Testing for Structural Breaks in Covariance: Exchange Rate Pass-Through in Canada*

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Abstract

Empirical studies looking for changes over time in exchange rate pass-through to consumer prices generally consider the context of a changing inflation mean to examine this issue. This paper allows for endogeneity in exchange rate movements and proposes a new method to test this hypothesis for Canada: A correlated VAR is proposed, and its covariance matrix is tested for breaks. For the latter purposes, we extend the test method proposed in Anderson (1971) to breaks in covariates and to unknown break dates. Our test accounts for breaks in mean, and is exact for fixed regressors.

We find strong evidence of structural changes, and a decline over time in pass-through. Nevertheless, we also find that the covariance between Canadian inflation and exchange rates changes has actually increased in the recent period.

*JEL classification:*

*Bank classification:* Econometric and statistical methods;
1. Introduction

Recent studies suggest that the pass-through of exchange rate changes into consumer and import prices is incomplete in industrialized countries. In addition, other empirical work suggests that this pass-through appears to have also declined over time. With some exceptions, most of the evidence for the latter is based on finding significant subsample dummy variables, when these are applied to the coefficient on the exchange rate in a univariate inflation equation. Examples are Gagnon and Ihrig (2001), and Baillu and Fujii (2004), the former imposing a dummy based on a known break date, and the latter, incorporating dummies based on estimated break dates.

However, most of the above studies do not directly address: (i) the evolution of the second moments of the variables of interest, and (ii) the fact that exchange rate changes may be endogenous over some part of the sample period, despite evidence on declining inflation variance and rising exchange rate variance over time. Clearly, if these factors are relevant, as theory suggests that they are, they are likely to affect the precision and consistency of the obtained pass-through estimates.

One frequently-cited reason for incomplete pass-through is pricing-to-market behaviour, and, in particular, local currency pricing (LCP) by exporting firms. If, as Devereux and Engel (2002) suggest, the volatility of real and nominal exchange rates is largely due to LCP, then exchange rate changes should be treated as endogenous to inflation. Similarly, as suggested by Betts and Devereux (1996), stickiness in the consumer price index likely also plays a role in the degree of pass-through; in which case, changes in inflation variability should also be important for declines in pass-through. Finally, Devereux, Engel, and Storgaard (2003), using an open-economy model of endogenous exchange rate pass-through, show that the relationship between exchange rate volatility and economic structure can be importantly affected by the degree of pass-through, and that the latter is related, among other things, to the relative stability of monetary policy between trading countries. In this case, if price stickiness increases in a country (for example, due to the implementation of inflation-targeting that, in turn, causes inflation expectations to be anchored), then the relationship between exchange rates and inflation will also change. Therefore, based on Devereux, Engel, and Storgaard (2003), when trying to measure pass-

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1 See, for example, Engel (1993), Goldberg and Knetter (1997), Parsley and Wei (2001), and Ambler, Dib, and Rebei (2003).

2 See, for example, Kichian (2001), Gagnon and Ihrig (2001), Leung (2003), and Baillu and Fujii (2004).
through, not only is it important to treat exchange rate changes as endogenous, but it is also necessary to account for shifting variances and covariances over time.

In this paper, we estimate exchange rate pass-through to consumer prices, and, more generally, examine the relationship between exchange rate changes and inflation, accounting for the above-stated factors. An intuitive and simple way is to consider a bivariate vector autoregression (VAR) for the variables of interest and estimate its parameters accounting for changes over time in the mean and the covariance of the system. This is where our methodological contribution intervenes: we develop a break test applicable to the covariance matrix of a multivariate linear system. The test extends the method proposed in Anderson (1971) to an unknown change point and accounts for any breaks in the mean. In addition, if regressors are strongly exogenous, it is exact in finite samples.

Applications of the break test to the VAR with Canadian data reveals evidence of breaks, at the 5 per cent level, in 1984Q2 and in 1991Q1. Accordingly, we estimate the VAR and calculate impulse responses for the period ending in 1984Q1, and for the period starting a little after the second break. The results show that pass-through has indeed changed over time: from a relatively high and long-lasting phenomenon in the first subperiod, to essentially no pass-through in the last subperiod. Furthermore, we show that imposing dummies for the exchange rate variable (for the different subsamples) only in the mean of the system would have led us to conclude differently; namely, that pass-through is not significant over the entire sample.

In the next section we summarize the literature on the declining pass-through in Canada, and present our simple VAR model. Section 3 explains the break test and its application to our data. Section 4 presents the VAR estimation results and discusses the corresponding impulse-response functions. The last section concludes.

2. The Canadian Evidence and Our Econometric Model

In the case of Canada, several studies produced at the Bank of Canada suggest that the effect of a change in the exchange rate on Canadian CPI inflation seems to have decreased after 1983-84, though precise estimates vary from study to study. Kichian (2001) uses a backward-looking Phillips curve with time-varying parameters (and, therefore, changing conditional variances) and measures pass-through as the coefficient on US inflation relative to Canadian inflation. She shows that this coefficient drops from an average value of 0.2 to
essentially zero after 1983–84. Leung (2003) also uses a backward-looking Phillips curve specification, but with fixed coefficients, and measures pass-through as the coefficient on the lag of the first difference in the exchange rate. Finding a structural break in the inflation series in the first quarter of 1984, he estimates the model over 1974Q1–1984Q1 and 1984Q2–2003Q2, respectively. He shows that the coefficient on the exchange rate is about 7 per cent in the first sample, but statistically not different from zero in the second. Finally, Baillu and Fujii (2004) also consider a backward-looking Phillips curve specification, but within a multi-country panel context. In this case, pass-through is defined as the coefficient on the contemporaneous change in the exchange rate. Finding breaks in the eighties and the nineties in the inflation series of most of the countries considered, they add two terms to their estimation equations: a dummy variable for each of the two decades, multiplied by the change in the exchange rate. They conclude that, while the coefficient on the eighties interaction term is not significant, average pass-through across countries declines from 11 percent in the seventies to somewhere in the vicinity of 5–6 per cent over the nineties.

Many explanations have been proposed for such a decline. These range from evolving industrial conditions (such as the adoption of free-trade between Canada and the United States), to changes in various institutional factors, to deliberate changes in policy. In particular, shifts in monetary policy towards a larger weight on inflation control has been emphasized—a hypothesis advanced by Taylor (2000). The latter proposition is validated in a simulation exercise conducted within a small calibrated structural model by Gagnon and Ihrig (2001). These authors also estimate a backward-looking Phillips curve equation for various countries over two different sub-samples using actual data. In almost all cases, the coefficient on import price inflation, which is taken to be their measure of pass-through, is found to be higher before the mid-eighties than after.\(^3\) In other sets of regressions, Gagnon and Ihrig find significant relations between changes in pass-through coefficients, and changes in first and second moments of inflation. In particular, the change in inflation variability is found to have a stronger impact on changes in pass-through. Thus, their reasoning is that, if monetary policy affects the inflation environment (and specially, inflation stability), it will cause a decline in pass-through.\(^4\) The latter line of thought

\(^3\)In the case of Canada, the long-run pass-through coefficient has a value of 0.3 before 1985, and 0.01 thereafter.

\(^4\)Regressions of changes in pass-through on estimated long-run coefficients on the inflation gap in a
is also considered in Murchison (2004). That study presents a DSGE model for Canada that is partly estimated and partly calibrated, and with various types of real and nominal rigidities. Nonetheless, pass-through is considered within the context of a Phillips curve, and it is reported that sufficient increases in policy aggressiveness can eliminate it.

Interestingly, evidence shows that the economic environment in developed countries, in general, has changed over time. In particular, the variability of output growth and of inflation in these countries have decreased importantly. For example, in the case of Canada, Debs (2001), and Chacra and Kichian (2004) document structural breaks in the volatility of output growth (in 1991Q2), as well as in that of some of the components of output. Similarly, in Dodge (2002), the governor of the Bank of Canada explains how the Canadian inflation series evolved from a highly-volatile and unpredictable process to a more stable and predictable one. He points out that a year after establishing a policy of inflation-targeting, inflation reached 2 per cent (the midpoint of the announced target bands), and that, given the credibility of the announced policy, inflation expectations soon fell in line with the targets.

The issue of whether changes in policy affect the comovement in macroeconomic variables has also been examined, though mainly from a theoretical perspective. For example, using a general-equilibrium model, Betts and Devereux (1996) show that LCP and stickiness in CPI prices affect the volatility of real and nominal exchange rates. Similarly, Devereux, Engel, and Storgaard (2003), using an open-economy model of endogenous exchange rate pass-through, show that the relationship between exchange rate volatility and economic structure can be importantly affected by pass-through, and that the latter is related, among other things, to the relative stability of monetary policy between trading countries. Thus, if price stickiness increases in a country, (for example, due to the implementation of inflation-targeting that causes inflation expectations to be anchored), the relationship between exchange rates and inflation will also likely change.

Thus, it appears not only that pass-through may have declined over time, but also that the general economic environment may have changed. Whether this is due to deliberate shifts in policy, or to changes in the types of shocks hitting the economies in question,

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6 This observation was based on surveys of forecasters, and on the difference between 30-year yields on conventional and index-linked bonds.
is an open question. But, for the purposes of measuring declines in pass-through, the above discussion
emphasizes the importance of (i) treating inflation and the exchange rate as endogenous variables, and (ii) accounting for changing variances and covariances among macroeconomic variables. Existing empirical studies, however, do not necessarily integrate all of these desired features. For example, each of Gagnon and Ihrig (2001), Leung (2003), and Baillu and Fujii (2004), consider the idea that a change in the inflation environment can affect the degree of pass-through. However, even though Leung (2003) tests for breaks in the inflation series, and estimates his univariate model over each sub-sample (effectively allowing for a change in the variance of inflation, as well as in its mean), he does not explicitly account for the endogeneity of the exchange rate. Similarly, while Baillu and Fujii (2004) allow for the mean process of inflation to change in a specific fashion—namely, by allowing for a different effect of the exchange rate change on inflation during the seventies, eighties, and nineties—they account neither for changing variances of inflation, nor for endogenous breaks in the covariance between the exchange rate change and inflation. As for Gagnon and Ihrig (2001), while they propose a structural model that shows how the different variables in the system interact, they do not use this same model to measure pass-through. The model that they use instead unfortunately has the same drawbacks as in Leung (2003) and Baillu and Fujii (2004).

Based on the above discussion, we propose the simplest empirical model that could be used to examine the evolution of the simultaneous relationship between the exchange rate and inflation; namely a correlated and unrestricted bivariate VAR of order one.\(^7\) Thus, while a fully-articulated general equilibrium model would have been more suitable for understanding the reasons behind, and the mechanics of, any changes in pass-through, the parsimonious nature of the VAR more readily accommodates estimations and tests over subsamples, and is thus an appropriate choice for examining statistically the issue of declines in pass-through.

The system is given by:

\[
\begin{align*}
\pi_t &= \alpha_{10} + \alpha_{11} \pi_{t-1} + \alpha_{12} \Delta e_{t-1} + \alpha_{13} \Delta p^c_t + \alpha_{14} \Delta p^*_t + \epsilon_{1t} \\
\Delta e_t &= \alpha_{20} + \alpha_{21} \pi_{t-1} + \alpha_{22} \Delta e_{t-1} + \alpha_{23} \Delta p^c_t + \alpha_{24} \Delta p^*_t + \epsilon_{2t}. \tag{1}
\end{align*}
\]

Here, inflation is given by \(\pi_t\), the nominal exchange rate is \(e_t\), commodity prices are given by the index \(p^c_t\), while foreign price is given by \(p^*_t\). The first difference operator is \(\Delta\),

\(^7\)The lag of order one is selected statistically.
so that $\Delta x_t = x_t - x_{t-1}$. In this context, pass-through can be defined very intuitively as the effect of a shock in the exchange rate change on domestic inflation. Impulse responses can thus be constructed, showing both the impact and the duration of such a shock on inflation.

3. Testing for Breaks in the Variance-Covariance Matrix of the VAR

The bivariate VAR system above shows that the same regressors appear in both equations, and that the system can be estimated by ordinary least squares. But the residuals of the two equations are contemporaneously correlated. Since impulse responses will be used to describe pass-through, and since the Choleski decomposition of the variance-covariance matrix enters these response functions (along with moving average coefficients of the system), any breaks in the moments of the VAR have to be properly accounted for.

In this section we apply a new test to examine whether there are any changes in the mean or the variance-covariance matrix of the non-orthogonalized VAR. The test extends the LR procedure of Anderson (1971) to: (i) models with covariates, and (ii) an unknown break date. In addition, the method generalizes the break-in-scale tests of Dufour, Khalaf, Bernard and Genest (2003) to the multivariate context. In simple terms, the test looks for a break in the variance-covariance matrix of a multivariate linear system while accounting for a break in the mean. Furthermore, the break point is considered unknown, and, if the regressors are fixed or exogenous, the test is exact (valid in finite samples). More generally, if lagged autoregressive terms enter the equations, bootstrapping can be used to construct an appropriate test p-value. Finally, the test also allows for the possibility of non-normal errors. In the next section, we provide the general outline of the test. Formal theorems and proofs can be found in the Appendix.

Test Framework

Our break test procedure generalizes the procedure of Anderson (1971, chapter 10) for testing equality of several covariance matrices. The test, as originally proposed, does not allow for covariates (the only regressor is a constant) and is valid for a given break date; our extension allows for covariates and an unknown break point. The test, in its general
form, is presented in the Appendix; we also provide a proof of its exactness in the case of fixed regressors. Here, we summarize the steps involved in its parametric bootstrap version as we apply it to our VAR model (1)-(2).

To test our model for a break-in-variance (or scale) that occurs at time $T_B$, obtain OLS estimates of the VAR covariance matrix over the full sample, and over two sub-samples: the first including the observations on the regressors and dependent variables prior to date $T_B$, and the second including observations from date $T_B$ to the sample endpoint. Conforming with the notation adopted in the Appendix, denote these estimated covariances $\hat{\Sigma}$, $\hat{\Sigma}_{(T_B,1)}$ and $\hat{\Sigma}_{(T_B,2)}$; in addition, let $T$ denote the size of the full sample, and $T_{T_B,i}$, $i = 1, 2$, the sizes of the first and second subsamples. Then the test statistic we consider is:

$$LR(T_B) = T \ln(\det(\hat{\Sigma})) - \sum_{i=1}^{2} T_{T_B,i} \ln(\det(\hat{\Sigma}_{(T_B,i)})).$$

In order to implement the test with unknown break point, consider a number of potential break points. Based on these, obtain a supremum statistic, sweeping over all potential break dates (as in Andrews (1993); this statistic retains its usual justification: the date which yields the highest test statistic is the likeliest break date).

In the Appendix, we show that in multivariate regressions, the null distribution of the individual and sup-type statistics are invariant to the regression coefficient and the error covariance; this allows us to derive an exact MC $p$-value (or a parametric bootstrap) by simulation. The procedure we apply here may be summarized as follows.

1. Calculate the sup-statistic from the observed data; in the process, save the constrained estimates of the VAR coefficients and covariance matrix (imposing stability).

2. Using the latter estimates, and drawing VAR errors from the normal distribution, obtain $N$ simulated samples from model (1)-(2); since the parameter estimates used to derive these samples impose stability, then by construction, the $N$ simulated samples satisfy the null hypothesis under test.

3. For each simulated sample, derive the associated sup-LR statistic; it is important to sweep the same potential break dates considered for the observed sample. Be-
cause the simulated samples impose the null hypothesis, this algorithm leads to \( N \) simulated samples conformable with the null hypothesis.

4. Obtain a MC \( p \)-value based on the rank of the observed statistic relative to the simulated one; the exact formula is given in the Appendix. The latter \( p \)-value is then referred to the desired significance level.

For regular VAR models, the latter procedure can provide a good approximate level correct \( p \)-value. If the model at hand is not dynamic (i.e. includes only fixed regressors), then we show in the Appendix that a \( p \)-value obtained using the latter algorithm is numerically invariant to the parameters chosen to construct the simulated samples (in step 2). This is why we can show its finite sample exactness. Exact extensions in dynamic models (such as the ones proposed in Dufour and Kiviet 1996, 1998) are conceptually feasible, and are a worthy research objective beyond the scope of this paper. Nevertheless, we report here a small-scale simulation study to document the size and power of this test in multivariate regressions.

We consider two designs: a model with dimensions close to our empirical VAR, with two equations, 60 observations and 12 regressors, and another model with larger dimensions: five equations, 100 observations and 12 regressors; for presentation purposes, we denote these models MLR(2) and MLR(5). The regressors include an intercept and 11 variates drawn as standard normal. The regression coefficient is set to zero (because of location-scale invariance, there is no loss of generality).

The errors are drawn as in Dufour and Khalaf (2002) to allow a simple design beyond 2-3 equations as follows: under the null hypothesis, the errors are independently generated as i.i.d. \( N(0, \Sigma) \) with \( \Sigma = GG' \) where the elements of \( G \) are drawn (once) from a normal distribution. Under the alternative, the errors in the first subsample are drawn (once) as i.i.d. \( N(0, \Sigma_1) \) with \( \Sigma_1 = G_1G_1' \) where the elements of \( G_1 \) are drawn (once) from a normal distribution; the errors in the second subsample are independently drawn (once) as i.i.d. \( N(0, \Sigma_2) \) with \( \Sigma_2 = g \times G_2G_2' \) where the elements of \( G_2 \) are drawn (once) from a normal distribution (independently from \( G_1 \)), and \( g \) is a scale term which serves to assess the power of the test, as a response to varying scale deviations across the samples.

In all cases, breaks at the first third, mid-point and last third of the sample are considered. The potential break dates (as in Dufour, Khalaf, Bernard and Genest 2004) sweep a window of 11 observations, centered at the break date ± up to five observations.
The results (empirical rejections for a test with 100 MC replications and 1000 simulations) are summarized in the following Table. Even with the small samples we considered, the test displays very good power.

<table>
<thead>
<tr>
<th>Break Window</th>
<th>MLR(2)</th>
<th>MLR(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T/3$</td>
<td>$T/5$</td>
</tr>
<tr>
<td>$\Sigma_1 = \Sigma_2 = \Sigma$</td>
<td>.032</td>
<td>.046</td>
</tr>
<tr>
<td>$\Sigma_1 \neq \Sigma_2$</td>
<td>$g = 1$</td>
<td>.097</td>
</tr>
<tr>
<td></td>
<td>$g = 10$</td>
<td>.400</td>
</tr>
<tr>
<td></td>
<td>$g = 20$</td>
<td>.968</td>
</tr>
</tbody>
</table>

**Test Application**

We apply the above-described break test to our model. Casual inspection of the inflation series suggests the existence of two episodes when inflation seems to shift to a lower-mean and lower-variance situation. The first occurs around 1983–84, where inflation appears to transit from a high to a moderate level, and the second, around 1990–91, after the adoption of inflation-targeting, and when the level of inflation drops even further. We therefore test break point around these two episodes.

Our data is at quarterly frequency and at annualized rates, and extends from 1972Q1 to 2003Q3. Canadian (domestic) and US (foreign) inflation measures are calculated using CPI prices. Our nominal exchange rate variable is the bilateral exchange rate between Canada and the United States (defined as the Canadian price of 1 USD). Finally, the Bank of Canada commodity price index is used to construct the variable $p_c$.

The model in equations (1) and (2) is tested and the results are reported in Tables 1a and 1b. One autoregressive lag is included for each endogenous variable, and dummies are added for each of the coefficients of the mean, comformable with the periods before and after a potential break date.

Because of a likely break in the early nineties, we first test over the sample ending in 1989Q4, and the interval over which a break may have occurred is the 1982–1984 period. Thus, the sample size consists of 68 observations, and the number of regressors is 10 (including the 5 dummy variables). The results are reported in Table 1a. These show that the LR value is highest in 1984Q2, and that the Monte Carlo sup-LR test $p$-value associated with a break at this date is 0.0270, which is significant at the 5 per cent level.
Next, we consider the sample 1985Q1 to 2003Q3, with a break date possibly occurring over the 1990–1991 period. In this sample, the total number of observations is 71, and the same regressors are considered, so that their number, including dummy variables for possible breaks in the mean, is again 10. The results reported in Table 1b show that the LR value is highest in 1991Q1 (at a value of 31.66), and that the associated Monte Carlo sup-LR test p-value is 0.001, which is significant, even at the 1 per cent level.

Based on the above results, we consider the two subsamples: 1972Q1 to 1984Q1, and 1992Q1 to 2003qQ3; the first ending prior to the first break point, and the second starting a little after the second break point. The interim period is discarded since it is too short to consider running meaningful estimations on.

4. VAR Estimation and Pass-Through

Given the test results above, the VAR model is orthogonalized using a Cholesky decomposition, and estimated with ordinary least squares over the two selected subperiods. In each case, the appropriate explanatory variables are included such that there is no autocorrelation or heteroskedasticity in the residuals of the equations at the 5 per cent level.\(^8\) Thus, in the first subsample, we found that, instead of the contemporaneous value of the commodity price inflation, the second and fourth lag of that variable needed to be included in the regression equations. At the same time, omitting the contemporaneous value, but including the third lag of the US inflation variable improved the regression fit. In the second subsample, in addition to the contemporaneous values of both the commodity price inflation and US inflation, including the fourth lag of the former, and a dummy variable for exogenous tax shocks (with a value of one in 1991Q1, 1994Q1, and 1994Q2, and zero otherwise) improved the adjusted R-squares\(^9\)

The results in Table 2 generally show fairly high adjusted R-squares (0.60 and 0.54 for the inflation equation over the first and second subsamples, and 0.17 and 0.19 for the exchange rate equation, respectively). There are also some marked differences between the two estimated outcomes. First, the constant terms for both variables decline from the first sample to the second (2.72 to 0.53 in the case of inflation, and 5.69 to 3.23

\(^8\)The tests are an LM test for autocorrelation in the residuals, and White’s heteroscedasticity test; each at 4 lags.

\(^9\)Gagnon and Ihrig (2001) also consider such an indirect tax change dummy variable.
for the exchange rate first difference term). More interestingly, the coefficients on own autoregressive terms also fall dramatically (from a significant 0.38 to zero for inflation, and from 0.32 to 0.19 for the exchange rate case). Different, also, is the effect of a change in the exchange rate on inflation (the short-run pass-through), which falls from 9 per cent in the first subsample, and significant at the 10 per cent level, to zero, in the second. The long-run impact of the variable (defined as the coefficient on the lagged difference of the exchange rate, divided by one minus the coefficient on the lagged inflation rate) thus also changes from 0.15 to 0.01.10

Interestingly, the effect of lagged inflation on changes in the exchange rate goes from being insignificant in the first subsample, to being significant at the 10 per cent level in the second. In addition, the initial impact of a change in commodity price on inflation is much faster during the nineties than it was in the early sample, whereas its second-round effect is felt later in 1992Q1–2003Q3. Commodity price changes also affect the exchange rate more quickly in the second subsample. Finally, US prices have a much quicker impact on Canadian inflation in the later sample than they did before.

These changes can also be summarized by looking at first and second moments of fitted means. Thus, the inflation mean is 8.79, and that of the exchange rate change, 2.09, in the first subsample. These decline to 1.81 and 1.67, respectively, in the later subsample. As for their variances, they change from 3.34 and 29.14, to 1.30 and 62.75, respectively, while the covariance between the two variables shifts from -0.14 over 1972Q1–1984Q1 to 1.24 over the period 1992Q1–2003Q3. Thus, the surprising outcome obtains that the correlation between consumer price inflation and changes in the nominal exchange rate increases from virtually zero (more precisely, -0.015) to 0.14.

Thus, the results reveal these main factors: (i) despite the fact that exchange rate changes are much more volatile in the nineties than they were before, they have virtually no effect on inflation over this period, (ii) despite the fact that inflation is much more stable over the nineties, it starts to have an impact on changes in the exchange rate, and, (iii) the relationship between the exchange rate and inflation is higher in the nineties compared to what it was in the 1972Q1–1984Q1 period, through this effect clearly manifests itself through indirect channels. These outcomes seem to lend some support to Betts and

These findings are similar to Leung (2003). He finds that the short-run impact of the exchange rate passes from 7 per cent in the first subsample, to insignificant in the second, and the long-run impact, from 0.17 to zero, respectively.
Devereux (1996) and Taylor (2000), who suggest that the inflation environment renders the exchange rate more volatile and lowers pass-through. Similarly, our results are supportive of Devereux, Engel, and Storgaard (2003) who suggest that a more stable money growth process (which can effectively lead to increased price stickiness), in conjunction with more local currency pricing, renders the exchange rate more volatile, and that, in turn, the high variability of the exchange rate will cause more of a “disconnect” between itself and various macroeconomic variables such as inflation.

Impulse response functions are also calculated for the two subperiods. Figures 1 to 2 show the responses to a one unit non-factorized shocks to each variable\textsuperscript{11} Two standard-error (i.e., 90 per cent) Monte Carlo confidence bands are also included.\textsuperscript{12} From these it is clear that pass-through has declined over time in Canada. Thus, over the first subsample, pass-through is significant at the 10 per cent level and has a duration of about 5–6 quarters, with an initial impact of about 9 per cent on inflation. In contrast, over the second subsample, pass-through is insignificant. Also notable is the changed response of inflation to a shock in that same variable, in that the shock disappears much faster in the second subsample compared to the time needed in the early sample. Finally, the response of the exchange rate movement to a shock to inflation, which was insignificant in the 1972Q1–1984Q1 period, starts to become significant (at the 10 per cent level) in the later sample, with a duration of 3–4 quarters and a sizeable negative impact.

To complete the analysis, we report estimation results over the full sample period (see Table 3). From the top panel, and ignoring any breaks, it appears that only own dynamics and the exogenous variables play a role for the dependent variables. In particular, pass-through is not significant over the full sample. However, despite the high adjusted R-square, tests strongly reject the hypothesis of normality, mainly due to residual heteroskedasticity.\textsuperscript{13}

Similar to other studies, we next included two dummy terms for the break periods 1984Q2–1990Q4 and 1991Q1–2003Q3, to capture a different impact of exchange rate changes on the dependent variables, and, in particular, on inflation. As can be seen in

\textsuperscript{11}Impulse responses to Cholesky one standard deviation shocks to each equations are not reported, but available upon request.

\textsuperscript{12}For the Monte Carlo exercise, 1000 replications were used.

\textsuperscript{13}The Jarque-Bera test for the hypothesis of joint normality is rejected with a $p$-value of 0.000, while the $p$-value for the joint test for no heteroskedasticity in the two residual series, and which includes cross-terms, is 0.0145.
the bottom panel of Table 3, doing so does not add anything of value to the analysis. Furthermore, the resulting impulse responses would have led us to erroneously conclude that inflation shocks have a much more long-lasting impact on both dependent variables, even in the post 1991Q1 period. Finally, had we included interaction terms between all of the coefficients of the model and dummy variables for the two break periods, we would still have concluded that pass-through was never significant, that inflation shocks die out after 4-5 quarters, and that they affect only inflation. Indeed, the estimated correlation between the two dependent variables is 0.013. Of course, the normality hypothesis is still strongly rejected, indicating, again, that to obtain precise parameter estimates, the break in the covariance matrix should be taken into account.

Clearly, then, the interaction between inflation and changes in the exchange rate is very different in the more recent period compared to before 1984. In addition, our study has shown the importance of accounting for changes in both the mean and the variance-covariance matrix of the system when measuring pass-through. Finally, our results provide evidence on the fact that the so-called “disconnect” between exchange rate changes and prices has increased.\footnote{disconnect in the sense that the variability of the exchange rate does not matter for the behaviour of macroeconomic variables.} We have learned, in particular, that univariate and multivariate estimates of inflation dynamics over the two subsamples reveal: (i) similar results for the effect of the exchange rate on inflation, (ii) different results for the effect of the rest of the variables on inflation, and (iii) switching impacts of US inflation and Canadian inflation on the behaviour of the exchange rate. Taken together, our results seem to provide support for Taylor (2000) and Devereux, Engel, and Storgaard (2003), in that a more stable monetary environment affects the exchange rate (specifically, its volatility), and that the latter, in turn (and under certain conditions, such as optimal choice of invoice currency), reinforces the negligible effect of the exchange rate on prices.

5. Conclusion

We proposed a new test by extending the Anderson (1971) test to covariates and allowing for unknown break points. The test also allows for shifts in the mean over the sample. Proofs were provided to show exactness of the test for the case of fixed regressors. We also showed how to apply the test to models including autoregressive terms by using
bootstrap methods. We then applied this test to a bivariate correlated VAR model for inflation and changes in the exchange rate to measure any declines in pass-through over our sample. Studies in the past have looked for such declines, but only within the context of a changing inflation mean. Yet, there is good reason to believe that the relationship between exchange rates and inflation, in general, may have changed.

We applied the test to Canadian data and found evidence of breaks, at the 5 per cent level, in 1984Q2 and in 1991Q1. Accordingly, we estimated our VAR model, and calculated impulse responses, for the subsamples 1972Q1–1984Q1, and 1992Q1–2003Q3. The results showed that pass-through has indeed changed over time: from 9 per cent, and with an impact over several quarters in the first subperiod, to no significant pass-through in the later subperiod. In addition, we showed that, as suspected, the relationship between inflation and exchange rate, in general, has changed.
References


Appendix

The Break Test:
Derivation and Finite sample Theory

Consider the multivariate linear regression (MLR) model:

\[ Y = XB + U \]  \hfill (1)

where \( Y = [Y_1, \ldots, Y_n] \) is a \( T \times n \) matrix of observations on \( n \) dependent variables, \( X \) is a \( T \times k \) full-column rank matrix of fixed regressors, and \( U = [U_1, \ldots, U_n] = [V_1, \ldots, V_T]' \) is the \( T \times n \) matrix of error terms. Here, our main statistical results require that the rows of \( U \), i.e. the vectors \( V_t, t = 1, \ldots, T \), satisfy the following

\[ V_t = JW_t, \ t = 1, \ldots, T, \]  \hfill (2)

where \( J \) is an unknown, non-singular matrix and the distribution of the vector \( w = vec(W_1, \ldots, W_T) \): (i) is fully specified, i.e. does not depend on any unknown parameter, or (ii) is specified up to an unknown nuisance-parameter. Let

\[ \Sigma = JJ'. \]

In particular, assumption (2) is satisfied when

\[ W_t \ i.i.d. \ N(0, I_n), \ t = 1, \ldots, T, \]  \hfill (3)

in which case \( \Sigma \) gives the variance/covariance of \( V_t \). In matrix form, and setting \( W = [W_1, \ldots, W_T]' \), (2) may be rewritten as \( W = U(J^{-1})' \) i.e. \( U = WJ' \). We present general distributional results which require no further regularity assumptions on the error terms.

In this set-up, the break test procedure may be described as follows. Partition the observed \( X \) and \( Y \) matrix conforming to the break date, into \( X_{(T_B,1)} \) and \( Y_{(T_B,1)} \), and \( X_{(T_B,2)} \) and \( Y_{(T_B,2)} \). In other words, \( X_{(T_B,1)} \) and \( Y_{(T_B,1)} \) include the observations on the regressors and dependent variables prior to date \( T_B \), and \( X_{(T_B,2)} \) and \( Y_{(T_B,2)} \) include the observation from date \( T_B \) to the sample endpoint. For further reference, let \( T_{T_B,1} \) and \( T_{T_B,2} = T - T_{T_B,1} \) denote the size of each subsample, respectively.
As will become clear from our presentation, this test is applicable when $T_{B,1}$ and $T_{B,2}$ both exceed $k$ to allow running the MLR over both sub-periods. In this context, the QLR-based procedure of Anderson leads to the statistic

$$LR(T_B) = T \ln(\det(\hat{\Sigma})) - \sum_{i=1}^{2} T_{B,i} \ln(\det(\hat{\Sigma}(T_{B,i})))$$  \hspace{1cm} (4)$$

where

$$\hat{\Sigma} = \hat{U}'\hat{U}/T, \quad \hat{U} = Y - X\hat{B}, \quad \hat{B} = (X'X)^{-1}X'Y$$

$$\hat{\Sigma}(T_{B,i}) = \hat{U}'_{(T_{B,i})}\hat{U}_{(T_{B,i})}/T_{B,i},$$
$$\hat{U}_{(T_{B,i})} = Y_{(T_{B,i})} - X_{(T_{B,i})}\hat{B}_{(T_{B,i})},$$
$$\hat{B}_{(T_{B,i})} = (X'_{(T_{B,i})}X_{(T_{B,i})})^{-1}X'_{(T_{B,i})}Y_{(T_{B,i})},$$
$$i = 1, 2.$$ 

In location-scale models (with no-covariates), an asymptotic $p$-value for the statistic may be obtained if $T_B$ is known, using the approximation for its null distribution due to Anderson (1971):

$$LR(T_B) \sim \chi^2((M(M+1)/2) + M).$$  \hspace{1cm} (5)$$

Let first examine the null distribution of $LR(T_B)$ when $T_B$ is known.

Under (2) and (1), the statistic defined by (4) is distributed like

$$LR(T_B) = T \ln(\det(\hat{S})) - \sum_{i=1}^{2} T_{B,i} \ln(\det(\hat{S}(T_{B,i})))$$  \hspace{1cm} (6)$$

where

$$\hat{S} = W'MW/T$$
$$\hat{S}(T_{B,i}) = W'_{(T_{B,i})}M_{(T_{B,i})}W_{(T_{B,i})}/T_{B,i}$$

$$M = I - X(X'X)^{-1}X;$$
$$M_{(T_{B,i})} = I - X_{(T_{B,i})}(X'_{(T_{B,i})}X_{(T_{B,i})})^{-1}X_{(T_{B,i})},$$

and $W_{(T_{B,i})}$ are obtained by partitioning the matrix $W$ in (2) conforming to the subsamples $T_{B,i}$.  

19
The latter distributional result obtains as follows. Under the null hypothesis,

\[ T \ln(\det(\hat{S})) = T \ln(\det(U'MU)) = T \ln(\det((J)(J^{-1})U'MU(J^{-1}'(J)'))) = T \ln(\det((J)W'MW(J'))) = T \ln [\det(J) \det(W'MW) \det(J')] = T [\ln(\det(J)) + \ln(\det(W'MW)) + \ln(\det(J'))]. \]

Similarly,

\[ T_{T_B,1} \ln(\det(\hat{S}_{(T_B,1)})) = T_{T_B,1} \ln(\det(U'_{(T_B,1)}M_{(T_B,1)}U_{(T_B,1)})) = T_{T_B,1} [\ln(\det(J)) + \ln(\det(W'_{(T_B,1)}M_{(T_B,1)}W_{(T_B,1)})) + \ln(\det(J'))] \]

so that

\[ T_{T_B,1} \ln(\det(\hat{S}_{(T_B,1)})) + T_{T_B,2} \ln(\det(\hat{S}_{(T_B,2)})) = T_{T_B,1} [\ln(\det(W'_{(T_B,1)}M_{(T_B,1)}W_{(T_B,1)}))] + T_{T_B,2} [\ln(\det(W'_{(T_B,2)}M_{(T_B,2)}W_{(T_B,2)}))] + (T_{T_B,1} + T_{T_B,2}) \ln(\det(J)) + (T_{T_B,1} + T_{T_B,2}) \ln(\det(J')). \]

On recalling that \(T_{T_B,1} + T_{T_B,2} = T\), we see that \(\ln(\det(J))\) and \(\ln(\det(J'))\) are evacuated by subtraction from the expression for the test statistic, to yield the pivotal quantity (6).

This shows that the null distribution of \(LR(T_B)\) does not depend on \(B\) nor \(J\) (and thus not on \(\Sigma\)) and may easily be simulated if draws from the distribution of \(W_1, \ldots, W_T\) are available. Thus, a Monte Carlo exact test procedure may be easily applied using the above theorem, and the procedures from Dufour (2002). The simulation-based algorithm which allows to obtain a MC size-correct exact \(p\)-value may be summarized as follows.

Let \(LR^0(T_B)\) denote the observed value of test statistic, calculated from the observed data set. Draw \(W^j = [W^j_1, \ldots, W^j_T], j = 1, \ldots, N\), as in (2), and compute the pivotal quantity (6)

\[ LR^j(T_B) = T \ln(\det(\hat{S}^j)) - \sum_{i=1}^{2} T_{T_B,1} \ln(\det(\hat{S}_{(T_B,1)}^j)) \]

where

\[
\hat{S} = W^{j'}MW^{j}/T, \quad \hat{S}_{(T_B,1)} = W^{j'}_{(T_B,1)}M_{(T_B,1)}W^{j}_{(T_B,1)}/T_{T_B,1}.
\]
This leads to $N$ simulated values of the test statistic $LR^j(T_B)$, $j = 1, \ldots, N$. Under the null hypothesis, $LR^0(T_B)$, $LR^1(T_B)$, $LR^2(T_B)$, $\ldots$, $LR^N(T_B)$ are exchangeable. Given the latter series, compute

$$
\hat{p}_N(LR(T_B)) = \frac{N \hat{G}_N(LR^0(T_B)) + 1}{N + 1},
$$

where $N \hat{G}_N(LR^0(T_B))$ is the number of simulated values greater than or equal to $LR^0(T_B)$. The MC critical region is: $\hat{p}_N(LR(T_B)) \leq \alpha$. Since the distribution of the statistic is continuous, then

$$
P[\hat{p}_N(LR(T_B)) \leq \alpha] = \alpha
$$

under the null hypothesis when $\alpha(N + 1)$ is an integer; see Dufour (2002). This formally demonstrates that the test so described will be size correct.

To obtain a test for an unknown break date, it is usual practice to run the latter test over a window of possible break-dates; denote this subset of potential break dates $J_B$. Then a combined statistic can be derived as in Dufour, Khalaf, Bernard, and Genest (2004) as follows:

$$
LR_{sup} = \max_{T_B \in J_B} \{LR(T_B)\}
$$

where $LR(T_B)$ is the statistic defined in (4). Under the null hypothesis, the statistics corresponding to each $T_B$ are jointly pivotal, so the above defined MC test procedure can be applied to the sup-test and will also yield an exact test procedure. For completion, we summarize the simulation-based algorithm associated with the joint test. Let $LR^0_{sup}$ denote the observed value of test statistic, calculated from the observed data set. Draw $W^j = [W_{1j}, \ldots, W_{Tj}], j = 1, \ldots, N$, as in (2), and for each draw, compute the pivotal quantity

$$
LR_{sup}^j = \max_{T_B \in J_B} \left\{ \frac{1}{T} \ln(\det(\tilde{S})) - \sum_{i=1}^{2} T_{T_B,i} \ln(\det(\tilde{S}_{(T_B,i)})) \right\}
$$

where

$$
\tilde{S} = W^j'MW^j/T,
$$

$$
\tilde{S}_{(T_B,i)} = W_{(T_B,i)}^jM_{(T_B,i)}W_{(T_B,i)}^j/T_{T_B,i}
$$

This leads to $N$ simulated values of the test statistic $LR_{sup}^j$, $j = 1, \ldots, N$. Given the latter series, the MC $p$-value of for the sup-test is

$$
\hat{p}_N(LR_{sup}) = \frac{N \hat{G}_N(LR^0_{sup}) + 1}{N + 1},
$$

21
where $N\hat{G}_N(LR_{sup}^0)$ is the number of simulated values greater than or equal to $LR^0(T_B)$. A test based on latter $p$-value is exact because under the null hypothesis, the combined statistics are jointly pivotal, so $LR_{sup}^0, LR_{sup}^1, \ldots, LR_{sup}^N$ are exchangeable. The reader may refer to Dufour, Khalaf, Bernard, and Genest (2004), for related results on sup-type break tests in univariate models (the multivariate case is not considered by Dufour et al. (2004)).
Table 1a: Testing for breaks over 1983–1984
Sample: 1972Q1 to 1989Q4

<table>
<thead>
<tr>
<th>Break Date</th>
<th>LR value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1983Q1</td>
<td>7.0277</td>
</tr>
<tr>
<td>1983Q2</td>
<td>5.8906</td>
</tr>
<tr>
<td>1983Q3</td>
<td>7.6054</td>
</tr>
<tr>
<td>1983Q4</td>
<td>7.4805</td>
</tr>
<tr>
<td>1984Q1</td>
<td>7.4881</td>
</tr>
<tr>
<td>1984Q2</td>
<td>13.6118</td>
</tr>
<tr>
<td>1984Q3</td>
<td>12.2751</td>
</tr>
<tr>
<td>1984Q4</td>
<td>12.1217</td>
</tr>
</tbody>
</table>

MC sup p-val = 0.0270

Table 1b: Testing Breaks within 1990–1991
Sample: 1985Q1 to 2003Q3

<table>
<thead>
<tr>
<th>Break Date</th>
<th>LR value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990Q1</td>
<td>23.7258</td>
</tr>
<tr>
<td>1990Q2</td>
<td>26.3336</td>
</tr>
<tr>
<td>1990Q3</td>
<td>28.2035</td>
</tr>
<tr>
<td>1990Q4</td>
<td>30.1081</td>
</tr>
<tr>
<td>1991Q1</td>
<td>31.6584</td>
</tr>
<tr>
<td>1991Q2</td>
<td>9.9838</td>
</tr>
<tr>
<td>1991Q3</td>
<td>8.4816</td>
</tr>
<tr>
<td>1991Q4</td>
<td>8.3393</td>
</tr>
</tbody>
</table>

MC sup p-val = 0.0010
Table 2: VAR Estimation Results (continued)

Sample: 1973Q2 to 1984Q1

<table>
<thead>
<tr>
<th></th>
<th>$\pi_t$ Equation</th>
<th>$\Delta e_t$ Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>2.720935 (1.10522)</td>
<td>5.687010 (3.26589)</td>
</tr>
<tr>
<td>$\pi_{t-1}$</td>
<td>0.375708 (0.12145)</td>
<td>-0.399138 (0.35888)</td>
</tr>
<tr>
<td>$\Delta e_{t-1}$</td>
<td>0.091795 (0.04970)</td>
<td>0.323033 (0.14685)</td>
</tr>
<tr>
<td>$\Delta p^c_{t-3}$</td>
<td>0.219398 (0.10571)</td>
<td>-0.075371 (0.31236)</td>
</tr>
<tr>
<td>$\Delta p^c_{t-2}$</td>
<td>0.060975 (0.02097)</td>
<td>-0.145174 (0.06196)</td>
</tr>
<tr>
<td>$\Delta p^c_{t-4}$</td>
<td>0.023542 (0.02742)</td>
<td>0.125061 (0.08102)</td>
</tr>
<tr>
<td>Adjusted R-square</td>
<td>0.602</td>
<td>0.171</td>
</tr>
</tbody>
</table>

Sample: 1991Q1 to 2003Q3

<table>
<thead>
<tr>
<th></th>
<th>$\pi_t$ Equation</th>
<th>$\Delta e_t$ Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.527909 (0.53984)</td>
<td>3.228772 (3.74384)</td>
</tr>
<tr>
<td>$\pi_{t-1}$</td>
<td>0.004243 (0.10315)</td>
<td>-1.356453 (0.71535)</td>
</tr>
<tr>
<td>$\Delta e_{t-1}$</td>
<td>0.011502 (0.01964)</td>
<td>0.187666 (0.13622)</td>
</tr>
<tr>
<td>$\Delta p^c_{t}$</td>
<td>0.542318 (0.19565)</td>
<td>0.250724 (1.35681)</td>
</tr>
<tr>
<td>$\Delta p^c_{t}$</td>
<td>0.024630 (0.01097)</td>
<td>-0.145818 (0.07609)</td>
</tr>
<tr>
<td>$\Delta p^c_{t-4}$</td>
<td>-0.017636 (0.00913)</td>
<td>-0.085779 (0.06330)</td>
</tr>
<tr>
<td>$tax_t$</td>
<td>-3.934808 (0.85562)</td>
<td>3.845537 (5.93375)</td>
</tr>
<tr>
<td>Adjusted R-square</td>
<td>0.544</td>
<td>0.186</td>
</tr>
</tbody>
</table>

**$tax_t$ is a tax dummy: 1 for 1991Q1, 1994Q1, 1994Q2, 0 elsewhere. Standard errors are in parentheses.**
Table 3: VAR Estimation Results

Sample: 1972Q2 to 2003Q3, No breaks in the Mean

<table>
<thead>
<tr>
<th></th>
<th>$\pi_t$ Equation</th>
<th>$\Delta e_t$ Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.134923 (0.30624)</td>
<td>0.373663 (1.07466)</td>
</tr>
<tr>
<td>$\pi_{t-1}$</td>
<td>0.536962 (0.06412)</td>
<td>-0.514761 (0.22500)</td>
</tr>
<tr>
<td>$\Delta e_{t-1}$</td>
<td>0.018371 (0.02326)</td>
<td>0.293804 (0.08163)</td>
</tr>
<tr>
<td>$\Delta p^e_t$</td>
<td>0.439038 (0.07830)</td>
<td>0.714641 (0.27478)</td>
</tr>
<tr>
<td>$\Delta p^c_t$</td>
<td>0.011596 (0.01011)</td>
<td>-0.129808 (0.03548)</td>
</tr>
<tr>
<td>Adjusted R-square</td>
<td>0.751</td>
<td>0.180</td>
</tr>
</tbody>
</table>

Sample: 1972Q2 to 2003Q3b, Dummies in the Mean

<table>
<thead>
<tr>
<th></th>
<th>$\pi_t$ Equation</th>
<th>$\Delta e_t$ Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.143697 (0.31162)</td>
<td>0.527639 (1.09140)</td>
</tr>
<tr>
<td>$\pi_{t-1}$</td>
<td>0.539788 (0.06530)</td>
<td>-0.541847 (0.22872)</td>
</tr>
<tr>
<td>$\Delta e_{t-1}$</td>
<td>0.041227 (0.04684)</td>
<td>0.391039 (0.16405)</td>
</tr>
<tr>
<td>$\Delta p^e_t$</td>
<td>0.430533 (0.07981)</td>
<td>0.708110 (0.27951)</td>
</tr>
<tr>
<td>$\Delta p^c_t$</td>
<td>0.010813 (0.01027)</td>
<td>-0.128007 (0.03598)</td>
</tr>
<tr>
<td>Dum1* $\Delta e_{t-1}$</td>
<td>-0.050052 (0.07465)</td>
<td>-0.020017 (0.26145)</td>
</tr>
<tr>
<td>Dum2* $\Delta e_{t-1}$</td>
<td>-0.024238 (0.05611)</td>
<td>-0.166899 (0.19651)</td>
</tr>
<tr>
<td>Adjusted R-square</td>
<td>0.745</td>
<td>0.172</td>
</tr>
</tbody>
</table>

Figure 1: Impulse Responses, 1973Q2–1984Q1

Response to Nonfactorized One Unit Innovations ± 2 S.E.

**Note: CANCPIINF is inflation and DCANE is the first-differenced nominal exchange rate.**
Figure 2: Impulse Responses, 1992Q1–2003Q3

Response to Nonfactored One Unit Innovations - 2 S.E.