Measuring the NAIRU with Reduced Uncertainty: A Multiple Indicator-Common Component Approach

by

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Abstract:

Standard estimates of the NAIRU or natural rate of unemployment are subject to considerable uncertainty. We show in this paper that using multiple indicators to extract an estimated NAIRU cuts in half uncertainty as measured by variance. The inclusion of an Okun's Law relation is particularly valuable. We estimate the NAIRU as an unobserved component in a state-space model and show that using multiple indicators reduces both parametric uncertainty and filtering uncertainty. Additionally, our multivariate approach overcomes the "pile-up" problem observed by other investigators.

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1. Introduction

A prerequisite for the conduct of countercyclical macroeconomic policy is to know where we are in the cycle – loosely, are we above or below the NAIRU? Measuring the natural rate and the corresponding cyclical fluctuations of the US economy with reasonable precision poses a significant challenge. Our aim in this paper is to reduce uncertainty about the NAIRU by employing a multiple indicator approach. The essential notion is that the concept of "a business cycle" is meaningful in that there is a single "gap" which drives cyclical behavior across sectors and across variables. By employing a number of indicators jointly we are able to significantly improve the precision of estimates of the NAIRU, reducing uncertainty by about half.

While our primary interest is in reducing uncertainty about the NAIRU, we confront two related issues along the way. The first issue we discuss is the need for care in measuring uncertainty for a target, such as the NAIRU, which is itself unobserved. For our purposes the resolution is to use standard models from the literature as benchmarks. The second issue is the so-called "pile-up" problem, where the Kalman filter puts too little weight on the variance of the permanent component. Where the goal is to find point estimates of the NAIRU as an unobserved component, the pile-up problem is an annoyance that has been dealt with by imposing reasonable values on the variance parameters. This solution is unsatisfactory when the goal is to measure uncertainty, because picking a value for the variance comes too close to picking a value for total uncertainty. Fortunately, our multivariate approach seems to eliminate the pile-up problem.

We treat the NAIRU as an unobserved component to be estimated by the Kalman filter. Staiger, Stock and Watson (1997a, hereafter SSW (1997a)) points out three sources of uncertainty in a state-space setup. The first source of uncertainty is the model uncertainty arising out incomplete knowledge about the true model. The second source is the parametric uncertainty due to estimation of the parameters of the model from a sample. The final source is unpredictable stochastic shocks to the NAIRU, also called filtering uncertainty.

We begin with a discussion of model uncertainty and identification in the context of trying to predict an unobserved component. Measuring uncertainty about an unobserved component adds a twist that isn't present in the discussion about uncertainty in forecasting an ex post observable variable. Suppose the econometrician is attempting to predict an ex post observable variable. For an observable variable, it is only a mild exaggeration to say that whatever model gives the tightest forecast confidence intervals is the best model. For an unobservable variable the econometrician still wants a tight confidence interval, but confidence intervals are comparable only across models using the same stochastic specification for the unobserved NAIRU and gap. One needs to separate arguments about improved prediction from arguments that are really about appropriate models. In other words, a tight confidence interval for an incorrectly defined NAIRU isn't very useful.[1] However, for any given specification a multiple indicator approach has the potential to improve precision.[2] Our solution is to start with standard models in the literature and show that using multiple indicators can significantly reduce the variance.

[1] If tight confidence intervals alone were a sufficient criterion, then further research to identify the NAIRU would be unnecessary, at least for the United States. The Humphrey-Hawkins Act declared full employment to be exactly 4 percent.
[2] This approach has, of course, been employed in other settings. See for example, Avery (1979).

There are two genres of identification restrictions used in the literature. One set of restrictions describes the statistical behavior of the natural rate, for example that the gap averages zero. The other set of restrictions uses the Phillips curve and identifies the gap as the variable that drives a wedge between expected and realized inflation. In particular, we follow SSW (1997a) in making this distinction. Laubach (2001) showed that the NAIRU uncertainty can be reduced by using both kinds of identifying restrictions. We use Laubach's models as a launching point.

As a first model, suppose the natural rate is constant. A constant natural rate is a straw man rather than a seriously tenable model. (See, for examples of time-varying NAIRU, Summers (1986), Juhn, Topel and Murphy (1991), Gordon (1997, 1998), Shimer (1998), and Ball and Mankiw (2002), to name only a few.) Nonetheless, two different estimates of a constant NAIRU reinforce SSW (1997a)'s conclusion that it is very difficult to estimate the natural rate precisely. First, suppose the identifying restriction is that the gap, unemployment minus the NAIRU, averages zero. Then the estimate of the NAIRU is the sample mean of unemployment. In our data a regression of unemployment on a constant with an AR(2) error process gives an estimated NAIRU of is 5.90 with a standard error of 0.41. Using the Phillips curve method, reported in Table 1 below, the estimate is similar, 5.99 with a standard error of 0.50.

In what follows we decompose total uncertainty within a given model into the components due to parametric uncertainty and filtering uncertainty. In general the portion due to parametric uncertainty is fairly large, a result which is not surprising given the large degree of uncertainty seen in the constant NAIRU model – where there is no filtering uncertainty. We show that moving to a multiple indicator model reduces both parametric and filtering uncertainty as cross-equation correlations improve the efficiency of estimates.

In section 2 we lay out our benchmark model, provide the estimates and show the results on reduction of filtering uncertainty when moving from a univariate to a bivariate approach. We augment our benchmark model in section 3 by estimating the NAIRU shock variance and thereafter extending our model to a multivariate framework adding four more variables; real GDP, GDP inflation, wage inflation and employment level. We summarize and conclude in section 4.

2. Reduction of filtering uncertainty using multiple indicators

2.1 Specifying the benchmark model

In modeling the NAIRU, we follow the standard set-up of Gordon (1997), SSW (1997a), Laubach (2001). The following equations form the basic model of NAIRU:

(1) $$\Delta\pi_{C,t} = \beta_C(L)\Delta\pi_{C,t-1} + \gamma_C(L)g_{U\,t-1} + \delta_C X_t + \varepsilon_{C,t}$$

(2) $$U_t = N_t + g_{U\,t}$$

where L is the lag operator, $\Delta\pi_{C,t}$ is the first difference of inflation (calculated using the CPI-all items), U_t and N_t are the observed civilian unemployment rate and the NAIRU at time t. The term X_t denotes a vector of supply shocks and $g_{U\,t}$ stands for the unemployment gap. The supply shocks used throughout this paper are a dummy variable for the Nixon price control era and the supply shocks measured by the difference between CPI inflation and food and energy price inflation[3].

[3] We follow Gordon (1990) to construct the dummies for Nixon era price control and King and Watson (1994) construct the supply shocks. The procedures are also mentioned in SSW (1997a).

We specify the NAIRU to be a simple random walk (as in SSW (1997a), Gordon (1997) and Laubach (2001)) and specify the gap as following an autoregressive process of order two to allow for periodicity in the cycle measure:

(3) $N_t = N_{t-1} + \varepsilon_{N,t}$

(4) $g_{U\,t} = \phi_{U1} g_{U\,t-1} + \phi_{U2} g_{U\,t-2} + \varepsilon_{g_U,t}$

A special case of the random walk NAIRU is the constant NAIRU, where the variance in equation (3) equals zero. Equations (1) and (3) form the univariate model of NAIRU whereas equations (1) – (4) form the bivariate model of NAIRU. This approach of using information from inflation about output/unemployment gap in a bivariate set-up was initiated by Kuttner (1994) and was used to reduce NAIRU uncertainty by Laubach (2001). We assume $\varepsilon_{N,t} \sim iidN(0, \sigma_N^2)$ and $\varepsilon_{g_U,t} \sim N(0, \sigma_{g_U}^2)$, $\varepsilon_{g_U,t}$ uncorrelated with $\varepsilon_{N,t}$ but correlated with $\varepsilon_{C,t}$ [4].

In the univariate model of NAIRU, the maximum likelihood estimation of σ_N suffers from the 'pile-up[5]' problem. To maintain consistency with the earlier studies, we specify σ_N = 0.2 in the univariate model.

2.2 Data, estimation and uncertainty in the univariate NAIRU model

We use quarterly data from the first quarter of 1955 to the third quarter of 2003, taken from Fred-II data base of the Federal Reserve Bank of St. Louis and the DRI database. We use maximum likelihood method to estimate the parameters of the models and the Kalman filter to extract the estimates of the state variables[6]. Two lags of

[4] This structure is consistent with Laubach (2001) but we will generalize the covariance matrix of the shocks in the next section.
[5] See Stock (1994), Stock and Watson (1998) and Laubach (2001) for discussions on this problem.
[6] Algorithms for the procedure are outlined in Hamilton (1994), and Kim and Nelson (1999).

unemployment gap and two lags of CPI inflation difference proved to be sufficient in the models. (The selection of lags is based on significance of the last lag.) The starting value of the state variable NAIRU was the actual unemployment rate in the fourth quarter of 1954, specified with a diffuse prior. The calculation of the total uncertainty – sum of parametric uncertainty and filtering uncertainty (in variances)[7], conditional on the model, was based on 2000 Monte Carlo simulations as outlined in Hamilton (1986, 1994).

We start with estimating the constant NAIRU model and its uncertainty as a simple case. Note that the uncertainty in the constant NAIRU is just the parametric uncertainty – there is no filtering uncertainty. In Table 1, Panel A – we report the NAIRU estimate as 5.99 percent. The asymptotic standard error is 0.50, but because the estimate of the natural rate is derived by dividing the intercept by $\gamma_C(1)$ the asymptotic approximation is likely to be poor. Following SSW (1997a), we report the 'Gaussian' confidence interval of the NAIRU at the 95 percent level to be between 4.8 percent and 7.5 percent. In Figure 1, we show the corresponding F-statistic values for our estimated model.

A time-varying, univariate, NAIRU model shows much higher reported total variance than does the constant NAIRU model. Filtering uncertainty is the dominant source of uncertainty about the NAIRU estimates. In Table 1, Panel B we present the estimation results[8]. The estimates show that the parametric variance is only about six

[7] We concentrate only on two-sided filtering uncertainty in this paper.
[8] State-space models using the Kalman filter generally assume normal errors. In principle this is problematic because it implies that the unemployment rate is unbounded. One approach would be to model the log of unemployment and then back out estimate of the level of the natural rate. As a practical matter we found this to be an issue only in the univariate model. In calculating the filtering uncertainty using Monte Carlo methods, we resampled if the standard deviation of the filtering uncertainty turned out to be greater than 3 – which would put a less than zero NAIRU value within the 95 percent confidence interval based on a 6 percent NAIRU. This meant 9 percent resampling in the univariate model but no resampling in

percent of the total variance. In the table we also show the point estimates of NAIRU at the beginning of last three decades along with its total standard deviation, parametric standard deviation and filtering standard deviation.

In Figure 2, we present the two-sided estimates of the time-varying NAIRU, the unemployment gap and the 95 percent confidence interval of the NAIRU. The graph shows that the gap estimates pick up the shaded NBER recessions efficiently. The estimates of the NAIRU show a rise from the mid 1970s and a decline starting in mid 1980s and keeping low throughout 1990s. These features of the natural rate estimate are consistent with studies like Ball and Mankiw (2002), Gordon (1997, 1998), Juhn, Topel and Murphy (1991), SSW (1997a, 1997b), Laubach (2001), Salemi (1999), Shimer (1998) Katz and Kruger (1999). It also illustrates the main point – that the NAIRU is very imprecisely estimated in the univariate model – by showing the large confidence interval of the NAIRU.

2.3 A bivariate model reduces filtering uncertainty

We now add equations (2) and (4) to equations (1) and (3) to make a bivariate model of NAIRU. Estimates of the model are in Table 2. We observe a dramatic decrease in the average total variance coming from a decline in both average parameter variance and average filtering variance. Parametric uncertainty is reduced by a factor of five from the univariate model. But the drop in filtering uncertainty is even greater and most of the uncertainty in the previous model came from filtering, so reduction in filtering uncertainty dominates by being approximately 95 percent of the decline in total variance.

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the all the following multivariate models. So, the uncertainty in the univariate model might be downward biased, but there is no such bias in the multivariate models.

The unemployment gap is fairly persistent; the sum of the autoregressive coefficients is 0.92.

In Figure 3, we show the estimates of NAIRU from the bivariate model along with the 95 percent confidence interval. The NAIRU estimates are quite similar to the previous ones showing a similar rise from late 1960s – a result consistent with Summers (1986), and a decline from the early 1980s. The confidence interval bands are much narrower now – the key result of Laubach (2001) – coming primarily from the decline in the filtering uncertainty. This decline is due to the bivariate – common factor approach since the NAIRU shock variance had the same value in both the models.

3. The estimated NAIRU shock variance, its standard error and the NAIRU uncertainty

3.1 The bivariate model with the estimated NAIRU shock variance

We now use the bivariate model described above in equations (1) – (4), generalize the variance covariance matrix of the three shocks, $\varepsilon_{C,t}$, $\varepsilon_{N,t}$ and $\varepsilon_{g_U,t}$, and estimate the matrix. There are two motivations for this exercise. Firstly, we estimate the variance of the shock to the NAIRU. The bivariate model gives us enough cross-equation information to estimate the shock variance covariance matrix without the pile-up problem Estimation of the shock variance provides a better estimate of filtering uncertainty. The estimation also allows us to have the standard error of the estimated variance of the NAIRU shock – which enters the calculation of parametric uncertainty. The second motivation is due to the Morley, Nelson and Zivot (2003) (hereafter MNZ) result that the estimates of the trend and cycle can be very sensitive to the correlation structure of the shocks.

We faced a computational issue regarding parametric uncertainty while estimating the above model. Estimation of the parameters did not pose any problems but the Hessian of the parameter estimates turned out to be very unstable with respect to some covariance parameter terms between the shocks (log likelihood function very flat for those parameters). We took the following approach to address this problem: we estimated the model with the generalized variance covariance matrix. We noted the off-diagonal parameters with estimated values being close to zero and imprecisely estimated. Then we restricted those off-diagonal parameters to zero and re-estimated the model. The restricted model was used if it was not significantly different at the 90 percent after comparing the log likelihood values. We follow this approach for the rest of the paper.

This effectively meant two restrictions in our model and the log-likelihood difference was not significant at the 75 percent even for one restriction. The results are in Table 3. The standard deviation of the shock to the NAIRU is 0.24, quite close to Laubach and ours imposed value of 0.20. The NAIRU – unemployment gap shock correlation is -0.77, precisely estimated and supports the MNZ result. The average total standard error is now 0.57 – a 20 percent rise over the bivariate model in section 2. The average parametric standard error doubles, from 0.14 in section 2 to 0.28 in this model, since we now incorporate uncertainty about σ_N which was previously omitted. The filtering uncertainty increases marginally due to a higher value of the variance of the shock to the NAIRU. The NAIRU estimates along with the 95 percent confidence interval are shown in Figure 4. The estimates confirm our previous observations.

3.2 The multivariate model

We now augment the bivariate model in section 3.1 to a multivariate model by using four more variables. We have two more inflationary measures – GDP (chained) deflator and wage (hourly compensation of labor in non-farm business), real GDP and civilian employment level (16 years or over, in millions). The real GDP (in natural logs), Y_t, equation, following Watson(1986), Kuttner (1994), MNZ (2003), is specified as a sum of a permanent stochastic trend, $T_{Y\,t}$ and the output gap, g_{y_t}:

(5) $$Y_t = T_{Y\,t} + g_{Y\,t}.$$

The permanent stochastic trend follows a random walk with a constant drift and the output gap follows a second order autoregressive process:

(6) $$T_{Y\,t} = \mu_Y + T_{Y\,t-1} + \varepsilon_{T_Y\,t}$$

(7) $$g_{Y\,t} = \phi_{Y1} g_{Y\,t-1} + \phi_{Y2} g_{Y\,t-2} + \varepsilon_{g_Y,t}$$

Following Clark (1989), we link the output gap and the unemployment gap by a dynamic version of Okun's Law:

(8) $$g_{U\,t} = \sum_{k=0}^{K} \theta_{Y,k} g_{Y\,t-k}$$

Similarly, the employment level (in logs), L_t, equation is also a sum of a permanent, stochastic trend, $T_{L\,t}$, and an employment gap variable driven by the current and lagged output gap:

(9) $$L_t = T_{L\,t} + \sum_{m=0}^{M} \theta_{L,m} g_{Y\,t-m}.$$

We also assume the permanent, stochastic trend of the employment variable, $T_{L\,t}$, follows a random walk with a constant drift:

(10) $T_{L\,t} = \mu_L + T_{L\,t-1} + \varepsilon_{T_L\,t}$

Note that the lags chosen equations (8) and (9) are on the basis of significance of the last lag.

The GDP deflator inflation equation is quite similar to the CPI inflation equation except that we use the output gap instead of the unemployment gap. The wage inflation equation also has a similar structure as our equation (1) in section 2.

(11) $\Delta\pi_{G,t} = \beta_G(L)\Delta\pi_{G,t-1} + \gamma_G(L)g_{Y\,t-1} + \delta_G X_t + \varepsilon_{G,t}$

(12) $\Delta\pi_{W,t} = \beta_W(L)\Delta\pi_{W,t-1} + \gamma_W(L)(U_{t-1} - N_{t-1}) + \delta_W X_t + \varepsilon_{W,t}$

In the above equations, $\Delta\pi_{G,t}$ is the first difference of the GDP inflation rate and $\Delta\pi_{W,t}$ is the first difference of the wage inflation rate. Equations (1) – (12) now form our new multivariate model. We start our estimation with a generalized variance – covariance matrix of the seven shocks and then restrict the off-diagonal parameters as described previously.

Based on the significance of the last lag, we used one lag (along with the contemporaneous) for equations (8) and (9). This is consistent with the Clark (1989) framework. For equations (11) and (12) we had to use three lags of their respective inflation differences and two lags of the respect gaps. The estimates of the model are in Table 4. The parameter estimate of the standard deviation of the shock to the NAIRU is 0.17 and much more precisely estimated. The estimate of the standard deviation of the GDP trend shock is large and precise. The standard deviation of the employment trend shock is lower than that of the GDP trend shock, but still precise. The correlation of the GDP trend shock to the gap shock is negative. The drift terms imply a 3.2 percent annual GDP growth and 1.7 percent annual employment growth.

Comparisons of Tables 3 and 4 show the large increase in precision due to the use of multiple indicators. Overall variance drops in half, from 0.33 to 0.16. This overall drop comes from roughly equal proportional decreases in each component, the parametric variance dropping from 0.08 to 0.03 and the filtering variance dropping from 0.24 to 0.13. Since filtering variance was considerably larger in terms of absolute level, most of the total decrease is due to the reduction in filtering uncertainty. Note that a considerable part of the improvement in the filtering uncertainty is due to the lower estimate of the NAIRU shock variance. In Figure 5, the NAIRU estimates are quite similar to previous estimates but the 95 percent confidence interval is narrower.

4. Conclusion

We show in this paper that using multiple indicators to extract a common unobserved factor helps to reduce the filtering uncertainty and parametric uncertainty around the extracted point estimates. We use this method to estimate the NAIRU and reduce its uncertainty. Specifically, we find that four variables, the GDP deflator, average wage, real GDP, and civilian employment level are valuable indicators of the gap in the business cycle. The improvement in precision cuts in half the uncertainty as measured by total variance. We chose these additional indicators because they did a good job and are consistent with theory. Use of this method opens the possibility for further research which might suggest yet more such useful indicators.

Table 1: Parameter and NAIRU Estimates from the Univariate Models

Panel A: Constant NAIRU

NAIRU Estimate	Confidence Interval (95 percent)	$\gamma_C(1)$
5.99	4.80, 7.47	-0.20

Panel B: Time-varying NAIRU

Log L	$\gamma_C(1)$	Average Total Variance
-340.11	-0.14	1.72

Average Parametric Variance	Average Filtering Variance
0.10	1.62

Date	1980:1	1990:1	2000:1
NAIRU	6.79	5.96	5.49
Total Std. Dev.	1.20	1.20	1.32
Parametric Std. Dev.	0.23	0.17	0.23
Filtering Std. Dev.	1.17	1.18	1.29

Table 2: Parameter and NAIRU Estimates from the Bivariate Model

Time-varying NAIRU

Log L	$\gamma_C(1)$	$\phi_U(1)$	Average Total Variance
-196.00	-0.35	0.92	0.22

Average Parametric Variance	Average Filtering Variance
0.02	0.20

Date	1980:1	1990:1	2000:1
NAIRU	7.22	6.19	4.99
Total Std. Dev.	0.45	0.44	0.47
Parametric Std. Dev.	0.10	0.06	0.13
Filtering Std. Dev.	0.44	0.44	0.45

Table 3: Parameter and NAIRU Estimates from the Bivariate Model with Estimated NAIRU Shock Variance

Time-varying NAIRU				
Log L	$\gamma_C(1)$	$\phi_U(1)$	σ_N	ρ_{Ng_U}
-186.35	-0.22	0.92	0.24 (0.07)	-0.78 (0.12)

Average Total Variance	Average Parametric Variance	Average Filtering Variance
0.33	0.08	0.24

Date	1980:1	1990:1	2000:1
NAIRU	7.71	6.41	5.42
Total Std. Dev.	0.62	0.51	0.59
Parametric Std. Dev.	0.41	0.21	0.19
Filtering Std. Dev.	0.47	0.47	0.56

Note: The standard errors of the parameter estimates are in the parentheses.

Table 4: Parameter and NAIRU Estimates from the Multivariate Model with Estimated NAIRU Shock Variance

Time-varying NAIRU				
Log L	$\phi_Y(1)$	σ_N	σ_{T_Y}	σ_{T_L}
-372.94	0.90	0.17 (0.02)	0.78 (0.07)	0.28 (0.02)
μ_Y	μ_L	ρ_{Ng_Y}	$\rho_{T_Y g_Y}$	ρ_{NT_Y}
0.82 (0.05)	0.43 (0.02)	0.51 (0.19)	-0.50 (0.13)	-0.56 (0.10)

Average Total Variance	Average Parametric Variance	Average Filtering Variance
0.16	0.03	0.13

Date	1980:1	1990:1	2000:1
NAIRU	7.58	6.72	5.24
Total Std. Dev.	0.39	0.37	0.40
Parametric Std. Dev.	0.16	0.11	0.09
Filtering Std. Dev.	0.35	0.35	0.39

Note: The standard errors of the parameter estimates are in the parentheses.

Figure 1: The Constant NAIRU and Its 95 Percent "Gaussian" Confidence Interval

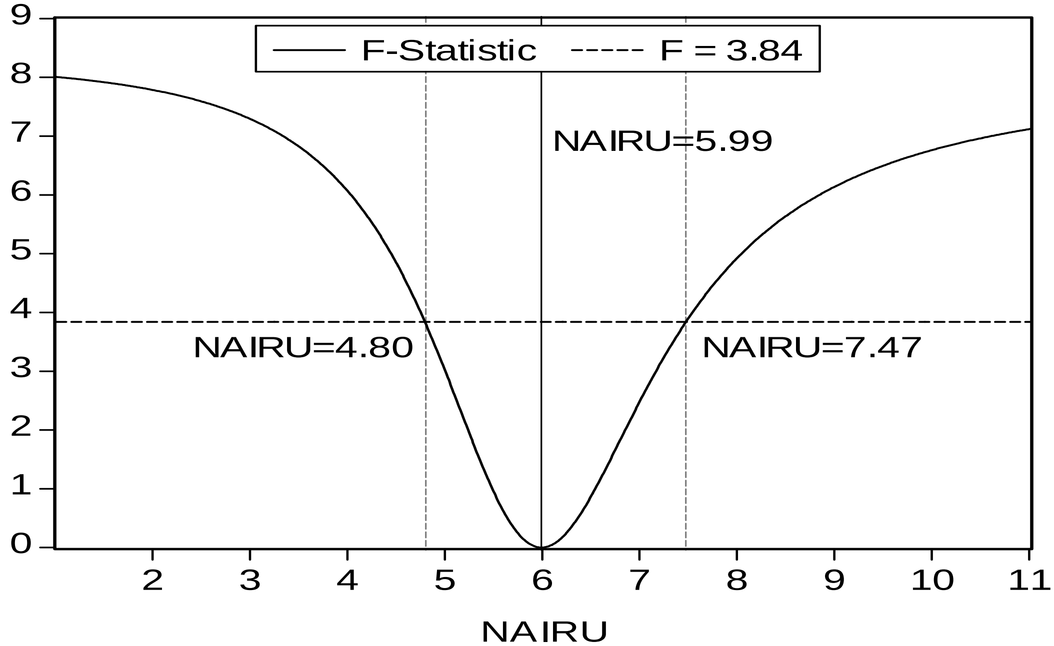


Figure 2: The Time-Varying NAIRU and Its 95 Percent Confidence Interval from the Univariate Model

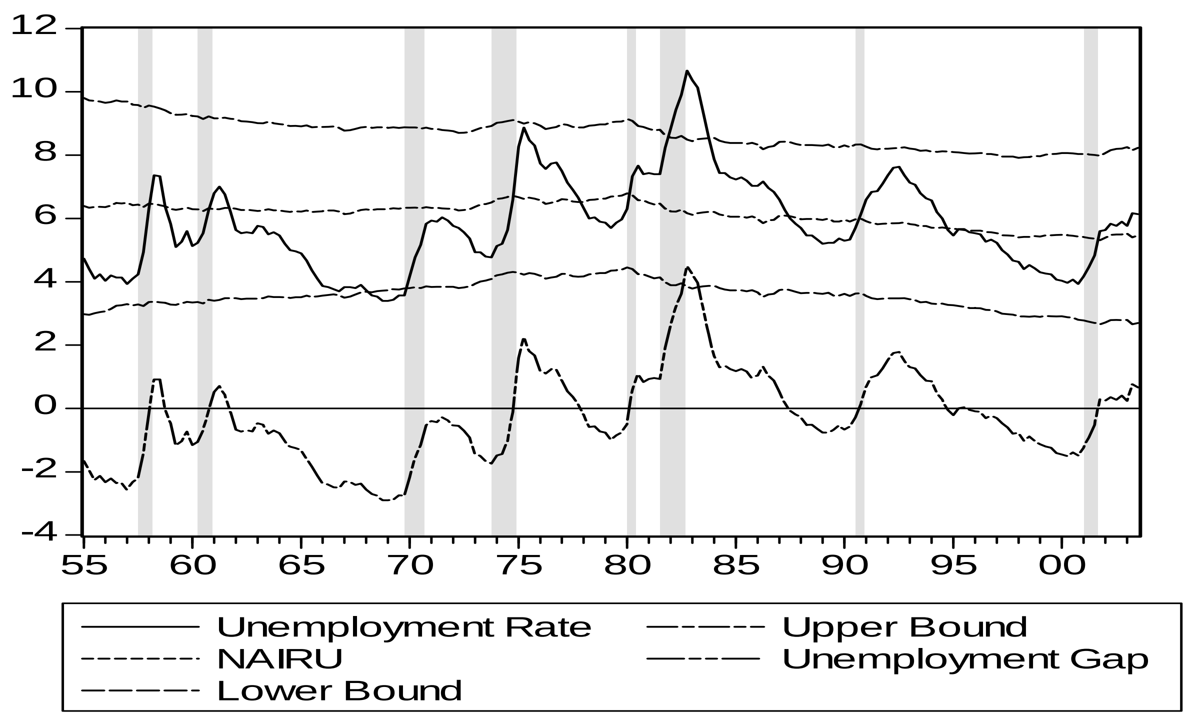


Figure 3: The Time-Varying NAIRU and Its 95 Percent Confidence Interval from the Bivariate Model

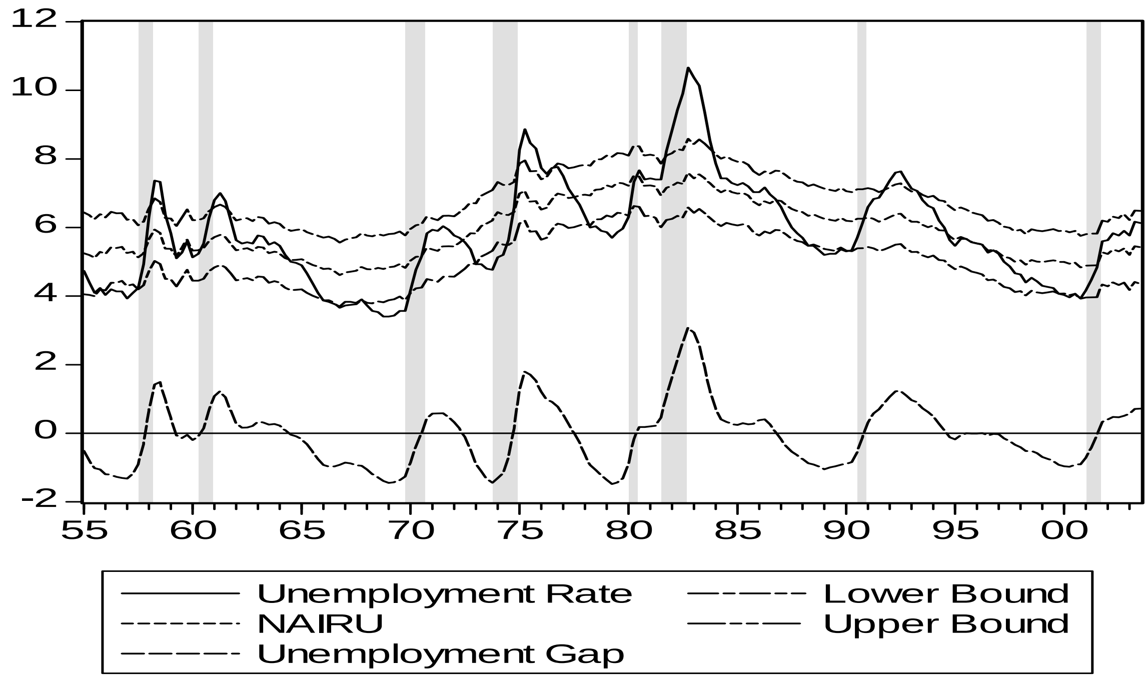


Figure 4: The Time-Varying NAIRU and Its 95 Percent Confidence Interval from the Bivariate Model with Estimated NAIRU Shock Variance

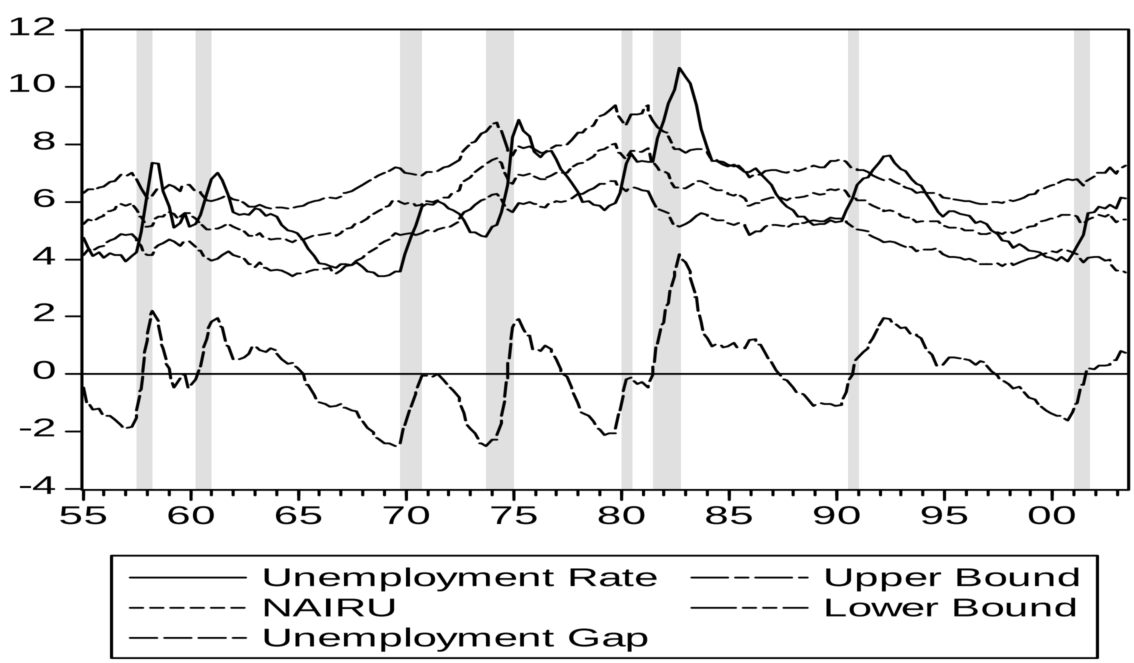
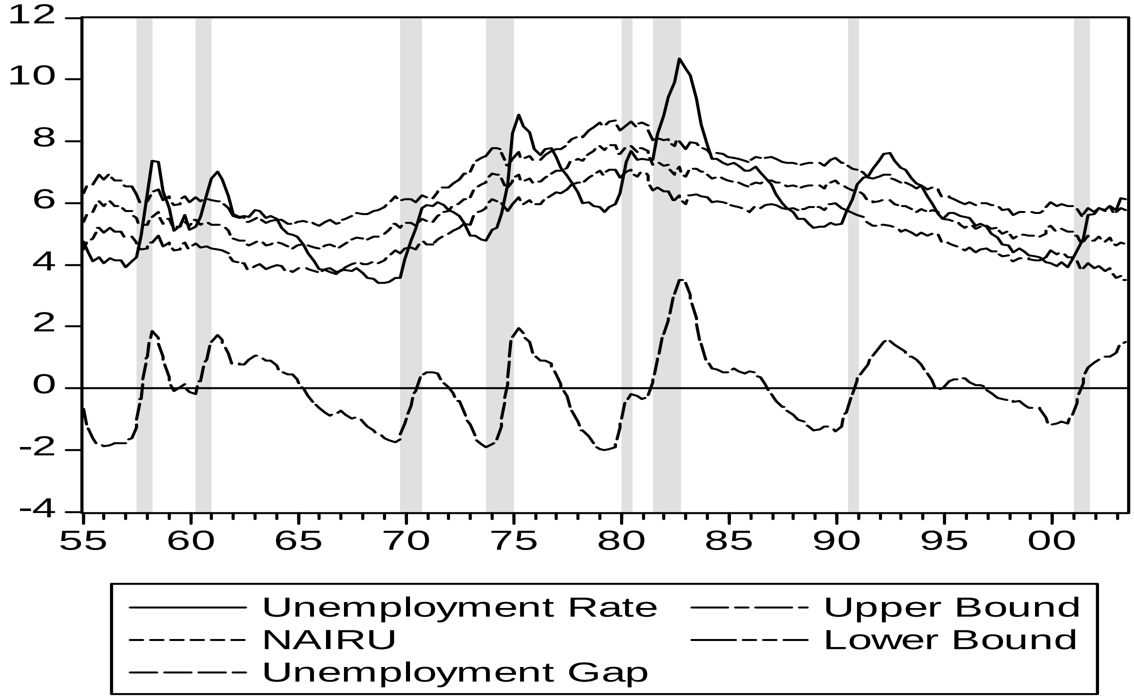


Figure 5: The Time-Varying NAIRU and Its 95 Percent Confidence Interval from the Multivariate Model with Estimated NAIRU Shock Variance



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Appendix: The NAIRU and its Uncertainty Estimates from the Multivariate Model

Year	Nairu	TU	PU	FU	Year	Nairu	TU	PU	FU
1955:1	5.40	0.46	0.16	0.43	1965:4	4.56	0.39	0.16	0.35
1955:2	5.64	0.46	0.17	0.42	1966:1	4.51	0.39	0.17	0.35
1955:3	5.72	0.45	0.17	0.41	1966:2	4.62	0.39	0.17	0.35
1955:4	6.06	0.44	0.18	0.41	1966:3	4.67	0.39	0.17	0.35
1956:1	5.90	0.44	0.17	0.40	1966:4	4.54	0.40	0.18	0.35
1956:2	6.06	0.43	0.18	0.40	1967:1	4.58	0.40	0.19	0.35
1956:3	5.90	0.42	0.16	0.39	1967:2	4.70	0.40	0.19	0.35
1956:4	5.90	0.42	0.17	0.39	1967:3	4.80	0.40	0.19	0.35
1957:1	5.72	0.42	0.17	0.38	1967:4	4.87	0.40	0.20	0.35
1957:2	5.73	0.42	0.17	0.38	1968:1	4.82	0.42	0.22	0.35
1957:3	5.50	0.40	0.14	0.38	1968:2	4.82	0.42	0.22	0.35
1957:4	5.27	0.39	0.12	0.37	1968:3	4.91	0.43	0.24	0.35
1958:1	5.29	0.40	0.15	0.37	1968:4	4.90	0.44	0.26	0.35
1958:2	5.51	0.40	0.15	0.37	1969:1	5.01	0.44	0.26	0.35
1958:3	5.70	0.38	0.11	0.37	1969:2	5.12	0.44	0.26	0.35
1958:4	5.34	0.38	0.10	0.37	1969:3	5.33	0.45	0.28	0.35
1959:1	5.47	0.38	0.11	0.36	1969:4	5.21	0.44	0.26	0.35
1959:2	5.19	0.37	0.09	0.36	1970:1	5.32	0.41	0.21	0.35
1959:3	5.23	0.37	0.09	0.36	1970:2	5.35	0.40	0.19	0.35
1959:4	5.47	0.38	0.12	0.36	1970:3	5.25	0.39	0.16	0.35
1960:1	5.27	0.38	0.13	0.36	1970:4	5.49	0.38	0.15	0.35
1960:2	5.44	0.38	0.12	0.36	1971:1	5.42	0.39	0.17	0.35
1960:3	5.30	0.38	0.14	0.36	1971:2	5.38	0.39	0.16	0.35
1960:4	5.36	0.40	0.17	0.36	1971:3	5.57	0.39	0.16	0.35
1961:1	5.31	0.40	0.18	0.36	1971:4	5.72	0.39	0.18	0.35
1961:2	5.28	0.40	0.17	0.36	1972:1	5.75	0.39	0.16	0.35
1961:3	5.22	0.39	0.16	0.36	1972:2	5.87	0.39	0.17	0.35
1961:4	5.09	0.38	0.13	0.36	1972:3	5.99	0.40	0.18	0.35
1962:1	4.84	0.38	0.14	0.36	1972:4	6.14	0.41	0.21	0.35
1962:2	4.78	0.38	0.14	0.36	1973:1	6.13	0.42	0.23	0.35
1962:3	4.79	0.38	0.15	0.36	1973:2	6.48	0.43	0.24	0.35
1962:4	4.61	0.39	0.16	0.35	1973:3	6.60	0.44	0.26	0.35
1963:1	4.72	0.39	0.16	0.35	1973:4	6.67	0.43	0.25	0.35
1963:2	4.76	0.39	0.15	0.35	1974:1	6.95	0.43	0.24	0.35
1963:3	4.61	0.39	0.15	0.35	1974:2	6.91	0.44	0.26	0.35
1963:4	4.67	0.39	0.16	0.35	1974:3	6.86	0.41	0.22	0.35
1964:1	4.72	0.38	0.14	0.35	1974:4	6.47	0.37	0.12	0.35
1964:2	4.69	0.38	0.14	0.35	1975:1	6.69	0.37	0.12	0.35
1964:3	4.55	0.39	0.16	0.35	1975:2	6.91	0.37	0.12	0.35
1964:4	4.52	0.39	0.17	0.35	1975:3	6.70	0.37	0.11	0.35
1965:1	4.62	0.38	0.15	0.35	1975:4	6.80	0.37	0.11	0.35
1965:2	4.66	0.39	0.15	0.35	1976:1	6.66	0.37	0.10	0.35
1965:3	4.62	0.38	0.15	0.35	1976:2	6.68	0.37	0.10	0.35

Year	Nairu	TU	PU	FU	Year	Nairu	TU	PU	FU
1976:3	6.83	0.37	0.10	0.35	1987:4	6.55	0.38	0.14	0.35
1976:4	6.96	0.36	0.09	0.35	1988:1	6.57	0.38	0.13	0.35
1977:1	7.04	0.36	0.09	0.35	1988:2	6.48	0.37	0.12	0.35
1977:2	7.02	0.37	0.10	0.35	1988:3	6.54	0.37	0.11	0.35
1977:3	7.15	0.37	0.11	0.35	1988:4	6.57	0.37	0.11	0.35
1977:4	7.34	0.37	0.12	0.35	1989:1	6.57	0.37	0.11	0.35
1978:1	7.42	0.38	0.14	0.35	1989:2	6.57	0.37	0.10	0.35
1978:2	7.36	0.38	0.15	0.35	1989:3	6.49	0.37	0.09	0.35
1978:3	7.59	0.39	0.17	0.35	1989:4	6.67	0.37	0.10	0.35
1978:4	7.68	0.40	0.18	0.35	1990:1	6.72	0.37	0.11	0.35
1979:1	7.83	0.40	0.18	0.35	1990:2	6.56	0.37	0.11	0.35
1979:2	7.70	0.40	0.18	0.35	1990:3	6.49	0.37	0.11	0.35
1979:3	7.85	0.40	0.19	0.35	1990:4	6.39	0.37	0.11	0.35
1979:4	7.88	0.41	0.20	0.35	1991:1	6.33	0.37	0.11	0.35
1980:1	7.58	0.39	0.16	0.35	1991:2	6.27	0.37	0.11	0.35
1980:2	7.77	0.38	0.14	0.35	1991:3	6.09	0.38	0.13	0.35
1980:3	7.85	0.40	0.18	0.35	1991:4	6.03	0.39	0.16	0.35
1980:4	7.66	0.41	0.21	0.35	1992:1	6.05	0.40	0.20	0.35
1981:1	7.78	0.41	0.22	0.35	1992:2	6.10	0.42	0.22	0.35
1981:2	7.70	0.42	0.22	0.35	1992:3	6.08	0.42	0.22	0.35
1981:3	7.22	0.41	0.21	0.35	1992:4	5.97	0.40	0.19	0.35
1981:4	7.31	0.41	0.21	0.35	1993:1	5.85	0.40	0.18	0.35
1982:1	7.20	0.42	0.22	0.35	1993:2	5.90	0.39	0.17	0.35
1982:2	7.22	0.43	0.25	0.35	1993:3	5.79	0.39	0.16	0.35
1982:3	6.93	0.47	0.30	0.35	1993:4	5.70	0.39	0.17	0.35
1982:4	7.16	0.46	0.30	0.35	1994:1	5.80	0.39	0.16	0.35
1983:1	6.86	0.46	0.29	0.35	1994:2	5.68	0.38	0.14	0.36
1983:2	7.11	0.44	0.26	0.35	1994:3	5.75	0.38	0.12	0.36
1983:3	7.11	0.42	0.22	0.35	1994:4	5.69	0.37	0.11	0.36
1983:4	7.01	0.40	0.19	0.35	1995:1	5.48	0.38	0.14	0.36
1984:1	6.95	0.39	0.17	0.35	1995:2	5.45	0.38	0.14	0.36
1984:2	6.80	0.40	0.18	0.35	1995:3	5.38	0.38	0.13	0.36
1984:3	6.84	0.40	0.18	0.35	1995:4	5.28	0.38	0.13	0.36
1984:4	6.80	0.39	0.17	0.35	1996:1	5.20	0.38	0.14	0.36
1985:1	6.72	0.40	0.19	0.35	1996:2	5.29	0.38	0.12	0.36
1985:2	6.67	0.40	0.19	0.35	1996:3	5.16	0.39	0.14	0.36
1985:3	6.65	0.40	0.18	0.35	1996:4	5.23	0.38	0.13	0.36
1985:4	6.56	0.40	0.20	0.35	1997:1	5.26	0.38	0.11	0.36
1986:1	6.50	0.41	0.22	0.35	1997:2	5.18	0.38	0.11	0.36
1986:2	6.68	0.41	0.21	0.35	1997:3	5.18	0.38	0.11	0.36
1986:3	6.62	0.40	0.20	0.35	1997:4	5.02	0.38	0.12	0.36
1986:4	6.69	0.40	0.18	0.35	1998:1	5.05	0.38	0.11	0.37
1987:1	6.73	0.39	0.16	0.35	1998:2	4.86	0.39	0.12	0.37
1987:2	6.67	0.38	0.15	0.35	1998:3	4.95	0.39	0.11	0.37
1987:3	6.60	0.38	0.15	0.35	1998:4	4.96	0.38	0.10	0.37

Year	Nairu	TU	PU	FU
1999:1	4.95	0.39	0.10	0.37
1999:2	4.91	0.39	0.10	0.38
1999:3	4.91	0.39	0.10	0.38
1999:4	5.02	0.40	0.10	0.38
2000:1	5.24	0.40	0.09	0.39
2000:2	5.11	0.40	0.09	0.39
2000:3	5.16	0.41	0.10	0.40
2000:4	5.07	0.42	0.12	0.40
2001:1	5.06	0.43	0.12	0.41
2001:2	4.96	0.44	0.13	0.42
2001:3	4.68	0.44	0.12	0.43
2001:4	4.93	0.45	0.12	0.44
2002:1	4.79	0.47	0.13	0.45
2002:2	4.91	0.48	0.14	0.46
2002:3	4.76	0.50	0.16	0.47
2002:4	4.88	0.51	0.16	0.49
2003:1	4.61	0.53	0.18	0.50
2003:2	4.73	0.56	0.20	0.52
2003:3	4.65	0.58	0.20	0.54

Not for Publication Appendix:

A Simple Example of How to Reduce the Filtering Uncertainty of the State Estimates

Let us consider the following unobserved components model where we observe y_{1t}, y_{2t} and y_{3t}. Our problem is to get estimates and the filtering uncertainty of the unobserved components i_{1t}, i_{2t}, i_{3t} and i_{4t} given that we know the parameters of the model. We know $i_{jt} \sim iidN(0,1),\ j=1,3,4$ and $i_{2t} \sim iidN(0,\sigma^2)$.

$$\begin{aligned} y_{1t} &= i_{1t} + i_{2t} \\ y_{2t} &= i_{3t} + \gamma_1 i_{2t} \\ y_{3t} &= i_{4t} + \gamma_2 i_{2t} \end{aligned}$$

The common component in the three observed series is the unobserved component i_{2t} and this makes the estimates of i_{2t} to have lower filtering variance. A special case is when $\gamma_1 = \gamma_2 = 0$ and the resulting estimates of i_{2t} will be based on only y_{1t}.

We can write the model in the state-space representation as:

Measurement equations:

$$y_t = HI_t$$

$$y_t = \begin{bmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \end{bmatrix}, H = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & \gamma_1 & 1 & 0 \\ 0 & \gamma_2 & 0 & 1 \end{bmatrix}, I_t = \begin{bmatrix} i_{1t} \\ i_{2t} \\ i_{3t} \\ i_{4t} \end{bmatrix}$$

Transition equations:

$$\begin{bmatrix} i_{1t} \\ i_{2t} \\ i_{3t} \\ i_{4t} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} i_{1t-1} \\ i_{2t-1} \\ i_{3t-1} \\ i_{4t-1} \end{bmatrix} + \begin{bmatrix} i_{1t} \\ i_{2t} \\ i_{3t} \\ i_{4t} \end{bmatrix}$$

$$i.e., I_t = FI_{t-1} + I_t$$

$$Let, \quad F = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sigma^2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In the above transition equations, F is the transition matrix of the state variables and Q is the variance covariance matrix of the shocks to the state variables (which are variances of the states themselves in this special case since the states are all white noise). To start the Kalman filter iteration, we need the steady-state values of I_t as $I_{0|0}$ and uncertainty around I_t at time zero as $P_{0|0}$. We specify those initial values as

$$I_{0|0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$P_{0|0} = FP_{0|0}F' + Q = Q$$

Starting the Kalman filter iteration, we have

$$I_{t|t-1} = FI_{t-1|t-1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$P_{t|t-1} = FP_{t-1|t-1}F' + Q = Q$$

We denote $I_{t|t-1}$ as the linear projection of the state variables based on time $t-1$ information. The uncertainty (or the variance-covariance matrix) around the projections is denoted as $P_{t|t-1}$, also based on time $t-1$. The forecast errors are denoted as $\eta_{t|t-1}$, and $f_{t|t-1}$ is the conditional variance of the forecast errors. In the case we are considering

$$\eta_{t|t-1} = y_t - HI_{t|t-1} = y_t$$

$$f_{t|t-1} = HP_{t|t-1}H' = HQH' = \begin{bmatrix} 1+\sigma^2 & & \\ \gamma_1\sigma^2 & 1+\gamma_1^2\sigma^2 & \\ \gamma_2\sigma^2 & \gamma_1\gamma_2\sigma^2 & 1+\gamma_2^2\sigma^2 \end{bmatrix}$$

Updating the iterations to include time t information, we have the Kalman gain component, K_t:

$$K_t = P_{t|t-1}H'f_{t|t-1}^{-1}$$

Therefore

$$I_{t|t} = I_{t|t-1} + K_t\eta_{t|t-1} = K_t y_t$$
$$P_{t|t} = P_{t|t-1} - K_t HP_{t|t-1} = Q - K_t HQ$$

Since the F matrix is a zero matrix, we have $P_{t|t} = P_{t|T}$, where

$$P_{t|t} = P_{t|T} = \begin{bmatrix} \frac{\sigma^2}{1+\gamma_1\sigma^2+\gamma_2\sigma^2+\sigma^2} & & & \\ \frac{-\sigma^2}{1+\gamma_1\sigma^2+\gamma_2\sigma^2+\sigma^2} & \frac{\sigma^2}{1+\gamma_1\sigma^2+\gamma_2\sigma^2+\sigma^2} & & \\ \frac{\gamma_1\sigma^2}{1+\gamma_1\sigma^2+\gamma_2\sigma^2+\sigma^2} & \frac{-\gamma_1\sigma^2}{1+\gamma_1\sigma^2+\gamma_2\sigma^2+\sigma^2} & \frac{\gamma_1^2\sigma^2}{1+\gamma_1\sigma^2+\gamma_2\sigma^2+\sigma^2} & \\ \frac{\gamma_2\sigma^2}{1+\gamma_1\sigma^2+\gamma_2\sigma^2+\sigma^2} & \frac{-\gamma_2\sigma^2}{1+\gamma_1\sigma^2+\gamma_2\sigma^2+\sigma^2} & \frac{\gamma_1\gamma_2\sigma^2}{1+\gamma_1\sigma^2+\gamma_2\sigma^2+\sigma^2} & \frac{\gamma_2^2\sigma^2}{1+\gamma_1\sigma^2+\gamma_2\sigma^2+\sigma^2} \end{bmatrix}$$

It is obvious from the above matrix that for $\gamma_1 = \gamma_2 = 0$, the variance of $i_{2t|T} = \dfrac{\sigma^2}{1+\sigma^2}$ is maximum. So, non-zero values of γ_1 and γ_2 will reduce the variance – filtering uncertainty improves with a multiple indicator - common factor approach. Moreover, the marginal effect of σ^2 is

$$\frac{\partial(\mathrm{var}(i_{2t|T}))}{\partial\sigma^2} = \frac{1}{(1+\gamma_1^2\sigma^2+\gamma_2^2\sigma^2+\sigma^2)^2} \geq 0\,.$$

As evident from the analysis above, the same argument applies to the precision of i_{1t}. This example highlights, ceteris paribus, the role a common factor approach can play by extracting information from multiple indicators in improving its precision.

The above model also shows that impact of an additional indicator on improving filtering uncertainty goes down with increasing number of indicators if everything else is same. The reduction in filtering uncertainty when we augment the univariate model to a bivariate system is $\frac{\gamma_1^2\sigma^4}{(1+\sigma^2)(1+\gamma_1^2\sigma^2+\sigma^2)}$. Similarly, when we extend the bivariate to a trivariate system, the decline in filtering uncertainty is $\frac{\gamma_2^2\sigma^4}{(1+\gamma_1^2\sigma^2+\sigma^2)(1+\gamma_1^2\sigma^2+\gamma_2^2\sigma^2+\sigma^2)}$. To simplify algebra, let us assume that $\gamma_1=\gamma_2=\gamma$ - thereby making the assumption that y_{2t} and y_{3t} individually contain same amount of information about i_{2t}. Then,

$\frac{\gamma^2\sigma^4}{(1+\sigma^2)(1+\gamma^2\sigma^2+\sigma^2)} \geq \frac{\gamma^2\sigma^4}{(1+\gamma^2\sigma^2+\sigma^2)(1+2\gamma^2\sigma^2+\sigma^2)}$. So, in the above model, the biggest reduction in the filtering uncertainty comes from extending the model from the univariate to the bivariate setup.

The Bivariate Model with Generalized Covariance Matrix of the Shocks

In this section we re-estimated the bivariate model of the Section 3 (eqs. (1) – (4)) with a generalized variance covariance matrix of the three shocks. Specifically,

$$Var - Cov(\varepsilon_{C,t}, \varepsilon_{N,t}, \varepsilon_{g_U,t}) = \begin{bmatrix} \sigma_C^2 & & \\ \sigma_{CN} & \sigma_N^2 & \\ \sigma_{Cg_U} & \sigma_{Ng_U} & \sigma_{g_U}^2 \end{bmatrix}$$

is the new variance – covariance matrix of the shocks. This implies estimation of two more parameters and examining the effects of their standard errors on the total uncertainty.

In Table A1, we show the estimation results of the new bivariate model with generalized covariance matrix. The estimate of the standard deviation of the shock to the NAIRU is 0.22, very similar to what studies like Gordon (1997), Laubach (2001) used before and what we have in our Table 3. The estimate of the correlation between the NAIRU and the unemployment gap, ρ_{Ng_U}, is -0.77 – strongly negative like the MNZ result. The comparison of the log likelihood values with Table 3 indicates that the inclusion of the two new parameters were insignificant at the 90 percent level. The estimate of persistence of the unemployment gap is quite similar to the previous estimate, 0.92.

The surprising element of the generalized covariance matrix is in its effect on the total uncertainty. The average filtering variance remain almost the same as in Table 3, not a surprising result given the estimate of the standard deviation of the shock to the NAIRU is quite similar to what used before. However, the average total variance now has risen to 1.33, primarily due to the big rise in the average parametric variance. In Figure A1, we show the new NAIRU estimates along with the new 95 percent confidence bands. The

NAIRU estimates are similar to our previous estimates but the confidence bands are a lot wider. This was happening because the log likelihood function is very flat with respect to the two new covariance parameters, making the Hessian and the variance – covariance matrix of the estimated parameters very unstable and resulting in a large increase in the parametric uncertainty.

Table A1: Parameter and NAIRU Estimates from the Bivariate and Trivariate Models with Generalized Variance Covariance Matrix of the Shocks

Time-varying NAIRU (Bivariate)				
Log L	$\gamma_C(1)$	$\phi_U(1)$	σ_N	ρ_{Ng_U}
-185.78	-0.22	0.92	0.22	-0.77

Average Total Variance	Average Parameter Variance	Average Filtering Variance
1.33	1.12	0.21

Date	1980:1	1990:1	2000:1
NAIRU	7.36	6.31	5.41
Total Std. Dev.	1.46	0.74	0.95
Parametric Std. Dev.	1.39	0.61	0.82
Filtering Std. Dev.	0.42	0.43	0.49

Figure A1: The Time-Varying NAIRU and Its 95 Percent Confidence Interval from the Bivariate Model with Generalized Covariance Matrix of the Shocks