

# General Equilibrium Implications of the Capital Adequacy Regulation for Banks

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*Preliminary version. Comments welcome.*

## **Abstract**

The capital adequacy regulation sets a lower limit for the ratio of capital to the risk-weighted sum of assets banks can hold in their portfolios. The banking literature has studied its effects on banks' behavior mainly from a microeconomics viewpoint. However, new insights are obtained when a general equilibrium perspective is adopted.

This paper proposes a dynamic stochastic general equilibrium model that allows to investigate how the stringency of the regulation changes endogenously over the business cycle, how banks respond optimally to it and what are the implications for firms' investment and production decisions as well as for households' consumption-saving decisions.

The model is solved numerically using a finite-element approximation method. Numerical simulations suggest that banks try to anticipate aggregate shocks by accumulating a buffer of capital over the regulatory minimum. Nevertheless, if the negative shock is strong enough banks will run up against the constraint and the only possibility left to them will be to cut back on lending. Thus, the interest rate spread increases in the downturn and households reduce savings more than in standard models. This suggests the existence of a financial accelerator, since the supply of bank credit shrinks together with the demand during recessions.

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# 1 Introduction

The 1990s witnessed the raise of bank capital requirements as one of the most important banking regulations of the decade, as more than 100 countries world-wide adopted the Basel Accords of 1988. Originally, the accords were designed for international banks only, but soon after they were extended to all credit institutions in the European Union. In the US, they were also extended to all banks through the Federal Deposit Insurance Corporation Improvement Act (FDICIA) in 1991. The accords emerged as a reaction to the solvency crises experienced by banks in the 1980s. The primary goal of the Basel Committee was to strengthen the soundness and stability of the international banking system by promoting banking institutions to boost their capital positions (BIS, 1999).

At a theoretical level, the existence of this regulation has been rationalized by considering the presence of informational asymmetries in financial contracts. For example, Dewatripont and Tirole (1994) argue that the reason for requiring a minimum capital-to-assets (CA) ratio is the reduction of bank failure risk as a way to protect small unsophisticated (uninformed) depositors. Other authors believe that this regulation is closely linked to the existence of an insurance deposit scheme, since it would correct bank incentives to 'misbehave'. For example, theoretical models show that these solvency standards can limit the risk-taking behavior by banks (Kohen and Santomero, 1980 and Kim and Santomero, 1988).

Despite the fact that it has been more than 15 years since the regulation began to be enforced and that thousands of studies on banking, banking regulations and contract theory has appeared since then, there are still no clear-cut answers to several very important questions regarding the operation of the bank capital requirement.

One question that is of critical importance for the regulator is if the intro-

duction of capital requirements has effectively led banks to choose higher CA ratios than would have otherwise been the case. The answer to this question is not obvious in light of the evidence showing that banks set CA ratios to levels well above the legal minimum. Addressing this question empirically is difficult because it requires to compare bank's behavior in times of capital regulation with its behavior in a no regulation period while controlling for every other aspect that may have changed in the meantime. With the adoption of the Basel standards, the average ratio of capital to risk-weighted assets of major banks in G-10 countries rose from 10% in 1988 to 11% in 1996 (BIS, 1999). However, is hard to identify all the variables that explains this change and their relative importance, especially if those relationships change over time. Another challenge in answering this question is to explain why do banks choose CA ratios well above the required minimum. As long as this 'excess capital' held by banks cannot be accounted for there is little hope in understanding the effects of the regulation on CA ratios.

Another question that empirical research has tried to answer is whether banks adjust their CA ratios to meet the required minimum by increasing bank capital or by cutting-back on lending. Bank's response to a situation in which the regulatory constraint is hit may vary according to many aspects related to either macroeconomic conditions (i.e. the phase of the business cycle) or to the bank's own financial situation. The effect of the business cycle on bank's behavior is particularly difficult to capture in these empirical models because of identification problems: it is not clear if a fall in bank lending is caused by a reduction in loan supply or by a demand for credit that becomes weaker during recessions.

Finally, a question that has generated a big debate in the literature is if the adoption of Basel Accords, and the subsequent increase in capital requirements, have created a 'credit crunch'. For example, some authors believe that this may have happened in the US in the early 1990s. There

is some circumstantial evidence showing that banks assets portfolio shifted away from commercial and industrial loans toward government securities just when the US economy was entering a recession. However, there has been no definitive conclusion in the empirical literature to this respect (Bernanke and Lown, 1991; Hall, 1993; Bergen and Udell, 1994; Hancock and Wilcox, 1995 and Peek and Rosengren, 1995 among others). In general, this empirical papers run a regression of bank lending growth on capital growth. However, even when some of them find a significant coefficient for this relationship, it is not clear from those results what is specifically the impact of changing the capital requirement and what is the difference between changes in bank capital and bank capital regulation (Furfine, 2000). A structural model of bank behavior would be necessary to address these issues.

Moreover, the structural model needed should impose restrictions from theory not only as regards bank behavior, but also in the relationships between the bank and the rest of the economy. For example, the correlation found in the reduced-form regressions between bank lending and capital, if any, does not necessarily means causality. Empirically, it is difficult to prove that a fall in bank lending is caused by a tightening of the bank capital requirement and not simply explained by the fact that periods when loan demand is weak coincide with times during which banks are making large write-offs or specific provisions (i.e. reducing capital) (BIS, 1999).

This paper proposes a model that imposes an economic structure into the problem, and in that way can suggest answers to all these questions from a theoretical standpoint. The dynamic, stochastic, general equilibrium (DSGE) model developed here allows to analyze all the issues described earlier in an internally consistent manner. Specifically, the model answers the questions posed in the empirical literature from a macroeconomic perspective. That is, it studies not only the microeconomics of the banking firm in relation to the capital regulation, but also it pays attention to the rest of the

economic agents and how the regulation affects their optimizing behavior as well as markets equilibria.

The general equilibrium analysis undertaken through this model also offers a new insight into the assessment of the effects of capital regulation that has never been considered before, at least in a formal setting. Since bank's profitability and thus bank's capital depend on the financial health of the borrowers, a negative shock (either a TFP shock or an AD shock) that makes production firms to incur in losses and to demand less credit during the recession will also make bank capital to shrink. The tight link between bank capital and credit induced by the regulation require banks to cut-back on lending right when the economy is entering a recession. Thus, the presence of capital requirements will work as a financial accelerator of macroeconomic fluctuations.

The rest of the paper is outlined as follows. Next section includes a brief discussion on the mechanics of the model, an evaluation of the assumptions needed and a review of theoretical literature related to this paper. A formal setting of the model, composed of agents optimization conditions, market clearing conditions and regulatory constraints is laid out in the following section. Finally, the last section presents briefly the numerical strategy followed to approximate the model solution. This approximate solution and the numerical simulations based on them are used there to do a qualitative analysis of the model dynamics.

## **2 Capital Regulation and Macroeconomic Shocks**

One of the main goals of the solvency standard introduced through the Basel Accord of 1988 was to limit the risk-taking behavior by banks (i.e. to limit 'credit risk'). For that purpose it was established that bank equity should not fall below 8% of the risk-weighted sum of bank assets.

In their first version the accords did not take into account 'market risk', such as interest rate risk<sup>1</sup>. Basel II agreements introduce some changes in the design of the regulation so that it can capture not only the individual risk of each type of asset held by banks but also the correlation among them within the assets portfolio. The required CA ratio however has not been changed from its original 8% level.

A big challenge that still remains in the design of the regulation concerns the response of banks to a generalized undercapitalization of the sector due to adverse macroeconomic shocks, such as a wave of failures in the production sector or a crash in stock markets. It may be the case that the strict enforcement of a minimum CA ratio produces undesired effects such as making banks more vulnerable to 'aggregate risk' than they would be otherwise. Even more important for the goal of this paper, the imposition of capital requirements for banks in the context of a negative macroeconomic shock may have consequences for the rest of the economy if the reduction in bank credit affects investment and production.

An adverse macro shock will make all banks in the system to experience low return realizations simultaneously. For example, a wave of failures in the production sector will result in higher bankruptcy rates and lower repayment of bank loans. Bank equity will be affected as bank profitability decreases. Under capital requirements, banks may all run up against the regulatory constraint at the same time and if that occurs they will be left with only two courses of action: either to recapitalize or to cut-back on lending.

Suppose that banks cannot recapitalize all at the same time. Suppose further that firms cannot easily replace bank loans by other forms of financing, such as issuing commercial paper or bonds or retaining earnings. Under those circumstances, the negative shock will propagate itself automatically

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<sup>1</sup>Interest rate risk arises due to the volatility of the term structure of interest rates and the mismatch of maturities of bank assets and liabilities

through the reduction in credit, investment and production.

According to Blum and Hellwig (1995) these two assumptions are not unreasonable:

- Firms in general and banks in particular are reluctant to issue equity during bad times because of the negative inferences that may be drawn as regards how solvent is the institution. This is specially true for banks which are constantly evaluating how their actions affects depositors confidence on the system.
- Firms use predominantly bank lending. For example, in the US around 60% of external financing is represented by bank loans while 30% and 2% are bond and stocks respectively<sup>2</sup>. So it would be very costly if not impossible for the economy to undergo a massive substitution of bank lending by other forms of financing.

This idea of capital requirements working as an automatic amplifier to macroeconomic fluctuations has been discussed informally in various empirical papers (CITE HERE). Blum and Hellwig (1995) are the only ones that have studied this topic before in a formal setting though. However, no one to my knowledge has carried out the study using a DSGE model as here. Blum and Hellwig (1995) set an AD-AS model in which aggregate uncertainty is driven by exogenous AD shocks. In that model, investment demand depends on bank loans which in turn depends on bank deposits, reserves and bank equity.

They find that conditional on a binding regulatory constraint, further increases in the required CA ratio lead to a fall in lending and investment. Moreover, they find that the sensitivity of equilibrium production level to

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<sup>2</sup>Half of the stock and almost all the bonds are sold to some kind of financial intermediary (Dewatripont and Tirole, 1994).

demand shocks increases when they compare the case of a binding regulation against a non-binding regulation.

However the analysis that can be done with this type of models is very limited. Its static nature and the lack of micro foundations for the model economy are two well known criticisms to AD-AS models. The problem in this specific context is that the model cannot explain how the optimal profit-maximizing CA ratio chosen by the bank changes over time and it occasionally hits the constraint imposed by the regulation. Without this analytical tool there is no hope in studying the automatic amplifier described earlier.

The model in this paper also builds on Aiyagari and Gertel (1998). They use a dynamic model to explain over-reaction of asset prices. That is, they try to explain why asset prices in stock markets tend to decrease below their fundamental values.

For that purpose they augment a standard Lucas-Tree model by including a trader firm who uses leverage plus equity to finance investments in risky securities. However, this trader firm is limited in the amount of debt it can use<sup>3</sup>. With an inelastic supply for securities and under the assumption of no recapitalization (no equity issue by the trader) they find that stock prices overreact.<sup>4</sup>

Their main result hinges on the fact that issuing equity is not a possibility for the traders and thus the best way they can recapitalize after and adverse shock is through retained earnings (i.e. driving down dividend payments to

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<sup>3</sup>The trader behaves exactly the same than a bank who uses leverage and equity to finance risky loans and is limited by the regulatory constraint.

<sup>4</sup>They do not model a production economy and there is no capital accumulation. If they did it, the supply for securities (i.e. demand for bank loans) would not be inelastic anymore, and thus we could have 'over-reaction' in quantities rather than in prices. That is, with an elastic supply for securities the model could explain the amplification of aggregate fluctuations due to the extra volatility of lending and investment.



the trader firm owners to zero if possible). The trader (or bank) will avoid by all means to get to the point in which the regulation binds because in that case profitable investment opportunities should be given up in order to not violate the regulation. Therefore, even if the regulation does not bind today, the trader (or bank) will build as much equity as it can when faced to the probability of the constraint binding sometime in the future.

During the transition it may happen that a large negative realization of the shock makes the regulatory constraint to bind. If that is the case, the trader (or bank) would adjust downwards its asset portfolio and asset prices would fall below their fundamentals. However, over the long-run traders (or banks) will accumulate enough equity to rule out the possibility of the constraint binding in any state of nature. That is, they will retain earnings until the point in which they do not use debt anymore (i.e. non-stationary steady-state).

In order to avoid this unrealistic long-run prediction of their model, Aiyagari and Gertel suggest to introduce some kind of benefit from holding debt (or bank deposits). The interaction of such benefit with the cost of using debt (or bank deposits), which is given by the probability of the constraint ever binding in the future, should result in an stationary steady-state.

Following this line of reasoning Van de Huevel (2003) simply introduces a tax on corporate profits with interest payments on debt being exempt from the tax. This exemption constitutes a benefit of being leveraged. Of course, this is not enough to produce an interior solution for the deterministic steady-state CA ratio. Moreover, the only deterministic steady-state possible in this case is with the constraint always binding<sup>5</sup>.

This paper builds on Aiyagari and Gertel (1998) and Van de Huevel (2003) models. However, both them are partial equilibrium (PE) models in which

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<sup>5</sup>The benefit of using debt is always present while the cost of using debt is larger than zero only in the deterministic steady-state in which the constraint is binding

interest rate is exogenous. Van de Huevel (2003) has also exogenous default rate for loans. In both models there is neither production sector nor capital accumulation.

In this paper I extend those models to general equilibrium by adding capital accumulation, a production sector and endogenous default on loans. The model also incorporates the tax exemption idea of Van de Huevel (2003).

## 3 The Model

### 3.1 Households

Representative household maximize lifetime utility by choosing optimally the lifetime profile of consumption ( $c_t$ ), labor ( $l_t$ ), bank deposits ( $D_t$ ) and bank stock shares ( $s_t$ ). Households have access also to a storage technology that pays no return. They use it just occasionally when the return on alternative investment opportunities decreases to zero.

$$\begin{aligned}
 & \max_{\{c_t, l_t, D_{t+1}, s_{t+1}, Z_{t+1}\}_0^\infty} E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t, l_t)] \\
 & \text{s.t.} \\
 & (1 + r_t)D_t + Z_t + w_t l_t + \left[ \Delta_t \left( 1 - \phi\left(\frac{\Delta_t}{s_t}\right) \right) + p_t s_t \right] + TR_t \geq \\
 & \quad c_t + D_{t+1} + Z_{t+1} + p_t s_{t+1} \quad (1) \\
 & \quad Z_t \geq 0
 \end{aligned}$$

Flow budget constraint (1) indicates that family income is made of interest payments from deposits, wages, bank dividends ( $\Delta_t$ ) net of personal income tax ( $\Delta_t \phi(\frac{\Delta_t}{s_t})$ ) and a lump-sum government transfer financed with this tax plus a corporate income tax paid by banks ( $TR_t$ ). The tax function  $\phi(\cdot)$  depends on the dividend level reflecting the progressivity built in the

tax code (see J.J. Seater, 198.)<sup>6</sup>. Households also receive whatever resources they had stored last period ( $Z_t$ ).

Solving the dynamic programming problem for the representative household

$$-\frac{u_l(c_t, l_t)}{u_c(c_t, l_t)} = w_t \quad (2)$$

$$u_c(c_t, l_t) = \beta(1 + r_{t+1})E_t \left[ u_c(c_{t+1}, l_{t+1}) \right] \quad (3)$$

$$u_c(c_t, l_t) \geq \beta E_t \left[ u_c(c_{t+1}, l_{t+1}) \right] \quad \Rightarrow Z_{t+1} = 0 \quad (4)$$

$$u_c(c_t, l_t) = \beta E_t \left[ u_c(c_{t+1}, l_{t+1}) \left( \frac{p_{t+1} + \delta_{t+1}(1 - \phi(\delta_{t+1}))}{p_t} \right) \right] \quad (5)$$

Equation (2) is the intra-temporal tangency condition by which the marginal rate of substitution between consumption and leisure is equalized to the relative price. Equations (3) and (5) are the Euler conditions describing the optimal inter-temporal choice between current and future consumption by allocating resources to bank deposits and bank equity respectively<sup>7</sup>. Equation (4) governs the allocation of resources to the non-interest asset  $Z_t$ . Only if this equation holds with equality will the household store a positive amount.

## 3.2 Banks

Banks are competitive and they are all alike. As any other profit maximizing firm, banks objective consists of maximizing its market value. The bank's objective can be derived from first principles by using equation (5). The idea

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<sup>6</sup>Without loss of generality we assume that only bank dividends are subject to income tax. We could make all sources of income to pay tax without affecting the main mechanism at work in the model

<sup>7</sup>The variable  $\delta_t$  is simply the dividend rate, i.e.  $\delta_t \equiv \frac{\Delta_t}{s_t}$ .

is to find the value of the bank shares to their owners by solving (5) forward and by using the fact that in this model there is no issue of shares ( $s_t = \bar{s}$ ) (see appendix for derivation). As a result banks maximize the present value of the expected stream of dividends payments (net of taxes) to their owners discounted at the households intertemporal marginal rate of substitution.

$$\begin{aligned}
\max_{e_{t+1}, D_{t+1}, L_{t+1}} \quad & E_0 \sum_{t=0}^{\infty} \prod_{j=0}^t q_j \Delta_t (1 - \phi(\Delta_t)) & q_j &= \beta \frac{u_c(c_j, l_j)}{u_c(c_{j-1}, l_{j-1})} & q_0 &= 1 \quad s.t. \\
\pi_t^{bank} &= i_t L_t + \pi_t^{firm} \\
\pi_t^{bank} &= r_t D_t + \Delta_t + RE_t + T_t \\
e_{t+1} &= RE_t + e_t \\
\Delta_t &\geq 0 \\
L_t &= D_t + e_t \\
T_t &= \tau(\pi_t^{bank} - r_t D_t) \\
e_{t+1} &\geq \gamma L_{t+1}
\end{aligned}$$

The two first constraints are sources and uses of funds respectively. Since firms only source of financing is bank lending ( $L_t$ ) the bank can claim the full amount of firm's cash flow  $\pi^{firm}$ . The uses of bank's cash flow are interest payments on deposits, dividends payments, retained earnings and corporate income tax. The third restriction is the equation of motion for bank equity ( $e_t$ ) and the non-negativity constraint on dividends puts an upper limit on retained earnings. Since by assumption banks cannot issue equity, the only way they can change the stock of equity is through dividend policy. Negative dividends would in fact operate as if the bank issued equity, so the non-negativity constraint on dividends is introduced to eliminate this possibility. The next equation is the bank's balance sheet constraint. The corporate income tax formula indicates that interest payments on debt are exempt from the tax. As in Van den Huevel (2003), the tax exemption implies that banks

will use as much debt as they can to finance their loans. This guarantees that the bank problem is stationary and that the financial structure will not drift toward an only-equity financing steady state (see Aiyagari and Gertler, 1998). Finally, the last inequality represents the regulation indicating that bank equity cannot be less than a certain proportion ( $\gamma$ ) of bank lending.

Combining the equality restrictions, the bank's budget constraint is obtained

$$\Delta_t = \left[ (1 - \tau)(1 + i_t) + \tau \right] L_t - \left[ (1 + r_t)(1 - \tau) + \tau \right] D_t - L_{t+1} + D_{t+1} + (1 - \tau)\pi_t^{firm}$$

Solving the dynamic programming problem, the following FOCs are derived

$$\Delta_t \eta_t = 0 \tag{6}$$

$$\left[ (1 - \gamma)L_{t+1} - D_{t+1} \right] \mu_t = 0 \tag{7}$$

$$\begin{aligned} (w(\Delta_t) + \eta_t) - (1 - \gamma)\mu_t &= \left[ (1 - \tau)(1 + i_{t+1}) + \tau \right] \\ &E_t \left[ q_{t+1} (w(\Delta_{t+1}) + \eta_{t+1}) \right] \end{aligned} \tag{8}$$

$$\begin{aligned} (w(\Delta_t) + \eta_t) - \mu_t &= \left[ (1 - \tau)(1 + r_{t+1}) + \tau \right] \\ &E_t \left[ q_{t+1} (w(\Delta_{t+1}) + \eta_{t+1}) \right] \end{aligned} \tag{9}$$

where  $w(\Delta_t) \equiv [1 - \phi(\Delta_t) - \Delta_t \phi'(\cdot)]$ .

Equations (6) and (7) are the two complementarity conditions for the dividends and regulatory constraints respectively. Euler equations (8) and (9) describe the optimal intertemporal decisions of the bank as regards loans and deposits respectively. Equation (8) show that banks balance the marginal cost of an additional unit of loans against the expected marginal benefit

discounted at the market interest rate. Equation (9) equates the marginal benefit of one unit of deposits against the expected marginal cost. Combining (8) and (9) by subtracting the latter from the former:

$$\gamma\mu_t = (1 - \tau)(i_{t+1} - r_{t+1})E_t \left[ q_{t+1} (w(\Delta_{t+1}) + \eta_{t+1}) \right]$$

Which gives an expression for the interest rate spread. The only reason for a competitive bank to charge an interest rate on loans that is higher than the marginal cost of funds is the existence of the capital regulation. With no regulation ( $\gamma = 0$ ), there will be no spread. Moreover, due to tax exemption on interest payments, banks would prefer to hold no equity in this case. From the equation of motion for bank equity it is clear that the only intertemporal problem for the bank is the choice between dividend payments and retained earnings. So without equity the bank optimization problem becomes a static one.

### 3.3 Firms

The representative firm maximizes the present value of the stream of expected future cash flows. For that purpose it chooses the optimal profile of investment, labor and bank lending. The discount rate used here is related to the opportunity cost of funds for the firms' owners: the banks. The best alternative available to the bank is to save in deposits at the rate of  $(1 + r_t)$  per unit.

$$\begin{aligned} \max_{\{I_t, l_t, L_{t+1}\}_0^\infty} \quad & E_0 \sum_{t=0}^{\infty} \left[ \prod_{j=0}^t \frac{1}{1 + r_j} \right] \pi_t^{firm} \quad r_0 = 0 \\ \text{s.t.} \quad & \\ \pi_t^{firm} = \quad & A_t F(K_t, l_t) - w_t l_t - I_t + L_{t+1} - (1 + i_t) L_t \end{aligned}$$

$$K_{t+1} = I_t + (1 - \delta)K_t \quad (10)$$

$$L_{t+1} \geq K_{t+1}$$

$$\log A_{t+1} = \rho \log A_t + \epsilon_{t+1}, \quad \epsilon_{t+1} \sim N(0, \sigma) \quad (11)$$

Equation (10) gives the law of motion for the economy's capital stock and (11) is the equation of motion for the exogenous state. The inequality constraint on loans imposes the need for bank financing in the model. Since interest rate on loans is greater or equal than the discount rate, firms prefer to use internal sources (i.e. cash flows) rather than external financing. Thus, the constraint will hold with equality which means that capital depreciation is paid out of firm's cash flow and net investment is entirely financed with debt. The implication of this assumption is that we can eliminate one state variable from the problem:  $K$ . Solving the dynamic programming problem we get,

$$A_t F_l(L_t, l_t) = w_t \quad (12)$$

$$\frac{1}{(1 + r_{t+1})} E_t \left[ A_{t+1} F_K(L_{t+1}, l_{t+1}) + (1 - \delta) - (1 + i_{t+1}) \right] = 0 \quad (13)$$

Equation (12) is the static condition for optimal labor input and equation (13) is the Euler equation indicating the optimal intertemporal decision of the firm as regards capital accumulation.

### 3.4 The Recursive Competitive Equilibrium

The decentralized stationary recursive competitive equilibrium implies that each decision-making unit solves an independent dynamic programming problem. We distinguish between aggregate per capita state variables ( $\Upsilon$ ) and the individual agents state variables over which they have control. In equilibrium it will be true that aggregate per capita state variables will coincide with their individual counterparts (Cooley and Prescott, 1995).

The state variables for households are  $v_t^h = (A_t, D_t, Z_t, s_t, r_t; \Upsilon_t)$  where  $\Upsilon_t$  stands for the economy wide per capita counterparts of all state variables in the model  $(K_t, L_t, D_t, Z_t, s_t, r_t, i_t)$ . For banks and firms the states are given by  $v_t^b = (A_t, D_t, L_t, r_t, i_t; \Upsilon_t)$  and  $v_t^f = (A_t, K_t, L_t, i_t; \Upsilon_t)$  respectively.

The recursive competitive equilibrium in this economy consists of:

- Decision-making units value functions:  $V^h(v_t^h)$ ;  $V^b(v_t^b)$  and  $V^f(v_t^f)$ .
- A set of optimal decision rules:  $c(v_t^h), l(v_t^h), D(v_t^h), Z(v_t^h), s(v_t^h)$  for households;  $D(v_t^b), L(v_t^b), \Delta(v_t^b)$  for banks; and  $l(v_t^f), I(v_t^f), L(v_t^f)$  for firms.
- The corresponding set of aggregate per capita decision rules.
- Price functions:  $i(A_t, \Upsilon_t), r(A_t, \Upsilon_t), p(A_t, \Upsilon_t)$  for financial assets,  $w(A_t, \Upsilon_t)$  and shadow prices  $\eta(A_t, \Upsilon_t), \mu(A_t, \Upsilon_t)$ .

such that this functions satisfy:

- Households, banks and firms intertemporal optimization problems.
- Market clearing conditions (i.e. labor, bank deposits, loans and bank shares markets).
- The consistency of individual and the corresponding aggregate decisions.
- Household's budget constraint, bank's budget constraint, capital regulation constraint, non-negativity of dividends, non-negativity of  $Z$  and  $L = K$ .



## 4 Numerical Solution and Results

The optimal response functions mapping the state space  $(A_t, \Upsilon_t)$  into the decisions are the objects of interest. After imposing  $K_{t+1} = L_{t+1}$ ,  $TR_t = T_t + \Delta_t \phi(\Delta_t)$ ,  $s_{t+1} = 1$  and using the bank's budget constraint to eliminate  $\Delta_t$ , these functions are the solution to the functional equation problem (1)-(9) and (11)-(13). These functions cannot be obtained analytically, they can only be approximated numerically to a certain degree of accuracy.

The computational task is to find a close enough approximation to the policy functions such that an appropriately defined residual function is approximately equal to zero. For this purpose we use a finite-element method (see McGrattan, 1999 and Fackler, 2003). The method requires first to choose some discretization of the state space. Each partition of the state space is an  $m$ -dimensional element (where  $m$  is the dimension of the state space). There are  $n$  partitions or nodes.

The approximation to the policy function is done through a linear combination of  $n$  known basis functions (for example, low-order polynomial or polynomial splines). The  $n^m$  coefficients on the linear combination are the objects to be computed to obtain the approximate solution (McGrattan, 1999). These coefficients are found by collocation method, that is solving the non-linear system of equations that arises from setting an appropriately defined residual function to zero (for example, the value of equations (1)-(9) and (11)-(13) evaluated at the approximate solution). If there are  $p$  policy functions to be approximated the number of unknowns to solve for is  $p \times n^m$ . Finally, we can solve the non-linear system through generic root-finding algorithms such as Newton's method or Quasi-Newton methods. Alternatively, the structure of the problem suggests to solve for the coefficients through a fixed-point iteration scheme that demands far less computer effort and memory requirements than the previous ones (Fackler, 2003).

A complete discussion on the practical issues involved in the implementation of the method can be found in Fackler (2003), including Matlab codes<sup>8</sup>. The appendix also includes a short discussion comparing this method with alternative algorithms commonly used to obtain numerical solutions in DSGE models.

## 4.1 Results

It is not the purpose of this study to do a calibration exercise. That is, the approximate solutions and the numerical simulations of the model are used just to examine the qualitative dynamics of the system in response to the exogenous TFP process.

In order to approximate the solutions numerically, values must be assigned to the parameters as well as functional forms to the production and utility functions. I assume that households behave according to the Constant Relative-Risk Aversion (CRRA) type of functions,  $u(c_t, l_t) = \frac{(c_t - \frac{l_t^\omega}{\omega})^{1-\theta}}{1-\theta}$ . As regards production technology, a Cobb-Douglas function is assumed  $A_t F(k_t, l_t) = A_t k_t^\alpha l_t^{1-\alpha}$ .

The personal income tax function is  $\phi(\Delta_t) = a\Delta_t^b$ . This reflects the progressivity of the income tax system. The parameters  $a$  and  $b$  are calibrated to match the average and marginal tax rates in the US (J.J. Seater, 198.).

The model period is specified to be one year. The parameters values for  $\alpha$ ,  $\beta$ ,  $\delta$ ,  $\omega$  and  $\theta$ <sup>9</sup> are standard in the literature of RBC for the US post-

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<sup>8</sup>The Matlab implementation was programmed by Fackler (2003) and is called *resolve*. Many other utilities included in the *CompEcon* toolbox (Fackler and Miranda, 2002) are also used here.

<sup>9</sup>It is common to find in the literature values of the coefficient of relative risk aversion  $\theta$  ranging from 1 (i.e. log utility) to 2. Without loss of generality, I choose a value close to the lower end (1.1) because it makes consumption and thus investment more responsive to changes in the interest rate. Of course, results would not change with a coefficient of 2.

war annual data (Prescott, 1986). The autocorrelation coefficient  $\rho$  and the standard deviation of the shocks  $\sigma$  are in the range of estimations from TFP process arising from the US business cycle measured at annual frequency<sup>10</sup>. The required CA ratio  $\gamma$  and the corporate income tax rate  $\tau$  are calibrated to real world values. The parameters  $a$  and  $b$  of the bank’s objective are picked so that the average income tax rate in steady-state is around 25%.

Table 1: Parameter Values

$\alpha$	$\beta$	$\gamma$	$\delta$	$\omega$	$\tau$	$\theta$	$\rho$	$\sigma$	$a$	$b$
0.36	0.96	0.08	0.1	2	0.25	1.1-2	0.9	0.01	4	1

#### 4.1.1 A General Equilibrium Perspective to Capital Requirements

The numerical simulations of the model can be used to examine many of the unresolved issues in the empirical literature on bank capital regulation.

One first problem the model allows to address is the explanation for the fact that the representative bank finds it optimal to hold ‘excess capital’ (i.e. capital in excess of the minimum required). Figure 1 shows the stationary distribution of several variables belonging to the bank’s problem<sup>11</sup>. In particular, the mean of the optimal CA ratio is well above the minimum required of 8%. Comparing the expected values of these variables (vertical solid lines) with the deterministic steady-state counterparts (vertical dashed lines) it can be seen that, when faced with uncertainty, banks decide to hold more equity and less deposits.

<sup>10</sup>As it is made clear in Prescott (1986), the HP filtered log of the TFP for the US economy in the period III-1955 to I-1984 “displays considerably serial correlation, with their first differences nearly serially uncorrelated”. Kydland and Prescott find that a highly persistent AR(1) (for example, with  $\rho = 0.9$ ) results in essentially the same fluctuations than a random walk process.

<sup>11</sup>The distributions were computed from 500 simulations for each variable 1000 periods long.

From Euler equation (9), banks balance the benefit of using more debt financing with the fact that, associated to this, there is a fall in bank capital and thus an increase in the probability of the non-negativity constraint on dividends binding next period (i.e. an expected increase in the shadow price of the non-negativity constraint on dividends  $E[\eta_{t+1}] > 0$ ). The regulatory constraint on bank capital is closely linked to the non-negativity constraint of dividends because if the former is binding then, in order to rebuild equity, the bank will retain earnings as long as the later does not bind. Thus, a too low CA ratio will make the bank to face a high probability of the regulation binding next period and thus a high probability of retaining earnings. Additionally, risk-averse-like bank managers<sup>12</sup> have incentives to smooth dividend payments  $\Delta_t$  over time. That is, as the likelihood of retaining earnings and thus reducing dividend payments next period increases, bank managers expected "marginal utility" (i.e.  $w(\Delta_{t+1})$  goes up. This in turns increase the cost of using debt financing. Of course, the importance of this second effect will be reduced if the curvature of the bank's objective function is sufficiently low.

The reason why bank behavior ends up in overaccumulation of capital above the regulation limit is that the bank must acquire self-insurance. Self-insurance arises when there is a nonnegativity restriction on asset holdings. Aiyagari and McGrattan (1998) and Ljungqvist and Sargent (2000) have analyzed the issue of self-insurance for households (i.e. borrowing constraints) in the context of idiosyncratic uncertainty and incomplete markets. The key result they obtain is that in their models the stationary equilibrium interest rate falls short of the rate of time preference  $\beta^{-1}$ . The lower interest is consistent with a finite overaccumulation of assets above the credit limit. If interest rate were equal to  $\beta^{-1}$  agents would accumulate infinite amount of assets.

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<sup>12</sup>In effect, they behave like risk-averse agents due to the curvature of the bank's objective introduced through the progressive income tax.

The constraint on the net worth of the bank operates in the same manner here. With aggregate uncertainty the bank desires to accumulate a buffer of equity. The bank, however, does not accumulate an infinite amount of equity (i.e. all-equity financing) because in equilibrium the return on equity obtained by the bank falls short to the discount rate (which is approximately equal to  $(1 + r_{t+1})$ ). This is shown in Figure 1 (middle right panel) by the gap between the solid and dashed lines<sup>13</sup>. Note that a return on equity lower than  $(1 + r_{t+1})$  is consistent with a risk-premium on bank shares. Risk-averse households will hold bank shares only if they are compensated for risk. With an inelastic supply of bank shares, the demand  $s_{t+1}^d$  and the price of shares  $p_t$  will both adjust to ensure a premium on the risky asset.

A second issue that is not well understood in the banking regulation literature is how do banks respond to an increase of capital requirements. Do they increase equity or they cut-back on lending. Figure 2 show the expected path followed by bank equity and lending as banks adjust to an unexpected and permanent change in capital adequacy ratios, from 6% to 8%<sup>14</sup>. The simulations were performed by setting starting values of the variables at the mean of the stochastic steady-state corresponding to  $\gamma = 0.06$ . Thus the figure shows the transitional dynamics of the model from a low- $\gamma$  to a high- $\gamma$  steady-state. The policy change is introduced in period 10 of the simulation.

Banks respond to a change in the regulation mainly by increasing equity holdings. They retain earnings until the higher level of equity is reached. That is, both equity and dividends increase over the transition (see top right and bottom left panels), but dividends first falls on impact as retained earn-

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<sup>13</sup>The true return on equity obtained by the bank is  $\frac{\Delta_t + RE_t}{e_t}$ . However, on average  $RE_t = 0$ .

<sup>14</sup>These figures roughly resemble the change in the capital regulation occurred in the US in 1991. The US had implemented solvency regulations since 1980, setting the legal minimum to 6%. After the adoption of the Basel Committee standards, the ratio was increased to 8%.

ings increase enough. The bottom right panel shows the ratio of bank dividends to equity. As usual the bank finances the excess equity holding by paying a lower return on it. Bank lending does not change significantly and, therefore, the optimal CA ratio increases from around 8.7% to 10.1%.

Bank's behavior can be rationalized again by using equation (9) which describes the financing decisions of the bank. The change in the required ratio essentially squeezes the excess capital on impact. Since the probability of the constraints binding increases the bank will retain earnings and move to a higher CA ratio. The bank balances the tax exemption on deposits with the expected cost of a binding constraint. Thus, after the change in the regulation the bank adjusts the debt/equity financing mix so that expected marginal cost equalizes the marginal benefit.

The expected value of loans in the simulations falls just 0.5% while interest rate increases in no more than 1 basic point. These changes are really small, however, they are qualitatively important. They imply that banks also cut-back on lending as a way to rebuild equity. That is, previous papers have guessed that banks would retain earnings for this purpose and that they would cut-back on lending only when the capital regulation kicks-in (Blum and Hellwig,1995; van den Huevel,2003; Aiyagari and Gertel, 1998). However, according to my numbers 3% of the percentage increase in the optimal CA ratio is due to reduction of lending even when the ratio is not even close to the minimum required.

The fact that bank lending is only slightly affected by the regulatory change implies that bank's financing decisions have just a mild real effect in the long run. This result have direct implications for the hypothesis of a 'credit crunch' after the increase in bank capital requirements. The empirical literature have strived to obtain a quantitative measure of the causal relationship between higher capital requirements and a slowdown of the economy. The empirical findings are mixed and, in general, they refer to the

relationship between bank capital and bank lending. However, changes in bank capital can arise for several reasons other than a change in the capital adequacy regulation. This model provides an artificial laboratory that allows to isolate just the effect of the later. Also, those empirical estimations may suffer identification problems, as they infer the coefficients from market equilibrium quantities of loans and capital without controlling for simultaneous shifts of supply and demand for credit. The structural model proposed here allows to disentangle credit demand effects arising from the TFP shocks from credit supply changes coming from a 'shock' to the regulation parameter. As it was explained above, Figure 2 shows that the bank's optimal CA ratio increases after the increase in the minimum required without significantly affecting the amount of credit given out by banks.

Finally, it is worth noting that these empirical studies use for the estimations cross-sectional data on bank loan growth and bank capital growth around the time the regulation was imposed. If they did find a significant relationship (as some of them actually do), any inference based on them would be just a comparative statics analysis (i.e. comparison of pre- and post-regulation steady-states). However, this contrasts with the commonly accepted idea that finding evidence on a 'credit crunch' amounts to explaining the cause of the US recession of early 1990s (i.e. a temporary deviation from steady-state). This approach to the problem is misleading because a permanent change in the capital requirement will make the macroeconomic variables to adjust to a new level at which they will remain permanently as the economy moves from one steady-state to another. Therefore, we cannot look in the regulatory changes for the primary cause of a temporary deviation of output from trend.

Furfine (2000) arrives to quantitative results for the 'credit crunch' hypothesis that roughly resemble the qualitative analysis carried out here. He estimates an structural dynamic partial equilibrium model of the bank and

he finds evidence of a very weak link between bank capital regulation and bank loans. In addition he finds that a change in the capital regulation produces a permanent change in bank variables rather than a temporary deviation from steady-state. He sees this finding as evidence against the ‘credit crunch’ explanation for the US recession of early 1990s.

Of course, this does not mean that the capital adequacy regulation does not play any role during a downturn of the economy. The stringency of bank capital requirements may well influence the dynamics of the macroeconomic variables as the economy heads toward a recession. This link is investigated in the next section.

#### **4.1.2 A Financial Accelerator of Aggregate Fluctuations**

In this section the model is used to explore the extent to which bank capital requirement can work as an automatic amplifier of aggregate fluctuations. This hypothesis has not been studied before in a formal theoretical setting. Empirical tests are not easy to implement due to the fact that both demand and supply for credit shift over the business cycle so it is hard to achieve identification.

In a first version, the model is solved for  $\gamma = 0$ . That is, the idea is to capture how the economy behaves in a no-regulation scenario and later compare it to the regulated economy. Due to the tax exemption on deposits, when there is no capital requirement banks will choose to hold no equity. With no equity and thus no dividend payments, inequality constraints (6) and (7) become irrelevant for the problem. Therefore, the model collapses to a standard closed-economy RBC model with firms making investment and production decisions and households making consumption-saving decisions. Banks are completely redundant in this setting.

As usual in standard RBC models, the interest rate is positively related to TFP shocks, as firms demand for credit changes. Since banks are perfectly



competitive, the bank interest rate spread is zero. The responses of output, consumption, labor and investment are all the expected ones. All them are positively linked to the TFP process, the only source of fluctuations in the model. The cyclical behavior of investment implies that if the regulation is going to work as an amplification mechanism of fluctuations, then there must be an additional fall in credit coming from the supply side of the market. That is, in addition to the direct effect of the TFP shock over demand for credit, an extra indirect effect of the TFP shock over the supply of credit (operating through the capital requirement) is needed.

Figure 3 displays bank optimal responses in the regulated environment. The capital requirement graph in the bottom right panel shows that for a big enough negative shock the regulation starts to have force. The middle left panel shows that the bank will cut dividend payments and retain earnings as a way to recapitalize the bank. But, eventually, dividends hit the non-negativity constraint (middle right) and the only possibility is to cut-back on lending. When all banks in the sector reduce lending, interest rate spread ( $i_{t+1} - r_{t+1}$ ) increases at the same time interest rate on deposits falls by an extra amount (see top panels). The counter-cyclical interest rate spread for competitive banks shows up as a consequence of the regulation.

The counter-cyclical spread constitutes an empirical test for the existence of a financial accelerator related to banking regulations. That is, instead of trying to overcome the usual identification problems for supply and demand for credit, one can simply look at the spread. After controlling by market power and monetary policy any change in spreads should be explained by the relative value of the bank's CA ratio respect to the minimum required.

The analysis of bank's behavior based on its optimal policy functions is not very informative of the dynamics of the simulated economy since the state variables of the system evolve endogenously over time. Figure 4 shows the impulse-response functions for the bank's variables. They were derived

by perturbing the system with a TFP shock big enough to make the capital requirement to bind in the regulated economy (see bottom left panel). In the current calibration such a shock is around 18% of the deterministic steady-state value of TFP.

Banks respond as expected during the recession. They first retain earnings in an attempt to avoid cutting-back on loans (see middle left panel), but eventually either the non-negativity constraint on dividends is hit or the marginal utility on dividends goes up so much that banks prefer to reduce credit. In any case, comparing equations (8) and (9) it becomes clear that with the regulation binding ( $\mu_t > 0$ ) the interest rate spread increases (see top right panel), making the interest rate on deposits to decrease by more than in the no regulation case. With a counter-cyclical interest spread and an extra fall in deposits interest rate, households reduce consumption by less and savings by more than in the economy with no capital requirement. Due to the reduced demand deposits and to the fall of equity, bank loans decreases more than in the economy with no regulation. As the capital stock decreases one to one with bank loans, output, consumption and investment all will display more persistence than in the economy with no capital requirement.

The fact that such a big shock (-18%) is needed for the constraints to kick-in is consistent with the stationary distributions shown in Figure 1. There, the probability of such a realization of the shocks and thus the probability of the constraint binding is almost zero. One could conclude from this that the probability of the financial accelerator working as described above is also zero. However, as it was made clear in Figure 2, the presence of the constraints reshape bank's optimal behavior even when those constraints do not actually bind. As it was the case with an increase in the capital requirement, a smaller negative shock (say 1.7%) will make banks to start cutting-back on loans right away, at the same time they retain earnings.

This is shown in Figure 5, which displays the responses of output, capital,

consumption and investment to a negative shock (thicker lines correspond to the regulated economy). Again, on impact consumption decreases by less and investment by more than in the non-regulation scenario. After that, all macroeconomic variables remain below their no regulation counterparts as the economy returns to its steady-state. It is worth noting that output is the only variable that on impact does not behave differently than in the unregulated model. However, as the differential effect on investment builds up over time and capital stock recovers at a slower pace, output starts to lag behind.

As can be seen in the dynamic response of the system, the size of the financial accelerator effect is small compared to the size of the TFP shock. This remains true for several parameterizations of the model<sup>15</sup>. This result is disappointing, given the rich dynamics involved in the mechanism by which the regulation affects the system, but it is not surprising. As it was described before, along the stochastic steady-state banks will keep a buffer of excess capital to cushion the effect of negative aggregate shocks. A unexpected large shock may make equity to fall enough to make the constraint to bind on impact, but immediately after the shock banks will try to restore the buffer of capital to its normal level. Thus, the financial accelerator will be very short-lived; it just operates on impact. The TFP process governing the dynamics of the demand for credit, on the other hand, is highly persistent. The reduction in the demand for credit of course relaxes the regulation constraint. And due to the persistence of TFP, this effects not only operates on impact but also it builds up over time.

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<sup>15</sup>The model was solved for different values of  $\omega$ , governing the labor supply elasticity,  $\alpha$  for demand elasticity of capital and  $\gamma$  the level of the minimum CA ratio. The comparison of the two models, with and without regulation, seems to be robust to all these changes.

## 5 Conclusions

The capital adequacy regulation has been designed as an incentive mechanism to limit the risk-taking behavior of the bank. The Basel Accords have set the standards by stating that banks capital should not fall below 8% of their risk-weighted portfolio of assets. The last revision to the accords, called Basel II, introduces some other changes aimed at controlling the amount of market risk in banks operations. However, several basic questions as regards how exactly this capital regulation affects bank's behavior in the context of aggregate risk and how the induced bank actions reshape in turn the dynamics of the main macroeconomic variables are still unanswered. On the one hand, the large empirical literature on the subject cannot reach definitive conclusions about them. On the other, there are not many theoretical models proposing explanations from a general equilibrium perspective.

A DSGE model is used here to suggest possible answers to some of these questions. Numerical simulations of the model provides a qualitative characterization of the general equilibrium dynamics that arises from the interaction among the capital regulation, bank's behavior and the other economic agents. First, uncertainty combined with rational forward-looking behavior make banks to hold capital in excess of the minimum required by the regulation. This is true even when issuing equity is a more expensive financing method than bank deposits. Second, an increase in the capital requirement like the one implemented in the US in the early 1990s make optimal CA ratios to increase. Banks change the equity/debt financing mix by accumulating more equity rather than cutting-back on loans. Since issuing equity would be too costly when the banking system is undergoing a generalized under-capitalization, banks prefer to retain earnings. Third, this paper does not give much support to the hypothesis of a credit crunch, at least from a theoretical point of view. The model predicts a slow reaction of bank loans

to a change in the regulation. This seems to be confirmed by the weak evidence on a credit crunch found in the empirical literature. Additionally, the dynamic analysis makes it clear that it is incorrect to explain a recession with the credit crunch hypothesis. This would amount to relate a permanent change in a parameter of the model with a temporary deviation of output from steady-state, when in fact such a change will make the economy to move permanently from one steady-state to another.

Finally, no formal thought have been given in the previous literature to the issue of the macroeconomic implications of a regulation restricting the amount of bank credit and becoming more stringent during recessions. The model is used to study the presence of an automatic amplifier of macroeconomic fluctuations. When the economy is hit by a negative aggregate shock, investment and demand for credit fall. At the same time firms make net losses and they cannot pay back their debt plus interest. Banks must absorb these losses and banks equity shrink. It was previously believed that banks would prefer to accommodate the shock by rebuilding capital and retaining earnings rather than by cutting-back on loans. Only when dividends have been reduced to zero banks will have no alternative other than reducing lending as a way to maintain capital above the legal minimum. Although this last part of the explanation is still true, the simulations in this paper show that banks will in fact cut-back lending from the very beginning, even when dividends are still positive. The sole presence of the capital regulation and the non-negativity constraint on dividends in the problem will reshape bank's behavior, even if they do not actually bind in any state of nature. As a consequence, investment and production fall an extra amount as banks cut-back on credit.

There are many dimensions over which further theoretical research would be interesting. There are several features of reality that were omitted here. If included, they could either reinforce or weaken the conclusions of this model

in a significant manner. For example, making dividends per unit of equity dependent on the scale of operation of the bank would make the financial accelerator more persistent and thus its effects more important. This would be the case of an oligopolistic banking industry (and under certain conditions as regards the cyclicalities of the demand elasticity for credit). Modelling economies of scale in the intermediation services of the bank would alternatively enhance the financial accelerator in the same way. A different way would be to introduce bank assets of maturity longer than one period. As explained in Blum and Hellwig (1995), as the value of long-lived assets fall when market rates of interest decrease in the downturn, bank equity falls and the CA ratio gets closer to the minimum. Moreover, given the illiquidity of bank loans, the stock adjustment required by the regulation will rely mainly on cutting-back the new loans that finance investment.

There are other extensions to the model that would likely decrease the importance of the financial accelerator effects. For example, introducing bank heterogeneity by considering that the degree of capitalization is different across banks would break the rigid link between bank capital and aggregate lending. As firms switch from poorly capitalized banks to healthier banks during the economic recession, bank lending and investment would fall by less than in the representative bank model. Of course, it would be necessary to consider how does the distribution of banks change over the cycle and also what are the costs for firms of switching among banks. Another extension to consider is the fact that banks have developed different strategies to overcome the restriction implied by the capital requirements. The practice of securitization of bank's risky assets and other forms of artificially increasing the CA ratio are regulated in the Basel II guidelines. By making use of these instruments bank could avoid decreasing loans as the stringency of the regulation increases during a downturn of the cycle.

Finally, it would be interesting to carry out a welfare analysis of the

regulation. Explicitly modelling the negative externality that gives birth to the capital regulation would allow to measure its social benefit. Moreover, balancing this benefit with the kind of costs studied in this paper one could come up with an optimal (welfare maximizing) level for the bank capital requirement.

Figure 1: Stationary Distribution of Bank Variables

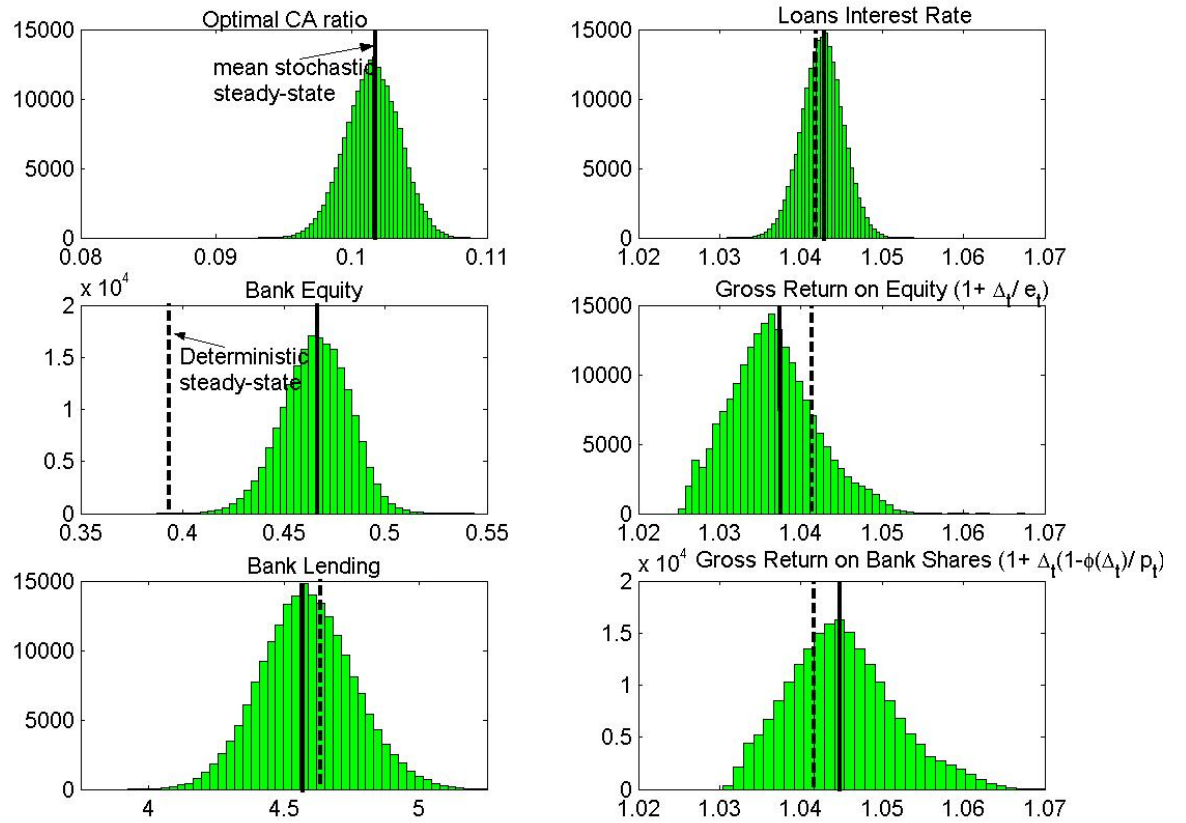




Figure 2: Transitional Dynamics for a Permanent Change in  $\gamma$

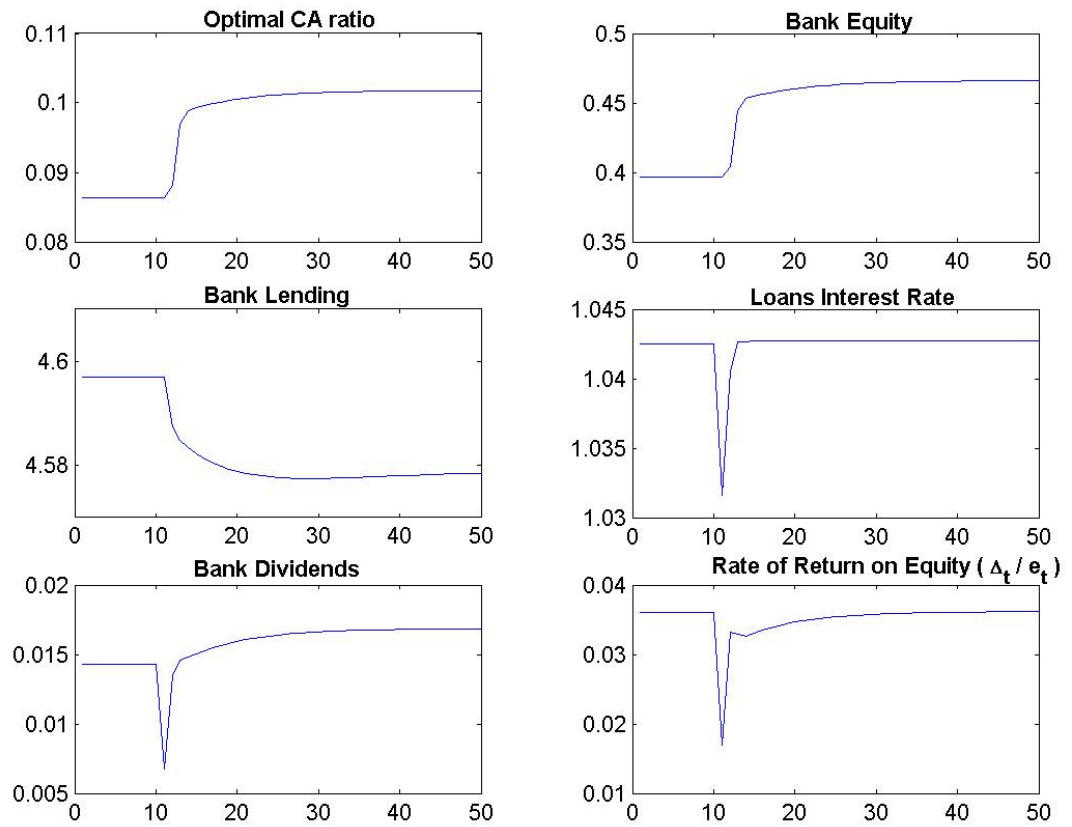


Figure 3: Bank Optimal Response Functions

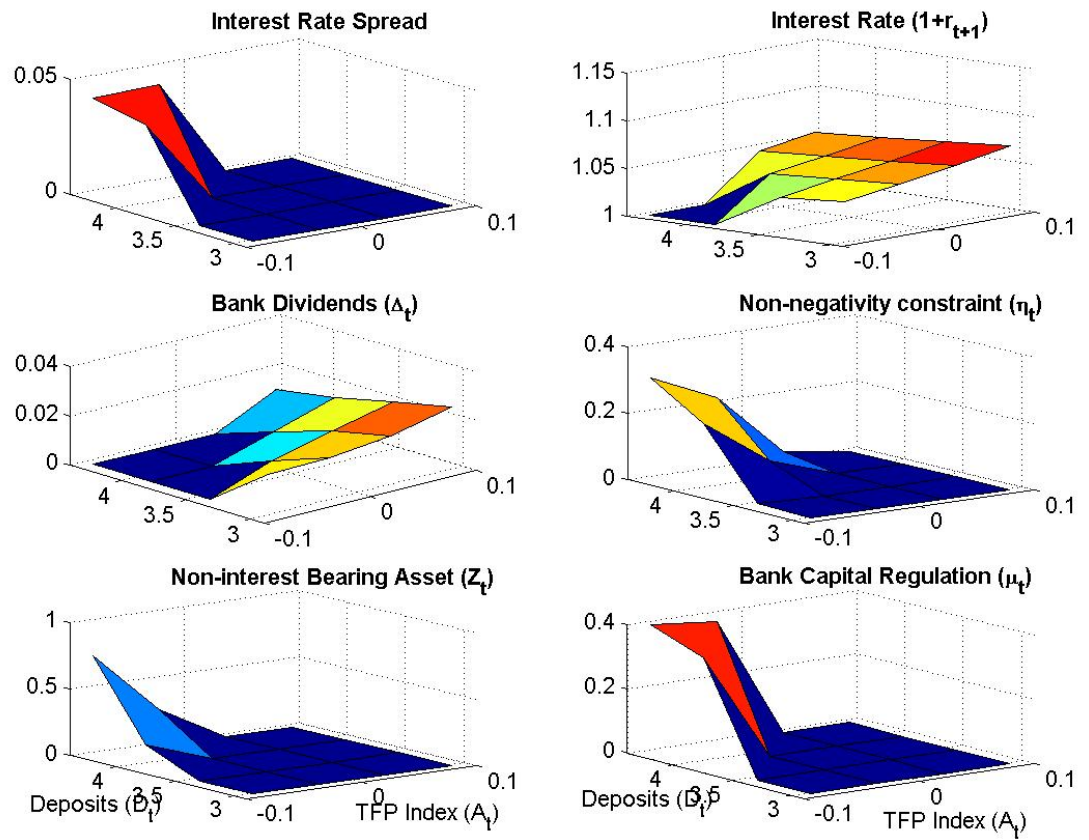


Figure 4: Impulse-Response Functions for Bank Variables

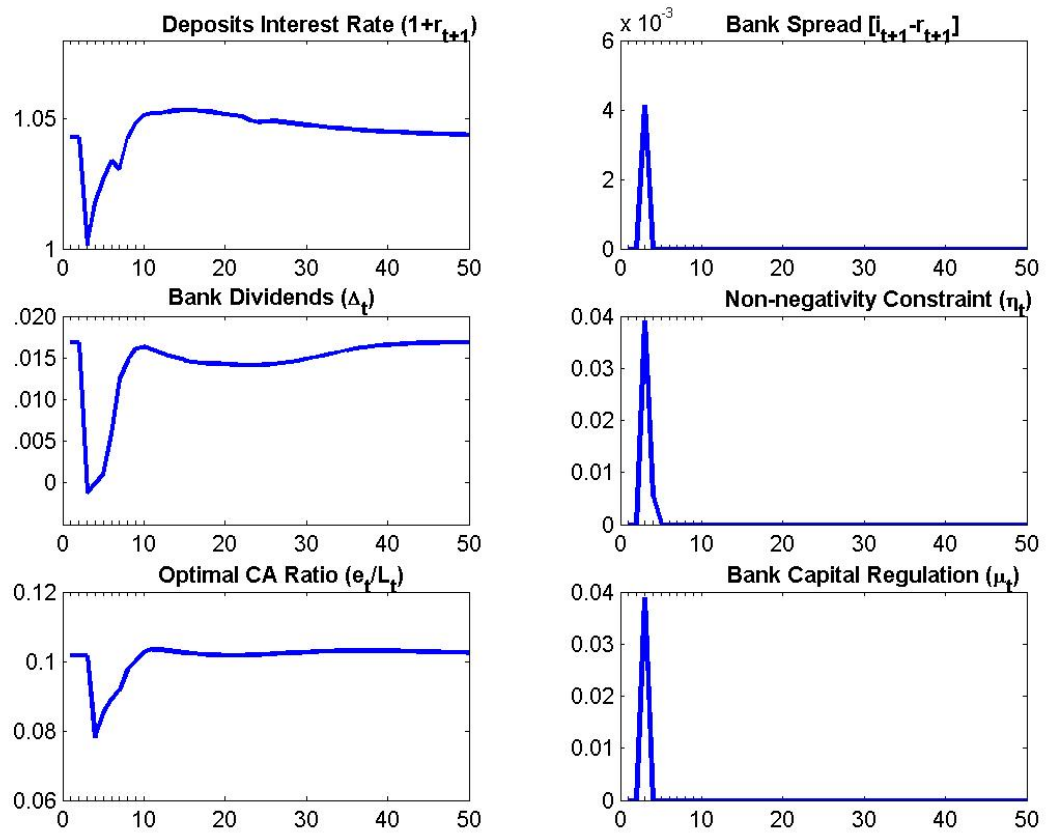
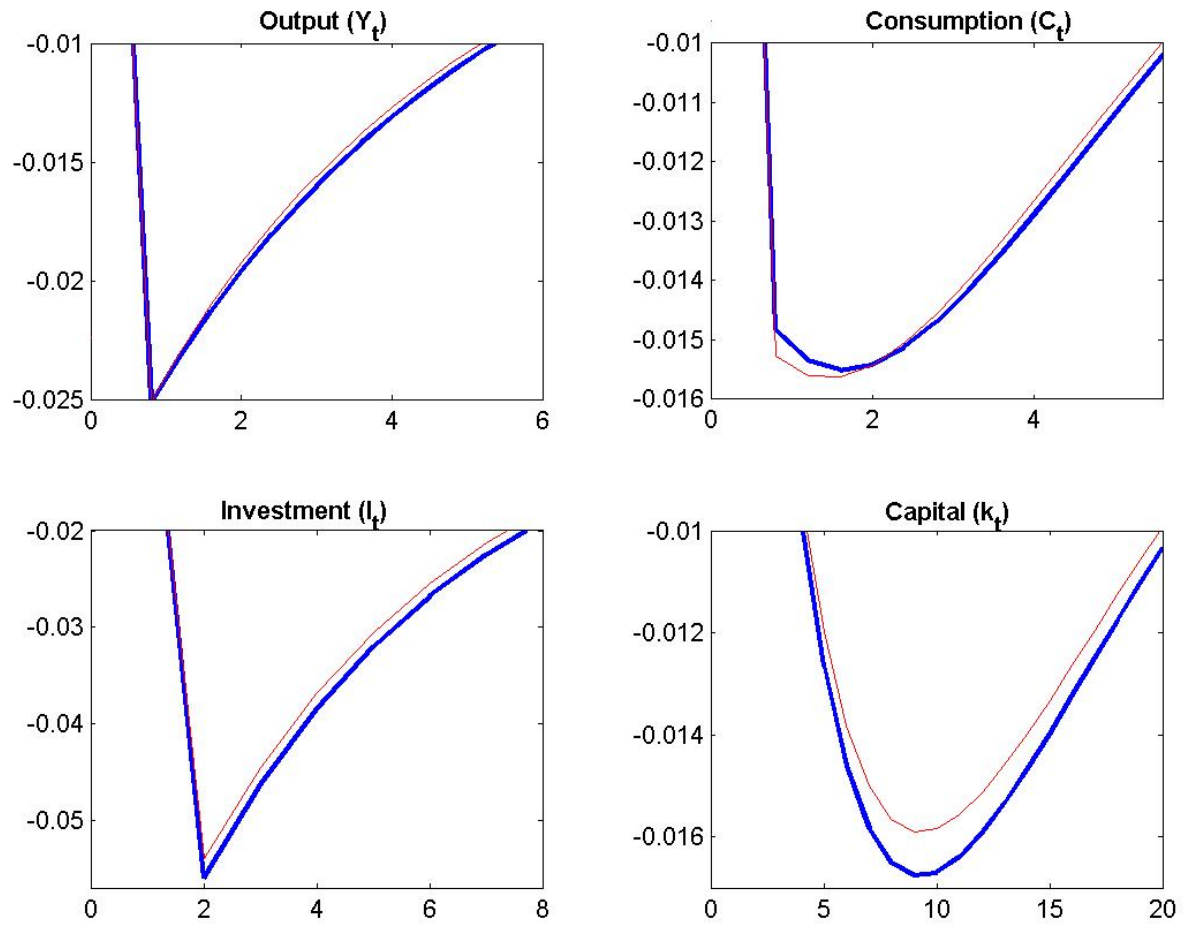


Figure 5: Impulse-Response Functions for Regulated Economy



## Appendices

### A Bank's Objective Function

The derivation of the bank's objective in the main text relies on the standard formulation of asset pricing models for valuation of a particular asset. That is, using equation (5) for optimal holding of bank's shares, dividing by current marginal utility, multiplying both sides for  $p_t \bar{s}$  and calling  $v_t \equiv p_t \bar{s}$  to the market value of the bank:

$$v_t = \beta E_t \left[ \frac{u_c(c_{t+1}, l_{t+1})}{u_c(c_t, l_t)} \left( \Delta_{t+1} (1 - \phi(\Delta_{t+1})) + v_{t+1} \right) \right] \quad (14)$$

Finally, solving this expression forward

$$v_t = E_t \left[ \sum_{j=t}^{\infty} \beta^{j-t} \frac{u_c(c_j, l_j)}{u_c(c_t, l_t)} \Delta_j (1 - \phi(\Delta_j)) \right] \quad (15)$$

### B Numerical Method

Solving nonlinear rational expectations models present difficulties because their solutions are functions of unknown form and because the equilibrium conditions involve integrals with no explicit solutions (Fackler, 2003). Optimal policy functions cannot be obtained analytically except in some very particular cases. Methods for solving linear rational expectation models, however, are well developed. Thus, one strategy could be first to use linear or log-linear approximation methods to obtain the numerical solutions. However, in this model it is the presence of occasionally binding constraints what makes any linearization technique useless as they introduce non-convexities into the optimal responses.

The problem of occasionally binding constraints could be handled well by value function (VFI) or policy function iteration (PFI) methods. However, this model cannot be reformulated in terms of a Central Planner's problem. Using either of these methods in the context of a decentralized competitive environment is not practical because solving for market-clearing prices adds an extra loop to the algorithm<sup>16</sup>.

The method of parameterized expectations approach (PEA) is another alternative. It basically produces a numerical approximation to the policies by solving simultaneously from the set of first-order conditions arising from agents optimization, market clearing conditions and inequality constraints. The method relies on approximating the expectation functions in the system by using some polynomial or other known functional form. The idea is to obtain the parameters that minimize the difference between the expectation function and the approximating function. As described by Marcet and Lorenzoni (1999), the parameters are estimated by non-linear least squares from data generated through Monte-Carlo simulation of the dynamic system. Finally, the routine goes over a fixed-point iteration scheme which converges when the estimated parameters are not different than the ones used to simulate the model. However, there are a number of problems associated to this technique. First of all, the system must be highly stationary for the estimation of the parameters to be (consistent?). Second, convergence to a fixed-point depends greatly on how "good" the initial guess is, which in turn is very difficult to obtain when there are inequality constraints<sup>17</sup>. Third,

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<sup>16</sup>Roughly speaking, it would be necessary to iterate over the Bellman equation for each agent in the economy taking prices as given and then check if markets clear at those prices. If they do not, the algorithm should update prices and then solve all over again. See Mendoza and K. Smith (2002) for an application of VFI with occasionally binding constraints in a decentralized economy setting.

<sup>17</sup>Without inequality constraints, the solution obtained through log-linear approximation would be close enough to the solutions from PEA. However, with inequality constraint

even with reasonable good guesses convergence is not guaranteed. Finally, the inequality constraints introduce kinks into the functions being approximated that are difficult to replicate with the approximate solution. This is especially true if we are using spectral methods as described above (i.e. the approximant function is non-zero almost everywhere in the domain).

The method used in this paper to obtain the numerical solutions is a finite-element method. The general idea in the so-called weighted residual methods (see McGrattan, 1999) is to represent the approximate solution to the functional equation problem with a linear combination of known basis functions such as polynomials. The method consists of finding the coefficients of the combination that minimize an appropriately defined residual function evaluated at the approximate solution. The finite-element method can be understood as a piecewise application of the weighted residual method. That is, the domain of the state space is divided into no-overlapping sub-domains and low-order polynomials are fitted to each one of them. The local approximations are then pieced together to give the global approximation.

Following Fackler (2003) there are several choices to make related the implementation of this method: first, how the expectation operators in the model are approximated; second, what family of basis functions are used to represent the solution; third, what method is used to find the coefficients of the linear combination and finally, what algorithm is used to solve for these parameters.

First, it has been shown that expectation operators can be approximated well by a discrete distribution (see Miranda and Fackler, 2002 and Burnside, 1999).

$$E[f(e)] \approx \sum_j w_j f(e_j)$$

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it is necessary to use the idea of homotopy (see Marcet and Lorenzoni, 1999).

Where  $e$  is the random variable and  $w_j$  are the weight or probabilities associated to each realization of  $e$ . The idea is to approximate numerically the integral involved in the expectation. In this paper we use a five-point Gaussian quadrature approach.

Second, the optimal response functions are of unknown form so they must be approximated numerically. The optimal policy is a function of the state variables both directly and also indirectly through the conditional expectation function (which is also of unknown form). Thus, there are two possibilities: one can directly approximate the policy functions or one can first approximate numerically the expectations as a function of the states and then solve for the optimal policy from the equilibrium conditions. This second alternative is close to the Parameterized Expectations Approach (see Marcet and Lorenzoni, 1999).

As regards the approximant functions used in either method, it is convenient to work with families of functions that are linear in a set of coefficients (Fackler, 2003). For example, functions of the form  $\phi(\Upsilon)\theta$ , where  $\Upsilon$  represents the state space,  $\phi(\Upsilon)$  is a vector of basis functions and  $\theta$  is a matrix of coefficients. Specifically, polynomials and polynomial splines (including piecewise linear functions) fall into this category. In this paper I use piecewise linear functions. This tends to give better approximation when there are kinks in the approximate solutions such as those corresponding to inequality constraints.

Once the approximant function has been selected one needs to select a criterion to determine the weights of the basis functions given by the matrix of coefficients  $\theta$ . One possibility among others is the Collocation Method (Miranda and Fackler ,2002 explain it in detail). The idea is to partition the state space at  $n$  points, called the collocation nodes. The coefficients can be found by requiring the approximant to make an appropriately defined residual function (such as the functional equation itself) equal to zero at those nodes.



Since the approximant consists of  $n$  basis functions and  $n$  coefficients, the collocation method amounts to replace the infinite-dimensional functional equation problem with a system of  $n$  nonlinear equations<sup>18</sup>.

Finally, one must choose the algorithm to solve the system of equations for the coefficient values. Some possibilities discussed in Fackler (2003) are Newton's method and a more efficient Quasi-Newton Method called Broyden's Method. Fackler (2003) also suggests an alternative to these memory-consuming root-finding methods that consists on a fixed-point iteration scheme. The iteration starts with some guess on the parameter values and then it computes optimal policies for next period  $t + 1$  for each and every state of nature by using the transition rule for the states. With these next period policies and the shocks one can approximate numerically the integral corresponding to the expectation function. Once the values of the expectation functions are known, one can re-compute the optimal policy and update the initial guess. The iterations continues until the change in the policies or in the parameters is sufficiently small.

The choice of initial guess turns out to be critical in this fixed-point iteration, and even with good initial values convergence is not guaranteed. This contrasts with the quadratic and superlinear rates of convergence of Newton's and Broyden's methods, respectively (see Fackler, 2003 for details). Due to the curse of dimensionality and because of memory limitations, I have to use the fixed-point iteration in this paper.

The Matlab implementation for all these steps including different choices of family of basis functions, the Collocation Method and the fixed-point iteration scheme to solve for the coefficients was programmed by Fackler (2003) and is called `resolve`. The command allows for great flexibility in specifying all the options discussed above. Many other utilities included in

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<sup>18</sup>The system would be  $n^m$  equations in  $n^m$  unknowns for a state-space of dimension  $m$ , and it would increase to  $p \times n^m$  if there are  $p$  response functions being approximated.

the CompEcon toolbox (Fackler and Miranda, 2002) are also used here.

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