Economic Growth in a World of Ideas: Some Pleasant Arithmetic

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Persistent trends in R&D intensity and educational attainment, in conjunction with the absence of any trend in per capita income growth, are inconsistent with the predictions of most growth models. Jones (*American Economic Review,* **92**(1):220-39, 2002)*, has shown that the data are consistent with out-of-steady state predictions of his semi-endogenous growth model. This paper presents an alternative explanation: R&D intensity and educational attainment are rising because passive learning has become more difficult in the face of increasing technological complexity. We construct a model in which R&D and learning are substitutes and education facilitates on-the-job learning. The model presents an endogenous explanation for the observed increases in the inputs into knowledge creation, along with a rise in the skill premium. In contrast to Jones, our model does not predict that a dramatic decline in the growth rate of per capita income must follow the transition period.*(*JEL* O40, E10)

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Figure 1 contains some US aggregate time-series that are now very familiar. Panel (a) shows a dramatic rise since 1950 in the intensity of R&D, whether measured as a proportion of the labor force or as a proportion of aggregate expenditure. Panel (b) shows a more modest, but nonetheless marked, increase in educational attainment. One may quibble about the significance of these data. R&D has no doubt become more formal and, consequently, more broadly defined in the official statistics, and much of the increase in educational attainment may be a consumption good that contributes little to measured productivity growth (Klenow and Rodriguez-Clare, 1997; Dinopoulos and Thompson, 1999). Nonetheless, given the strength of the observed trends, the underlying real changes in R&D and educational attainment must be considerable. However, panels (c) and (d) show that, despite these dramatic changes in the key inputs of the knowledge production function, there has been no corresponding rise in either per capita income growth or labor productivity growth.

The evidence in Figure 1 is inconsistent with many endogenous growth models. However, Jones (2002) has made a strong case that it is broadly consistent with out-of-steady state predictions of his semi-endogenous growth model (Jones, 1995). In his model, per capita income growth is proportional to population growth in the steady state, but can be sustained at a constant, higher, rate when input intensity is rising. Applying traditional growth accounting techniques, Jones concludes that rising input intensity accounts for 80 percent of post-war growth. Jones' analysis contains some unpleasant arithmetic. Eventually, the secular increases in R&D intensity and educational attainment must end. When they do, income growth can be expected to decline dramatically, perhaps to no more than one-fifth of its post-war trend.

In this paper we suggest an alternative explanation for the evidence in Figure 1 that predicts no such collapse. We assume that increases in productivity can result from formal R&D effort and from learning by doing. However, during the latter half of the 20th century, increased technological complexity has made passive learning more difficult. We argue that firms have consequently

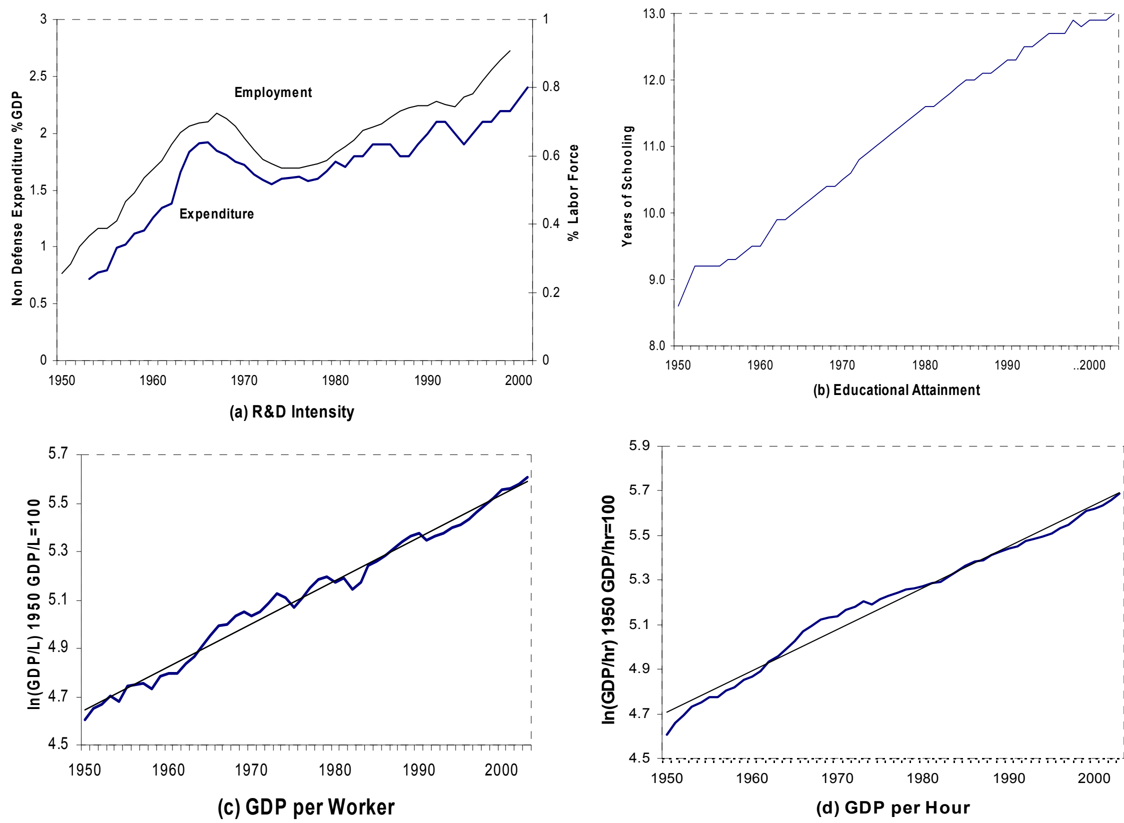

FIGURE. 1. R&D, education and economic performance in the US. For sources, see Appendix B.

substituted R&D for learning and, because skilled workers can overcome the challenges of learning in a more complex environment more readily than can unskilled workers, the relative demand for skill has also risen. The consequent increase in the returns to skill in turn has induced an increase in educational attainment. Our theory explains how increases in R&D intensity and educational attainment can be equilibrium responses to changing conditions that make growth more difficult. Despite greater complexity, R&D and educational attainment must, as in Jones (2002), eventually cease to grow. But, in stark contrast to Jones, our theory does not imply that income and productivity growth will collapse once the new steady state is reached.

In Section 3, we formalize these ideas with a general equilibrium model of R&D and learning in the spirit of earlier work by Young (1991, 1993), Lucas (1993), and Parente (1994). For simplicity we assume that R&D is not necessary to develop new product generations, which arrive at an exogenous rate. Instead, R&D is assumed to influence the productivity of a new product at the time it is launched, and the more R&D that is conducted, the less there is left to learn. Skilled labor is a necessary input into R&D, and it also enhances a firm's ability to learn in production. As a consequence, the immediate effect is to increase the price of skill. The initial increase in wages of skilled workers is offset over time by an induced rise in the supply of skills. Of course, in order to sustain an increased supply of skills in the long run wage inequality must remain higher than before the increase in the difficulty of learning, These dynamic responses are obtained in a setting in which the aggregate rate of growth is constant. Thus, a reversal in the difficulty of learning would induce a decline in R&D and in the returns to skill, but no decline in economic growth.

Our theory rests on some precise assumptions, and it is worth fixing ideas on these immediately: technology has become more complex over time; learning by doing is more difficult in complex environments; skill is more valuable in complex learning environments; and research is a substitute for learning by doing. None of these assumptions seems particularly contentious, but it turns out to be quite difficult to produce direct evidence for them. We do not have any easy way to measure complexity, and attempts to measure rates of passive learning have proved to be rather unreliable (Mishina, 1999; Lazonick and Brush, 1985; Sinclair, Klepper, and Cohen, 2000; Thompson, 2001). Nonetheless, there is a body of indirect evidence consistent with our assumptions, which is briefly reviewed here.

A. *Learning and complexity*

Jovanovic and Nyarko's (1995) Bayesian model of learning is perhaps the best-known study of the interaction between complexity and learning. They define complexity in terms of the number of independent tasks that must be undertaken in the production process. Their model predicts that in more complex technologies there will be more to learn, but the rate of learning is slower. Parameter estimates obtained from fitting their model to a dozen data sets are consistent with these predictions. In a series of papers (Argote, Beckman and Epple, 1990; Darr, Argote and Epple, 1995; Epple, Argote and Devadas, 1991), Argote, Epple and colleagues obtained similar results from estimating learning curves from three distinct activities – the operation of pizza franchises, an automotive assembly plant, and wartime shipbuilding. Figure 2 plots the learning curves implied by their parameter estimates. [1] If we are willing to entertain the notion that shipbuilding is a more complex task than automotive assembly, and automotive assembly is more complex than operating a pizza franchise, the learning curves yield half-lives of learning consistent with the predictions of Jovanovic and Nyarko.

Our ranking of Argote and Epple's three technologies is inevitably subjective. Unfortunately, Jovanovic and Nyarko's inferences about the relative complexity of different activities are even more problematic for our purposes, as they are obtained from the learning curves themselves. Galbraith (1990) took perhaps a more objective approach by allowing senior project engineers learning to work with new technologies to evaluate their complexity. He studied 32 instances in which high-technology companies transferred core manufacturing technology to plants located at least 100 miles from where the technology was originally in use. The senior project engineer at each recipient location was asked to rate on a five-point scale the complexity of the transferred technology relative to the recipient's

[1] Epple and Argote assume that knowledge rises log-linearly with cumulative output and declines as a function of time. They interpret their results as evidence of organizational forgetting. Thompson (2004) has argued that forgetting may be a spurious result of assuming a learning curve in which, absent forgetting, productivity must rise without bound. In Figure 2 we simply plot the predicted productivity levels implied by the regression estimates.

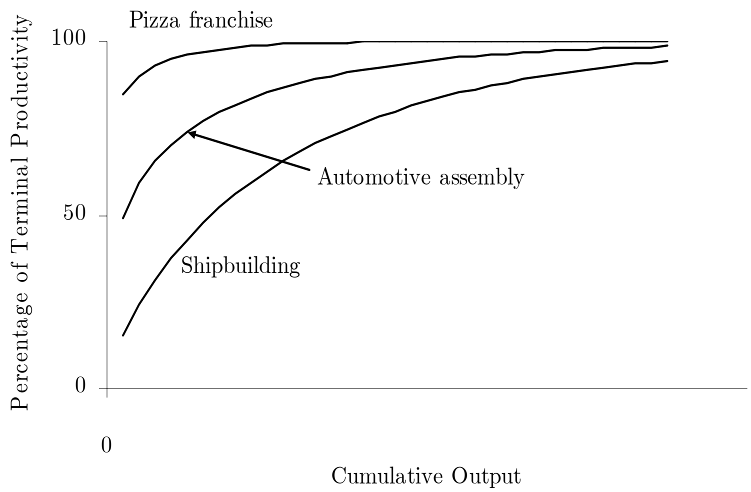

Figure 2. Learning curves from three industries. Curves plot the function q_t/q^*, where $q_t = K_t^\gamma$, $K_t = \lambda K_{t-1} + 1$. and $q^* = (1-\lambda)^\gamma$. Parameter estimates are: $\gamma = 0.71$, $\lambda = 0.93$ for shipbuilding (from Argote, Beckman and Epple, 1990, table 1, column 1); $\gamma = 0.28$, $\lambda = 0.92$ for automotive assembly (from Epple, Argote and Devadas, 1991, table 1, column 4); $\gamma = 0.104$, $\lambda = 0.80$ for pizza franchises (from Darr, Argote and Epple, 1995, table 1, column 4).

existing technologies. Galbraith shows that the time it took the recipient site to reach the level of productivity at the donor site increased significantly with the complexity of the technology, even controlling for an initial loss in productivity that was higher in the more complex transfers. An increase of one on the five-point scale led to an increase in the initial productivity loss of about 16.7% and an increase in the recovery time of the lost productivity of about 15 percent.

B. *Skill and Learning*

An extensive literature on wage inequality and technology is consistent with our assumption that skilled labor has an advantage in learning more complex technologies and that, as technology became more complex in recent decades, the returns to education and unobservable skills have increased.

The sharp rise in the return to schooling since the early 1980s (Blackburn, Bloom and Freeman, 1990; Katz and Murphy, 1992), in the premium for unobserved ability (Juhn, Murphy and Pierce, 1993; DiNardo and Pischke, 1997), and in the premium for observable indicators of cognitive ability (Murnane, Willet and Levy, 1995) are all consistent with our assertion that education and ability have become more valuable as complexity has increased. Evidence that earnings profiles are steeper for educated workers (Psacahropoulos and Layard, 1979; Knight and Sabot, 1981; Altonji and Dunn, 1995; Altonji and Pierret, 1997; Brunello and Comi, 2004; Low et al., 2004) is consistent with our assertion that educated workers are more able to learn.

If newer technologies are more complex than older technologies, our assumptions imply a positive correlation between wages and use of new technology, and this is again consistent with empirical evidence. Autor, Katz, and Krueger (1998) document an increased demand for skilled labor during the last five decades, and especially since 1970. They argue that the diffusion of computers and related technologies contributed significantly to this phenomenon and show that skill upgrading occurred more rapidly in industries that are computer intensive. Berman, Bound and Griliches (1994) and Berman, Bound and Machin (1998) find large within–industry increases in the share of non-production workers in manufacturing, both in the US and in a sample of OECD countries, despite the rise in their relative wages during the 1980s and 1990s. They also show that the increase in the share of non-production workers is associated with R&D and computer investment. Allen (2001), focusing on the timeframe 1979-1989, shows that wage gaps by schooling increased the most in industries with rising R&D intensity and accelerating growth in the capital-labor ratio.[2]

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[2] Further evidence relating the wage structure to technology use can be found in Krueger (1993), Dunne and Schmitz (1995), Doms, Dunne and Troske (1997), and Thompson (2003).

Despite the wealth of wage data consistent with our assertions, we must acknowledge that the evidence is only circumstantial. Rising wage inequality is also predicted by models in which skilled individuals have an advantage simply in adopting or working with new technologies (Caselli, 1999; Galor and Moav; 2000, Lloyd-Ellis, 2002). Chari and Hopenhayn (1993) predict that workers employed on newer technologies will exhibit steeper earnings profiles, even though all workers learn at the same rate. R&D intensive industries, and industries and plants using newer technologies are likely to be more capital intensive, and capital-skill complementarity may be sufficient to explain their higher wages.

C. *Learning and R&D*

Our assumption that R&D may substitute for time spent learning is supported by the pioneering work of Cohen and Levinthal (1989), who provide evidence that R&D effort is a significant determinant of a firm's "absorptive capacity", by which they mean a firm's ability to appropriate knowledge from other firms and industries, and from basic science. Cohen and Levinthal conjectured that R&D is most important when assimilation is made more difficult by the complexity or quality of outside knowledge. Using independent, survey-based measures of complexity, they found support for their conjecture.

Subsequent researchers have provided further evidence that R&D is a major factor influencing absorptive capacity. Using industry-level data, Griffith, Redding and Van Reenen (2003, 2004) find that it matters for cross-border assimilation of knowledge among 12 OECD countries; Cockburn and Henderson (1998) find that R&D is closely associated with pharmaceutical companies' "connectedness" with basic research; and Szulanski (1996) concludes that the ability to transfer best practices even within firms depends on recipient absorptive capacity.

Finally, our model predicts a positive correlation between R&D intensity and the skill-content of non-R&D workers. We would therefore expect, taking our basic assumptions together, that R&D intensive industries will have higher wages even among non-R&D workers. This is consistent with the evidence (Hodson and England, 1986; Dickens and Katz, 1987; Loh, 1992).

The Model

Models that combine R&D, new products and bounded learning have tended to be rather complicated. As a result, they have also tended to be rather stylized. We do not depart from that "tradition" here. We construct a stylized model in which there are two types of skill – one is a necessary input into R&D, the other enhances learning by doing. In Section A, we assume that both skills are in fixed supply and show that, if learning becomes harder, the wages of both skills must rise. In Section B we endogenize the supply of skill in an overlapping generations setting. When the returns to skill rise, agents are naturally induced to accumulate more. The increased supply facilitates an increase in R&D intensity, as well as an increase in the employment of skilled labor outside of R&D. Section C describes the transitional response to a change in the difficulty of learning.

A representative agent's intertemporal utility is given by

$$U = \int_0^\infty e^{-\rho t} \ln D(t) dt \,, \tag{1}$$

where

$$D(t) = \left[\int_0^1 q(i,t)^{1/\theta} x(i,t)^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)} \,, \tag{2}$$

is a quality-adjusted Dixit-Stiglitz consumption index defined over a continuum of goods of unit mass. The parameter $q(i,t)$ is an index of the quality of good i, while $x(i,t)$ denotes its quantity.

The familiar Euler equation,

$$\frac{\dot{E}(t)}{E(t)} = r(t) - \rho \,, \tag{3}$$

where $E(t)$ is the agent's nominal expenditure on consumption goods, solves the consumer's intertemporal optimization problem. Nominal expenditure is the numeraire, so that $r(t)=\rho$, and instantaneous consumer demands satisfy

$$x(i,t) = \frac{q(i,t)p(i,t)^{-\theta}}{\int_0^1 q(i,t)p(i,t)^{1-\theta}di}. \tag{4}$$

Production is carried out by unskilled labor, one unit of which produces one unit of output. Let $w(t)$ be the wage of the unskilled. Each good is produced by a monopolistic firm i which chooses a constant markup over marginal cost, setting a price $p(i,t)=w(t)\theta/(\theta-1)$, and consequently facing demand

$$x(i,t) = \frac{(\theta-1)\alpha(i,t)}{\theta w(t)}, \tag{5}$$

where $\alpha(i,t) = q(i,t) / \int_0^1 q(i,t)di = q(i,t) / Q(i,t)$ is the relative quality of firm i's product. Let $L(t)$ denote the supply of unskilled workers, and $G(\alpha,t)$ the distribution of relative quality. Full employment of unskilled workers requires that

$$\begin{aligned} L(t) &= \frac{(\theta-1)}{\theta w(t)} \int_0^1 \alpha dG(\alpha,t) \\ &= \frac{(\theta-1)}{\theta w(t)}, \end{aligned} \tag{6}$$

which identifies the wage, $w(t)=(\theta-1)/(\theta L(t))$, demands, $x(i,t)=\alpha(i,t)L(t)$, and profits from manufacturing, $\pi(i,t)= \alpha(i,t)/\theta$.

New generations of the product arrive to each firm randomly according to an exogenous Poisson process with mean intensity μ. Let $\bar{\alpha}(i,t)$ denote the initial relative quality of the current generation of i's product line, and let $T(i)<t$ denote the date it was introduced. If the firm's next generation, arrives at time t, it yields an improvement in relative quality of magnitude $\tilde{\lambda}$. It turns out to be useful to choose the normalization $\tilde{\lambda} = \lambda\int_0^1 \bar{\alpha}(i,t)e^{-g(t-T(i))}di$. The integral will prove to be constant in the steady state, and the normalization will yield a steady-state growth rate of $g = \dot{Q}(t) / Q(t) = \lambda\mu$.

Upon arrival of a new generation, the firm may further enhance its initial quality through R&D. The firm must maintain a permanent R&D laboratory with constant employment to secure any given increase in product quality. After the product is launched, further improvements are obtained through learning by doing. Skilled labor is central to both processes, but it is convenient to assume that R&D and learning by doing involve distinct skills. R&D is conducted by engineers, while the speed of learning by doing is enhanced by the employment of supervisors.

The evolution of firm i's relative quality for product generation j launched at time 0 satisfies

$$e^{gt}\alpha(i,t) = \left(\bar{\alpha}^j + \phi(l_e)\right)\left(1 - \Gamma(t;l_s,\psi)\right) + \left(\bar{\alpha}^j + a\right)\Gamma(t;l_s,\psi) \tag{7}$$

where $\phi(l_e)$ is the R&D production function and l_e is employment of engineers; it is increasing, concave and satisfies the Inada conditions. The learning function $\Gamma(t;l_s,\psi)$ depends on the employment of supervisors and the difficulty, ψ, of learning. $\Gamma : R_+ \to [0,1]$ is increasing in t, with $\Gamma(0;l_s,\psi) = 0$ and $\lim_{t\to\infty} \Gamma(t;l_s,\psi) = 1$. Relative quality is a weighted average of the product's initial quality, $\bar{\alpha}^j + \phi(l_e)$, and its terminal quality, $\bar{\alpha}^j + a$. To ensure that learning is positive, it is assumed that $\sup\phi < a$. To avoid tedious discussion of no growth traps (c.f. Jovanovic and Nyarko, 1996), we assume that $a<\lambda$. Thus the initial productivity of the next generation is always high enough to make switching worthwhile.

Let $\gamma = d\Gamma / dt$ denote the rate of learning. We make the following assumptions about the learning function:

$$\gamma_{l_s} > 0, \quad \gamma_{\psi} < 0, \quad \gamma_{l_s\psi} > 0, \quad \gamma_{l_s l_s} < 0, \tag{8}$$

where subscripts denote derivatives. The rate of learning is an increasing concave function of l_s, and is decreasing in the difficulty of learning. The marginal productivity of supervisors rises as learning becomes more difficult. These key assumptions were laid out in the introduction.

Given that $\Gamma(0;l_s,\psi) = 0$, the assumptions in equation (8) apply also to Γ:

$$\Gamma_{l_s} > 0, \quad \Gamma_{\psi} < 0, \quad \Gamma_{l_s\psi} > 0, \quad \Gamma_{l_s l_s} < 0\,. \tag{9}$$

Upon arrival of a new generation, the firm must choose its intensity of R&D, as well as a time path for l_s. Given a hazard rate μ for obsolescence of the current generation, an interest rate ρ, and profits $\pi(i,t)= \alpha(i,t)/\theta$, the dynamic problem is

$$\max_{l_e, \bar{l}_s(t)} \int_0^{\infty} \Big\{ \theta^{-1} \Big[\big(\bar{\alpha}^j + \phi(l_e)\big)\big(1 - \Gamma(t; l_s(t), \psi)\big) + \big(\bar{\alpha}^j + a\big)\Gamma(t; l_s(t), \psi) \Big] e^{-(\rho+g+\mu)t} - \big(l_s(t) w_s(t) + l_e w_e(t)\big) e^{-(\rho+\mu)t} \Big\}\, dt\,. \tag{10}$$

Pointwise maximization yields an optimal path for $l_s(t)$,

$$l_s(t) = \eta\left(\frac{\theta w_s(t) e^{gt}}{a - \phi(l_e)}; t, \psi\right), \tag{11}$$

where η is the inverse of the marginal productivity of l_s, a decreasing function of its first argument. Given our assumptions about Γ, it is easy to see that η is increasing in ψ, but no predictions can be made about the time path for l_s. For given R&D intensity, $l_s(t)$ is increasing in a, and decreasing in θ, $w_s(t)$ and g. An autonomous increase in R&D effort reduces l_s, because it leaves less to learn.

The optimal R&D intensity satisfies

$$l_e = (\phi^{/})^{-1}\left(\frac{\theta \int_0^{\infty} w_e(t) e^{-(\rho+\mu)t} dt}{\int_0^{\infty} (1 - \Gamma(t; l_s(t), \psi)) e^{-(\rho+g+\mu)t} dt}\right). \tag{12}$$

where $(\phi^{/})^{-1}$ is the inverse of the marginal productivity of l_e, a decreasing function of its argument. Given our assumptions about Γ, $(\phi^{/})^{-1}$ is increasing in ψ,. For a constant level of $l_s(t)$, l_e is also decreasing in θ and the time-path of w_e.

For a given set of parameters, (11) and (12) define optimal employment of the two types of skilled labor as functions of g and of wages. Note that neither l_s nor l_e depend on $\alpha(i,t)$. The arrivals of new generations are Poisson, so the equilibrium distribution across firms of the ages of current product generations is exponential with mean μ^{-1}. Hence, the employment constraint for supervisors is

$$\mu \int_0^1 l_s(\tau, w_s(t)) e^{-\mu\tau} d\tau = L_s(t), \tag{13}$$

where the right hand side is the economy's endowment of this skill. The left hand side is decreasing in $w_s(t)$, with unlimited demand at $w_s(t)=0$. Hence, (13) defines a unique equilibrium wage for supervisors. The employment constraint for engineers satisfies

$$\mu \int_0^1 l_e(\vec{w}_e(t-\tau)) e^{-\mu\tau} d\tau = L_e(t), \tag{14}$$

where $\vec{w}_e(t-\tau)$ denotes the entire future path of wages beginning at the launch of a firm's current product generation at time $t-\tau$.

A. *Fixed supply of skills*

If the supplies of skilled labor are constant, then so are their wages. Thus, all firms undertake the same intensity of R&D, so (12) and (14) simplify to

$$L_e = l_e = (\phi')^{-1} \left(\frac{\theta w_e}{(\rho+\mu)\int_0^\infty (1-\Gamma(t; l_s(t), \psi)) e^{-(\rho+g+\mu)t} dt} \right), \tag{15}$$

while (11) and (13) simplify to

$$L_s = \mu \int_0^1 \eta\left(\frac{\theta w_s e^{g\tau}}{a-\phi(L_e)}; \tau, \psi \right) e^{-\mu\tau} d\tau\,. \tag{16}$$

The steady state is characterized by a constant growth rate equal to the product of the proportional quality improvement brought about by each new product generation, λ, and the arrival rate of new generations, μ (claim 1).[3] There is a stable, non-degenerate distribution of firm relative quality and hence firm size, with finite variance (claim 2). The wage of unskilled workers is given by (6). Equation (16) defines the equilibrium wage for supervisors. A rise in ψ

[3] The proof of this claim, and of claim 2 below, are given in Appendix A.

clearly raises w_s. From (15), we see that the denominator on the right hand side rises in response to an increase in ψ. This may induce changes in the time path of l_s for individual firms, but the net effect will be to raise the demand for engineers. In consequence, an increase in the difficulty of learning also drives up the wages for engineers.

An increase in the difficulty of learning has a level effect on income across steady states, but not a growth effect. The aggregate growth rate, $g=\lambda\mu$, depends only on exogenous parameters, so growth is unaffected by changes in ψ. The age distribution of current product generations is also invariant to increases in ψ. However, given τ, each firm will have made less progress along the learning curve, so that its absolute quality will be lower at any point in time.

B. *Endogenous supply of skills*

We now replace the representative agent with a continuum of agents with a constant death rate δ. When an agent dies, she is immediately replaced by a new agent. Upon birth, new agents can choose to pay a cost, c, to obtain education. Each agent has ability in two dimensions, engineering ability, $\xi_e\in[0,1]$, and supervisory ability, $\xi_s\in[0,1]$, which are draws from the joint distribution $F(\xi_e, \xi_s)$. If the cost is paid, and the agent chooses to train as an engineer, she becomes skilled as an engineer with probability ξ_e, and remains unskilled otherwise. If she chooses to train as a supervisor, she becomes a skilled supervisor with probability ξ_s, and remains unskilled otherwise.

Indirect utility is separable in expenditure, so the agent is concerned only to make the education choice that maximizes the discounted present value of her lifetime earnings. As an unskilled worker, this is $\int_0^\infty e^{-(\rho+\delta)t}w(t)dt$. Restricting attention to the steady state with constant wages, the expected lifetime earnings of an unskilled worker is therefore $w/(\rho+\delta)$. For a worker with abilities $\{\xi_e, \xi_s\}$ expected earnings are $-c+(1-\xi_e)w/(\rho+\delta)+\xi_e w_e/(\rho+\delta)$ if she chooses to train as an engineer, and $-c+(1-\xi_s)w/(\rho+\delta)+\xi_s w_s/(\rho+\delta)$ if she chooses to train as a supervisor.

Given wages $\{w, w_e, w_s\}$, it is easy to see that the fraction of agents that choose not to train is increasing in w and decreasing in w_e and w_s. Thus, the steady-state supply of unskilled workers can be written as $L(w, w_e, w_s)$. Similarly, the steady-state supply of engineers is increasing in w_e and decreasing in w and w_s, while the steady-state supply of supervisors is increasing in w_s, and decreasing in w and w_e. With the fixed labor supplies in (6), (14), and (15) replaced by these functions, general equilibrium is defined by a system of three labor supply and demand curves.

C. *Transitory responses to an increase in the difficulty of learning*

When a technological revolution raises the difficulty, ψ, of learning, there can be no immediate response in skill supply. Existing workers have already chosen their training, and they are replaced only gradually, at the rate δdt, by new workers that have yet to choose whether to undertake training. The immediate response to an increase in ψ, then, is an increase in the wages of both types of skilled workers. These wage increases induce a greater fraction of new workers to seek training. The supply of engineers gradually rises, and this facilitates a steady rise in the intensity of R&D. The supply of supervisors gradually rises, and this facilitates an increase in the ratio of skilled to unskilled workers employed in production. The increased supply will mute the initial increased wage inequality, and moderate the demand for education. Thus, the initial response is to overshoot the long-run equilibrium change in wage inequality and in the demand for education by young workers.

It is possible, but by no means necessary, that the *average* educational attainment of the workforce also overshoots the long-run equilibrium. Whether it does depends on assumptions about functional forms and parameter values. However, if overshooting in average educational attainment occurs, it will be associated with cyclical behavior in wage inequality.

What happens to growth along this transition? We have already seen that the aggregate growth of the economy in the steady-state is given by $\lambda\mu$, regardless of the difficulty of learning. There will be some transitional dynamics occurring for each firm's quality within product generations, but whether we would observe a transitory rise or fall in the growth rate is unclear. First, an increase in ψ induces a rise in R&D effort, so that new product generations will be launched at a higher initial quality. This will contribute to a transitory rise in the aggregate growth rate which decays with time as the fraction of firms in the economy that have introduced their next generation increases. Offsetting this, existing product generations will see a slower improvement in quality. The former [latter] effect dominates if μ is sufficiently large [small].

Conclusions

In this paper we offer an explanation for the paradox presented by the coexistence of secular increases in R&D expenditure and educational attainment alongside a constant growth rate. As Jones (2002) pointed out, these observations are inconsistent with most endogenous growth models. Jones showed that the data are consistent with out-of-steady-state behavior in what has become known as the semi-endogenous model of R&D-driven growth. He allows exogenous trend growth in R&D and educational attainment and, using traditional growth-accounting techniques, concludes that the secular trends account for about 80 percent of post-war growth. As such secular trends must eventually end, Jones predicts a startling collapse in future income growth rates.

We construct a quality-ladders model developed based on Thompson and Waldo's (1994) characterization of Schumpeterian trustified capitalism. New product generations arrive stochastically at an exogenous rate. Formal R&D and learning by doing influence the productivity of the new product. Skilled labor is a necessary input into R&D, and it enhances the rate of learning. We claim that learning became more difficult during the latter half of the 20^{th} century, as a result of the increased complexity of the technologies that firms have to work with.

In this setting, rising R&D expenditure and rising educational attainment are shown to be *equilibrium* responses to greater complexity. In our model, however, greater complexity has no consequences for the steady-state growth rate of income.

The model is stylized, and we assume that new product generations are launched by firms at an exogenous rate. The aggregate steady-state growth rate is consequently also exogenous, depending only on the product of two parameters, the proportional quality improvement brought about by each new product generation and the arrival rate of new generations. At the cost of greater complexity, it would of course be possible also to construct a model in which product arrivals depend on R&D expenditure. Doing so, however, would serve only to strengthen our conclusions. The model would then contain two types of R&D, one that affects the arrival rate of new product generations and one that determines each generation's initial productivity. When passive learning becomes more difficult and the demand for R&D intended to raise initial quality rises in response, skilled labor will be drawn away from R&D aimed at securing new product generations. Consequently, a rise in aggregate R&D intensity and educational attainment will be associated with a short-run decline in income growth. Eventually, as the supply of skill increases, at least some of this decline will be reversed. To an even greater extent than our stylized model predicts, the secular rise in R&D and educational attainment do not presage a decline in future income growth. The contrast with the pessimistic prediction embodied in Jones' (2002) analysis could not be stronger.

Appendix A

Claim 1. *The steady state growth rate, g, is equal to $\lambda\mu$.*

Proof. Write equation (7) as

$$\alpha(i,t) = \overline{\alpha}(i,t)e^{-g(t-T(i))} + h(t-T(i))e^{-g(t-T(i))}\,, \tag{A.1}$$

where

$$h(t-T(i)) = \phi(l_e)\big(1-\Gamma(t-T(i),l_s,\psi)\big) + a\Gamma(t-T(i),l_s,\psi)\,.$$

The function $h(t-T(i)) \in [\phi(l_e), a]$ is a continuous, bounded function, monotonically increasing with t. It then follows that

$$\int_0^1 h(t-T(i))e^{-g(t-T(i))}di \equiv \int_0^\infty \mu h(\tau)e^{-(g+\mu)\tau}d\tau \tag{A.2}$$

is well-defined and constant for any given g.

Integrate (A.1) over all firms,

$$\int_0^1 \alpha(i,t)di = \int_0^1 \overline{\alpha}(i,t)e^{-g(t-T(i))}di + \int_0^1 h(t-T(i))e^{-g(t-T(i))}di \tag{A.3}$$

The L.H.S. of (A.3) is one by definition, while the second term on the R.H.S. is constant from (A.2). Hence, the term $\int_0^1 \overline{\alpha}(i,t)e^{-g(t-T(i))}di$ is also constant in the steady state. Differentiating this term with respect to time therefore yields:

$$-g(1-\mu dt)dt\int_0^1 \overline{\alpha}(i,t)e^{-g(t-T(i))}di + \mu\tilde{\lambda}dt = 0\,, \tag{A.4}$$

The first term is the change over the interval dt contributed by the fraction $(1-\mu dt)$ of firms that do not launch a new production generation. The second term is the change contributed by the firms that do innovate. All these firms see their relative quality rise by an amount $\tilde{\lambda}$. Dividing (A.4) throughout by dt and letting $dt \to 0$ yields

$$\begin{aligned} g\int_0^1 \overline{\alpha}(i,t)e^{-g(t-T(i))}di &= \mu\tilde{\lambda} \\ &= \mu\lambda\int_0^1 \overline{\alpha}(i,t)e^{-g(t-T(i))}di\,, \end{aligned}$$

and hence g=$\mu\lambda$ as claimed.

Claim 2. *There is a stable steady-state distribution of firm size, with finite variance.*

Proof. Assume for the moment that $h(t-T(i)) \equiv 0 \quad \forall i$, so we can concentrate on the stochastic process $\tilde{\alpha}(i,t) = \overline{\alpha}(i,t)e^{-g(t-T(i))}$. $\tilde{\alpha}$ is a shot-noise process. It experiences Poisson jumps of intensity μ and magnitude $\tilde{\lambda} = \lambda\int_0^1 \overline{\alpha}(i,t)e^{g(t-T(i))}di$, and decays at the exponential rate g. When $h(t-T(i)) \equiv 0 \quad \forall i$, $\tilde{\lambda} = \lambda\int_0^1 \overline{\alpha}(i,t)e^{g(t-T(i))}di = \lambda\int_0^1 \alpha(i,t)di = \lambda$. Let

$\tau_j(i)$ denote the arrival time of the jth product generation for firm i, and let $n(i,t)$ denote the number of product generations that have been launched by firm i by time t. At time t, the current contribution to relative quality of a product generation of vintage $t-\tau$ is $\lambda e^{-g(t-\tau)}$. Hence, we can write

$$\tilde{\alpha}(i,t) = \overline{\alpha}(i,0)e^{-gt} + \sum_{j=1}^{n(i,t)} x_j(i,t) ,$$

where $x_j(i,t) = \lambda e^{-g(t-\tau_j(i))}$. The $\tau_j(i)$ are i.i.d. random variables, uniformly distributed on $[0,t]$. Using the method of transformations to obtain the pdf of x, we have

$$f(x,t) = \begin{cases} (gxt)^{-1}, & \lambda e^{-gt} \le x \le \lambda \\ 0, & \text{otherwise} \end{cases} .$$

The characteristic function for x is

$$\begin{aligned} \phi_x(s,t) &= \int_{\lambda e^{-gt}}^{\lambda} e^{\hat{i}sx}(gxt)^{-1}dx \\ &= \int_{\lambda e^{-gt}}^{\lambda} \cos(sx)(gxt)^{-1}dx + \hat{i}\int_{\lambda e^{-gt}}^{\lambda} \sin(sx)(gxt)^{-1}dx , \end{aligned}$$

where $\hat{i} = \sqrt{-1}$. The second inequality comes from Euler's formula. Let $z(i,t) = \tilde{\alpha}(i,t) - \overline{\alpha}(i,0)e^{-gt}$. As the τ_j are i.i.d., the characteristic function for $z(i,t)$ is simply the expectation of the $n(i,t)$-fold product of $\phi_x(s,t)$, where $n(i,t)$ is a Poisson r.v. with mean μt.

$$\begin{aligned} \phi_z(s,t) &= E\left[\phi_x(s,t)^{n(i,t)}\right] \\ &= \sum_{n=0}^{\infty} \frac{\phi_x(s,t)^n(\mu t)^n e^{-\mu t}}{n!} \\ &= e^{\mu t[\phi_x(s,t)-1]} . \end{aligned}$$

The last line used the series expansion $e^y = \sum_{n=0}^{\infty} y^n / n!$. The kth moment is found by differentiating $\phi_z(s,t)$ k times with respect to s, multiplying by $-\hat{i}^k$, and evaluating the resulting expression at s=0:

$$m_1(z) = E\left[z(i,t)\right] = \frac{\lambda\mu}{g}\left(1 - e^{gt}\right)$$

$$m_2(z) = E\left[z(i,t)^2\right] = \frac{\lambda^2\mu(g+2\mu) - \lambda^2\mu^2 e^{-gt} - \lambda^2\mu e^{-2gt}(g-2\mu)}{2g^2}.$$

Noting that $g=\lambda\mu$, the mean and variance of $\tilde{\alpha}(i,t)$ are $E[\tilde{\alpha}(i,t)] = 1 + [\overline{\alpha}(i,0) - 1]e^{-gt}$ and $v[\tilde{\alpha}(i,t)] = \lambda\left[1 - e^{-2gt}\right]/2$. In the steady state, $t \to \infty$, so the steady-state mean and variance are $E[\tilde{\alpha}] = 1$ and $V[\tilde{\alpha}] = \lambda/2$.

In the presence of learning, $\alpha(i,t)$ differs from $\tilde{\alpha}(i,t)$ as a result of the amount of quality improvement in i's product brought about by R&D and learning relative to the average amount secured by other firms. We can write this as

$$\alpha(i,t) = \tilde{\alpha}(i,t) + \delta\left(h(t-T(i))e^{-g(t-T(i))}, E\left[h(t-T)e^{-g(t-T)}\right]; \tilde{\alpha}(i,t)\right),$$

where $\delta\left(E\left[h(t-T)e^{-g(t-T)}\right], E\left[h(t-T)e^{-g(t-T)}\right]\right) = 0$. Moreover, as we have shown that $E(\tilde{\alpha}) = 1$, and $E(\alpha) = 1$ by assumption, it must be the case that $E[\delta] = 0$. Finally, δ is a continuous function, $h(t-T(i))$ is a bounded continuous function, and $t-T(i)$ is exponentially distributed with parameter μ. Thus, δ has a well-defined stationary distribution with finite variance, $v(\delta)$, and $v[\alpha(i,t)] = v[\tilde{\alpha}(i,t)] + v[\delta] + 2\operatorname{cov}[\tilde{\alpha}(i,t), \delta]$ is finite. From the relations between $\alpha(i,t)$, profits, and demands given in the main text, we conclude that the distributions of relative quality, profits, and firm size are stationary, with finite variance.

Appendix B

Sources of data for Figure 1. The number of scientists and engineers engaged in R&D for the period 1950-1980 are taken from Jones (2002), and for the rest of the series (1981-1999) from the National Patterns of R&D Resources: 2002 provided by the National Science Foundation at http://www.nsf.gov/sbe/srs/nsf03313/tables/tab8.xls. Missing data are derived from averages of adjacent years. Labor force data are from the Bureau of Labor Statistics. Expenditure on non-defense R&D as a percentage of GDP is from Jones for the period 1953-1980, and for the rest of the series from the National Patterns of R&D Resources: 2002, provided by the National Science Foundation at http://www.nsf.gov/sbe/srs/nsf03313/tables/tab10.xls.

Average years of educational attainment in the population among persons 25 years and older are from Jones for 1950-1980. The remaining years are estimated using Jones' method. The US Census Bureau reports interval data on educational attainment at http://www.census.gov/population/socdemo/education/tabA-1.xls. In computing the average we assume that every person in a given interval had schooling equal to the interval mean. Persons that have four or more years of college are assumed to have 4 years.

GDP per worker in 1996 dollars is from the Penn World Data for the period 1950-2000 and from the Bureau of Economic Analysis (http://www.bea.gov/bea/newsrel/gdpnewsrelease.htm) for 2001-2003. For real GDP per hour, we use real GDP in 2000 chained dollars (1950-2003) from the BEA, National Income and Products Account Table (http://www.bea.gov.doc/bea/dn/nipaweb/SetlectTables.asp?Popular=Y). Employment of the civilian population over age 16 is from Jones for 1950-1979 and from Labor Force Statistics (Current Population Survey of the BLS) at http://www.bls.gov/cps/cpsatabs for the rest of the series. Average weekly hours of production are from the Current Population Survey (ftp://ftp.bls.gov/pub/suppl/ empsit.ceseeb2.txt). We assume an average work load of 50 weeks a year to estimate real GDP per hour.

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