Wojciech W. Charemza[*)] and Svetlana Makarova[**)]

Stochastic and deterministic unit root models: problem of dominance

An empirical investigation of world-wide stock market indices

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*) National Bank of Poland and the University of Leicester, UK

**) European University at St. Petersburg, Russia and the National Bank of Poland

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Abstract

The paper considers the question of dominance, in the context of financial markets, of the deterministic unit root processes with a structural break by the bilinear unit root model without such break or *vice versa*. In the deterministic unit root process breaks are usually interpreted as exogenous, while the unit root bilinearity is mostly attributed to speculation. A series of Monte Carlo experiments show substantial size distortions in testing for the deterministic unit root process in the presence of unit root bilinearity and *vice versa*. To eliminate this problem, two additional tests are proposed here: one for the joint testing of the process with a structural break and unit root bilinearity, and the other for testing the unit root bilinearity conditional on the break. The asymptotic properties of these tests have been analysed. The tests are applied for the daily stock price indices for 63 countries, for the period 1992-2005. It has been found out that in 34 cases the bilinearity is present in the series, and in only two cases a structural break was discovered without the presence of bilinearity. Since for most of the series a possible break occurs either in 2000 or 2001, it sheds some new light on the reasons for the stock market breakdown at the beginning of the 21st century.

Wojciech W. Charemza and Svetlana Makarova

Stochastic unit roots or deterministic breaks?

An empirical investigation of word-wide stock market indices*)

1. Introduction

The unit root tests developed within the last two decades have often failed to determine the nature of nonstationarity for a wide number of macroeconomic and financial time series. This can be seen as a substantial drawback of empirical economics, since the issue of stationarity (or non-stationarity) usually has to be resolved prior to modelling, hypothesis testing and forecasting. In view of this failure, there has been a recent tendency towards developing more general, nonlinear models of nonstationary economic time series. It is possible to identify two main streams here. The first approach is based on models with deterministic unit root(s) and elaborated nonlinear deterministic part of the process (e.g. structural breaks). The other approach uses stochastic rather than deterministic unit roots, without paying much attention to the deterministic part.

So far, in the literature, more attention has been given the deterministic rather than stochastic unit root processes. There have been a substantial number of papers devoted to testing the former in the presence of a complex nonlinear deterministic part of the model, usually describing 'structural breaks' in the series. The readings start from the seminal paper by Perron (1989), with further milestone papers by Perron (1990), (1997), Perron and Vogelsang (1992), Vogelsang and Perron (1998), Zivot and Andrews (1992), Harvey and Mills (2004) and numerous applications. In contrast, the literature related to the stochastic unit root models is more modest (see the theoretical papers by Subba Rao, 1997, McCabe and Tremayne, 1995, Leybourne, McCabe and Tremayne, 1996, Granger and Swanson, 1997, Francq, Lifshits and Zakoian, 2005, and, for some financial applications, Sollis, Leybourne and Newbold, 2000). In fact, it appears to be difficult to establish the superiority of one of these approaches over the other one without further thorough investigations. The approach based on deterministic models seems to be simpler in application, and especially in testing, than some of the stochastic models. Moreover, the fact that changes in the deterministic part of the process are usually modelled as 'jumps' or 'spikes' makes these

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models suitable for modelling the series affected by clearly defined single policy decisions. On the other hand the deterministic models are highly parameterised and changes of these parameters in the future make their forecasting properties questionable. On the other hand, the stochastic unit root models are usually lowly parameterised, which makes them less dependent on the *a-priori* assumptions regarding policy regime changes. They are, however, more complicated in terms of the asymptotic properties of estimators and tests statistics and their finite sample properties have not yet been fully investigated.

This study aims at a comparison of these two approaches by investigating the slowdown at the word-wide financial markets, which appeared at the beginning of the 21st century. From the beginning of 2000, for some period of time, markets around the world incurred heavy losses. This is illustrated by Figure 1, which presents the dynamics of the *Datastream* World Stock Price Index for the period from the 23rd of March 1992 until the 9th of March 2005.

Figure 1. Aggregate world stock price indices

Figure 1 indicates that, although the development of the world prices clearly exhibits a nonstationary pattern, this pattern is not of a familiar random walk nature. The process is either subject to a deterministic break in the year 2000/2001 (and possibly another one in 2002/2003), or is affected by another type of nonlinearity, possibly of a stochastic nature.

In this paper, we attempt to decide about the nature of this phenomenon for stock market indices in 63 countries. In order to do this, we started with some basic definitions in Section 2 and, in Section 3, we analyse possible effects of misspecification in testing of the underlying data generating process (*DGP*). We check to what extent mistaking of a deterministic unit root process with a break for a stochastic unit root process and *vice versa* might affect testing. Next, in Section 4, we propose a set of the marginal, joint and conditional tests aiming at separation of both effects. Finally, in Section 5, we proceed to the empirical analysis of price indices, individually and also as a panel of data. Section 6 presents conclusions. The paper is accompanied by two Appendices. Appendix A contains proofs of asymptotic properties for the tests and Appendix B gives detailed empirical results and also abbreviations used and country codes.

2. Deterministic and stochastic unit root processes

The paper considers two particular stochastic processes embedded within the following general process:

$$u_t = d_t + y_t \quad , \quad y_t = r_t y_{t-1} + v_t \quad , \tag{1}$$

where d_t is a deterministic part, $E(y_0^p)^2 < \infty$, $v_t = d(L)e_t = \sum_{j=0}^{\infty} d_j e_{t-j}$ and $\sum_{j=0}^{\infty} j\,|d_j| < \infty$ (see Ng and Perron, 2001), r_t is series of (possibly degenerated) random variables, $e_t \sim IID(0, s^2)$ and $t = 1, 2, \ldots, T$. A simple example of the deterministic part in (1) is:

$$d_t = const. + g B_t \ , \tag{2}$$

where B_t is a variable indicating a nonlinear change in the deterministic part of the process. Such change is usually described as a structural break. The commonly used form of such break is:

$$B_t = \begin{cases} 0, & \text{for} \quad t < T_B \\ 1, & \text{otherwise} \end{cases} \ , \tag{3}$$

and T_B indicates the position of the beginning of the structural break in the series. A number of other, more complex, specifications of B_t is widely discussed in the literature (see Vogelsang and Perron, 1998, and Harvey and Mills, 2004).

As far as the unit root part in (1) is concerned, the following terminology is introduced:

(a) The unit root process has a *deterministic* unit root and is *linear* if in (1) $r_t = 1$ and in (2) $g = 0$.

(b) The unit root process is *deterministic* and *nonlinear* if $r_t = 1$ and $g \neq 0$.

(c) The unit root process has a *stochastic* unit root if r_t is a non-degenerated random variable with the expected value of one. In this paper we only consider the stochastic unit root process without a deterministic part, that is where $g = 0$.

The first two processes, (a) and (b) traditionally denoted as $I(1)$, constitute a family of the deterministic unit root processes. The literature on testing and evaluation of the $I(1)$ processes is well developed. In particular, the nonlinear $I(1)$ processes are analysed in Kim, Leybourne and Newbold (2000), Ng and Perron (2001), Perron, (1989), (1990), (1997), Vogelsang and Perron (1998), Zivot, and Andrews (1992). Among the stochastic unit root processes, this paper focuses on the bilinear unit root process of the first order (see Charemza, Lifshits and Makarova, 2005), where r_t in (1) is defined as:

$$r_t = a + be_{t-1}, \quad b \neq 0 \text{ and } a = 1. \tag{4}$$

For alternative specifications see e.g. Granger and Swanson (1997), Leybourne, McCabe and Tremayne (1996), McCabe and Tremayne (1995), Sollis, Leybourne and Newbold (2000). We consider a simple case of (1) and (4) where $d_t = 0$, and $v_t = e_t$, which gives:

$$y_t = (1 + be_{t-1})y_{t-1} + e_t \quad . \tag{5}$$

Possible advantages of using (5) in the financial analysis is its interpretation since, if y_t is a series of logarithms of prices, it becomes a process allowing for endogenous speculative bubbles. Moreover, returns (differences of y_t) are no longer normal even if e_t is normally distributed. For $b = 0$, (5) becomes a simple $I(1)$ process, without any deterministic part (the random walk). For b ? 0, the main problem with (5) is its explosive nature since, for $a = 1$, the stationarity condition, that is $a^2 + b^2 s^2 < 1$, is not fulfilled (see Granger and Andersen, 1978). In Charemza, Lifshits and Makarova (2005) it is shown that, under the null hypothesis that b=0 the Student-t ratio for $\hat{b}$ in the regression equation:

$$\Delta y_t = \hat{b} y_{t-1} \Delta y_{t-1} + e_t \quad , \tag{6}$$

where e_t are the regression residuals, has an asymptotic standard normal distribution. It is also shown that the similar test statistics can be formulated for a regression containing an intercept and for demeaned series of first differences of y_t. If the relationship between v_t and e_t is more complex, that is, where v_t is described by a fully defined moving average process, it is possible to add augmentations to (6), as in the Augmented Dickey-Fuller test. In all these cases the distribution of the Student-t statistic for b is also asymptotically standard normal.

The rationale of enquiry about the b parameter depends crucially on the assumption that $a = 1$ in (4), that is, that under the null of $b = 0$, the process is a random walk. A two-step procedure is suggested, where the existence of a unit root under the null of $b = 0$ is first confirmed by the usual unit root tests aiming at detecting a deterministic unit root, and next the inquiry regarding b is made using (6). It clearly depends on the performance of the unit root tests.

3. Effects of unit root misspecification: Monte Carlo experiments

The problem considered here more complex, since the possible *DGP* might contain nonlinearities in a form of a structural break. In order to show a possible impact of such nonlinearity on the inquiry regarding b, a simulation experiment is set up, where the *DGP* is given by (1), (2) and (3), with $r_t = 1$, $g = 10$, $v_t = e_t \sim IIDN(0,1)$ and varying T_B. The unit root hypothesis: $r_t = 1$ is at first tested by a battery of well-known tests: the Phillips-Perron Z_a test, modified Z_a test, MZ_a test, Augmented Dickey-Fuller test (*ADF*), Point Optimal test (*PT*), Modified Point Optimal test (*MPT*), and Modified Sargan-Bhargava test (*MSB*). The Z_a, MZ_a and *ADF* tests were used in two versions, with detrending done by ordinary least squares method (*OLS*) and also by generalised least squares method (*GLS*) as suggested by Ng and Perron (2001)[1)]. These tests do not explicitly consider an existence of a deterministic nonlinear part in the *DGP* and are called herein the *linear unit root tests.* Once a series survived a unit root test at the 5% significance level (that is the null hypothesis of a unit root has not been rejected), it is subjected to the b-test, that is testing for $b = 0$ using the Student-t ratio in regression (6). The sample size we use was respectively $T = 100$, 500 and 1,000 and T_B was set in such a way that:

$$l = T_B / T = 0.89, 0.85, 0.80, 0.70, \cdots, 0.20, 0.15, 0.11.$$

In another words, we were simulating an impact of a break, which appears within the 80% middle of sample observations, with a particular interest in the breaks occurring close to the edges of this selection. For each T and l we simulated 10,000 replications.

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1) We are grateful to Serena Ng and Pierre Perron for making their code available at: http://econ.bu.edu/perron/code.html, which we used in our computations. They bear no responsibility for possible errors resulting from application of their program.

We are interested here in size distortions for the b-test, that is the differences between the expected frequency of rejection of the null of no bilinearity and the corresponding empirical frequency. On the second stage we are using 10% rather than 5% level of significance here (in order to evaluate the test size on the basis of more information), so that the expected frequency of rejection is $0.95 \times 0.10 = 0.095$. Since it turned out that the differences between outcomes obtained by all the linear unit root tests listed above are negligible (identical to the third decimal point), in Table 1 we present results averaged for all these tests. Further detailed results are not reported here and available on request.[2)]

Table 1: Frequencies of rejection of the hypothesis $b = 0$, where the *DGP* is a nonlinear deterministic unit root process, averaged for all linear unit root tests

l	$T = 100$	$T = 500$	$T = 1{,}000$
No break	0.090	0.102	0.099
0.89	0.068	0.099	0.104
0.85	0.063	0.098	0.102
0.80	0.069	0.097	0.098
0.70	0.063	0.098	0.098
0.60	0.062	0.099	0.096
0.50	0.058	0.092	0.098
0.40	0.048	0.098	0.097
0.30	0.038	0.088	0.096
0.20	0.032	0.086	0.095
0.15	0.028	0.082	0.091
0.11	0.024	0.080	0.091

Table 1 shows a marked influence of the deterministic nonlinearities on the efficiency of testing for the bilinearity. In small samples, size of the b-test is biased downward substantially, which is due to a decreased probability of committing the type I error by the unit root test at the first stage of the procedure. For larger samples, size of the b-test is distorted in a different way. For $T = 500$

[2)] All computations presented in this paper have been made in GAUSS. Computational programs used here are available on request.

and T = 1,000 it is too low for small values of l (that is, for breaks appearing close to the beginning of the sample) and too high for high values of l (breaks close to the end of the sample). It is approximately right for breaks being close to the middle of the sample. As the result, and regardless of the method of detecting the unit root, distorted size of the *b*-test is likely to cause additional uncertainty at the second stage, especially when the null hypothesis is rejected.

Different distortions can be observed when the unit root tests concern about the nonlinear processes, explicitly allowing for deterministic nonlinearities of the type (3). Table 2 presents the results of the Vogelsang and Perron (1998) 'Additive Outlier' (AO) Model 1 (using their terminology), that is:

$$u_t = d_t + y_t \ , \quad y_t = r y_{t-1} + v_t \ ,$$

where d_t is estimated as:

$$\hat{d}_t = \hat{m} + \hat{q} \hat{B}_t \ , \ \text{and:}$$

$$\hat{B}_t = \begin{cases} 0, & \text{for} \quad t < \hat{T}_B \\ 1, & \text{otherwise} \end{cases} \ ,$$

and $\hat{T}_B$ is a break estimated in the series using a particular search criterion. The search criteria usually applied here are (*i*) $\min(t_{\hat{r}})$, (*ii*) $\max(t_{\hat{q}})$, where $t_{\hat{r}}$, $t_{\hat{q}}$ are the Student-*t* ratios for the *OLS* estimates of r and q, and (*iii*) $\max(|t_{\hat{q}}|)$. In each case a full search is conducted over the sample period reduced by a certain percentage of observations from top and bottom. The null hypothesis here is that the process is *I*(1), with the alternative *I*(0), with the mean changed by a jump to a new level at point $\hat{T}_B$ (the 'step' process). Since these methods are computationally expensive, the number of replications is reduced here to 5,000. In this case the dependence of size distortions on the position of the break in the series is also evident. Additionally, and unlike in the linear unit root cases, some difference between the expected and actual frequency of the rejection of the null hypothesis is also detected in the 'no break' case, that is where the *DGP* is the linear deterministic unit root process.

In the reverse experiment, when the *DGP* is that of a bilinear unit root process, the results vary among the particular unit root tests. In this case, the *DGP* becomes (5) with $e_t \sim IIDN(0,1)$. The parameter b is changed in such a way that d = 0.1 , 0.9, where $d = b\sqrt{T}$. For each d, the number of replications is 10,000.

Table 2: Frequencies of rejection of the hypothesis *b* = 0, where the *DGP* is a nonlinear deterministic unit root process, *AO* test, model 1, criterion $\min(t_{\hat{r}})$

l	$T = 100$	$T = 500$	$T = 1{,}000$
No break	0.094	0.110	0.111
0.85	0.035	0.095	0.103
0.80	0.034	0.085	0.095
0.70	0.030	0.090	0.099
0.50	0.036	0.091	0.099
0.30	0.028	0.087	0.095
0.20	0.021	0.078	0.097
0.15	0.021	0.081	0.093

In this case, the maximal attainable power of the *b*-test is 0.95, since the significance level at the first stage of testing is 5%. It appears that the best, in terms of maximising the *b*-test power, is the MZ_a^{OLS} test followed by PT and ADF^{OLS} (for high values of $d = b\sqrt{T}$), which perform well for both small and large samples and *b*'s. Among the worst linear unit root tests test are here $AO\min(t_{\hat{r}})$, ADF^{OLS} (for low values of $d = b\sqrt{T}$), and MZ_a^{GLS}. It should be noted that the nonlinear unit root test applied here, $AO\min(t_{\hat{r}})$, performs markedly worse than the linear tests.

Overall, the results indicate that, despite visual similarities of the processes, mistaking a nonlinear deterministic unit root process with a bilinear one or *vice versa* might cause severe distortions and lead to false decisions. In particular, in these cases the probability of detecting the bilinear unit root process is diminishing, size of the *b*-test is distorted and there is an increased chance that such a process can be mistaken for an *I*(0) process with a 'step'.

Table 3: Frequencies of rejection of the hypothesis b = 0, where the *DGP* is a bilinear unit root process

Det. unit root tests	T = 100		T = 500		T = 1,000	
	d = 0.1	d = 0.9	d = 0.1	d = 0.9	d = 0.1	d = 0.9
Z_a	0.166	0.800	0.389	0.927	0.533	0.931
MZ_a	0.165	0.804	0.388	0.923	0.532	0.927
MSB	0.166	0.806	0.389	0.929	0.533	0.935
ADF^{GLS}	0.166	0.813	0.388	0.927	0.533	0.931
PT	0.167	0.817	0.389	0.931	0.533	0.933
MPT	0.166	0.807	0.388	0.928	0.533	0.932
MZ_t	0.166	0.807	0.389	0.927	0.532	0.931
Z_a^{OLS}	0.166	0.810	0.386	0.932	0.527	0.935
MZ_a^{OLS}	0.169	0.833	0.392	0.950	0.533	0.950
MZ_a^{GLS}	0.165	0.801	0.389	0.923	0.532	0.927
ADF^{OLS}	0.163	0.833	0.378	0.949	0.518	0.950
$AO\min(t_{\hat{r}})$	0.160	0.789	0.388	0.877	0.526	0.891

4. Joint and conditional testing

The disappointing performance of the deterministic unit root tests in the presence of bilinearity and of the b-test in the presence of structural breaks call for some development of joint and conditional, rather than marginal, testing. In addition to the (marginal) b-test applied above, we suggest the following:

(1) Joint test for the bilinearity and structural break (the *BSB* test). Suppose that, under the null hypothesis, y_t is $I(1)$ process. Consider the test equation:

$$\Delta y_t = c + b y_{t-1}\Delta y_{t-1} + g\, B_t + n_t \quad , \tag{7}$$

where B_t is defined by (3) and the properties of n_t have been described in Section 2 above. Its *OLS* estimator is given by:

$$\Delta y_t = \hat{c} + \hat{b} y_{t-1}\Delta y_{t-1} + \hat{g}\, B_t + \sum_{i=1}^{k} \hat{c}_i \Delta y_{t-i} + e_t \quad , \tag{8}$$

where k is selected in such a way that e_t approximates a white noise process (see Ng and Perron, 2001).

(2) Conditional test for the unit root bilinearity (the *CB* test). Suppose now, that the null hypothesis is more complex and the *DGP* allows for a 'spike' in the unit root process:

$$y_t = y_{t-1} + g\, P_t + n_t \quad ,$$

where:

$$P_t = \begin{cases} 0, & t \neq T_B \\ 1, & t = T_B \end{cases} \quad ,$$

Consider the following procedure:

i) Estimate by the *OLS* regression:

$$\Delta y_t = \hat{g}\, P_t + \hat{v}_t \quad , \tag{9}$$

ii) Define a new process z_t as: $\Delta z_t = \hat{v}_t$, and recover z_t as: $z_t = \sum_{k=1}^{t} \hat{v}_t$,

iii) Consider the *OLS* estimator of b in the regression equation:

$$\Delta z_t = \hat{b} z_{t-1}\Delta z_{t-1} + \sum_{i=1}^{k} \hat{c}_i \Delta z_{t-i} + e_t \quad , \tag{10}$$

with k and e_t is selected as in (8) and test $H_0 : b = 0$ against $H_1 : b \neq 0$.

Asymptotic properties of the *BSB* test statistics are given by the following Theorem 1.

Theorem 1. *Let the series y_t be generated by the process* (1) *with $d_t \equiv 0$, $y_0 = 0$, $v_t = e_t$ and $r_t = 1$. Denote by l the ratio of time of break T_B to the sample size T and assume that it is constant (as $T \to \infty$), that is:*

$$\frac{T_B}{T} = l = cons. \quad . \tag{11}$$

Let W_1, W_2 *are two independent Wiener processes on* $[0,1]$, $c^2(2)$ *is chi-squared distributions with two degrees of freedom and* $\Rightarrow$ *denotes weak convergence when* $T \to \infty$. *Then, for the regression model* (8)*:*

1. *Under the null of* $b = g = 0$, *as* $T \to \infty$, *the F-type-statistic has a limit distribution of the form*[3)]*:*

$$F \Rightarrow \frac{1}{2}\left[\left(\frac{\int_0^1 W_1(t)dW_2(t)}{\sqrt{\int_0^1 W_1^2(t)dt}}\right)^2 + \left(\frac{l W_1(1) - W_1(l)}{\sqrt{l(1-l)}}\right)^2\right] \sim \frac{1}{2}c^2(2) \ .$$

2. *Under the null of* $g = 0$, *as* $T \to \infty$, *the t-ratio for* $\hat{g}$, *that is:* $t_{\hat{g}} = \hat{g} / s.e.(\hat{g})$, *has a limit distribution of the form:*

$$t_{\hat{g}} = \frac{l W_1(1) - W_1(l)}{\sqrt{l(1-l)}} \sim N(0,1) \quad .$$

3. *Under the null hypothesis of* $b = 0$ *as* $T \to \infty$ *the t-ratio for* $\hat{b}$ *has a limit distribution of the form:*

$$t_{\hat{b}} = \frac{\int_0^1 W_1(t)dW_2(t)}{\sqrt{\int_0^1 W_1^2(t)dt}} \sim N(0,1) \quad .$$

Proof of this Theorem follows the Gihman and Skorohod (1979) technique and is given in Appendix A.

For the *CB* test, the following Theorem 2 is needed:

Theorem 2. *Let the series* y_t *be generated by* (1), (2) *and* (3) *with* $y_0 = 0$, $v_t = e_t$ *and* $r_t = 1$. *For the regression model* (10), *under the null of* $b = 0$, *as* $T \to \infty$, *the t-ratio for* $\hat{b}$ *has a limit distribution of the form:*

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3) We are grateful to Mikhail Lifshits for additional comment on the independence of the chi squared variables in this theorem.

$$t_{\hat{b}} \Rightarrow \frac{\int_0^1 W_1(t)dW_2(t)}{\sqrt{\int_0^1 W_1^2 dt}} \sim N(0,1) \quad ,$$

where W_1, W_2 *are two independent Wiener processes on* $[0,1]$.

Proof of the Theorem 2 is also given in Appendix A.

Both *BSB* and *CB* tests depend on the existence of the unit root in the series, so that their application should be preceded by testing for a unit root by one of the deterministic unit root tests. Results given in Section 2 suggest MZ_a^{OLS} here, which is relatively robust for possible nonlinearities, and ADF^{OLS} which works well in cases of high bilinearity. Moreover, the *BSB* test requires knowledge of T_B, the possible breakpoint in the series. Since this is usually unknown, it has to be estimated prior to testing, for instance by one of the search criteria used in the *AO* testing. The Monte Carlo results (available on request) show that convergence of both statistics to normality is fast and the critical values of standard normal distribution can be used for samples greater than 200.

In applications, the principal difference between the *BSB* and *CB* tests is such that the *BSB* evaluates the possibility of the existence of a bilinear component independently from a possible existence of nonlinear deterministic part of the process. This test verifies the presence of a nonlinear deterministic component within a relatively weak null hypothesis of a linear $I(1)$ process, with no bilinearity. Hence, the *BSB* test is rather restrictive and its conclusions are not strong. On the other hand, the *CB* test evaluates the bilinearity of a nonstationary process taking explicitly into account the possible existence of a nonlinear deterministic part. The price paid here are some computational complications and an additional assumption that the break date in the process is known.

5. Empirical analysis of worldwide stock market indices

The aim of the empirical part here is to identify the stochastic nature of prices and distributions of returns on the world stock markets. If prices are found to contain a unit root and returns are found to be non-normal, this non-normality can be explained either by the nonlinear, but deterministic, nature of the price series or, alternatively, by the fact that their unit root process is bilinear rather than linear.

We have checked, for deterministic nonlinearities (structural breaks) and unit root bilinearity, series of stock market indices from 63 countries. We used session-to-session (daily) observations from the period from 23 March, 1992 to 9 March, 2005. Most series (45 of them) were of full length of 3,383 observations. Other series, mainly for newly independent East European countries were of a shorter length, the shortest (for Bulgaria) containing 1,144 observations. All data have been retrieved from *Datstream* and unadjusted.

Plan of the research is as follows. First we evaluate the distribution of returns here (first differences of logs of the analysed series) and the deterministic unit root hypothesis for the level of prices (in logarithms). If the returns are found to be non-normal and prices contain a unit root, further enquiry regarding structural breaks and bilinearity is conducted, with estimations of dates of the breaks, bilinear parameters and joint and conditional testing described above.

Table B1 in Appendix B shows basic descriptive statistics for the first differences of logs of the analysed series. It reveals substantial non-normality of the distribution of session-to-session returns. With few exceptions, either significantly positive or significantly negative skewness is observed throughout. Abnormal kurtosis of returns is also evident for all the series. Table B2 gives the McCulloch (1986) estimates of the parameters of the stable (Pareto-Levy) distribution. When the characteristic exponent of this distribution (the parameter *alpha* of the stable distribution) is equal to 2, the stable distribution becomes normal. Here we find the estimates of the parameters for nearly all series markedly below the value of 2. Another parameter of the stable distribution, *beta*, indicating skewness is, in general, negligible.

Tables B3a and B3b show the results of deterministic unit root testing, for levels and first differences respectively. In addition to two tests found superiour by the Monte Carlo analysis described in Section 3, MZ_a^{OLS} and ADF^{OLS}, the augmented *RMK* test which explicitly allows for a the stable distribution of the residuals rather than normal (see Rachev, Mittnik and Kim, 1998, Greszta, 2003). In this case the parameter *alpha* of the stable distribution is first estimated by the McCulloch (1986) method, and then used for testing in a Dickey-Fuller style procedure. Since the returns have been found non-normal, application of this test seems to be plausible here. For levels (table B3a), the only country for which there is some confirmation, at the 0.05 level of significance, of a stationarity, is South Africa, for which two tests out of three rejected the null of a unit root. Weak evidence of stationarity (rejection by one test only at 0.05 or 0.10 level of significance) can also be found for Bulgaria, Hong Kong, India, New Zealand, Philippines and Poland. All other results indicate a presence of a unit root in the series. For the returns

(Table B3b) all test for all countries show, without exception, that the null hypothesis of a unit root should be rejected at the 0.01 level of significance. Hence, from this analysis, on the grounds of the linear deterministic unit root theory it can be concluded that the series have one unit root.

This is additionally supported by the results of the *IPS* unit root test, in which the entire set of 63 series is treated as a heterogeneous panel (see Im, Pesaran and Shin, 2003). Particular *IPS* statistics are 1.37 for levels of indices and -55.20 for returns. Since the asymptotic distribution of the *IPS* statistic under the null of a unit root in the panel is standard normal, it supports the previous finding of the existence of a unit root in the data.

Table 4 presents the number of breaks in the series discovered in particular sub-periods by the different search criteria: $\min(t_{\hat{r}})$, $\max(t_{\hat{q}})$ and $\max(|t_{\hat{q}}|)$. It is assumed that only one break might appear in the series. The table indicates that the frequency of negative breaks was indeed the greatest in 2000 and 2001, while the positive breaks, indicated by the $\max(t_{\hat{q}})$ criterion, which selects only positive breaks, shows that they cumulate prior to, and before that period.

Table 4: Number of breaks discovered in the series in particular sub-periods

Period	Break selection criteria		
	$\min(t_{\hat{r}})$	$\max(t_{\hat{q}})$	$\max(\lvert t_{\hat{q}}\rvert)$
before 2000	22	45	23
2000-2001	30	2	24
after 2001	11	16	16

Table B4 in the Appendix B presents the results of the estimation of the nonlinear deterministic unit root *AO* model and break identification with the selection criterion $\min(t_{\hat{r}})$. Large, in absolute values, Student-*t* statistics for the ‘break’ variable does not, in fact, indicate their significance. The selection of a break by successively estimating the model and searching for the best outcome resembles ‘data mining’ and this affects the true probability of type I error and causes selection bias. It should be noted, however, that the number of countries for which the unit root hypothesis can be rejected has increased to 11. However, the evidence of price stationarity is still weak; only for Zimbabwe and China it can be rejected at the 0.01 level of significance. For the remaining countries (Canada, Croatia, Estonia, Indonesia, Latvia, Luxemburg, Poland, Russia and South Africa) the level of significance has to be higher.

Concluding on non-normality of the series and overwhelming evidence for the existence of a deterministic unit root, it seems plausible to proceed into testing for unit root bilinearity. First, the bilinear parameter b in (5) is estimated using the *OLS* and the recursive Kalman Filter (*KF*) technique (see Hristova[4]), 2005). Unlike testing, consistency of the *OLS* and *KF* estimators of the parameter b has not been fully proven so far. Nevertheless, some theoretical support (see Lifshits, 2004) and a number of Monte Carlo experiments (available on request) show means squared error convergence for $0 < d < 1$, $d = s^{-1}b\sqrt{T}$ for $d \to 1$. The experiments also indicate that the *KF* estimates of b are more mean-square efficient than the *OLS*.

Table B5 in Appendix B contains the estimates obtained by the both methods. Unfortunately, for 25 countries the estimation of the covariance matrix of the *KF* estimates failed by all three of the methods applied (Hessian, cross-product of Jacobian and quasi-maximum likelihood). In this case the t-ratios have been computed using the estimated *KF* parameters and the *OLS* standard errors. Overall, for 53 series (*OLS*) and 52 series (*KF*) the hypothesis of no bilinearity has to be rejected, in most cases strongly. An unexpected outcome here was the fact that for 16 significant and negative *OLS* estimates of the bilinear coefficients 14 turned out to be positive and significant, when the *KF* method was applied. According to the *KF* results, only two series exhibit negative bilinearity.

Joint and conditional test results, *BSB* and *CB*, are given in Table B6 in Appendix B. The break variables, defined by (3), were regarded as known, and identified previously by the sequential search with the $\min(t_{\hat{r}})$ criterion. The poolability of the panel was evaluated in an *ad hoc* manner, by the analysis of covariances of the residuals of the estimated equation (8) for all countries. The fraction of significant, at the 0.05 level of significance, correlations is 0.14. There is, therefore, some distortion in the results due to the inter-sample dependence but it does not seem to affect the bulk of the results in a substantial way. The results show strong evidence for the dominance of the stochastic bilinearity in the unit root over the deterministic nonlinearity. For 12 countries the F test did not discover any significance of neither the break nor a bilinearity in the unit root. According to the *BSB* test results, the structural break seems to dominate over the unit root bilinearity in two cases, for Austria and Pakistan. The *CB* test results are less

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4) We are grateful to Daniela Hristova for sharing her code with us. She bears no responsibility of any errors here.

overwhelming. Here the bilinear unit root hypothesis is confirmed for 34 countries, in most cases, strongly.

6. Conclusions

Stochastic unit root modelling and, in particular, the bilinear unit root approach presented here, offers an attractive alternative to the traditional (deterministic) unit root analysis. The concept of the bilinear unit roots can substantially enrich the analysis traditionally conducted within the deterministic unit root framework. More specifically, the speculative bubble interpretation of the unit root bilinear processes and the computationally simple nature of its tests create an interesting tool for the analysis of ups and downs on financial markets. Results presented here also reveal that a substantial number of the empirical financial time series exhibits unit root bilinearity, which clearly dominates over simple forms of a deterministic structural break. Testing is feasible here and can be done without the need for developing specialised software. Finally the bilinear unit root process, being lowly parameterised, does not require specific assumptions or additional tests regarding the nature or timing of the structural breaks.

It is beyond this work to identify a pattern regarding the dependence of the nonlinear and bilinear characteristics on external economic characteristics. Nevertheless, it is interesting to note the relatively frequent absence of the unit root bilinearity for the most developed countries. Among the richest 25 countries (in terms of *GDP* per capita) 12 are in the group of 29 countries for which the bilinearity was not discovered. In the same group there are aspiring countries like China and Korea. Another 8 are in the group of 18 countries for which significant negative bilinearity was detected. Although the evidence is very weak and by no means conclusive, the hypothesis that the emerging rather than developed markets are prone to unit root bilinearity might be further investigated in the future.

Appendix A: Proofs of the theorems

Proof of the Theorem 1.

Proof of the Theorem 1 is based on the following Lemma:

Lemma. *Let the series* y_t *be generated by the process* (1) *with* $d_t \equiv 0$, $y_0 = 0$, $v_t = e_t$ *and* $r_t = 1$. *Denote the ratio of time of break T_B to the sample size T by* (11) *and assume that it is constant as* $T \to \infty$. *For the regression model* (7) *with B_t defined by* (3) *under the null hypothesis that* $b = g = 0$, *as* $T \to \infty$ *the OLS estimates of parameters in* (7) *have the following asymptotic:*

1) $\sqrt{T}\hat{c} \Rightarrow \frac{s}{l} W_1(l)$,

2) $T\hat{b} \Rightarrow \dfrac{\int_0^1 W_1(t)dW_2(t)}{s\int_0^1 W_1^2(t)}$,

3) $\sqrt{T}\hat{g} \Rightarrow \dfrac{s}{l(1-l)}[lW_1(1) - W_1(l)]$,

where $\Rightarrow$ *denotes weak convergence,* W_1, W_2 *are two independent Wiener processes on* [0,1].

Proof of the Lemma. Consider the data generating process (*DGP*) as described by the assumption of the Lemma, that is given by equation:

$$y_t = y_{t-1} + e_t,\ e_t \sim IID(0, s^2),\ y_0 = 0,\ t = 1,2, \ldots, T. \tag{A1}$$

For the parameters of the equation of interest (7) the usual *OLS* estimator is given as:

$$\left[\hat{c}, \hat{b}, \hat{g}\right]' = (X'X)^{-1} X'Y, \tag{A2}$$

where:

$$X = \begin{bmatrix} 1 & y_1\Delta y_1 & 0 \\ 1 & y_1\Delta y_1 & 0 \\ \ldots & \ldots & \ldots \\ 1 & y_{T_B-2}\Delta y_{T_B-2} & 0 \\ 1 & y_{T_B-1}\Delta y_{T_B-1} & 1 \\ \ldots & \ldots & \ldots \\ 1 & y_{T-1}\Delta y_{T-1} & 1 \end{bmatrix}, \quad \text{and} \quad Y = \begin{bmatrix} \Delta y_2 \\ \Delta y_3 \\ \ldots \\ \Delta y_T \end{bmatrix}, \tag{A3}$$

and T_B is a moment of a possible break. To derive the asymptotic distribution for the matrices $X'X$ and $X'Y$ (with X and Y defined by (A3)) under the *DGP* of (A1) and the null hypothesis of $b = g = 0$, let us apply the Donsker 's theorem (see e.g. Maddala and Kim 1998) and some results of Charemza, Lifshits and Makarova (2005), namely of the Statement, section 1 and Lemma C, sections (3) and (4), which give:

$$T^{-1}(XX')_{11} \Rightarrow 1, \quad T^{-1}(XX')_{12} \Rightarrow s^2\left(\int_0^1 W_1(t)dW_1(t)+1\right), \quad T^{-1}(XX')_{13} \Rightarrow 1-l,$$
$$T^{-2}(XX')_{22} \Rightarrow s^4\int_0^1 W_1^2(t)dt, \quad T^{-1}(XX')_{23} \Rightarrow s^2\left(\int_l^1 W_1(t)dW_1(t)+1-l\right), \quad T^{-1}(XX')_{31} \Rightarrow 1-l,$$

(A4)

and:

$$T^{-1/2}(X'Y)_{11} \Rightarrow s\,W(1), \quad T^{-1}(X'Y)_{21} \Rightarrow s^3\int_0^1 W_1(t)dW_2(t), \quad T^{-1/2}(X'Y)_{31} \Rightarrow s\,\big(W(1)-W(l)\big),$$

(A5)

where $(XX')_{ij}$ and $(X'Y)_{i1}$ ($i,j = 1,2,3$) are corresponding elements of matrices $X'X$ and $X'Y$.

Combination of (A2), (A4) and (A5) after some usual algebra complete the proof of the Lemma. •

Proof of the Theorem 1. The test statistics for $b = g = 0$ in (7) has a form of the *F*-test:

$$F = \frac{(RSS_{(R)} - RSS_{(UR)})/2}{RSS_{(UR)}/(T-3)}, \tag{A6}$$

where $RSS_{(R)}$ is sum of squared *OLS*-residuals from the restricted regression, that is:

$$\Delta y_t = c + w_t, \quad t = 2,3,\dots,T, \tag{A7}$$

and $RSS_{(UR)}$ is sum of squared *OLS*-residuals from the unrestricted regression, that is from (7). For the restricted model (A7) the sum of squared *OLS*-residuals are:

$$RSS_{(R)} = \sum_{t=2}^{T}\hat{w}_t^2 = \sum_{t=2}^{T}(\Delta y_t)^2 - T\Big(\frac{1}{T}\sum_{t=2}^{T}\Delta y_t\Big)^2, \tag{A8}$$

and, under the *DGP* of (A1) and the null hypothesis of $b = g = 0$, we get:

$$T\sum_{t=2}^{T}(\Delta y_t)^2 = T\sum_{t=2}^{T} e_t^2 \Rightarrow s^2 \quad , \tag{A9}$$

and:

$$T\left(\frac{1}{T}\sum_{t=2}^{T}\Delta y_t\right)^2 = \left(\frac{1}{\sqrt{T}}\sum_{t=2}^{T} e_t\right)^2 \Rightarrow [s\, W_1(1)]^2 . \tag{A10}$$

Sum of squared *OLS*-residuals from the unrestricted regression of (7) with the use of (A2) and (A3) may be decomposed as:

$$\begin{aligned} RSS_{(UR)} = \sum_{t=2}^{T} e_t^2 = \sum_{t=2}^{T}(\Delta y_t)^2 + T\hat{c} + \hat{b}^2\sum_{t=2}^{T}(y_{t-1}\Delta y_{t-1})^2 - 2\hat{c}\sum_{t=2}^{T}\Delta y_{t-1} \\ - 2\hat{b}\sum_{t=2}^{T} y_{t-1}\Delta y_{t-1}\Delta y_t + 2\hat{b}\hat{c}\sum_{t=2}^{T} y_{t-1}\Delta y_{t-1} - 2\hat{g}\sum_{t=T_B}^{T}\Delta y_{t-1} \\ + 2\hat{g}\,\hat{c}(T - T_B) + 2\hat{g}\hat{b}\sum_{t=T_B}^{T} y_{t-1}\Delta y_{t-1} + \hat{g}^2(T - T_B) . \end{aligned} \tag{A11}$$

From (A8), (A9) and (A10) we obtain that, under the null hypothesis and as $T \to \infty$, the denominator in (A6) will converge to s^2, that is:

$$(T-3)^{-1} RSS_{(UR)} = (T-3)^{-1}\left(\sum_{t=2}^{T}(e_t)^2 - T\left(\frac{1}{T}\sum_{t=2}^{T} e_t\right)^2\right) \Rightarrow s^2 . \tag{A12}$$

For the nominator in (A6), applying the Lemma for the decomposition (A11), we get:

$$RSS_{(R)} - RSS_{(UR)} \Rightarrow s^2 \frac{\left(\int_0^1 W_1(t)\,dW_2(t)\right)^2}{\int_0^1 W_1^2(t)\,dt} + s^2\left(\frac{\sqrt{l}\,W_1(1) - \frac{1}{\sqrt{l}}W_1(l)}{\sqrt{1-l}}\right)^2 . \tag{A13}$$

Combination of (A6), (A12) and (A13) complete the proof of Theorem 1. •

Proof of the Theorem 2.

Consider the data generating process (*DGP*) given by equation:

$$y_t = y_{t-1} + g\,P_t + e_t , \quad e_t \sim IID(0, s^2), \quad y_0 = 0 , \quad t = 1,2, \ldots, T , \tag{A14}$$

and denote cumulative sums of e_t as S_t, that is:

$$S_t = \sum_{k=1}^{t} e_k . \tag{A15}$$

Estimation of (9) by *OLS* gives:

$$\hat{g} = \Delta y_{T_B} . \tag{A16}$$

Under the *DGP* of (A14) we obtain that $\hat{g} = d + e_{T_B}$. Combining (9) and (A16) we obtain that:

$$\Delta z_t = \Delta y_t - \hat{g} \cdot P_t = \begin{cases} \Delta y_t, & t \neq T_B \\ 0, & t = T_B \end{cases},$$

which under the *DGP* of (A14) gives: $\Delta z_t = \begin{cases} e_t, & t \neq T_B \\ 0, & t = T_B \end{cases}$ and, further on:

$$z_t = \begin{cases} S_t, & t < T_B \\ S_{T_B - 1}, & t = T_B \\ S_t - e_{T_B}, & t > T_B \end{cases}, \tag{A17}$$

where S_t is defined by (A15).

To perform now marginal *b*-test for z_t estimate by *OLS* the following test equation:

$$\Delta z_t = \hat{b} z_{t-1} \Delta z_{t-1} + u_t \ . \tag{A18}$$

The *t*-ratio for parameter $\hat{b}$ is

$$t_{\hat{b}} = \frac{\hat{b}}{s.e.(\hat{b})} = \frac{\sum_{t=2}^{T} z_{t-1} \Delta z_{t-1} \Delta z_t}{\hat{s}_u \cdot \sqrt{\sum_{t=2}^{T} \left(z_{t-1} \Delta z_{t-1} \right)^2}} = \frac{A}{\hat{s}_u \sqrt{B}}, \tag{A19}$$

where $A = \sum_{t=2}^{T} z_{t-1} \Delta z_{t-1} \Delta z_t$, $B = \sum_{t=2}^{T} \left(z_{t-1} \Delta z_{t-1} \right)^2$ and $\hat{s}_u$ is consistent estimate of standard deviation of residuals in (A18). Consider the nominator and denominator in (A19) separately under the *DGP* of (A14) and the null of $b = 0$.

Nominator. Under the *DGP* of (A14), the null of $b = 0$ and with the use of (A17) we obtain:

$$\begin{aligned} A &= \sum_{t=1}^{T_B - 2} S_t e_t e_{t+1} + \sum_{t=T_B+1}^{T-1} \left(S_t - e_{T_B} \right) e_t e_{t+1} \\ &= \sum_{t=1}^{T-1} S_t e_t e_{t+1} - \left(S_{T_B - 1} e_{T_B - 1} e_{T_B} + S_{T_B} e_{T_B} e_{T_B + 1} + e_{T_B} \sum_{t=T_B+1}^{T-1} e_t e_{t+1} \right) \end{aligned} . \tag{A20}$$

In Charemza, Lifshits and Makarova (2005) at the Statement, section 1 it is shown that

$$T^{-1} \sum_{t=1}^{T-1} S_t e_t e_{t+1} \Rightarrow s^3 \int_0^1 W_1(t) dW_2(t), \tag{A21}$$

where W_1, W_2 are two independent Wiener processes on [0,1]. Applying Donsker` s theorem to the sum in round brackets of (A20) and bearing in mind that $\|e_t\| = s$ for each *t*, we obtain that:

$$\left(S_{T_B-1} e_{T_B-1} e_{T_B} + S_{T_B} e_{T_B} e_{T_B+1} + e_{T_B} \sum_{t=T_B+1}^{T-1} e_t e_{t+1} \right) = O(\sqrt{T}) . \tag{A22}$$

Combining (A20), (A21) and (A22) we get:

$$A = s^3 T \int_0^1 W_1(t) dW_2(t) + O(\sqrt{T}) . \tag{A23}$$

Denominator. Under the *DGP* of (A14), the null of $b = 0$ and with the use of (A17) we obtain:

$$\begin{aligned} B &= \sum_{t=1}^{T_B-1} S_t^2 e_t^2 + \sum_{t=T_B+1}^{T-1} \left(S_t - e_{T_B} \right)^2 e_t^2 \\ &= \sum_{t=1}^{T-1} S_t^2 e_t^2 - \left(S_{T_B}^2 e_{T_B}^2 + 2 e_{T_B} \sum_{t=T_B+1}^{T-1} S_t e_t^2 - e_{T_B}^2 \sum_{t=T_B+1}^{T-1} e_t^2 \right) \end{aligned} . \tag{A24}$$

Applying sections 3 and 4 of Lemma C from Charemza, Lifshits and Makarova (2005) we obtain that

$$\sum_{t=1}^{T-1} S_t^2 e_t^2 = T^2 s^4 \int_0^1 W_1^2(t) dt + O\left(T\sqrt{T} \right), \tag{A25}$$

and similarly to (A22):

$$\left(S_{T_B}^2 e_{T_B}^2 + 2 e_{T_B} \sum_{t=T_B+1}^{T-1} S_t e_t^2 - e_{T_B}^2 \sum_{t=T_B+1}^{T-1} e_t^2 \right) = O\left(T\sqrt{T} \right). \tag{A26}$$

As a result of (A24)-(A26) we have:

$$B = s_e^4 T^2 \int_0^1 W_1^2(t) dt + O(T\sqrt{T}) \tag{A27}$$

Combining (A19), (A23), (A27) and bearing in mind that under the null of $b = 0$ variances of error terms in (A14) and (A18) are equal to each other, we obtain the statement of Theorem 2. •

Appendix B. Empirical results

Table B1. Basic descriptive statistics for first differences of logs of share price indices

code	No.obs.	mean	st.dev	skewness		kurtosis	
				coef	p-value	coef	p-value
ARG	3383	0.000	0.008	0.154	0.000	6.834	0.000
AUS1	3383	0.000	0.003	-0.310	0.000	5.302	0.000
AUS2	3383	0.000	0.003	-0.611	0.000	5.469	0.000
BEL	3383	0.000	0.004	0.162	0.000	6.177	0.000
BRA	2788	0.000	0.007	0.269	0.000	12.088	0.000
BUL	1144	0.001	0.009	-0.451	0.000	27.665	0.000
CAN	3383	0.000	0.004	-0.668	0.000	6.724	0.000
CHIL	3383	0.000	0.004	0.173	0.000	4.441	0.000
CHIN	3383	0.000	0.010	1.569	0.000	20.787	0.000
COL	3383	0.000	0.004	0.091	0.030	16.124	0.000
CROA	2135	0.000	0.008	0.120	0.024	12.371	0.000
CYPR	3186	0.000	0.007	3.197	0.000	66.090	0.000
CZE	2957	0.000	0.006	1.984	0.000	34.551	0.000
DEN	3383	0.000	0.004	-0.408	0.000	8.733	0.000
EGY	2658	0.000	0.006	0.837	0.000	444.996	0.000
EST	2288	0.000	0.008	-1.210	0.000	20.027	0.000
FIN	3383	0.000	0.009	-0.456	0.000	7.126	0.000
FRA	3383	0.000	0.005	-0.155	0.000	2.890	0.000
GER	3383	0.000	0.005	-0.347	0.000	3.218	0.000
GRE	3383	0.000	0.007	0.003	0.946	4.239	0.000
HK	3383	0.000	0.007	-0.058	0.169	9.211	0.000
HUN	3383	0.000	0.007	-0.746	0.000	14.375	0.000
ICE	3180	0.000	0.003	-0.235	0.000	8.764	0.000
INDIA	3383	0.000	0.007	-0.842	0.000	13.862	0.000
INDO	3383	0.000	0.008	0.112	0.008	9.082	0.000
IRE	3383	0.000	0.004	-0.408	0.000	5.844	0.000
ISR	3179	0.000	0.006	-0.338	0.000	4.505	0.000
ITA	3383	0.000	0.006	-0.130	0.002	2.476	0.000
JAP	3383	0.000	0.005	0.055	0.193	3.164	0.000
KOR	3383	0.000	0.009	0.078	0.066	3.623	0.000
LAT	2187	0.000	0.009	-0.875	0.000	13.082	0.000
LEBAN	2383	0.000	0.005	-0.076	0.131	7.808	0.000
LIT	2399	0.000	0.005	-1.672	0.000	64.688	0.000
LUX	3383	0.000	0.004	0.107	0.011	10.339	0.000
MAL	3383	0.000	0.007	0.618	0.000	37.222	0.000
MAU	3383	0.000	0.002	0.260	0.000	15.236	0.000
MEX	3383	0.000	0.006	0.051	0.224	4.630	0.000
MOR	3383	0.000	0.003	-0.252	0.000	37.934	0.000
NETH	3383	0.000	0.005	-0.250	0.000	4.513	0.000
NOR	3383	0.000	0.005	-0.152	0.000	5.592	0.000
NZEL	3383	0.000	0.004	-0.914	0.000	21.643	0.000
PAK	3300	0.000	0.008	-0.333	0.000	17.868	0.000
PER	2918	0.000	0.005	0.314	0.000	12.080	0.000
PHI	3383	0.000	0.006	0.724	0.000	11.534	0.000
POL	2877	0.000	0.008	-0.177	0.000	5.962	0.000
POR	3383	0.000	0.004	-0.562	0.000	7.956	0.000
ROM	2155	0.001	0.011	1.775	0.000	53.817	0.000
RUS	2798	0.000	0.012	0.394	0.000	20.667	0.000
SAFR	3383	0.000	0.005	-1.164	0.000	14.595	0.000
SING	3383	0.000	0.005	0.017	0.695	6.263	0.000
SLOVE	1593	0.000	0.004	0.775	0.000	7.940	0.000
SPA	3383	0.000	0.005	-0.248	0.000	2.739	0.000
SRIL	3383	0.000	0.006	-2.753	0.000	96.220	0.000
SUE	3383	0.000	0.004	-0.343	0.000	4.694	0.000

code	No.obs.	mean	st.dev	skewness		kurtosis	
				coef	p-value	coef	p-value
SWE	3383	0.000	0.006	0.127	0.003	4.092	0.000
TAI	3383	0.000	0.007	0.069	0.100	2.244	0.000
THA	3383	0.000	0.008	0.529	0.000	4.904	0.000
TUN	1876	0.000	0.002	0.973	0.000	13.327	0.000
TUR	3383	0.001	0.013	-0.038	0.370	3.481	0.000
UK	3383	0.000	0.004	-0.174	0.000	3.438	0.000
US	3383	0.000	0.004	-0.133	0.002	4.278	0.000
VEN	3383	0.000	0.009	0.830	0.000	14.328	0.000
ZIMB	3383	0.001	0.021	-0.361	0.000	1347.234	0.000

Table B2. Stable distribution estimates

code	No.obs.	*alpha*	sd(*alpha*)	*beta*	sd(*beta*)
ARG	3383	1.462	0.033	-0.048	0.057
AUS1	3383	1.684	0.045	0.071	0.096
AUS2	3383	1.565	0.038	-0.024	0.072
BEL	3383	1.467	0.034	-0.141	0.056
BRA	2788	1.480	0.037	0.011	0.065
BUL	1144	1.191	0.049	0.136	0.085
CAN	3383	1.523	0.036	-0.097	0.062
CHIL	3383	1.637	0.043	0.229	0.089
CHIN	3383	1.331	0.031	0.091	0.053
COL	3383	1.248	0.029	0.090	0.051
CROA	2135	1.285	0.037	0.049	0.066
CYPR	3186	1.112	0.027	0.042	0.052
CZE	2957	1.431	0.035	-0.006	0.061
DEN	3383	1.493	0.034	-0.041	0.058
EGY	2658	1.222	0.033	0.164	0.055
EST	2288	1.325	0.037	0.016	0.066
FIN	3383	1.486	0.034	0.004	0.059
FRA	3383	1.547	0.037	-0.104	0.068
GER	3383	1.507	0.035	-0.094	0.059
GRE	3383	1.466	0.034	0.094	0.057
HK	3383	1.451	0.033	-0.021	0.058
HUN	3383	1.425	0.033	0.111	0.055
ICE	3180	1.381	0.033	0.063	0.057
INDIA	3383	1.453	0.033	-0.002	0.058
INDO	3383	1.375	0.031	0.002	0.056
IRE	3383	1.514	0.035	0.018	0.062
ISR	3179	1.586	0.040	-0.046	0.078
ITA	3383	1.528	0.036	-0.010	0.065
JAP	3383	1.556	0.037	0.029	0.070
KOR	3383	1.443	0.033	0.025	0.057
LAT	2187	1.206	0.035	0.065	0.063
LEBAN	2383	1.190	0.033	0.056	0.061
LIT	2399	1.355	0.037	0.074	0.064
LUX	3383	1.352	0.031	0.041	0.054
MAL	3383	1.383	0.032	-0.031	0.056
MAU	3383	1.055	0.027	0.144	0.048
MEX	3383	1.532	0.036	0.099	0.064
MOR	3383	1.113	0.027	0.071	0.050
NETH	3383	1.470	0.034	-0.128	0.056
NOR	3383	1.588	0.039	-0.052	0.076
NZEL	3383	1.596	0.040	0.028	0.078
PAK	3300	1.328	0.031	-0.012	0.055
PER	2918	1.350	0.034	0.097	0.057
PHI	3383	1.481	0.034	0.025	0.058

code	No.obs.	*alpha*	sd(alpha)	*beta*	sd(beta)
POL	2877	1.433	0.035	0.007	0.062
POR	3383	1.392	0.032	0.041	0.056
ROM	2155	1.277	0.038	0.117	0.064
RUS	2798	1.371	0.034	-0.006	0.061
SAFR	3383	1.582	0.039	0.054	0.075
SING	3383	1.480	0.034	-0.016	0.059
SLOVE	1593	1.395	0.047	0.089	0.080
SPA	3383	1.594	0.040	-0.096	0.078
SRIL	3383	1.308	0.030	0.041	0.053
SUE	3383	1.538	0.037	-0.157	0.065
SWE	3383	1.590	0.039	-0.033	0.077
TAI	3383	1.420	0.033	0.095	0.055
THA	3383	1.472	0.034	0.060	0.057
TUN	1876	1.467	0.045	0.070	0.077
TUR	3383	1.581	0.039	0.111	0.075
UK	3383	1.550	0.037	-0.068	0.069
US	3383	1.432	0.033	-0.072	0.056
VEN	3383	1.271	0.029	0.035	0.052
ZIMB	3383	1.178	0.029	0.156	0.049

Table B3a. Deterministic unit root results, levels

	RMK		MZ_a^{OLS}		ADF^{OLS}
code	**signif.**	**lags**	**signif.**	**lags**	**signif.**
ARG	(0)	10	(0)	10	(0)
AUS1	(0)	0	(0)	6	(0)
AUS2	(0)	11	(0)	16	(0)
BEL	(0)	29	(0)	16	(0)
BRA	(0)	36	(0)	12	(0)
BUL	(0)	24	(0)	16	(2)
CAN	(0)	33	(0)	4	(0)
CHIL	(0)	36	(0)	5	(0)
CHIN	(0)	27	(0)	27	(0)
COL	(0)	6	(0)	12	(0)
CROA	(0)	35	(0)	10	(0)
CYPR	(0)	30	(0)	27	(0)
CZE	(0)	15	(0)	15	(0)
DEN	(0)	11	(0)	4	(0)
EGY	(0)	5	(0)	5	(0)
EST	(0)	30	(0)	15	(0)
FIN	(0)	25	(0)	1	(0)
FRA	(0)	15	(0)	15	(0)
GER	(0)	24	(0)	14	(0)
GRE	(0)	1	(0)	2	(0)
HK	(0)	3	(0)	7	(1)
HUN	(0)	12	(0)	13	(0)
ICE	(0)	20	(0)	20	(0)
INDI	(0)	31	(0)	14	(2)
INDO	(0)	22	(0)	25	(0)
IRE	(0)	35	(0)	1	(0)
ISR	(0)	10	(0)	12	(0)
ITA	(0)	20	(0)	22	(0)
JAP	(0)	25	(0)	6	(0)
KOR	(0)	29	(0)	5	(0)
LAT	(0)	30	(0)	16	(0)
LEBA	(0)	23	(0)	1	(0)
LIT	(0)	31	(0)	25	(0)
LUX	(0)	16	(0)	16	(0)
MAL	(0)	15	(0)	20	(0)

code	RMK signif.	RMK lags	MZ_a^{OLS} signif.	MZ_a^{OLS} lags	ADF^{OLS} signif.
MAU	(0)	25	(0)	15	(0)
MEX	(0)	31	(0)	2	(0)
MOR	(0)	34	(0)	13	(0)
NETH	(0)	18	(0)	20	(0)
NOR	(0)	25	(0)	4	(0)
NZEL	(0)	20	(0)	2	(1)
PAK	(0)	35	(0)	3	(0)
PER	(0)	28	(0)	19	(0)
PHI	(0)	12	(0)	12	(1)
POL	(0)	17	(0)	13	(1)
POR	(0)	32	(0)	25	(0)
ROM	(0)	35	(0)	16	(0)
RUS	(0)	10	(0)	1	(0)
SAFR	(0)	8	(2)	2	(2)
SING	(0)	34	(0)	1	(0)
SLOV	(0)	25	(0)	6	(0)
SPA	(0)	1	(0)	3	(0)
SRIL	(0)	35	(0)	7	(0)
SUE	(0)	30	(0)	22	(0)
SWE	(0)	24	(0)	15	(0)
TAI	(0)	13	(0)	15	(0)
THA	(0)	30	(0)	15	(0)
TUN	(0)	20	(0)	20	(0)
TUR	(0)	1	(0)	5	(0)
UK	(0)	18	(0)	19	(0)
US	(0)	35	(0)	28	(0)
VEN	(0)	28	(0)	18	(0)
ZIMB	(0)	3	(0)	13	(0)

Description of symbols in Tables B3a and B3b:
Significance:
(3): significance at 0.01 level
(2): significance at 0.05 level
(1): significance at 0.10 level

lags:
maximum number of lags selected by the general to specific methodology at 0.05 level of significance (*RMK* test) or Ng and Perron (2001) selection criteria (other tests)

Table B3b. Deterministic unit root results, returns

code	RMK signif.	RMK lags	MZ_a^{OLS} signif.	MZ_a^{OLS} lags	ADF^{OLS} signif.
ARG	(3)	0	(3)	28	(3)
AUS1	(3)	0	(3)	27	(3)
AUS2	(3)	0	(3)	28	(3)
BEL	(3)	0	(3)	28	(3)
BRA	(3)	35	(3)	27	(3)
BUL	(3)	0	(3)	21	(3)
CAN	(3)	0	(3)	27	(3)
CHIL	(3)	35	(3)	28	(3)
CHIN	(3)	21	(3)	20	(3)
COL	(3)	5	(3)	27	(3)
CROA	(3)	0	(3)	25	(3)
CYPR	(3)	9	(3)	15	(3)
CZE	(3)	9	(3)	26	(3)
DEN	(3)	0	(3)	28	(3)

	RMK		MZ_a^{OLS}		ADF^{OLS}
code	**signif.**	**lags**	**signif.**	**lags**	**signif.**
EGY	(3)	9	(2)	27	(3)
EST	(3)	8	(3)	22	(3)
FIN	(3)	0	(3)	28	(3)
FRA	(3)	0	(3)	28	(3)
GER	(3)	0	(3)	28	(3)
GRE	(3)	0	(3)	28	(3)
HK	(3)	0	(3)	24	(3)
HUN	(3)	35	(3)	19	(3)
ICE	(3)	19	(3)	28	(3)
INDI	(3)	0	(3)	28	(3)
INDO	(3)	34	(3)	28	(3)
IRE	(3)	0	(3)	28	(3)
ISR	(3)	0	(3)	28	(3)
ITA	(3)	0	(3)	28	(3)
JAP	(3)	0	(3)	28	(3)
KOR	(3)	0	(3)	28	(3)
LAT	(3)	0	(3)	25	(3)
LEBA	(3)	0	(3)	26	(3)
LIT	(3)	4	(3)	16	(3)
LUX	(3)	6	(3)	28	(3)
MAL	(3)	14	(3)	25	(3)
MAU	(3)	36	(3)	27	(3)
MEX	(3)	0	(3)	27	(3)
MOR	(3)	4	(3)	27	(3)
NETH	(3)	0	(3)	28	(3)
NOR	(3)	0	(3)	27	(3)
NZEL	(3)	0	(3)	21	(3)
PAK	(3)	0	(3)	25	(3)
PER	(3)	0	(3)	27	(3)
PHI	(3)	0	(3)	17	(3)
POL	(3)	0	(3)	12	(3)
POR	(3)	3	(3)	28	(3)
ROM	(3)	0	(3)	25	(3)
RUS	(3)	0	(3)	27	(3)
SAFR	(3)	0	(3)	27	(3)
SING	(3)	35	(3)	28	(3)
SLOV	(3)	0	(3)	22	(3)
SPA	(3)	0	(3)	28	(3)
SRIL	(3)	0	(3)	14	(3)
SUE	(3)	0	(3)	14	(3)
SWE	(3)	33	(3)	28	(3)
TAI	(3)	0	(3)	28	(3)
THA	(3)	0	(3)	28	(3)
TUN	(3)	19	(3)	18	(3)
TUR	(3)	0	(3)	13	(3)
UK	(3)	0	(3)	28	(3)
US	(3)	33	(3)	28	(3)
VEN	(3)	0	(3)	27	(3)
ZIMB	(3)	6	(3)	28	(3)

Table B4. Results of deterministic unit root testing, AO, Model 1

Break selection criterion: $\min(t_{\hat{r}})$

code	No.obs	$t_{\hat{r}}$	Snfce $t_{\hat{r}}$	Break date	$t_{\hat{q}}$
ARG	3383	-3.80	(0)	28/03/03	38.21
AUS1	3383	-4.23	(0)	08/03/02	-70.84
AUS2	3383	-3.06	(0)	19/11/03	54.05
BEL	3383	-3.17	(0)	12/02/01	-55.18
BRA	2788	-3.57	(0)	25/11/98	-8.18
BUL	1144	-3.79	(0)	19/12/02	17.81
CAN	3383	-4.60	(1)	07/11/00	-48.31
CHIL	3383	-3.20	(0)	02/09/97	-34.41
CHIN	3383	-5.47	(3)	20/03/96	57.94
COL	3383	-3.09	(0)	22/10/97	-35.45
CROA	2135	-5.03	(2)	23/03/98	-50.79
CYPR	3186	-3.22	(0)	25/08/00	-29.51
CZE	2957	-2.57	(0)	03/07/03	40.23
DEN	3383	-4.07	(0)	13/07/01	-61.85
EGY	2658	-1.87	(0)	17/11/00	-29.44
EST	2288	-4.78	(1)	02/04/98	-48.17
FIN	3383	-3.62	(0)	25/09/00	-34.11
FRA	3383	-3.70	(0)	23/05/01	-52.43
GER	3383	-3.87	(0)	16/05/01	-70.25
GRE	3383	-3.74	(0)	31/07/00	-38.88
HK	3383	-3.72	(0)	26/01/01	-39.17
HUN	3383	-3.65	(0)	29/12/95	61.05
ICE	3180	-3.51	(0)	10/03/00	-38.54
INDIA	3383	-3.43	(0)	09/06/03	31.89
INDO	3383	-4.60	(1)	04/02/00	-57.16
IRE	3383	-3.69	(0)	11/07/01	-71.80
ISR	3179	-3.15	(0)	27/02/01	-24.11
ITA	3383	-3.64	(0)	04/06/01	-59.97
JAP	3383	-3.68	(0)	07/05/01	-41.94
KOR	3383	-3.73	(0)	30/04/96	-28.76
LAT	2187	-5.41	(2)	15/05/98	-84.36
LEBAN	2383	-2.99	(0)	03/03/04	51.04
LIT	2399	-3.32	(0)	03/04/03	62.53
LUX	3383	-4.61	(1)	28/08/00	-46.13
MAL	3383	-4.56	(0)	23/07/97	-49.82
MAU	3383	-3.49	(0)	02/09/98	-22.45
MEX	3383	-4.32	(0)	01/02/00	-24.91
MOR	3383	-2.99	(0)	11/11/99	-42.45
NETH	3383	-3.41	(0)	08/06/01	-71.28
NOR	3383	-3.46	(0)	13/11/00	-49.81
NZEL	3383	-4.13	(0)	16/04/98	-35.42
PAK	3300	-2.63	(0)	06/03/03	54.86
PER	2918	-3.17	(0)	29/05/98	-33.06
PHI	3383	-3.39	(0)	16/06/99	-27.20
POL	2877	-4.83	(2)	27/12/95	39.01
POR	3383	-3.51	(0)	16/01/01	-62.37
ROM	2155	-3.30	(0)	20/04/98	-33.05
RUS	2798	-4.95	(2)	20/04/98	-55.60
SAFR	3383	-5.09	(2)	22/09/93	36.85
SING	3383	-2.74	(0)	13/02/97	-18.27
SLOVE	1593	-3.71	(0)	28/01/00	-30.93
SPA	3383	-3.73	(0)	29/01/01	-62.16
SRIL	3383	-3.60	(0)	13/08/01	37.01
SUE	3383	-3.61	(0)	21/05/01	-75.53

code	No.obs	$t_{\hat{r}}$	Snfce $t_{\hat{r}}$	Break date	$t_{\hat{q}}$
SWE	3383	-4.24	(0)	12/01/01	-65.62
TAI	3383	-4.47	(0)	10/08/00	-62.88
THA	3383	-3.79	(0)	27/01/97	-50.49
TUN	1876	-3.09	(0)	10/12/98	27.47
TUR	3383	-3.70	(0)	07/11/00	-57.25
UK	3383	-3.80	(0)	07/02/01	-65.75
US	3383	-4.19	(0)	15/11/00	-61.01
VEN	3383	-3.41	(0)	13/04/98	-34.57
ZIMB	3383	-7.01	(3)	16/12/02	69.65

Description of symbols column 4:
Significance:
(3): significance at 0.01 level
(2): significance at 0.05 level
(1): significance at 0.10 level

Table B5. Estimates of the bilinear parameter in unit root model

		OLS			*ML* Kalman Filter	
country	**No.obs.**	***b*-test**	**signif.**	**max.aug.**	***b*-test**	**signif**
ARG	3383	7.774	(3)	27	7.127	(3)
AUS1	3383	-2.928	(3)	1	7.086	(3)
AUS2	3383	4.813	(3)	20	7.123	(3)
BEL	3383	9.981	(3)	34	7.125	(3)
BRA	2788	5.746	(3)	36	6.301	(3)
BUL	1144	4.821	(3)	24	-0.324	(0)
CAN	3383	-2.773	(3)	33	0.318	(0)
CHIL	3383	16.256	(3)	36	17.456	(3)
CHIN	3383	1.302	(0)	32	1.645.	(1)
COL	3383	-2.885	(3)	6	17.458	(3)
CROA	2135	-3.465	(3)	35	-0.489	(0)
CYPR	3186	11.180	(3)	30	12.347	(3)
CZE	2957	4.703	(3)	15	714.200 .	(3)
DEN	3383	-3.235	(3)	36	10.062	(3)
EGY	2658	-12.466	(3)	8	-8.729	(3)
EST	2288	-1.367	(0)	30	11.269	(3)
FIN	3383	-3.997	(3)	29	5.302	(3)
FRA	3383	1.883	(3)	15	2.004.	(2)
GER	3383	3.332	(3)	26	3.450	(3)
GRE	3383	8.187	(3)	27	10.079	(3)
HK	3383	3.027	(3)	36	3.223	(3)
HUN	3383	-7.087	(3)	36	7.121	(3)
ICE	3180	4.819	(3)	20	7.116	(3)
INDIA	3383	7.453	(3)	31	7.136	(3)
INDO	3383	7.773	(3)	35	7.127	(3)
IRE	3383	-3.456	(3)	35	7.122	(3)
ISR	3179	2.233	(3)	10	10.115	(3)
ITA	3383	2.169	(3)	22	2.108	(2)
JAP	3383	4.134	(3)	25	4.695	(3)
KOR	3383	2.774	(3)	30	3.003	(3)
LAT	2187	3.958	(3)	30	7.117	(3)
LEBAN	2383	4.549	(3)	23	5.044	(3)
LIT	2399	7.084	(3)	31	12.340	(3)
LUX	3383	7.139	(3)	16	7.130	(3)
MAL	3383	5.290	(3)	15	0.328	(0)
MAU	3383	4.265	(3)	15	10.079	(3)
MEX	3383	9.062	(3)	31	7.127	(3)
MOR	3383	4.785	(3)	34	4.710	(3)
NETH	3383	1.386	(0)	35	10.088	(3)

country	No.obs.	*OLS* *b*-test	signif.	max.aug.	*ML* Kalman Filter *b*-test	signif
NOR	3383	3.449	(3)	25	3.551	(3)
NZEL	3383	0.083	(0)	20	0.130	(0)
PAK	3300	0.757	(0)	35	0.813	(0)
PER	2918	-2.053	(2)	28	5.044	(3)
PHI	3383	10.961	(3)	12	10.091	(3)
POL	2877	-2.778	(3)	13	7.136	(3)
POR	3383	-2.312	(2)	32	7.127	(3)
ROM	2155	4.692	(3)	35	7.096	(3)
RUS	2798	1.516	(0)	23	1.642	(0)
SAFR	3383	7.933	(3)	36	12.336	(3)
SING	3383	5.163	(3)	36	5.302	(3)
SLOVE	1593	-2.696	(3)	25	15.529	(3)
SPA	3383	-2.350	(3)	29	0.254	(0)
SRIL	3383	-1.532	(0)	35	10.069	(3)
SUE	3383	3.228	(3)	30	7.127	(3)
SWE	3383	-4.004	(3)	35	0.342	(0)
TAI	3383	0.905	(0)	21	4.287	(3)
THA	3383	6.608	(3)	33	6.470	(0)
TUN	1876	-6.487	(3)	20	10.693	(3)
TUR	3383	-5.200	(3)	10	9.813	(0)
UK	3383	1.537	(0)	33	2.224	(2)
US	3383	0.434	(0)	35	0.701	(0)
VEN	3383	4.137	(3)	28	12.322	(3)
ZIMB	3383	29.871	(3)	1	-23.607	(3)

Description of symbols: as in earlier tables

Underlined are such KF test results (and their significance) where the *t*-ratios were computed for the KF estimates of parameters and the OLS standard errors.

Table B6. Joint and conditional tests results

	Joint model						Conditional model		
Code	***t*(*b*)**	**Snfce**	***t*(Break)**	**Snfce**	***F*-test**	**Snfce**	***b*-test**	***t*-stat**	**Snfce**
ARG	7.75	(3)	0.884	(0)	30.568	(3)	-0.05	-0.388	(0)
AUS1	-2.93	(3)	0.204	(0)	4.305	(2)	-0.43	-2.892	(3)
AUS2	4.64	(3)	2.907	(3)	15.846	(3)	-0.18	-0.862	(0)
BEL	9.98	(3)	-0.014	(0)	49.661	(3)	0.00	-0.031	(0)
BRA	5.74	(3)	0.433	(0)	16.556	(3)	0.36	5.960	(3)
BUL	4.86	(3)	2.909	(3)	15.980	(3)	0.61	5.295	(3)
CAN	-2.77	(3)	-0.109	(0)	3.842	(2)	-0.33	-2.810	(3)
CHIL	16.25	(3)	-0.404	(0)	131.977	(3)	-0.13	-0.842	(0)
CHIN	1.30	(0)	-0.461	(0)	0.949	(0)	0.03	1.021	(0)
COL	-2.91	(3)	0.425	(0)	4.244	(2)	-0.31	-2.796	(3)
CROA	-3.48	(3)	1.003	(0)	6.486	(3)	-0.49	-3.693	(3)
CYPR	11.17	(3)	-0.870	(0)	62.615	(3)	0.03	0.653	(0)
CZE	4.68	(3)	1.125	(0)	11.677	(3)	-0.20	-1.455	(0)
DEN	-3.23	(3)	-0.110	(0)	5.228	(3)	-0.35	-3.595	(3)
EGY	-12.59	(3)	2.069	(2)	79.761	(3)	-1.30	-12.576	(3)
EST	-1.37	(0)	0.198	(0)	0.947	(0)	-0.18	-1.562	(0)
FIN	-3.98	(3)	-1.071	(0)	8.544	(3)	-0.19	-4.074	(3)
FRA	1.88	(1)	-0.536	(0)	1.915	(0)	0.07	1.380	(0)
GER	3.32	(3)	-0.661	(0)	5.755	(3)	0.17	3.091	(3)
GRE	8.18	(3)	-0.438	(0)	33.574	(3)	0.03	0.499	(0)
HK	3.03	(3)	-0.176	(0)	4.590	(2)	0.17	3.190	(3)
HUN	-7.08	(3)	0.057	(0)	25.052	(3)	-0.45	-7.039	(3)
ICE	4.79	(3)	0.117	(0)	11.586	(3)	0.22	7.211	(3)
INDIA	7.42	(3)	1.119	(0)	28.345	(3)	-0.06	-0.511	(0)
INDO	7.77	(3)	0.129	(0)	30.106	(3)	-0.03	-0.201	(0)

Code	Joint model $t(b)$	Snfce	t(Break)	Snfce	F-test	Snfce	Conditional model b-test	t-stat	Snfce
IRE	-3.44	(3)	-0.774	(0)	6.259	(3)	-0.31	-3.630	(3)
ISR	2.23	(2)	0.290	(0)	2.533	(1)	0.11	1.309	(0)
ITA	2.16	(2)	-0.481	(0)	2.463	(1)	-0.18	-2.024	(3)
JAP	4.13	(3)	0.055	(0)	8.537	(3)	0.30	1.324	(0)
KOR	2.77	(3)	-0.073	(0)	3.842	(2)	-0.06	-0.541	(0)
LAT	3.96	(3)	0.408	(0)	7.877	(3)	0.47	4.020	(3)
LEBAN	4.56	(3)	1.951	(1)	12.234	(3)	0.67	4.545	(3)
LIT	6.97	(3)	1.649	(1)	26.355	(3)	0.92	7.163	(3)
LUX	7.16	(3)	-0.967	(0)	25.897	(3)	0.56	6.978	(3)
MAL	5.29	(3)	-0.402	(0)	14.042	(3)	0.62	6.729	(3)
MAU	5.18	(3)	-3.836	(3)	16.763	(3)	0.56	4.193	(3)
MEX	9.06	(3)	0.154	(0)	41.015	(3)	-0.08	-0.989	(0)
MOR	4.75	(3)	-1.407	(0)	12.416	(3)	0.18	4.887	(3)
NETH	1.30	(0)	-1.047	(0)	1.417	(0)	0.05	1.405	(0)
NOR	3.45	(3)	0.176	(0)	5.954	(3)	-0.13	-1.427	(0)
NZEL	0.06	(0)	-1.021	(0)	0.523	(0)	0.00	0.015	(0)
PAK	0.65	(0)	1.994	(2)	2.248	(0)	-0.03	-0.295	(0)
PER	-2.05	(2)	0.012	(0)	2.100	(0)	-0.28	-1.784	(3)
PHI	10.90	(3)	-1.593	(0)	61.328	(3)	-0.12	-0.795	(0)
POL	-2.77	(3)	0.563	(0)	4.011	(2)	-0.32	-2.736	(3)
POR	-2.30	(2)	-0.593	(0)	2.839	(1)	-0.26	-2.410	(3)
ROM	4.69	(3)	-0.410	(0)	11.052	(3)	0.19	4.719	(3)
RUS	1.52	(0)	0.072	(0)	1.150	(0)	0.04	1.362	(0)
SAFR	7.93	(3)	0.108	(0)	31.386	(3)	0.08	0.732	(0)
SING	5.16	(3)	-0.537	(0)	13.452	(3)	0.08	0.437	(0)
SLOVE	-2.72	(3)	0.567	(0)	3.782	(2)	-0.31	-2.441	(3)
SPA	-2.34	(2)	-0.401	(0)	2.836	(1)	-0.21	-2.417	(3)
SRIL	-1.58	(0)	1.161	(0)	1.858	(0)	-0.16	-1.491	(0)
SUE	3.21	(3)	-0.900	(0)	5.605	(3)	0.09	2.582	(3)
SWE	-4.00	(3)	-0.632	(0)	8.191	(3)	-0.25	-4.062	(3)
TAI	0.90	(0)	-0.590	(0)	0.582	(0)	0.03	0.493	(0)
THA	6.61	(3)	-0.116	(0)	21.778	(3)	0.10	0.983	(0)
TUN	-6.48	(3)	-0.304	(0)	21.018	(3)	-3.00	-6.548	(3)
TUR	-5.16	(3)	-0.846	(0)	13.864	(3)	-0.11	-5.280	(3)
UK	1.53	(0)	-0.758	(0)	1.465	(0)	0.08	1.507	(0)
US	0.42	(0)	-0.893	(0)	0.492	(0)	0.01	0.316	(0)
VEN	4.14	(3)	0.020	(0)	8.539	(3)	0.20	4.196	(3)
ZIMB	29.78	(3)	-0.145	(0)	445.743	(3)	0.87	29.918	(3)

Description of symbols: as in earlier tables.

Table B7. Country Codes and abbreviations

(codes for countries which belong to the *G25* countries are marked by *)

ARGENTINA	ARG
AUSTRALIA	AUS1*
AUSTRIA	AUS2*
BELGIUM	BEL*
BRAZIL	BRA
BULGARIA	BUL
CANADA	CAN*
CHILE	CHIL
CHINA	CHIN
COLUMBIA	COL
CROATIA	CROA
CYPR	CYPR*
CZECH REPUBLIC	CZE

DENMARK	DEN*
EGYPT	EGY
ESTONIA	EST
FINLAND	FIN*
FRANCE	FRA*
GERMANY	GER*
GREECE	GRE
HONG KONG	HK*
HUNGARY	HUN
ICELAND	ICE*
INDIA	INDIA
INDONESIA	INDO
IRELAND	IRE*
ISRAEL	ISR*
ITALY	ITA*
JAPAN	JAP*
KOREA, REPUBLIC OF	KOR
LATVIA	LAT
LEBANON	LEBAN
LITHUANIA	LIT
LUXEMBOURG	LUX*
MALAYSIA	MAL
MAURITIUS	MAU
MEXICO	MEX
MOROCCO	MOR
NETHERLANDS	NETH*
NORWAY	NOR*
NEW ZEALAND	NZEL*
PAKISTAN	PAK
PERU	PER
PHILIPPINES	PHI
POLAND	POL
PORTUGAL	POR
ROMANIA	ROM
RUSSIA	RUS
SOUTH AFRICA	SAFR
SINGAPORE	SING*
SLOVENIA	SLOVE
SPAIN	SPA*
SRI LANKA	SRIL
SWITZERLAND	SUE*
SWEDEN	SWE*
TAIWAN, PROVINCE OF CHINA	TAI
THAILAND	THA
TUNISIA	TUN
TURKEY	TUR
UNITED KINGDOM	UK*
UNITED STATES	US*
VENEZUELA	VEN
ZIMBABWE	ZIMB

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