Reconciling the Effects of Monetary Policy Actions on Consumption within a Heterogeneous Agent Framework*

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Abstract

This paper incorporates heterogeneous agents into a NNS model with nominal inertia. Heterogeneous households are introduced into NNS models to try and reconcile the movements in interest rates, consumption and inflation. The key findings here are that heterogeneity and wage inertia are needed to help reconcile these observations. Aggregate consumption and its expected growth rate responds much more to myopic households than compared to optimizing households when myopic households set wages one periods in advance. When myopic households set wages in the current period, aggregate consumption and its expected growth rate is found to respond much more to the respective profiles for optimizing households.

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1 Introduction

The recent literature on monetary policy has exploited the New Neoclassical Synthesis.[1] These models incorporate optimizing behaviour and rational expectations. They incorporate nominal rigidities that allow monetary policy to have real and persistent effects in the short run, whilst remaining consistent with the proposition of long run neutrality. This paper examines the transmission mechanism of monetary policy within a new neoclassical framework that incorporates heterogeneous households. In particular, the paper examines the effects of a monetary policy action on consumption, and tries to reconcile the responses of consumption of heterogeneous households with the transmission mechanism of monetary policy within New Neoclassical Synthesis models.

New Neoclassical Synthesis models (NNS models for short) have the theoretical prediction that real interest rates and the expected growth rate of consumption should be perfectly positively correlated. Within this literature, a monetary policy action affects aggregate demand primarily through its effect on households' consumption expenditures. Central banks are assumed to target interest rates when setting monetary policy, and these models typically equate the interest rate within the consumption Euler equation to a money market rate. Hence, a monetary policy action impacts households' consumption-savings decisions, and has an impact on the economy through its impact on expected consumption growth. A monetary expansion that lowers interest rates, is thought to lower expected consumption growth as households increase consumption today relative to the next period. This transmission mechanism leads to the observed increase in consumption and output.

The prediction that real interest rates and consumption growth rates move together, has not been bourne out in the empirical literature. This literature on monetary policy documents a 'hump-shaped' response of aggregate consumption to a monetary policy action, implying a negative correlation between real interest rates and aggregate consumption growth. Namely, a monetary expansion that lowers interest rates raises consumption this period, but increases consumption in

[1] The term "New Neoclassical Synthesis" was coined by Goodfriend and King (1997). Monetary models that incorporate this framework include Rotemberg and Woodford (1997), Clarida, Gali and Gertler (1999), Erceg, Henderson and Levin (2000) and Woodford (2003).

the next period by more. Hence the growth rate of consumption increases. Canzoneri, Cumby and Diba (2002a) and Ahmad (2004) have found the correlation between real interest rates and aggregate consumption growth to be low, and often negative, across most of the G7 countries. These observations pose a serious problem for NNS models that utilize this monetary transmission mechanism, and equate the interest rate in the consumption Euler equation to a money market rate.

The heart of the problem lies in an inability to reconcile the time series properties of interest rates, consumption and inflation with the consumption Euler equation. Although both the asset pricing, consumption and monetary literatures have identified a number of problems with the consumption Euler equation using aggregate data, more recent investigations have found more favourable results using micro level data. Attanasio and Weber (1993, 1995) have found evidence that the consumption Euler equation fits micro level consumption data better than aggregate data. More recently, Vissing-Jorgensen (2002) and Brav, Constantinides and Geczy (2002) have found more reasonable estimates of the intertemporal elasticity of substitution within the consumption Euler equation, using models that incorporate limited asset market participation.

This paper attempts to examine the role of heterogeneity within a NNS model. Rule of thumb, or myopic households are introduced into a standard NNS framework with optimizing agents, in a similar fashion to that of Campbell and Mankiw (1989). The benchmark version of the model incorporates nominal inertia in the form of sticky prices, sticky wages and preset wages (on the part of myopic households). However, results are reported for differing assumptions of price and wage stickiness. There are four main findings.

First, the introduction of a small number of rule of thumb households into the benchmark NNS model is able to yield a low correlation between the real interest rate and expected aggregate consumption growth. In this case, myopic households set wages one period in advance and are unable to observe the current period monetary policy and productivity shocks. I find that even when there is a small number of agents who behave myopically, expected aggregate consumption growth responds much more to the consumption profile of the myopic agents as their consumption responses dominate those of forward-looking or optimizing agents.

Second, the correlations between interest rates and expected consumption growth rates are ordered as follows. The correlation of either the nominal or real interest rate with the expected consumption growth rate for myopic households is less than the correlation of either interest rate with the expected aggregate consumption growth rate. Both of these correlations are, in turn, less than the correlation of either interest rate with the expected consumption growth rate for optimising households.

Third, I find that heterogeneity alone is unable to reconcile consumption, inflation and interest rates with the monetary transmission mechanism in NNS models. The correlation between interest rates and expected aggregate consumption growth depends to a large extent on whether myopic households set their wages one period in advance. The correlation between interest rates and expected aggregate consumption growth is very close to one when myopic households are able to set wages after they observe current shocks. In this case, expected aggregate consumption growth is dominated by the response of optimizing households, even when there are a large number of myopic households present.

Finally, the results here find that wage inertia plays a greater role in generating persistence from a monetary policy shock as compared to price inertia, and this is consistent with what is seen in the literature, e.g. Christiano, Eichenbaum and Evans (2001). However, aggregate consumption does not display a hump-shaped response under any assumptions of price or wage inertia. This is consistent with the literature, which finds that models with time seperable preferences are unable to generate a hump-shaped response, e.g. Fuhrer, 2000.

The remainder of the paper is organized as follows. Section 2 outlines a NNS model that incorporates informational inertia as well as sticky prices and wages. Section 3 outlines the calibration methodology and examines the effects of productivity and monetary policy shocks and its implications for correlations between interest rates and expected consumption growth rates. Section 4 eliminates the informational inertia and provides some evidence on how the correlations are affected by changing the assumption on whether myopic households are able to observe shocks. Section 5 examines the role that price and wage inertia play within the model. Finally, section 6 concludes.

2 The Economic Environment

The objective within this paper is to examine the effects of introducing heterogeneous households into a NNS model, with a view towards reconciling the transmission mechanism of monetary policy. In a NNS model where agents are only forward looking (i.e. do not incorporate any myopic agents), a monetary policy action affects real variables in the economy through its impact on household's consumption-savings decisions. Consider the consumption Euler equation below arising in a standard NNS model that incorporates power utility. NNS models link the stance of monetary policy to the interest rate found in the consumption Euler equation. With nominal inertia, a change in the interest rate arising from a monetary policy action impacts expected consumption growth, which leads to changes in actual consumption and output.

$$\frac{1}{1+i_t} = E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)\right]$$

However, problems arise in attempting to reconcile the time series properties of consumption, inflation and interest rates with the transmission mechanism outlined above. The empirical literature on monetary policy documents a hump-shaped response of aggregate consumption to a monetary policy shock. In addition the correlations between interest rates and expected consumption growth have been found to be low, and sometimes negative across many of the industrialized countries (see Ahmad, 2004). I introduce rule of thumb households into a standard NNS model with price and wage inertia to try and reconcile two facts. First, the consumption Euler equation holds for optimizing, or forward looking households, implying a perfect positive correlation between the real interest rate and the expected growth rate of consumption for optimizing households. Second, the model exhibits a low correlation between interest rates and expected aggregate consumption as seen in the empirical literature.

This section outlines the key players within the model. The economic environment consists of a perfectly competitive industry producing a final good, a continuum of firms producing differentiated intermediate goods, heterogeneous households, and a central bank setting monetary policy. The objectives and the constraints faced by these different agents are outlined next. All the key equations are derived in the technical appendix (Appendix A) at the end of the paper.

2.1 The Firms

I assume that there is a continuum of monopolistically competitive firms, each producing a differentiated intermediate good. These intermediate goods are then used as inputs by a perfectly competitive industry who produces a single final good.

2.1.1 Final Good Firms

Final goods firms produces a final consumption good, Y_t, at time t, using the intermediate goods produced by other firms as an input. They combine the continuum of intermediate goods produced by the intermediate good's firms, $j \in [0,1]$ using a constant returns to scale production technology:

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}} \tag{1}$$

where $\theta > 1$, and Y_{jt} denotes the amount of the intermediate good j, used at time t. The final goods firms maximize profits (or minimizes costs), taking the final goods price, P_t and the prices of the intermediate goods, P_{jt} as given. This yields the set of demand schedules for the individual intermediate goods:

$$Y_{j,t}^d = \left(\frac{P_{j,t}}{P_t} \right)^{-\theta} Y_t \tag{2}$$

along with the zero profit condition:

$$P_t = \left(\int_0^1 P_{j,t}^{1-\theta} dj \right)^{\frac{1}{1-\theta}} \tag{3}$$

Equation (2) suggests that the demand for the intermediate good j, is a decreasing function of the relative price of the good, and an increasing function of aggregate output, Y_t. Equation (3) can be simply obtained by using equations (2) and (1). More detailed derivations can be found in the technical appendix (Appendix A).

2.1.2 Intermediate Goods Firms

As mentioned above, the continuum of intermediate good firms produce differentiated products and behave monopolistically. The production function for the representative intermediate goods firm, j, is given by:

$$Y_{j,t} = Z_t K_{j,t}^{\alpha} N_{j,t}^{1-\alpha} \tag{4}$$

where Z_t is a productivity shock, K_{jt} and N_{jt} are the amounts of capital and composite labour services employed by firm j. These intermediate firms are assumed to rent capital and the composite labour in perfectly competitive factor markets and hence, take wages and the rental cost of capital as given when choosing the optimal amounts of capital and labour to employ. They solve a cost minimization problem which yields the following optimality condition:

$$R_t^k K_{j,t} = \left(\frac{\alpha}{1-\alpha}\right) W_t N_{j,t} \tag{5}$$

where R_t^k and W_t are the nominal rental rate on capital services and a wage index (to be defined later) comprising of an appropriately weighted sum of household wages. Firm $j's$ total costs at time t are given by $R_t^k K_{jt} + W_t N_{jt}$ and this yields the following real marginal cost, s_t:

$$s_t = \left(\frac{1}{\alpha}\right)^{\alpha} \left(\frac{1}{1-\alpha}\right)^{1-\alpha} Z_t^{-1} \left(r_t^k\right)^{\alpha} (\omega_t)^{1-\alpha} \tag{6}$$

where $r_t^k = \frac{R_t^k}{P_t}$ and $\omega_t = \frac{W_t}{P_t}$. The firm's profits at time t are:

$$(P_{j,t} - P_t s_t) Y_{j,t}$$

Price Setting Intermediate goods firms are assumed to set prices by a method similar to the one proposed by Calvo (1983). In each period, a firm faces a constant probability, $1 - \xi_p$ of being able to reoptimise its price, and this is independent across firms and time. This Calvo price-setting mechanism captures firm's responses to a variety of costs of being able to change prices. Hence, in any given period, a measure $1 - \xi_p$ of firms are able to reoptimise their prices whilst a fraction ξ_p are unable to do so.

Consider first, firms who are unable to reoptimise their prices. In this case, prices are updated according to a simple rule. Here, following Erceg, Henderson & Levin (2000) and Yun (1996), this simple rule is assumed to take the form that the old price is simply adjusted by steady state inflation, i.e.:

$$P_{j,t} = \Omega P_{j,t-1} \tag{7}$$

where $\Omega = P/P_{-1}$ is used to denote gross steady state inflation.[2]

A firm with the ability to reoptimise prices maximizes:

$$\max_{\widetilde{P}_t} E_t \sum_{k=0}^{\infty} \xi_p^k \lambda_{t+k} Y_{j,t+k} \left(\widetilde{P}_t \Omega^k - P_{t+k} s_{t+k} \right) \tag{8}$$

subject to equations (2) and (6). In the equation above, $\widetilde{P}_t$ represents the price chosen by firms who are able to reset prices and with probability ξ_p^k, the price $\Omega^k \tilde{P}_t$ will be in effect in period $t+k$. Also, λ_t is the marginal value of a dollar to the households who own the firms, and this is treated as exogenous by the firm. Hence, the equation above transforms the profits into utility terms and so one interpretation of the equation above is that the firm maximizes the expected utility derived from profits for its owners. The first order condition associated with the choice of $\widetilde{P}_t$ for the problem above is:

$$\begin{aligned} E_t \left\{ \sum_{k=0}^{\infty} \xi_p^k \lambda_{t+k} Y_{j,t+k} \left(\widetilde{P}_t \Omega^k - \mu_p P_{t+k} s_{t+k} \right) \right\} &= 0 \\ \text{where } \mu_p &\equiv \frac{\theta}{\theta - 1} \end{aligned} \tag{9}$$

[2] An alternative specification for Ω could take the form: $\Omega = \pi_{t-1}$ as in Christiano, Eichenbaum & Evans (2001), but this is not considered with this paper.

This equation depicts the individual firm's pricing behaviour. When prices are fully flexible, i.e. $\xi_p = 0$, equation (9) reduces to the condition that the firm sets its price, $\widetilde{P}_t$ equal to a markup over its expected marginal cost, $P_t s_t$. When some degree of price stickiness exists, i.e. $\xi_p > 0$, the firm sets $\tilde{P}_t$ to a markup over weighted marginal costs over time. Rearranging (9) yields:

$$\tilde{P}_t = \mu_p \frac{E_t \sum_{k=0}^{\infty} \xi_p^k \lambda_{t+k} Y_{j,t+k} P_{t+k} s_{t+k}}{\sum_{k=0}^{\infty} \xi_p^k \lambda_{t+k} Y_{j,t+k} \Omega^k} = \mu_p \frac{PB_t}{PA_t} \tag{10}$$

where

$$\begin{aligned} PB_t &\equiv E_t \sum_{k=0}^{\infty} \xi_p^k \lambda_{t+k} \left(\Omega^k\right)^{-\theta} P_{t+k}^{\theta} Y_{t+k} P_{t+k} s_{t+k} \\ &= \lambda_t P_t^{\theta} Y_t NMC_t + \xi_p \Omega^{-\theta} E_t PB_{t+1} \end{aligned} \tag{11}$$

$$PA_t \equiv E_t \sum_{k=0}^{\infty} \xi_p^k \lambda_{t+k} \left(\Omega^k\right)^{-\theta} P_{t+k}^{\theta} Y_{t+k} \Omega^k = \lambda_t P_t^{\theta} Y_t + \xi_p \Omega^{1-\theta} E_t PA_{t+1} \tag{12}$$

Finally, the equation that describes the dynamics for the aggregate price level is obtained from (3) and is given by:

$$P_t = \left[\left(1-\xi_p\right) \tilde{P}_t^{1-\theta} + \xi_p \left(\Omega P_{t-1}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{13}$$

2.2 Households

There is a continuum of infinitely lived households, indexed by $i \in (0,1)$. Households are heterogenous and belong to one of two different types, in a setup similar to that proposed by Campbell and Mankiw (1989). Some recent papers have introduced rule of thumb agents into the New Keynesian framework.[3] The setup here is similar to that of Gali, López-Salido and Vallés (2004). I assume that a fraction of households, $(1-\nu)$ has access to capital markets where they can trade a full set of contingent securities. In addition, they can accumulate physical capital, which they rent out to firms. This subset of households are henceforth referred to as the set of *optimizing* households.

[3] See for example, Gali, López-Salido and Valles (2004) and Erceg, Guerrieri and Gust (2003).

The remaining fraction, ν, are *rule-of-thumb* or *myopic* households, and these labels are used interchangeably throughout the paper. They are assumed not have access to capital markets, do not own any assets and make consumption expenditures based upon their current labour income.[4] The objectives and constraints faced by these two different types of households are outlined next.

2.2.1 Optimizing Households

Denoting optimizing households by the superscript 'o' to represent $i \in (\nu, 1)$, the measure $(1-\nu)$ of optimizing households maximizes their expected discounted utility over their lifetime:

$$\max E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} U\left(C_\tau^o, L_\tau^o\right) = \max E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[\frac{(C_\tau^o)^{1-\sigma}}{1-\sigma} - \kappa \frac{(L_\tau^o)^{1+\chi}}{1+\chi} \right] \tag{14}$$

where C_t^o and L_t^o represent consumption and differentiated labour services for these households. Each period, the optimizing household decides how much to consume C_t^o; the amount to spend in adjusting a portfolio of state contingent bonds, B_{t+1}; how much capital, K_{t+1}^o to accumulate; the amount of capital services to spend in supplying capital goods to the intermediate goods firms, I_t^o and its utilization rate, u_t. In addition, they also choose a wage, W_t^o to post to the intermediate goods firms for their differentiated labour service.

These optimizing households receive income from labour earnings $W_t^o L_t^o$, renting out capital services $R_t^k K_t^o$, dividends from their ownership of the firms D_t and the nominal return on asset holdings, B_t. W_t^o and R_t^k represent the nominal wage and the (nominal) rental cost of capital in period t, respectively. They also face a cost $a\left(u_t\right) K_t^o$, in terms of consumption goods, of employing a utilization rate, u_t. $a\left(.\right)$ represents an increasing convex function. Hence, the optimizing household chooses $C_t^o, B_{t+1}^o, K_{t+1}^o, I_t^o, u_t$ and W_t^o to maximize (14) subject to the labour demand schedule and equations (15) and (16) below.

[4] There are a variety of reasons for the existence of these rule-of-thumb, or myopic households. Household's may consume out of their current income because of myopic behaviour, or due to their inability to access capital markets, or because of binding borrowing constraints, or simply because of their ignorance about the possiblities to smooth their consumption patterns over time.

$$P_t\left(C_t^o+I_t^o\right)+E_\tau\left(\Delta_{t,t+1}B_{t+1}^o\right)$$
$$=W_t^oL_t^o+B_t^o+\left(R_t^k u_t-P_t a\left(u_t\right)\right)K_t^o+D_t \tag{15}$$

$$K_{t+1}^o=(1-\delta)K_t^o+\psi\left(\frac{I_t^o}{K_t^o}\right)K_t^o \tag{16}$$

where $\Delta_{t,t+1}$ is the stochastic discount factor, and $\psi(.)K_t^o$ represents capital adjustment costs with the properties: $\psi'(.)>0,\psi''(.)\leq 0,\psi(\delta)=\delta,\psi'(\delta)=1$. All optimizing households are assumed to face the same set of asset prices and have the same subjective probabilities of the states of the world that can occur. Hence the stochastic discount factor is the same across all the households.[5] The portfolio is assumed to contain a riskless asset, which is a bond that costs one dollar in τ, and pays out R dollars in all states in $\tau+1$, i.e.:

$$1=E_t\left[\Delta_{t,t+1}R_t\right]\ \Rightarrow E_t\left[\Delta_{t,t+1}\right]=\frac{1}{R_t} \tag{17}$$

The remainder of this section discusses the first order conditions of the households, bar the wage decision, which is discussed later in the section. The first order conditions for C_t^o and B_{t+1}^o are:

$$\begin{aligned}
\left(C_t^o\right)^{-\sigma} &= \lambda_t P_t \ \ \Rightarrow \lambda_t=\frac{\left(C_t^o\right)^{-\sigma}}{P_t} && (18)\\
\lambda_t E_t\left(\Delta_{t,t+1}\right) &= \beta E_t\lambda_{t+1}\ \Rightarrow E_t\left(\Delta_{t,t+1}\right)\equiv\frac{1}{R_t}=E_t\left(\beta\frac{\lambda_{t+1}}{\lambda_t}\right) && (19)
\end{aligned}$$

The variable λ_t is the lagrange multiplier pertaining to the period budget constraint given by (15). Combining the two yields:

$$1=R_tE_t\left[\Lambda_{t,t+1}\right] \tag{20}$$

[5] Following Cochrane (2001, Chp 3), the price of a portfolio B of contingent claims, $P(B)=\sum_\xi \varsigma(\xi)B(\xi)$, where ξ denotes the states of the world, $\varsigma(\xi)$ is the price of an asset which pays out one dollar in state ξ in $\tau+1$, and $B(\xi)$ is the number of such assets in the portfolio. Under the assumption that all optimising households have access to the complete set of contingent claims, the payoffs in the portfolio can be written in a state-price density form: $P(B)=E[\Delta(\xi)B(\xi)]$,where Δ is the stochastic discount factor.

where $\Lambda_{t,t+k} \equiv \beta^k \left(\frac{C^o_{t+k}}{C^o_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+k}}\right)$. The equation characterizes the consumption-savings decision made by the optimizing households. They have access to capital markets and assets which provides them additional avenues to smooth fluctuations in their income over time, as compared to the rule of thumb households.

Investment The first order conditions for I^o_t and K^o_{t+1} are:

$$-P_t\lambda_t + \zeta_t\psi^{'}_t(.) = 0 \;\Rightarrow P_t\lambda_t = \zeta_t\psi^{'}\left(\frac{I^o_t}{K^o_t}\right) \tag{21}$$

$$\begin{aligned}\zeta_t = \beta E_t \Bigg\{ & \left[R^k_{t+1}u_{t+1} - a\left(u_{t+1}\right)\right]\lambda_{t+1} \\ & + \zeta_{t+1}\left[(1-\delta) + \psi_{t+1}(.) - \psi^{'}_{t+1}(.)\left(\frac{I^o_{t+1}}{K^o_{t+1}}\right)\right]\Bigg\}\end{aligned} \tag{22}$$

where the variable ζ_t is the lagrange multiplier pertaining to the capital accumulation equation, equation (16). The Euler equation (21), for investment, equates the marginal cost of a unit of investment goods, $P_t\lambda_t$ (in the sense of additional utility lost from lowered consumption), to the marginal benefit from investing in an extra unit of capital, i.e. $\zeta_t\psi'(.)$. Similarly, the Euler equation for capital, (22) equates the marginal cost of spending an extra unit on K^o_{t+1}, i.e. ζ_t, to the return from installing an extra unit of capital in terms of the consumption good - the right hand side of (22). Combining these two equations yields:

$$\begin{aligned}P_tQ_t = E_t \Bigg\{ & \Lambda_{t,t+1}\Bigg[\left(R^k_{t+1}u_{t+1} - a\left(u_{t+1}\right)\right) \\ & + P_{t+1}Q_{t+1}\left[(1-\delta) + \psi_{t+1}(.) - \psi^{'}_{t+1}(.)\left(\frac{I^o_{t+1}}{K^o_{t+1}}\right)\right]\Bigg]\Bigg\}\end{aligned} \tag{23}$$

where $Q_t \equiv \frac{1}{\psi^{'}_t(.)}$ is Tobin's Q.

Capital Utilization In the model, only the optimizing households have access to capital markets and face a capital accumulation decision. Hence, all the available stock of physical capital is owned by the optimizing households. Capital services, K_t are related to this physical stock of capital by:

$$K_t = (1-\nu)\, u_t K_t^o \tag{24}$$

The first order condition for the optimizing household's capital utilization decision is:

$$\lambda_t \left(R_t^k - P_t a'(u_t) \right) = 0 \;\Rightarrow R_t^k = P_t a'(u_t) \tag{25}$$

This equations states that the (nominal) marginal benefit from increasing the utilization rate, R_t^k must equal the (nominal) marginal cost from doing so, $P_t a'(u_t)$, at the optimum. It is easy to see from here that changes in the utilization rate, u_t, affect the real rental rate of capital, r_t^k and hence real marginal costs in equation (6).

2.2.2 Rule of Thumb Households

Denoting rule of thumb households by the superscript 'r' to represent $i \in (0, v)$, the measure v of rule of thumb households do not have (or are simply unaware of their) access to asset markets.[6] As a result, they are unable to smooth their consumption patterns over time when faced with fluctuations in their labour income. Following the setup in Gali, López-Salido & Vallés (2004), these rule of thumb households simply solve a static problem from period to period, i.e.:

$$\max U(C_t^r, L_t^r) = \max \frac{(C_t^r)^{1-\sigma}}{1-\sigma} - \kappa \frac{(L_t)^{1+\chi}}{1+\chi} \tag{26}$$

where C_t^r and L_t^r represent consumption and differentiated labour services for the rule of thumb households. Since they are assumed to be unable to access capital markets, their only source of income is their labour income. Each period, they simply consume C_t^r, which equals their labour income in that period. Rule of thumb households are also assumed to belong to a union which posts

[6] As mentioned before, there are a variety of reasons why agents might undertake consumption expenditures based upon their current labour income. The interpretation that corresponds most closely to the formal setup of this chapter is that households have heterogenous discount factors, i.e. $\{\beta : \beta_i = 0,\ \forall\ i \in (0, v)\ \text{ and } \beta_i = \beta > 0,\ \forall\ i \in (v, 1)\}$.

a wage, W_t^r, to intermediate firms for their labour services and this is outlined next within this section. Hence the problem for the rule of thumb households boils down to choosing consumption to satisfy their period budget constraint:

$$P_t C_t^r = W_t^r L_t \tag{27}$$

The wage posting decision of both the rule of thumb households and the optimizing households follows next.

2.2.3 The Wage Decision

Labour Aggregator Following Erceg, Henderson and Levin (2000), all the optimizing households are assumed to be monopolistic suppliers of differentiated labour services. The rule of thumb households are assumed to be members of a union, who sets the wage on their behalf, taking into account the firm's labour demand schedule. Furthermore, households of both types are assumed to sell their labour services to a representative competitive firm. This competitive firm combines the labour services from both optimizing households, L_t^o, and myopic households, L_t^r and then transforms it into an aggregate composite labour input, N_t using the following technology:

$$\begin{aligned} N_t &= \left[\int_0^{\nu} L_{i,t}^{\frac{\phi-1}{\phi}}\, di + \int_{\nu}^{1} L_{i,t}^{\frac{\phi-1}{\phi}}\, di\right]^{\frac{\phi}{\phi-1}} \\ &= \left[\int_0^{1} L_{i,t}^{\frac{\phi-1}{\phi}}\, di\right]^{\frac{\phi}{\phi-1}} \end{aligned} \tag{28}$$

where $\phi > 1$, and $L_{i,t}$ are the individual amounts of labour services, where the superscripts 'r' and 'o' refer to $i \in (0, v)$ and $i \in (v, 1)$ respectively. The perfectly competitive firm which aggregates labour services faces an analogous problem to the final goods firm. The demand curve for household's labour services are given by:

$$L_{i,t}^d = \left(\frac{W_{it}}{W_t}\right)^{-\phi} N_t, \quad i \in (0,1) \tag{29}$$

where W_t is the aggregate wage rate. Under the assumption that this perfectly competitive firm takes both the price of its output (i.e. the composite labour) and individual household wages, $W_{i,t}$ as given, the aggregate wage rate can be written as:

$$W_t = \left[\int_0^1 W_{i,t}^{1-\phi} di \right]^{\frac{1}{1-\phi}} \tag{30}$$

All the households takes both N_t and W_t as given. The wage setting behaviour of both the optimizing households and rule of thumb households are outlined next.

Wage Setting The wage setting behaviour of the optimizing households differ to that of the rule of thumb households. Optimizing households set their wage similarly to the mechanism by which the intermediate goods firms set prices. In each period, an optimizing household faces a constant probability $1-\xi_w$ of being able to reoptimise its nominal wage, and this is independent across time and households. Thus in any given period, a fraction $(1-\xi_w)$ of optimizing households are able to reoptimise its wage, whilst a fraction ξ_w are unable to.

Optimizing Households:

As before in the case of the intermediate firms, consider first the measure of optimizing households that are unable to reoptimise their wages. Their nominal wage is assumed to be updated period to period according to a simple rule, where the old nominal wage is indexed by steady state inflation:

$$W_{i,t} = \Omega W_{i,t-1} \tag{31}$$

The problem for an optimizing household is outlined in section (2.2.1). Focusing on the wage decision, an optimizing household with the ability to reoptimise its nominal wage, picks $\tilde{W}_t^o$ to maximize (14) subject to the firm's labour demand schedule (29), and equations (15) and (16). This yields the following FOC:

$$E_t \sum_{\tau=t}^{\infty} (\xi_w \beta)^{\tau-t} \left\{ \left[-\kappa(-\phi) L_\tau^\chi \left(\frac{\Omega^{\tau-t}\tilde{W}_t^o}{W_\tau} \right)^{-\phi} \frac{N_\tau}{\tilde{W}_t^o} \right] + \lambda_\tau \left[(1-\phi)\left(\Omega^{\tau-t}\right) \left(\frac{\Omega^{\tau-t}\tilde{W}_t^o}{W_\tau} \right)^{-\phi} N_\tau \right] \right\} = 0$$

Rearranging this expression for $\tilde{W}_t^o$ yields the following wage setting equation for optimizing households:

$$\begin{aligned} \left(\tilde{W}_t^o\right)^{1+\chi\phi} &= \kappa\mu_w \frac{E_t \sum_{\tau=t}^{\infty} (\xi_w \beta)^{\tau-t} \left[\left(\frac{\Omega^{\tau-t}}{W_\tau} \right)^{-\phi(1+\chi)} N_\tau^{1+\chi} \right]}{E_t \sum_{\tau=t}^{\infty} (\xi_w \beta)^{\tau-t} \left\{ \lambda_\tau \left[\left(\Omega^{\tau-t}\right)^{1-\phi} W_\tau^\phi N_\tau \right] \right\}} \\ &= \kappa\mu_w \frac{WB_t}{WA_t} \qquad (32) \end{aligned}$$

where

$$\begin{aligned} WB_t &\equiv E_t \sum_{\tau=t}^{\infty} (\xi_w \beta)^{\tau-t} \left[\left(\frac{\Omega^{\tau-t}}{W_\tau} \right)^{-\phi(1+\chi)} N_\tau^{1+\chi} \right] \\ &= \left(N_t W_t^\phi\right)^{1+\chi} + \xi_w \beta \Omega^{-\phi(1+\chi)} E_t WB_{t+1} \qquad (33) \\ WA_t &\equiv E_t \sum_{\tau=t}^{\infty} (\xi_w \beta)^{\tau-t} \left\{ \lambda_\tau \left[\left(\Omega^{\tau-t}\right)^{1-\phi} W_\tau^\phi N_\tau \right] \right\} \\ &= \lambda_t N_t W_t^\phi + \xi_w \beta \Omega^{1-\phi} E_t WA_{t+1} \qquad (34) \\ \mu_w &\equiv \frac{\phi}{\phi-1} \end{aligned}$$

This equation depicts the wage setting behaviour of the optimizing households. When wages are fully flexible, i.e. $\xi_w = 0$, equation (32) reduces down to the usual labour-leisure tradeoff faced by households. In this case, the households sets its real wage equal to a constant markup over the expected marginal rate of substitution between consumption and leisure. When some degree of wage rigidity exists, i.e. $\xi_w > 0$, household's analogously set the real wage based upon their expected weighted discounted stream of their marginal rate of substitution between consumption and leisure over time.

The Union's Problem:

The wage setting process for rule of thumb households is slightly different. Rule of thumb households are assumed to belong to a union which picks a wage to maximize the utility of their members. The union is assumed to have similar preferences to the myopic household it represents and behave in a static fashion. Since all the rule of thumb households are identical from the point of view of the union, the union maximizes the utility for a representative rule of thumb household by posting a wage every period. In this benchmark version of the model, I assume that the union sets wages one period in advance and that they are unable to observe any shocks that hit the economy in the current period. Later, in section (4), they are assumed to be able to observe the current shocks when posting the wage.

The union's problem is to try to maximize period utility subject to the budget constraint of the rule of thumb households and the firm's labour demand schedule, using lagged information. Let $\tilde{W}_t^r$ be the wage posted by the union. They solve the problem posed in section (2.2.2), i.e. substituting (27) and (29) into (26):

$$\max U\left(\tilde{W}_t^r\right) = \max E_{t-1}\left\{\left(\frac{1}{1-\sigma}\right)\left[\left(\frac{\tilde{W}_t^r}{P_t}\right)\left(\frac{\tilde{W}_t^r}{W_t}\right)^{-\phi} N_t\right]^{1-\sigma} - \frac{\kappa}{1+\chi}\left[\left(\frac{\tilde{W}_t^r}{W_t}\right)^{-\phi} N_t\right]^{1+\chi}\right\}$$

Differentiating with respect to $\tilde{W}_t^r$ and rearranging the FOC yields the following wage setting equation for rule of thumb households:

$$\left(\tilde{W}_t^r\right)^{1+\phi\chi+\sigma(\phi-1)} = \mu_w \kappa \frac{E_{t-1}\left[W_t^{\phi(1+\chi)} N_t^{1+\chi}\right]}{E_{t-1}\left[W_t^{\phi(1-\sigma)} P_t^{\sigma-1} N_t^{1-\sigma}\right]} \tag{35}$$

In this benchmark (- henceforth referred to as PSW) case, the union posts a wage $\tilde{W}_t^r$ to the firm based upon its expectations aggregate wages, employment and prices. The equation that governs the dynamics of aggregate wages is determined by aggregating (29) across households and imposing (28). It is given by:

$$
\begin{aligned}
W_t &= \left(\int_0^1 W_{i,t}^{1-\phi} di\right)^{\frac{1}{1-\phi}} = \left(v\left(\tilde{W}_t^r\right)^{1-\phi} + (1-v)\left(W_t^*\right)^{1-\phi}\right)^{\frac{1}{1-\phi}} && (36)\\
\text{where } \left(W_t^*\right)^{1-\phi} &= (1-\xi_w)\left(\tilde{W}_t^o\right)^{1-\phi} + \xi_w \Omega^{1-\phi}\left(W_{t-1}^*\right)^{1-\phi} && (37)
\end{aligned}
$$

and where $W_t^* \equiv \left(\frac{1}{1-v}\int_v^1 \left(W_{i,t}^o\right)^{1-\phi} di\right)^{\frac{1}{1-\phi}}$

2.2.4 Aggregation

Aggregation of the key variables across the two types of households can be achieved as follows. Aggregate consumption and labour services are simply the weighted average of the two household types:

$$
\begin{aligned}
C_t &= vC_t^r + (1-v)\,C_t^o && (38)\\
L_t &= vL_t^r + \int_v^1 L_{i,t}^o di
\end{aligned}
$$

Using equation (29) and defining $(1-v)\left(WC_t^*\right)^{-\phi} \equiv \frac{1}{1-v}\int_v^1 \left(W_{i,t}^o\right)^{-\phi} di$, L_t can be expressed as:

$$
\begin{aligned}
L_t &= vL_t^r + N_t W_t^{\phi}\int_v^1 \left(W_{i,t}^o\right)^{-\phi} di\\
&= vL_t^r + (1-v)\,N_t W_t^{\phi}\left(WC_t^*\right)^{-\phi} && (39)\\
\text{where } \left(WC_t^*\right)^{-\phi} &= (1-\xi_w)\left(\tilde{W}_t^o\right)^{-\phi} + \xi_w\Omega^{-\phi}\left(WC_{t-1}^*\right)^{-\phi}
\end{aligned}
$$

For aggregate investment, I_t and aggregate capital stock, only optimizing households have access to capital markets and only they contribute towards the aggregate. Hence the aggregate capital stock in given by equation (24), i.e. $K_t \equiv (1-v)\,u_t K_t^o$, and aggregate investment is similarly defined: $I_t = (1-v)\,I_t^o$.

2.3 Monetary Policy

The monetary authority targets interest rates in setting monetary policy. They do this by setting the nominal interest rate every period according to a variant of the Taylor rule. I assume that the monetary authority is a strict inflation targeter and does not care about the output gap:

$$r_t = (1 - \gamma_r)\,\bar{r} + \gamma_r r_{t-1} + (1 - \gamma_r)\,\gamma_\pi \pi_t + \varepsilon_t \tag{40}$$

In the equation above, $\bar{r}$ represents the steady state real interest rate and $\varepsilon_t \sim iid\ N(0, \sigma_\varepsilon^2)$ is an interest rate, or monetary shock.[7] Hence, equation (40) states that the monetary authority sets interest rates based upon what interest rates were set at the previous period and the level of inflation today. Interest rates also move due to monetary shocks. Section 3.3.2 presents impulse response functions of key variables arising from these monetary shocks.

2.4 Market Clearing

The final element of the model involves the market clearing conditions in the goods market and factor markets. Market clearing in the factor markets implies that the following conditions hold, for all t:

$$\begin{aligned} N_t &= \int_0^1 N_{j,t} dj \\ K_t &= \int_0^1 K_{j,t} dj = (1 - v) u_t K_t^o \\ Y_{j,t} &= Y_{j,t}^d \ \text{ for all } j \in [0, 1] \end{aligned}$$

The first and second conditions respectively state that the total supply of composite labour (from the labour aggregating firm) equals the total labour demanded at each firm and that the total supply of capital equals the total amount of capital services demanded at each firm. The third equation states that the supply of intermediate good j is equal to the demand for intermediate good j, for all $j \in [0, 1]$, at the final goods firm. The last equation says that the total supply of

[7] Similar results are obtained to those within the paper if there is no interest rate smoothing incorporated within the Taylor rule.

labour services offered by households must equal the amount of labour employed by the labour aggregating firm.

The final market clearing conditions are the labour market clearing condition, given by equation (28) and the goods market condition:

$$Y_t = C_t + I_t + (1 - v)a\left(u_t\right) K_t^o \tag{41}$$

The equation above shows that aggregate output, Y_t is allocated between consumption, investment and resources that are put towards capital utilization.[8]

2.5 Functional Forms

The following functional forms are assumed for the adjustment costs to investment and capital utilization. For investment, the function $\psi\left(.\right)$ is given by:

$$\psi\left(\frac{I_t^o}{K_t^o}\right) = \frac{I_t^o}{K_t^o} - \frac{1}{2}h\left(\frac{I_t^o}{K_t^o} - \delta\right)^2$$

where h is a constant. At the non-stochastic steady state, optimizing households only undertake investment to replace depreciation of the stock of physical capital and it is easy to verify that ψ satisfies the properties outlined previously.

The function capturing the costs to capital utilization, $a\left(u_t\right)$, must satisfy two restrictions. First, u_t is required to be 1 in the steady state, and this value is pinned down by equation (25). Second, it is assumed that $a\left(1\right) = 0$. The particular functional form is given by:

$$a\left(u_t\right) = \frac{1}{2}\eta u_t\left(u_t - 1\right)$$

where $\eta > 0$, is a constant set to satisfy $u = 1$ in the steady state equilibrium.

[8] It should be noted that the presence of price and wage distributions means that exact aggregation is not possible. This is because total output involves price and wage dispersion terms that enter in Y_t due to the Calvo price and wage setting assumptions. However, as shown by Erceg, Henderson and Levin (2000), Yun (1996) and Christiano, Eichenbaum and Evans (2001), these dispersion terms do not appear in a linear approximation of the resource constraint about the steady state.

3 Model Simulation and Results

The objective of this paper is to examine the transmission mechanism of monetary policy within a NNS model that incorporates heterogeneous households with a view towards reconciling the movements of interest rates, consumption and inflation. The benchmark version of the model incorporates both price and wage inertia. This section of the paper discusses the calibration of parameters used in the model and evaluates the model. Correlations are reported for different assumptions of price and wage stickiness, although I focus on the sticky price and sticky wage (SPSW) case. Correlations and descriptive statistics are reported within tables (2) - (4), whilst figures (1) - (17) depict the effects of monetary policy and productivity shocks. However, the calibration methodology is outlined first and this follows next.

3.1 Calibration of Parameters

The model is calibrated at a quarterly frequency. Table (1) summarizes the values used for the parameters. The benchmark calibration incorporates no rule of thumb households. This is done so that parameters may be chosen to replicate some stylized facts, although no serious attempt is made to calibrate the model to any particular economy. The parameters are chosen such that in the steady state, under this benchmark case with no rule of thumb households, the investment to output ratio is approximately 15 percent. The consumption to output ratio, is approximately 85 percent.[9] In addition the ratio of capital to quarterly output is set to be approximately 6.

Considering the parameters associated with the household's problem first, the discount factor β is set equal to 0.99, which implies a steady state real annual return of approximately 4 percent. The intertemporal elasticity of substitution over consumption expenditures, σ, is set equal to 2. The parameter in the utility function corresponding to labour services, χ, is set equal to unity, implying a Frisch (or constant λ_t) elasticity of labour supply equal to 1. κ, the weight that households assign to the disutility arising from labour relative to consumption, is set equal to one. In addition, the share of rule of thumb households, v, is set to be 0.25. This value is greater than 0.19, found by

[9] Since the model abstracts from the fiscal authority and the government sector, one interpretation of consumption expenditures here, is that it is the sum of private consumption and government consumption expenditures.

Jappelli (1990) for credit constrained individuals, but less than the value of 0.5 used by Campbell and Mankiw (1989). As mentioned previously, rule of thumb households consist of both liquidity constrained individuals and others who are either simply unaware of the opportunities to smooth consumption intertemporally, or unwilling to do so. Hence I utilize a value of 0.25 to include both these different types of myopic agents.

For the firm side, the share of capital is set equal to 0.25, which is a little less than what is typically used in the literature, but due to the presence of monopoly rents makes the labour share of income approximately equal to two thirds. The depreciation rate of capital is set to 0.025, which is widely used in the literature and implies an annual rate of depreciation on capital approximately equal to 10 percent. The inertia in the log productivity process is set as in Canzoneri, Cumby and Diba (2004a) to 0.923, who estimate it for the US between 1960:1 and 2003:2. The parameter ψ, representing adjustment costs to investment is set to 4. The parameter η, representing the sensitivity of the costs of capital utilization to changes in the utilization rate, is calibrated to ensure that $a'(1) = r^k$.

The elasticity of substitution across goods, θ, is set to 7 implying a markup, μ_p, of about 17 percent, which is greater than the 15 percent markup used by Rotemberg and Woodford (1997). However this value falls within the range estimated by Bayoumi, Laxton and Pesenti (2004), who find it ranges from 11 percent to 23 percent across sectors. The elasticity of substitution across workers, ϕ, is similarly set to 7. The fraction of firms unable to reoptimise their prices in any given period, ξ_p, is set at 0.6 in the sticky price case. This implies that firms are able to reoptimise their prices on average in under three quarters. This value lies between the value of 0.67 set by Rotemberg and Woodford (1997) in earlier work, and the value of 0.5 set by Begnino and Woodford (2003) more recently. The fraction of optimizing households unable to reoptimise their wages in any given period, ξ_w, is set at 0.85, which is higher than the value of 0.75 typically used in the literature (see Taylor, 1999).

With regards to the coefficients in the monetary policy rule, the weight on the lagged interest rate (γ_r) and inflation (γ_π) are set to be 0.824 and 2.5 respectively. The weight on inflation is set a little higher than what is typically found in the literature, due to recent results by Gali, López-Salido and Vallés (2004). They find that the Taylor Principle is insufficient to guarantee the uniqueness of the equilibrium. Setting a large weight on inflation allows us to abstract from the issue of ensuring uniqueness of the equilibrium. Finally, the steady state gross inflation, Ω, is set to 1.

3.2 Results

The model is solved numerically using Dynare (see Juillard, 2003) by taking first order Taylor approximations to the relevant model equations near a deterministic steady state. Two normalizations are made to the nominal model outlined above due to convergence issues in finding a steady state. First, the nominal model is converted into a “real” version by normalizing nominal variables with respect to aggregate prices. Second, the wage setting equations, (31) - (37), are also normalized by aggregate wages so that the variables are relative to aggregate wages. Details of the normalizations can be found in the technical appendix (Appendix A) and the particular equations used to solve the model can be found in Appendix A.7.

As mentioned before, the benchmark case presented within the paper incorporates both sticky prices and sticky wages. The shocks faced by rule of thumb households here are unexpected. The fraction of myopic households, v, is set to 0.25. I focus on interest rates and consumption since the transmission mechanism of monetary policy (outlined earlier) works primarily through the consumption Euler equation within NNS models. Table (2) reports the correlations between the consumption growth rates of aggregate consumption, of optimizing households and of rule of thumb households, with nominal and real interest rates. There are two main results which are outlined below.

First, consider the correlations in Table (2) for the PSW case. The correlations between interest rates and expected consumption growths are ordered as follows. The correlation for myopic households is less than the correlation with expected aggregate consumption growth, which in turn is less than the correlation for optimizing households. This holds true for both nominal and real rates. Furthermore, in the case of real rates, the correlation with the expected consumption growth rate of optimizing households is 1, whilst the correlation with expected aggregate consumption growth is nearly half that at 0.628. The correlation of the real interest rate with the expected consumption growth rate for myopic households is 0.525

In order to gain some intuition behind the correlations, it is useful to examine the impulse responses of interest rates and expected consumption growth rates from monetary policy and productivity shocks. These are plotted in figures (1) and (2). Consider the results of an expansionary monetary policy shock depicted in figure (1). There it is possible to see that the response of nominal and real interest rates is negative to an expansionary monetary policy shock, whilst the response of inflation is positive. Consumption for both types of households increase, with myopic households increasing consumption much more than optimizing households in the initial period.

The intution behind this result is as follows. An expansionary monetary policy shock that lowers nominal interest rates, increases prices only a little, due to the presense of price inertia. Myopic households are unable to reoptimise their wages since wages are set one period in advance for them. As a result, real wages for myopic households fall by a greater amount when compared to the fall in real wages for optimizing households, some of whom are able to reoptimize their wages. However, the magnitude of the fall in real wages for either group of households is small, due to the presence of price inertia. The fall in real wages leads firms to hire more workers and employment (for both type of workers) and output increases. For both types of households, the increase in employment offsets the decline in real wages, and hence their labour income increases.

Thus the observed difference in the consumption responses for the two types of households can be easily rationalized. Myopic households simply consume the additional labour income. The relative difference between the observed consumption responses of the two types of households, simply arises because optimizing households are able to smooth consumption intertemporally and hence allocate any increase in labour income between consumption and savings. Finally, the expected consumption growth rates of both types of households fall with regards to a expansionary monetary policy shock. Hence the results from a monetary policy shock imply that the correlations between interest rates and expected consumption growths should all be positive.

Figure (2) shows the results of a (positive) productivity shock. A productivity shock leads to a negative response for inflation. The central bank lowers nominal interest rates to stabilize prices, but this raises real interest rates. With regards to the expected growth rates of consumption, the initial response is positive for optimizing households, whilst it is negative for myopic households. The response of aggregate consumption growth is also negative as it appears to track the response of myopic households. The intuition in the case of a productivity shock can be seen as follows.

A positive productivity shock raises the marginal product of labour, increasing real wages a little and employment and output a lot more. Real wages increase only a little due to the presence of wage and price inertia, but they do increase for both optimizing and myopic households, since both types of workers are perfect substitutes. They initially increase more for optimizing households compared to myopic households, since a fraction of optimizing households are able to reoptimize their wages, whilst the nominal wage is set in advance for myopic households. As a result, the income effect of the increase in real wages means that both types of households consume more. The effects on consumption and employment for the rule of thumb households are both greater than that of optimizing households for the same reason as in the case of a monetary policy shock. Optimizing households trade off consumption and leisure and smooth consumption by allocating the increase in labour earnings between consumption and savings. Myopic households view the productivity shock as unexpected, work more and simply consume the additional labour income it generates.

The results here under a productivity shock have different implications for the correlations than in the case for a monetary policy shock. Using real interest rates as an example, the results imply that the correlations between the real interest rate with expected aggregate consumption growth, and with the expected consumption growth of myopic households, should be negative. By contrast, productivity shocks lead to a positive correlation between interest rates and the consumption growth rate of optimizing households.

Table (3) allows the results from the impulse response functions to be reconciled with the correlations in Table (2).[10] The correlations at the bottom of Table (3) correspond with the impulse response functions in figures (1) and (2). As can be seen, a monetary policy shock yields a high correlation between the consumption growth rates and the real interest rate. However, when only the productivity shock is present, the correlation between the real interest rate and the expected consumption growth rates is actually negative for aggregate consumption growth and myopic households. Only the expected consumption growth rate of optimizing households yields a perfect positive correlation. Examining the variance decomposition shows that the real interest rate, the expected aggregate consumption growth rate and the expected consumption growth rate for myopic households responds primarily to a monetary shock. The expected consumption growth rate for optimizing households responds more evenly between the two types of shocks. Hence, the overall correlation between the real interest rate and the respective consumption growth rates in Table (2) can be rationalized.

To summarize the results in this section, a low correlation is found in the SPSW case between aggregate consumption growth and real interest rates when unions set wages one period in advance and the effects of shocks are unexpected. The correlation with respect to the expected consumption growth rate for myopic households is lower than that of the aggregate consumption growth, whilst the correlation for optimizing households is correspondingly higher. Myopic households respond to shocks by changing consumption to a greater extent than optimizing households. The following section considers an alternative to the benchmark scenario examined within this section, by assuming that the union is able to observe any shocks that hit the economy, prior to setting wages.

4 Contemporaneous Wage Setting

This section removes the structural wage inertia present within the model by examining the scenario where the union is able to observe any shocks, prior to picking a wage, $\tilde{W}_t^r$, to post to firms. Denoting this case as the contemporaneous wage setting (- henceforth CWS) case, the problem they face is analogous to the previous case where they were unable to observe any shocks when

[10] Note: All the variables are in logs. With the exception of the growth rates, the remaining variables are HP-Filtered, using a smoothing parameter of 1600, consistent with what is used in the literature for quarterly data.

picking $\tilde{W}_t^r$. However, now they pick the wage using contemporaneous information. Hence they maximize:

$$\max U\left(\tilde{W}_t^r\right) = \max\left\{\left(\frac{1}{1-\sigma}\right)\left[\left(\frac{\tilde{W}_t^r}{P_t}\right)\left(\frac{\tilde{W}_t^r}{W_t}\right)^{-\phi} N_t\right]^{1-\sigma} - \frac{\kappa}{1+\chi}\left[\left(\frac{\tilde{W}_t^r}{W_t}\right)^{-\phi} N_t\right]^{1+\chi}\right\} \tag{42}$$

Differentiating with respect to $\tilde{W}_t^r$ and rearranging the FOC yields the following wage setting equation for rule of thumb households:

$$\left(\tilde{W}_t^r\right)^{1+\phi\chi+\sigma(\phi-1)} = \mu_w \kappa P_t^{1-\sigma} W_t^{\phi(\chi+\sigma)} N_t^{\chi+\sigma} \tag{43}$$

The interpretation here for rule of thumb households is equivalent to the a flexible wage setting case for optimizing households. In essence these rule of thumb households behave as if they are flexible wage setters, thus reducing the total degree of wage inertia present within the economy. The remaining equations within the model remain the same, since the assumption that unions set wages in the current period means that only their wage setting behaviour has changed.

The results for this version of the model are depicted under the contemporaneous wage setting column in Table (2) and (4) and in figures (3) and (4). Considering the correlations presented in Table (2) first, they provide a similar picture as in the PSW case. The correlations between interest rates and expected consumption growth rates have the same ordering. That is, the correlation for myopic households is less than the correlation with expected aggregate consumption growth, which again is less than the correlation for optimizing households. However, unlike the benchmark case, when examining real interest rates, the correlation between the ex-ante real rate and expected aggregate consumption growth is much closer to one. It appears that even when a quarter of the population behaves myopically, optimizing households have a greater impact on the consumption profile for aggregate consumption.

The impulse response functions in figures (3)-(4) provide some insight as to why this is the case. Figure (3) depicts the responses of interest rates, consumption levels and their growth rates arising from an expansionary monetary policy shock. The responses are very similar to the benchmark case. That is the response of both the nominal and real interest rate is negative to an expansionary monetary policy shock, whilst the response of inflation is positive. Similarly, consumption increases for both types of households, whilst the growth rate of consumption is also negative. However, although the hump-shaped response of consumption is more pronounced under the CWS case compared to the PSW case, the overall increase in consumption for rule of thumb households is less than in the PSW case. Rule of thumb households increase their consumption in the CWS case less than optimizing households do.

When examining the impulse response from a productivity shock in figure (4), the results are mostly similar to the PSW case, with two exceptions. First, although the responses of interest rates and consumption profiles for both types of households are similar to the PSW case, the overall change in the consumption of myopic households is once again less than the response of optimizing households. As with the results of the monetary policy shock under CWS, this is despite a much more pronounced hump-shaped response for consumption. Under the CWS case, the consumption response for optimizing households dominate in determining the profile for aggregate consumption. The results imply a positive correlation between all three consumption growth rates and interest rates under a monetary policy shock that is expansionary.

Second, the initial response of aggregate consumption growth and the consumption growth of myopic households are actually positive under CWS, as compared to the PSW case, where they were initially negative. In this CWS case, the results here imply a *positive* correlation between consumption growth rates and interest rates. The correlations that arise only under a monetary policy shock, or only under a productivity shock, are presented at the bottom of Table (4). Focusing once again on the real interest rate, the correlation of all the consumption growth rates are close to one under a monetary policy shock. Under a productivity shock, the consumption growth rate of myopic households is low, and it appears to have negligible impact on the aggregate consumption growth rate.

The results under the CWS case can be rationalized with the results under the PSW case as follows. Consider the aggregate consumption equation, given by (38):

$$C_t = vC_t^r + (1-v)\,C_t^o$$

Forwarding this equation once, and dividing by C_t yields:

$$\begin{aligned}\frac{C_{t+1}}{C_t} &= v\left(\frac{C_t^r}{C_t}\right)\left(\frac{C_{t+1}^r}{C_t^r}\right) + (1-v)\left(\frac{C_t^o}{C_t}\right)\left(\frac{C_{t+1}^o}{C_t^o}\right) \\ &\Rightarrow\ E_t\left(\frac{C_{t+1}}{C_t}\right) = Z_{1,t}E_t\left(\frac{C_{t+1}^r}{C_t^r}\right) + Z_{2,t}E_t\left(\frac{C_{t+1}^o}{C_t^o}\right) \qquad (44)\end{aligned}$$

where $Z_{1,t} = v\left(\frac{C_t^r}{C_t}\right)$ and $Z_{2,t} = (1-v)\left(\frac{C_t^o}{C_t}\right)$ are the respective weights on the expected consumption growth rates of myopic and optimizing households. Equation (44) links the expected aggregate consumption growth to the expected consumption growth rates of myopic households and to the expected consumption growth rate of optimizing households. It is easy to see from the impulse response functions in figures (1) to (4) and equation (44) why expected aggregate consumption growth responds the way it does to shocks, under the two versions of the model.

In the PSW case, although $v = 0.25$, the impact of myopic households on expected aggregate consumption growth, $Z_{1,t}$, is much larger than its counterpart for optimizing households, $Z_{2,t}$, under both types of shocks. Figures (1) and (2) show that under the expansionary monetary policy shock, the increase in consumption for myopic households is approximately five times the increase for optimizing households, whilst under a productivity shock myopic households increase consumption nearly three times as much as optimizing households. Hence the response of the expected consumption growth of myopic households dominates the response of the expected consumption growth of optimizing households in determining the response for aggregate consumption growth. However, under the CWS case, figures (3) and (4) show that the optimizing households increase their consumption approximately two and a half times (averaged between the two types of shocks) as much, compared to myopic households. Hence, it is clear to see that impact of optimizing households on expected aggregate consumption growth, $Z_{2,t}$, clearly dominates in equation (44).

The magnitude of the change in the levels of consumption also shed some light on the overall correlations. The large overall correlation in the sticky price and sticky wage case in Table (2) can clearly be attributed to the difference in the magnitude of the changes in consumption, arising from the shocks, between the two types of households. Under the CWS case, it appears that the growth rate of consumption of rule of thumb households has relatively little impact on aggregate consumption. The growth rate of consumption of optimizing households clearly dominates. The results within this section appear to suggest that merely introducing heterogeneous agents is insufficient to reconcile consumption, inflation and interest rates in terms of the transmission mechanism of monetary policy. Heterogeneity by itself cannot reconcile the low correlation observed between real interest rates and expected aggregate consumption growth. The key features are wage and informational inertia that help to reconcile this low correlation.

5 Importance of Wage Rigidity

This section attempts to gauge the sensitivity of the results to specific assumptions made within the model. More precisely, it examines the relative importance of nominal inertia on the correlations within the model. The relative importance of wage inertia compared to price inertia can be seen from the correlations in Table (2). Comparing the correlations under the alternative cases of price and wage rigidity highlights an interesting point. Cases (II) and (III) show the correlations between the interest rates and expected consumption growth rates when wages are flexible. Case (IV) shows the results in the case where wages are sticky, but prices are flexible.

The results appear to suggest that wage inertia is very important for obtaining a low correlation between interest rates and expected consumption growth for myopic agents. Both the CWS and PSW cases yield a low correlation between interest rates and expected consumption growth for myopic households, only when wage rigidity is present. When the only source of nominal rigidity is price rigidity (SPFW - as in case III), the correlations are very close to one. Moreover, price stickiness does not yield persistent responses to a monetary policy shock, and this can be seen in figure (16) which plots the response of output, investment, employment and consumption to monetary policy shocks under the benchmark and contemporaneous wage setting cases. This result is consistent with the findings in the monetary policy literature, e.g. Christiano, Eichenbaum and Evans (2001)..

In addition, a negative correlation is observed between interest rates and the expected consumption growth rate of myopic households, when wages are sticky and prices flexible under the contemporaneous wage setting case.[11] However, no case yields a negative correlation between interest rates and expected aggregate consumption growth, as was found by Canzoneri, Cumby and Diba (2002a) and Ahmad (2004). The result in the previous section provide an explanation for why this is true. When unions set wages contemporaneously, the negative correlation found for myopic households in the flexible price, sticky wage (FPSW) case does not translate through to the correlation for expected aggregate consumption growth, since the expected consumption growth for optimizing households dominates the response for the expected consumption growth for myopic households. It is for this reason that expected aggregate consumption growth mirrors the expected consumption growth of optimizing households in all four cases: FPFW, SPFW, FPSW and SPSW.

6 Conclusions

This paper examines the transmission mechanism of monetary policy within a NNS model that incorporates heterogeneous households. Standard NNS models without heterogeneous households, link the stance of monetary policy to the interest rate in the consumption Euler equation. They equate the interest rate in the Euler equation to the instrument of monetary policy. A monetary policy action then has an impact on real variables in the economy through its impact on household's consumption-savings decisions. This monetary transmission mechanism has the implication that real interest rates and expected consumption growth should be perfectly correlated. Recent results

[11] When the Taylor rule does not incorporate interest rate smoothing, wage inertia is sufficient to generate a negative correlation between the consumption growth rate of myopic households and both nominal and real interest rates under the full information case. However, those results are not presented here.

by Canzoneri, Cumby and Diba (2002a) and Ahmad (2004) find the correlation to be low, and often negative across the majority of the G7 countries. Moreover, the movements of interest rates, consumption and inflation implied by the monetary transmission mechanism cannot be reconciled with the hump-shaped response of aggregate consumption, documented in the literature.

The motivation for incorporating rule of thumb households within an NNS model is due to recent findings that the consumption Euler equation holds much better using micro level consumption data (see Attanasio and Weber, 1993, 1995). Other findings by Vissing-Jorgensen (2002) and Brav, Constantinides, and Geczy (2002) report similar results where they obtain plausible values for the parameter for intertemporal substitution by fitting the consumption Euler equation using households that carry assets. This paper focuses on the correlations between real interest rates and expected consumption growth rates to try and reconcile consumption, inflation and interest rates with the consumption Euler equation.

There are three main findings within the paper. First, I obtain a correlation of 0.628 between the real interest rate and expected aggregate consumption growth in the benchmark version of the model with nominal inertia. Second, the corresponding correlation between interest rates and expected consumption growth for myopic and optimizing households are lower and higher than this, respectively. Third, heterogeneity by itself is unable to reconcile movements in interest rates, consumption and inflation with the monetary transmission mechanism. The key features of the model that yields a lower correlation is wage inertia.

These three results can be rationalized with the following observations. When monetary policy shocks are unexpected in the presence of nominal inertia and preset wages, both aggregate consumption and its expected growth rate respond much more to the corresponding profiles for myopic households, leading to a low correlation between interest rates and expected aggregate consumption growth. However, when myopic households are able to observe monetary policy shocks prior to setting wages, aggregate consumption and its expected growth rate respond much more to the corresponding profiles of optimizing households, and the correlation between interest rates and expected aggregate consumption growth is close to one.

The results within the paper go some way towards reconciling the transmission mechanism of monetary policy in NNS models with observed empirical facts. Incorporating heterogeneity provides a greater understanding of how different households respond to shocks. The results here appear to suggest that heterogeneity can compliment the existing literature and may provide a new avenue to achieve a greater understanding of the transmission mechanism of monetary policy.

Tables

Parameter	Value	Description
α	0.25	- Share of capital
β	0.99	- discount factor
γ_r	0.824	- smoothing coefficient on lagged interest rate
γ_π	2.5	- inflation coefficient in monetary policy rule
δ	0.025	- depreciation rate
θ	7	- elasticity in goods aggregator
κ	1	- coefficient for disutility of work
ξ_p	{0,0.6}	- Calvo price setting parameter
		(fraction of sticky price firms)
ξ_w	{0,0.85}	- Calvo wage setting parameter
		(fraction of sticky wage optimizing households)
ρ	0.923	- inertia in productivity process
σ	2	- inverse elasticity of substitution
ϕ	7	- elasticity in labour aggregator
h	4	- investment adjustment cost parameter
χ	1	- inverse Frisch elasticity
υ	0.25	- share of myopic agents
η	0.0702	- parameter in capital utilization cost function
Ω	1	- (gross) steady state inflation

Table 1: Parameters Used In The Model

	PSW		CWS	
	Nominal Interest Rate	Real Interest Rate	Nominal Interest Rate	Real Interest Rate
Case I: Sticky Prices and Wages				
Expected Aggregate Consumption Growth	0.6175	0.6278	0.7679	0.9797
Expected Consumption Growth - Optimizers	0.8813	1	0.879	1
Expected Consumption Growth - Myopic	0.5329	0.525	0.2515	0.6759
Case II: Flexible Prices and Wages				
Expected Aggregate Consumption Growth	0.8563	0.9881	0.9976	0.9971
Expected Consumption Growth - Optimizers	0.9256	1	1	1
Expected Consumption Growth - Myopic	0.8515	0.9866	0.9892	0.9881
Case III: Sticky Prices and Flexible Wages				
Expected Aggregate Consumption Growth	0.9097	0.9693	0.8154	0.9925
Expected Consumption Growth - Optimizers	0.9501	1	0.8799	1
Expected Consumption Growth - Myopic	0.9056	0.9656	0.7932	0.9872
Case IV: Flexible Prices and Sticky Wages				
Expected Aggregate Consumption Growth	0.5977	0.6618	0.9965	0.9959
Expected Consumption Growth - Optimizers	0.9965	1	0.9999	1
Expected Consumption Growth - Myopic	0.5214	0.5901	-0.0459	-0.0538

Notes:
1. Share of myopic agents, v = 0.25
2. PSW - Benchmark Case (Union sets wages 1 period in advance)
3. CWS - Contemporaneous Wage Setting Case

Table 2: Correlations Between Expected Consumption Growth Rates And Nominal And Real Interest Rates

MOMENTS

VARIABLE	MEAN	STD. DEV.	VARIANCE
Nominal Interest Rate	0.0101	0.0025	0.0000
Inflation Rate	0	0.0026	0.0000
Ex ante Real Interest Rate	0.0101	0.0028	0.0000
Agg. Consumption	0.1517	0.0155	0.0002
Consumption - Optimizers	0.1868	0.0128	0.0002
Consumption - Myopic	0.038	0.0327	0.0011
Exp. Agg. Cons Growth	0	0.0079	0.0001
Exp. Cons Growth - Optimizers	0	0.0021	0.0000
Exp. Cons Growth - Myopic	0	0.0314	0.001

VARIANCE DECOMPOSITION (in percent)

	ε_z	ε_m
Nominal Interest Rate	47.46	52.54
Inflation Rate	61.08	38.92
Ex ante Real Interest Rate	21.49	78.51
Agg. Consumption	58.97	41.03
Consumption - Optimizers	74.6	25.4
Consumption - Myopic	33.64	66.36
Exp. Agg. Cons Growth	22.41	77.59
Exp. Cons Growth - Optimizers	43.36	56.64
Exp. Cons Growth - Myopic	26.94	73.06

CORRELATION

	Nominal Interest Rate	Real Interest Rate
Only Monetary Policy Shock Present		
Exp. Agg. Cons Growth	0.8104	0.8738
Exp. Cons Growth - Optimizers	0.9883	1
Exp. Cons Growth - Myopic	0.7607	0.8313
Only Productivity Shock Present		
Exp. Agg. Cons Growth	0.2997	-0.3065
Exp. Cons Growth - Optimizers	0.7721	1
Exp. Cons Growth - Myopic	0.1723	-0.4343

Table 3: Descriptive Statistics of Consumption Growth Rates Under The Benchmark Case

MOMENTS

VARIABLE	MEAN	STD. DEV.	VARIANCE
Nominal Interest Rate	0.0101	0.0025	0.0000
Inflation Rate	0	0.0027	0.0000
Ex ante Real Interest Rate	0.0101	0.0028	0.0000
Agg. Consumption	0.1517	0.0114	0.0001
Consumption - Optimizers	0.1868	0.0128	0.0002
Consumption - Myopic	0.038	0.0071	0.0001
Exp. Agg. Cons Growth	0	0.0019	0.0000
Exp. Cons Growth - Optimizers	0	0.0021	0.0000
Exp. Cons Growth - Myopic	0	0.0019	0.0000

VARIANCE DECOMPOSITION (in percent)

	ε_z	ε_m
Nominal Interest Rate	47.78	52.22
Inflation Rate	59.69	40.31
Ex ante Real Interest Rate	22.08	77.92
Agg. Consumption	77.53	22.47
Consumption - Optimizers	74.74	25.26
Consumption - Myopic	91.75	8.25
Exp. Agg. Cons Growth	41.9	58.1
Exp. Cons Growth - Optimizers	43.83	56.17
Exp. Cons Growth - Myopic	60.82	39.18

CORRELATION

	Nominal Interest Rate	Real Interest Rate
Only Monetary Policy Shock Present		
Exp. Agg. Cons Growth	0.9758	0.9977
Exp. Cons Growth - Optimizers	0.9882	1
Exp. Cons Growth - Myopic	0.8852	0.9456
Only Productivity Shock Present		
Exp. Agg. Cons Growth	0.4508	0.9188
Exp. Cons Growth - Optimizers	0.7653	1
Exp. Cons Growth - Myopic	-0.3365	0.3446

Table 4: Descriptive Statistics of Consumption Growth Rates Under The Contemporaneous Wage Setting Case

Figures

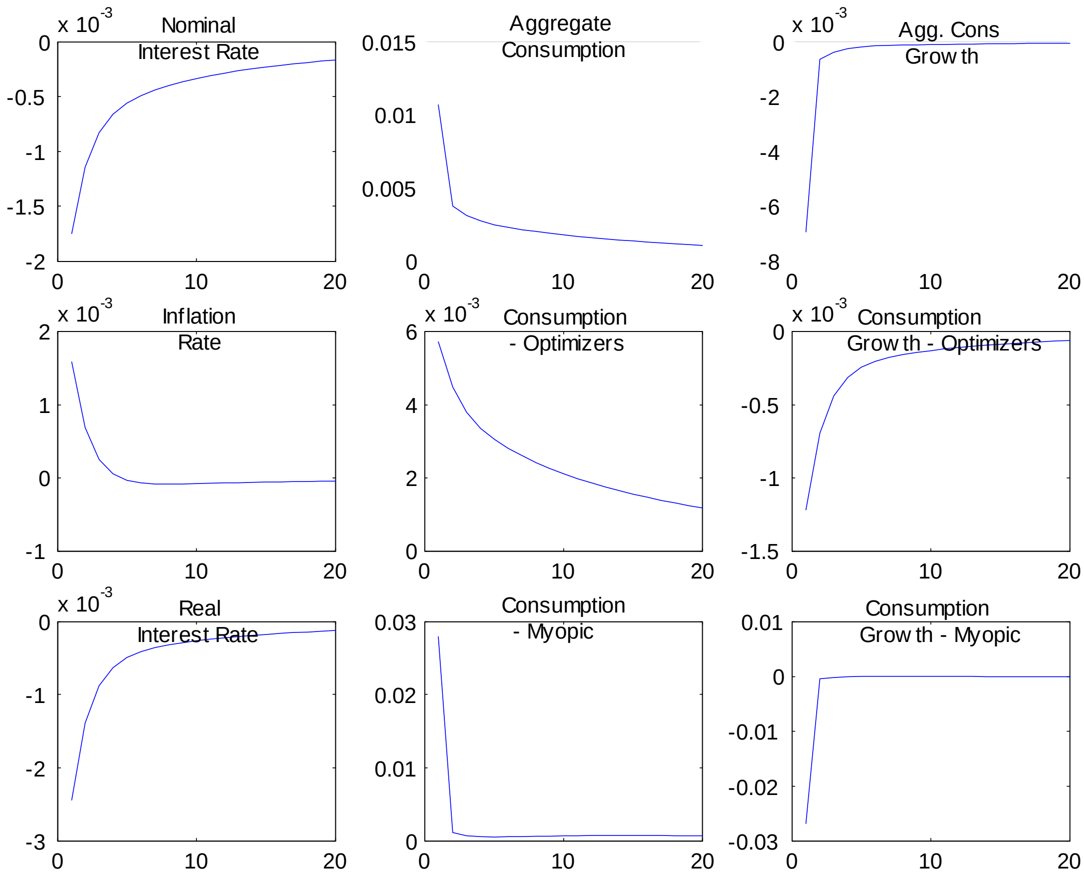

Figure 1: Impulse Responses From An Expansionary Monetary Policy Shock Under The Benchmark Case With Sticky Prices And Sticky Wages

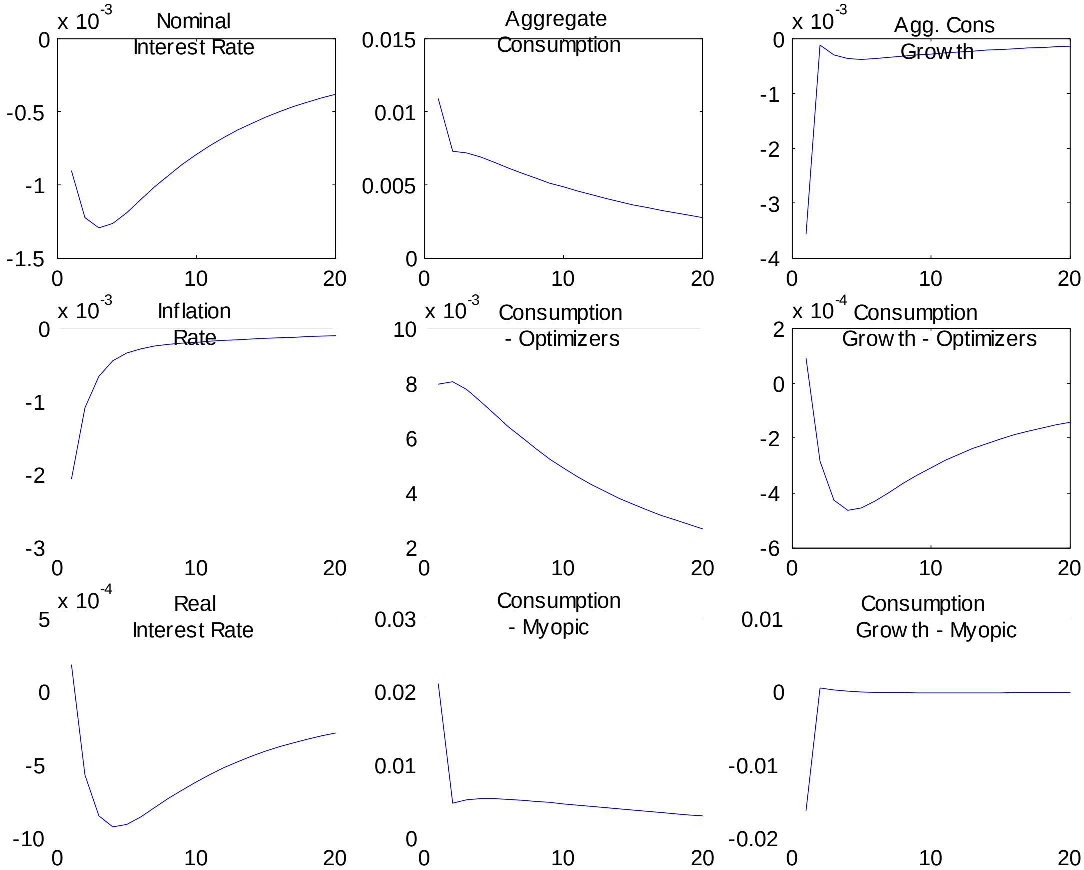

Figure 2: Impulse Responses From A Productivity Shock Under The Benchmark Case With Sticky Prices And Sticky Wages

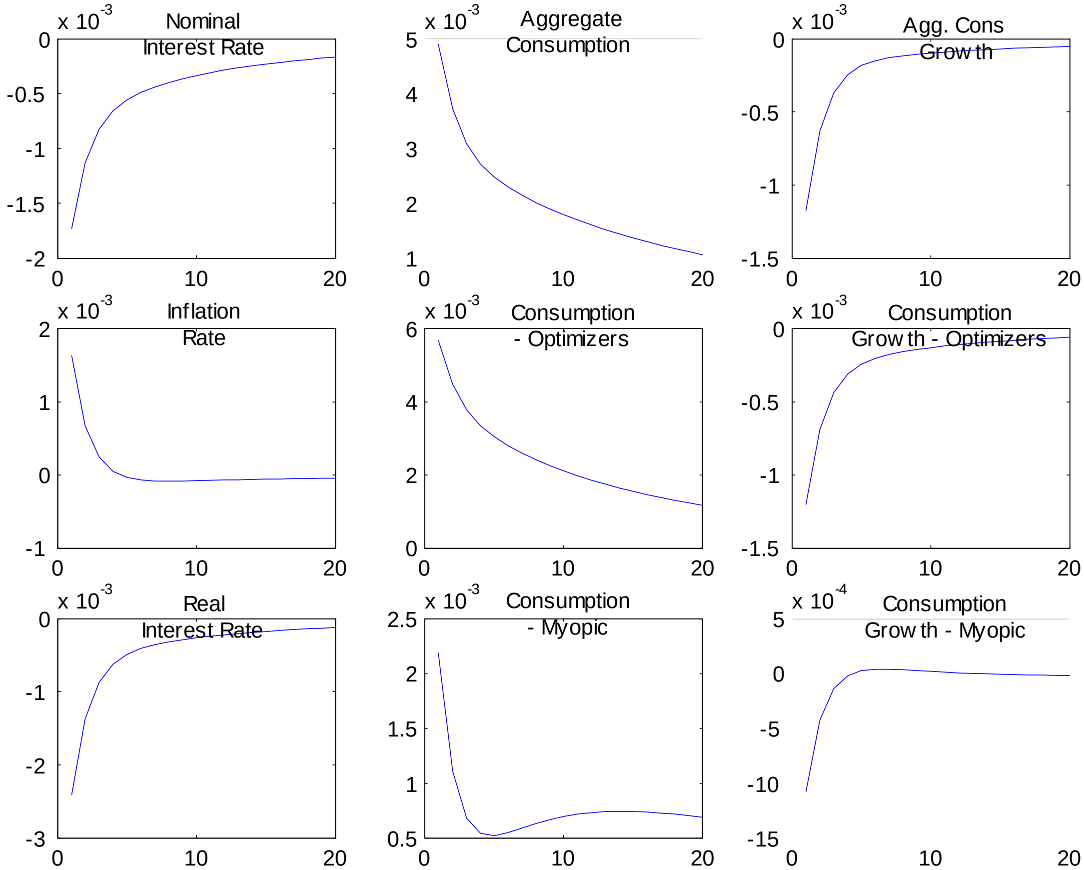

Figure 3: Impulse Responses From An Expansionary Monetary Policy Shock Under The Contemporaneous Wage Setting Case With Sticky Prices And Sticky Wages

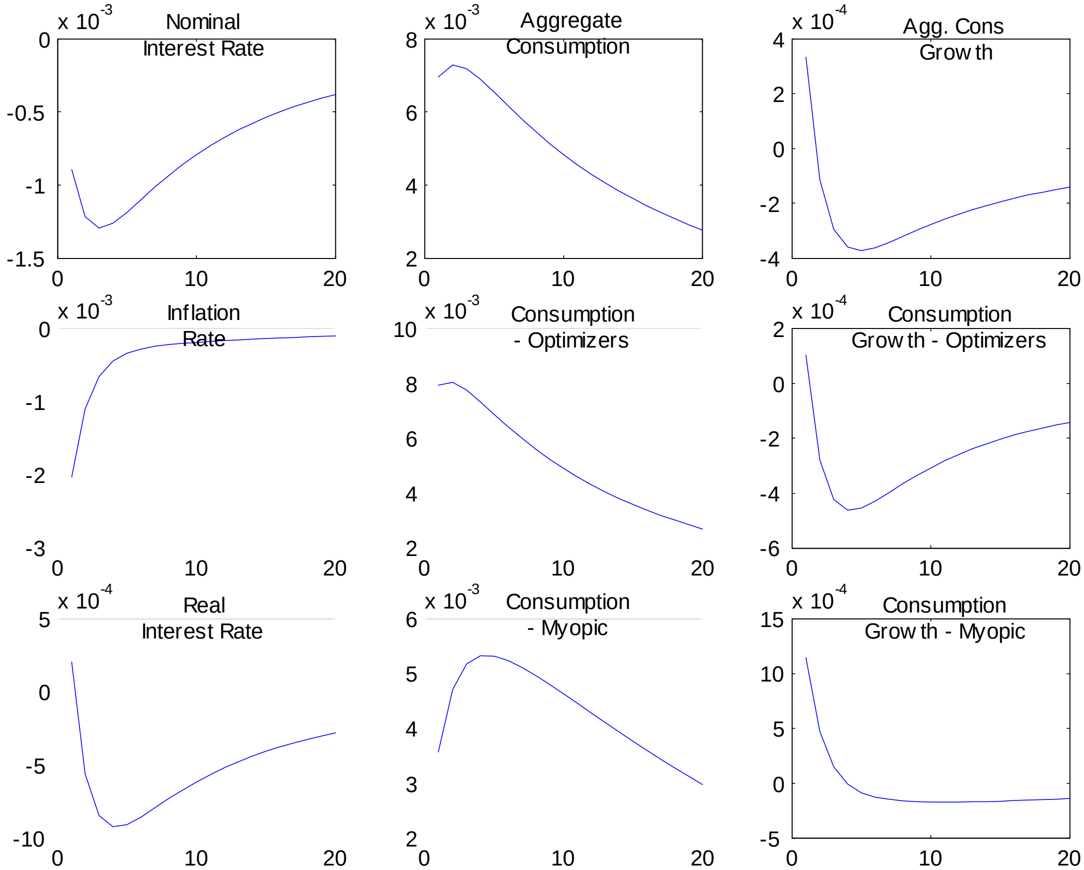

Figure 4: Impulse Responses From A Productivity Shock Under The Contemporaneous Wage Setting Case With Sticky Prices And Sticky Wages

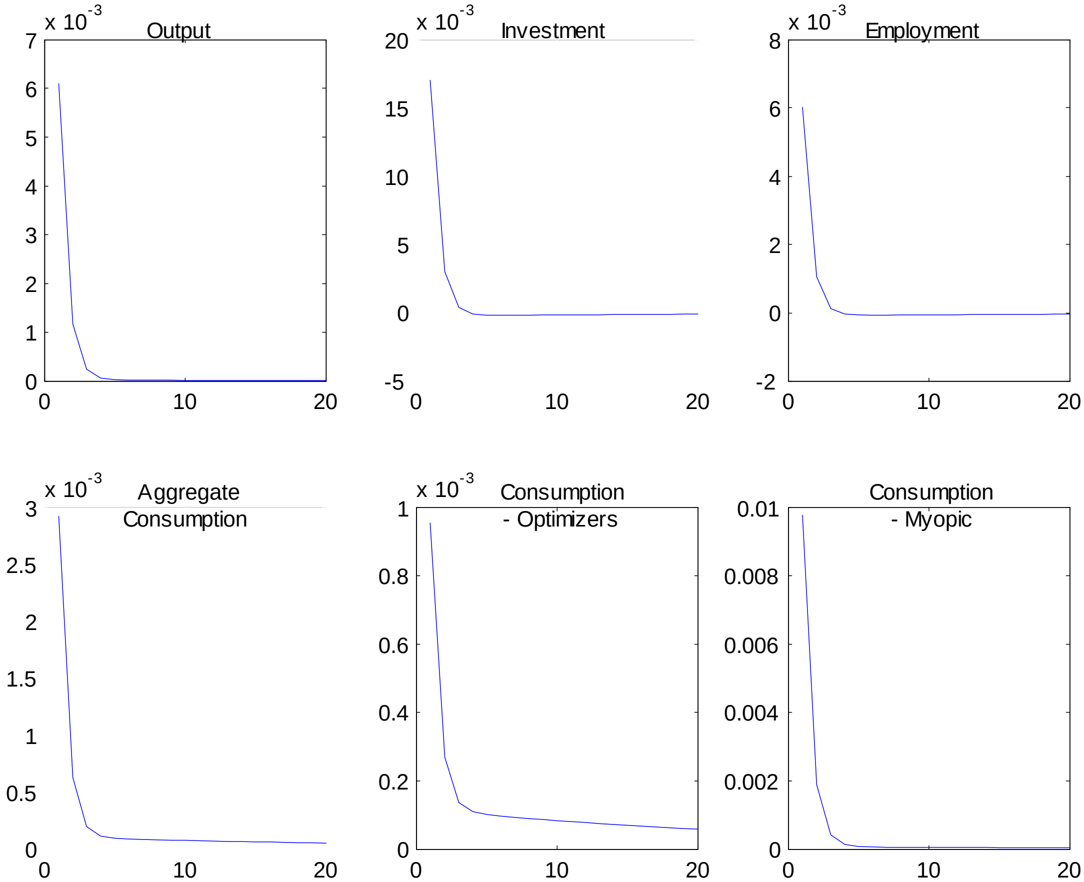

Figure 16.a: Impulse Responses From An Expansionary Monetary Policy Shock With Sticky Prices And Flexible Wages Under The Contemporaneous Wage Setting Case

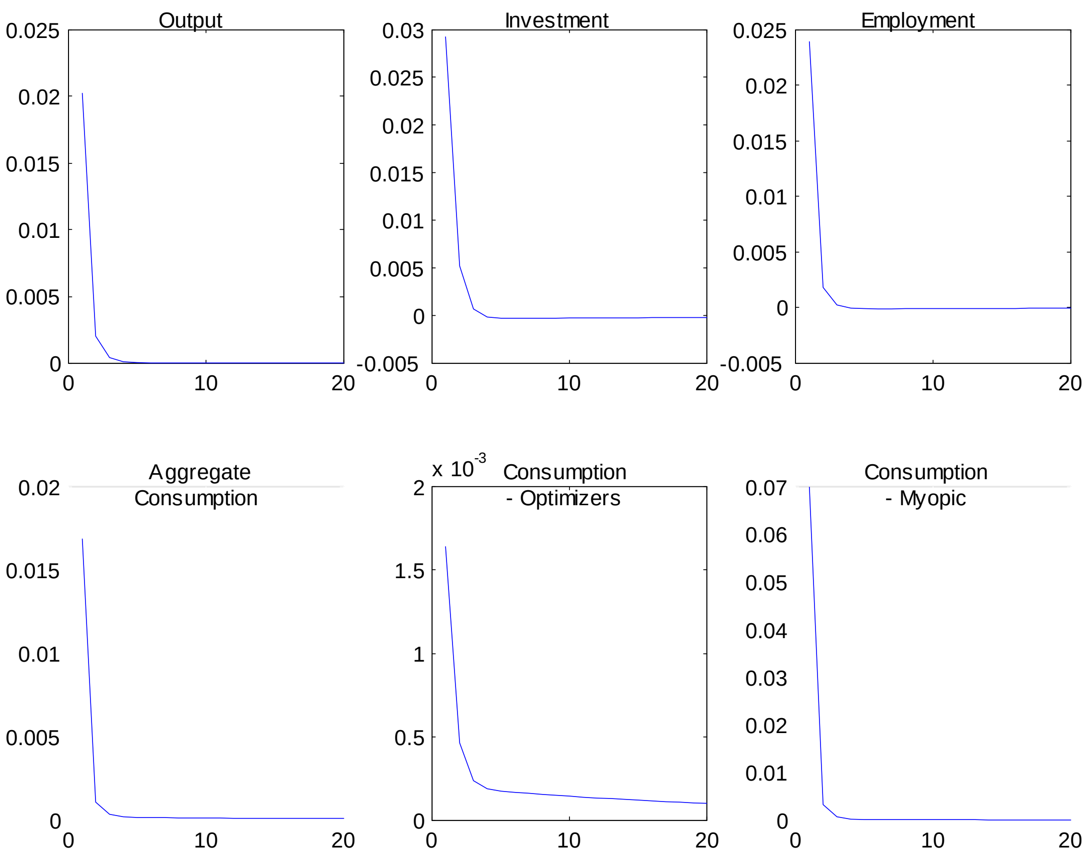

Figure 16.b: Impulse Responses From An Expansionary Monetary Policy Shock With Sticky Prices And Flexible Wages Under The Benchmark Case

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A Technical Appendix - Derivation of Key Equations

The technical appendix here derives the key equations in the model. The notes for the firm's problem, and the Calvo (1983) price and wage setting follow the notes prepared by Canzoneri, Cumby and Diba (2002b, 2003).

A.1 Firms

- Continuum of monopolistically competitive firms producing a differentiated (intermediate) good.
- Perfectly competitive final goods industry producing "bundled" final consumption good using intermediate goods as inputs.
- Calvo Price Setting - a fraction $\left(1-\xi_p\right)$ have the opportunity to reoptimise their prices.

A.1.1 Final Goods Firms Problem

Final goods firms produce a final consumption good, Y_t, at time t, using the intermediate goods produced by other firms as an input. It combines the continuum of intermediate goods, $j \in [0,1]$ using a constant returns to scale production technology:

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}} \tag{A.2}$$

where $Y_{j,t}$ is the quantity of intermediate good j used as the input. The final goods firms maximizes profits (or minimizing costs), taking the final goods price, P_t and the prices of the intermediate goods, P_{jt} as given. This yields the set of demand schedules for the individual intermediate goods:

$$Y_{jt} = \left(\frac{P_{j,t}}{P_t}\right)^{-\theta} Y_t$$

along with the zero profit condition:

$$P_t = \left(\int_0^1 P_{j,t}^{1-\theta} dz\right)^{\frac{1}{1-\theta}}$$

For a given firm, j at some point in time s, solve the dual problem first to derive the price index, P_s. Show that, by definition, P_s is the minimum cost (or price) of a unit of the "bundled" good, $Y \equiv \left(\int_0^1 Y_j^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}}$. Let the *total cost* or *expenditure*, $E \equiv \int_0^1 P_j Y_j dj$. The firm then minimizes (dropping time subscripts):

$$\min_{Y_j} E \equiv \int_0^1 P_j Y_j dj \text{ subject to } Y \equiv \left(\int_0^1 Y_j^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}} = 1 \tag{A.3}$$

The first order condition is:

$$\begin{aligned} P_j - \upsilon Y_j^{\left(\frac{\theta-1}{\theta}-1\right)} \left(\int_0^1 Y_j^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}-1} &= 0 \\ \Longrightarrow P_j = \upsilon Y_j^{-\frac{1}{\theta}} \left(\int_0^1 Y_j^{\frac{\theta-1}{\theta}} dj\right)^{\frac{1}{\theta-1}} & \end{aligned}$$

From the constraint, since, $Y = 1$, it implies $\int_0^1 Y_j^{\frac{\theta-1}{\theta}} dj = 1$. Hence:

$$\begin{aligned} P_j &= \upsilon Y_j^{-\frac{1}{\theta}} \\ P_j Y_j^{\frac{1}{\theta}} &= \upsilon \text{ or equivalently } P_j Y_j = \upsilon Y_j^{\frac{\theta-1}{\theta}} \end{aligned} \tag{A.4}$$

Hence, P is the minimum cost/expenditure *(- Value function)* of one unit of the consumption good's bundle, i.e.:

$$P \equiv \min_{Y_j} E = \int_0^1 P_j Y_j dj = \upsilon \int_0^1 C(j)^{\frac{\theta-1}{\theta}} dj = \upsilon$$

To show that P fits its definition:

$$\begin{aligned} \upsilon &= P_j Y_j^{\frac{1}{\theta}} \\ &\Rightarrow P_j^{1-\theta} Y_j^{\frac{1-\theta}{\theta}} = \upsilon^{1-\theta} \\ &\Rightarrow P_j^{1-\theta} = \upsilon^{1-\theta} Y_j^{\frac{\theta-1}{\theta}} \end{aligned}$$

Integrating over all the different types of goods:

$$
\begin{aligned}
\upsilon^{1-\theta}\int_0^1 Y_j^{\frac{\theta-1}{\theta}} dj &= \int_0^1 P_j^{1-\theta} dj \\
&\Rightarrow \upsilon^{1-\theta} = \int_0^1 P_j^{1-\theta} dj \\
&\Rightarrow P \equiv \upsilon = \left(\int_0^1 P_j^{1-\theta} dj\right)^{\frac{1}{1-\theta}}
\end{aligned}
$$

Next, solve the Primal problem to derive the individual and aggregate demand curves:

$$
\max_{Yj} Y \equiv \left(\int_0^1 Y_j^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}} \text{ subject to } \int_0^1 P_j Y_j dj \equiv E \tag{A.5}
$$

where E is an arbitrary, but fixed level of expenditure. Writing the Lagrangian as:

$$
\mathcal{L}(P_j, Y_j) = \left(\int_0^1 Y_j^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}} + \varphi\left(E - \int_0^1 P_j Y_j dj\right)
$$

Hence the FOC are the same as before, but $\varphi = \frac{1}{\upsilon} = \frac{1}{P}$. However, here, $Y \neq 1$:

$$
\begin{gathered}
Y_j^{[\frac{\theta-1}{\theta}-1]}\left(\int_0^1 Y_j^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}-1} - \varphi P_j Y_j = 0 \\
\Longrightarrow \varphi P_j = Y_j^{-\frac{1}{\theta}}\left(\int_0^1 Y_j^{\frac{\theta-1}{\theta}} dj\right)^{\frac{1}{\theta-1}}
\end{gathered}
$$

So, since $\varphi = \frac{1}{P}$

$$
\begin{aligned}
\frac{P_j}{P} &= Y_j^{-\frac{1}{\theta}}\left(\int_0^1 Y_j^{\frac{\theta-1}{\theta}} dj\right)^{\frac{1}{\theta-1}} \\
&\Longrightarrow \left(\frac{P_j}{P}\right)^{-\theta} = Y_j\left(\int_0^1 Y_j^{\frac{\theta-1}{\theta}} dj\right)^{-\frac{\theta}{\theta-1}} \\
&\Longrightarrow Y_{j,s}^d = \left(\frac{P_{j,s}}{P_s}\right)^{-\theta} Y_s \qquad \text{(A.6)}
\end{aligned}
$$

A.1.2 Intermediate Goods Firms

Intermediate good $j \in (0,1)$, is produced by a monopolist who utilizes the following production technology:

$$Y_{j,t} = Z_t K_{j,t}^{\alpha} N_{j,t}^{1-\alpha} \tag{A.7}$$

where Z_t is a productivity shock, $K_{j,t}$ and $N_{j,t}$ are the amounts of capital and composite labour services employed by firm j. These intermediate firms act competitively with regards to input prices, and take wages and the rental cost of capital as given when choosing the optimal amounts of capital and labour. Dropping the j subscripts:

$$TC \equiv \min_{K_t N_t} R_t^k K_t + W_t N_t \text{ subject to } Z_t K_t^{\alpha} N_t^{1-\alpha} \geq \overline{Y}_t$$

This yields the following FOCs:

$$\begin{aligned} R_t^k &= \alpha\mu Z_t K_t^{\alpha-1} N_t^{1-\alpha} && \text{(A.8)} \\ W_t &= (1-\alpha)\mu Z_t K_t^{\alpha} N_t^{-\alpha} && \text{(A.9)} \\ \overline{Y}_t &= Z_t K_t^{\alpha} N_t^{1-\alpha} && \text{(A.10)} \end{aligned}$$

Equations (A.8) and (A.9) imply that:

$$R_t^k K_t = \left(\frac{\alpha}{1-\alpha}\right) W_t N_t \tag{A.11}$$

Using equation (A.10), this yields the optimal K_t^*, N_t^* as functions of α, W, R^k and $\bar{Y}$, i.e.:

$$N_t = \left(\frac{\bar{Y}_t}{Z_t K_t^{\alpha}}\right)^{\frac{1}{1-\alpha}}$$

$$R_t^k K_t = \left(\frac{\alpha}{1-\alpha}\right) W_t \left(\frac{\bar{Y}_t}{Z_t K_t^{\alpha}}\right)^{\frac{1}{1-\alpha}}$$

$$K_t^* = \left[\left(\frac{\alpha}{1-\alpha}\right)\frac{W_t}{R_t^k}\right]^{1-\alpha} \frac{\bar{Y}_t}{Z_t} \text{ and } N_t^* = \left[\left(\frac{1-\alpha}{\alpha}\right)\frac{R_t^k}{W_t}\right]^{\alpha} \frac{\bar{Y}_t}{Z_t} \tag{A.12}$$

Plugging K_t^* and N_t^* back into the objective function, yields the Total (nominal) Cost function

$TC(R_t^k, W_t, \bar{Y}_t)$:

$$
\begin{aligned}
TC_j &= R_t^k K_{jt}^* + W_t N_{jt}^* \\
&= R_t^k \left[\left(\frac{\alpha}{1-\alpha} \right) \frac{W_t}{R_t^k} \right]^{1-\alpha} \frac{\bar{Y}_t}{Z_t} + W_t \left[\left(\frac{1-\alpha}{\alpha} \right) \frac{R_t^k}{W_t} \right]^{\alpha} \frac{\bar{Y}_t}{Z_t} \\
&= \left(R_t^k\right)^{\alpha} W_t^{1-\alpha} \left[\left(\frac{\alpha}{1-\alpha} \right)^{1-\alpha} + \left(\frac{1-\alpha}{\alpha} \right)^{\alpha} \right] \frac{\bar{Y}_t}{Z_t} \\
&= \left(R_t^k\right)^{\alpha} W_t^{1-\alpha} \left(\frac{\alpha + 1 - \alpha}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \right) \frac{\bar{Y}_t}{Z_t} \\
&= \frac{\left(R_t^k\right)^{\alpha} W_t^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} . \frac{\bar{Y}_t}{Z_t}
\end{aligned}
$$

Hence nominal marginal costs are obtained by varying $\bar{Y}_t$, i.e. $\frac{\partial TC_t}{\partial \bar{Y}_t}$:

$$
P_t s_t = \frac{1}{\Phi} Z_t^{-1} \left(R_t^k\right)^{\alpha} W_t^{1-\alpha} \quad \text{(A.13)}
$$

$$
\text{and real marginal costs } s_t = \frac{1}{\Phi} Z_t^{-1} \left(r_t^k\right)^{\alpha} \omega_t^{1-\alpha} \quad \text{(A.14)}
$$

where $\Phi = \alpha^{\alpha}(1-\alpha)^{1-\alpha}$, $r_t^k = \frac{R_t^k}{P_t}$ and $\omega_t = \frac{W_t}{P_t}$.

Note:

- Intermediate firms make zero profits in the steady state. So WLOG, set $Z_t = 1$ or you can introduce a fixed cost which can be calibrated to ensure profits are zero in the steady state.
- The labour demand curve is derived in a similar way when looking at the wage setting scenario.

A.1.3 Calvo Price Setting

- Staggered price setting: Within each period, every firm faces a constant probability $(1 - \xi_p)$ of being able to reoptimise its nominal price.
- The ability to reoptimise its price is independent across firms and time.
- Each period, a measure $(1 - \xi_p)$ of firms are able to reset prices whereas a fraction ξ_p are not able to.

A firm with the ability to reoptimise its price, maximizes:

$$\max_{\widetilde{P}_t} E_t \sum_{k=0}^{\infty} \xi_p^k \lambda_{t+k} Y_{j,t+k} \left(\Omega^k \widetilde{P}_t - P_{t+k} s_{t+k} \right) \tag{A.15}$$

$$\begin{aligned} \text{subject to} \quad &: \\ Y_{j,t} &= \left(\frac{P_{j,t}}{P_t} \right)^{-\theta} Y_t & \text{(A.16)} \\ s_t &= \frac{1}{\Phi} \left(r_t^k \right)^{\alpha} \omega_t^{1-\alpha} & \text{(A.17)} \end{aligned}$$

The first order condition associated with the choice of $\widetilde{P}_t$ for the problem above is:

$$(1-\theta) E_t \sum_{k=0}^{\infty} \xi_p^k \lambda_{t+k} \left(\Omega^k \right)^{1-\theta} \left(\frac{\tilde{P}_t}{P_{t+k}} \right)^{-\theta} Y_{t+k} + ...$$

$$... + \theta E_t \sum_{k=0}^{\infty} \xi_p^k \lambda_{t+k} \left(\Omega^k \right)^{-\theta} \tilde{P}_t^{-\theta-1} P_{t+k}^{1+\theta} Y_{t+k} s_{t+k} = 0$$

This implies

$$\begin{aligned} E_t \sum_{k=0}^{\infty} \xi_p^k \lambda_{t+k} \Omega^k Y_{j,t+k} + \frac{\theta}{1-\theta} E_t \sum_{k=0}^{\infty} \xi_p^k \lambda_{t+k} \left(\frac{\Omega^k \tilde{P}_t}{P_{t+k}} \right)^{-\theta} \left(\frac{P_{t+k}}{\tilde{P}_t} \right) Y_{t+k} s_{t+k} &= 0 \quad (*) \\ \Rightarrow \frac{1}{\tilde{P}_t} \left(E_t \sum_{k=0}^{\infty} \xi_p^k \lambda_{t+k} \Omega^k Y_{j,t+k} \tilde{P}_t - \frac{\theta}{\theta-1} E_t \sum_{k=0}^{\infty} \xi_p^k \lambda_{t+k} Y_{j,t+k} P_{t+k} s_{t+k} \right) &= 0 \\ \Rightarrow E_t \left\{ \sum_{k=0}^{\infty} \xi_p^k \lambda_{t+k} Y_{j,t+k} \left(\Omega^k \widetilde{P}_t - \mu_p P_{t+k} s_{t+k} \right) \right\} &= 0 \qquad \text{(A.18)} \end{aligned}$$

$$\text{where } \mu_p \equiv \frac{\theta}{\theta-1}$$

Rearranging (*) above, the expression for $\tilde{P}_t$ can also be written (following Canonzeri Cumby and Diba, 2003) as a ratio of difference equations and in terms of the final goods Y_{t+k} and P_{t+k}. This is done by dividing (*) through by $\tilde{P}_t^{-\phi}$ and rearranging. This yields:

$$\tilde{P}_t = \mu_p \frac{E_t \sum_{k=0}^{\infty} \xi_p^k \lambda_{t+k} \left(\Omega^k \right)^{-\theta} P_{t+k}^{\theta} Y_{t+k} P_{t+k} s_{t+k}}{E_t \sum_{k=0}^{\infty} \xi_p^k \lambda_{t+k} \left(\Omega^k \right)^{-\theta} P_{t+k}^{\theta} Y_{t+k} \Omega^k} = \mu_p \frac{PB_t}{PA_t} \tag{A.19}$$

where

$$
\begin{aligned}
PB_t &\equiv E_t \sum_{k=0}^{\infty} \xi_p^k \lambda_{t+k} \left(\Omega^k\right)^{-\theta} P_{t+k}^{\theta} Y_{t+k} P_{t+k} s_{t+k} \\
&= \lambda_t P_t^{\theta} Y_t NMC_t + \xi_p \Omega^{-\theta} E_t PB_{t+1} && \text{(A.20)} \\
PA_t &\equiv E_t \sum_{k=0}^{\infty} \xi_p^k \lambda_{t+k} \left(\Omega^k\right)^{-\theta} P_{t+k}^{\theta} Y_{t+k} \Omega^k \\
&= \lambda_t P_t^{\theta} Y_t + \xi_p \Omega^{1-\theta} E_t PA_{t+1} && \text{(A.21)}
\end{aligned}
$$

The dynamics of the aggregate price level are given by:

$$
\begin{aligned}
P_t &= \left[\int_0^1 P_{j,t}^{1-\theta} dj\right]^{\frac{1}{1-\theta}} \\
&= \left[\sum_{k=0}^{\infty} \left(1-\xi_p\right) \xi_p^k \left(\Omega^k\right)^{1-\theta} \tilde{P}_{t-k}^{1-\theta}\right]^{\frac{1}{1-\theta}}
\end{aligned}
$$

Lagging P_t by one period yields:

$$
P_{t-1} = \left[\sum_{k=0}^{\infty} \left(1-\xi_p\right) \xi_p^k \left(\Omega^k\right)^{1-\theta} \tilde{P}_{t-1-k}^{1-\theta}\right]^{\frac{1}{1-\theta}}
$$

Hence,

$$
\begin{aligned}
P_t &= \left[\left(1-\xi_p\right) \tilde{P}_t^{1-\theta} + \sum_{k=1}^{\infty} \left(1-\xi_p\right) \xi_p^k \left(\Omega^k\right)^{1-\theta} \tilde{P}_{t-k}^{1-\theta}\right]^{\frac{1}{1-\theta}} \\
&= \left[\left(1-\xi_p\right) \tilde{P}_t^{1-\theta} + \xi_p \Omega^{1-\theta} \sum_{s=0}^{\infty} \left(1-\xi_p\right) \xi_p^s \left(\Omega^s\right)^{1-\theta} \tilde{P}_{t-s-1}^{1-\theta}\right]^{\frac{1}{1-\theta}} \\
&= \left[\left(1-\xi_p\right) \tilde{P}_t^{1-\theta} + \xi_p \left(\Omega P_{t-1}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} && \text{(A.22)}
\end{aligned}
$$

A.2 Households

- Continuum of infinitely lived households, indexed by $i \in (0, 1)$.
- Fraction $(1-\nu)$ of households are optimizing households: are able to access asset markets, own the capital stock in the economy which they rent to firms.

- Fraction ν of households are rule-of-thumb, or myopic households: maximize period utility each time period.

- Optimizing households offer differentiated labour services, and hence are monopolistic wage setters.

- Myopic households are part of a union, which negotiates a wage on their behalf (period to period).

A.2.1 Optimizing Households $i \in (\nu, 1)$

Denoting optimizing households by the superscript 'o' to represent $i \in (\nu, 1)$, the optimizing household:

$$\max_{\substack{C_\tau^o, B_{\tau+1}^o, K_{\tau+1}^o \\ I_\tau^o, u_t, W_\tau^o}} E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} U\left(C_\tau^o, L_\tau^o\right) = \max E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[\frac{\left(C_\tau^o\right)^{1-\sigma}}{1-\sigma} - \kappa \frac{\left(L_\tau^o\right)^{1+\chi}}{1+\chi} \right] \tag{A.23}$$

subject to:

$$P_t\left(C_t^o + I_t^o\right) + E_\tau\left(\Delta_{t,t+1} B_{t+1}^o\right) = W_t^o L_t^o + B_t^o + \left(R_t^k u_t - P_t a\left(u_t\right)\right) K_t^o + D_t$$

$$K_{t+1}^o = (1-\delta) K_t^o + \psi\left(\frac{I_t^o}{K_t^o}\right) K_t^o \tag{A.24}$$

$$L_t^o = \left(\frac{W_t^o}{W_t}\right)^{-\phi} N_t \tag{A.25}$$

where u_t is the capacity utilization rate, $\psi'(.) > 0, \psi''(.) \leq 0, \psi(\delta) = \delta, \psi'(\delta) = 1$. The optimizing household chooses $C_t^o, B_{t+1}^o, K_{t+1}^o, I_t^o, u_t$ and W_t^o to maximize (A.23) subject to (**??**)-(A.25). Writing the Lagrangian as:

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ \left[\frac{\left(C_\tau^o\right)^{1-\sigma}}{1-\sigma} - \kappa \frac{\left(\left(\frac{W_\tau^o}{W\tau}\right)^{-\phi} N_\tau\right)^{1+\chi}}{1+\chi} \right] \right.$$

$$+ \lambda_\tau \left[W_\tau^o L_\tau + B_\tau^o + \left(R_\tau^k u_\tau - P_\tau a\left(u_\tau\right)\right) K_\tau^o + D_\tau \right.$$

$$\left. \left. - P_\tau \left(C_t^o + I_t^o\right) - E_\tau \left(\Delta_{t,t+1} B_{t+1}^o\right) \right] + \zeta_\tau \left[(1-\delta) K_\tau^o + \psi \left(\frac{I_\tau^o}{K_\tau^o}\right) K_\tau^o - K_{\tau+1}^o \right] \right\}$$

This yields the following first order conditions:

$$C_t^o \quad : \quad \left(C_t^o\right)^{-\sigma} = \lambda_t P_t \quad \Rightarrow \lambda_t = \frac{\left(C_t^o\right)^{-\sigma}}{P_t} \tag{A.26}$$

$$B_{t+1}^o \quad : \quad E_t\left(-\lambda_t \Delta_{t,t+1} + \beta \lambda_{t+1}\right) = 0 \;\Rightarrow E_t\left(\Delta_{t,t+1}\right) = E_t\left(\beta \frac{\lambda_{t+1}}{\lambda_t}\right) \tag{A.27}$$

$$I_t^o \quad : \quad -P_t \lambda_t + \zeta_t \psi_t^{'}(.) = 0 \;\Rightarrow P_t \lambda_t = \zeta_t \psi^{'}\left(\frac{I_t^o}{K_t^o}\right) \tag{A.28}$$

$$\begin{aligned}
K_{t+1}^o \quad : \quad & E_t \left\{ -\zeta_t + \beta\left(R_{t+1}^k u_{t+1} - a\left(u_{t+1}\right)\right) \lambda_{t+1} + \beta \zeta_{t+1}(1-\delta) \right. \\
& \left. + \beta \zeta_{t+1} \left[\psi_{t+1}(.) + \psi_{t+1}^{'}(.) \left(\frac{-I_{t+1}^o}{\left(K_{t+1}^o\right)^2}\right) .K_{t+1}^o \right] \right\} \\
\Rightarrow \quad & \zeta_t = \beta E_t \left\{ \left(R_{t+1}^k u_{t+1} - P_{t+1} a\left(u_{t+1}\right)\right) \lambda_{t+1} \right. \\
& \left. + \zeta_{t+1} \left[(1-\delta) + \psi_{t+1}(.) - \psi_{t+1}^{'}(.) \left(\frac{I_{t+1}^o}{K_{t+1}^o}\right) \right] \right\}
\end{aligned} \tag{A.29}$$

$$u_t \quad : \quad \lambda_t \left[R_t^k - P_t a'\left(u_t\right)\right] = 0 \;\Rightarrow \frac{R_t^k}{P_t} = a'\left(u_t\right) \tag{A.30}$$

where $E_t\left(\Delta_{t,t+1}\right) \equiv \frac{1}{R_t}$. Combining equations A.26)&(A.27):

$$\begin{aligned}
\frac{1}{R_t} \quad &= \quad E_t \left[\frac{\beta \left(C_{t+1}^o\right)^{-\sigma} P_{t+1}^{-1}}{\left(C_t^o\right)^{-\sigma} P_t^{-1}} \right] = E_t \left[\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right) \right] \\
\Rightarrow \quad & 1 = R_t E_t \left[\Lambda_{t,t+1}\right]
\end{aligned} \tag{A.31}$$

and (A.28)&(A.29):

$$
\begin{aligned}
\frac{P_t \lambda_t}{\psi_t'(.)} &= \beta E_t \Bigg\{ \left(R_\tau^k u_\tau - a(u_\tau) \right) \lambda_{t+1} \\
&\quad + \frac{P_{t+1}\lambda_{t+1}}{\psi_{t+1}'(.)} \left[(1-\delta) + \psi_{t+1}(.) - \psi_{t+1}'(.) \left(\frac{I_{t+1}^o}{K_{t+1}^o} \right) \right] \Bigg\} \\
\text{i.e. } P_t Q_t &= E_t \Bigg\{ \Lambda_{t,t+1} \Bigg[\left(R_{t+1}^k u_{t+1} - P_{t+1} a(u_{t+1}) \right) \\
&\quad + P_{t+1} Q_{t+1} \left[(1-\delta) + \psi_{t+1}(.) - \psi_{t+1}'(.) \left(\frac{I_{t+1}^o}{K_{t+1}^o} \right) \right] \Bigg] \Bigg\} \qquad \text{(A.32)}
\end{aligned}
$$

where $\Lambda_{t,t+k} \equiv \beta^k \left(\frac{\lambda_{t,t+k}}{\lambda_t} \right) = \beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+k}} \right)$ and $Q_t \equiv \frac{1}{\psi_t'(.)}$ is the shadow value of installed capital, namely, Tobin's Q.

A.2.2 Rule Of Thumb Households $i \in (0, v)$

Rule of thumb households are unable to access capital/asset markets. As a result they solve a static problem. Each period they choose C_t^r (taking W_t^r and L_t^r as given) to maximize:

$$
\begin{aligned}
\max U(C_t^r, L_t^r) &= \max \frac{(C_t^r)^{1-\sigma}}{1-\sigma} - \kappa \frac{(L_t^r)^{1+\chi}}{1+\chi} && \text{(A.33)} \\
\text{subject to} &: \; P_t C_t^r = W_t^r L_t^r && \text{(A.34)} \\
&: \; L_t^r = \left(\frac{W_t^r}{W_t} \right)^{-\phi} N_t && \text{(A.35)}
\end{aligned}
$$

The solution to this static problem for C_t^r is:

$$
C_t^r = \left(\frac{W_t^r}{P_t} \right) L_t^r
$$

A.3 The Wage Decision

A.3.1 Labour Aggregator

All optimizing households are assumed to be monopolistic suppliers of differentiated labour services. The rule of thumb households are assumed to be members of a union, who sets the wage on their behalf. Furthermore, households of both types are assumed to sell their labour services to a representative competitive firm. This competitive firm combines the labour services from both optimizing households, L_t^o, and myopic households, L_t^r and then transforms it into an aggregate composite labour input, N_t using the following technology:

$$\begin{aligned}
N_t &= g\left(L_t^r, L_t^o\right) \\
&= \left[\int_0^{\nu} L_{i,t}^{\frac{\phi-1}{\phi}} di + \int_{\nu}^{1} L_{i,t}^{\frac{\phi-1}{\phi}} di\right]^{\frac{\phi}{\phi-1}} \\
&= \left[\int_0^{1} L_{i,t}^{\frac{\phi-1}{\phi}} di\right]^{\frac{\phi}{\phi-1}} \quad i \in (0,1)
\end{aligned} \tag{A.36}$$

where $\phi > 1$, and $L_{i,t}$ are the individual amounts of labour services, where the superscripts 'r' and 'o' refer to $i \in (0, v)$ and $i \in (v, 1)$ respectively. The perfectly competitive firm which aggregates labour services faces an analogous problem to the final goods firm. The firm minimizes its total expenditure,

$$\min_{L_i} E_L \equiv \int_0^1 W_{i,t} L_{i,t} di \text{ subject to } N_t = \left(\int_0^1 L_{i,t}^{\frac{\phi-1}{\phi}} di\right)^{\frac{\phi}{\phi-1}}$$

This yields the FOC:

$$\begin{aligned}
W_{i,t} &= \gamma L_{i,t}^{-\frac{1}{\phi}} \left(\int_0^1 L_{i,t}^{\frac{\phi-1}{\phi}} di\right)^{\frac{\phi}{\phi-1}-1} \\
&= \gamma L_{i,t}^{-\frac{1}{\phi}} \left(\int_0^1 L_{i,t}^{\frac{\phi-1}{\phi}} di\right)^{\frac{1}{\phi-1}} \\
&= \gamma L_{i,t}^{-\frac{1}{\phi}} \left[\left(\int_0^1 L_{i,t}^{\frac{\phi-1}{\phi}} di\right)^{\frac{\phi}{\phi-1}}\right]^{\frac{1}{\phi}} \\
&= \gamma L_{i,t}^{-\frac{1}{\phi}} N_t^{\frac{1}{\phi}}
\end{aligned}$$

where γ is the Lagrange multiplier. Since the bundler is competitive, the minimum expenditure $E_L^* \equiv W_t = \gamma$:

$$W_{i,t} = W_t \left(\frac{L_{i,t}}{N_t}\right)^{-\frac{1}{\phi}}$$
$$\Rightarrow L_{i,t} = \left(\frac{W_{i,t}}{W_t}\right)^{-\phi} N_t \tag{A.37}$$

which is the demand curve for household's labour services. Integrating (A.37) across i, and imposing (A.36) yields:

$$\begin{aligned}
\text{i.e. } L_{i,t}^{\frac{\phi-1}{\phi}} &= \left(\frac{W_{i,t}}{W_t}\right)^{-(\phi-1)} N_t^{\frac{\phi-1}{\phi}} \\
&\Rightarrow \int_0^1 L_{i,t}^{\frac{\phi-1}{\phi}} di = \left(\frac{1}{W_t}\right)^{1-\phi} N_t^{\frac{\phi-1}{\phi}} \int_0^1 W_{i,t}^{1-\phi} di \\
&\Rightarrow W_t^{1-\phi} N_t^{\frac{\phi-1}{\phi}} = N_t^{\frac{\phi-1}{\phi}} \left(\int_0^1 W_{i,t}^{1-\phi} di\right) \\
&\Rightarrow W_t = \left(\int_0^1 W_{i,t}^{1-\phi} di\right)^{\frac{1}{1-\phi}} && \text{(A.38)}
\end{aligned}$$

where W_t is the aggregate wage rate. This assumes a perfectly competitive firm that takes both the price of its output (i.e. the composite labour) and individual household wages, $W_{i,t}$ as given.

A.3.2 Calvo Wage Setting

- Staggered wage setting: Each period, an optimizing household faces a constant probability $(1 - \xi_w)$ of being able to reoptimise its nominal wage.
- Ability to reoptimise their wage is independent across time and other households.
- Each period, a measure $(1 - \xi_w)$ are able to reoptimise wages, and a measure ξ_w are unable to.
- Wages are indexed for optimizing households who are unable to reoptimise their nominal wage. The indexation is: $W_t^o = \Omega W_{t-1}^o$

Optimizing Households

Let $\tilde{W}_t^o$ represent the nominal wage chosen by an optimizing household with the ability to reoptimise its nominal wage. They pick $\tilde{W}_t^o$ to maximize (A.23) subject to equations (**??**), (A.24) and the firm's labour demand schedule (A.25), i.e.:

$$\begin{aligned}
& \max_{\tilde{W}_t^o} E_\tau \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[\frac{(C_\tau^o)^{1-\sigma}}{1-\sigma} - \kappa \frac{(L_\tau^o)^{1+\chi}}{1+\chi} \right] \\
\equiv \quad & \max_{\tilde{W}_t^o} E_\tau \sum_{\tau=t}^{\infty} (\xi_w \beta)^{\tau-t} \left[-\kappa \frac{\left(\left(\frac{\Omega^{\tau-t} \tilde{W}_t^o}{W_\tau} \right)^{-\phi} N_\tau \right)^{1+\chi}}{1+\chi} \right] \\
\text{st} \quad : \quad & P_t \left(C_t^o + I_t^o \right) + E_\tau \left(\Delta_{t,t+1} B_{t+1}^o \right) \\
= \quad & \tilde{W}_t^o \left(\frac{\tilde{W}_t^o}{W_t} \right)^{-\phi} N_t + B_t^o + \left(R_t^k u_t - a\left(u_t\right) \right) K_t^o + D_t \\
\text{and} \quad : \quad & K_{t+1}^o = (1-\delta) K_t^o + \psi \left(\frac{I_t^o}{K_t^o} \right) K_t^o
\end{aligned}$$

Writing the Lagrangian as:

$$\begin{aligned}
\mathcal{L} \quad : \quad & E_\tau \sum_{\tau=t}^{\infty} (\xi_w \beta)^{\tau-t} \left\{ \left[-\kappa \frac{\left(\left(\frac{\Omega^{\tau-t} \tilde{W}_t^o}{W_\tau} \right)^{-\phi} N_\tau \right)^{1+\chi}}{1+\chi} \right] \right. \\
& \left. + \lambda_\tau \left[\left(\Omega^{\tau-t} \right)^{1-\phi} \left(\tilde{W}_t^o \right)^{1-\phi} W_\tau^\phi N_\tau + \ldots \right] \right\}
\end{aligned}$$

This yields the FOC:

$$\begin{aligned}
E_\tau \sum_{\tau=t}^{\infty} (\xi_w \beta)^{\tau-t} & \left\{ \left[-\kappa \left(-\phi\right) L_\tau^\chi \left(\frac{\Omega^{\tau-t} \tilde{W}_t^o}{W_\tau} \right)^{-\phi} \frac{N_\tau}{\tilde{W}_t^o} \right] \right. \\
& \left. + \lambda_\tau \left[\left(1-\phi\right) \left(\Omega^{\tau-t} \right) \left(\frac{\Omega^{\tau-t} \tilde{W}_t^o}{W_\tau} \right)^{-\phi} N_\tau \right] \right\} = 0
\end{aligned}$$

$$\text{i.e. } E_\tau \sum_{\tau=t}^{\infty} (\xi_w \beta)^{\tau-t} \left\{ \left(\frac{\Omega^{\tau-t} \tilde{W}_t^o}{W_\tau} \right)^{-\phi} N_\tau \left[\kappa \phi \frac{L_\tau^\chi}{\tilde{W}_t^o} - \lambda_\tau \left[(\phi - 1) \Omega^{\tau-t} \right] \right] \right\} = 0$$

where L_τ is given by (A.37). Dividing by $(\phi - 1)$ and $\left(\tilde{W}_t^o\right)^{-\phi}$ and rearranging yields:

$$E_\tau \sum_{\tau=t}^{\infty} (\xi_w \beta)^{\tau-t} \left[\kappa \left(\frac{\phi}{\phi - 1} \right) \left(\left(\frac{\Omega^{\tau-t}}{W_\tau} \right)^{-\phi} N_\tau \right)^\chi \left(\frac{\Omega^{\tau-t}}{W_\tau} \right)^{-\phi} N_\tau \left(\tilde{W}_t^o \right)^{-(1+\chi\phi)} \right] = E_\tau \sum_{\tau=t}^{\infty} (\xi_w \beta)^{\tau-t} \left\{ \lambda_\tau \left[\left(\Omega^{\tau-t} \right) \left(\frac{\Omega^{\tau-t}}{W_\tau} \right)^{-\phi} N_\tau \right] \right\}$$

Rearranging and solving for $\tilde{W}_t^o$:

$$\left(\tilde{W}_t^o \right)^{1+\chi\phi} = \kappa \mu_w \frac{E_\tau \sum_{\tau=t}^{\infty} (\xi_w \beta)^{\tau-t} \left[\left(\frac{\Omega^{\tau-t}}{W_\tau} \right)^{-\phi(1+\chi)} N_\tau^{1+\chi} \right]}{E_\tau \sum_{\tau=t}^{\infty} (\xi_w \beta)^{\tau-t} \left\{ \lambda_\tau \left[\left(\Omega^{\tau-t} \right)^{1-\phi} W_\tau^\phi N_\tau \right] \right\}} \tag{A.39}$$

where $\mu_w \equiv \frac{\phi}{\phi-1}$. Following Canonzeri, Cumby and Diba (2003), this expression can be re-written as a ratio of two difference equations:

$$\left(\tilde{W}_t^o \right)^{1+\phi\chi} = \kappa \mu_w \left(\frac{WB_t}{WA_t} \right) \tag{A.40}$$

where

$$\begin{aligned} WB_t &\equiv E_\tau \sum_{\tau=t}^{\infty} (\xi_w \beta)^{\tau-t} \left[\left(\frac{\Omega^{\tau-t}}{W_\tau} \right)^{-\phi(1+\chi)} N_\tau^{1+\chi} \right] \\ &= \left(N_t W_t^\phi \right)^{1+\chi} + \xi_w \beta \Omega^{-\phi(1+\chi)} E_t WB_{t+1} && \text{(A.41)} \\ WA_t &\equiv E_\tau \sum_{\tau=t}^{\infty} (\xi_w \beta)^{\tau-t} \left\{ \lambda_\tau \left[\left(\Omega^{\tau-t} \right)^{1-\phi} W_\tau^\phi N_\tau \right] \right\} \\ &= \lambda_t N_t W_t^\phi + \xi_w \beta \Omega^{1-\phi} E_t WA_{t+1} && \text{(A.42)} \end{aligned}$$

Union's Problem

Rule of thumb households are assumed to belong to a union which picks a wage to maximize the utility of their members. Let $\tilde{W}_t^r$ be the wage posted by the union in maximizing:

$$\max U\left(\tilde{W}_t^r\right) = \max E_{t-1}\left\{\left(\frac{1}{1-\sigma}\right)\left[\left(\frac{\tilde{W}_t^r}{P_t}\right)\left(\frac{\tilde{W}_t^r}{W_t}\right)^{-\phi} N_t\right]^{1-\sigma} - \frac{\kappa}{1+\chi}\left[\left(\frac{\tilde{W}_t^r}{W_t}\right)^{-\phi} N_t\right]^{1+\chi}\right\}$$

The FOC for $\tilde{W}_t^r$ is:

$$E_{t-1}\left[(1-\phi)\left(C_t^r\right)^{-\sigma} P_t^{-1}\left(\frac{\tilde{W}_t^r}{W_t}\right)^{-\phi} N_t - \kappa\left(L_t^r\right)^{\chi}(-\phi)\left(\frac{\tilde{W}_t^r}{W_t}\right)^{-\phi}\frac{N_t}{\tilde{W}_t^r}\right] = 0 \tag{A.43}$$

$$\text{i.e. } E_{t-1}\left[(1-\phi)\left(\tilde{W}_t^r\right)^{-\phi-\sigma(1-\phi)} P_t^{-1+\sigma} W_t^{\phi(1-\sigma)} N_t^{1-\sigma} + \kappa\phi\left(\tilde{W}_t^r\right)^{-1-\phi(1+\chi)} W_t^{\phi(1+\chi)} N_t^{1+\chi}\right] = 0$$

Dividing through by $\phi - 1$ and rearranging yields the wage setting equation for the rule of thumb households:

$$\begin{aligned} \left(\tilde{W}_t^r\right)^{1+\phi\chi+\sigma(\phi-1)} &= \mu_w \kappa \frac{E_{t-1} WD_t}{E_{t-1} WE_t} \\ E_{t-1} WD_t &= E_{t-1}\left[W_t^{\phi(1+\chi)} N_t^{1+\chi}\right] \\ E_{t-1} WE_t &= E_{t-1}\left[W_t^{\phi(1-\sigma)} P_t^{\sigma-1} N_t^{1-\sigma}\right] \end{aligned} \tag{A.44}$$

This can be re-written in real terms by dividing through by $P_{t-1}^{1+\phi\chi+\sigma(\phi-1)}$ which yields:

$$\left(\tilde{w}_t^r\right)^{1+\phi\chi+\sigma(\phi-1)} = \left(\frac{\tilde{W}_t^r}{P_{t-1}}\right)^{1+\phi\chi+\sigma(\phi-1)} = \mu_w \kappa \frac{E_{t-1} wd_t}{E_{t-1} we_t} \tag{A.45}$$

where

$$
\begin{aligned}
E_{t-1} wd_t &= \frac{E_{t-1} WD_t}{P_{t-1}^{\phi(1+\chi)}} \\
&= E_{t-1}\left[\left(\frac{W_t}{P_{t-1}}\right)^{\phi(1+\chi)} N_t^{1+\chi}\right] \\
&= E_{t-1}\left[\left(\frac{W_t}{P_t}\right)^{\phi(1+\chi)} \left(\frac{P_t}{P_{t-1}}\right)^{\phi(1+\chi)} N_t^{1+\chi}\right] \\
&= E_{t-1}\left[w_t^{\phi(1+\chi)} \pi_t^{\phi(1+\chi)} N_t^{1+\chi}\right]
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
E_{t-1} we_t &= \frac{E_{t-1} WE_t}{P_{t-1}^{-1+\phi+\sigma(1-\phi)}} \\
&= \frac{E_{t-1}\left[W_t^{\phi(1-\sigma)} P_t^{\sigma-1} N_t^{1-\sigma}\right]}{P_{t-1}^{\phi(1-\sigma)} . P_{t-1}^{-(1-\sigma)}} \\
&= E_{t-1}\left[\left(\frac{W_t}{P_{t-1}}\right)^{\phi(1-\sigma)} \left(\frac{P_t}{P_{t-1}}\right)^{\sigma-1} N_t^{1-\sigma}\right] \\
&= E_{t-1}\left[w_t^{\phi(1-\sigma)} \pi_t^{(\phi-1)(1-\sigma)} N_t^{1-\sigma}\right]
\end{aligned}
$$

Aggregate Wages

The aggregate wage level is given by equation (A.38):

$$
\begin{aligned}
W_t &= \left(\int_0^1 W_{i,t}^{1-\phi} di\right)^{\frac{1}{1-\phi}} = \left(\int_0^v W_{i,t}^{1-\phi} di + \int_v^1 W_{i,t}^{1-\phi} di\right)^{\frac{1}{1-\phi}} \\
&= \left(v\left(\tilde{W}_t^r\right)^{1-\phi} + \int_v^1 \left(W_t^o\right)^{1-\phi} di\right)^{\frac{1}{1-\phi}}
\end{aligned}
$$

Let

$$
(1-v)\left(\tilde{W}_t^o\right)^{1-\phi} = \int_v^1 \left(W_t^o\right)^{1-\phi} di \tag{A.46}
$$

Then

$$
\begin{aligned}
W_t &= \left(v \left(\tilde{W}_t^r\right)^{1-\phi} + (1-v) \sum_{k=0}^{\infty} (1-\xi_w)\, \xi_w^k \left(\Omega^k\right)^{1-\phi} \left(\tilde{W}_{t-k}^o\right)^{1-\phi} \right)^{\frac{1}{1-\phi}} \\
&= \left(v \left(\tilde{W}_t^r\right)^{1-\phi} + (1-v) \left(W_t^*\right)^{1-\phi} \right)^{\frac{1}{1-\phi}} \qquad \text{(A.47)}
\end{aligned}
$$

where

$$
\begin{aligned}
\left(W_t^*\right)^{1-\phi} &\equiv \sum_{k=0}^{\infty} (1-\xi_w)\, \xi_w^k \left(\Omega^k\right)^{1-\phi} \left(\tilde{W}_{t-k}^o\right)^{1-\phi} \\
&= (1-\xi_w) \left(\tilde{W}_t^o\right)^{1-\phi} + \xi_w \Omega^{1-\phi} \left(W_{t-1}^*\right)^{1-\phi} \qquad \text{(A.48)}
\end{aligned}
$$

A.4 Aggregation

Following reasoning to that above aggregate labour supply can be expressed as follows:

$$
\begin{aligned}
L_t &= \int_0^1 L_{i,t} di \\
&= \int_0^v L_{i,t}^r di + \int_v^1 L_{i,t}^o di \\
&= vL_t^r + \int_v^1 \left(\frac{W_{i,t}^o}{W_t}\right)^{-\phi} N_t di \\
&= vL_t^r + W_t^{\phi} N_t \int_v^1 \left(W_{i,t}^o\right)^{-\phi} di \\
&= vL_t^r + (1-v)\, W_t^{\phi} N_t \left(WC_t^*\right)^{-\phi}
\end{aligned}
$$

where

$$
\begin{aligned}
(1-v)\left(WC_t^*\right)^{-\phi} &\equiv \int_v^1 \left(W_t^o\right)^{-\phi} di \\
\text{and } \left(WC_t^*\right)^{-\phi} &= (1-\xi_w) \left(\tilde{W}_t^o\right)^{-\phi} + \xi_w \Omega^{-\phi} \left(WC_{t-1}^*\right)^{-\phi} \qquad \text{(A.49)}
\end{aligned}
$$

A.5 Real Version of the Model

The nominal variables in the model can be converted to real terms to avoid the price indeterminacy issue that arises with the monetary authority pursuing an interest rate rule. Let $\tilde{\Lambda}_t = P_t\lambda_t$, $\tilde{p}_t = \frac{\tilde{P}_t}{P_t}$ and $\tilde{\mu}_t \equiv P_t\mu_t$. The following equations can be rewritten utilizing the substitutions above. For the price setting, equations (A.19)-(A.22), $\tilde{P}_t$ is given by

$$\tilde{P}_t = \mu_p \kappa \frac{PB_t}{PA_t}$$

where

$$\begin{aligned} PB_t &\equiv E_t \sum_{k=0}^{\infty} \xi_p^k \lambda_{t+k} \left(\Omega^k\right)^{-\theta} P_{t+k}^{\theta} Y_{t+k} P_{t+k} s_{t+k} \\ PA_t &\equiv E_t \sum_{k=0}^{\infty} \xi_p^k \lambda_{t+k} \left(\Omega^k\right)^{-\theta} P_{t+k}^{\theta} Y_{t+k} \Omega^k \end{aligned}$$

Dividing $\tilde{P}_t$ through by P_t yields:

$$\tilde{p}_t = \frac{\tilde{P}_t}{P_t} = \mu_p \kappa \frac{pb_t}{pa_t}$$

where

$$\begin{aligned} pb_t &= \frac{PB_t}{P_t^{\theta}} = P_t^{-\theta} E_t \sum_{k=0}^{\infty} \xi_p^k \lambda_{t+k} \left(\Omega^k\right)^{-\theta} P_{t+k}^{\theta} Y_{t+k} P_{t+k} s_{t+k} \\ &= E_t \sum_{k=0}^{\infty} \xi_p^k \tilde{\Lambda}_{t+k} \left(\Omega^k\right)^{-\theta} \left(\frac{P_{t+k}}{P_t}\right)^{\theta} Y_{t+k} s_{t+k} \\ &= \tilde{\Lambda}_t Y_t s_t + E_t \sum_{k=1}^{\infty} \xi_p^k \tilde{\Lambda}_{t+k} \left(\Omega^k\right)^{-\theta} \left(\frac{P_{t+k}}{P_t}\right)^{\theta} Y_{t+k} s_{t+k} \\ &= \tilde{\Lambda}_t Y_t s_t \\ &\quad + \xi_p \Omega^{-\theta} E_t \sum_{k=0}^{\infty} \left(\xi_p^k \tilde{\Lambda}_{t+1+k} \left(\Omega^k\right)^{-\theta} \left(\frac{P_{t+1+k}}{P_{t+1}}\right)^{\theta} Y_{t+1+k} s_{t+1+k} \right) \left(\frac{P_{t+1}}{P_t}\right)^{\theta} \\ &= \tilde{\Lambda}_t Y_t s_t + \xi_p \Omega^{-\theta} E_t \left[\left(\frac{P_{t+1}}{P_t}\right)^{\theta} pb_{t+1} \right] \end{aligned}$$

Similarly, pa_t can be written as:

$$
\begin{aligned}
pa_t &= P_t \frac{PA_t}{P_t^{\theta}} = \frac{P_t}{P_t^{\theta}} E_t \sum_{k=0}^{\infty} \xi_p^k \lambda_{t+k} \left(\Omega^k\right)^{-\theta} P_{t+k}^{\theta} Y_{t+k} \Omega^k \\
&= E_t \sum_{k=0}^{\infty} \xi_p^k \tilde{\Lambda}_{t+k} \left(\Omega^k\right)^{1-\theta} \left(\frac{P_{t+k}}{P_t}\right)^{\theta} \left(\frac{P_t}{P_{t+k}}\right) Y_{t+k} \\
&= E_t \sum_{k=0}^{\infty} \xi_p^k \tilde{\Lambda}_{t+k} \left(\Omega^k\right)^{1-\theta} \left(\frac{P_{t+k}}{P_t}\right)^{\theta-1} Y_{t+k} \\
&= \tilde{\Lambda}_t Y_t + \xi_p \Omega^{1-\theta} E_t \left[\left(\frac{P_{t+1}}{P_t}\right)^{\theta-1} \sum_{k=0}^{\infty} \xi_p^k \tilde{\Lambda}_{t+1+k} \left(\Omega^k\right)^{1-\theta} \left(\frac{P_{t+1+k}}{P_{t+1}}\right)^{\theta-1} Y_{t+1+k} \right] \\
&= \tilde{\Lambda}_t Y_t + \xi_p \Omega^{1-\theta} E_t \left[\left(\frac{P_{t+1}}{P_t}\right)^{\theta-1} pa_{t+1} \right]
\end{aligned}
$$

The aggregate price setting equation can be divided through by $P_t^{1-\theta}$ and written as:

$$
\begin{aligned}
\frac{P_t^{1-\theta}}{P_t^{1-\theta}} &= \frac{\left(1-\xi_p\right) \tilde{P}_t^{1-\theta} + \xi_p \left(\Omega P_{t-1}\right)^{1-\theta}}{P_t^{1-\theta}} \\
&\Rightarrow \quad 1 = \left(1-\xi_p\right) \tilde{p}_t^{1-\theta} + \xi_p \left(\Omega \frac{P_{t-1}}{P_t}\right)^{1-\theta}
\end{aligned}
$$

The price level can be eliminated from the FOC's within the household's problems as follows:

$$
\left(c_t^o\right)^{-\sigma} = \tilde{\Lambda}_t
$$

$$
R_t \beta E_t \left[\left(\frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t}\right) \left(\frac{P_t}{P_{t+1}}\right) \right] = 1
$$

$$
\tilde{\Lambda}_t = \zeta_t \psi_t^{'} (.)
$$

$$
\zeta_t = \beta E_t \left\{ \left(r_\tau^k u_\tau - a\left(u_\tau\right) \right) \tilde{\Lambda}_{t+1} + \zeta_{t+1} \left[(1-\delta) + \psi_{t+1}(.) - \psi_{t+1}^{'}(.) \left(\frac{I_{t+1}^o}{K_{t+1}^o}\right) \right] \right\}
$$

$$
\left(c_t^r\right)^{-\sigma} = \tilde{\mu}_t
$$

$$
\left(\tilde{w}_t^r\right)^{1+\phi\chi} = \frac{\mu_w \kappa}{\tilde{\mu}_t} \left(N_t w_t^{\phi}\right)^{\chi}
$$

The wage picked by optimizing households can be rewritten in a similar fashion to the price set by firms, i.e. given

$$\left(\tilde{W}_t^o\right)^{1+\phi\chi} = \kappa\mu_w\left(\frac{WB_t}{WA_t}\right)$$

where

$$\begin{aligned}
WB_t &\equiv E_t\sum_{\tau=t}^{\infty}(\xi_w\beta)^{\tau-t}\left[\left(\frac{\Omega^{\tau-t}}{W_\tau}\right)^{-\phi(1+\chi)}N_\tau^{1+\chi}\right] \\
WA_t &\equiv E_t\sum_{\tau=t}^{\infty}(\xi_w\beta)^{\tau-t}\left\{\lambda_\tau\left[\left(\Omega^{\tau-t}\right)^{1-\phi}W_\tau^{\phi}N_\tau\right]\right\}
\end{aligned}$$

Dividing $\left(\tilde{W}_t^o\right)^{1+\phi\chi}$ through by $P_t^{1+\phi\chi}$:

$$(\tilde{w}_t^o)^{1+\phi\chi} = \left(\frac{\tilde{W}_t^o}{P_t}\right)^{1+\phi\chi} = \kappa\mu_w\left(\frac{wb_t}{wa_t}\right)$$

where

$$\begin{aligned}
wb_t &= \frac{WB_t}{P_t^{\phi(1+\chi)}} = P_t^{-\phi(1+\chi)}E_t\sum_{\tau=t}^{\infty}(\xi_w\beta)^{\tau-t}\left[\left(\frac{\Omega^{\tau-t}}{W_\tau}\right)^{-\phi(1+\chi)}N_\tau^{1+\chi}\right] \\
&= E_t\sum_{\tau=t}^{\infty}(\xi_w\beta)^{\tau-t}\left[\left(\Omega^{\tau-t}\right)^{-\phi(1+\chi)}N_\tau^{1+\chi}\left(\frac{w_tP_t}{P_t}\right)^{\phi(1+\chi)}\right] \\
&= \left(N_tw_t^{\phi}\right)^{1+\chi} \\
&\quad +\xi_w\beta\Omega^{-\phi(1+\chi)}E_t\left[\left(\frac{P_{t+1}}{P_t}\right)\sum_{\tau=t+1}^{\infty}(\xi_w\beta)^{\tau-t}\left(\Omega^{\tau-t}\right)^{-\phi(1+\chi)}\left(N_\tau w_t^{\phi}\right)^{1+\chi}\right] \\
&= \left(N_tw_t^{\phi}\right)^{1+\chi} + \xi_w\beta\Omega^{-\phi(1+\chi)}E_t\left[\left(\frac{P_{t+1}}{P_t}\right)wb_{t+1}\right]
\end{aligned}$$

Similarly, wa_t can be rewritten as:

$$
\begin{aligned}
wa_t &= P_t^{1+\phi\chi}\frac{WA_t}{P_t^{\phi(1+\chi)}} = P_t^{1+\phi\chi}E_t\sum_{\tau=t}^{\infty}(\xi_w\beta)^{\tau-t}\left\{\frac{\lambda_\tau\left[\left(\Omega^{\tau-t}\right)^{1-\phi}w_\tau^{\phi}P_\tau^{\phi}N_\tau\right]}{P_t^{\phi(1+\chi)}}\right\} \\
&= P_t^{1+\phi\chi}E_t\sum_{\tau=t}^{\infty}(\xi_w\beta)^{\tau-t}\left\{\frac{\tilde{\Lambda}_\tau\left[\left(\Omega^{\tau-t}\right)^{1-\phi}w_\tau^{\phi}P_\tau^{\phi-1}N_\tau\right]}{P_t^{\phi(1+\chi)}}\right\} \\
&= E_t\sum_{\tau=t}^{\infty}(\xi_w\beta)^{\tau-t}\left\{\tilde{\Lambda}_\tau\left[\left(\Omega^{\tau-t}\right)^{1-\phi}w_\tau^{\phi}\left(\frac{P_\tau}{P_t}\right)^{\phi-1}N_\tau\right]\right\} \\
&= \tilde{\Lambda}_t w_t^{\phi}N_t + \xi_w\beta\Omega^{1-\phi}E_t\left[\left(\frac{P_{t+1}}{P_t}\right)^{\phi-1}wa_{t+1}\right]
\end{aligned}
$$

The other key equations are the aggregate wage equation, and the respective labour supply equations for the optimizing and rule of thumb households. Consider the aggregate wage equation:

$$W_t^{1-\phi} = v\left(\tilde{W}_t^{r}\right)^{1-\phi} + (1-v)\left(W_t^{*}\right)^{1-\phi}$$

Dividing through by $P_t^{1-\phi}$ yields:

$$w_t^{1-\phi} = v\left(\tilde{w}_t^{r}\right)^{1-\phi} + (1-v)\left(w_t^{*}\right)^{1-\phi}$$

where

$$
\begin{aligned}
\left(w_t^{*}\right)^{1-\phi} &= \left(\frac{W_t^{*}}{P_t}\right)^{1-\phi} \\
&= P_t^{-(1-\phi)}\sum_{k=0}^{\infty}(1-\xi_w)\,\xi_w^{k}\left(\Omega^{k}\right)^{1-\phi}\left(\tilde{W}_{t-k}^{o}\right)^{1-\phi} \\
&= (1-\xi_w)\left(\tilde{w}_t^{o}\right)^{1-\phi} + \xi_w\Omega^{1-\phi}\left(\frac{P_{t-1}}{P_t}\right)^{1-\phi}\left(w_{t-1}^{*}\right)^{1-\phi}
\end{aligned}
$$

With regards to the labour supply for the two types of households:

$$
\begin{aligned}
L_t^{r} &= N_t w_t^{\phi}\left(\tilde{w}_t^{r}\right)^{-\phi} \\
L_t^{o*} &= N_t w_t^{\phi}\left(wc_t^{*}\right)^{-\phi}
\end{aligned}
$$

where

$$
\begin{aligned}
(wc_t^*)^{-\phi} &= \left(\frac{WC_t^*}{P_t}\right)^{-\phi} \\
&= P_t^{\phi}\left(1-\xi_w\right)\left(\tilde{W}_t^o\right)^{-\phi} + \xi_w \Omega^{-\phi}\left(WC_{t-1}^*\right)^{-\phi} \\
&= \left(1-\xi_w\right)\left(\tilde{w}_t^o\right)^{-\phi} + \xi_w \Omega^{-\phi}\left(\frac{P_{t-1}}{P_t}\right)^{-\phi}\left(wc_{t-1}^*\right)^{-\phi}
\end{aligned}
$$

The "real" wage setting equation for rule of thumb is given above, by equation (A.45).

A.6 Price/Wage Inflation Version Of The Model

The model can also be written in a price/wage inflation version. This was done due to nonlinearities in the wage nexus which meant that the model was not able to converge to a steady state. This version of the model involves the wage nexus. Dividing the wage setting equations through by W_t instead of P_t so that wages for the optimizing and myopic households are written as relative wages (- relative to the aggregate).

The wage picked by optimizing households can be rewritten in a similar fashion to the price set by firms, i.e. given

$$
\left(\tilde{W}_t^o\right)^{1+\phi\chi} = \kappa\mu_w\left(\frac{WB_t}{WA_t}\right)
$$

where

$$
\begin{aligned}
WB_t &= \left(N_t W_t^{\phi}\right)^{1+\chi} + \xi_w \beta \Omega^{-\phi(1+\chi)} E_t WB_{t+1} \\
WA_t &= \lambda_t N_t W_t^{\phi} + \xi_w \beta \Omega^{1-\phi} E_t WA_{t+1}
\end{aligned}
$$

Dividing $\left(\tilde{W}_t^o\right)^{1+\phi\chi}$ through by $W_t^{1+\phi\chi}$:

$$
\left(\tilde{w}_t^o\right)^{1+\phi\chi} = \left(\frac{\tilde{W}_t^o}{W_t}\right)^{1+\phi\chi} = \kappa\mu_w\left(\frac{wb_t}{wa_t}\right)
$$

where

$$
\begin{aligned}
wb_t &= \frac{WB_t}{W_t^{\phi(1+\chi)}} = W_t^{-\phi(1+\chi)} E_t \sum_{\tau=t}^{\infty} (\xi_w \beta)^{\tau-t} \left[\left(\frac{\Omega^{\tau-t}}{W_\tau} \right)^{-\phi(1+\chi)} N_\tau^{1+\chi} \right] \\
&= E_t \sum_{\tau=t}^{\infty} (\xi_w \beta)^{\tau-t} \left[\left(\Omega^{\tau-t} \right)^{-\phi(1+\chi)} N_\tau^{1+\chi} \left(\frac{W_\tau}{W_t} \right)^{\phi(1+\chi)} \right] \\
&= N_t^{1+\chi} + \xi_w \beta \Omega^{-\phi(1+\chi)} E_t \left[\left(\frac{W_{t+1}}{W_t} \right) \sum_{\tau=t+1}^{\infty} (\xi_w \beta)^{\tau-t} \left(\Omega^{\tau-t} \right)^{-\phi(1+\chi)} N_\tau^{1+\chi} \right] \\
&= N_t^{1+\chi} + \xi_w \beta \Omega^{-\phi(1+\chi)} E_t \left[\left(\frac{P_{t+1}}{P_t} \right) wb_{t+1} \right]
\end{aligned}
$$

Similarly, wa_t can be rewritten as:

$$
\begin{aligned}
wa_t &= \frac{WA_t}{W_t^{\phi-1}} = E_t \sum_{\tau=t}^{\infty} (\xi_w \beta)^{\tau-t} \left\{ \frac{\lambda_\tau \left[\left(\Omega^{\tau-t} \right)^{1-\phi} W_\tau^{\phi} N_\tau \right]}{W_t^{\phi-1}} \right\} \\
&= E_t \sum_{\tau=t}^{\infty} (\xi_w \beta)^{\tau-t} \left\{ \lambda_\tau \left[\left(\Omega^{\tau-t} \right)^{1-\phi} \left(\frac{W_\tau}{W_t} \right)^{\phi} W_t N_\tau \right] \right\} \\
&= E_t \sum_{\tau=t}^{\infty} (\xi_w \beta)^{\tau-t} \left\{ \tilde{\Lambda}_\tau \left[\left(\Omega^{\tau-t} \right)^{1-\phi} \left(\frac{W_\tau}{W_t} \right)^{\phi} \left(\frac{W_t}{P_t} \right) N_\tau \right] \right\} \\
&= \tilde{\Lambda}_t w_t N_t + \xi_w \beta \Omega^{1-\phi} E_t \left[\left(\frac{W_{t+1}}{W_t} \right)^{\phi} wa_{t+1} \right]
\end{aligned}
$$

The other key equations are the aggregate wage equation, and the respective labour supply equations for the optimizing and rule of thumb households. Consider the aggregate wage equation:

$$
W_t^{1-\phi} = v \left(\tilde{W}_t^r \right)^{1-\phi} + (1-v) \left(W_t^* \right)^{1-\phi}
$$

Dividing through by $W_t^{1-\phi}$ yields:

$$
1 = v \left(\frac{\tilde{w}_t^r}{w_t} . \frac{1}{\pi_t} \right)^{1-\phi} + (1-v) \left(w_t^* \right)^{1-\phi}
$$

where $\tilde{w}_t^r \equiv \left(\frac{\tilde{W}_t^r}{P_t} \right)$ and

$$
\begin{aligned}
\left(w_t^*\right)^{1-\phi} &= \left(\frac{W_t^*}{W_t}\right)^{1-\phi} \\
&= W_t^{-(1-\phi)} \sum_{k=0}^{\infty} \left(1-\xi_w\right) \xi_w^k \left(\Omega^k\right)^{1-\phi} \left(\tilde{W}_{t-k}^o\right)^{1-\phi} \\
\left(w_t^*\right)^{1-\phi} &= \left(1-\xi_w\right) \left(\tilde{w}_t^o\right)^{1-\phi} + \xi_w \Omega^{1-\phi} \left(\frac{W_{t-1}}{W_t}\right)^{1-\phi} \left(w_{t-1}^*\right)^{1-\phi}
\end{aligned}
$$

With regards to the labour supply for the two types of households:

$$
\begin{aligned}
L_t^r &= N_t \left(\frac{\tilde{w}_t^r}{w_t}\right)^{-\phi} \\
L_t^{o*} &= N_t \left(wc_t^*\right)^{-\phi}
\end{aligned}
$$

where

$$
\begin{aligned}
\left(wc_t^*\right)^{-\phi} &= \left(\frac{WC_t^*}{W_t}\right)^{-\phi} \\
&= W_t^{\phi} \left(1-\xi_w\right) \left(\tilde{W}_t^o\right)^{-\phi} + \xi_w \Omega^{-\phi} \left(WC_{t-1}^*\right)^{-\phi} \\
&= \left(1-\xi_w\right) \left(\tilde{w}_t^o\right)^{-\phi} + \xi_w \Omega^{-\phi} \left(\frac{W_{t-1}}{W_t}\right)^{-\phi} \left(wc_{t-1}^*\right)^{-\phi}
\end{aligned}
$$

A.7 Contemporaneous Wage Setting Case

In the contemporaneous wage setting case, the union is assumed to be able to observe any shocks prior to setting the wage. Let $\tilde{W}_t^r$ be the wage posted by the union in maximizing:

$$
\begin{aligned}
& \max U\left(\tilde{W}_t^r\right) \\
= \;& \max \left(\frac{1}{1-\sigma}\right) \left[\left(\frac{\tilde{W}_t^r}{P_t}\right) \left(\frac{\tilde{W}_t^r}{W_t}\right)^{-\phi} N_t\right]^{1-\sigma} - \frac{\kappa}{1+\chi} \left[\left(\frac{\tilde{W}_t^r}{W_t}\right)^{-\phi} N_t\right]^{1+\chi}
\end{aligned}
$$

The FOC for $\tilde{W}_t^r$ is:

$$
(1-\phi)\left(C_t^r\right)^{-\sigma} P_t^{-1}\left(\frac{\tilde{W}_t^r}{W_t}\right)^{-\phi} N_t-\kappa\left(L_t^r\right)^\chi(-\phi)\left(\frac{\tilde{W}_t^r}{W_t}\right)^{-\phi} \frac{N_t}{\tilde{W}_t^r}=0 \tag{A.50}
$$

$$
\text{i.e. } \left(\frac{\tilde{W}_t^r}{W_t}\right)^{-\phi} N_t\left((1-\phi)\left[\left(\frac{\tilde{W}_t^r}{P_t}\right)\left(\frac{\tilde{W}_t^r}{W_t}\right)^{-\phi} N_t\right]^{-\sigma}\right.
$$

$$
\left.+\kappa\phi\left[\left(\frac{\tilde{W}_t^r}{W_t}\right)^{-\phi} N_t\right]^\chi\left(\tilde{W}_t^r\right)^{-1}\right)=0
$$

Dividing through by $\phi-1$ and rearranging yields the wage setting equation for the rule of thumb households:

$$
\left(\tilde{W}_t^r\right)^{1+\phi\chi+\sigma(\phi-1)}=\mu_w \kappa P_t^{1-\sigma} W_t^{\phi(\chi+\sigma)} N_t^{\chi+\sigma} \tag{A.51}
$$

This can be re-written in real terms by dividing through by $P_t^{1+\phi\chi+\sigma(\phi-1)}$ which yields:

$$
\begin{aligned}
\left(\tilde{w}_t^r\right)^{1+\phi\chi+\sigma(\phi-1)} & = \left(\frac{\tilde{W}_t^r}{P_t}\right)^{1+\phi\chi+\sigma(\phi-1)} \\
& = \mu_w \kappa\left(\frac{W_t}{P_t}\right)^{\phi(\chi+\sigma)} N_t^{\chi+\sigma} \\
& \Rightarrow \left(\tilde{w}_t^r\right)^{1+\phi\chi+\sigma(\phi-1)}=\mu_w \kappa\left(w_t\right)^{\phi(\chi+\sigma)} N_t^{\chi+\sigma}
\end{aligned} \tag{A.52}
$$

The wage inflation version of this equation is obtained by dividing equation (A.51) through by $W_t^{1+\sigma(\phi-1)+\phi\chi}$:

$$
\begin{aligned}
\left(\tilde{w}_t^r\right)^{1+\sigma(\phi-1)+\phi\chi} & = \left(\frac{\tilde{W}_t^r}{W_t}\right)^{1+\sigma(\phi-1)+\phi\chi} \\
& = \frac{\mu_w \kappa P_t^{1-\sigma} W_t^{\phi(\chi+\sigma)} N_t^{\chi+\sigma}}{\left(W_t\right)^{1+\sigma(\phi-1)+\phi\chi}} \\
& = \mu_w \kappa \frac{N_t^{\chi+\sigma}}{w_t^{1-\sigma}}
\end{aligned}
$$

B Steady State Equations

Let $\tilde{\Lambda}_t \equiv P_t \lambda_t$, $\tilde{p}_t = \frac{\tilde{P}_t}{P_t}$, $\tilde{w}_t^r \equiv \frac{\tilde{W}_t^r}{W_t}$, $\tilde{w}_t^o = \frac{\tilde{W}_t^o}{W_t}$ and $w_t^* = \frac{W_t^*}{W_t}$. Equations D1-D8, F1-F8, H1-H12 and E1-E14 are the key equations used to find the steady state:

B.1 Definitional Equations (D)

	Model equation	Steady State
D1.	$\mu_p = \frac{\theta}{\theta - 1}$	$\mu_p = \frac{\theta}{\theta - 1}$
D2.	$\mu_w = \frac{\phi}{\phi - 1}$	$\mu_w = \frac{\phi}{\phi - 1}$
D3.	$AC_t = \left(\frac{I_t^o}{K_t^o}\right) - \delta$	$AC_f = 0$
D4.	$Q_t = \frac{1}{(1 - h.AC_t)}$	$Q_f = 1$
D5.	$AQ_t = \left(\frac{I_t^o}{K_t^o}\right) AC_t$	$AQ_f = 0$
D6.	$\tilde{\Lambda}_{t+1}^R = \tilde{\Lambda}_{t+1}(r_t^k u_t - \frac{1}{2}\eta u_t \left(u_t - 1\right)))$	$\tilde{\Lambda}_f^R = r^k$
D7.	$\tilde{\Lambda}_t^Q = \tilde{\Lambda}_t Q_t$	$\tilde{\Lambda}_f^Q = 1$
D8.	$\Phi = (\alpha)^\alpha (1 - \alpha)^{1-\alpha}$	$\Phi = (\alpha)^\alpha (1 - \alpha)^{1-\alpha}$

B.2 Firm Side (F)

	Model Equation	Steady State
F1.	$Y_t = Z_t K_t^\alpha N_t^{1-\alpha}$	$Y_f = K_f^\alpha N_f^{1-\alpha}$
F2.	$\ln Z_t = \rho \ln Z_{t-1} + \varepsilon_{z,t}$	$Z_f = 1$ with $\ln Z_0 = 0$;
F3.	$r_t^k K_t = \left(\frac{\alpha}{1-\alpha}\right) w_t N_t$	$r^k K = \left(\frac{\alpha}{1-\alpha}\right) wN$
F4.	$s_t = \Phi^{-1} Z_t^{-1} \left(\frac{R_t^k}{P_t}\right)^\alpha \left(\frac{W_t}{P_t}\right)^{1-\alpha}$	$s_f = \Phi^{-1} \left(r^k\right)^\alpha (w)^{1-\alpha}$

Firm Side (F) Cont...

	Model Equation	Steady State
F5.	$\tilde{p}_t = \mu_p \frac{pb_t}{pa_t}$	$\tilde{p} = \mu_p \frac{pb}{pa}$
F6.	$pb_t = \tilde{\Lambda}_t Y_t s_t + \xi_p \Omega^{-\theta} E_t \left[\left(\frac{P_{t+1}}{P_t} \right)^{\theta} pb_{t+1} \right]$	$pb = \left(\frac{\tilde{\Lambda} Y s}{1-\xi_p} \right)$
F7.	$pa_t = \tilde{\Lambda}_t Y_t + \xi_p \Omega^{1-\theta} E_t \left[\left(\frac{P_{t+1}}{P_t} \right)^{\theta-1} pa_{t+1} \right]$	$pa = \frac{\tilde{\Lambda} Y}{1-\xi_p}$
F8.	$1 = \left(1 - \xi_p\right) \tilde{p}_t^{1-\theta} + \xi_p \left(\Omega \frac{P_{t-1}}{P_t} \right)^{1-\theta}$	$1 = \tilde{p}$

B.3 Optimizing Households (H)

	Model Equation	Steady State
H1.	$(C_t^o)^{-\sigma} = \tilde{\Lambda}_t$	$(C^o)^{-\sigma} = \tilde{\Lambda}$
H2.	$R_t \beta E_t \left[\frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t} \frac{P_t}{P_{t+1}} \right] = 1$	$R\beta = 1$
H3.	$\tilde{\Lambda}_t = P_t \lambda_t$	$\tilde{\Lambda} = \lambda = (C^o)^{-\sigma}$
H4.	$Q_t^{-1} \zeta_t = \tilde{\Lambda}_t$	$\zeta = \lambda = (C^o)^{-\sigma}$
H5.	$\tilde{\Lambda}_t Q_t = \beta E_t \left\{ \tilde{\Lambda}_{t+1}^R + \tilde{\Lambda}_t^Q [(1-\delta) - \frac{1}{2} hAC_{t+1}^2 \right.$	$\left. + hAC_{t+1} \left(\frac{I_{t+1}^o}{K_{t+1}^o} \right) \right] \Big\}$
	and in the steady state, this equals...	$1 = \beta \left(r^k + 1 - \delta \right)$
H6.	$(\tilde{w}_t^o)^{1+\phi\chi} = \kappa \mu_w \left(\frac{wb_t}{wa_t} \right)$	$(\tilde{w}^o)^{1+\phi\chi} = \kappa \mu_w \left(\frac{wb}{wa} \right)$
H7.	$wb_t = N_t^{1+\chi} + \xi_w \beta \Omega^{-\phi(1+\chi)} E_t \left[\left(\frac{P_{t+1}}{P_t} \right) wb_{t+1} \right]$	$wb = \frac{1}{1-\xi_w \beta} N^{1+\chi}$
H8.	$wa_t = \tilde{\Lambda}_t w_t N_t + \xi_w \beta \Omega^{1-\phi} E_t \left[\left(\frac{W_{t+1}}{W_t} \right)^{\phi} wa_{t+1} \right]$	$wa = \frac{1}{1-\xi_w \beta} (C^o)^{-\sigma} Nw$
H9.	$K_{t+1}^o = (1-\delta) K_t^o + \left(\frac{I_t^o}{K_t^o} - \frac{1}{2} \psi AC_t^2 \right) K_t^o$	$1 = 1$
H10.	$R_t^k = P_t \eta u_t \left(u_t - \frac{1}{2} \right)$	$R_t^k = a' (1)$

B.4 Rule of Thumb Households/ Union (H)

	Model Equation	Steady State
H11.	$C_t^r = \tilde{w}_t^r L_t^r$	$C_t^r = \tilde{w}_t^r L_t^r$
H12.	$(\tilde{w}_t^r)^{1+\phi\chi+\sigma(\phi-1)} = \mu_w \kappa \frac{E_{t-1}\left[w_t^{\phi(1+\chi)} \pi_t^{\phi(1+\chi)} N_t^{1+\chi}\right]}{E_{t-1}\left[w_t^{\phi(1-\sigma)} \pi_t^{(\phi-1)(1-\sigma)} N_t^{1-\sigma}\right]}$	

and in the Steady State this equals...

$$(\tilde{w}_t^r)^{1+\phi\chi+\sigma(\phi-1)} = \mu_w \kappa \frac{E_{t-1}\left[w_t^{\phi(1+\chi)} \pi_t^{\phi(1+\chi)} N_t^{1+\chi}\right]}{E_{t-1}\left[w_t^{\phi(1-\sigma)} \pi_t^{(\phi-1)(1-\sigma)} N_t^{1-\sigma}\right]}$$

where $\tilde{w}_t^r \equiv \frac{\tilde{W}_t^r}{P_{t-1}}$

B.5 Aggregate Equations, Monetary Policy and Market Clearing Conditions (E)

	Model Equation	Steady State
E1.	$1 = v\left(\frac{\tilde{w}_t^r}{w_t}.\frac{1}{\pi_t}\right)^{1-\phi} + (1-v)\left(w_t^*\right)^{1-\phi}$	$1 = v\left(\tilde{w}^r\right)^{1-\phi} + (1-v)\left(w^*\right)^{1-\phi}$
E2.	$\left(w_t^*\right)^{1-\phi} = (1-\xi_w)\left(\tilde{w}_t^o\right)^{1-\phi} + ...$	$w_t^* = \tilde{w}_t^o$
	$... + \xi_w \Omega^{1-\phi}\left(\frac{W_{t-1}}{W_t}\right)^{1-\phi}\left(w_{t-1}^*\right)^{1-\phi}$	
E3.	$C_t = vC_t^r + (1-v)\,C_t^o$	$C = vC^r + (1-v)\,C^o$
E4.	$N_t = L_t = vL_t^r + (1-v)\,L_t^{o*}$	$N = vL^r + (1-v)\,L^{o*}$
E5.	$L_t^r = N_t W_t^{\phi}\left(\tilde{W}_t^r\right)^{-\phi}$	$L^r = NW^{\phi}\left(\tilde{W}^r\right)^{-\phi}$
E5.	$L_t^r = N_t\left(\tilde{w}_t^r\right)^{-\phi}$	$L^r = N\left(\tilde{w}^r\right)^{-\phi}$
E7.	$L_t^{o*} = N_t\left(wc_t^*\right)^{-\phi}$	$L^o = N\left(wc^*\right)^{-\phi}$
E8.	$\left(wc_t^*\right)^{-\phi} = (1-\xi_w)\left(\tilde{w}_t^o\right)^{-\phi} + ...$	$wc_t^* = \tilde{w}_t^o$
	$... + \xi_w \Omega^{-\phi}\left(\frac{W_{t-1}}{W_t}\right)^{-\phi}\left(wc_{t-1}^*\right)^{-\phi}$	
E9.	$K_t = (1-v)\,u_t K_t^o$	$K = (1-v)\,K^o$
E10.	$I_t = (1-v)\,I_t^o$	$I = (1-v)\,I^o$
E11.	$\Omega = \frac{P}{P_{-1}}$	$\Omega = 1$
E12.	$\pi_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$	$\pi = 0$
E13.	$r_t = (1-\gamma_r)\,\bar{r} + \gamma_r r_{t-1} + ...$	$r = \frac{1}{\beta} - 1$
	$... + (1-\gamma_r)\,\gamma_\pi\left(\pi_t - \pi^*\right) + \varepsilon_{m,t}$	
E14.	$Y_t = C_t + I_t + (1-v)a(u_t)K_t^o$	$Y = C + I$

B.6 Full Information Version

In the full information version, the following equations are replaced:

	Model Equation	Steady State
H12.	$(\tilde{w}_t^r)^{1+\sigma(\phi-1)+\phi\chi} = \mu_w \kappa w_t^{\sigma-1} N_t^{\chi+\sigma}$	$(\tilde{w}_t^r)^{1+\sigma(\phi-1)+\phi\chi} = \mu_w \kappa w_t^{\sigma-1} N_t^{\chi+\sigma}$
E1.	$1 = v\,(\tilde{w}_t^r)^{1-\phi} + (1-v)\,(w_t^*)^{1-\phi}$	$1 = v\,(\tilde{w}^r)^{1-\phi} + (1-v)\,(w^*)^{1-\phi}$

where $\tilde{w}_t^r \equiv \frac{\tilde{W}_t^r}{W_t}$.