford (1997), Clarida, Gali and Gertler (2000), Carlstrom and Fuerst (1999), and Woodford (2003), among others, have used New Keynesian models and pursued determinacy equilibrium analysis for interest rate rules.[3] They have found that these rules may generate aggregate instability in the economy by inducing multiple equilibria (real indeterminacy).[4] From the economic policy-design perspective, this result implies that the aforementioned rules may open the possibility of sunspot equilibria and lead the economy to equilibria with undesirable properties such as a large degree of volatility. This implication in turn suggests that a determinacy of equilibrium analysis can be used to differentiate among rules favoring those that guarantee a unique equilibrium with a lower degree of volatility. Although appealing this argument is still far from complete and may suffer from some drawbacks. The reason is that in the typical determinacy of equilibrium analysis, it is implicitly assumed that agents can coordinate their actions and learn the equilibria (unique or multiple) induced by the rule. But relaxing this assumption may have interesting consequences for the design of interest rate rules rules. On one hand, if agents cannot learn the unique equilibrium targeted by the rule then the economy may end up diverging from this equilibrium. But if this is the case then it is clear that there are some rules that although guaranteeing a unique equilibrium, do not ensure that the economy will reach it, as pointed by Bullard and Mitra (2002). On the other hand, if agents cannot learn sunspot equilibria then one may doubt the relevance of characterizing rules that lead to multiple equilibria as "bad" ones. After all, if agents cannot learn sunspot equilibria then they are less likely to occur.

Therefore, it seems clear that a determinacy of equilibrium analysis should in principle be accompanied by a learnability of equilibrium analysis. Both analyses might help policy makers to distinguish and design

[1] See Taylor (1999) and Woodford (2003) among others for closed economy models and Ball (1999), Svensson (2000) and Gali and Monacelli (2004) among others for open economy models..

[2] See Clarida et al. (1998, 2000), Corbo (2000), Schorfheide and Lubik (2004) and Orphanides (1997) among others.

[3] See also Dupor (1999) and Taylor (1999).

[4] From now on we will use the terms "multiple equilibria" and "real indeterminacy" (a "unique equilibrium" and "real determinacy") interchangeably. By real indeterminacy we mean a situation in which the behavior of one or more (real) variables of the model is not pinned down by the model. This situation implies that there are multiple equilibria and opens the possibility of the existence of sunspot equilibria.

interest rate rules satisfying two criteria or requirements: uniqueness and learnability of the equilibrium.

In this paper we use these two criteria to answer the following question: *which measure of inflation should the central bank respond to in the interest rate rule?* In order to provide an answer we consider a "typical" New Keynesian closed economy general equilibrium model with two goods: a flexible-price good and a sticky-price good. Hence there are three possible measures of inflation that the central bank can target in the rule: the flexible-price inflation, the sticky price inflation and a convex combination of them (a core inflation). We also consider three different timings for the rule: a forward-looking rule, a contemporaneous rule and a backward-looking rule. In the first one, the central bank responds to the expected future (measure of) inflation. In the second one the monetary authority responds to the current inflation whereas in the last one it reacts to the past inflation.

The specific criteria of selection are the following: 1) the rule must guarantee a unique equilibrium and 2) the Minimal State Variable (MSV) representation of this unique equilibrium must be learnable in the E-Stability sense proposed by Evans and Honkapohja (1999, 2001).[5,6]

We find that the conditions under which these two criteria are satisfied depend not only on the interest rate response coefficient to inflation but also on the measure of inflation included in the rule and on wether the flexible-price good and the sticky-price good are Edgeworth complements, substitutes or utility separable.

The main result of the paper is that under the aforementioned criteria the central bank should follow a backward looking rule that responds exclusively to the sticky-price inflation. Responding to the flexible-price inflation or to the core inflation is more prone to induce multiple equilibria (real indeterminacy) especially when the rule is forward-looking or contemporaneous and when the two goods are either Edgeworth complements or utility separable. Under these cases targeting the flexible-price inflation or the core inflation makes also the MSV solution E-unstable.

More importantly backward-looking rules with respect to the flexible-price inflation or the core inflation may guarantee a unique equilibrium but in these cases the MSV solution is not learnable in the E-stability sense.

Although our selection criteria are different, our findings agree with previous results from the Optimal Monetary Policy literature that suggest that the central bank should target the sticky-price inflation.[7] In addition, this paper is also related to Zanna (2003) and Carlstrom, Fuerst and Ghironi (2004). The first work argues that in the context of a small open economy the interest rate rule should respond to the sticky

[5]Henceforth we will use the terms "learnability", "E-stability" and "expectational stability" interchangeably in this paper.

[6]Evans and Honkapoja (1999, 2001) have argued that a unique equilibrium and sunspot equilibria are not "fragile" if they are learnable in the sense of E-stability. Technically what they propose is to assume that agents in the model initially do not have rational expectations but are endowed with a mechanism to form forecasts using recursive learning algorithms and previous data from the economy. Then they develop some E-stability conditions which govern whether or not a given rational expectations equilibrium is aymptotically stable under least squares learning.

[7]See Aoki (2001) and Mankiw and Reis (2002) among others.

price (non-traded) inflation in order to avoid real indeterminacy problems. In fact rules that respond to the flexible-price (traded) inflation are more prone to induce self-fulfilling equilibria. The second work focuses on the a closed economy model where labor is immobile between two sectors that produce two sticky-price goods. Its main conclusion is that a contemporaneous rule that responds actively to only one of the sticky price goods inflations is sufficient to guarantee real determinacy in the whole economy. Both works focus exclusively on the determinacy of equilibrium analysis without pursuing a learnability analysis.

The remainder of this paper is organized as follows. Section 2 presents the set-up with its main assumptions of a closed economy model with one composite sticky-price good and one flexible-price good. Section 3 pursues the determinacy of equilibrium and learning (E-stability) analyses for the three timings of the rule (forward-looking, contemporaneous and backward-looking) that respond exclusively to one of the previously mentioned measures of inflation. Finally Section 4 concludes.

2 The Model

2.1 The Household-Firm Unit

Consider a closed economy populated by a large number of identical and infinitely lived household-firm units, each of whom derives utility from consumption of two goods, from liquidity services of money, and from not working in the production of goods. Its preferences are described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t [U(c_t^F, c_t^S) + L(m_t) + V(h_t^F, h_t^S)] \tag{1}$$

where c_t^F and c_t^S denote consumptions of a flexible-price good and a sticky-price good respectively, m_t corresponds to real money balances measured with respect to the flexible-price good, P_t^F, and h_t^F and h_t^S denote labor efforts required to produce the aforementioned goods. $\beta \in (0,1)$ represents the subjective discount factor and E_0 is the expectational operator conditional on the set of information available at time 0.

By the specification in (1) it is clear that we assume separability in the single period utility function among consumption, real money balances and labor. By doing this we remove the distortionary effects of transactions money demand.[8] More formally we assume the following.

Assumption 1: *The functions specified in (1) satisfy: a)* $U_F \equiv \frac{\partial U}{\partial c_t^F} > 0$, $U_{FF} < 0$, $U_S > 0$, $U_{SS} < 0$, $U_{FS} = U_{SF}$, $U_{SS} - U_{SF}\frac{U_S}{U_F} < 0$ *and* $U_{FF}U_{SS} - (U_{FS})^2 > 0$; *b)* $L_m > 0$ *and* $L_{mm} < 0$; *and c)* $V_F \equiv \frac{\partial V}{\partial h_t^F} < 0$, $V_{FF} < 0$, $V_S < 0$, $V_{SS} < 0$ *and* $V_{SF} = V_{FS} = 0$.

[8] This assumption also allows us to write the real money balances that enter the utility of the agent in terms of the price of the flexible-price good, $m_t \equiv \frac{M_t}{P_t^F}$, without consequences for our results.

In particular it is important to notice that we assume that the instantaneous utility function of consumption, $U(.,.)$, is strictly increasing and concave in both arguments and that both goods are normal. However we are not imposing any sign restrictions in the cross derivatives U_{FS} and U_{SF}. We assume that they are equal but in this paper we will consider three cases: c_t^F and c_t^S can be Edgeworth complements ($U_{FS} > 0$), Edgeworth substitutes ($U_{FS} < 0$) or they may be utility separable ($U_{FS} = 0$).[9]

The representative household-firm unit is engaged in the production of the flexible-price good and the sticky-price good by employing labor from a perfectly competitive market. The technologies are described by

$$y_t^F = z_t^F f\left(\breve{h}_t^F\right) \qquad and \qquad y_t^S = z_t^S g\left(\tilde{h}_t^S\right) \tag{2}$$

where $\breve{h}_t^T$ and $\tilde{h}_t^N$ denote the labor hired by the household-firm unit for the production of the flexible-price and the sticky-price goods respectively; z_t^F and z_t^S are productivity shocks whose logarithms, $\hat{z}_t^F = \ln(z_t^F)$ and $\hat{z}_t^S = \ln(z_t^S)$, follow stationary AR(1) stochastic processes:

$$\hat{z}_t^F = \phi^F \hat{z}_{t-1}^F + \xi_t^F \qquad\qquad \hat{z}_t^S = \phi^S \hat{z}_{t-1}^S + \xi_t^S \tag{3}$$

with $\phi^F, \phi^S \in (0,1)$ and $\xi_t^F \sim \mathcal{N}(0, \sigma^F)$ and $\xi_t^S \sim \mathcal{N}(0, \sigma^S)$. Technology shocks are the only source of fundamentals uncertainty and that they are not cross-correlated. The structure of the production functions is completed by the following assumption.

Assumption 2: *$f_h > 0$, $f_{hh} < 0$, $g_h > 0$, and $g_{hh} < 0$.*

In other words the production functions are strictly increasing and strictly concave in their arguments.

The consumption of the stick-price good, c_t^S, is assumed to be a composite good made of a continuum of intermediate differentiated goods. The aggregator function is described of the Dixit-Stiglitz type. We assume that each household-firm unit is the monopolistic producer of one variety of sticky-price intermediate goods. The demand for the intermediate good is of the form $C_t^S d\left(\frac{\tilde{P}_t^S}{P_t^S}\right)$ satisfying $d(1) = 1$ and $d'(1) = -\mu$, where C_t^S denotes the level of aggregate demand for the sticky-price good, $\tilde{P}_t^S$ is the nominal price of the intermediate sticky-price good produced by the household-firm and P_t^S is the price of the composite sticky-price good. The household-firm unit that behaves as a monopolist in the production of the sticky-price good, sets the price of the good it supplies, $\tilde{P}_t^S$, taking the level of aggregate demand for the sticky-price good as given. The monopolist is constrained to satisfy demand at that price. That is

[9] For instance the commonly used utility function

$$U(c_t^F, c_t^S) = \frac{\left[(\alpha_p)^{\frac{1}{a}}\left(c_t^F\right)^{\frac{a-1}{a}} + (1-\alpha_p)^{\frac{1}{a}}\left(c_t^S\right)^{\frac{a-1}{a}}\right]^{\left(\frac{a}{a-1}\right)(1-\sigma)} - 1}{1-\sigma}$$

with $\alpha_p \in (0,1)$ and $\sigma, a > 0$ satisfies Assumption 1a) and the sign of U_{FS} is determined by the values of the intratemporal elasticity of substitution, a, and the intertemporal elasticity of substitution, $\frac{1}{\sigma}$. Specifically $U_{FS} \gtreqless 0$ if and only if $\frac{1}{\sigma} \gtreqless a$.

$$z_t^S g\left(\tilde{h}_t^S\right) \geq C_t^S \left(\frac{\tilde{P}_t^S}{P_t^S}\right)^{-\mu} \tag{4}$$

Following Rotemberg (1982) we introduce sluggish price adjustment in the production of the intermediate sticky-price good. We assume the household firm unit faces a quadratic resource cost (Q_t^S) in the inflation rate of the sticky-price good it produces. Hence

$$Q_t^S = \frac{\gamma}{2}\left(\frac{\tilde{P}_t^S}{\tilde{P}_{t-1}^S} - \bar{\pi}\right)^2$$

where the parameter γ measures the degree of price stickiness. If $\gamma = 0$ then prices are flexible. The higher γ is the more sluggish is the adjustment of nominal prices.

Moreover we assume that the representative household-firm unit can invest in a bond issued by the government, B_t, that pays a gross nominal interest rate, R_t. The real value of this asset is denoted by $b_t = B_t/P_t^F$ This representative agent receives a wage income from working, $W_t^F h_t^F + W_t^S h_t^S$, and pays labor costs, $W_t^F \check{h}_t^F$ and $W_t^S \tilde{h}_t^S$, for producing goods. Letting denote $n_t \equiv m_t + b_t$ denote the real financial wealth, the household-firm unit's flow budget constraint is then given by

$$\begin{aligned} n_t \quad \leq \quad & \frac{R_{t-1}}{\pi_t^F} n_{t-1} + \frac{(1-R_{t-1})}{\pi_t^F} m_{t-1} + \frac{W_t^F}{P_t^F} h_t^F + \frac{W_t^S}{P_t^F} h_t^S - c_t^F - \frac{c_t^S}{q_t} + P_t^F T_t^g \\ & + \left[z_t^F f\left(\check{h}_t^F\right) - \frac{W_t^F}{P_t^F}\check{h}_t^F\right] + \frac{1}{q_t}\left[C_t^S d\left(\frac{\tilde{P}_t^S}{P_t^S}\right) - \frac{W_t^S}{P_t^S}\tilde{h}_t^S - \frac{\gamma}{2}\left(\frac{\tilde{P}_t^S}{\tilde{P}_{t-1}^S} - \bar{\pi}\right)^2\right] \end{aligned} \tag{5}$$

where $q_t \equiv \frac{P_t^F}{P_t^S}$ refers to the relative price between the flexible-price good and the sticky-price of the composite goods, $\pi_t^F \equiv \frac{P_t^F}{P_{t-1}^F}$ denotes the gross flexible-price good inflation, T_t^g is the lump-sum transfers from the government and the expressions in square brackets correspond to profits that the representative agent receives from selling the flexible-price good and the sticky-price composite good.

In addition the household-firm unit is subject to an Non-Ponzi game condition of the form

$$\lim_{t\to\infty} \frac{n_t}{\prod_{j=0}^{t-1}\left(R_j/\pi_{j+1}^F\right)} \geq 0 \tag{6}$$

The representative household-firm unit chooses the set of processes $\{c_t^F, c_t^S, h_t^F, h_t^S, \check{h}_t^F, \tilde{h}_t^S, \tilde{P}_t^S\}$ and $\{m_t, n_t\}_{t=0}^{\infty}$ in order to maximize (1) subject to (4), (5) and (6), given the initial condition n_0 and the set of processes $\{R_t,\ P_t^F,\ P_t^S,\ W_t^F,\ W_t^S, T_t^g\ ,\ C_t^S,\ z_t^F,\ z_t^S\}$. Note that since the utility function specified in (1) implies that the preferences of the agent display non-sasiation then constraints (5) and (6) both hold with equality.

The first order conditions correspond to (5) and (6) with equality and

$$U_F(c_t^F, c_t^S) = \lambda_t \tag{7}$$

$$\frac{U_F(c_t^F, c_t^S)}{U_S(c_t^F, c_t^S)} = q_t \tag{8}$$

$$-\frac{V_F(c_t^F, c_t^S)}{U_F(c_t^F, c_t^S)} = \frac{W_t^F}{P_t^F} \tag{9}$$

$$-\frac{V_S(c_t^F, c_t^S)}{U_F(c_t^F, c_t^S)} = \frac{W_t^S}{P_t^F} \tag{10}$$

$$1 = \frac{W_t^F/P_t^F}{z_t^F f_h\left(\check{h}_t^F\right)} \tag{11}$$

$$\frac{\varpi_t}{\lambda_t} = \frac{W_t^S/P_t^S}{q_t z_t^S g_h\left(\tilde{h}_t^S\right)} \tag{12}$$

$$\begin{aligned} 0 \quad = \quad & \frac{\lambda_t C_t^S}{q_t} d\left(\frac{\tilde{P}_t^S}{P_t^S}\right) + \frac{\lambda_t \tilde{P}_t^S C_t^S}{q_t P_t^S} d'\left(\frac{\tilde{P}_t^S}{P_t^S}\right) - \gamma \frac{\lambda_t}{q_t}\left(\frac{\tilde{P}_t^N}{\tilde{P}_{t-1}^N} - \bar{\pi}\right)\frac{\tilde{P}_t^S}{\tilde{P}_{t-1}^S} - \varpi_t C_t^S d'\left(\frac{\tilde{P}_t^S}{P_t^S}\right) \\ & + \beta\gamma E_t\left[\frac{\lambda_{t+1}}{q_{t+1}}\left(\frac{\tilde{P}_{t+1}^S}{\tilde{P}_t^S} - \bar{\pi}\right)\frac{\tilde{P}_{t+1}^S}{\tilde{P}_t^S}\right] \end{aligned} \tag{13}$$

$$\frac{1}{L_m(m_t)} = \frac{1}{U_F(c_t^F, c_t^S)}\left(\frac{R_t}{R_t - 1}\right) \tag{14}$$

$$\lambda_t = \beta E_t\left(\frac{\lambda_{t+1} R_t}{\pi_{t+1}^F}\right) \tag{15}$$

where ϖ_t and λ_t correspond to the Lagrange multipliers of (4) and (5) respectively.

The interpretation of the first order conditions is straightforward. Condition (7) is the usual intertemporal envelope condition that makes the marginal utility of consumption of flexible-price goods equal to the marginal utility of wealth (λ_t). Condition (8) implies that the marginal rate of substitution between flexible-price and the sticky-price goods must be equal to the relative price between these goods. In addition conditions (9) makes the marginal rate of substitution between labor supplied for the production of the flexible-price good and consumption of the flexible-price good equal to the real wage paid for producing the flexible-price good. Condition (10) does the same for labor supplied for the production of the sticky-price good but in this case the real wage corresponds to the ratio between the nominal wage paid for producing the sticky-price good and the flexible price. Conditions (11) (respectively 12) equalizes the marginal cost to the ratio between the real wage and the marginal product of labor for producing the flexible-price good (sticky-price good). Equation (14) represents the demand for real balances of money that, under Assumption 1, is as an increasing function of consumption of the flexible-price good and a decreasing function of the gross nominal interest rate. And finally condition (15) implies a standard asset pricing equation.

2.2 The Government

The government issues two nominal liabilities: money, M_t^g, and a domestic bond, B_t^g, that pays a gross nominal interest rate R_t. It also makes transfers , $P_t^F T_t^g$, pays interest on its debt, $(R_t - 1)B_t^g$, and receives revenues from seigniorage. By letting $n_t^g \equiv (M_t^g + B_t^g)/P_t^F$ denote the real government liabilities at the beginning of period we can write the government budget constraint in real terms as

$$n_t^g = \frac{R_{t-1}}{\pi_t^F} n_{t-1}^g - \left[\frac{(R_{t-1} - 1)}{\pi_t^F} m_{t-1} - T_t^g \right] \tag{16}$$

We assume that the government follows a generic Ricardian fiscal policy. Under this policy, it picks the path of transfers, T_t^g, satisfying the intertemporal version of (16) in conjunction with the transversality condition

$$\lim_{t \to \infty} \frac{n_t^g}{\prod_{j=0}^{t-1} \left(R_j / \pi_{j+1}^F \right)} = 0 \tag{17}$$

We define monetary policy as an interest rate feedback rule whereby the government sets the gross nominal interest rate, R_t, as an increasing and continuous function of the deviation of inflation with respect to a target

$$R_t = R^* \rho \left(E_t \left(\frac{\pi_{t+j}}{\pi^*} \right) \right) \qquad with \;\; j = -1, 0, 1 \tag{18}$$

where π^* corresponds to the inflation target and $R^* = \pi^*/\beta$. We will consider three different timings. In other words the rule may be forward-looking responding to the expected inflation deviation, $E_t \left(\frac{\pi_{t+1}}{\pi^*} \right)$; contemporaneous, when the rule reacts to the current inflation deviation, $\left(\frac{\pi_t}{\pi^*} \right)$, or backward-looking, when it responds to the past inflation deviation $\left(\frac{\pi_{t-1}}{\pi^*} \right)$. The measure of inflation that we consider in (18) corresponds to the core inflation, and it is a convex combination of the flexible-price inflation, $\pi_t^F \equiv \frac{P_t^F}{P_{t-1}^F}$ and the sticky-price inflation, $\pi_t^S \equiv \frac{P_t^S}{P_{t-1}^S}$. In other words,

$$\pi_t = w \pi_t^F + (1 - w) \pi_t^S \tag{19}$$

where $w \in [0, 1]$ is the weight that the government puts on the flexible-price inflation. Note that when the government sets $w = 1$ then the rule responds exclusively to the flexible-price inflation, $\pi_t = \pi_t^F$. On the other hand if the government sets $w = 0$ then the rule responds solely to the sticky-price inflation, $\pi_t = \pi_t^S$.

We will assume that the government responds aggressively to inflation. That is, at the inflation rate target, in response of a one percent of inflation the government will raise the nominal interest by more that one percent. In the terminology of Leeper (1991) this rule is called an "active" rule.

2.3 The Equilibrium

We will focus on a symmetric equilibrium in which all the monopolistic producers of sticky-price goods pick the same price. Hence $\tilde{P}_t^S = P_t^N$. Since all the monopolists face the same wage rate, W_t^S, the same technology shock , z_t^S and the same production function, $z_t^S g\left(h_t^S\right)$, then they will demand the same amount of labor $\tilde{h}_t^S = h_t^S$. In equilibrium the money market, the labor markets, the flexible-price good market and the sticky-price goods market clear. Therefore

$$m_t = m_t^g \tag{20}$$

$$h_t^F = \check{h}_t^F \tag{21}$$

$$h_t^S = \tilde{h}_t^S \tag{22}$$

$$c_t^F = z_t^F f\left(h_t^F\right) \tag{23}$$

and

$$c_t^S = C_t^S = z_t^S g\left(h_t^S\right) - \frac{\gamma}{2}\left(\pi_t^S - \bar{\pi}\right)^2 \tag{24}$$

Utilizing the notion of a symmetric equilibrium, conditions (7)-(13), (22), (24) and the definitions $\pi_t^S = P_t^S / P_{t-1}^S$, $d(1) = 1$ and $d'(1) = -\mu$ we can rewrite the first order condition (13) as

$$E_t\left[U_S(c_{t+1}^F, c_{t+1}^S)(\pi_{t+1}^S - \bar{\pi})\pi_{t+1}^S\right] = \frac{U_S(c_t^F, c_t^S)(\pi_t^S - \bar{\pi})\pi_t^S}{\beta} - \frac{(1-\mu)c_t^S U_S(c_t^F, c_t^S)}{\beta\gamma} + \frac{\mu c_t^S V_S(h_t^F, h_t^S)}{\beta\gamma z_t^S g_h\left(h_t^S\right)} \tag{25}$$

that corresponds to the augmented Phillips curve for the sticky-price goods inflation.[10] Furthermore using conditions (7), (8), (15) and the definition of q_t we obtain

$$U_F(c_{t+1}^F, c_{t+1}^S) = \beta E_t\left[\frac{U_F(c_{t+1}^F, c_{t+1}^S)R_t}{\pi_{t+1}^F}\right] \tag{26}$$

and

$$U_S(c_{t+1}^F, c_{t+1}^S) = \beta E_t\left[\frac{U_S(c_{t+1}^F, c_{t+1}^S)R_t}{\pi_{t+1}^S}\right] \tag{27}$$

that correspond to the Euler equations for consumption of the flexible-price and the sticky-price goods respectively. In addition from (9), (11) and (21) we can derive the equation

$$-\frac{V_F(c_t^F, c_t^S)}{U_F(c_t^F, c_t^S)} = z_t^F f_h\left(h_t^F\right) \tag{28}$$

[10] We would have derived a similar augmented Phillips curve if we had follow Calvo's (1983) approach.

We are ready to provide a definition of an equilibrium in an economy where the government follows a Ricardian fiscal policy and implements an interest rate rule.

Definition 1 *Given the initial condition n^g_{-1}, the inflation target, $\bar{\pi}$ and the exogenous stochastic processes $\left\{z^F_t, z^S_t\right\}^{\infty}_{t=0}$, a symmetric competitive equilibrium under a Ricardian fiscal policy is defined as a set of stochastic processes $\{c^F_t,\ c^S_t,\ h^F_t,\ h^S_t,\ n^g_t,\ T^g_t,\ \pi^F_t,\ \pi^S_t,\ R_t\}^{\infty}_{t=0}$ satisfying 1) conditions (25)-(28), 2) the market clearing conditions (23) and (24), 3) the intertemporal version of (16) together with (17), 4) the interest rate rule (18) and 5) the definition of core-inflation (19).*

Note that once we solve for $\{c^F_t, c^S_t, h^F_t, h^S_t, \tau_t, \pi^F_t, \pi^S_t,\ R_t\}^{\infty}_{t=0}$ it is possible to derive the set of processes $\{\lambda_t,\ q_t,\ m_t\}^{\infty}_{t=0}$ using (7), (8) and (14) whereas $n_t = n^g_t$.

For our determinacy analysis and learning analysis we can reduce the model further. In particular note that since the fiscal policy is assumed to be Ricardian, we know that the intertemporal version of the government's budget constraint in conjunction with its transversality condition will be always satisfied. Hence for the aforementioned analyses we will not consider (16) and (17).

2.4 The Log-linearized Economy

In this subsection we derive a log-linearized version of the system of equations that describe the competitive equilibrium of the economy, as stated in Definition 1. As explained before we exclude (16) and (17) given that the fiscal policy is Ricardian.

Specifically we log-linearize the rule (18), the definition of core-inflation (19) and the conditions (23), (24), and (25)-(28) around a non-stochastic steady state. In particular note that at the steady-state we have that $\bar{\pi} = \bar{\pi}^F = \bar{\pi}^S$. Manipulating the log-linearized equations we obtain the following set of equations

$$\bar{c}^F \hat{c}^F_t = \alpha \bar{c}^S \hat{c}^S_t + \varkappa_1 \hat{z}^F_t \tag{29}$$

$$\hat{c}^F_t = E_t\left(\hat{c}^F_{t+1}\right) + \frac{1}{\chi}\left[\hat{R}_t - E_t\left(\hat{\pi}^F_{t+1}\right)\right] + \varkappa_2 \hat{z}^F_t \tag{30}$$

$$\hat{c}^S_t = E_t\left(\hat{c}^S_{t+1}\right) + \frac{1}{\epsilon}\left[\hat{R}_t - E_t\left(\hat{\pi}^S_{t+1}\right)\right] - \varkappa_3 \hat{z}^F_t \tag{31}$$

$$\hat{\pi}_t^S = \beta E_t(\hat{\pi}_{t+1}^S) + \beta\delta\hat{c}_t^S + \beta\varkappa_4\hat{z}_t^S + \beta\varkappa_5\hat{z}_t^F \tag{32}$$

$$\hat{R}_t = \rho_\pi E_t\hat{\pi}_{t+j} \qquad with\ j = -1, 0, 1 \tag{33}$$

$$\hat{\pi}_t = w\hat{\pi}_t^F + (1-w)\hat{\pi}_t^S \tag{34}$$

where

$$\alpha = -\frac{(f_h)^2 U_{SF}}{f_{hh}U_F + (f_h)^2 U_{FF} + V_{FF}} \qquad \varkappa_1 = \frac{fV_{FF} - (f_h)^2 U_F + f_{hh}\bar{c}^F U_{SF}}{f_{hh}U_F + (f_h)^2 U_{FF} + V_{FF}} \tag{35}$$

$$\epsilon = \frac{(\alpha_1 U_{FS} + U_{SS})\,\bar{c}^S}{U_S} \qquad \chi = \frac{(\alpha_1 U_{FF} + U_{SF})\,\bar{c}^F}{\alpha_1 U_F} \tag{36}$$

$$\varkappa_2 = \frac{\alpha_2(1-\varphi^F)U_{SF}}{\alpha_1\chi U_F}\varkappa_3 = \frac{\alpha_2(1-\varphi^F)U_{FS}}{\epsilon U_S}$$

$$\delta = \frac{\mu\bar{c}^S}{\beta\gamma\bar{\pi}^2 g_h U_S}\left[\frac{V_S(U_{SS}+\alpha_1 U_{FS})\bar{c}^S}{U_S} - \frac{V_{SS}\bar{c}^S}{g_h} + \frac{g_{hh}V_S\bar{c}^S}{(g_h)^2}\right] \tag{37}$$

$$\varkappa_4 = \frac{\mu\bar{c}^S}{\beta\gamma\bar{\pi}^2 g_h U_S}\left[\frac{V_{SS}\bar{c}^S}{g_h} - \frac{g_{hh}V_S\bar{c}^S}{(g_h)^2} + V_S\right] \qquad \varkappa_5 = \frac{\mu\bar{c}^S}{\beta\gamma\bar{\pi}^2 g_h U_S}\left(\frac{\alpha_2 V_S U_{FS}}{U_S}\right) \tag{38}$$

and all the derivatives of the functions $U(.,.)$, $V(.,.)$, $f(.)$ and $g(.)$ are evaluated at the steady state; $\bar{c}^F$ and $\bar{c}^S$ denote the steady state levels of consumption for the flexible-price and the sticky-price goods, respectively, $\bar{\pi} = \pi^*$ and $\hat{x}_t = \log(\frac{x_t}{\bar{x}})$. To complete the system of log-linearized equations that describes the dynamics of the economy we have to consider the processes (3).

In addition for subsequent analysis it is useful to characterize the sign of some of the coefficients of the log-linearized system. The following Lemma accomplishes this goal.

Lemma 1 *Under Assumptions 1 and 2 it follows that a)* $\alpha \gtreqless 0$ *if and only if* $U_{SF} \gtreqless 0$; *b)* $\epsilon < 0$; *c)* $\chi > 0$; *and d)* $\delta > 0$

Proof. See Appendix. ■

We are now ready to pursue the determinacy and learnability of equilibrium analyses.

3 The Determinacy and Learning of Equilibrium Analyses

In this section we will study the determinacy and learnability of equilibrium properties for three different rules (forward-looking, contemporaneous and backward-looking rules) that may react to different measures of inflation. The reason of pursuing determinacy of equilibrium and learnability analyses for these rules lies on two intertwined arguments. The first one points out that interest rate rules may generate aggregate instability in the economy by inducing multiple equilibria (real indeterminacy).[11] The second one emphasizes that some rules that although guaranteeing a unique equilibrium, do not ensure that the economy will reach it.[12]

Both arguments are important from an economic policy-design perspective. The first one implies that the aforementioned rules may open the possibility of sunspot equilibria and lead the economy to equilibria with undesirable properties such as a large degree of volatility. This implication in turn suggests that a determinacy of equilibrium analysis can be used to differentiate among rules favoring those that guarantee a unique equilibrium. However this suggestion although appealing may suffer from some drawbacks that lead to the second argument. In the typical determinacy of equilibrium analysis, it is implicitly assumed that agents can coordinate their actions and learn the equilibria (unique or multiple) induced by the rule. But if agents cannot learn the unique equilibrium targeted by the rule then the economy may end up diverging from this equilibrium. If this is the case then it is clear that there are some rules that although inducing real determinacy do not ensure that the economy will be driven into the targeted equilibrium.[13]

[11] See Benhabib, Schmitt-Grohé and Uribe (2001), Bernanke and Woodford (1997), Clarida, Gali and Gertler (2000), Carlstrom and Fuerst (1999), Taylor (1999) and Woodford (2003), among others.

[12] See Bullard and Mitra (2002).

[13] Moreover if agents cannot learn sunspot equilibria then one may doubt the relevance of characterizing rules that lead to multiple equilibria as "bad" ones. After all, if agents cannot learn sunspot equilibria then they are less likely to occur.

Therefore, as pointed by Bullard and Mitra (2002), an interest rate rule should not only guarantee a unique equilibrium but also a "learnable" equilibrium. This suggests that a determinacy of equilibrium analysis for interest rate rules should in principle be accompanied by a learnability of equilibrium analysis. Both analyses may help policy makers to distinguish and design rules satisfying two requirements: uniqueness and learnability of the equilibrium.

For both the determinacy and learnability analyses we will focus on a log-linearized version of the system of equations that describe the competitive equilibrium in this economy, as stated in Definition 1. Before finding this log-linearized version, it is useful to describe the methodology that we will apply to pursue the aforementioned analyses.

3.1 The Methodology

For the determinacy of equilibrium analysis we will apply the results for Blanchard and Kahn (1980) that help to characterize whether a rational expectations linear system of equations such as

$$\hat{x}_t = \Lambda E_t(\hat{x}_{t+1}) \tag{39}$$

has a unique equilibrium, multiple equilibria or no equilibrium, where $\hat{x}_t$ is a system of $s \times 1$ vector of endogenous variables, and Λ is a $s \times s$ matrix of constants. As is well known the analysis consists on comparing the number of non-explosive eigenvalues of the matrix, Λ, with the number of non-predetermined variables. Then in the determinacy of equilibrium of a specific rule that responds to a particular measure of inflation we reduce the log-linearized version of the economy described by (29)-(34) and (3) to the representation in (39) in order to apply Blanchard and Kahn's results.[14]

On the other hand as a criterion of "learnability" of an equilibrium we will use the concept of "E-stability" proposed by Evans and Honkapohja (1999, 2001). That is, an equilibrium is "learnable" if it is "E-Stable". Consequently we start by assuming that agents in our model no longer are endowed with rational expectations. Instead they have adaptive rules whereby agents form expectations using recursive

[14] See also Benhabib and Farmer (1999) and Farmer (1999).

least squares updating and data from the system. Then we derive the conditions for expectational stability (E-stability). We will focus on the E-stability concept for the following reason. In models that display a unique equilibrium (real determinacy models), Marcet and Sargent (1989) and Evans and Honkapohja (1999, 2001) have shown that under some general conditions, the notional time concept of expectational stability of a rational expectation equilibrium governs the local convergence of real time adaptive learning algorithms. Specifically they have shown that under E-stability, recursive least-squares learning is locally convergent to the rational expectations equilibrium.

Then we need to define the concept of E-stability. To do so it is useful to explain the methodology proposed by Evans and Honkapohja (1999, 2001). Consider the model

$$\hat{y}_t = \eta + \Omega E_t \hat{y}_{t+1} + \Gamma \hat{y}_{t-1} + \Psi \hat{z}_t \qquad \textit{and} \qquad \hat{z}_t = \Phi \hat{z}_{t-1} + \xi_t \tag{40}$$

where $\hat{y}_t$ is a $s \times 1$ vector of endogenous variables, η is a $s \times 1$ vector of constants vector, Ω, Γ, Ψ, and Φ are $s \times s$ matrices of constants, and $\hat{z}_t$ is a $s \times 1$ vector of exogenous variables which follows a stationary VAR whose $s \times 1$ vector of shocks consists of white noise terms. In addition E_t denotes in general (non-rational) expectations. Next, assume that the agents follow a perceived law of motion (PLM) that in the case of real determinacy corresponds to the fundamental solution or Minimal State Variable (MSV) representation[15]

$$\hat{y}_t = k + P \hat{y}_{t-1} + Q \hat{z}_t \tag{41}$$

where k, P, Q are conformable vectors and matrices and are to be derived by the method of undetermined coefficients. Iterating forward this law of motion and using it to eliminate all the forecasts ($E_t \hat{y}_{t+1} = k + P \hat{y}_t + Q \Phi \hat{z}_t$) in the model specified in (40) we can derive the implied actual law of motion (ALM)

$$\hat{y}_t = k^A + P^A \hat{y}_{t-1} + Q^A \hat{z}_t \tag{42}$$

Then we obtain the T-mapping $T(k, P, Q) = (k^A, P^A, Q^A)$, whose fixed points correspond to the rational expectations equilibrium. An equilibrium described by the MSV representation is said to be E-stable if this mapping is stable at the equilibrium in question. More formally a fixed point of the T-mapping is E-stable

[15] See McCallum (1983) and Uhlig (1999).

provided that the differential equation

$$\frac{d(k,P,Q)}{d\tau_n} = T(k,P,Q) - (k,P,Q) \tag{43}$$

is locally asymptotically stable at that particular fixed point, where τ_n is defined as the "notional" time.

More specifically, the ALM of the system (40) corresponds to

$$\hat{y}_t = (I - \Omega P)^{-1}\left[\eta + \Omega k + \Gamma \hat{y}_{t-1} + (\Omega Q \Phi + \Psi)\hat{z}_t\right]$$

where I is the identity matrix. Using this ALM and the PLM in (41) we find the T-Mapping

$$T(k,P,Q) = (k^A, P^A, Q^A) = [(I-\Omega P)^{-1}(\eta + \Omega k), (I-\Omega P)^{-1}\Gamma, (I-\Omega P)^{-1}(\Omega Q \Phi + \Psi)] \tag{44}$$

whose fixed point correspond to the rational expectation equilibrium and can be used to determine the coefficients matrices $\bar{k}, \bar{P}$, and $\bar{Q}$ of the MSV solutions. That is $\bar{k}, \bar{P}$, and $\bar{Q}$ are the solutions to

$$(I - \Omega P - \Omega)k = \eta \qquad \Omega P^2 - P + \Gamma = 0 \qquad and \qquad (I - \Omega P)Q - \Omega Q \Phi = \Psi \tag{45}$$

Using (44) the conditions under which the differential equation (43) is locally asymptotically stable are derived and stated in Proposition 10.3 in Evans and Honkapohja (2001). They basically say that an MSV solution of the form (41) to the system (40) is E-stable if all the eigenvalues of the matrices evaluated at $\bar{k}, \bar{P}$, and $\bar{Q}$,

$$DT_k = (I - \Omega \bar{P})^{-1}\Omega \qquad DT_P = \left[(I - \Omega \bar{P})^{-1}\Gamma\right]' \otimes [(I - \Omega \bar{P})^{-1}\Omega] \qquad and \qquad DT_Q = \Phi' \otimes [(I - \Omega \bar{P})^{-1}\Omega] \tag{46}$$

have real parts less than one. Moreover the solution is not E-stable if any of the eigenvalues has real part larger than one.

It is important to observe that a fundamental part in the learnability analysis consists of making explicit what agents know when they form their forecasts. In the E-stability analysis literature it is common to assume that when agents form their expectations $E_t\hat{y}_t$, they do not know $\hat{y}_t$. In this paper this assumption may be inconsistent with the assumptions that we use to derive the equations of the model.[16] Henceforth for the learnability analysis we will assume that when forming expectations agents know $\hat{y}_t$.

To conclude and summarize, in order to determine whether a specific rule that responds to a particular measure of inflation induces a learnable MSV representation of the equilibrium we proceed as follows. First we reduce the log-linearized version of the economy described by (29)-(34) and (3) to a similar system as in (40). Then we calculate the MSV solution of this system and check if all the eigenvalues of the matrices in (46) have real parts less than one.

3.2 Forward-Looking Rules

In this section we focus on interest rate rules reacting to one-period ahead (expected) inflation, i.e. forward looking rules. Our analysis is motivated by the forward-looking rules estimations of Clarida Gali and Gertler (1998,2000) for the US, United kingdom, Germany, France, Italy, and Japan; and the estimations by Corbo(2000) for Chile, Colombia, Peru, Costa Rica and El Salvador. Specifically we focus on rules of the following type

$$\hat{R}_t = \rho_\pi E_t \hat{\pi}_{t+1} \qquad with\ \rho_\pi > 1 \tag{47}$$

whereby the government responds actively to the expected core-inflation rate, $\hat{\pi}$.

Using equations (29)-(32), (33) and (47) we obtain the system

$$\hat{y}_t = \eta + \Omega E_t \hat{y}_{t+1} + \Psi \hat{z}_t \qquad and \qquad \hat{z}_t = \Phi \hat{z}_{t-1} + \xi_t \tag{48}$$

where $\hat{y}_t = [\hat{\pi}_t^S, \hat{c}_t^S]'$, $\hat{z}_t = [\hat{z}_t^F, \hat{z}_t^S]', \xi_t = [\xi_t^F, \xi_t^S]'$, $\eta = 0$,

[16] In particular notice that for the derivation of the first order conditions of the representative agent we assume that $E_t P_t^N(j) = P_t^N(j)$ (or in a symmetric equilibrium $E_t P_t^N = P_t^N$). Therefore assuming in the learnability analysis that the agents do not know P_t^N when forming expectations would have some implications for the specification of the model. Specifically it would require to replace $\hat{\pi}_t^S$ in (32) with the expectations of $\hat{\pi}_t^S$.

$$\Omega = \begin{bmatrix} \beta - \frac{\beta\delta(1-\rho_\pi)}{\epsilon(1-\tau w \rho_\pi)} & \beta\delta \\ -\frac{(1-\rho_\pi)}{\epsilon(1-\tau w \rho_\pi)} & 1 \end{bmatrix} \tag{49}$$

and $\Phi = \begin{bmatrix} \phi^F & 0 \\ 0 & \phi^S \end{bmatrix}$. The form of Ψ is omitted since it is not required for the following analysis. Moreover, β corresponds to the subjective discount factor, ρ_π is the interest response coefficient to core inflation, w is the weight that the government puts on the flexible-price inflation to construct the core inflation (see 34), α, ϵ and δ are constants defined in (35), (36) and (37) and characterized by Lemma 1 and

$$\tau = 1 - \frac{\theta}{\epsilon} \qquad \theta = \alpha\chi\left(\frac{\bar{c}^S}{\bar{c}^F}\right) \tag{50}$$

The solution of (48) will pin down the dynamics of $\hat{y}_t = [\hat{\pi}_t^S, \hat{c}_t^S]'$. Then using these and equations (29), (30), (34), (47) and processes (3) we can find the set of processes $\{\hat{c}_t^F, \hat{\pi}_t^F, \hat{\pi}_t, R_t\}$.

To complete the specification of the system we study the properties of θ and τ in the following Lemma.

Lemma 2 *Under Assumptions 1 and 2 a)* $\theta \gtreqless 0$ *if and only if* $U_{SF} \gtreqless 0$; *b) if* $U_{SF} \gtreqless 0$ *then* $\tau \gtreqless 1$; *c)* $\tau > 0$.

Proof. See Appendix. ■

The following Proposition studies active forward-looking rules that responds to the core inflation. It provides necessary and sufficient conditions for these rules to deliver a unique and learnable equilibrium.

Proposition 1 *Consider the system defined in (48). Let* $\rho_\pi^w = \frac{[\beta\delta - 2(1+\beta)\epsilon]}{\beta\delta - 2(1+\beta)\epsilon\tau w}$ *and assume that the government follows an active forward-looking rule in terms of the core inflation* ($\hat{\pi}$) *described by* $\hat{R}_t = \rho_\pi E_t \hat{\pi}_{t+1}$ *with* $\rho_\pi > 1$.

a) *If* $U_{SF} \leq 0$ ($\hat{c}_t^F$ *and* $\hat{c}_t^S$ *are either Edgeworth substitutes or utility separable) then a sufficient and necessary condition for the existence of a unique equilibrium is that* $1 < \rho_\pi < \rho_\pi^w$ *for any* $w \in (0,1)$. *Moreover this is a sufficient condition for the MSV solution* $\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t$ *of this equilibrium to be E-stable, where* $\bar{k} = 0$ *and* $\bar{Q}$ *solves* $Q - \Omega Q \Phi = \Psi$.

b) *If* $U_{SF} > 0$ ($\hat{c}_t^F$ *and* $\hat{c}_t^S$ *are Edgeworth complements) then a sufficient and necessary condition for the existence of a unique equilibrium is that* $1 < \rho_\pi < \rho_\pi^w$ *for any* $w \in (0, 1/\tau)$. *Moreover this is a sufficient condition for the MSV solution* $\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t$ *of this equilibrium to be E-stable, where* $\bar{k} = 0$ *and* $\bar{Q}$ *solves* $Q - \Omega Q \Phi = \Psi$.

Proof. See the Appendix. ■

The conditions stated by Proposition 1 under which active forward-looking rules deliver a unique and learnable equilibrium depend not only on the interest rate response coefficient to inflation, but also on the weight w and on whether are $\hat{c}_t^F$ and $\hat{c}_t^S$ are Edgeworth substitutes, Edgeworth complements or utility separable. The following paragraphs elaborate on this statement.

The importance of w, the weight that the government puts in the flexible-price inflation to construct the core inflation, can be grasped by realizing that $\frac{\partial \rho_\pi^w}{\partial w} < 0$. Which means that as the government puts more weight on flexible-price inflation then the range $1 < \rho_\pi < \rho_\pi^w$ under which an active forward-looking rule will deliver a unique learnable and unique equilibrium will be reduced. In fact it is possible to prove that as long as $\rho_\pi > \rho_\pi^w$ then the forward-looking rule will always deliver multiple equilibria. This observation has dramatic consequences for the performance of forward-looking rules when the goods $\hat{c}_t^F$ and $\hat{c}_t^S$ are either Edgeworth complements ($U_{SF} > 0$) or utility separable ($U_{SF} = 0$). To understand this we can study thoroughly the properties of the function ρ_π^w and construct Figures 1,2 and 3 that correspond to the cases when the goods are Edgeworth substitutes, utility separable or Edgeworth complements, repectively. These Figures show not only the values for ρ_π and w for which forward-looking rules that respond to the core inflation will deliver unique and learnable equilibria (real determinacy and E-stability) but also the values for which there exists multiple equilibria (real indeterminacy). Within the value regions for which there exist multiple equilibria for active rules, we also characterize whether the MSV solution is E-stable or E-unstable. In the figures "D" and "I" stand for real determinacy (unique equilibrium) and real indeterminacy (multiple equilibria) respectively. Whereas "ES" stands for E-stable and "EU" for E-unstable.

In all these figures it is clear that ρ_π^w is a decreasing function of w and that its intercept with the ρ_π-axis when $w = 0$ is equal to ρ_π^0. Figure 1 corresponds to the case in which $\hat{c}_t^F$ and $\hat{c}_t^S$ are Edgeworth complements ($U_{SF} > 0$). In this case it is clear that active forward-looking rules will always lead to real indeterminacy for either $w > \frac{1}{\tau}$ and any $\rho_\pi > 1$ or for $w \in (0, \frac{1}{\tau})$ whenever that $\rho_\pi > \rho_\pi^w$. Only rules satisfying $1 < \rho_\pi < \rho_\pi^w$ and $w \in (0, \frac{1}{\tau})$ (characterized by part b of Proposition 1) will deliver unique and learnable equilibria. Figure 2 corresponds to the case in which $\hat{c}_t^F$ and $\hat{c}_t^S$ are utility separable ($U_{SF} = 0$). In this case the region of real

Forward-Looking Rule ($U_{SF} < 0$ and $\tau > 1$)
Inflation Coefficient (ρ_π) vs. Weight (w)

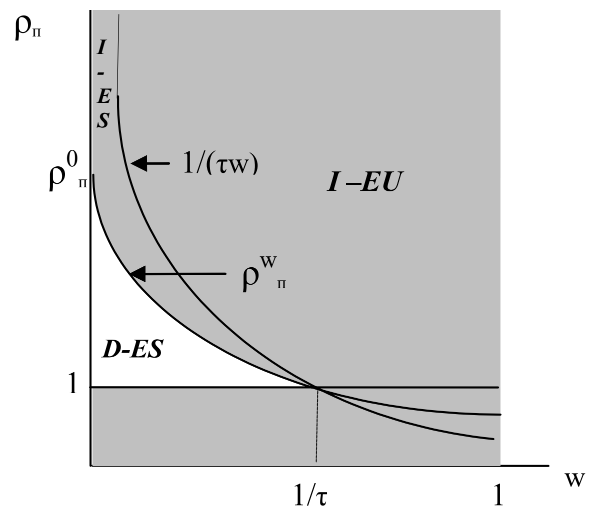

Figure 1:

determinacy and E-stability is increased with respect to the previous case. The reason is that the intercept of ρ_π^w with the ρ_π-axis is still ρ_π^0. However the sufficient and necessary condition for ρ_π of part a) of Proposition 1 will hold for any $w \in (0,1)$. Nevertheless if w is very closed to one then active rules will still induce multiple equilibria. Figure 3 is associated with the case when $\hat{c}_t^F$ and $\hat{c}_t^S$ are Edgeworth substitutes ($U_{SF} < 0$). In this case the possible problems of real indeterminacy seem to subside although it is still true that the region of real determinacy and E-stability characterized by part a) of Proposition 1 decreases as w increases.

This analysis suggests that it is important to study active forward-looking rules that react exclusively to either the flexible-price inflation, $\hat{\pi}^F$, or the sticky-price inflation, $\hat{\pi}^S$. After all these two cases correspond to react to the core inflation and set $w = 1$ and $w = 0$ respectively. By studying these two cases we will also be able to provide an economic intuition of the results from Proposition 1 and depicted in the previous Figures.

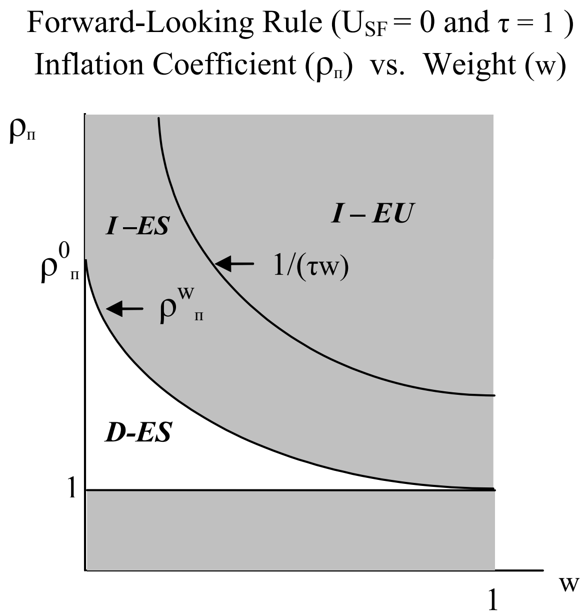

Figure 2:

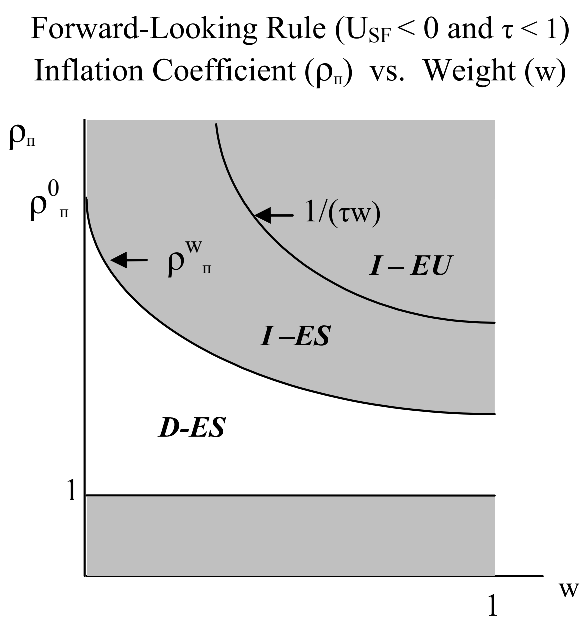

Figure 3:

The following Proposition summarizes the determinacy of equilibrium and learnability results for an active rule that responds solely to the flexible-price inflation.

Proposition 2 *Consider the system defined in (48) with* $w = 1$. *Let* $\rho_\pi^1 = \frac{[\beta\delta - 2(1+\beta)\epsilon]}{\beta\delta - 2(1+\beta)\epsilon\tau}$ *and assume that the government follows an active forward-looking rule in terms of the flexible-price inflation* ($\hat{\pi}^F$) *described by* $\hat{R}_t = \rho_\pi E_t \hat{\pi}^F_{t+1}$ *with* $\rho_\pi > 1$.

> **a)** *If* $U_{SF} < 0$ ($\hat{c}^F_t$ *and* $\hat{c}^S_t$ *are Edgeworth substitutes*) *then a sufficient and necessary condition for the existence of a unique equilibrium is that* $1 < \rho_\pi < \rho_\pi^1$. *Moreover this is a sufficient condition for the MSV solution* $\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t$ *of this equilibrium to be E-stable, where* $\bar{k} = 0$ *and* $\bar{Q}$ *solves* $Q - \Omega Q \Phi = \Psi$.
>
> **b)** *If* $U_{SF} \geq 0$ ($\hat{c}^F_t$ *and* $\hat{c}^S_t$ *are either Edgeworth complements or utility separable*) *then there is NOT a value of* $\rho_\pi > 1$ *for which there exists a unique equilibrium and the MSV solution* $\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t$ *is E-stable, where* $\bar{k} = 0$ *and* $\bar{Q}$ *solves* $Q - \Omega Q \Phi = \Psi$. *More specifically for any* ρ_π *such that* $\rho_\pi > 1$ the *rule will always induce multiple equilibria and more importantly the MSV solution is NOT learnable in the E-stability sense.*

Proof. See the Appendix. ■

The results of Proposition 2 show that conditions under which active forward-looking rules that respond exclusively to the flexible-price inflation deliver a unique and learnable equilibrium depend strongly on whether are $\hat{c}^F_t$ and $\hat{c}^S_t$ are Edgeworth substitutes, Edgeworth complements or utility separable. In particular when $\hat{c}^F_t$ and $\hat{c}^S_t$ are either Edgeworth complements or utility separable, then the aforementioned rules will always induce aggregate instability by generating multiple equilibria. More interestingly even if we focus on the MSV solution, we find that this representation is never E-stable.

To understand this it is useful to derive the following equation from (29)-(31), (33) and (34)

$$E_t \hat{\pi}^F_{t+1} = \left(\frac{1-\tau}{1-\tau\rho_\pi} \right) E_t \hat{\pi}^S_{t+1} \tag{51}$$

where we have ignored the terms associated with $\hat{z}^F_t$. With this equation and equations (31) and (32), it is possible to construct a self-fulfilling equilibrium when $\hat{c}^F_t$ and $\hat{c}^S_t$ are either Edgeworth complements or utility separable. We pursue this goal.

Assume that agents expect a higher sticky-price inflation. When $\hat{c}^F_t$ and $\hat{c}^S_t$ are Edgeworth complements then by Lemma 2 we know that $\tau > 1$. If in addition the rule is active $\rho_\pi > 1$, then by (51) we see that higher expectations of the sticky price-inflation will be associated with with higher expectations of the flexible price inflation but satisfying $E_t\hat{\pi}^F_{t+1} < E_t\hat{\pi}^S_{t+1}$. If the rule responds actively and exclusively to the flexible-price inflation $\hat{R}_t = \rho_\pi E_t\hat{\pi}^F_{t+1}$ then the real interest rate measured with respect to the expected sticky-price inflation, $\hat{R}_t - E_t\hat{\pi}^S_{t+1}$, will decrease stimulating consumption of the sticky-price good (see 31). As a response to this increase in consumption firms will raise the price of the sticky-price good inducing a higher sticky-price inflation (see 33). Then the original expectations of a higher sticky-price inflation are validated.

When $\hat{c}^F_t$ and $\hat{c}^S_t$ are utility separable, then by Lemma 2 we know that $\tau = 1$. But this and (51) imply that $E_t\hat{\pi}^F_{t+1} = 0$. Now assume that agents expect a higher sticky-price inflation. Since the rule responds actively and only to the flexible-price inflation $\hat{R}_t = \rho_\pi E_t\hat{\pi}^F_{t+1}$ then the real interest rate measured with respect to the expected sticky-price inflation, $\hat{R}_t - E_t\hat{\pi}^S_{t+1}$ will decrease and the same mechanism explained above self-validates the original expectations.

Next consider the case when $\hat{c}^F_t$ and $\hat{c}^S_t$ are Edgeworth substitutes. From Lemma 2 we know that $0 < \tau < 1$. If in addition the rule is active $\rho_\pi > 1$ and satisfies $\rho_\pi < \frac{1}{\tau}$, then by (51) we see that higher expectations of the sticky price-inflation will be associated with higher expectations of the flexible price inflation but in this case we have that $E_t\hat{\pi}^F_{t+1} > E_t\hat{\pi}^S_{t+1}$. If the rule responds actively and exclusively to the flexible-price inflation $\hat{R}_t = \rho_\pi E_t\hat{\pi}^F_{t+1}$ then it is possible that the real interest rate measured with respect to the expected sticky-price inflation, $\hat{R}_t - E_t\hat{\pi}^S_{t+1}$ will increase reducing consumption of the sticky-price good (see 31). As a response to this decline in consumption firms will decrease the price of the sticky-price good inducing a lower sticky-price inflation (see 33). Then the original expectations of a higher sticky-price inflation are not validated.

In a similar way it is possible to grasp the intuition of why the MSV solution is not learnable (E-stable) when $\hat{c}^F_t$ and $\hat{c}^S_t$ are Edgeworth complements or utility separable whereas it is learnable if $\hat{c}^F_t$ and $\hat{c}^S_t$ are Edgeworth substitutes. In order to do so we use (51) and the rule $\hat{R}_t = \rho_\pi E_t\hat{\pi}^F_{t+1}$ to derive the real interest rate measured with respect to the expected sticky-price inflation

$$\hat{R}_t - E_t\hat{\pi}^S_{t+1} = -\left(\frac{1-\rho_\pi}{1-\tau\rho_\pi}\right)E_t\hat{\pi}^S_{t+1} \tag{52}$$

Consider the case in which they are Edgeworth complements or utility separable ($\tau \geq 1$) and recall that $\rho_\pi > 1$. According to (52) a deviation of people's expected sticky-price inflation from the rational expectations value will always lead to a decrease in the real interest rate measured with respect to the expected sticky-price inflation. But this will stimulate consumption of the sticky-price good by (31) which in turn will increase the sticky-price inflation by (33). Over time this mechanism leads to upward revisions of both the expected sticky-price inflation and the expected consumption of the sticky-price goods. Therefore the policy of targeting actively the flexible-price inflation will not off-set the initial deviation from the rational expectations equilibrium. It will move the economy further away from it.

On the contrary assume that the goods are Edgeworth substitutes. In this case $0 < \tau < 1$. Using (52) we can see that a deviation of people's expected sticky-price inflation from the rational expectations value may induce an increase in the real interest rate measured with respect to the expected sticky-price inflation, as long as $1 < \rho_\pi < \frac{1}{\tau}$. But this will decrease consumption of the sticky-price good by (31) which in turn will decrease the sticky-price inflation by (33). Over time this mechanism leads to upward revisions of both the expected sticky-price inflation and the expected consumption of the sticky-price goods. Hence in this case the policy of targeting actively the flexible-price inflation is able to lead the original people's expectations towards the the rational expectations value.

The analysis of rules that respond to either the core-inflation or the flexible-price inflation poses the question of whether it is possible to design a rule that induces a unique and learnable equilibrium without depending on the joint characteristics of $\hat{c}^F_t$ and $\hat{c}^S_t$. The answer to this can be found in active forward-looking rules that respond solely to the sticky-price inflation. The following proposition makes this point.

Proposition 3 *Consider the system defined in (48) with $w = 0$. Let $\rho^0_\pi = 1 - \frac{2(1+\beta)\epsilon}{\beta\delta}$ and assume that the government follows an active forward-looking rule in terms of the sticky-price inflation ($\hat{\pi}^S$) described by $\hat{R}_t = \rho_\pi E_t\hat{\pi}^S_{t+1}$ with $\rho_\pi > 1$. Then regardless of whether $U_{SF} \gtreqless 0$ (that is whether $\hat{c}^F_t$ and $\hat{c}^S_t$ are Edgeworth* *substitutes, complements or utility separable),a sufficient and necessary condition for the existence of a unique equilibrium is that* $1 < \rho_\pi < \rho_\pi^0$. *Moreover this is a sufficient and a necessary condition for the MSV solution* $\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t$ *of this equilibrium to be E-stable, where* $\bar{k} = 0$ *and* $\bar{Q}$ *solves* $Q - \Omega Q \Phi = \Psi$.

Proof. See the Appendix. ■

The results of Proposition 3 are equivalent to the ones in Bullard and Mitra (2002) and their intuition is straightforward.

3.3 Contemporaneous Rules

In this subsection we study thoroughly active contemporaneous rules that may respond to the core inflation, the flexible price-inflation or the sticky-price inflation. The motivation for studying contemporaneous rules stems from empirical evidence such as Lubik and Schorfheide (2003). Specifically we focus on rules of the following type

$$\hat{R}_t = \rho_\pi \hat{\pi}_t \qquad with\ \rho_\pi > 1 \tag{53}$$

whereby the government responds actively to the current core-inflation rate, $\hat{\pi}$.

Using equations (29)-(32), (33) and (53) we obtain the system

$$\hat{y}_t = \eta + \Omega E_t \hat{y}_{t+1} + \Psi \hat{z}_t \qquad and \qquad \hat{z}_t = \Phi \hat{z}_{t-1} + \xi_t \tag{54}$$

where $\hat{y}_t = [\hat{\pi}_t^F, \hat{\pi}_t^S, \hat{c}_t^S]'$, $\hat{z}_t = [\hat{z}_t^F, \hat{z}_t^S]'$, $\xi_t = [\xi_t^F, \xi_t^S]'$, $\eta = 0$,

$$\Omega = \begin{bmatrix} \frac{1}{\tau w \rho_\pi} - \frac{\beta(1-w)\delta}{\epsilon \tau w} & -\frac{1-\tau}{\tau w \rho_\pi} - \beta\left(\frac{1-w}{w}\right)\left(1 - \frac{\delta}{\epsilon} - \frac{(1-\tau)\delta}{\epsilon\tau}\right) & -\frac{\beta(1-w)\delta}{w} \\ \frac{\beta\delta}{\epsilon\tau} & \beta\left(1 - \frac{\delta}{\epsilon} - \frac{(1-\tau)\delta}{\epsilon\tau}\right) & \beta\delta \\ -\frac{1}{\epsilon\tau} & -\frac{1}{\epsilon\tau} & 1 \end{bmatrix} \tag{55}$$

and $\Phi = \begin{bmatrix} \phi^F & 0 \\ 0 & \phi^S \end{bmatrix}$. The form of Ψ is omitted since it is not required for the following analysis. Moreover, β corresponds to the subjective discount factor, ρ_π is the interest response coefficient to core inflation, w is the weight that the government puts on the flexible-price inflation to construct the core inflation (see 34),

α, ϵ and δ are constants defined in (35), (36) and (37) and characterized by Lemma 1, and τ is a constant defined in (50) and characterized in Lemma 2.

Surprisingly active contemporaneous rules that react to the current core-inflation always deliver real indeterminacy regardless of whether $\hat{c}_t^F$ and $\hat{c}_t^S$ are Edgeworth substitutes, complements or utility separable. Moreover although the MSV solution maybe learnable when $\hat{c}_t^F$ and $\hat{c}_t^S$ are Edgeworth substitutes, it is not E-stable when these goods are complements or utility separable, for any $\rho_\pi > 1$. The following Proposition formalizes these statements.

Proposition 4 *Consider the system defined in (54). Assume that the government follows an active contemporaneous rule in terms of the core inflation ($\hat{\pi}$) described by $\hat{R}_t = \rho_\pi \hat{\pi}_t$ with $\rho_\pi > 1$.*

> **a)** *If $U_{SF} \geq 0$ ($\hat{c}_t^F$ and $\hat{c}_t^S$ are either Edgeworth complements or utility separable) then there are no values of $\rho_\pi > 1$ and $w \in (0,1)$ for which there exists a unique equilibrium and the MSV solution $\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t$ is E-stable, where $\bar{k} = 0$ and $\bar{Q}$ solves $Q - \Omega Q \Phi = \Psi$. More specifically for any ρ_π, such that $\rho_\pi > 1$, and $w \in (0,1)$ the rule will always induce multiple equilibria and the MSV solution is NOT learnable in the E-stability sense.*
>
> **b)** *If $U_{SF} < 0$ ($\hat{c}_t^F$ and $\hat{c}_t^S$ are Edgeworth substitutes) then there are no values of ρ_π and $w \in (0,1)$ for which there exists a unique equilibrium. However the MSV solution $\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t$, where $\bar{k} = 0$ and $\bar{Q}$ solves $Q - \Omega Q \Phi = \Psi$, may be E-stable for some $\rho_\pi > 1$ and $w \in (0,1)$.*

Proof. See Appendix. ■

It is surprising that active contemporaneous rules with respect to the core-inflation will always deliver multiple equilibria. To understand this point it is useful to study rules that respond exclusively to either the flexible-price inflation or the sticky-price inflation. Note that although we can use the system(54) with Ω defined in (55) in order to analyze rules that react to the flexible-price inflation, we cannot use the same system to investigate rules that respond to the sticky-price inflation. For the former rules we can use the aforementioned system and matrix with $w = 1$. However for the latter rules some entries of Ω will not be defined when $w = 0$.

Proposition 5 *Consider the system defined in (54) with $w = 1$. Assume that the government follows an active contemporaneous rule in terms of the flexible-price inflation ($\hat{\pi}^F$) described by $\hat{R}_t = \rho_\pi \hat{\pi}_t^F$ with $\rho_\pi > 1$.*

> **a)** *If $U_{SF} \geq 0$ ($\hat{c}_t^F$ and $\hat{c}_t^S$ are either Edgeworth complements or utility separable) then there is not a value of ρ_π for which there exists a unique equilibrium and the MSV solution $\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t$ is E-stable, where $\bar{k} = 0$ and $\bar{Q}$ solves $Q - \Omega Q \Phi = \Psi$. More specifically for any ρ_π, such that $\rho_\pi > 1$, and $w \in (0,1)$ the rule will always induce multiple equilibria and the MSV solution is not learnable in the E-stability sense..*

b) *If* $U_{SF} < 0$ *(*$\hat{c}_t^F$ *and* $\hat{c}_t^S$ *are Edgeworth substitutes) then there is not a value of* ρ_π *for which there exists a unique equilibrium. However the MSV solution* $\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t$*, where* $\bar{k} = 0$ *and* $\bar{Q}$ *solves* $Q - \Omega Q \Phi = \Psi$*, may be E-stable for some* $\rho_\pi > 1$.

Proof. See Appendix. ■

Proposition 5 suggests that the previous results for active contemporaneous rules that respond to the core-inflation are mainly explained by the fact that targeting the core inflation indirectly implies targeting the flexible-price inflation. In particular, it is important to emphasize that when $\hat{c}_t^F$ and $\hat{c}_t^S$ are either Edgeworth complements or utility separable, an active contemporaneous rule that responds to the flexible-price inflation will induce multiple equilibria. In addition in this case the MSV solution is not learnable.

To verify that the results for rules that respond to the core inflation are mainly explained by the results for rules that react to the flexible-price inflation we proceed by studying contemporaneous rules that respond exclusively to the sticky-price inflation. As argued before we need to find the system that describes the economy. Using equations (29)-(32), (33) and the rule $\hat{R}_t = \rho_\pi \hat{\pi}_t^S$ we obtain the system

$$\hat{y}_t = \eta + \Omega E_t \hat{y}_{t+1} + \Psi \hat{z}_t \qquad \textit{and} \qquad \hat{z}_t = \Phi \hat{z}_{t-1} + \xi_t \tag{56}$$

where $\hat{y}_t = [\hat{\pi}_t^S, \hat{c}_t^S]'$, $\hat{z}_t = [\hat{z}_t^F, \hat{z}_t^S]'$, $\xi_t = [\xi_t^F, \xi_t^S]'$, $\eta = 0$,

$$\Omega = \begin{bmatrix} \frac{(\epsilon-\delta)\beta}{\epsilon-\beta\delta\rho_\pi} & \frac{\beta\delta\epsilon}{\epsilon-\beta\delta\rho_\pi} \\ \frac{\beta\rho_\pi-1}{\epsilon-\beta\delta\rho_\pi} & \frac{\epsilon}{\epsilon-\beta\delta\rho_\pi} \end{bmatrix} \tag{57}$$

and $\Phi = \begin{bmatrix} \phi^F & 0 \\ 0 & \phi^S \end{bmatrix}$. The form of Ψ is omitted since it is not required for the following analysis. Moreover, β corresponds to the subjective discount factor, ρ_π is the interest response coefficient to core inflation, α, ϵ and δ are constants defined in (35), (36) and (37) and characterized by Lemma 1.

Proposition 6 *Consider the system defined in (56). Assume that the government follows an active contemporaneous rule in terms of the sticky-price inflation (*$\hat{\pi}^S$*) described by* $\hat{R}_t = \rho_\pi \hat{\pi}_t^S$ *with* $\rho_\pi > 1$*. Then regardless of whether* $U_{SF} \gtreqless 0$ *(that is whether* $\hat{c}_t^F$ *and* $\hat{c}_t^S$ *are Edgeworth substitutes, complements or utility separable), a sufficient and necessary condition for the existence of a unique equilibrium is that* $\rho_\pi > 1$*. Moreover this is a sufficient and a necessary condition for the MSV solution* $\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t$ *of this equilibrium to be E-stable, where* $\bar{k} = 0$ *and* $\bar{Q}$ *solves* $Q - \Omega Q \Phi = \Psi$.

Proof. See the Appendix. ■

The results of Proposition 6 confirm our previous assertion about how the results for rules that respond to the core inflation are practically explained by the effect of the flexible-price inflation on the core inflation. In particular if the rule reacts solely to the current sticky-price inflation, that is when the weight w on the flexible-price inflation is zero and consequently the core inflation coincides with the sticky-price inflation then active rules will deliver a unique and learnable equilibrium.

Our analyses for contemporaneous and forward-looking rules with different measures of inflation have some important policy implications. First forward-looking and contemporaneous rules that respond to either the core-inflation or the flexible-price inflation are more prone to deliver real indeterminacy than rules that respond exclusively to the sticky price inflation. This result is very clear when $\hat{c}_t^F$ and $\hat{c}_t^S$ are either Edgeworth complements or utility separable. More importantly under these assumptions about $\hat{c}_t^F$ and $\hat{c}_t^S$, the MSV solution is never learnable for active forward-looking and contemporaneous rules that respond to the flexible-price inflation. These results in tandem with the results of Proposition 6 suggest that the measure of inflation that should be included in the rules is the sticky-price inflation. The natural question that arises is wether this policy recommendation is still valid for backward-looking rules. We proceed pursuing the analysis of these rules.

3.4 Backward-Looking Rules

In this subsection we pursue the determinacy of equilibrium and learning analyses for active backward-looking rules that may react to the core inflation, the flexible price-inflation or the sticky-price inflation. The motivation for studying these rules comes not only from an empirical motivation such as Taylor (1993) but also form a theoretical motivation. In fact works by Benhabib et al. (2001), Bernanke and Woodford (1997) and Carlstrom and Fuerst (1999) suggest that backward-looking rules are less prone to induce multiple equilibria.

We start our analysis by studying rules that respond exclusively and actively to the core-inflation. That is

$$\hat{R}_t = \rho_\pi \hat{\pi}_{t-1} \qquad with\ \rho_\pi > 1 \tag{58}$$

For the determinacy of equilibrium analysis we use equations (29)-(32), (33) and (58) to obtain the system

$$\hat{x}_t = \Lambda E_t \hat{x}_{t+1} + \Sigma \hat{z}_t \qquad and \qquad \hat{z}_t = \Phi \hat{z}_{t-1} + \xi_t \tag{59}$$

where $\hat{x}_t = [\hat{R}_t, \hat{\pi}_t^F, \hat{\pi}_t^S, \hat{c}_t^S]'$, $\hat{x}_t = [\hat{z}_t^F, \hat{z}_t^S]', \xi_t = [\xi_t^F, \xi_t^S]'$, $E_t \hat{R}_{t+1} = \hat{R}_{t+1}$ since $\hat{R}_t$ is a predetermined variable,

$$\Lambda = \begin{bmatrix} 0 & \frac{1}{\tau} & \frac{\tau-1}{\tau} & 0 \\ \frac{1}{w\rho_\pi} & -\frac{(1-w)\beta\delta}{w\tau\epsilon} & -\beta\left(\frac{1-w}{w}\right)\left(1-\frac{\delta}{\epsilon\tau}\right) & -\frac{\beta(1-w)\delta}{w} \\ 0 & \frac{\beta\delta}{\epsilon\tau} & \beta\left(1-\frac{\delta}{\epsilon\tau}\right) & \beta\delta \\ 0 & \frac{1}{\epsilon\tau} & -\frac{1}{\epsilon\tau} & 1 \end{bmatrix} \tag{60}$$

and $\Phi = \begin{bmatrix} \phi^F & 0 \\ 0 & \phi^S \end{bmatrix}$. The form of Ψ is omitted since it is not required for the following analysis. Moreover, β corresponds to the subjective discount factor, ρ_π is the interest response coefficient to core inflation, w is the weight that the government puts on the flexible-price inflation to construct the core inflation (see 34), α, ϵ and δ are constants defined in (35), (36) and (37) and characterized by Lemma 1, and τ is a constant defined in (50) and characterized in Lemma 2.

On the other hand since there is a predetermined variable (an endogenous state variable) then for the E-stability analysis we utilize (29)-(32), (33) and (58) to derive the system

$$\hat{y}_t = \eta + \Omega E_t \hat{y}_{t+1} + \Gamma \hat{y}_{t-1} + \Psi \hat{z}_t \qquad and \qquad \hat{z}_t = \Phi \hat{z}_{t-1} + \xi_t \tag{61}$$

where $\hat{x}_t = [\hat{R}_t, \hat{\pi}_t^F, \hat{\pi}_t^S, \hat{c}_t^S]'$, $\hat{x}_t = [\hat{z}_t^F, \hat{z}_t^S]', \xi_t = [\xi_t^F, \xi_t^S]'$, $\eta = 0$, $E_t \hat{R}_{t+1} = \hat{R}_{t+1}$ (since $\hat{R}_t$ is a predetermined variable),

$$\Omega = \begin{bmatrix} 0 & 1 & \tau - 1 & 0 \\ \frac{1}{w\rho_\pi} & 0 & -\beta\left(\frac{1-w}{w}\right)\left(1-\frac{\delta}{\epsilon}\right) & -\frac{\beta(1-w)\delta}{w} \\ 0 & 0 & \beta\left(1-\frac{\delta}{\epsilon}\right) & \beta\delta \\ 0 & 0 & -\frac{1}{\epsilon} & 1 \end{bmatrix} \tag{62}$$

$$\Gamma = \begin{bmatrix} 0 & (1-\tau)w\rho_\pi & (1-\tau)(1-w)\rho_\pi & 0 \\ 0 & -\frac{(1-w)\beta\delta\rho_\pi}{\epsilon} & -\frac{(1-w)^2\beta\delta\rho_\pi}{w\epsilon} & 0 \\ 0 & \frac{\beta\delta w\rho_\pi}{\epsilon} & \frac{(1-w)\beta\delta\rho_\pi}{\epsilon} & 0 \\ 0 & \frac{w\rho_\pi}{\epsilon} & \frac{(1-w)\rho_\pi}{\epsilon} & 0 \end{bmatrix} \tag{63}$$

$$\Psi = \begin{bmatrix} \frac{\varkappa_1 \bar{c}^S U_{FF}(1-\phi^F)}{\bar{c}^F U_F} & \frac{(\tau-1)\varkappa_1 U_{FS}(1-\phi^F)}{U_S} \\ -\frac{(1-w)\beta\varkappa_5}{w} & -\beta\left(\frac{1-w}{w}\right)\left[\varkappa_4 - \frac{\delta\varkappa_1 U_{FS}(1-\phi^F)}{\epsilon U_S}\right] \\ \beta\varkappa_5 & \beta\left[\varkappa_4 - \frac{\delta\varkappa_1 U_{FS}(1-\phi^F)}{\epsilon U_S}\right] \\ 0 & \frac{\varkappa_1 U_{FS}(1-\phi^F)}{\epsilon U_S} \end{bmatrix} \tag{64}$$

and $\Phi = \begin{bmatrix} \phi^F & 0 \\ 0 & \phi^S \end{bmatrix}$. $\varkappa_1$, $\varkappa_4$, and $\varkappa_5$ are constants defined in (35) and (38).

For the determinacy of equilibrium analysis and for the learning analysis it is not possible to derive analytical results. Therefore we have to simulate. We will assume some functional forms and assign some reasonable values for the parameters associated with these forms. Then we will apply the methodology described in 3.1 taking into account that for the E-stability analysis we have to find the the MSV solution described by $\hat{y}_t = k + P\hat{y}_{t-1} + Q\hat{z}_t$.

The functional forms are the following. For consumption and labor preferences

$$U(c_t^F, c_t^S) = \frac{\left[(\alpha^p)^{\frac{1}{a}}\left(c_t^F\right)^{\frac{a-1}{a}} + (1-\alpha^p)^{\frac{1}{a}}\left(c_t^S\right)^{\frac{a-1}{a}}\right]^{\left(\frac{a}{a-1}\right)(1-\sigma)} - 1}{1-\sigma} \qquad V(h_t^F, h_t^S) = -\frac{\left(h_t^F\right)^{1+\xi^p}}{1+\xi^p} - \frac{\left(h_t^S\right)^{1+\xi^p}}{1+\xi^p}$$

where $\alpha^p \in (0,1)$, $\sigma, a > 0$ and $\xi^p \geq 0$. Note that they satisfy Assumption 1. In particular the sign of U_{FS} is determined by the values of the intratemporal elasticity of substitution, a, and the intertemporal elasticity of substitution, $\frac{1}{\sigma}$. That is $U_{FS} \gtreqless 0$ if and only if $\frac{1}{\sigma} \gtreqless a$.

The technologies are described by

$$y_t^F = (h_t^F)^{\theta^F} \qquad y_t^S = (h_t^S)^{\theta^S}$$

where $\theta^F, \theta^S \in (0,1)$. They satisfy Assumption 2.

The time unit is a quarter. Then we set $\beta = 0.98$. We will assume that the share of sticky-price goods is bigger than the share of flexible-price goods. Hence we set $\alpha^p = 0.2$. We set $\sigma = 1$ and $a = 0.8$. However since the relative magnitudes of σ and a determine the sign of U_{FS}, that is whether c_t^F and c_t^S are Edgeworth substitutes, complements or utility separable, we will vary a in some of the simulations. We also set $\theta^F = \theta^S = 0.5$. In addition we use the following values $\bar{\pi} = 1.01$, $\xi^p = 0.5$, $\mu = 6$, $\gamma = 17.5$, $\phi^F = \phi^S = 0.82$ and $\xi^p = 0.5$ that agree with some of the parameter values used in the monetary rules literature that use New Keynesian models.[17]

The results of the simulations are presented in Figure 4. that shows the combinations of the interest rate response coefficient to inflation, ρ_π, and weight on the flexible-price inflation, $w \in (0,1)$, that lead to either real determinacy (D) or real indeterminacy (I). In addition it shows the combinations for these parameters under which the MSV solution is either E-stable (ES) or E-unstable (EU). In the figure a circle with an "x" inside stand for indeterminacy and E-instability of the MSV solution. Whereas a circle with anything inside represents determinacy and E-instability of the MSV solution. The right panel corresponds to the case of Edgeworth complements whereas the left panel corresponds to the case of substitutes.

The figure shows that regardless of the type of goods active backward-looking rules are more prone to induce indeterminacy as the weight, w is reduced. In addition whether the goods are complements or substitutes matter for the determinacy of equilibrium. In general if they are substitutes active backward-looking rules with respect to the core inflation are more prone to deliver multiple equilibria than if they are complements. These results contrast with our previous results for forward-looking rules. More interestingly the figure also shows that regardless of the type of goods, the interest rate response coefficient to inflation ρ_π and the weight w the MSV solution is not learnable (E-stable).

These results suggests that it is important to analyze active backward-looking rules that respond solely to the flexible-price inflation. To accomplish this goal we can use the systems in (59) and (61) taking into account that targeting explicitly the flexible-price inflation implies that $w = 1$. It is possible to derive analytical results for the determinacy of equilibrium analysis when the goods are utility separable. We they

[17] See Schmitt-Grohé and Uribe (2004) and Woodford (2003) among others.

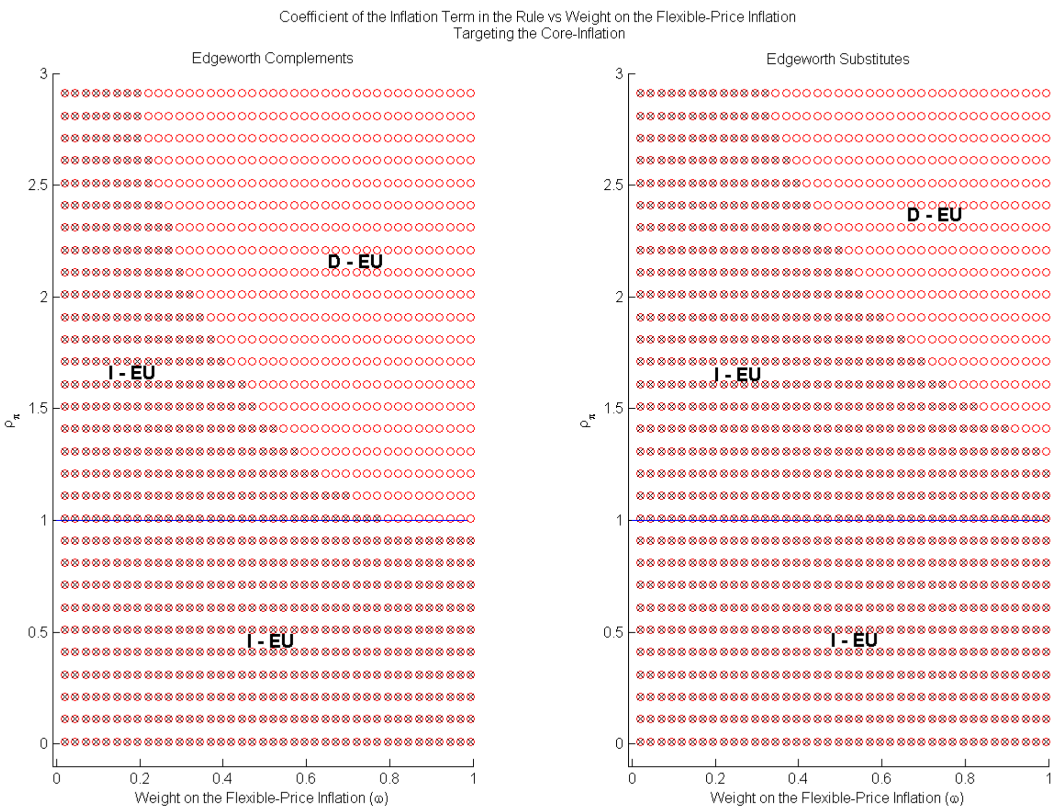
Coefficient of the Inflation Term in the Rule vs Weight on the Flexible-Price Inflation
Targeting the Core-Inflation
Edgeworth Complements
Edgeworth Substitutes
D - EU
I - EU
I - EU
D - EU
I - EU
I - EU
Weight on the Flexible-Price Inflation (ω)
Weight on the Flexible-Price Inflation (ω)

Figure 4:

are either Edgeworth complements or substitutes we have to simulate. On the contrary for the learning analysis, it is feasible to derive analytical results for all the cases. The following proposition summarizes the analytical results.

Proposition 7

a) *Consider the system (59) with* $w = 1$ *and assume that the government follows an active backward-looking rules in terms of the flexible-price inflation* ($\hat{\pi}^F$) *described by* $\hat{R}_t = \rho_\pi \hat{\pi}^F_{t-1}$ *with* $\rho_\pi > 1$. *If* $U_{SF} = 0$ ($\hat{c}^F_t$ *and* $\hat{c}^S_t$ *are utility separable) then the rule induces a unique equilibrium.*

b) *Consider the system (61) with* $w = 1$ *and assume that the government follows an active backward-looking rules in terms of the flexible-price inflation* ($\hat{\pi}^F$) *described by* $\hat{R}_t = \rho_\pi \hat{\pi}^F_{t-1}$ *with* $\rho_\pi > 1$. *Then regardless of whether* $U_{SF} \gtreqless 0$ *(that is whether* $\hat{c}^F_t$ *and* $\hat{c}^S_t$ *are Edgeworth substitutes, complements or utility separable), the MSV solution* $\hat{y}_t = k + P\hat{y}_{t-1} + Q\hat{z}_t$ *where* $\bar{k}, \bar{P}$ *and* $\bar{Q}$ *solve (45)is NOT learnable in the E-stability sense..*

Proof. See Appendix. ■

Proposition 7 states one of the most important results of the paper: even if the rule is backward-looking, reacting to the flexible-price inflation will make the MSV solution E-unstable. Therefore it is not learnable. The proposition also points out that as long as the two goods are utility separable then the rule will guarantee a unique equilibrium. In order to complete the determinacy analysis for these rules, we use the aforementioned parametrization to simulate the model when the goods are substitutes and complements. The results are presented in Figure 6.

This figure shows the combinations of the interest rate response coefficient to inflation (ρ_π) and the intratemporal elasticity of substitution (a) that lead to either real determinacy (D) or real indeterminacy (I). In addition it shows the combinations for these parameters under which the MSV solution is either E-stable (ES) or E-unstable (EU). In the figure a circle with an "x" inside stands for indeterminacy and E-instability of the MSV solution whereas circles with anything inside represent determinacy and E-instability of the MSV solution. As argued before the relation between a and σ determines the type of goods under consideration. For our parametrization we have that $\sigma = 1$. Hence the case of Edgeworth complements, substitutes and utility separable correspond to $a < 1$, $a > 1$ and $a = 1$, respectively.

Figure 6 confirms our results from Proposition 7: the type of goods under consideration (complements,

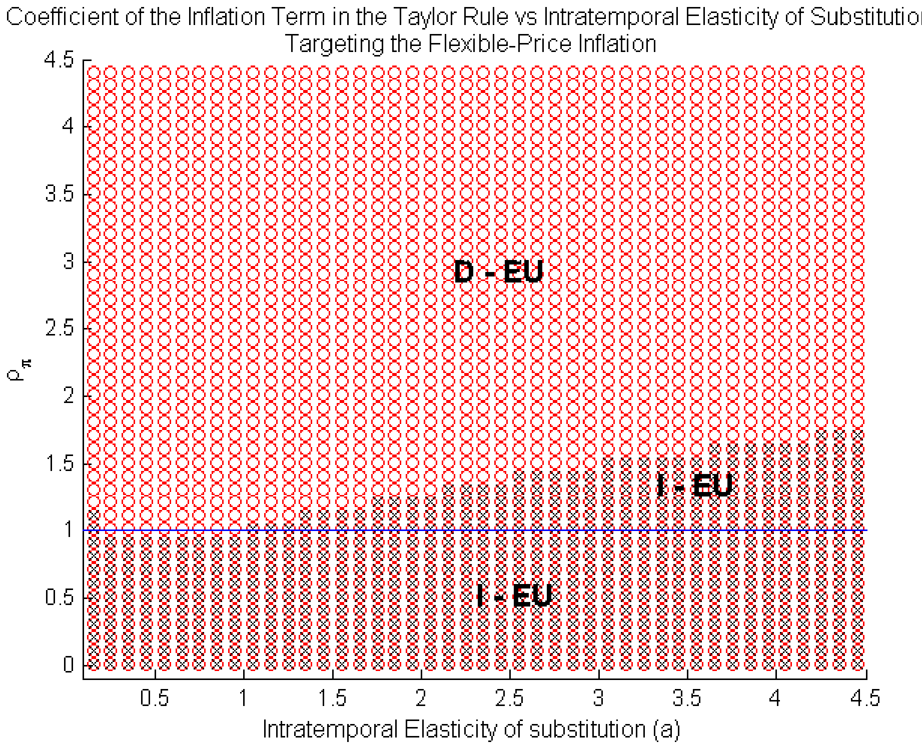

Figure 5:

substitutes and utility separable) does not affect the E-stability characterization of the MSV solution. This solution is never learnable. However it also shows that the type of goods under consideration may affect the determinacy results for active backward-looking rules. Interestingly when the goods are Edgeworth substitutes, active backward-looking rules may induce multiple equilibria.

To conclude our analysis we focus on active backward-looking rules that respond to the sticky-price inflation. In this case it is not possible to set $w = 0$ and use the systems (59) and (61). The reason is that by doing this some of the coefficients of Λ and Ω in (60) and (62) become indeterminate. Hence we have to derive new systems to pursue the determinacy and learning analyses. Furthermore we cannot derive analytical results for the leaning analysis (only for the determinacy) analysis. Hence we prefer to present some simulations that combine both analyses. Figure 7 presents and summarizes our results for active backward-looking rules that respond exclusively to the sticky-price inflation. As in Figure 6, this

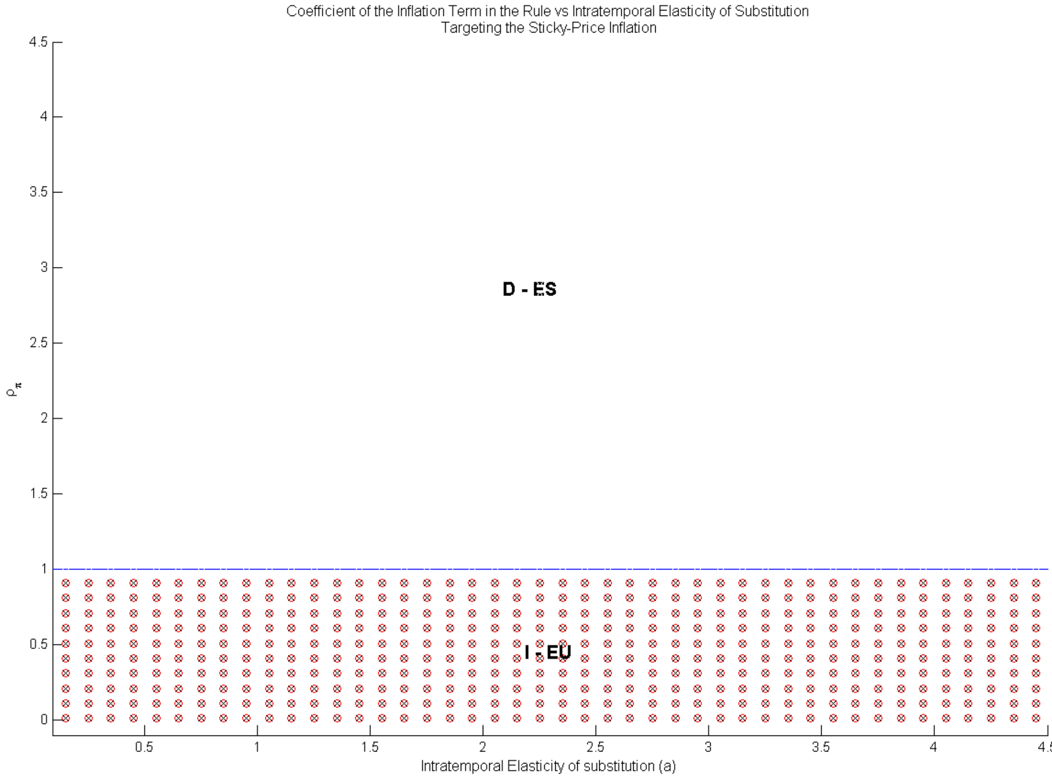

Figure 6:

figure shows the combinations of the interest rate response coefficient to inflation (ρ_π) and the intratemporal elasticity of substitution (a) that lead to either real determinacy (D) o real indeterminacy (I). In addition it shows the combinations for these parameters under which the MSV solution is either E-stable (ES) or E-unstable (EU). In the figure a circle with an "x" inside stands for indeterminacy and E-instability of the MSV solution whereas no marker depicted represents determinacy and E-stability of the MSV solution. As argued before the relation between a and σ determines the type of goods under consideration. For our parametrization we have that $\sigma = 1$. Hence the case of Edgeworth complements, substitutes and utility separable correspond to $a < 1$, $a > 1$ and $a = 1$, respectively.

It is clear from Figure 7 that active backward-looking rules with respect to the sticky-price inflation will deliver a unique and learnable equilibrium regardless of the type of goods under consideration. It is important to emphasize that this result cannot be derived setting $w = 0$ and using the systems (59) and (61) that were used for the analysis of rules that respond to the core inflation. In fact it is clear from Figure 5 that when $w = 0$, the determinacy and learning results do not coincide with those in Figure 7. The reason is that there is a discontinuity for the aforementioned systems at results $w = 0$.

4 Conclusions

In this paper we develop a closed economy model with a flexible-price good and a sticky-price good to answer the following question: which measure of inflation should the government target in the rule in order to guarantee a unique equilibrium whose MSV solution is learnable in the E-stability sense proposed by Evans and Honkapojha (2001)?. We find that the answer corresponds to the sticky-price inflation.

In order to find this answer we study how the conditions under which interest rate rules lead to real determinacy and to E-stability of the MSV solution may depend not only on the interest rate response coefficient of the rule but also on other factors. Besides the timing of the rule, we show that the measure of inflation included in the rules and the type of goods under consideration may affect significantly the aforementioned conditions.

Specifically responding to either the flexible-price inflation or the core inflation is more prone to induce multiple equilibria (real indeterminacy). This is particularly valid under forward-looking and contemporaneous rules and when the two goods are either Edgeworth complements or utility separable. Under these timings of the rule and characteristics of the goods targeting the flexible-price inflation or the core inflation makes also the MSV solution E-unstable.

More importantly backward-looking rules with respect to the flexible-price inflation or the core inflation may guarantee a unique equilibrium but in these cases the MSV solution is not learnable in the E-stability sense. In order to guarantee a unique and learnable equilibrium only responding actively to the sticky-price inflation seems to be a robust policy recommendation across timings and types of goods.

Although our selection criteria are different, our findings agree with previous results from the Optimal Monetary Policy literature that suggest that the central bank should target the sticky-price inflation such as Aoki (2001) and Mankiw and Reis (2002).

5 Appendix

5.1 Proof of Lemma 1

Proof. The proof is straightforward. a) follows from Assumptions 1 and 2 and the definition of α in (35). b) follows from using the definition of ϵ in (36) and Assumptions 1 and 2, $\bar{c}^S > 0$ and $\alpha U_{FS} + U_{SS} = \frac{\left[f_{hh}U_F U_{SS} + (f_h)^2(U_{FF}U_{SS} - U_{FS}^2) + V_{FF}U_{SS}\right]\bar{c}^S}{f_{hh}U_F + (f_h)^2 U_{FF} + V_{FF}} < 0$. Part c) can be proved by using the definition of α and the definition of χ in (36) in order to rewrite $\chi = -\frac{(f_{hh}U_F + U_{FF})\bar{c}^F}{(f_h)^2 U_F}$. Then the result follows from using this in tandem with Assumptions 1 and 2, and $\bar{c}^S > 0$. Finally d) follows from using Assumptions 1 and 2, with $\mu, \bar{c}^S, \bar{c}^F, \beta, \gamma, \bar{\pi} > 0$. ■

5.2 Proof of Lemma 2

Proof. The proof is straightforward. $\theta \gtreqless 0$ follows from parts a) and c) of Lemma 1 and $U_{SF} \gtreqless 0$, $\bar{c}^S > 0$, and $\bar{c}^F > 0$. To prove parts b) and c) we use part a) of the present Lemma and part b) of Lemma 1 ($\epsilon < 0$). Clearly if $U_{SF} \geqq 0$ then $\theta \geqq 0$ which together with $\epsilon < 0$ and the definition of τ imply $\tau \geqq 1$. On the other hand if $U_{SF} < 0$, we use the definitions of θ, α and χ provided in (50), (35) and (36) respectively to write $\tau = 1 - \frac{\theta}{\epsilon} = 1 - 1/\left[\frac{(U_{SS}U_{FF} - U_{SF}^2)f_h^2 U_F}{(U_F f_{hh} + V_{FF})U_{SF}U_S} + \frac{U_{SS}U_F}{U_{SF}U_S}\right]$. From Assumptions 1 and 2, and $U_{SF} < 0$ we have that $\frac{U_{SS}U_F}{U_{SF}U_S} > 1$ and $\frac{(U_{SS}U_{FF} - U_{SF}^2)f_h^2 U_F}{(U_F f_{hh} + V_{FF})U_{SF}U_S} > 0$. Then $0 < 1/\left[\frac{(U_{SS}U_{FF} - U_{SF}^2)f_h^2 U_F}{(U_F f_{hh} + V_{FF})U_{SF}U_S} + \frac{U_{SS}U_F}{U_{SF}U_S}\right] < 1$ and therefore $0 < \tau < 1$ if $U_{SF} < 0$. ■

5.3 Proof of Proposition 1

Proof. The characteristic polynomial for Ω defined in (49) is given by $\mathcal{P}(v) = v^2 - Trace(\Omega)v + Det(\Omega)$ where $Det(\Omega)$ refers to the determinant of Ω and

$$Trace(\Omega) = 1 + \beta - \frac{\beta\delta(1-\rho_\pi)}{\epsilon(1-\tau w\rho_\pi)} \qquad and \qquad Det(\Omega) = \beta \tag{65}$$

Using these and the characteristic polynomial $\mathcal{P}(v)$ we can derive that

$$\mathcal{P}(1) = \frac{\beta\delta(1-\rho_\pi)}{\epsilon(1-\tau w\rho_\pi)} \qquad and \qquad \mathcal{P}(-1) = \left[\frac{\beta\delta - 2(1+\beta)\epsilon\tau w}{\epsilon(1-\tau w\rho_\pi)}\right](\rho_\pi - \rho_\pi^w) \tag{66}$$

Recall from Lemma 1 that $\delta > 0$ and $\epsilon < 0$.

We first prove a). By Lemma 2 we know that $\tau > 0$ and if $U_{SF} \leq 0$ then $\tau \leq 1$. Using this and $\delta > 0$, $\epsilon < 0$, $\beta \in (0,1)$ and $w \in (0,1)$ we can infer that $\rho_\pi^w = \frac{[\beta\delta - 2(1+\beta)\epsilon]}{\beta\delta - 2(1+\beta)\epsilon\tau w} > 1$ and that $\frac{[\beta\delta - 2(1+\beta)\epsilon]}{\beta\delta - 2(1+\beta)\epsilon\tau w} < \frac{[\beta\delta - 2(1+\beta)\epsilon]}{[\beta\delta - 2(1+\beta)\epsilon]\tau w}$. Hence $1 < \rho_\pi^w < \frac{1}{\tau w}$. This in tandem with $1 < \rho_\pi < \rho_\pi^w$, (66), $\delta > 0$, $\epsilon < 0$, $\beta \in (0,1)$ and $w \in (0,1)$ imply that $\mathcal{P}(1) > 0$ and $\mathcal{P}(-1) > 0$. Moreover since $Det(\Omega) = \beta$ and $\beta \in (0,1)$ then $0 < Det(\Omega) < 1$. This together with $\mathcal{P}(1) > 0$ and $\mathcal{P}(-1) > 0$ imply that the two eigenvalues of Ω are inside the unique circle. Hence the steady state is a sink (See Azariadis, 1993). Since $\hat{\pi}_t^S$ and $\hat{c}_t^S$ are non-predetermined variables then by Blanchard and Kahn (1980) we conclude that there exists a unique equilibrium.

To prove that $1 < \rho_\pi < \rho_\pi^w$ for any $w \in (0,1)$ is also a sufficient condition for the MSV solution $\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t$ to be E-stable we first find the E-stability conditions. For the system in (48), the MSV solution corresponds to $\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t$. Then following the description of the methodology that we provided in the subsection 3.1 we can deduce that E-stability conditions are reduced to verify that all the eigenvalues of $DT_k = \Omega$ and $DT_Q = \Phi' \otimes \Omega$ have real parts less than one. However by assumption the eigenvalues of $\Phi = \begin{bmatrix} \phi^F & 0 \\ 0 & \phi^S \end{bmatrix}$ are less than one. This implies that the E-stability conditions will be satisfied whenever that all the eigenvalues of Ω have real parts less than one. To check this it is sufficient to verify that all the eigenvalues of $\Omega - I$ have negative real parts. We do so in the following way. By theorem 1.2.12 from Horn and Johnson (1985) the eigenvalues υ_1 and υ_2 of $\Omega - I$ satisfy $Trace(\Omega - I) = \upsilon_1 + \upsilon_2$ and $Det(\Omega - I) = \upsilon_1\upsilon_2$. Then sufficient and necessary conditions for all the eigenvalues of $\Omega - I$ to have negative real parts are $Trace(\Omega - I) < 0$ and $Det(\Omega - I) > 0$. We calculate

$$Trace(\Omega - I) = \beta - \frac{\beta\delta(1-\rho_\pi)}{\epsilon(1-\tau w\rho_\pi)} - 1 \qquad and \qquad Det(\Omega - I) = \frac{\beta\delta(1-\rho_\pi)}{\epsilon(1-\tau w\rho_\pi)} \tag{67}$$

As was proved before $\rho_\pi^w < \frac{1}{\tau w}$ which together with the assumption $1 < \rho_\pi < \rho_\pi^w$, the expression for $Det(\Omega - I)$ in (67) and $\delta > 0$, $\epsilon < 0$, $\beta \in (0,1)$ and $w \in (0,1)$ imply that $Det(\Omega - I) > 0$. On the other hand, using the expression for $Trace(\Omega - I)$ in (67) we can observe that $Trace(\Omega - I) < 0$ is equivalent to $-(1-\beta) < \frac{\beta\delta(1-\rho_\pi)}{\epsilon(1-\tau w\rho_\pi)} = Det(\Omega - I)$. This last inequality is trivially satisfied provided that $Det(\Omega - I) > 0$ and $\beta \in (0,1)$. Then E-stability of the MSV follows.

Second we prove b). By Lemma 2 we know that $\tau > 0$ and if $U_{SF} > 0$ then $\tau > 1$. Define the function $\hat{\rho}_\pi = \frac{1}{w\tau}$. Using this definition and the definition of ρ_π^w it is trivial to show that $\frac{\partial\hat{\rho}_\pi}{\partial w} < 0$, $\frac{\partial\rho_\pi^w}{\partial w} < 0$, $\lim_{w\to 0}\hat{\rho}_\pi = +\infty$, $\lim_{w\to 0}\rho_\pi^w = 1 - \frac{2(1+\beta)\epsilon}{\beta\delta} > 1$, $\lim_{w\to\frac{1}{\tau}}\hat{\rho}_\pi = \lim_{w\to\frac{1}{\tau}}\rho_\pi^w = 1$ which together imply that $\rho_\pi^w < \hat{\rho}_\pi = \frac{1}{w\tau}$ for any $w \in (0, 1/\tau)$. Then using this together with the assumption $1 < \rho_\pi < \rho_\pi^w$ for any $w \in (0, 1/\tau)$ we can proceed as we did to prove the existence of a unique equilibrium and the E-stability of the MSV representation in a). ■

5.4 Proof of Proposition 2

Proof. We just need to consider (48) with $w = 1$. The characteristic polynomial for Ω defined in (49) is given by $\mathcal{P}(v) = v^2 - Trace(\Omega)v + Det(\Omega)$ where $Det(\Omega)$ refers to the determinant of Ω. By replacing $w = 1$ into Ω we can obtain

$$Trace(\Omega) = 1 + \beta - \frac{\beta\delta(1-\rho_\pi)}{\epsilon(1-\tau\rho_\pi)} \qquad and \qquad Det(\Omega) = \beta \tag{68}$$

Using these and the characteristic polynomial $\mathcal{P}(v)$ we can derive that

$$\mathcal{P}(1)=\frac{\beta\delta(1-\rho_\pi)}{\epsilon(1-\tau\rho_\pi)} \qquad and \qquad \mathcal{P}(-1)=\left[\frac{\beta\delta-2(1+\beta)\epsilon\tau}{\epsilon(1-\tau\rho_\pi)}\right](\rho_\pi-\rho_\pi^1) \tag{69}$$

Moreover we recall from Lemma 1 that $\delta>0$ and $\epsilon<0$.

We first prove a). By Lemma 2 we know that $\tau>0$ and if $U_{SF}<0$ then $\tau<1$. Using this and $\delta>0$, $\epsilon<0$, and $\beta\in(0,1)$ we can infer that $\rho_\pi^1=\frac{[\beta\delta-2(1+\beta)\epsilon]}{\beta\delta-2(1+\beta)\epsilon\tau}>1$ and $\frac{[\beta\delta-2(1+\beta)\epsilon]}{\beta\delta-2(1+\beta)\epsilon\tau}<\frac{[\beta\delta-2(1+\beta)\epsilon]}{[\beta\delta-2(1+\beta)\epsilon]\tau}$. Hence $\rho_\pi^1<\frac{1}{\tau}$. This in tandem with $1<\rho_\pi<\rho_\pi^1$,(69), $\delta>0$, $\epsilon<0$, and $\beta\in(0,1)$ imply that $\mathcal{P}(1)>0$ and $\mathcal{P}(-1)>0$. Moreover since $Det(\Omega)=\beta$ and $\beta\in(0,1)$ then $0<Det(\Omega)<1$. This together with $\mathcal{P}(1)>0$ and $\mathcal{P}(-1)>0$ imply that the two eigenvalues of Ω are inside the unique circle. Hence the steady state is a sink (See Azariadis, 1993). Since $\hat{\pi}_t^S$ and $\hat{c}_t^S$ are non-predetermined variables then by Blanchard and Kahn (1980) we conclude that there exists a unique equilibrium.

To prove that $1<\rho_\pi<\rho_\pi^1$ is also a sufficient condition for the MSV solution $\hat{y}_t=\bar{k}+\bar{Q}\hat{z}_t$ to be E-stable we first find the E-stability conditions. For the system in (48) with $w=1$, the MSV solution corresponds to $\hat{y}_t=\bar{k}+\bar{Q}\hat{z}_t$. Then following the description of the methodology that we provided in the subsection 3.1 we can deduce that E-stability are reduced to verify that all the eigenvalues of $DT_k=\Omega$ and $DT_Q=\Phi'\otimes\Omega$ have real parts less than one. Moreover there the MSV is not E-stable if any of the eigenvalues of DT_k and DT_Q are bigger than one. Hence we proceed to characterize the eigenvalues of these matrices.

By assumption the eigenvalues of $\Phi=\begin{bmatrix}\phi^F & 0\\ 0 & \phi^S\end{bmatrix}$ are less than one. This implies that the E-stability conditions will be satisfied whenever that all the eigenvalues of Ω have real parts less than one. To check this it is sufficient to verify that all the eigenvalues of $\Omega-I$ have negative real parts. We do so in the following way. By theorem 1.2.12 from Horn and Johnson (1985) the eigenvalues υ_1 and υ_2 of $\Omega-I$ satisfy $Trace(\Omega-I)=\upsilon_1+\upsilon_2$ and $Det(\Omega-I)=\upsilon_1\upsilon_2$. Then a sufficient and necessary conditions for the eigenvalues of $\Omega-I$ to have negative real parts are $Trace(\Omega-I)<0$ and $Det(\Omega-I)>0$. We calculate them taking into account that $w=1$. Hence

$$Trace(\Omega-I)=\beta-\frac{\beta\delta(1-\rho_\pi)}{\epsilon(1-\tau\rho_\pi)}-1 \qquad and \qquad Det(\Omega-I)=\frac{\beta\delta(1-\rho_\pi)}{\epsilon(1-\tau\rho_\pi)} \tag{70}$$

As was proved before $\rho_\pi^1<\frac{1}{\tau}$ which together with the assumption $1<\rho_\pi<\rho_\pi^1$, the expression for $Det(\Omega-I)$ in (70) and $\delta>0$, $\epsilon<0$, and $\beta\in(0,1)$ imply that $Det(\Omega-I)>0$. On the other hand, using the expression for $Trace(\Omega-I)$ in (70) we can observe that $Trace(\Omega-I)<0$ is equivalent to $-(1-\beta)<\frac{\beta\delta(1-\rho_\pi)}{\epsilon(1-\tau\rho_\pi)}=Det(\Omega-I)$. This last inequality is trivially satisfied provided that $Det(\Omega-I)>0$ and $\beta\in(0,1)$. Then E-stability follows.

Second we prove b). By Lemma 2 we know that $\tau>0$ and if $U_{SF}\geq 0$ then $\tau\geq 1$. Therefore $\frac{1}{\tau}\leq 1$. Suppose that $\tau=1$ then from (69), $\delta>0$, $\epsilon<0$, $\beta\in(0,1)$ we can infer that $\mathcal{P}(1)<0$. Now consider the case in which $\tau>1$. Since the rule is active ($\rho_\pi>1$) and $1>\frac{1}{\tau}$ then we can see that $\rho_\pi>\frac{1}{\tau}$. Using this, $\rho_\pi>1$, (69), $\delta>0$, $\epsilon<0$, and $\beta\in(0,1)$ we derive that $\mathcal{P}(1)>0$. Hence regardless of whether $\tau=1$ or $\tau>1$ (equivalently $U_{SF}=0$ or $U_{SF}>0$) we have that $\mathcal{P}(1)<0$. It is easy to prove that

$\mathcal{P}(-1) = 2(1 + Det(\Omega)) - \mathcal{P}(1)$. Then using this, $\mathcal{P}(1) < 0$ and $Det(\Omega) = \beta > 0$ we can deduce that $\mathcal{P}(-1) > 0$. This and $\mathcal{P}(1) < 0$ are sufficient to conclude that Ω has one eigenvalue inside the unit circle and one eigenvalue outside the unit circle. Hence the steady state is a saddle path (See Azariadis, 1993). Since $\hat{\pi}_t^S$ and $\hat{c}_t^S$ are non-predetermined variables then by Blanchard and Kahn (1980) we conclude that there exists multiple equilibria.

Furthermore to prove that the MSV solution is not E-stable we start by recalling there exists E-instability if any of the eigenvalues of DT_k and DT_Q have real parts bigger than one. Hence we proceed to characterize the eigenvalues of these matrices. By assumption the eigenvalues of Φ are less than one. Then we can just focus on the eigenvalues of Ω. We want to prove that Ω has some eigenvalues with real parts bigger than one, or equivalently that $\Omega - I$ has some eigenvalues with positive real parts. When $\tau = 1$ then from (70) we can deduce that $Det(\Omega - I) = \frac{\beta\delta}{\epsilon}$. And using this and the facts that $\delta > 0$, $\epsilon < 0$, $\beta \in (0,1)$, we deduce that $Det(\Omega - I) < 0$. On the other hand if $\tau > 1$ we already derived that in this case $\rho_\pi > \frac{1}{\tau}$. Using this, with $\rho_\pi > 1$, $\delta > 0$, $\epsilon < 0$, $\beta \in (0,1)$, and the expression of $Det(\Omega - I)$ in (70) allows to conclude that $Det(\Omega - I) < 0$. Hence regardless of whether $\tau = 1$ or $\tau > 1$ (equivalently $U_{SF} = 0$ or $U_{SF} > 0$) we have that $Det(\Omega - I) < 0$. By theorem 1.2.12 from Horn and Johnson (1985) the eigenvalues υ_1 and υ_2 of $\Omega - I$ satisfy $Det(\Omega - I) = \upsilon_1 \upsilon_2$. Therefore $Det(\Omega - I) < 0$ implies that there exists one eigenvalue with a positive real part and the E-instability of the MSV solution follows. ■

5.5 Proof of Proposition 3

Proof. We just need to consider (48) with $w = 0$. The characteristic polynomial for Ω defined in (49) is given by $\mathcal{P}(\upsilon) = \upsilon^2 - Trace(\Omega)\upsilon + Det(\Omega)$ where $Det(\Omega)$ refers to the determinant of Ω. By replacing $w = 0$ into Ω we can obtain

$$Trace(\Omega) = 1 + \beta - \frac{\beta\delta(1-\rho_\pi)}{\epsilon} \qquad and \qquad Det(\Omega) = \beta \tag{71}$$

Using these and the characteristic polynomial $\mathcal{P}(\upsilon)$ we can derive that

$$\mathcal{P}(1) = \frac{\beta\delta(1-\rho_\pi)}{\epsilon} \qquad and \qquad \mathcal{P}(-1) = \left(\frac{\beta\delta}{\epsilon}\right)(\rho_\pi - \rho_\pi^0) \tag{72}$$

Moreover we recall from Lemma 1 that $\delta > 0$ and $\epsilon < 0$. Using this and $\beta \in (0,1)$ it is clear that $\rho_\pi^0 > 1$. Using $1 < \rho_\pi < \rho_\pi^0$ in tandem with (72), $\delta > 0$, $\epsilon < 0$, and $\beta \in (0,1)$ we can infer that $\mathcal{P}(1) > 0$ and $\mathcal{P}(-1) > 0$. Moreover since $Det(\Omega) = \beta$ and $\beta \in (0,1)$ then $0 < Det(\Omega) < 1$. This together with $\mathcal{P}(1) > 0$ and $\mathcal{P}(-1) > 0$ imply that the two eigenvalues of Ω are inside the unique circle. Hence the steady state is a sink (See Azariadis, 1993). Since $\hat{\pi}_t^S$ and $\hat{c}_t^S$ are non-predetermined variables then by Blanchard and Kahn (1980) we conclude that there exists a unique equilibrium.

Next we prove that $1 < \rho_\pi < \rho_\pi^0$ is also a sufficient and necessary condition for the MSV solution $\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t$ to be E-stable. In the the proofs of Propositions 1 and 2 we derived the E-stability conditions that the MSV solution must satisfy. More importantly we showed that these conditions will be satisfied whenever that all the eigenvalues of $\Omega - I$ have negative real parts which in turn is equivalent to check that $Trace(\Omega - I) < 0$ and $Det(\Omega - I) > 0$ hold. Using $w = 0$ we obtain

$$Trace(\Omega - I) = \beta - \frac{\beta\delta(1-\rho_\pi)}{\epsilon} - 1 \qquad and \qquad Det(\Omega - I) = \frac{\beta\delta(1-\rho_\pi)}{\epsilon} \tag{73}$$

Since by assumption $1 < \rho_\pi < \rho_\pi^0$, then the expression for $Det(\Omega - I)$ in (73) and $\delta > 0$, $\epsilon < 0$, and $\beta \in (0,1)$ imply that $Det(\Omega - I) > 0$. On the other hand, using the expression for $Trace(\Omega - I)$ in (73) we can observe that $Trace(\Omega - I) < 0$ is equivalent to $-(1-\beta) < \frac{\beta\delta(1-\rho_\pi)}{\epsilon} = Det(\Omega - I)$. This last inequality is trivially satisfied provided that $Det(\Omega - I) > 0$ and $\beta \in (0,1)$. Then E-stability of the MSV solution follows. ■

5.6 Proof of Proposition 4

Proof. We will first prove that regardless of $U_{SF} \gtreqless 0$ the rule always deliver real indeterminacy. In this case to prove determinacy is equivalent to prove that all the eigenvalues of Ω in (55) are inside the unit circle. The equivalence follows from the fact that $\hat{\pi}_t^F$, $\hat{\pi}_t^S$, and $\hat{c}_t^S$ are non-predetermined variables and the results from Blanchard and Kahn (1980).

Then to prove that all the eigenvalues of Ω in (55) are inside the unit circle we proceed as follows. Recall the Schur Theorem (See Lorenz, 1993) that states that the eigenvalues of a 3×3 matrix Ω are inside the unit circle if and only if having the characteristic polynomial $\mathcal{P}(v) = d_0v^3 + d_1v^2 + d_2v + d_3 = 0$ the following conditions are satisfied i) $d_0 + d_1 + d_2 + d_3 > 0$; ii) $d_0 - d_1 + d_2 - d_3 > 0$; iii) $d_0(d_0 + d_2) - d_3(d_1 + d_3) > 0$; iv) $d_0(d_0 - d_2) + d_3(d_1 - d_3) > 0$; v) $d_0 + d_3 > 0$, and vi) $d_0 - d_3 > 0$. Using the definition of Ω in (55) we can derive its characteristic polynomial obtaining that $d_0 = -1$ and $d_3 = \frac{\beta}{\tau w \rho_\pi}$. Using these it is simple to show that condition vi) of the Schur Theorem will be violated given that $d_0 - d_3 = -\left(1 + \frac{\beta}{\tau w \rho_\pi}\right) < 0$ provided that $\beta \in (0,1)$, $w \in (0,1)$, $\tau > 0$ (see Lemma 2) and $\rho_\pi > 1$.

Second we focus on the E-stability analysis. We need to find the E-stability conditions. For the system in (54), the MSV solution corresponds to $\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t$. Then following the description of the methodology that we provided in the subsection 3.1 we can deduce that E-stability conditions are reduced to verify that all the eigenvalues of $DT_k = \Omega$ and $DT_Q = \Phi' \otimes \Omega$ have real parts less than one. Moreover the MSV solution is not E-stable if any of the eigenvalues of DT_k and DT_Q have real parts bigger than one. Hence we proceed to characterize the eigenvalues of these matrices.

By assumption the eigenvalues of $\Phi = \begin{bmatrix} \phi^F & 0 \\ 0 & \phi^S \end{bmatrix}$ are less than one. Then we can just focus on the eigenvalues of Ω. We want to prove that Ω has some eigenvalues with real parts bigger than one, or equivalently that $\Omega - I$ has some eigenvalues with positive real parts. We do so in the following way. By theorem 1.2.12 from Horn and Johnson (1985) the eigenvalues v_1, v_2 and of v_3 of $\Omega - I$ satisfy $Det(\Omega - I) = v_1v_2v_3$. Then a necessary condition for all the eigenvalues of $\Omega - I$ to have negative real parts is that

$Det(\Omega - I) < 0$. Using Ω in (55) we obtain

$$Det(\Omega - I) = \frac{\beta\delta(1-\rho_\pi)}{\epsilon\tau w\rho_\pi} - \frac{\beta\delta}{\epsilon}\left(1 - \frac{1}{\tau^2}\right) \tag{74}$$

Consider the case $U_{SF} \geq 0$. By Lemma 2 we know that $\tau \geq 1$. This together with $\delta > 0$, $\epsilon < 0$, $\beta \in (0,1), w \in (0,1)$, $\rho_\pi > 1$, and (74) imply that $Det(\Omega - I) > 0$. Which means by theorem 1.2.12 from Horn and Johnson (1985) that $Det(\Omega - I)$ has at least one eigenvalue with a positive real part. Then E-instability of the MSV solution follows.

Consider $U_{SF} < 0$. By Lemma 2 we know that $\tau < 1$. This together with $\delta > 0, \epsilon < 0, \beta \in (0,1), w \in (0,1)$, $\rho_\pi > 1$ and (74) imply that there might be some values of $w \in (0,1)$ and $\rho_\pi > 1$ for which $Det(\Omega - I) < 0$. Which means by theorem 1.2.12 from Horn and Johnson (1985) that $Det(\Omega - I)$ may have either one eigenvalue or three eigenvalues with negative real parts. Then E-stability is possible. ■

5.7 Proof of Proposition 5

Proof. The proof is very simple. It is the same as the Proof for Proposition 4 taking into account that $w = 1$. ■

5.8 Proof of Proposition 6

Proof. The characteristic polynomial for Ω defined in (57) is given by $\mathcal{P}(v) = v^2 - Trace(\Omega)v + Det(\Omega)$ where $Det(\Omega)$ refers to the determinant of Ω and

$$Trace(\Omega) = \frac{(\epsilon - \delta)\beta + \epsilon}{\epsilon - \beta\delta\rho_\pi} \qquad and \qquad Det(\Omega) = \frac{\beta\epsilon}{\epsilon - \beta\delta\rho_\pi} \tag{75}$$

Using these and the characteristic polynomial $\mathcal{P}(v)$ we can derive that

$$\mathcal{P}(1) = \frac{\beta\delta(\rho_\pi - 1)}{\beta\delta\rho_\pi - \epsilon} \qquad and \qquad \mathcal{P}(-1) = \frac{\beta\delta\rho_\pi + \beta\delta - 2\epsilon(1+\beta)}{\beta\delta\rho_\pi - \epsilon} \tag{76}$$

Moreover we recall from Lemma 1 that $\delta > 0$ and $\epsilon < 0$. Using this, $\rho_\pi > 1$ in tandem with (76), and $\beta \in (0,1)$ we can infer that $\mathcal{P}(1) > 0$ and $\mathcal{P}(-1) > 0$. Moreover it is straightforward to prove that $0 < Det(\Omega) < 1$. This together with $\mathcal{P}(1) > 0$ and $\mathcal{P}(-1) > 0$ imply that the two eigenvalues of Ω are inside the unique circle. Hence the steady state is a sink (See Azariadis, 1993). Since $\hat{\pi}_t^S$ and $\hat{c}_t^S$ are non-predetermined variables then by Blanchard and Kahn (1980) we conclude that there exists a unique equilibrium.

Next we prove that $\rho_\pi > 1$ is also a sufficient and necessary condition for the MSV solution $\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t$ to be E-stable. We first find the E-stability conditions. For the system in (56), the MSV solution corresponds to $\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t$. Then following the description of the methodology that we provided in the subsection 3.1 we can deduce that E-stability conditions are reduced to verify that all the eigenvalues of $DT_k = \Omega$ and

$DT_Q = \Phi' \otimes \Omega$ have real parts less than one. However by assumption the eigenvalues of $\Phi = \begin{bmatrix} \phi^F & 0 \\ 0 & \phi^S \end{bmatrix}$ are less than one. This implies that the E-stability conditions will be satisfied whenever that all the eigenvalues of Ω have real parts less than one. To check this it is sufficient to verify that all the eigenvalues of $\Omega - I$ have negative real parts. We do so in the following way. By theorem 1.2.12 from Horn and Johnson (1985) the eigenvalues υ_1 and υ_2 of $\Omega - I$ satisfy $Trace(\Omega - I) = \upsilon_1 + \upsilon_2$ and $Det(\Omega - I) = \upsilon_1 \upsilon_2$. Then sufficient and necessary conditions for all the eigenvalues of $\Omega - I$ to have negative real parts are $Trace(\Omega - I) < 0$ and $Det(\Omega - I) > 0$. We calculate

$$Trace(\Omega - I) = \frac{\epsilon(\beta - 1) + \beta\delta(2\rho_\pi - 1)}{\epsilon - \beta\delta\rho_\pi} \qquad and \qquad Det(\Omega - I) = \frac{\beta\delta(1 - \rho_\pi)}{\epsilon - \beta\delta\rho_\pi} \tag{77}$$

The assumption $\rho_\pi > 1$, the expressions for $Trace(\Omega - I)$ and $Det(\Omega - I)$ in (75), and the facts that $\delta > 0$, $\epsilon < 0$, and $\beta \in (0, 1)$ imply that $Trace(\Omega - I) < 0$ and $Det(\Omega - I) > 0$. Hence E-stability of the MSV solution follows. ■

5.9 Proof of Proposition 7

Proof. First we prove a) by considering the representation (59). To do we derive the characteristic polynomial of the matrix Λ in (60) taking into account that $w = 1$ and that the two goods are utility separable. This means that $\tau = 1$ by Lemma 2. Then we obtain

$$\mathcal{P}(v) = \underbrace{\left(v^2 - \frac{1}{\rho_\pi}\right)}_{\mathcal{P}_1(v)} \underbrace{\left[v^2 - \left(1 + \beta - \frac{\beta\delta}{\epsilon}\right)v + \beta\right]}_{\mathcal{P}_2(v)} \tag{78}$$

that shows that the characteristic polynomial $\mathcal{P}(v)$ of Λ (with $w = 1$ and $\tau = 1$) corresponds to the product of the polynomial $\mathcal{P}_1(v)$ and $\mathcal{P}_2(v)$. Hence the roots of these two polynomials will determine the eigenvalues of Λ (with $w = 1$ and $\tau = 1$). Consider the roots of $\mathcal{P}_1(v)$. It is simple to see that if $\rho_\pi > 1$ then the two roots of $\mathcal{P}_1(v)$ are inside the unit circle. On the other hand the polynomial $\mathcal{P}_2(v)$ satisfies

$$\mathcal{P}_2(1) = \frac{\beta\delta}{\epsilon} \qquad \mathcal{P}_2(-1) = 2(1 + \beta) - \frac{\beta\delta}{\epsilon}$$

Using these expressions, $\beta \in (0, 1)$ and the facts that $\epsilon < 0$ and $\delta > 0$ (see Lemma 1) we can infer that $\mathcal{P}_2(1) < 0$ and $\mathcal{P}_2(-1) > 0$. Which in turn implies that one of the root of $\mathcal{P}_2(v)$ is inside the unit circle whereas the other one is outside of it (see Azariadis, 1993). Putting these results together we conclude that three of the roots of $\mathcal{P}(v)$ (or equivalently three of the eigenvalues of Λ with $w = 1$ and $\tau = 1$) are inside the unit circle while the fourth one is outside of it. Then since $\hat{\pi}_t^F$, $\hat{\pi}_t^S$ and $\hat{c}_t^S$ are the only non-predetermined variables we conclude that there exists a unique equilibrium (see Blanchard and Kahn 1980).

Next we prove part b) by considering the system (61). In this case $\tau \gtreqless 1$. We need to prove that the MSV solution $\hat{y}_t = k + P\hat{y}_{t-1} + Q\hat{z}_t$ is E-unstable. To do so we recall the description of the methodology that we provided in the subsection 3.1. We know by Evans and Honkapohja (2001) that the MSV solution is E-unstable if any of the eigenvalues of $DT_k = (I - \Omega\bar{P})^{-1}\Omega$, $DT_P = \left[(I - \Omega\bar{P})^{-1}\Gamma\right]' \otimes [(I - \Omega\bar{P})^{-1}\Omega]$, and $DT_Q = \Phi' \otimes [(I - \Omega\bar{P})^{-1}\Omega]$ have real parts bigger than one. Then we start by studying the eigenvalues of $DT_k = (I - \Omega\bar{P})^{-1}\Omega$. More specifically we will prove that $DT_k = (I - \Omega\bar{P})^{-1}\Omega$ has some eigenvalues with real parts bigger than one, or equivalently that $\left[(I - \Omega\bar{P})^{-1}\Omega - I\right]$ has some eigenvalues with real positive parts. To do so it is necessary to find the MSV solution. In particular we need to solve for $\bar{P}$ using the method of undetermined coefficients. From (45) we know that this matrix should satisfy $\Omega P^2 - P + \Gamma = 0$ or equivalently $(I - \Omega P)^{-1}\Gamma = P$. However since $w = 1$ then the matrix Γ in the system (61) becomes

$$\Gamma = \begin{bmatrix} 0 & (1-\tau)\rho_\pi & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{\beta\delta\rho_\pi}{\epsilon} & 0 & 0 \\ 0 & \frac{\rho_\pi}{\epsilon} & 0 & 0 \end{bmatrix} \tag{79}$$

which help us to have an "educated" guess for P. That is

$$P = \begin{bmatrix} 0 & p_1 & 0 & 0 \\ 0 & p_2 & 0 & 0 \\ 0 & p_3 & 0 & 0 \\ 0 & p_4 & 0 & 0 \end{bmatrix}$$

Using this expression, the expression for Ω (with $w = 1$) in (62), (79) and $(I - \Omega P)^{-1}\Gamma = P$ we can prove that in this case $\bar{P} = \Gamma$ Utilizing this and the expression for Ω (with $w = 1$) in (62) we can find the determinant of the matrix $\left[(I - \Omega\bar{P})^{-1}\Omega - I\right]$, which corresponds to $Det\left[(I - \Omega\bar{P})^{-1}\Omega - I\right] = \frac{\beta\delta(\rho_\pi - 1)}{\rho_\pi \epsilon \tau}$. Note that $Det\left[(I - \Omega\bar{P})^{-1}\Omega - I\right] < 0$ since by assumption $\rho_\pi > 1$, $\beta \in (0,1)$, $\epsilon < 0$, $\delta > 0$ and $\tau > 0$ (see Lemma 2).

Finally by theorem 1.2.12 from Horn and Johnson (1985) the eigenvalues υ_1, υ_2, υ_3, and υ_4 of $\left[(I - \Omega\bar{P})^{-1}\Omega - I\right]$ satisfy $Det\left[(I - \Omega\bar{P})^{-1}\Omega - I\right] = \upsilon_1\upsilon_2\upsilon_3\upsilon_4$. Given that $Det\left[(I - \Omega\bar{P})^{-1}\Omega - I\right] < 0$, we know that $\left[(I - \Omega\bar{P})^{-1}\Omega - I\right]$ has at least one eigenvalue with a positive real part. Hence the E-instability of the MSV solution follows. ■

References

[1] Aoki, K. (2001), "Optimal Monetary Policy Responses to Relative-Price Changes," *Journal of Monetary Economics, 48, 55-80.*

[2] Azariadis, C. (1993), *Intertemporal Macroeconomics*, Blackwell, Cambridge, Massachusetts.

[3] Ball, L. (1999), "Policy Rules for Open Economies," in *Monetary Policy Rules*, edited by John B. Taylor, Chicago, National Bureau of Economic Research, 127-144.

[4] Benhabib, J., S. Schmitt-Grohé and M. Uribe (2001a), "Monetary Policy Rules and Multiple Equilibria," *American Economic Review*, 91, 167-184.

[5] Benhabib, J., S. Schmitt-Grohé and M. Uribe (2001a), "Monetary Policy Rules and Multiple Equilibria," *American Economic Review*, 91, 167-184.

[6] Bernanke, B., and M. Woodford (1997), "Inflation Forecasts and Monetary Policy," *Journal of Money Credit and Banking,,* 29, 653-684.

[7] Blanchard, O. and C. Kahn (1980), "The Solution of Linear Difference Models Under Rational Expectations," *Econometrica,* 48, 1305-1312.

[8] Bullard, J. and K. Mitra, (2002), "Learning About Monetary Policy Rules," *Journal of Monetary Economics,* 49, 1105-1129.

[9] Carlstrom, C. and T. Fuerst (1999a), "Forward-Looking and Backward-Looking Rules," Federal Reserve Bank of Cleveland, Working Paper.

[10] Carlstrom, C., T. Fuerst and F. Ghironi (2004), ""Does It Matter (For Equilibrium Determinacy) What Price Index The Central Bank Targets," Forthcoming *JET*.

[11] Calvo, G. (1983), "Staggering Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics,* 12, 383-398.

[12] Clarida, J., J. Gali and M. Gertler (1998), "Monetary Policy Rules in Practice: Some International Evidence," *European Economic Review*, 42, 1033-1067.

[13] Clarida, J., J. Gali and M. Gertler (2000), "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," *Quarterly Journal of Economics*, 115, 147-180.

[14] Corbo, V. (2000), "Monetary Policy in Latin America in the 90s," Working Papers, No. 78, Central Bank of Chile.

[15] Evans, G, and S. Honkapohja (1999), "Learning Dynamics," in *Handbook of Macroeconomics*, eds. J. Taylor and M. Woodford, Elsevier, Amsterdam, Vol. 1, 449-542.

[16] Dupor, B. (2001), "Investment and Interest Rate Policy," *Journal of Economic Theory,* 98, 85-113.

[17] Evans, G, and S. Honkapohja (2001), *Learning and Expectations*, Princeton University Press, New Jersey.

[18] Gali, J. and T. Monacelli (2004), "Monetary Policy and Exchange Rate Volatility in a Small Open Economy," Forthcoming *Review of Economic Studies.*

[19] Lubik, T and F. Schorfheide (2003), "Do Central Banks Target Exchange Rates? A Structural Investigation", Manuscript, University of Pennsylvania.

[20] Lubik, T and F. Schorfheide (2004), "Testing for Indeterminacy: An Application to U.S. Monetary Policy", *American Economic Review*, 94(1), 190-217.

[21] Leeper, E. (1991), "Equilibria Under 'Active' and 'Passive' Monetary and Fiscal Policies," *Journal of Monetary Economics,* 27(1), 129-147

[22] Lorenz, H. (1993), *Nonlinear Dynamical Economics and Chaotic Motion*, Springer-Verlag, New York.

[23] Mankiw, G. and R. Reis (2002), "What Measure of Inflation Should a Central Bank Target?," NBER Working Paper, 9375. Forthcoming JEA.

[24] Marcet, A., and T. Sargent (1989), "Convergence of Least-Square Learning Mechanisms in Self-Referential Linear Stochastic Models," *Journal of Economic Theory*, 48, 337–368.

[25] McCallum, B. (1983), "On Nonuniqueness in Linear Rational Expectations Moldels: An Attempt at Perspective," *Journal of Monetary Economics,* 11, 134-168.

[26] Orphanides, A. (1997), "Monetary Policy Rules Based on Real-Time Data," Finance and Economic Discussion Series No. 1998-03, Federal Reserve Board.

[27] Rotemberg, J. (1982), "Sticky Prices in the United States," *Journal of Political Economy,* 90(6), 1187-1211.

[28] Schmitt-Grohé, S. and M. Uribe (2004), "Optimal Fiscal and Monetary Policy Under Sticky Prices," *Journal of Economic Theory*, 114, 198-230.

[29] Svensson, L. E. O. (2000), "Open-economy Inflation Targeting," *Journal of International Economics*, 50, 155-184.

[30] Taylor, J. B.(1999), *Monetary Policy Rules*, Chicago, National Bureau of Economic Research.

[31] Taylor, J. B. (1993), "Discretion Versus Policy Rules in Practice," *Carnegie-Rochester Conference Series on Public Policy*, 39, 195-214.

[32] Woodford, M. (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy,* Princeton, Princeton University Press.

[33] Zanna, L. F. (2003), "Interest Rate Rules and Multiple Equilibria in the Small Open Economy", International Finance Discussion Papers, No. 785, Board of Governors of the Federal Reserve System.