A Two-Sector Small Open Economy Model.  
Which Inflation to Target?*

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Abstract

This paper analyses the welfare implications of simple monetary policy rules in the context of an estimated, small open economy model for Canada with traded and non-traded goods and sticky prices and wages. We find statistically significant heterogeneity in the degree of price rigidity across sectors. We find welfare gains in targeting only the non-traded inflation since it is the production sector where prices are found more sticky.

We find that overall the higher welfare is achieved, given the estimated model for the Canadian economy, with a strict inflation targeting rule where the central bank reacts to next period’s expected deviation from the aggregate inflation target and does not target the output gap.

JEL classification: E31, E32, E52

Keywords: New Open Economy Macroeconomics; Optimal monetary policy; Inflation targeting.

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1 Introduction

This paper analyses the welfare implications of simple monetary policy reaction functions in the context of a new-Keynesian small open economy model with a traded and a non-traded sector and with imperfect competition and staggered prices in the product and labor markets, estimated for the case of Canada. The model belongs to the class of dynamic stochastic general equilibrium models with explicit microfoundations that constitute the so-called New Open Economy Macroeconomics (NOEM), pioneered by Obstfeld and Rogoff (1995) and that has become a substantial literature, part of whose results are summarized in Lane (2001), among others. Several such models have been estimated for Canada (for example, Ambler, Dib, Rebei (2003) and Bergin (2003)), none of which in a multisectoral setting.

The main objective of this paper is twofold. First, we want to evaluate quantitatively the response of the Canadian economy to domestic sectorial and aggregate shocks and to foreign shocks, given the past behavior of the monetary policy. For example, we want to be able to answer questions such as what are the sectorial reallocation effects of a real appreciation of the Canadian dollar such as the one observed in the recent past? Several dynamic stochastic general equilibrium models have been estimated for Canada (for example, Ambler, Dib, Rebei (2003) and Bergin (2003)), none of which in a multisectoral setting.

Second, we want to characterize the simple, Taylor-type monetary policy reaction function that would deliver higher welfare given the estimated model. Note that we consider simple reaction functions only, we do not compute the optimal monetary policy, i.e. we do not solve for the instrument value needed to bring inflation to target at each period given all model’s responses to realized shocks but rather derive the proportional reaction of interest rates to deviations of inflation from target and to the other arguments in the specified Taylor-type rule. For that purpose, we compare the welfare gain of the welfare-maximizing standard Taylor rule with alternative specifications of the nominal interest rate feedback rule that allow for different coefficients on the wage inflation as well as on the price inflation in the traded and non-traded sectors, since it may be the case that the preferences of households favor one sector over another.

To the best of our knowledge neither characterizing the welfare-maximizing simple inflation targeting rule nor evaluating the welfare gain of alternative specifications of the monetary policy reaction function have been explored yet in the context of a multi-sector small open economy NOEM model.1

The model economy aims at representing the main features needed for conducting monetary policy analysis in a tractable characterization of the Canadian economy. The main features of our model economy are that (i) there is monopolistic competition and staggered prices in the labor market as well as in all

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1Kollman (2002) and Smets and Wouters (2002) are recent examples of papers where the welfare implications of monetary policy are investigated for small open economy NOEM models.
product markets (domestic non-traded goods, domestic traded goods –for domestic consumption or for exports and imports); the degree of price rigidity can differ across sectors and with respect to wages, (ii) labor and capital are mobile across sectors and each sector has its own technology process, (iii) traded goods are priced to market and (iv) the systematic behavior of the monetary policy is represented by the standard Taylor rule where nominal interest rates respond to deviations of overall inflation from target and to the output gap. The economy is subject to eight shocks: three common domestic shocks –monetary policy shocks, to the money demand, and to the risk premium—, two sector specific technology shocks –to the non-traded sector and to the domestic traded one— and three foreign shocks –output, inflation and nominal interest rate—. The model is estimated using Bayesian techniques for quarterly Canadian data. Our estimates seem reasonable and are compatible with other small open economy estimated models in the NOEM literature for the Canadian case. We find statistically significant heterogeneity in the degree of nominal rigidity across sectorial prices, but wages are the more sticky prices of all.

We evaluate the welfare gains of alternative specifications of a simple inflation targeting rule using a second-order approximation of the expected permanent utility in each case as compared to that of the estimated rule. We also compare monetary policy rules computing the volatility they induce in the main macro variables. We find there would have been some welfare improvement with respect to the estimated rule for the last three decades in Canada had the central bank been a strict inflation targeter and a slightly more aggressive one than it has been the case in the last three decades, i.e. with no reaction to the output gap. Despite the fact that the nominal wage is the more sticky price, we find welfare losses if the central bank was to target wage inflation rather than CPI inflation. Impulse response functions show that pure CPI inflation targeting brings the main macroeconomic variables (in particular aggregate demand) closer to their reaction in the case of flexible prices and wages than targeting wage inflation. However, there is a substantial welfare gain of targeting sectorial rather than aggregate inflation, in particular of targeting only inflation deviations from target in the more sticky sector, i.e. the non-traded one. But this higher welfare comes at the expense of a higher volatility in the main macro variables, including inflation and output (while non-traded sector inflation is stabilized), than when targeting aggregate inflation.

Overall inflation targeting with moderate nominal interest rate smoothing and no output gap targeting is the simple rule that delivers higher welfare, when the central bank reacts to expected future deviations from target inflation instead of to contemporaneous inflation deviations.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 describes the estimation method and discusses the parameter estimates. The more relevant quantitative implications of the model are outlined in Section 4. Section 5 discusses the optimized parameterization for the monetary policy rule under alternative specifications of inflation targeting Taylor-type rules. Section 6 considers forward-looking monetary policy reaction functions and Section 7 concludes.
2 The model

2.1 Households

The $i$th household chooses consumption $c_t(i)$, investment $i_t(i)$, money balances $M_t(i)$, hours worked $h_t(i)$, local riskless bonds $Bd_t(i)$, and foreign bonds $Bd_{t}^{*}(i)$ that maximize its expected utility function, and it sets the wage rate constrained to a Calvo-type nominal rigidity in wages.

The preferences of the $i$th household are given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t U \left( c_t(i), \frac{M_t(i)}{P_t}, h_t(i) \right)$$

where $\beta \in (0, 1)$, $E_0$ is the conditional expectations operator, $M_t$ denotes nominal money balances held at the end of the period and $P_t$ is a price index that can be interpreted as the consumer price index (CPI). The functional form of time $t$ utility is given by

$$U(c) = \frac{\gamma}{\gamma - 1} \log \left( c_t(i)^{\frac{\gamma - 1}{\gamma}} + b_t^\frac{1}{\gamma} \left( \frac{M_t(i)}{P_t} \right)^{\frac{\gamma - 1}{\gamma}} \right) + \eta \log (1 - h_t(i)),$$

where $\gamma$ and $\eta$ are positive parameters. Total time available to the household in the period is normalized to one. The $b_t$ term is a shock to money demand. It follows the first-order autoregressive process given by

$$\log(b_t) = (1 - \rho_b) \log(b) + \rho_b \log(b_{t-1}) + \varepsilon_{bt},$$

with $0 < \rho_b < 1$ and where the serially uncorrelated shock $\varepsilon_{bt}$ is normally distributed with zero mean and standard deviation $\sigma_b$. The household’s budget constraint is given by:

$$P_t c_t + P_t i_t (1 + CAC_t(i)) + M_t + \frac{Bd_t(i)}{R_t} + \frac{e_t Bd_t^*(i)}{\kappa_t R_t^*} \leq \sum_{t=0}^{\infty} \beta^t U \left( c_t(i), \frac{M_t(i)}{P_t}, h_t(i) \right)$$

where $CAC_t(i) = \frac{\mu_t}{2} \left( \frac{i_t(i)}{\kappa_t(i)} - \delta \right)^2 k_t(i)$ is the cost faced each time the household adjusts its stock of capital $k_t(i)$, $i_t(i)$ is the investment, $W_t$ is the nominal wage rate, $R_t^k$ is the nominal interest on rented capital, $Bd_t^*(i)$ and $Bd_t(i)$ are foreign-currency and domestic-currency bonds purchased in $t$, and $e_t$ is the nominal exchange rate. Domestic-currency bonds are used by the government to finance its deficit. $R_t$ and $R_t^*$ denote, respectively, the gross nominal domestic and foreign interest rates between $t$ and $t + 1$. The household also receives nominal lump-sum transfers from the government $T_t$, as well as nominal profits $D_t = D_t^T + D_t^{NT} + D_t^{M}$ from domestic producers of traded and non-traded goods and from importers of intermediate goods.
We assume that each household $i$ sells in a monopolistically competitive market their labor supply, $h_t(i)$, to a representative, competitive firm which transforms it into aggregate labor input, $h_t$, using the following technology:

$$h_t = \left[ \int_0^1 h_t(i) \frac{\varphi^h t(i)}{\varphi^h} di \right]^{\frac{\varphi^h}{\varphi - 1}},$$

where $\varphi^h > 1$ is defined as the constant elasticity of substitution between differentiated labor skills. The demand for individual labor by the labor aggregator firm is

$$h_t(i) = \left( \frac{W_t(i)}{W_t} \right)^{-\varphi^h} h_t,$$

where $W_t$ is the aggregate wage rate which is related to individual household wages, $W_t(i)$, via the relationship:

$$W_t = \left[ \int_0^1 W_t(i)^{1-\varphi^h} di \right]^{\frac{1}{1-\varphi^h}}.$$

Households face a nominal rigidity coming from a Calvo-type contract on wages. When allowed to do so, with probability $(1 - d_h)$ each period, the household chooses the real wage contract $\tilde{w}_t(i) = W_t(i)/P_t$ to maximize its utility.\footnote{The risk premium reflects departures from uncovered interest parity. It depends on the ratio of net foreign assets to domestic output:}

$$\log(\kappa_t) = \varphi \left[ \exp \left( \frac{\epsilon_t B_d^*}{P_t y_t} \right) - 1 \right] + \omega_{\kappa t}$$

with $B_d^* = \int_0^1 B_d^t(i) di$. By following this functional form, the risk premium ensures that the model has a unique steady state.\footnote{We allow for an exogenous shock on the risk premium whose law of motion is}

$$\log(\omega_{\kappa t}) = (1 - \rho_\kappa) \log(\omega_\kappa) + \rho_\kappa \log(\omega_{\kappa t-1}) + \varepsilon_{\kappa t},$$

with serially uncorrelated disturbance $\varepsilon_{\kappa t}$ normally distributed with zero mean and standard deviation $\sigma_\kappa$, and with $0 < \rho_\kappa < 1$.

If domestic and foreign interest rates equal, the time paths of domestic consumption and wealth follow random walks. For an early discussion of this problem, see Giavazzi and Wyplosz (1984). Our risk premium equation is similar to the one used by Senhadji (1997). For alternative ways of ensuring that stationary paths exist for consumption in small open-economy models, see Schmitt-Grohö and Uribe (2003).
The foreign nominal interest rate, $R^*_t$, is exogenous and evolves according to the following stochastic process:

$$\log(R^*_t) = (1 - \rho R^*) \log(R^*_t) + \rho R^* \log(R^*_{t-1}) + \varepsilon R^*_{t},$$  \hspace{1cm} (10)

with $0 < \rho R^* < 1$ and where the serially uncorrelated shock, $\varepsilon R^*_{t}$, is normally distributed with zero mean and standard deviation $\sigma R^*$. 

Households also face a no-Ponzi-game restriction:

$$\lim_{T \to \infty} \left( \prod_{t=0}^{T} \frac{1}{\kappa_t R^*_t} \right) B d^*_T(i) = 0.$$

The first order conditions are as follows

$$\lambda_t(i) \left[ \frac{c_t(i)^{-\frac{\beta}{\gamma}}}{c_t(i)^{\frac{1}{\gamma}} + b_t^1 m_t(i)^{\frac{1}{\gamma}}} \right] = \lambda_t(i) \left( 1 - \frac{1}{R_t} \right)$$  \hspace{1cm} (11)

$$\lambda_t(i) \left[ \frac{b_t^1 m_t(i)^{-\frac{\beta}{\gamma}}}{c_t(i)^{\frac{1}{\gamma}} + b_t^1 m_t(i)^{\frac{1}{\gamma}}} \right] = \lambda_t(i) \left( 1 - \frac{1}{R_t} \right)$$  \hspace{1cm} (12)

$$\beta E_t \lambda_{t+1}(i) \frac{1}{\pi_{t+1}} = \beta E_t \lambda_{t+1}(i) \frac{1}{\pi_{t+1}}$$  \hspace{1cm} (13)

$$s_t E_t \frac{\pi_{t+1}}{\kappa_t R^*_t} = E_t s_{t+1} \frac{\pi_{t+1}}{R_t}$$  \hspace{1cm} (14)

$$\lambda_t(i) \left[ 1 + \chi \left( \frac{i_t(i)}{k_t(i)} - \bar{\delta} \right) \right] =$$

$$\beta E_t \lambda_{t+1}(i) \left[ 1 + \kappa_{t+1}(i) + \chi \left( \frac{i_{t+1}(i)}{k_{t+1}(i)} - \bar{\delta} \right) - \bar{\delta} + \chi \frac{1}{2} \left( \frac{i_{t+1}(i)}{k_{t+1}(i)} - \bar{\delta} \right)^2 \right]$$  \hspace{1cm} (15)

and the wage contract takes the form

$$\tilde{w}_t(i) = \frac{\partial h}{\partial h} E_t \sum_{\tau=0}^{\infty} \beta^\tau d^*_w \left[ \frac{h_{t+\tau}(i)}{1 - h_{t+\tau}(i)} \right] h_{t+\tau}(i)$$  \hspace{1cm} (16)

where $\tilde{w}_t$ is the real wage contract that a household chooses when it is allowed to reoptimize its wage.

2.2 Firms

Monopolistically competitive firms produce traded and non-traded goods. The traded goods are either imported or produced domestically, which in turn can either be sold home or exported.
2.2.1 Non-traded sector

There is a continuum of firms indexed by \( j \in [0, 1] \) in the non-traded sector. There is monopolistic competition in the market for non-traded goods, which are imperfect substitutes for each other in the production of the composite good \( y^N_t \), produced by a representative competitive firm. Aggregate non-traded output is defined using the Dixit and Stiglitz aggregator function

\[
y^N_t = \left( \int_0^1 y^N_t(j) \frac{y^N_{t-1}}{y^N_{t-1}} dj \right)^{\frac{\phi^N}{\phi^N - 1}}
\]

where \( \phi^N \) is the elasticity of substitution between differentiated non-traded goods. Given the prices \( P^N_t \) and \( P^N_t(j) \), the non-traded final good-producing firm chooses the production, \( y^N_t \), that maximizes its profits. The first order condition corresponds to the demand constraint for each intermediary firm \( j \)

\[
y^N_t(j) = \left( \frac{P^N_t(j)}{P^N_t} \right)^{-\phi^N} y^N_t \tag{17}
\]

where the price index for the composite imported goods is given by:

\[
P^N_t = \left( \int_0^1 P^N_t(j)^{1-\phi^N} dj \right)^{\frac{1}{1-\phi^N}} \tag{18}
\]

Each monopolistically competitive firm has a production function given by

\[
y^N_t(j) = A^N_t [k^N_t(j)]^{\alpha^N} [h^N_t(j)]^{1-\alpha^N}
\]

where \( A^N_t \) is the non-traded sector specific total factor productivity that follows the stochastic process

\[
\log(A^N_t) = (1 - \rho_{A^N}) \log(A^N) + \rho_{A^N} \log(A^N_{t-1}) + \varepsilon_{A^N_t}
\]

with \( \varepsilon_{A^N_t} \) a non-serially correlated technology shock normally distributed with zero mean and standard deviation \( \sigma_{A^N} \).

Firms face a nominal rigidity coming from a Calvo-type contract on prices. When allowed to do so, with probability \( (1 - d^N_t) \) each period, the producer of non-traded good \( j \) sets the price \( P^N_t(j) \) to maximize its weighted expected profits. Therefore, each individual firm chooses \( k^N_t(j) \), \( h^N_t(j) \), and \( P^N_t(j) \) through solving

\[
\max_{(k^N_t(j), h^N_t(j), P^N_t(j))} E_t \left[ \sum_{l=0}^{\infty} (\beta d^N)^l \left( \frac{\lambda_{t+l}}{\lambda_t} \right) D^N_{t+l}(j) \frac{P^N_{t+l}(j)}{P_{t+l}} \right]
\]

where \( \lambda_t \) is the marginal utility of wealth for a representative household, and time \( t + l \) profits of the firm changing price at time \( t \) are

\[
D^N_{t+l}(j) = P^N_t(j) y^N_{t+l}(j) - W_t + h^N_{t+l}(j) - R^N_{t+l+k^N_{t+l}(j)}
\]
The first-order conditions are:

\[
\frac{W_t}{P_t} = \xi_t(j)(1 - \alpha^N)\frac{y_t^N(j)}{h_t^N(j)} \tag{21}
\]

\[
\frac{R_t^k}{P_t} = \xi_t(j)\alpha^N\frac{y_t^N(j)}{k_t^N(j)} \tag{22}
\]

\[
\hat{P}_t^N(j) = \left( \frac{\partial N}{\partial N - 1} \right) E_t \sum_{i=0}^{\infty} (\beta d N)^i \left( \frac{\lambda_{t+i}}{\lambda_t} \right) \xi_{t+i}(j) y_{t+i}(j) \tag{23}
\]

where \(\xi_t(i)\) is the Lagrange multiplier associated with the production function constraint. It measures the non-traded sector firm’s real marginal cost.

### 2.2.2 Traded sector

Domestic firms producing good in the traded sector have very similar problem except the fact that each monopolistically competitive firm \(k\) produces two types of goods, \(y_t^{Td}(k)\) that will be consumed in the domestic market and \(y_t^X(k)\) that will be exported, for \(k \in [0, 1]\).

The production function is as follows

\[
y_t^T(k) = A_t^T \left[ k_t^T(k) \right]^{\alpha^T} \left[ h_t^T(k) \right]^{1-\alpha^T}
\]

where \(A_t^T\) is the traded-sector specific technology process

\[
\log(A_t^T) = (1 - \rho_{AT}) \log(A_{t-1}^T) + \rho_{AT} \log(A_{t-1}^T) + \varepsilon_{AT_t}
\]

and \(\varepsilon_{AT_t}\) the serially uncorrelated shock which is normally distributed with zero mean and standard deviation \(\sigma_{AT_t}\).

Each individual firm chooses \(k_t^T(k), h_t^T(k), P_t^{Td}(k), \) and \(P_t^X(k)\). We assume complete pricing to market for exports, i.e. \(P_t^X(k)\) is labelled in US dollars.\(^4\)

In addition, once the firm has the chance to update its price (with probability \((1 - d_T)\) each period) it will choose simultaneously \(P_t^{Td}(k)\) and \(P_t^X(k)\). The problem of each firm can be summarized by

\[
\max_{\{k_t^T(k), h_t^T(k), P_t^{Td}(k), P_t^X(k)\}} \quad E_t \left[ \sum_{i=0}^{\infty} (\beta d T)^i \left( \frac{\lambda_{t+i}}{\lambda_t} \right) D_{t+i}^T(k) / P_{t+i} \right] \tag{25}
\]

where time \(t + l\) profits of the firm changing price at time \(t\) are

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\(^4\)There is substantial evidence in favor of the pricing to market hypothesis in the Canada-US case. Engel and Rogers (1996) use CPI data for US and Canadian cities and find that deviations from the law of one price are much higher for two cities located in different countries than for two equidistant cities in the same country. Also, there is evidence suggesting the prevalence of invoicing in US dollars by foreign firms selling in the US market. Indeed, according to the ECU Institute (1995), over 80 per cent of US imports were invoiced in US dollars.
under the constraints dictating the local and foreign demand for traded goods:

\[ y_{Td}^t(k) = \left( \frac{P_{Td}^t(k)}{P_{Td}^t} \right)^{-\vartheta_T} y_{Td}^t \]  

(26)

and

\[ y_{X}^t(k) = \left( \frac{P_{X}^t(k)}{P_{X}^t} \right)^{-\vartheta_T} y_{X}^t \]  

(27)

where \( \vartheta_T \) is the elasticity of substitution between differentiated traded goods.

The first-order conditions are:

\[ \frac{W_t}{P_t} = \zeta_t(k)(1 - \alpha_T) \frac{y_{Td}^t(k)}{h_t^T(k)} \]  

(28)

\[ \frac{R_t^k}{P_t} = \zeta_t(k) \alpha_T \frac{y_{Td}^t(k)}{k_t^T(k)} \]  

(29)

\[ \tilde{P}_{Td}^t(k) = \left( \frac{\vartheta_T}{\vartheta_T - 1} \right) \frac{E_t \sum_{i=0}^{\infty} (\beta d_T)^i \left( \frac{\lambda_{t+i}}{\lambda_t} \right) \zeta_{t+i}(k) y_{Td}^{t+i}(k)}{E_t \sum_{i=0}^{\infty} (\beta d_T)^i \left( \frac{\lambda_{t+i}}{\lambda_t} \right) y_{Td}^{t+i}(k) \frac{1}{P_{Td}^{t+i}}} \]  

(30)

\[ \tilde{P}_{X}^t(k) = \left( \frac{\vartheta_T}{\vartheta_T - 1} \right) \frac{E_t \sum_{i=0}^{\infty} (\beta d_T)^i \left( \frac{\lambda_{t+i}}{\lambda_t} \right) \zeta_{t+i}(k) y_{X}^{t+i}(k)}{E_t \sum_{i=0}^{\infty} (\beta d_T)^i \left( \frac{\lambda_{t+i}}{\lambda_t} \right) e_{t+i} y_{X}^{t+i}(k) \frac{1}{P_{X}^{t+i}}} \]  

(31)

where \( \zeta_{t+i}(k) \) is the traded sector firm’s real marginal cost.

Similarly, the final traded good-producing sector has the following aggregate functions

\[ y_{Td}^t = \left( \int_0^1 y_{Td}^t(k) \frac{d\vartheta_T-1}{\vartheta_T} dk \right) \]  

(32)

and

\[ y_{X}^t = \left( \int_0^1 y_{X}^t(k) \frac{d\vartheta_T-1}{\vartheta_T} dk \right) \]  

(33)

with

\[ y_{t}^T = y_{Td}^t + y_{X}^t \]  

(34)

where \( y_{t}^T \) is the total production in the traded goods sector, \( y_{Td}^t \) and \( y_{X}^t \) are traded goods respectively for domestic and foreign markets.

The price indices for domestically consumed traded and exports are as follows
The foreign demand for locally produced goods is as follows

\[ P_t^d = \left( \int_0^1 P_t^{d}(k)^{1-\theta^d} dk \right)^{\frac{1}{1-\theta^d}} \] (35)

\[ P_t^X = \left( \int_0^1 P_t^{X}(k)^{1-\theta^X} dk \right)^{\frac{1}{1-\theta^X}} \] (36)

The foreign demand for locally produced goods is as follows

\[ y_t^X = \left( \frac{P_t^X}{P_t^*} \right)^{-\mu} y_t^* \] (37)

where \( \frac{\mu-1}{\mu} \) captures the elasticity of substitution between the exported goods and foreign-produced goods in the consumption basket of foreign consumers and \( y_t^* \) and \( P_t^* \) are, respectively, foreign output and price index. Both variables are exogenously given and foreign output and inflation follow the stochastic processes

\[ \log(y_t^*) = (1 - \rho_{y^*}) \log(y_t^*) + \rho_{y^*} \log(y_{t-1}^*) + \varepsilon_{y^*t} \]

\[ \log(\pi_t^*) = (1 - \rho_{\pi^*}) \log(\pi_t^*) + \rho_{\pi^*} \log(\pi_{t-1}^*) + \varepsilon_{\pi^*t} \] (38)

with 0 < \( \rho_{y^*}, \rho_{\pi^*} < 1 \) and where the serially uncorrelated shocks, \( \varepsilon_{y^*t}, \varepsilon_{\pi^*t} \) are normally distributed with zero mean and standard deviation \( \sigma_{y^*} \) and \( \sigma_{\pi^*} \), respectively.

### 2.2.3 Imported-goods sector

Finally, there is a continuum of intermediate good-importing firms indexed by \( i \in [0, 1] \). There is monopolistic competition in the market for imported intermediates, which are imperfect substitutes for each other in the production of the composite imported good, \( y_t^M \), produced by a representative competitive firm. We also assume Calvo-type staggered price setting in the imported goods sector in order to capture the empirical evidence on incomplete exchange rate pass-through into import prices\(^5\). Thus, when allowed to do so (with probability \( (1 - d_M) \) each period), the importer of good \( i \) sets the price, \( P_t^M(i) \), to maximize its weighted expected profits. It solves:

\[
\max_{(P_t^M(i))} E_t \left[ \sum_{l=0}^{\infty} (\beta d_M)^l \left( \frac{\lambda_{t+l}}{\lambda_t} \right) \frac{D_t^M(i)}{P_{t+l}} \right]
\] (39)

\(^5\)Campa and Goldberg (2001) find that they can reject the hypothesis of complete short-run pass-through in 22 of the 25 OCECD countries of their study for the period 1975-1999, but they find complete long-run pass-through. Ghosh and Wolf (2001) argue that sticky prices or menu cost are a preferable explanation for imperfect pass-through since it’s compatible with complete long-run pass-through, while that’s not the case of explanations based on international product differentiation. The evidence of incomplete exchange rate pass-through in Canada is well documented and seems to conclude that it’s moved towards almost zero pass-through in the recent past. See for example Bailliu and Bouakez (2004), Kichian (2001) and Leung (2003).
where time $t + l$ profits of the firm changing price at time $t$ are:

$$D_{t+l}^M(i) = \left( \hat{P}_{t}^M(i) - e_{t+l} P_{t+l}^* \right) \left( \frac{\hat{P}_{t}^M(i)}{P_{t+l}^*} \right)^{-\hat{\phi}^M} y_{t+l}^M$$  \hspace{1cm} (40)$$

with $\hat{\phi}^M$ representing the elasticity of substitution across differentiated imported goods. Note that the marginal cost of the importing firm is $e_{t} P_{t}^* 6$ and thus its real marginal cost is the real exchange rate $s_{t} \equiv e_{t} P_{t}^*/P_{t}^*$. The first-order condition is:

$$\hat{P}_{t}^M(i) = \left( \frac{\hat{\phi}^M}{\hat{\phi}^M - 1} \right) \frac{E_{t} \sum_{l=0}^{\infty} (\beta d_{M})^l \left( \frac{\lambda_{t+l}}{\lambda_t} \right) y_{t+l}^M(i) e_{t+l} P_{t+l}^*/P_{t+l}}{E_{t} \sum_{l=0}^{\infty} (\beta d_{M})^l \left( \frac{\lambda_{t+l}}{\lambda_t} \right) y_{t+l}^M(i)/P_{t+l}}$$  \hspace{1cm} (41)$$

As in the other cases, aggregate imported output is defined using the Dixit and Stiglitz aggregator function

$$y_{t}^M = \left( \int_{0}^{1} y_{t}^M(i)^{\frac{\phi^M - 1}{\phi^M - 1}} di \right)^{\frac{\phi^M}{\phi^M - 1}}$$

and the price index for the aggregated good is

$$P_{t}^M = \left( \int_{0}^{1} P_{t}^M(i)^{1 - \phi^M} di \right)^{\frac{1}{1 - \phi^M}}$$  \hspace{1cm} (42)$$

2.2.4 Final goods aggregators

The final domestically consumed good, $y_{t}^d$, is produced by a competitive firm that uses non-traded goods, $y_{t}^N$, and domestically consumed traded goods, $y_{t}^T$, as inputs subject to the following CES technology

$$y_{t}^d = \left[ n \frac{1}{\phi} (y_{t}^N)^{\frac{\phi - 1}{\phi}} + (1 - n) \frac{1}{\phi} (y_{t}^T)^{\frac{\phi - 1}{\phi}} \right]^{\frac{\phi}{\phi - 1}}$$  \hspace{1cm} (43)$$

where $n > 0$ is the share of non traded goods in the domestic goods basket at the steady state and $\frac{\phi - 1}{\phi} > 0$ is the elasticity of substitution between non-traded and non-exported traded goods. Profit maximization entails

$$y_{t}^N = n \left( \frac{P_{t}^N}{P_{t}^d} \right)^{-\phi} y_{t}^d$$  \hspace{1cm} (44)$$

and

$$y_{t}^T = (1 - n) \left( \frac{P_{t}^T}{P_{t}^d} \right)^{-\phi} y_{t}^d$$  \hspace{1cm} (45)$$

6For convenience, we assume that the price in foreign currency of all imported intermediates is $P_{t}^*$, which is also equal to the foreign price level.
Furthermore, the domestic final-good price, $P^d_t$, is given by

$$P^d_t = [n(P^N_t)^{1-\phi} + (1-n)(P^T_d)^{1-\phi}]^{1/(1-\phi)}$$

(46)

Finally, we aggregate domestic and imported goods using a CES function as follows

$$z_t = \left[ m^\frac{1}{\nu} (y^d_t) \frac{\nu-1}{\nu} + (1-m) \left( y^M_t \right)^{\frac{\nu-1}{\nu}} \right] \frac{\nu}{\nu-1}$$

(47)

where $m > 0$ is the share of domestic goods in the final goods basket at the steady state; and $\frac{\nu-1}{\nu} > 0$ is the elasticity of substitution between domestic and imported goods. The first order conditions are

$$y^d_t = m \left( \frac{P^d_t}{P_t} \right)^{-\nu} z_t$$

(48)

and

$$y^M_t = (1-m) \left( \frac{P^M_t}{P_t} \right)^{-\nu} z_t$$

(49)

The final-good price, $P_t$, which corresponds to the consumer price index or CPI, is given by

$$P_t = [m(P^d_t)^{1-\nu} + (1-m)(P^M_t)^{1-\nu}]^{1/(1-\nu)}$$

(50)

Aggregate output is used for consumption, investment, and covering the cost of adjusting capital

$$z_t = c_t + i_t(1 + CAC_t)$$

(51)

The gross domestic product is $y_t = z_t + y^X_t - y^M_t$. Finally, sectorial hours and capital simply sum to the aggregate hours and capital offered by households (i.e. $h^N_t + h^T_t = h_t$ and $k^N_t + k^T_t = k_t$).

2.3 The government

The government budget constraint is given by

$$T_t + Bd_{t-1} = M_t - M_{t-1} + \frac{Bd_t}{R_t}$$

(52)

which is combined with the no-Ponzi-game restriction:

$$\lim_{T \to \infty} \left( \prod_{t=0}^{T} \frac{1}{R_t} \right) Bd_T = 0$$

We consider a simple decision rule for nominal interest rate such as the standard Taylor rule

$$\log(R_t/R) = \rho_R \log(R_{t-1}/R) + \rho_\pi \log(\pi_t/\pi) + \rho_y \log(y_t/y) + \varepsilon_{Rt},$$

(53)

where $R$, $\pi$, and $y$ are the steady-state values of the gross nominal interest rate, CPI inflation, and real gross domestic output, and where $\varepsilon_{Rt}$ is a zero-mean, serially uncorrelated monetary policy shock with standard deviation $\sigma_R$. 

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3 Estimation

The above model is estimated using Bayesian estimation techniques that update prior distributions for the deep parameters of the model, which are defined according to a reasonable calibration, with the actual data. The estimation is done using recursive simulation methods, in particular the Metropolis-Hastings algorithm, which have been applied to estimate similar dynamic stochastic general equilibrium models in the literature, such as Smets and Wouters (2003).

The model has 8 shocks processes: three common domestic shocks—monetary policy shocks $\varepsilon_{Rt}$, to the money demand $\varepsilon_{bt}$, and to the risk premium $\varepsilon_{rt}$, two sector specific technology shocks—to the non-traded sector $\varepsilon_{ANt}$ and to the traded one $\varepsilon_{ATd}$, and three foreign shocks—output $\varepsilon_{yt}$, inflation $\varepsilon_{it}$ and nominal interest rate $\varepsilon_{Rt}$. In order to identify them in the estimation process we need to use the same number of actual series. We choose them to be as informative as possible. We use HP-filtered and seasonally adjusted quarterly series for Canada for the period 1972q1-2003q4. The series are real exchange rate (against the US dollar), real output, nominal interest rate on 3-month T-bills, real M2 per capita (deflated with the CPI), CPI inflation, US real output per capita, US CPI inflation and nominal US interest rate on 3-month T-bills.

Table 1 shows the prior distributions we have imposed for the deep parameters of the model as well as the median and 90 percent confidence interval for the posterior distributions. Figures 1 and 2 convey the same information by drawing the prior distributions, in green thick lines, together with the posterior ones, in thin blue lines.

We have borrowed some of the prior distributions from the literature but for those we didn’t have references we have used our best common sense while trying to construct little restrictive priors. We have selected beta distributions for those coefficients we wanted to restrict to lie between 0 and 1, such as the autocorrelation coefficients of the shock processes or the share parameters. Gamma and Inverted Gamma distributions are imposed when required to guarantee real positive values.

All three sectors, domestic traded, imports and non-traded and treated symmetrically a priori. They are given the same degree of nominal rigidity, in the form of an average prior probability of not changing prices of 0.67 which corresponds to changing prices every 3 quarters on average. The priors for the elasticities of substitution between differentiated goods are also equal across sectors, corresponding to equal steady state markups across sectors.

Some parameter values are taken as fixed rather than given a prior distribution that will be updated with the data; we calibrate them to values similar to the ones found in the literature. We have performed sensitivity analysis on their calibrated values and observed that the estimates of the rest of the model parameters were unchanged. These parameters are: the subjective discount rate, $\beta = 0.99$, which implies an annual real interest rate of 4 percent; the weight of leisure in the utility function, $\eta$, which is calibrated to yield a steady state share of time devoted to market activities of 30 percent; the quarterly depreciation rate of capital, $\delta = 0.025$; the gross steady state markups in all sectors,
\( \sigma_{\epsilon} \) = 1.14, which lies between the estimates of the empirical literature between 10 and 20 percent (see, for example, Basu (1995)) and the preference parameter governing the elasticity of substitution between consumption and real balances, \( \gamma = 0.1 \), for which we have taken a value close to Ireland (2003) who estimates a similar model for the US.

We find that data is most informative for the adequate parameterization of the price stickiness, the monetary policy reaction function and the shocks processes.

The prior of equal nominal rigidity across sectors does not hold, consistently with the findings of Bils and Klenow (2004), who document a high degree of heterogeneity in the frequency of price changes across retail goods and services. We find indeed significant heterogeneity in the degree of price stickiness across sectors, imports prices being the more flexible (with posterior median duration for prices of 2 quarters) and non-traded ones the more sticky (posterior median of almost 3 quarters). Domestic traded goods’ prices are estimated to have a posterior median duration of 2 \( \frac{1}{4} \) quarters.\(^7\) Table 1 shows that the 90 percent posterior confidence interval for \( d_M \) does not even overlap with those for \( d_N \) and \( d_T \). Similarly, Figure 2 shows how the equal prior distribution barely overlaps with the posterior distributions for \( d_M \). However, and consistently with virtually any study that studies wages and prices rigidities, the higher nominal stickiness of all is found for wages, with an estimated posterior duration of 5 quarters. In fact, one of the possible reasons behind the higher stickiness of non-traded versus traded prices can be the higher weight of wages in the cost of production of non-traded goods.

This heterogeneity in the nominal rigidity is an important finding and will condition many of the model implications for the dynamics as well as for the welfare improvement of alternative specifications of the monetary policy reaction function, especially when the central bank is willing to weight differently inflation stabilization in different sectors due to the consideration that agents may derive more utility from consumption from some particular sector that another.

The posterior estimates of the Taylor rule almost half the prior degree of interest rate smoothing (posterior median \( \rho_R = 0.46 \)), somewhat reduce the reaction to deviations of inflation from target to \( \rho_\epsilon = 1.19 \) and find a significant but low reaction to the output gap, with a posterior median coefficient \( \rho_y = 0.3 \). The historical estimated Taylor rule, therefore, is an inflation targeting one with moderate concern for output stabilization and with some sluggishness in the monetary policy instrument.

The actual data is also found very informative for estimating the volatility

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\(^7\)Our sectoral estimates bridge the gap between usual estimates of around 4 quarters for the aggregate price level and the microeconomic evidence of average duration of prices at the individual firm level of around one quarter. In a back-of-the-envelope calculation, if we weight the sectoral posterior median durations by the posterior median estimates of the steady state weights of the sectoral outputs in final consumption, we obtain an overall economy duration of prices of 2 \( \frac{1}{4} \) quarters, i.e. 7 months. Those estimated weights in the final consumption basket are 0.29 for the non-tradables, 0.25 for the tradables produced domestically and 0.46 for imports.
of shocks, which were given equal priors. Posterior estimates indicate that aggregate demand shocks, represented by the money demand one, are the more volatile—although the variance decomposition in the next section shows that they have very little role in explaining aggregate fluctuations in this model—, followed by shocks to the non-traded technology.

Data, however, is found little informative for some parameters whose posterior distributions are very coincident with their priors. In particular, this is the case of the one governing risk premium dynamics, $\varphi$, or the parameters governing the steady state shares of traded and non-traded goods in the domestic final composite good and those of domestic and imported content of the final consumption good, $n$ and $m$ respectively.

4 Quantitative Implications of the model

This section discusses the dynamics of the estimated model in terms of the variance decomposition of its main endogenous variables and in terms of their impulse responses to the shocks contemplated in the model.

4.1 Variance decomposition

Table 2 shows the decomposition of the long-run variance of the main endogenous variables of the model into the contribution of each of the eight shocks.

The business cycle volatility of the output in each production sector, traded and non-traded, is mainly explained by its corresponding sector-specific technology shock, but there is a substantial role for the monetary policy shocks as well, the domestic policy ones on the domestic traded production and the foreign ones on exports and imports. Aggregate inflation is found to be more explained by technology shocks—through their impact on the non-traded inflation—and by foreign interest rate and risk premium shock—through the impact of both on imports inflation—than by monetary policy shocks in the last three decades. Final spending, i.e. consumption and investment, are mainly explained by the non-traded technology shock, which is one of the shocks with higher estimated volatility, although the steady state share of the non-traded sector in final good is only $\frac{1}{3}$. Hours worked are also substantially explained by technology shocks in the two sectors but are also clearly affected by monetary policy shocks. Finally, the volatility of the real exchange rate is explained by technology, foreign monetary policy and the risk premium.

4.2 Responses to a foreign shock

Figure 3 represents the responses in terms of per cent deviations with respect to the steady state to a one-period increase of 100 basis points in the monetary policy instrument of the foreign economy, the U.S.

The uncovered interest parity yields a nominal and real impact depreciation of the Canadian dollar (2 per cent posterior median depreciation on impact of
the real exchange rate, $s_t$). The real depreciation rises directly the marginal cost of the importing firms and is therefore translated into a higher import prices and lower imports, $y_t^M$. It is important to note, however, due to the estimated sluggishness of import prices the exchange rate pass-through is not complete and imports inflation gets to rise only by 50 basis points.

Exports benefit from the depreciation. Because exports are priced in the foreign currency but traded-sector firms maximize their profits in Canadian dollars, the depreciation by itself increases the benefits from the part of the production that is exported. Because of that, traded-sector producers lower export prices and increase their exports on impact.

The increase of imports inflation makes aggregate inflation rise, which causes a monetary policy contraction. That in turn decreases demand ($c_t$ and $i_t$) that further reduces imports demand but also decreases demand of non-traded and of traded goods produced domestically.

### 4.3 Responses to a sectorial shock

Figure 4 represents the responses to a positive one-period technology shock of 1 per cent in the non-traded sector only.

Increased production in the non-traded sector\(^8\) rises demand all throughout the economy and therefore increases output in the traded and imports sectors as well.

Prices in the non-traded sector fall on impact leading to a mild fall in overall inflation, which in turn causes an expansionary reaction of the monetary policy that feeds into further increase of demand and also causes an impact nominal and real depreciation.

Increased demand increases imports as well as imports inflation, which helps undo the impact fall of aggregate inflation quite quickly.

As before, the depreciation increases the profits of the exported production in the traded sector but exports demand does not rise (foreign output being exogenous). Thus, traded-sector profit maximization makes firms lower export prices fixed in US dollars (pricing to market) and increase exports.

### 4.4 Responses to a common domestic shock

Figure 5 represents the responses to a temporary monetary policy contraction. The nominal interest rate shock increases by 100 basis points for one-period. On impact, the monetary policy instrument rises by less than 1 per cent because of the immediate fall in inflation. In fact, nominal interest rates rise by only a half of the 1 percent shock. Inflation falls on impact due to the impact decrease

\(^8\)As is well known in the sticky price literature, sticky prices prevent the 1 per cent increase in total factor productivity to be fully transformed into a 1 per cent increase in $y_t^{NT}$. Since capital is predetermined, the only possible way to generate that lower output increase is by reducing hours worked on impact, which is observed in Figure 4. $h_t^{NT}$ falls on impact but increases after 4 quarters.
in demand and consequently in activity in every sector, traded, non-traded and imports.

The monetary policy contraction causes a nominal and real impact appreciation of the Canadian dollar. Exports prices being set in US dollars, the appreciation reduces exporters’ profits and thus export prices rise, which causes a fall of exports.

5 Simple inflation targeting rules

In this section we search for the parameterization of feedback Taylor-type interest rate rule as in Eq.(53) that maximizes households’ welfare given our estimated model. In particular, we maximize the unconditional expectation of lifetime utility\(^9\) of households over the parameters of the Taylor rule. This implies:

$$\max_{\rho_r, \rho_y} E \{ u(c_t, m_t, h_t) \}.$$  

We measure the welfare gain associated with a particular monetary policy in terms of its compensating variation. That is, we calculate the percentage of lifetime consumption that should be added to that obtained under the estimated Taylor rule in order to give households the same unconditional expected utility than under the new monetary policy rule scenario:

$$E \{ u(c_t(1 + \text{welfare gain}), m_t, h_t) \} = E \{ u(\tilde{c}_t, \tilde{m}_t, \tilde{h}_t) \}$$

where variables without tildes are obtained under the estimated rule described before, and variables with tildes are under the optimized Taylor rule. Based on the results found in Kim and Kim (2003) and subsequent literature, we compute the long-run average utility by means of a second-order approximation around the steady state utility. In particular, we follow the approach of Schmitt-Grohé and Uribe (2004a).

$$E \left( u \left( \tilde{c}_t, \tilde{m}_t, \tilde{h}_t \right) \right) = u(c, m, h) + u' \left( \tilde{c}_t, \tilde{m}_t, \tilde{h}_t \right) + \frac{1}{2} u'' \left( \tilde{c}_t, \tilde{m}_t, \tilde{h}_t \right) \tilde{c}_t \tilde{m}_t \tilde{h}_t,$$

where \( u' \) and \( u'' \) are the first and second derivatives respectively of the utility function with respect to its arguments, evaluated at their deterministic steady state values, and variables with hats measure deviations from their levels in

\(^9\)Schmitt-Grohé and Uribe (2004b) adopt the conditional welfare optimization in their framework and they consider the non-stochastic steady state as an initial state of the economy. By computing the unconditional long-run utility we do not consider the effect of the initial state. Transition costs are crucially dependent on that initial state especially if the real state of the economy is never at the deterministic level. In addition, Schmitt-Grohé and Uribe (2004b) show that the optimal rule is robust to these definitions of welfare, but that the welfare improvement could be different in the sense that it is higher in the case of unconditional welfare given that no short term transition costs are incurred.
the deterministic steady state. The compensating variation in consumption, can therefore be decomposed into a first level effect and a second level or stabilization effect, i.e. into the welfare gains of the new parameterization of the monetary policy due to its effect on the average levels of consumption, real balances, and leisure and its effect on their volatilities. The first level effect is defined as:

\[ E \{ u(c_t(1 + 1^{\text{st level effect}}), m_t, h_t) \} = u(c, m, h) + u' E \left( \hat{c}_t, \hat{m}_t, \hat{h}_t \right), \]

and the second level effect as:

\[ E \{ u(c_t(1 + 2^{\text{nd level effect}}), m_t, h_t) \} = u(c, m, h) + \frac{1}{2} E \left( \hat{c}_t, \hat{m}_t, \hat{h}_t \right)' u'' \left( \hat{c}_t, \hat{m}_t, \hat{h}_t \right). \]

The overall effect in all cases is such that, approximately, \((1 + \text{welfare gain}) \approx (1 + 1^{\text{st level effect}})(1 + 2^{\text{nd level effect}})\). Table 3 reports the welfare gains, together with the unconditional long-run average values of the arguments of the utility function as well as that of the log-utility itself.

Table 4 reports another dimension for comparing alternative monetary policy reaction functions: the unconditional volatility they imply for the utility and its arguments as well as for some crucial macro variables, i.e. output, inflation and the nominal interest rate.

In what follows we limit our attention to the Taylor-type rules that guarantee the existence of a unique and stable equilibrium in the neighborhood of the deterministic steady state. We also restrict our search to monetary policy reactions to price and output deviations from target; we do this by keeping the degree of nominal interest smoothing unchanged and equal to the posterior median of the estimated value, i.e. \(\hat{R} = 0.46.10\)

Our reference interest rate feedback rule is the estimated one where, on top of that moderate nominal interest rate smoothing, the monetary authority has targeted inflation but not very aggressively (the posterior median estimate for the reaction to deviations of the aggregate CPI inflation from target is slightly above 1, \(\hat{\rho}_c = 1.19\) and there has been a significant although weak response of the monetary policy to the output gap (posterior median of \(\hat{\rho}_y = 0.31\)).

There are several reasons that motivate the choice of fixing \(\hat{R}\). One is that without interest rate smoothing there would be indeterminacy for values of the coefficient on inflation smaller than one. By keeping \(\hat{R}\) at its estimated value, we can compute the welfare gains of a wider range of values for \(\rho_c\), including those smaller than one.

Another important reason is that because the optimized rule would aim at maximizing inflation stabilization rather than instrument smoothing, the welfare-maximizing value of \(\hat{R}\) is very likely going to be zero. Indeed, Schmitt-Grohé and Uribe (2004b) find that the optimal degree of interest rate smoothing for Taylor rules in the Christiano, Eichenbaum and Evans (2001) model is zero. However, they also look, as we do, for the parameterization of the Taylor rule that delivers higher utility for degrees of interest rate smoothing closer to the observed ones. Precisely that, keeping our frame of analysis of alternative monetary policy reaction functions close to the observed features of monetary policy as it is implemented in practice, constitutes a further reason for keeping \(\hat{R}\) fixed as well as to stick to simple Taylor rules. A final reason is that maximizing welfare over several parameters is computationally expensive.
5.1 CPI inflation rate targeting

First, we consider the case where the central bank targets the same variables as in the historical rule, i.e. aggregate CPI inflation and output gap. The welfare-maximizing Taylor rule implies a very similar level of aggressiveness with respect to inflation deviations from target than the estimated historical rule, $\rho_\pi = 1.20$, but, contrary to the historical case, no response to the output gap, $\rho_y = 0$.

The historical rule entails a welfare cost of 0.08 per cent of the lifetime consumption associated with the optimized CPI inflation targeting rule (see second row in Table 3). Most of the welfare improvement of choosing $\rho_\pi = 1.20$ and $\rho_y = 0$ rather than the estimated parameters comes from the first level effect or improvement in long-run average utility, which amounts to a 0.11 per cent increase in lifetime consumption. This welfare-maximizing monetary policy reaction function implies slightly higher volatility in the utility arguments (see second row of Table 4), which is captured by a negative second order effect, as well as in output while it only very marginally improves inflation stabilization.

5.2 Targeting other inflation rates

Our model has different degrees of nominal inertia in the different sectorial prices and in wages. A welfare-maximizing central bank may prefer to target just one sectorial inflation instead of the aggregate CPI inflation, or wage inflation instead, or combinations of specific price inflations, depending on the sensitivity of households’ utility to specific price and wage developments.

In fact, several papers have found in recent years that the optimal monetary policy may entail such choices in the context of sticky price dynamic stochastic general equilibrium models with different sectors. Some prominent contributions to this literature are the following. Aoki (2001) shows that in a closed economy with a flexible-price sector and a sticky-price sector, the optimal monetary policy is to target the sticky-price inflation only. In an also closed economy but where both labor and product markets exhibit staggered prices, Erceg, Henderson and Levin (2000) find that strict price inflation targeting generates relatively large welfare losses with respect to the optimal flexible-price flexible-wage monetary policy, while combinations of wage and price inflation targeting or of price inflation and output gap targeting or even strict output gap targeting perform nearly as well as the optimal one. In a similar economy but with two sectors, durables and non-durables, Erceg and Levin (2002) find near-optimal to target a weighted average of aggregate price and wage inflations. Similarly, Huang and Liu (2005) find near optimal an interest rate rule that targets a combination of CPI and PPI inflations when there are nominal rigidities in both finished goods and intermediate goods markets.

In an open economy setting, Benigno (2004) shows in a model with different regions rather than sectors, that the monetary policy is near-optimal when the region with the higher nominal rigidity receives the higher weight in the inflation targeting strategy. Finally, Smets and Wouters (2002) estimate different degrees of domestic and import price stickiness and find the optimal monetary policy
minimizing a weighted average of both domestic and import price inflations.

We have applied our above specified welfare criterion to optimize over the parameters of the following varieties of the Taylor rule. First, we have considered aggregate CPI inflation, $\pi$, as well as wage inflation, $\pi^W$, and output gap targeting as in

$$\log(R_t/R) = \vartheta_R \log(R_{t-1}/R) + \vartheta_\pi \log(\pi_t/\pi) + \vartheta_{\pi^W} \log(\pi^W_t/\pi^W) + \vartheta_y \log(y_t/y).$$

(54)

Figure 6 represents the welfare surfaces with respect to $\vartheta_\pi$ and $\vartheta_{\pi^W}$ for different values of $\vartheta_y$ and holding constant the estimated degree of policy inertia, $\vartheta_R = 0.46$. As explained above, the welfare measure corresponds to a second order approximation of $E(\frac{1}{2} \mu_{c_t} \mu_{\tilde{m}_t} \mu_{\tilde{h}_t} - \beta_1 (c'_t - 1) \mu_{c_t} - \beta_1 (\frac{1}{2} m'_t - 1) \mu_{m_t} + \eta \log(1 - h_t))$. The welfare surfaces appear to be piecewise smooth in $\vartheta_\pi$, $\vartheta_{\pi^W}$ and $\vartheta_y$, except when approaching the zero inflation targeting area where the decay in welfare is very abrupt\footnote{Due to the possible flatness of the welfare function in some areas of the parameter space, we search for the welfare-maximizing interest rate rule using a grid search method over the policy parameters rather then relying on local optimizing routines. The intervals of the grid search on the coefficients are of size 0.2. The values for which there is indeterminacy, typically $\vartheta_\pi = 0$ and $\vartheta_{\pi^W} = 0$, are not plotted. Moreover, we restrict the search to values within the [0,5] interval, [0,4] when we search over the coefficients for several sectorial inflations at the same time.}. Figure 6 shows clearly that reacting aggressively to the output gap can be very damaging in terms of welfare losses. This is especially the case when the reaction to inflation deviations from target are low, where the welfare cost of the suboptimal rule is increasing in $\vartheta_y$.

The welfare-maximizing parameterization is the one explained above: strict CPI inflation targeting with coefficient $\vartheta_\pi = 1.2$ ($\vartheta_{\pi^W} = 0$ and $\vartheta_y = 0$). It is interesting to note that moving to a higher $\vartheta_\pi$ coefficient or to strict wage inflation targeting with $\vartheta_{\pi^W} > 1$ practically does not diminish welfare.

In the same spirit as Erceg, Henderson and Levin (2000), rows 4, 5 and 6 of Tables 3 and 4 explore the welfare and macroeconomic volatility implications of completely stabilizing one argument at a time in the above interest rate reaction function\footnote{Except that we compare our welfare results with the case of the historical estimated rule rather than with the flexible-price optimal rule. In order to guarantee complete stabilization of the target variable we impose very high coefficients one at a time: $\vartheta_\pi = 100$, $\vartheta_{\pi^W} = 100$ and $\vartheta_y = 2$. The latter is not that high because higher values for $\vartheta_y$ would cause indeterminacy unless they are coupled with high inflation reaction coefficients, which is impossible by definition in this particular exercise.}. We find that strict output gap stabilization reduces welfare with respect to the historical rule. Aggressive strict CPI inflation stabilization can improve welfare only very marginally with respect to $\vartheta_\pi = 1.2$ (a welfare gain of 0.085 versus 0.08 in the optimized CPI inflation targeting rule) while it significantly increases consumption and output volatility.

Strict wage inflation stabilization increases substantially all volatilities except consumption and hence cannot improve welfare with respect to CPI inflation targeting. This result seems to contradict part of the reported previous
research, which found that targeting the inflation rate of the more sticky price would be welfare superior. We have simulated the impulse responses of the main macro variables in our model using a strict CPI inflation targeting rule, a strict wage inflation targeting one and the responses under flexible prices and wages. Strict inflation targeting is the only one that gets similar responses to the flexible prices case in crucial variables like the aggregate final output $z_t$ after all shocks (except the monetary policy shock, which doesn’t affect real variables under flexible prices while it does under sticky prices).

We now explore the case in which the monetary authority can react differently to the different sectorial inflation rates: imports, traded, and non-traded in an interest rule of the type

$$
\log(R_t/R) = \varrho_R \log(R_{t-1}/R) + \varrho_{\pi^m} \log(\pi^m_t/\pi^m) + \varrho_{\pi^N} \log(\pi^N_t/\pi^N) + \varrho_{\pi^Td} \log(\pi^{Td}_t/\pi^{Td}).
$$

(55)

We compare the welfare gain and the volatility implications for different combinations of the monetary authority reactions to $\pi^m_t$, $\pi^N_t$, and $\pi^{Td}_t$. Again policy inertia is set to the estimated value, and we set $\rho_y = 0$ corresponding to the optimized value with CPI inflation Taylor rule. We do so in order to diminish considerably the time of optimizing the monetary rule over various coefficients for the different inflation rates.

Figures 7 and 8 show in two different ways a very clear result: aiming at stabilizing more the inflation rate in the imports sector or in the domestically produced traded sector does not increase welfare in any noticeable way. Only non-traded inflation targeting does. Row 7 in Tables 3 and 4 show the results for a coefficient on $\pi^N_t$ of 4, which is the higher depicted in Figures 7 and 8. The welfare gain of such rule is far superior to the other ones explored so far, and that is so because of the higher long-run average consumption and real money balances, despite the so much higher macroeconomic instability it causes. In fact, aggregate inflation and output volatilities are both more than 5 times those of the optimized CPI inflation targeting rule. Pushing to the limit non-traded inflation stabilization (row 8 in Tables 3 and 4) doubles the welfare gain but at the cost of a twice higher macroeconomic volatility.

Consequently, the central bank should react more aggressively to non-traded inflation and not at all to the other sectors. This is consistent with previous findings in the literature whereby the optimal monetary policy is to target exclusively the inflation rate of the sector with more nominal inertia, which in our case is the non-traded sector.

## 6 Targeting future price developments

To conclude this set of simple monetary policy rules optimization exercises, we explore the welfare and volatility impact of reacting to expected future deviations of the inflation rate from their targets rather than to contemporaneous deviations.
The top panel of Figure 9 shows the welfare surfaces for the cases in which we optimize over the CPI and wage inflation stabilization coefficients for different values of the output gap stabilization coefficient. In this case, all deviations from target to which the monetary policy reacts to are one-period ahead expected future deviations, that is, in the next quarter.

The two main results are (1) that the welfare-maximizing parameter set is exactly the same than when the central bank is not forward looking, i.e. \( \rho^t = 1.2, \rho^{t+1}_w = 0 \) and \( \rho^{t+1}_y = 0 \), and (2) that the welfare attained with a forward looking monetary policy rule is substantially higher. Row 3 of Table 3 shows that the welfare gain now is 0.11 per cent of the lifetime consumption, versus 0.08 per cent when optimizing a contemporary monetary policy rule. And this welfare gain comes together with an increased output and inflation volatility but a lower volatility in households’ utility (see row 3 in Table 4).

Of all the possible specifications explored in this paper, the one that achieves a higher welfare given the estimated model for the Canadian economy without causing a substantial excess macroeconomic volatility is a overall inflation targeting rule where the central bank reacts to next period’s expected deviation from the inflation target, does not target the output gap but allows for a moderate degree of nominal interest rate smoothing.

7 Conclusion

This paper analyses welfare-improving monetary policy reaction functions in the context of a new-Keynesian small open economy model with a traded and a non-traded sector and with sticky prices and wages. The model is estimated for the case of Canada and used to evaluate the welfare gains of alternative specifications of the feedback nominal interest rate rule.

The model is estimated using Bayesian techniques for quarterly Canadian data. We find statistically significant heterogeneity in the degree of price rigidity across sectors. We explore which would have been the optimal parameterization of a Taylor rule like the estimated one, where the central bank targets aggregate inflation. We find welfare gains in responding slightly more aggressively to aggregate inflation deviations from target than it has been the case in the last three decades and of not responding to the output gap, as opposed to what the Bank of Canada has done. We find further welfare gains of targeting sectorial rather than aggregate inflation. In particular, the gains are highest if the monetary authority reacts more aggressively to non-traded inflation since it is the production sector where prices are found more sticky.

We find that the higher welfare without inducing excess macroeconomic volatility is achieved, given the estimated model for the Canadian economy, with an overall inflation targeting rule where the central bank reacts to next period’s expected deviation from the inflation target, does not target the output gap but allows for a moderate degree of nominal interest rate smoothing.
References


Table 1: Parameter Estimation Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Type</th>
<th>Mean</th>
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<td>[−0.0307, −0.0166]</td>
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<td>Gamma</td>
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<td>[0.1643, 0.4142]</td>
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### Table 2: Variance Decomposition

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<th>$R_t$</th>
<th>$b_t$</th>
<th>$R_t^N$</th>
<th>$y_t^r$</th>
<th>$y_t^m$</th>
<th>$\pi_t^r$</th>
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<td>Average $m_t$</td>
<td>Average $h_t$</td>
<td>Average $u_t$</td>
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<td>Historical rule ( \tilde{R}<em>t = 0.46 \tilde{R}</em>{t-1} + 1.19 \tilde{\pi}_t + 0.31 \tilde{y}_t )</td>
<td>0.5337</td>
<td>0.2497</td>
<td>0.3005</td>
<td>-0.7929</td>
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<td>0.5345</td>
<td>0.2558</td>
<td>0.3013</td>
<td>-0.7921</td>
<td>0.0799</td>
<td>0.1112</td>
<td>-0.0311</td>
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<td>0.5349</td>
<td>0.2572</td>
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<td>-0.7918</td>
<td>0.1136</td>
<td>0.1549</td>
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<td>0.2618</td>
<td>0.3008</td>
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<td>0.0847</td>
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<td>0.2817</td>
<td>0.3012</td>
<td>-0.7923</td>
<td>0.0609</td>
<td>0.1719</td>
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<td>Output gap stabilization ( \tilde{R}<em>t = 0.46 \tilde{R}</em>{t-1} + \infty \tilde{y}_t )</td>
<td>0.5333</td>
<td>0.2462</td>
<td>0.3001</td>
<td>-0.7933</td>
<td>-0.0415</td>
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<td>Non-tradables inflation targeting ( \tilde{R}_t = 0.46 \tilde{R} + 4.00 \tilde{\pi}_t )</td>
<td>0.5413</td>
<td>0.7278</td>
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<td>-0.7833</td>
<td>0.9779</td>
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The welfare gain is expressed as a permanent percentage increase of consumption compared to its historical mean.
Table 4: Aggregate Volatility Induced by Alternative Monetary Regimes

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<th>Interest Rate rules</th>
<th>$\sigma_c$</th>
<th>$\sigma_m$</th>
<th>$\sigma_h$</th>
<th>$\sigma_u$</th>
<th>$\sigma_y$</th>
<th>$\sigma_\pi$</th>
<th>$\sigma_R$</th>
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<td>0.0552</td>
<td>0.0112</td>
<td>0.0226</td>
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<td>0.0596</td>
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<td>0.0301</td>
<td>0.0301</td>
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<td>Future CPI inflation targeting</td>
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<td>0.0277</td>
<td>0.0440</td>
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<td>CPI inflation stabilization</td>
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<td>0.0114</td>
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<td>0.0345</td>
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<td>Wage inflation stabilization</td>
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<td>0.0197</td>
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<td>0.0204</td>
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<td>0.0525</td>
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<td>0.0245</td>
<td>0.0097</td>
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<td>0.0615</td>
<td>0.0801</td>
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</table>

$\sigma$ denotes the unconditional standard deviation for the listed variables.
Figure 1: Prior and posterior distributions of the shocks parameters
Figure 2: Prior and posterior distributions of the behavioural parameters
Figure 3: Foreign Nominal Interest Rate Shock
Figure 4: Non-Tradables Technology Shock

[Graph showing various economic variables over time]
Figure 5: Local Nominal Interest Rate Shock
Figure 6: The Average Unconditional Utility with Respect to $\rho_\pi$ and $\rho_{\pi^w}$ ($\rho_y$ Changing)
Figure 7: The Average Unconditional Utility with Respect to $\rho_{\pi^N}$ and $\rho_{\pi^M}$ ($\rho_{\pi^Td}$ Changing)
Figure 8: The Average Unconditional Utility with Respect to $\rho_{\pi N}$ and $\rho_{\pi \tau d}$ ($\rho_{\pi M}$ Changing)
Figure 9: Period $t + 1$ Optimized Rules