Incentives to Cooperate in Network Formation

Haydée Lugo*

Universidad Simón Bolívar

Abstract

We propose a mechanism based on taxes and subsidies that enhances high cooperation in evolutionary networks. The interactions among agents are based on a Prisoners' Dilemma game in which each agent plays the same strategy with its local neighbors, collects an aggregate payoff and imitates the strategy of its best neighbor. The network can be adaptive if agents are able to change their local neighborhood according to their satisfaction level and the strategy played. The condition, in order to obtain highly cooperative non-taxed networks in the long-run time, is that the initial fraction of cooperators has to be sufficiently high. Focussing on this restriction, the implementation of our mechanism produces successful results, a highly cooperative network is reached. Additionally, we observe that the mechanism slightly affects the macrostructure of networks once they have reached a sufficiently high fraction of cooperative agents, this suggests that the mechanism could be implemented only for a short finite period of time.

1 Introduction

There is an increasing number of studies that examine agent-based models in which a social macrostructure can emerge from agents interactions. Researchers often try to find explanations of how such macrostructure emerges from the individual characteristics of the agents and how such characteristics can be influenced by incentives. The classic Prisoners' Dilemma game is perhaps the most popular game that has been used as the basis of such studies. The reason for this can be for its simplicity in showing how a dominant strategy equilibrium can be Pareto inefficient. The general result in these studies is that the incentives to defect can be very powerful. Despite of this, there is a growing number of experiments that examine mechanisms searching for Pareto efficient outcomes in a PD game. It is interesting to note that agents do respond to incentives to cooperate. For instance, Roth and Murninghan [3] found that in a game with an uncertain end point the agents were more cooperative the longer the anticipated horizon. Selten and Stoecker [4] and Andreoni and Miller [1] look at finitely repeated games and find that agents will be more cooperative when they can build reputations. Andreoni and Varian [2] consider a modified game in which each agent can offer to pay the other agent to cooperate. In their experimental game, they find an encouraging level of support for the mechanism.

Zimmermann *et al.* [5] tackle the problem of how cooperation arises in a dynamically evolving network of agents. In their adaptive network, agent are able to *adapt* their local neighborhood

*Universidad Simón Bolívar. Departamento de Computo Científico y Estadística, AP. 89000, Caracas 1080, Venezuela. Email: hlugo@usb.ve.

according to their satisfaction level and the strategy played. Their study shows how adaptability enhances a highly cooperative network. They pointed out that in order to obtain their results, the *initial* fraction of cooperators in the network must be sufficiently high. They argued that the cooperative strategy will replicate throughout the network, only if the approximate average payoff for agents that cooperate is larger than the one for agents that defect.

In this context, an interesting question arises, Can incentives to keep agents cooperating have a strong influence in the emerging macrostructure of networks?. It is this question that we want to address in this paper.

We propose a mechanism that incentives cooperative agents to keep cooperating. We are interesting in examining whether this mechanism may evolve a network of agent interactions. The implications of the proposed mechanism are studied with computational simulations that allow to show the dynamical evolution of the social structures. We find that this mechanism is very powerful either in adaptive or non-adaptive networks of low initial fraction of cooperative agents. Also, we find that the mechanism preserves the macrostructure of networks when they have a high fraction of cooperative agents.

The paper is organized as follows. In the next section, we make a brief description of the spatial version, discussed in [5], of the PD game in an adaptive network. In Section 3, we present the mechanism that incentives agents to keep cooperating. In Section 4, we describe our numerical simulations on the emergence of cooperation. Finally, the paper ends with a concluding section.

2 The adaptive network

We are going to approach a quite different version of the framework used by Zimmermann *et al* [5]:

N is the number of agents at nodes of an adaptive network Γ. We consider undirected or bidirectional links to define the neighborhoods. Thus, two agents are neighbors if they are connected by one link. The neighborhood of the agent i is a subset of Γ denoted by neigh(i) and K_i its cardinality. The coordination number K is defined as the expected average of neighbors per agent,

$$K = \mathbf{E}\left(\frac{\sum_{i=1}^{N} K_i}{N}\right)$$

The number of links in the network, $L = \sum_{i=1}^{N} K_i/2$, is a Poisson random variable with expected value $KN/2$. Random network Γ with coordination number K are formed by distributing L links without replacement on pairs of nodes (i, j), where $(i, j) = (j, i)$.

Each agent plays a PD game only with those agents directly connected by one link. The strategy of agent i at time step n is denoted by $s_i(n)$, where $s_i = 1$ corresponds to play cooperation (C), and $s_i = 0$ corresponds to defection (D). That refers to C-agents and D-agents, respectively. The payoff bi-matrix for a classic one-shot two-agent PD game is shown in table 1, where $b > \sigma > \delta > 0$ and $b/2 < \sigma$. For this PD game, there exists an unique Nash equilibrium (D, D) that is Pareto inefficient.

Based on the Prisoners Dilemma payoff matrix, Zimmermann *et al* consider the situation in which each agent plays the same strategy with all its neighbors neigh(i), and the strategy is

	C	D
C	σ, σ	$0, b$
D	$b, 0$	δ, δ

Table 1: Prisoners Dilemma payoff bi-matrix

updated by all the agents at the same time seeking for the largest possible benefit from their local interactions in the network Γ. The time step evolution of the adaptive network is as follows:

Step 1: Each agent i plays the PD game with each neighbor using the same strategy s_i and collecting a total individual payoff Π_i,

$$\Pi_i = s_i \mu_i \sigma + (1 - s_i)[\mu_i b + (K_i - \mu_i)\delta]$$

where K_i is the number of links of agent i and μ_i is the number of neighbors of agent i that are C-agents.

Step 2: Agent i revises its current strategy each time step and updates it by imitating the strategy of its neighbor with a highest payoff. The strategies are updated by all the agents at the same time in a synchronous update. Agent i is satisfied if it has the largest payoff in its neighborhood. Otherwise, agent i will be unsatisfied and it will revise its strategy.

Step 3: Each D-agent may adapt its local neighborhood. If a D-agent is unsatisfied then, with probability p, breaks a link with each D-neighbor $\in$ neigh(i), and replace it with a new agent chosen randomly from Γ.

For convenience, the adaptive network described by rules 1-3 will be called ZESM-network. In this network, the coordination number K remains constant: for each unsatisfied D-agent, it will replace on average D-neighbors by new neighbors randomly chosen for the whole set, and thus its number of neighbors K_i will not change. The total number of links in the network Γ is conserved since the replaced D-agents will lose one link and the new selected ones will gain one link. Therefore, a C-agent may increase its number of neighbors by receiving new links from searching D-agents. Destruction or spontaneous creation of links are not taking into account in this model. They also assume that links between satisfied agents do not change. Zimmermann *et al* justified this asymmetry situation in which only D-agents have searching capability, describing D-agents as *competitive*, while C-agents as *conservative*.

3 A mechanism of taxes and subsidies

The network rule describing in *step 3* only applies for D-agents, C-agents do not have preference to break links to D-agents. This induces the normal amount of defection and frustration among agents, specifically when the initial fraction of C-agents is not sufficiently high. We introduce a compensation mechanism to reverse this situation. The idea is that C-agents can be compensated for the cost that they incur for being conservatives. This compensation works as an incentive to keep cooperating. We find this mechanism to be quite successful specifically in networks of low initial fraction of cooperators.

We begin by describing the exact form of this compensation mechanism. Consider again the following two dynamical rules:

1. The *action rule* in which, at each time step, each agent plays the same strategy with all its local neighbors. Then the agents imitate the neighbors strategy with highest aggregate payoff.

2. The *network rule* in which, unsatisfied D-agents are allowed to change with a certain probability any link with others defectors and replace them with new neighbors randomly selected from the whole network.

We aggregate in the *action rule* a step in which C-agents are compensated by an extra payoff. Cooperators receive this compensation after they play with their neighbors but before they imitate the neighbors strategy with highest aggregate payoff. The motivation behind this, it is to avoid that exploited cooperators with low level of satisfaction switch their strategy to defect. We argue that agents could positively respond to incentives to cooperate.

Where this compensation come from?. There can be many answers to this question. We approach it by a tax-subsidy scheme. Let's imagine one individual outside the network acting as a central planner, government or state, collecting taxes and paying subsidies. Let it happens after step 1 but before step 2 of the time evolution of the adaptive network. The government rewards C-agents paying them a subsidy β that works as an incentive to cooperate. The cost of the government incentive βC, where C is the number of cooperators at that stage, is paid with a system of involuntary taxes. Therefore, each agent i is taxed for a fixed share t of his aggregate payoff, that is $t\Pi_i$, where $0 \leq t < 1$.

The mechanism works as follows, after step 1, the government observes the aggregate payoffs $\Pi_1, \ldots, \Pi_N$ and collects taxes $t\Pi_i$ from each i and pays a subsidy β to each C-agent. The government's total expenditure on subsidies βC is paid by $t\sum_{i=1}^{N}\Pi_i$. The net tax obligation of a C-agent is $t\Pi_i - \beta = t(\Pi_i - \Pi/f_C)$, where $\Pi = (\sum_{i=1}^{N}\Pi_i)/N$ and $f_C = C/N$. Each agent i collects a new total individual payoff Π_i,

$$\Pi_i = (1-t)(s_i\mu_i\sigma + (1-s_i)(\mu_i b + (K_i - \mu_i)\delta)) + s_i\beta.$$

As before, K_i is the number of links of agent i and μ_i is the number of neighbors of agent i that are C-agents. Since the parameter b has been considered the incentive to defect in the PD game, we consider the subsidy β the incentive to cooperate in the network formation.

It is important to notice that agents are not directly awake of the central planner's tastes for cooperation and equity. In fact, if an agent were aware of the central planner's tastes and could take the strategies of his neighbors as given then in order to maximize his payoff he could be playing a non-cooperative game and be reaching a Nash equilibrium. Actually, what each agent do in the network is to seek the largest possible benefit from local interactions imitating the strategy of his neighbor with a highest payoff.

In the next section, we present some results of computer simulations. We are going to look at the following statistics:

(i) the fraction of cooperators f_C, and

(ii) the average payoff per agent $\Pi = (\sum_{i=1}^{N}\Pi_i)/N$ of the whole network.

We examine our random networks with two initial fractions of C-agents:

(a) Initial fraction of C-agents equal and bigger than 0.6, where the ZESM-network achieves high fraction of cooperators in the steady state.

(b) One initial C-agent, the worst case of small initial fraction where the ZESM-network achieves a state of full defectors.

4 Dynamics of networks under taxes and subsidies

In [5], the authors compare the differences between non-adaptive and full adaptive networks. They observe that in the non-adaptive case, the fraction of cooperators fluctuates slightly, while in the adaptive case a steady state is reached with a highly cooperative level. They point out that their results are obtained only if the initial fraction of cooperators in the network is sufficiently high. In fact, an initial fraction of 0.6 of C-agents proved them to be a good number for the coordination numbers studied. They argue, and it is noticed in previous spatial games, that the cooperative strategy will replicate throughout the network, only if (the approximate) average payoff of C-agents is larger than the one of D-agents. We are going to show in the next subsections how our mechanism of taxes and subsidies makes such networks achieve proper levels of cooperation for any initial fraction of cooperators. Additionally, the mechanism preserves the asymptotic behavior observed in networks with highly enough fraction of cooperators in both cases, non-adaptive and full adaptive networks. Also, some aspects related with efficiency and equity are discussed.

In order to make suitable comparisons with Zimmerman *et al* results, we consider as well random initial network with coordination number $K = 4$ and $K = 8$ and full adaptive network $p = 1$. The parameter b which controls the incentive to defect varies in the range $1 < b < 2$. The other parameter were fixed to $\delta = 0.1$ and $\sigma = 1$.

4.1 The mechanism on full adaptive networks

Table 2 shows the averages of the fractions of C-agents, at the steady state, of full adaptive networks ($p = 1$) with and without taxation. The averages are taking over 10 different initial conditions after 30 time steps of evolution.

First, let us examine the case of initial $0.6N$ C-agents without taxation ($t = 0$). That is the case studied in [5]. We can observe how the average of the fractions of C-agents, at the steady state, decreases with an increasing value of the incentive to defect. The numerical results also show that increasing the coordination number, K, the average fraction of C-agents decreases faster with b. Now, if we apply the mechanism of taxes and subsidy described above with $t = 0.05$, we observe that an increasing value of the average number of links per agent K, does not significantly decrease the average fraction of C-agents. That means, the mechanism is quite robust from changes of K.

As we can see, the most remarkable result occurs in networks with only one C-agent in the initial time step. We observe how the mechanism of taxes and subsidies has a successful effect in such networks, the initial cooperator positively responds to the incentive to keep cooperating and produces the effect for defectors to change their strategies. The mechanism enhances highly cooperative networks for both initial fractions of C-agents.

Figure 1 illustrates the asymptotic behaviors of taxed and non-taxed networks for $b = 1.35$ and $K = 4$. We show in Figure 1 (a) the time series of f_C with initial fraction of $0.6N$ C-agents. We observe that the fraction of cooperators increases in both cases (taxed and non-taxed). Similar results were observed for the different values of b and K considered in Table 2. We

	0.6N C-agents at initial time				One C-agent at initial time			
	$K=4$		$K=8$		$K=4$		$K=8$	
b	$t=0$	$t=.05$	$t=0$	$t=.05$	$t=0$	$t=.05$	$t=0$	$t=.05$
1.05	0.951	0.966	0.898	0.995	0	0.968	0	0.996
1.15	0.943	0.943	0.697	0.992	0	0.942	0	0.996
1.35	0.819	0.948	0.395	0.884	0	0.920	0	0.990
1.55	0.526	0.777	0.196	0.785	0	0.812	0	0.685
1.75	0.443	0.639	0.100	0.578	0	0.630	0	0.598
1.95	0.346	0.514	0.000	0.371	0	0.335	0	0.396

Table 2: Averages of the fractions of C-agents of full adaptive networks.

conclude that the taxation enhances a highly cooperative network comparable with the non-taxed scheme. In the case of initial one C-agent, the results are extremely different between taxes and non-taxes case. Figure 1 (c) shows how the only cooperator switches to defect and a full defective network is reached in the non-taxes case, in contrast with the taxes case, where the mechanism enhances a highly cooperative network. The mechanism of taxation not only preserves the dynamics that settles onto a steady state after some transient time, also preserves the properties described in [5] such as the emergence of a leader and a cooperator with maximum number of connections.

The graphics (b) and (d), in Figure 1, show the effect that the mechanism of taxation has over the average payoff of the whole network. Although the mechanism could be seem to be socially unfair, because all agents are taxed but only C-agents receive the incentive, the increasing number of cooperators makes the incentive negligible and dispensable at short time. The taxation may be used as an ignition engine in networks with low initial fraction of C-agents. This mechanism of redistribution of wealth has a strong influence in the social macrostructure of the network because not only cooperators want to keep cooperating but also defectors want to change their strategies to cooperate.

4.2 On the non-adaptive case

We have seen the effect that our mechanism has over the macrostructure of full adaptive networks. Now, we are going to examine it in a non-adaptive case. In non-adaptive networks without taxation, the fraction of cooperators can either fluctuate slightly or the system can reach a full defect state where all agents play D strategy. In fact, for a fixed network size, if the characteristic coordination number K is large enough, there exists a critical value b^*, which depends on the network structure at initial time, such that for $b > b^*$ the system reaches a state of all D-agents and partial cooperation is supported for $b < b^*$. In Table 3, we consider $N = 100$, and the averages over 10 random initial networks. We observe that for the 10 studied cases, $K = 4$ was not large enough to observe $b^* > 1$. However, for $K = 8$, we observed the existence of critical values. Table 3 showed that the average of theses critical values lied in $(1.55, 1.75)$.

In the tax scheme, with $t = 0.05$, we did not observe a critical b^* for $N = 100$ and for any $K = 4$ or 8. Our numerical experiences, based in network sizes between 100 and 1000, show that we can choose a share value t such that the network, with at least one C-agent at initial time, never reaches a full defect state.

As in the adaptive case, the fraction of cooperators is quite robust from increasing value of

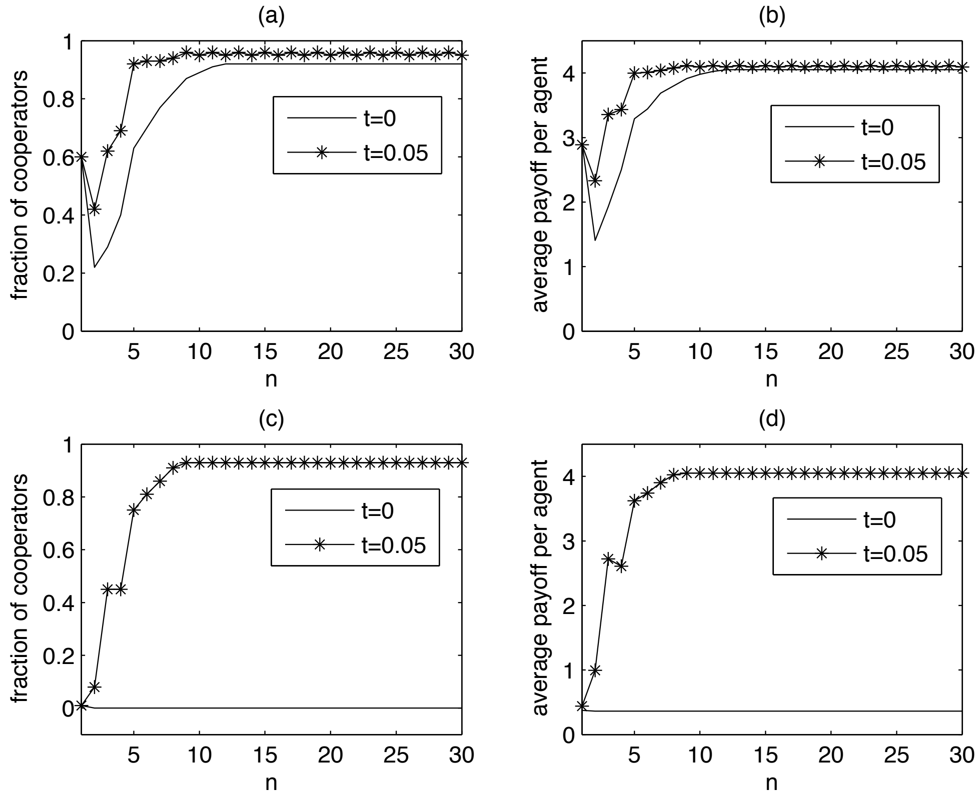

Figure 1: Time series of f_C and Π in full adaptive networks

the incentive to defect b and from increasing value of the average number of links per agent K.

	$0.6N$ C-agents at initial time				One C-agent at initial time			
	$K=4$		$K=8$		$K=4$		$K=8$	
b	$t=0$	$t=.05$	$t=0$	$t=.05$	$t=0$	$t=.05$	$t=0$	$t=.05$
1.05	0.920	0.944	0.972	0.992	0	0.937	0	0.968
1.15	0.887	0.910	0.710	0.929	0	0.948	0	0.923
1.35	0.626	0.789	0.325	0.754	0	0.753	0	0.652
1.55	0.322	0.457	0.057	0.383	0	0.596	0	0.489
1.75	0.177	0.352	0.000	0.238	0	0.375	0	0.219
1.95	0.046	0.264	0.000	0.215	0	0.198	0	0.155

Table 3: Averages of the fractions of C-agents of non-adaptive networks.

We illustrate in Fig 1(a) the asymptotic dynamics of the fraction of cooperators with a time series of f_C evolving for a non-taxed network ($t = 0$), and for a taxed network ($t = 0.05$). We observe that the fraction of cooperators fluctuates slightly after some transient time for both cases (tax and non-tax). Here, the initial fraction of C-agents is 0.6N and $b = 1.35$. Table 3 illustrates this behavior for other values of b. In the case of initial network of one C-agent, Fig

1(c) shows an extremely different result between taxes and non-taxes case. In the non-taxes case, the only cooperator switches to defect and a full defective network is reached, in contrast with the taxes case, where the mechanism enhances a highly cooperative network. The mechanism of taxation preserves the dynamics that settles onto a fluctuated fraction of cooperators after some transient time starting when the fraction of cooperators reaches a value greater than 0.6N.

Fig 2 (b) and (d), show the effect that the mechanism of taxation has over the average payoff of the whole network. When the initial fraction of C-agents is $0.6N$, the mechanism produces a slight effect over the average of payoff per agent. However, this mechanism has a strong influence in the social macrostructure of network with a small initial fraction of cooperators because not only cooperators want to keep cooperating but also defectors switch to imitate successful cooperators producing an increasing average payoff per agent.

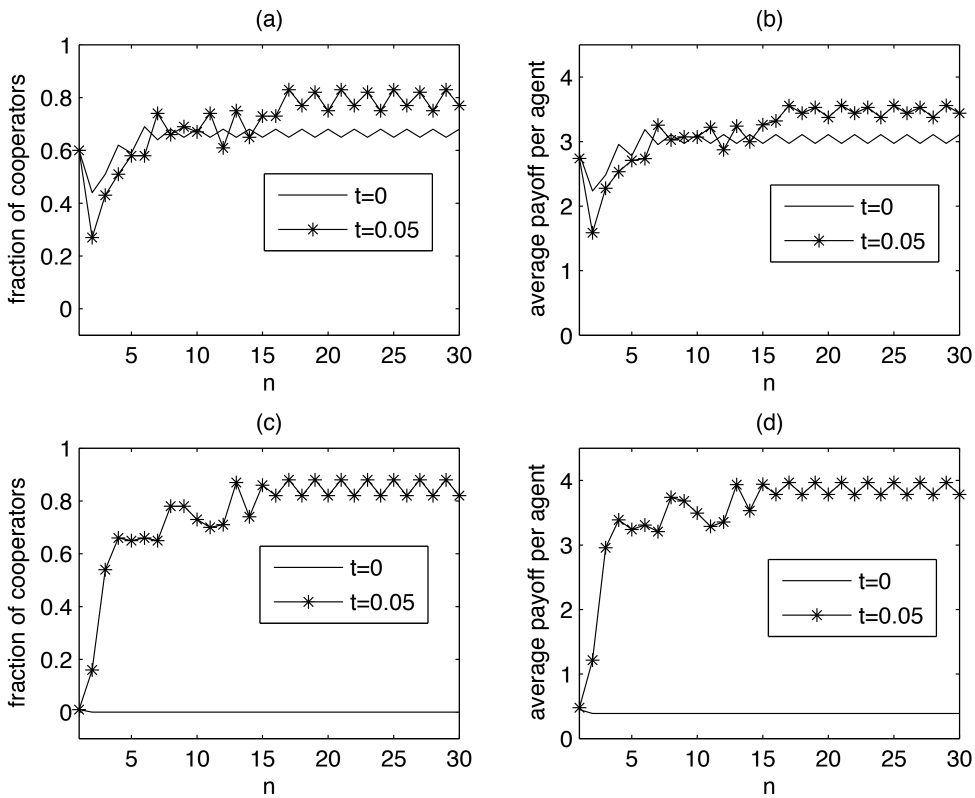

Figure 2: Time series of f_C and Π in full non-adaptive networks

5 Concluding remarks

In this paper, we have proposed a mechanism of taxes and subsidies that compensates cooperative agents for being conservative in adaptive networks. We also examined the mechanism in non-adaptive networks. Our results show that the mechanism works as an ignition engine to achieve highly cooperative level invariant from increasing value of the incentive to defect b and from increasing value of the average number of links per agent K in either case, adaptive or non-adaptive.

When the worst initial network consists of just one cooperative agent, the mechanism enhances a highly cooperative state contrasting with the results in the non-taxes scheme for either adaptive and non-adaptive network. The mechanism of taxation preserves the dynamics that settles onto a fluctuated fraction of cooperators after some transient time in the non-adaptive network or a steady state in the adaptive network.

In view of the mechanism of taxation has a very successful effect over the macrostructure of networks with a small initial fraction of C-agents and has a slightly effect in networks with a sufficiently high fraction of cooperative agents, it can be implement for a short finite period of time. That is, until the network achieves a proper level of cooperation, avoiding an infinite and unnecessary intervention of the central planner in the system.

We finally remark that the mechanism described above can be easily explained in an economic analysis approach, since we are assuming the existence of a benevolent central planner who redistributes wealth among agents searching for improving efficient outcomes in the long-run time of this network formation. Nevertheless, in a biological or other context, we can find a explanation for the mechanism if we assume the existence an individual in the population like a social leader or chief who is allowed for the society to act as a central planner. The idea is not to discuss over the existence or not of such leader, we just propose a solution to avoid inefficient macrostructure of in a network formation.

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