Dynamic Portfolio Optimization with Economic Uncertainty[1]

Xiaolou Yang[2]

June 2005

Abstract

The development and use of dynamic optimization model is extremely important in financial markets. The classical mean-variance portfolio model assumes the expected returns are known with perfect precision. In practice, however, it is extremely difficult to estimate precisely. While portfolios that ignore estimation error have very poor properties: the portfolio weights have extreme values and fluctuate dramatically over time. The Bayesian approach that is traditionally used to deal with estimation error assumes investors have only a single prior or is neutral to the risk. Further, the Bayesian approach has computational difficulty to incorporate future uncertainty into the model.

In this paper, I introduce Genetic algorithms technique in solving a dynamic portfolio optimization system, which incorporate economic uncertainties into a state dependent stochastic portfolio choice model. The advantage of GA is that it solves the model by forward-looking and backward-induction, which incorporates both historical information and future uncertainty when estimating the asset returns. It significantly improves the accuracy of mean return estimation and thus yields a superior model performance compared to the traditional methodologies. The empirical results showed that the portfolio weights using the GA model are less unbalanced and vary much less over time compared to the mean-variance portfolio weights. GA achieves a much higher Sharpe ratio and the out of sample returns generated by the GA portfolio model have a substantially higher mean and lower volatility compared to the classical mean-variance portfolio strategy and Bayesian approach.

JEL: E44

---

[1] I am very grateful to David Kendrick for extensive comments. I thank Russell Cooper, Lorenzo Garlappi, Douglas Dacy, and participants at presentations given at Economics department, the University of Texas at Austin, and the 2005 SCE conference for helpful suggestions.

[2] Author Information: Department of Economics, The University of Texas at Austin, Austin, TX, 78712; Email: xlyang@eco.utexas.edu; Tel: 512-626-6339.

1. Introduction

The development and use of dynamic optimization model is extremely important in financial markets. A dynamic portfolio optimization method is used to determine the percentage of the overall portfolio value allocated to each portfolio component by periodically rebalancing the portfolio in a constantly changing financial market, to achieve return maximization or risk minimization.

The classical mean-variance (MV) analysis assumes that the investor knows the true expected returns. However, in practice, the investor has to estimate expected returns from unknown probability distributions. Even if expected returns, variances, and covariances were known with certainty, MV optimized portfolios would not beat all other portfolios in every future investment periods, because return realizations will differ from their expected values. This is called the intrinsic risk. Further, estimating the unknown parameters involves an additional source of risk, estimation risk.

Estimation risk and the intrinsic risk are known to have a huge impact on Markowitz (1987, 1991) mean-variance optimized portfolios. It leads to unstable and extreme portfolio weights along portfolios on the MV efficient frontier. Only few assets are included in the optimal portfolio. They show sudden shifts in allocations along the efficient frontier and are also very unstable across time. MV optimized portfolios lack of diversification and show poor out-of-sample performance. These unintuitive and extreme solutions are a consequence of optimizers being "estimation error maximizers" (Michaud, 1989). MV optimizers overweight those assets that have large estimated expected returns, low estimated variances and low estimated correlations to other assets. These assets are the ones most likely to have large estimation errors. Due to estimation risk, portfolios on the efficient frontier are not unique as the MV optimization procedure suggests. Hence, estimation risk is one of the primary reasons to make standard MV optimization unfeasible in practice. Michaud (1998) summarizes “I believe that estimation risk is one of the great neglected areas of modern finance.”

The significance impact of estimation risk on optimal portfolios was explored by Chopra and ziemba (1993). They found that errors in means are about ten times as important as errors in variances, and errors in variances are about twice as important as errors in covariances. Best and Grauer (1991) showed that optimal portfolios are very sensitive to the level of expected returns. They note that “a surprisingly small increase in the mean of just one asset drives half the securities from the portfolio. Yet the portfolio expected return and standard deviation are virtually unchanged.” Therefore, how to improve the technique of mean return estimation becomes a key issue of the portfolio choice problem.

Several approaches to incorporate estimation risk into portfolio selection are suggested in the literature. These papers regularly discuss heuristic approaches (e.g., placing restrictions on portfolio weights or using an equally-weighted portfolio) and Bayesian estimators. The most popular one is the Bayesian estimators, developed by Jorion (1985, 1986). The idea of Bayesian inference is to combine extra-sample, or prior, information with sample returns. Returns are shrunk towards the prior, depending on the degree of noise in the sample. It shrinks the optimal portfolio towards the minimum-variance portfolio (MVP). The MVP is less vulnerable to estimation risk as it does not make use of any information about expected returns.

However, the Bayesian approach is a very problem-dependent approach and therefore lack of generality. Moreover, Bayesian approach assumes that the decision-maker has only a single prior or is neutral to the risk in the sense of Knight (1921). Further, Bayesian approach has difficulty to incorporate uncertainties into the model due to computational burden (Chopra, 1993). In this paper I will introduce Genetic algorithms (GA) technique in solving the dynamic version of portfolio optimization problem with variety of economic uncertainties.

A GA is an evolutionary optimization approach, which mimics operation in natural genetics to search for the optimal solution. Genetic algorithms are probabilistic search approaches, which are founded as an ideal optimization solver. Particularly in the last ten years, substantial research effort has been applied to the investigation and development of genetic algorithms. However, previous works on portfolio optimization model with Genetic Algorithms has been confined to the static-single-state of the world models. In other words, they ignore a fundamental type of future uncertainty when they apply GA methods in portfolio optimization problem. This will not only violate the realistic assumption of a dynamic asset pricing model, but will also affect the performance of the GA methodology. This paper develops a dynamic stochastic portfolio optimization model by incorporating future uncertainties. The advantage of GA is that it solves the model by forward-looking and backward-induction, which incorporates both historical information and future uncertainty when estimate the asset returns. It significantly improves the accuracy of mean return estimation and thus the model performance. In addition, GA could handle a large variety of future uncertainties, which overcome the computational difficulties in traditional Bayesian approach.

In order to compare the performance of different methodology, I apply the GA model to portfolio selection problem using MSCI data set. I consider the problem of a fund manager allocating wealth across eight international equity indices, who is uncertain about the expected returns on these equity indices. I characterize the properties of the portfolio weights under the GA approach and compare them to the standard mean-variance portfolio that ignores estimation error and the Bayesian portfolio that allows for estimation error but has a single prior or is uncertainty neutral. The empirical results showed that the performance of Genetic algorithm is more superior to the traditional methods such as mean-variance analysis and Bayesian approach. In particular the portfolio weights using the GA model are less unbalanced and vary much less over time compared to the mean-variance portfolio weights. The out of sample returns generated by the GA portfolio model have a substantially higher mean and lower volatility compared to the standard mean variance portfolio strategy and Bayesian approach. Further, GA achieves a much higher Sharpe ratio compared to the traditional methods.

The paper is organized as the following. In next section, I will describe the traditionally used methods, mean-variance analysis and Bayesian approach. Section 3 I will introduce the framework of Genetic algorithms in solving optimal portfolio selection problem. The empirical study is illustrated in section 4, and the paper is concluded in section 5.

2. The Traditional Methodologies

2.1 The Standard Mean-Variance Analysis Model

Markowitz (1987, 1991) mean-variance efficiency is the classic paradigm of modern finance for allocating capital among risky assets. The optimal portfolio of N risky assets, w, is given by the solution of the following optimization problem,

$$(1) \qquad \max_{w} w'm - \left(\frac{d}{2}\right) w'\Sigma w$$

where m is the N-vector of the true expected excess returns, Σ is the $N \times N$ covariance matrix, and the scalar d is the risk aversion parameter. The solution to this problem is

$$(2) \qquad w = \frac{1}{d}\Sigma^{-1} m$$

A fundamental assumption of the standard mean-variance portfolio selection model in (1) is that the investor knows the true expected returns. In practice, however, the investor has to estimated expected returns. Denoting the estimate of expected return by $\hat{m}$, the actual problem that the investor solves is

$$\text{(3)} \qquad \max_{w} w'\hat{m} - \left(\frac{d}{2}\right) w'\Sigma w$$

subject to $\sum_{i} w_i = 1$.

The problem in (3) coincides with (1) only if expected returns are estimated with infinite precision, that is, $\hat{m} = m$. In reality, however, expected returns are notoriously difficult to estimate. As a result, portfolio weights obtained from solving (3) tend to consist of extreme positions that swing dramatically over time. Moreover, these optimal portfolios often perform poorly out of sample even compared to portfolios selected according to some simple ad hoc rules, such as holding the value-weighted or equally-weighted market portfolio.

2.2 The Bayesian Approach

The foundation for the Bayesian approach was provided by Savage (1954). Early applications of this approach can be found in Klein and Bawa (1976), Jorion (1985, 1986). More recent applications include Pastor (2000) and Pastor and Stambaugh (2000).

Let U(R) be the utility function, where R is the return from the investment, and $g(R \mid q)$ the conditional density (likelihood) of asset returns given parameter q . In the setting of this paper, q is the vector of the expected returns of the risky assets. More generally, it can include the covariances of the asset returns. If the parameter q is known, then the conditional expected utility of the investor is

$$\text{(4)} \qquad E[U(R)|q] = \int U(R) g(R|q) dR$$

In practice, however, the parameter q is often unknown and needs to be estimated from data, i.e., there is parameter uncertainty. In the presence of such parameter uncertainty, Savage's expected utility approach is to introduce a conditional prior (posterior) $p(q|X)$, where $X = (r_1, \ldots, r_T)$ is the vector of past return, such that the expected utility is given by

(5) $$E[U(R)|X] = E[E[U(R)|q]|X] = \iint U(R) g(R|q) p(q|X) dR dq$$

Let $p(q)$ is the unconditional prior about the unknown parameter. Then the posterior density given X is

(6) $$p(q|X) = \frac{\prod_{t=1}^{T} g(r_t|q) p(q)}{\int \prod_{t=1}^{T} g(r_t|q) p(q) dq}$$

and the predictive density, given X, is

(7) $$g(R|X) = \int g(R|q) p(q|X) dq = \int g(R|q) \frac{\prod_{t=1}^{T} g(r_t|q) p(q)}{\int \prod_{t=1}^{T} g(r_t|q) p(q) dq} dq$$

Using the predictive density, the expected utility of the investor is given by

(8) $$E[U(R)|X] = \int U(R) \left(\int g(R|q, X) p(q|X) dq \right) dR = \int U(R) g(R|X) dR$$

Thus the key to the Bayesian approach is the incorporation of prior information and the information from data in the calculation of the posterior and predictive distributions. The effect of information on the investor's decision comes through its effect on the predictive distribution.

However, in the Bayesian approach the investor is implicitly assumed to be neutral to parameter and/or model uncertainty. That is, in the Bayesian approach the investor is uncertainty neutral is best seen through equation (8). The middle expression in the equation suggests that parameter and/or model uncertainty enters the investor's utility through the posterior $p(q|X)$, which can affect the investor's utility only through its effect on the predictive density $g(R|X)$. In other words, as far as the investor's utility maximization decision is concerned, it does not matter whether the overall uncertainty comes from the conditional distribution

$g(R|q)$ of the asset return or from the uncertainty about the parameter $p(q|X)$, as long as the predictive distribution $g(R|X)$ is the same. In other words, if the investor were in a situation where there is no parameter/model uncertainty, say, because the past data X could be used to identify the true parameter perfectly, and the distribution of asset returns is characterized by $g(R|X)$, then the investor would feel no different. In particular, there is no meaningful separation of risk aversion and uncertainty aversion. In this sense, the investor is uncertainty neutral.

The problem facing a Bayesian investor is to estimate the N-dimensional vector of means m from the i.i.d. population $y_t \sim N(m, \Sigma), \quad t = 1, \ldots, T$. The key result in Jorion (1986) can be summarized as follows. Assume the following two conditions: (i) Investors have an informative prior on m of the form

(9) $$p(m|\overline{m}, n_m) \propto \exp\left[-\frac{1}{2}(m-\overline{m})'(n_m \Sigma^{-1})(m-\overline{m})\right]$$

with $\overline{m}$ being the grand mean and n_m giving an indication of prior precision (or tightness of the prior); (ii) the density $p(n_m|m, \overline{m}, \Sigma)$ is a Gamma function. Then, the predictive density for the returns $p(r|y, \Sigma, n_m)$, conditional on Σ and the precision n_m is a multivariate normal with predictive mean, m, equal to

(10) $$m = (1 - l)\hat{m} + l m_{MIN} 1_N$$

where $\hat{m}$ is the sample mean, m_{MIN} is the minimum-variance portfolio,

(11) $$l = \left(\frac{n_m}{T + n_m}\right) = \frac{N+2}{(N+2) + T(\hat{m} - m_{MIN} 1_N)' \Sigma^{-1} (\hat{m} - m_{MIN} 1_N)}$$

and covariance matrix

(12) $$V[r]=\Sigma\left(1+\frac{1}{T+n_m}\right)+\frac{n_m}{T(T+1+n_m)}\frac{1_N 1_N^{'}}{1_N^{'}\Sigma^{-1}1_N}$$

note that for $n_m \to \infty$, the predictive mean is the common mean represented by the mean of the minimum variance portfolio.

We now ready to determine the optimal portfolio weights using the Bayesian estimators. Let us assume that we know the variance covariance matrix and that only the expected returns are unknown. In the case where a risk free asset is not available, we know that the classical mean-variance portfolio is given by (2). Substituting the empirical Bayesian estimator of mean in (2), one can show that the optimal weights can be written as follows:

(13) $$w_{BS} = l\,w_{MIN} + (1-l)w_{MV}$$

where the minimum-variance portfolio weights, which ignore expected returns altogether is

(14) $$w_{MIN} = \frac{1}{A}\Sigma^{-1}1_N$$

and the mean-variance portfolio weights formed using the maximum-likelihood estimates of the expected return are

(15) $$w_{MV} = \frac{1}{d}\Sigma^{-1}(\hat{m}-\overline{m})$$

3. The Genetic Algorithms Technique

The conception of GA in its current form is generally attributed to Holland (1975). GA starts with a *population* of randomly generated solutions called candidates to explore the solution space of a problem. An initial population is created containing a predefined number of individuals or solutions, each represented by a genetic string incorporating the variable information. Then GA searches for better solutions through a number of iterations called *generations*. Each individual has an associated fitness measure. The performance of each solution is evaluated by a fitness criterion, typically representing an objective value. The concept that fittest individuals in a population will produce fitter offspring is then implemented in order to reproduce the next population. In each generation, relatively good solutions have a higher chance to be selected for reproduction of offspring by genetic operators—crossover and mutation. Therefore, selected individuals are chosen for reproduction (or crossover) at each generation, with an appropriate mutation factor to randomly modify the genes of an individual, in order to develop the new population. The result is another set of individuals based on the original subjects leading to subsequent populations with better individual fitness. Therefore, the algorithm identifies the individuals with the optimal fitness values, and those with lower fitness will naturally get discarded from the population.

As indicated above, GA consists of four main stages: evaluation, selection, crossover and mutation. The *evaluation* procedure measures the fitness of each individual solution in the population and assigns it a relative value based on the defining optimization criteria. The *selection* procedure randomly selects individuals of the current population for development of the next generation. Various alternative methods have been proposed but all follow the idea that the fittest have a greater chance of survival. The *crossover* procedure takes two selected individuals and combines them about a crossover point thereby creating two new individuals. The *mutation* procedure randomly modifies the genes of an individual subject to a small mutation factor, introducing further randomness into the population. This iterative process continues until the termination criteria is met. For instance, a number of generations without fitness improvement occur, which applies that convergence slows to the optimal solution.

3.1 The Dynamic Genetic Algorithm Process

I will use a static-single-state model as a starting point. The optimization procedure for a single-state model will be illustrated by the following. For example, there are five assets in the portfolio and 10 candidates in each period. The candidates differ in terms of percentages of the five assets held. The candidate with the best performance in the first time period (the one with the highest return) "survives" as the parent to create a new group of populations in the second time period. This new set of candidates hold percentages which are related to the percentages held by the survivor. In the second time period, the new parent will be selected again according to the fitness criterion. The new survivor holds the best portfolio after the random return is generated. This procedure continues until the maximum iteration numbers is reached. For each period, the asset returns are randomly generated.

Single-state portfolio optimization model possesses several drawbacks. For examples, the risk is inconsistent over time. In reality, there could be tens or a hundred possibilities what tomorrow could turn out to be. In each possibility, asset returns could be different. For example, agents know there are ten possibilities that economy could become in the next period, and they know what the asset returns are in each possibility, but, they do not know which state will happen, rather they know only the probability of each state occurring. This illustrates a more realistic situation agents will face before they make their investment decision. Thus, it is vital to incorporate this kind of uncertainty into the model and develop a stochastic process for portfolio optimization problem. The multi-stage stochastic model (Mulvey (1997)) captures dynamic aspects of asset allocation problem. It is a quantitative model that integrates asset allocation strategies in a comprehensive fashion.

The stochastic nature incorporates scenario analysis into the model. For example, a model with only two states (or scenarios) in each time period (except the initial time period), each scenario depicts a single path over a multi-stage planning period, sharing the same history. In this system, scenarios are defined by the changes of market index. For example, we can simply set two scenarios as: (1)

the market index has dropped and (2) the market index has risen. Suppose we have to optimize a portfolio, with 1 denoting cash and the others may representing any investment instruments such as bonds, funds, futures and stocks. Let the entire planning horizon T be divided into a number of periods as $t=\{1, 2, 3, \ldots, T\}$. Investment decisions are made at each period. Each period may have different scenarios. A graphical scenario tree can be constructed to visualize the optimal dynamic balanced investment strategy for asset allocation. Figure 1 depicts a scenario tree with two scenarios and three time periods.

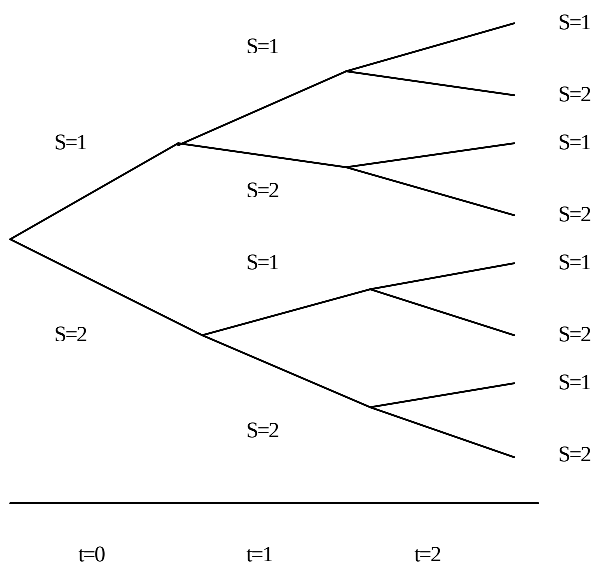

Figure 1: A Scenario Tree

In this paper, I incorporate GA methodology into a multiple-state of the world model. The solution procedure is the following: first, I will discrete the choice space into multiple subspaces. Each subspace represents one possibility that the world could occur. Again, I will use a random generating process to generate multiple possibilities in each time period, while guaranteeing that the sum of the possibilities in each period is equal to 1. Therefore, in each time period before an agent makes his decision, except for the initial time period, there are multi-states in front of them. They do not know which state will come to be true, they only know the probabilities that each state occurs. Notice that asset returns are different in different states.

3.2 The Robustness of the GA

A GA is a search technique, inspired by evolutionary mechanisms and theories, natural selection and genetics and presenting characteristics which, for specific problems, make this technique superior to the traditional heuristic methods based on calculus or random or enumerative procedures. (Grefenstette (1991)). A Genetic Algorithm is driven by the control parameters: number of generations, and size of population. Those parameters will directly affect the performance of GA. Therefore, how one is to decide the parameter values is an important issue in the use of GA method. In the previous work on GA, the parameter values are randomly assigned. However whether the results generated by the application of GA to a specific problem are conditioned by the value assigned to these parameters, becomes a main issue for research. Davis (1991) presents an excellent review of the mathematical foundations that support GA functioning. Other references to mathematical foundations of GA are: Whitley (1992); Stephens et al (1999); Grefenstette and Baker (1989). Their works showed that when GA is properly designed, they are better suited than other traditional techniques.

First, I shall test the speed of convergence to find a sufficient number of generations. Figure 2 shows the statistic for the generation size from zero to fifty. The horizontal is the number of generations and the vertical line is the value of the fitness (the measure of fitness are portfolio return at end of each period and standard deviation over all generations)

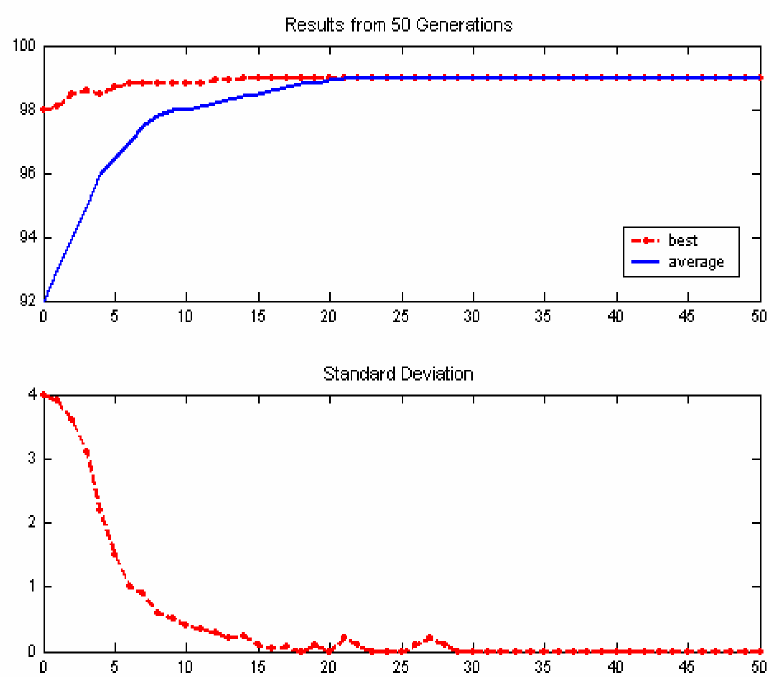

Figure 2: Effect of Generations

From the top one of Figure 2, we can see that the total value of the fitness converge after 18 generations. The bottom one shows that the standard deviation approaches to zero after 30 generations. Therefore, with 30 generations is more than enough in terms both measures of fitness. Next, I will test the effect of the population. I am going to try with 3 population size, 30, 40 and 50.

Figure 3 gives the statistic result for population size. In the figure, the horizontal line is the number of generations as in Figures 2, and the vertical is the fitness value. The dashed dotted line represents the population size with 30, which begin to converge after 8 generations. The solid line represents the population size with 40, which begins to converge after 18 generations. And the dashed line represents the population size with 50, which begins to converge after 10 generations. The GA seems quite robust to values assigned to the parameters "number of populations" and "number of generations" considered in the execution. The combination of 30 generations and 40 populations seems the most suitable combination for obtaining robust results. Therefore, for the empirical study in the next section, I will set the generation size at 30 and population size at 40 when applying GA approach.

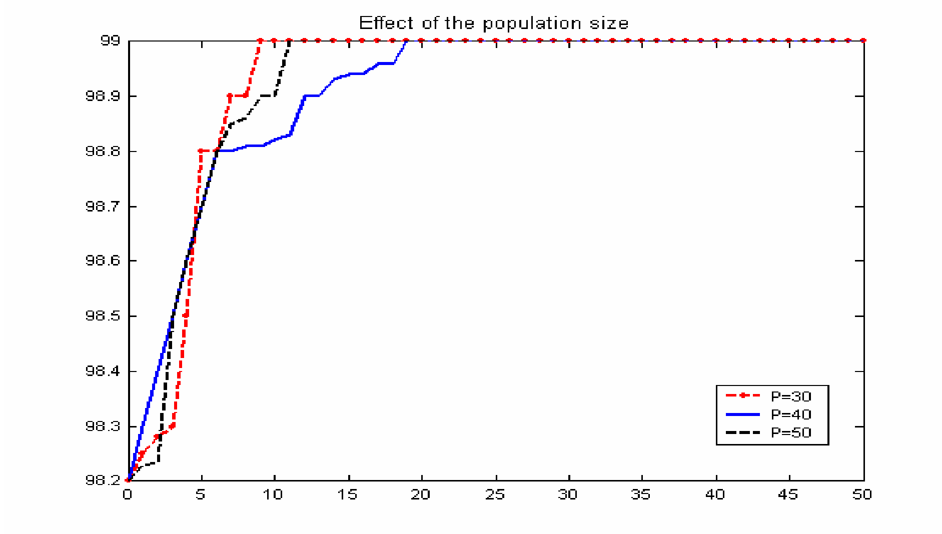

Figure 3: Effect of Populations

4. The Data and Empirical Results

The empirical study is based on the data set from MSCI (Morgan Stanley Capital International). It includes the total return equity indices of Canada, France, Germany, Japan, UK, and the USA. Equity returns are based on the month-end US-dollar value of the equity index for the period January 1970 to December 2004. Monthly excess returns are calculated using the 3 month T-Bill rate as the risk-free rate of return. To assess the performance of the different portfolio models, we determine the weights from each model based on a window of 60[3] months and then calculate the return from holding this portfolio in the 61st month. In each case, the out of sample period is from 1/1975 to 12/2004. A rolling

[3] I set T=60 because the estimation is done using a rolling-window of 60 months.

window of length T is used to estimate the optimization input parameters. E.g., for T=60, portfolio weights are first based on the estimation period from 1/70 to 12/74. Using the returns of 1/75, the first out of sample portfolio return can be calculated. Then the estimation period is rolled on month forward, and the next portfolio composition is based on 2/70 to 1/75. This procedure results in a total of 348 out of sample returns. Summary statistics for the indexes of the data are provided in Table 1.

Table 1: Summary Statistics of the Data

	Panel A: Summary Statistics	
	Mean	Std. Dev.
Canada	0.41	4.8
France	0.92	6.98
Germany	0.51	6.32
Japan	0.86	7.11
UK	0.78	6.28
USA	0.75	3.9
US Bonds	0.22	1.86
Euro Bonds	0.25	1.42

	Panel B: Unconditional Correlations of Excess Returns							
	Canada	France	Germ.	Japan	UK	USA	US Bs.	Euro Bs.
Canada	1	0.44	0.28	0.21	0.53	0.68	0.26	0.31
France		1	0.57	0.39	0.46	0.27	0.19	0.22
Germany			1	0.33	0.41	0.42	0.24	0.26
Japan				1	0.39	0.21	0.18	0.15
UK					1	0.55	0.22	0.23
USA						1	0.39	0.45
US Bs.							1	0.88
Euro Bs.								1

This table gives the summary statistics for the monthly returns on the eight indices and the unconditional correlations of excess returns.

To assess the performance of the different portfolio models, I compute the average out of sample means, volatilities and Sharpe ratios of each strategy— mean-variance analysis, Bayesian approach and Genetic algorithms. For the Genetic algorithms method, I consider both single-state GA and multi-states GA (I use 5 states represent the multi-states case).

The results are reported in Table 2. Compared to the mean-variance strategy in which historical mean returns $\hat{m}$ are taken to be the estimator of expected returns m, the portfolios constructed using the model that allows for parameter uncertainty exhibit uniformly higher means and lower volatility. Especially, the Genetic algorithms have higher returns and lower variance relative to both Mean-variance method and Bayesian approach. Genetic algorithms with multi-states dominate the single-state GA in terms of mean and variance. The Sharp ratio is 0.1816 for the multi-states GA, which is the highest value among the four. The Sharp ratios are very close for mean-variance and Bayesian approach, which is 0.1435 and 0.1428 respectively. The Sharp ratio for single-state GA is 0.1632 although lower than the multi-states GA, still outperform the traditional methods.

Table 2: Empirical Results

	Out of Sample Performances		
	Mean	Std.Dev.	Sharp Ratio
Mean-Variance	0.5238	4.3561	0.1435
Bayesian Approach	0.5122	4.3359	0.1428
GA-Single state	0.5847	3.9836	0.1632
GA-Multi-state	0.5936	3.6589	0.1816

To analyze the effect of uncertainty on the individual weights in the risky portfolio, I report in Figure 4 the percentage weight allocated to the US index from January 1975 to July2001. The dashed line refers to the percentage of wealth allocated to the US index implied by the mean-variance portfolio implemented using historical estimates. The other two lines refer to portfolios obtained by GA methods. The dashed dotted line represent the portfolio weights from the Genetic algorithms of the single-state of the world model, and the solid line represent the GA of the multi-states of the world model incorporating uncertainties. I find that the portfolio weights from the optimization incorporating future uncertainty has less extreme positions and the portfolio weights vary much less over time compared to the weights for the classical mean-variance portfolio. As a consequence, the more uncertainty, the less extreme are the portfolio weights. There exhibit a precaution effect when investors facing uncertainty.

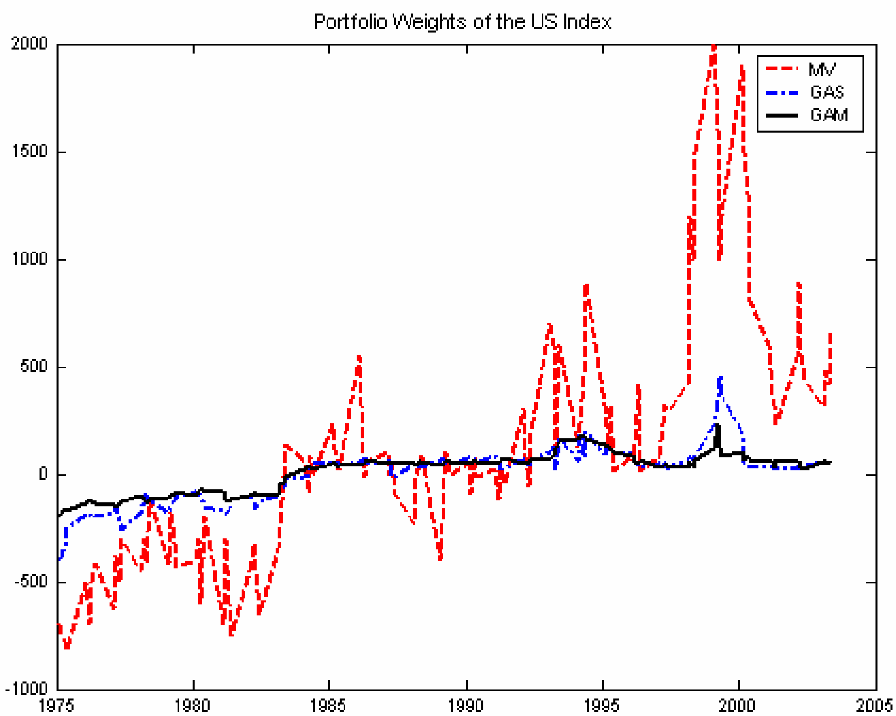

Figure 4: Portfolio Weights of the US over Time

5. Conclusions

In this paper, I presented a decision-making process that incorporates Genetic algorithms into a state dependent stochastic asset pricing model—a stochastic multi-sates of the world optimization system..

Traditional mean variance portfolio optimization assumes that the parameters that the expected returns used as inputs to the model and obtained using maximum likelihood estimation are known with perfect precision. In practice, however, it is extremely difficult to estimate expected returns precisely. And, portfolios that ignore estimation error have very poor properties: the portfolio weights have extreme values and fluctuate dramatically over time the Bayesian approach that is traditionally used to deal with estimation error assumes that investors have only a single prior and has computational difficulty to incorporate future uncertainty into the model.

In this paper, I have shown how one can extend the classical mean-variance portfolio optimization model and traditional Bayesian approach to allow for future uncertainties and reduce the estimation risk by Genetic algorithms method. The advantage of GA is that it solves the model by forward-looking and backward-induction, which incorporates both historical information and future uncertainty when estimate the asset returns. It significantly improves the accuracy of mean return estimation and thus the model performance. In addition, GA could handle a large variety of future uncertainties, which overcome the computational difficulties in traditional Bayesian approach.

Using the MSCI data set, I find that the portfolio weights using the GA model are less unbalanced and fluctuate much less over time compared to the standard mean-variance portfolio weights and also the portfolios from the Bayesian approach. Further, allowing for uncertainty about asst returns in the GA model leads to a higher out of sample Sharpe Ratio than otherwise. Empirical results showed that the stochastic multi-states Genetic Algorithm significantly improve the model performance over the static-single-state model. For the multi-state of the world model, an interesting feature is that risk tolerance decreases with uncertainties. In overall, the standard deviation of the portfolio weights gets smaller in the stochastic version of the model. There exists a precautionary effect when future uncertainty is introduced.

References

Bauer, R. (1994). "Genetic algorithms and investment strategies," John Wiley and Sons, Inc.

Berger, A. J. (1995). "Solving global optimization problems in long-term financial planning," *Statistics and Operations Research Technical Report*, Princeton University, January,1995.

Best, M. J., and R. R. Grauer (1991). "On the sensitivity of mean-variance-efficient portfolios to changes in asset means: some analytical and computational results," *Review of Financial Studies*, 4, 315-42.

Chopra, V. K., and W. T. Ziemba (1993). "The effect of errors in means, variances, and covariances on optimal portfolio choice," *Journal of Portfolio Management*, 19, 6-11.

Collin, M.(1994). "Genetic algorithms for financial modeling," John Wiley and Sons, Inc.

Fichter, D. P. (2000). "Application of genetic algorithms in portfolio optimization for the oil and gas industry," Proc. of the SPE Annual Technical Conference, Texas.

Goldberg, D. E. (1989). "Genetic algorithms in search, optimization and machine learning," Addison-Wesley, 1989.

Grefenstette, J. J. and J. J. Baker (1989). "How genetic algorithms work: a critical look at implicit parallelism," Proc. of the Third International Conference on Genetic Algorithms, 20-27.

Grefenstette, J. J. (1991). "Conditions for implicit parallelism," *Foundations of Genetic Algorithms*, 252-61.

Grinblatt, M. and Sheridan T. (1989). "Financial markets and corporate strategy," McGraw-ill Companies, Inc.

Holland, J. H. (1975). "Adaptation in natural and artificial systems: an introductory analysis with applications to biology, control and artificial intelligence," University of Michigan Press.

Jorion, P. (1985). “International portfolio diversification with estimation risk,” *Journal of Business*, 58, 259-78.

Jorion, P. (1986). “Bayes-stein estimation for portfolio analysis,” *Journal of Financial and Quantitative Analysis*, 21, 279-92.

Kendrick, D. A., Mercado, P. R., and H. M. Amman, “Computational economics.” The University of Texas, Austin, Texas, January 2004, http://www.eco.utexas.edu/compeco

Klein, R. W., and V. S. Bawa (1976). “The effect of estimation risk on optimal portfolio choice,” *Journal of Financial Economics*, 3, 215-31.

Knight, F. (1921). “Risk, uncertainty and profit,” Houghton Mifflin, Boston.

LeRoy, S. F. and Werner (2001). “Principles of financial economics,” Cambridge University Press 2001.

Marlowitx, H. M. (1952). “Portfolio selection,” *Journal of Finance*, 7, 77-91.

Markowitz, H. (1987). “Mean-variance analysis in portfolio choice and capital markets,” Cambridge Press.

Markowitz, H. (1991). “Portfolio selection: efficient diversification of investments,” New York Press, 1959.

Michaud, R. O. (1989). “The Markowitz optimization enigma: is optimized optimal,” *Financial Analysts Journal*, 45, 31-42.

Michaud, R. O. (1998). “Efficient Asset Management,” Harvard Business School Press, Boston.

Mulvey, J. M. (1997). “Strategic financial risk management and operations research,” *European Journal of Operational Research*, 1997, 1-16.

Pastor, L. (2000). “Portfolio selection and asset pricing models,” *Journal of Finance*, 55, 179-23.

Pastor, L., and R. F. Stambaugh (2000). “Comparing asset pricing models: an investment perspective,” *Journal of Financial Economics*, 56, 335-81.

Schaerf, A. (2002). "Local search techniques for constrained portfolio selection problems," *Computational Economics*, 20, 177-90.

Stephens, C., Waelbroeck, H. and R. Aguirre (1999). "Schemata as building blocks: does size matter?" *Foundations of Genetic Algorithms*, 117-133.

Whitley, D. (1992). "Deception, dominance and implicit parallelism in genetic search," *Annals of Mathematics and Artificial Intelligence*, 5, 1992, 49-78.