On-the-Job Search and Wage Rigidity in a General Equilibrium Business Cycle Model

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Abstract

This paper develops and solves a general equilibrium business cycle model with on-the-job search and wage rigidity arising from long-term labor contracts. Labor search models without these features have been criticized for failing to generate procyclical movements in job vacancies and insufficient volatility in the ratio of vacancies to unemployment. When calibrated to the match the magnitude and rates of labor turnover, the model successfully generates these patterns and matches other properties of labor turnover data.

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1   Introduction

The cyclical dynamics of the labor market are now commonly modeled with the search and matching approach of Mortensen and Pissarides (1994). However, Shimer (2004a) shows that such models do a poor job matching the dynamics of vacancies in employment data. They are unable to generate either the strongly procyclical behavior of vacancies or the high variance of the ratio of vacancies to unemployment. The source of this limitation within the model is that changes in payoffs from variations in aggregate productivity are not captured sufficiently by firms to create much variation in their incentive to post vacancies for new jobs. The expected value to firms of creating a new job does not increase sufficiently during booms nor fall during recessions.

Two types of modifications to the model have been suggested in the literature as potential sources for overcoming this limitation: wage rigidity and on-the-job search. In its standard form, the matching approach of Mortensen and Pissarides uses Nash bargaining with a fixed weight to divide the surplus of a job between the worker and the firm. Hall (2003) suggests that wage rigidity would make firms' share of the surplus fluctuate more procyclically with changes in aggregate productivity thereby creating greater procyclical variation in the incentive to post vacancies for new jobs. Shimer (2003) models wage rigidity and adds on-the-job search. He argues that on-the-job search also contributes to increasing a firm's payoff with an increase in aggregate productivity. A positive productivity shock gives employed workers greater incentive to search on the job thereby increasing the number of searching workers in the economy and raising the probability that a firm will be successful in meeting a worker in the search process. This lowers the cost of filling a vacancy during a boom thus creating the incentive for firms to post more vacancies.

This paper develops and solves a general equilibrium business cycle model with both on-the-job search and wage rigidity to evaluate the quantitative impact of those mechanisms on labor turnover data. The paper fits into the literature in which labor search models are embedded within general equilibrium models; Andolfatto (1996) and Merz (1995, 1999) are early examples. That literature has since been extended to make the models more realistic by incorporating endogenous job destruction decisions and persistence in firm-specific productivity levels (den Haan, Ramey, and Watson, 2000; Gomes, Greenwood, and Rebelo, 2001; Hussey, 2004).

A number of other papers have documented the impact of wage rigidity and on-the-job search on the dynamics of vacancies and unemployment, though they have done so in the context of deterministic models with exogenous job destruction (Hall, 2003; Nagyapál, 2004b; Shimer, 2003 and 2004b). Shimer (2004b) also introduces heterogeneity in the productivity of jobs, but the distribution is exogenous and fixed. Krause and Lubik (2004) is the only paper that I am aware of that models on-the-job search in a general equilibrium business cycle model. Those authors develop a model with two types of jobs, high productivity and low productivity. Workers in low productivity jobs search on-the-job for a higher productivity match, with their search intensity chosen endogenously. There is no wage rigidity in the model: the division of the surplus

of a match between a firm and a worker is renegotiated each period by Nash bargaining. Job destruction is exogenous. Krause and Lubik find that adding on-the-job search to a business cycle model is sufficient, even without wage rigidity, to generate strongly procyclical vacancies and variation in ratio of vacancies to unemployment that is comparable to that in the U.S. data, though their calibration seems to achieve this while underpredicting the volatility of employment. Because their model allows firms to choose whether to create high productivity or low productivity jobs, procyclical on-the-job search gives firms the incentive to create proportionately more high productivity jobs in response to an exogenous increase in the aggregate productivity of all jobs. This compositional effect implies that on-the-job search serves as a mechanism for propagating aggregate shocks, with greater persistence in output growth.

The model of this paper differs from that of Krause and Lubik in several ways. It contains both on-the-job search and wage rigidity. Wage rigidity comes from long-term labor contracts of the form found in Cahuc, Postel-Vinay, and Robin (2004) and Dey and Flinn (2003), with some modifications to accommodate the stochastic environment of this paper. Those modifications are similar to the wage setting mechanism in Hall (2003). Unlike Krause and Lubik, job separation is endogenous and firms cannot choose the productivity of a new job before making the decision to invest in job creation. The model is solved using a numerical method similar to that of Hussey (2004b).

Preliminary results are reported in the final section of the paper. The model is able to generate highly procyclical vacancies and large variation in the vacancy-unemployment ratio. Job-to-job transitions increase substantially in a boom. Correlations between various measures of the labor market are of similar magnitude to those in the data, though under the current calibration, the model may be generating excessive volatility in labor turnover. Impulse response functions suggest that fluctuations in the job finding rate are a more important component of cyclical variations in unemployment than are changes in the job separation rate, as has been observed by Shimer (2004b). The simulated data also indicate that on-the-job search and wage rigidity serve as a significant internal propagation mechanism, similar to the result of Krause and Lubik (2004).

The rest of the paper is organized as follows. Section 2 describes the general equilibrium model with on-the-job search and wage rigidity from long-term labor contracts. Section 3 discusses issues of specification and calibration. Results are presented in Section 4.

2   The Model

2.1   Production, Matching, and Separation

This economy is populated by a continuum of workers of unit mass and a continuum of potential firms of infinite mass. A match between a firm and worker produces output y_{it} in period t according to the production

function

$$y_{it} = z_t a_{it} k_{it}^{\alpha}, \tag{1}$$

where z_t is an aggregate productivity shock, a_{it} is the productivity level specific to match i, and k_{it} is the capital rented for the match. Each firm owns its productive process, or job, characterized by productivity level a_{it}. Productivity $a_{it} \in [\underline{a}, \overline{a}]$, follows a first-order Markov process with law of motion described by the conditional distribution $\tau(a_{it}|a_{it-1})$. If a match is separated, the firm decides whether to go out of existence or continue to exist with its productivity level a_{it} and search for another worker to refill the job. Thus, a firm and a job are synonymous, and i indexes existing jobs. If the job is filled (the firm is matched to a worker), the firm produces y_{it}. In this way, the model distinguishes between job destruction and match separation and can accommodate differences between jobs flows and worker flows, as exists in the data. In the notation that follows, time subscripts are dropped and primes indicate values in the subsequent period.

The model allows for both exogenous and endogenous separation of matches. At the beginning of each period, a fraction p of all matches are exogenously separated. For all other matches, the firm and worker observe z and their productivity level a_i and choose jointly whether to continue the match through the current period or to separate. The worker's payoff if the match continues is

$$w_i - I_i \kappa + W^{wm}(a_i, w_i, I_i, z, X), \tag{2}$$

where w_i is the wage received by the worker, I_i indicates whether the worker chooses to search on the job, κ is the cost of searching on the job, $W^{wm}(a_i, w_i, I_i, z, X)$ is the expected present value of future payoffs for a matched worker, and X_t is a vector of the endogenous aggregate state variables to be specified below. The worker's optimal choice of I_i is defined by

$$I_i = \begin{cases} 1 & \text{if } W^{wm}(a_i, w_i, 1, z, X) - W^{wm}(a_i, w_i, 0, z, X) \geq \kappa \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

For the solution procedure, the wage is bounded as $w_i \in [\underline{w}, \overline{w}]$ though in practice the limits are chosen such that the wages set by firms and workers always fall within the interval. The firm's payoff if the match continues is

$$\max_{k_i} z a_i (k_i)^{\alpha} - r k_i - w_i + W^{fm}(a_i, w_i, I_i, z, X), \tag{4}$$

where r is the rental rate of capital, k_i is amount of capital rented, and $W^{fm}(a_i, w_i, I_i, z, X)$ is the expected present value of future payoffs for the firm.

If the match is separated, the worker's payoff is

$$b + W^{wu}(z, X), \tag{5}$$

where b is the benefit obtained in the current period from being unemployed and $W^{wu}(z, X)$ is the expected present value of an unemployed worker's future payoffs. With separation, the firm chooses whether to post

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a vacancy and search or go out of existence. If it ceases to exist, its payoff is zero. If it searches, it pays a cost ν to post a vacancy. The expected present value of future payoffs to being unmatched with productivity level a_i is $W^{fu}(a_i, z, X)$. Thus, when a match is separated, the firm's payoff is

$$\max\left\{W^{fu}(a_i, z, X) - \nu, 0\right\}. \tag{6}$$

The joint payoff to firm and worker to continue their match is

$$za_i\left(k_i^*\right)^\alpha - rk_i^* + W^{fm}(a_i, w_i, I_i, z, X) - I_i\kappa + W^{wm}(a_i, w_i, I_i, z, X), \tag{7}$$

where k_i^* is the value of k_i that maximizes (4). Conditional on a wage w_i and a value of I_i, the payoff is increasing in a_i. Since the wage-setting mechanism described below allows the wage to be adjusted whenever it is advantageous to both firm and worker to do so, the monotonicity in a_i of the joint payoff implies there is a reservation value $\underline{a}^m$ such that the firm and worker will choose to maintain the match only if $a_i \geq \underline{a}^m$. A single wage, $\underline{w}$, is viable at the match destruction margin $\underline{a}^m$.[1] At $\underline{a}^m$ the payoff to maintaining the match is equal to the sum of the payoffs to the firm and worker if the match is separated:

$$\begin{aligned}
z\underline{a}^m\left(k_i^*\right)^\alpha - rk_i^* + W^{fm}(\underline{a}^m, \underline{w}, \underline{I}, z, X) - \underline{I}\kappa + W^{wm}(\underline{a}^m, \underline{w}, \underline{I}, z, X) \\
= b + W^{wu}(z, X) + \max\left\{W^{fu}(a_i, z, X) - \nu, 0\right\}.
\end{aligned} \tag{8}$$

After separation of matches at the beginning of the period, new firms come into existence, and searching firms and workers form new matches during the period. Vacancies can be posted by unmatched firms: those that existed in the previous period but were unmatched at the end of the period, those that were separated at the beginning of the current period, and new firms. Firms cannot post a vacancy while they are matched to a worker. Since $W^{fu}(a_i, z, X)$ is increasing in a_i, there is a reservation value $\underline{a}^u$ such that unmatched firms will choose to continue to exist and post a vacancy only if $a_i \geq \underline{a}^u$. The searching margin $\underline{a}^u$ satisfies

$$-\nu + W^{fu}(\underline{a}^u, z, X) = 0. \tag{9}$$

A potential new firm pays a cost χ to purchase a productive process. Upon paying that cost, it receives a productivity draw a_i from the time-invariant distribution $h(a)$. If $a_i \geq \underline{a}^u$, the firm chooses to come into existence and post a vacancy. Additional potential firms pay for a draw of a_i as long as the expected payoff exceeds the cost:

$$\int_{\underline{a}_t^u}^{\overline{a}}\left[-\nu + W^{fu}(a_i, z, X)\right]h(a_i)da_i \geq \chi. \tag{10}$$

If $\chi > 0$, in equilibrium $\underline{a}^u < \overline{a}$ so new firms come into existence and separated firms continue to exist at a range of productivity levels a_i.

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[1] $\underline{w}$ is the wage that will be set at the minimum productivity level $\underline{a}^m$, but it is not necessarily the minimum wage among matched workers.

Employment, unemployment, and the number of firms are measured as follows. Let $f(a_i, w_i)$ be the distribution of firms that remain matched after match separation has occurred in the current period. Aggregate employment is

$$N = \int_{\underline{a}^m}^{\overline{a}} \int_{\underline{w}}^{\overline{w}} f(a_i, w_i) dw_i da_i. \tag{11}$$

Since the quantity of workers is normalized to one, the mass of unemployed workers is $U = 1 - N$. Unemployed workers are those who were not matched at the end of the previous period and those whose match was separated at the beginning of the current period. The number of on-the-job searchers searchers is

$$Q = \int_{\underline{a}^m}^{\overline{a}} \int_{\underline{w}}^{\overline{w}} I_i f(a_i, w_i) dw_i da_i. \tag{12}$$

The total number of workers searching for a job is $S = U + Q$. For firms that post a vacancy, let $g(a_i)$ be the distribution of productivity levels a_i. The number of vacancies is

$$V = \int_{\underline{a}^u}^{\overline{a}} g(a_i) da_i. \tag{13}$$

If L is the number of potential new firms that pay the cost χ to get a productivity draw, then the part of vacancies that corresponds to new firms is $V^n = L \int_{\underline{a}^u}^{\overline{a}} h(a_i) da_i$.

The number of new matches $m(S, V)$ is increasing in the number of searching workers and the number of firms that post vacancies. This relationship is described by a Cobb-Douglas matching function (Mortensen and Pissarides, 1994)

$$m(S, V) = \ell S^{\psi} V^{1-\psi}. \tag{14}$$

The probability that a searching worker gets matched is $p^S = m(S, V)/S$; the probability that a searching firm gets matched is $p^V = m(S, V)/V$ Unemployed workers that get matched accept the match and negotiate a wage $\widehat{w}_i$ with the firm. (Variables with "hats," as is $\widehat{w}_i$, indicate values determined after matching has occurred.) A worker searching on the job who gets decides whether to switch firms and negotiates a new wage. Let $\widehat{w}_i$ be the new wage if the worker stays with her current firm and $\widehat{w}_j$ the wage if she switches firms. A firm whose worker accepts a new match enters the following period unmatched. Matches formed during the period begin producing in the following period. Because on-the-job searchers reject some matches, the probability that a vacancy gets filled q^V is

$$q^V = \frac{1}{V} \int_{\underline{a}^u}^{\overline{a}} \left\{ p^V \frac{Q}{S} \left[\frac{1}{Q} \int_{\underline{a}^m}^{a_i} \int_{\underline{w}}^{\overline{w}} I_j f(a_j, w_j) dw_j da_j \right] + p^V \frac{U}{S} \right\} g(a_i) da_i. \tag{15}$$

In this equation $p^V U/S$ is the probability and unmatched with productivity a_i is matched to an unemployed worker, $p^V Q/S$ is the probability the firm is matched to a worker searching on the job, and the term in square brackets is the probability that worker accepts the job. The job-to-job transition rate in the economy is

$$T = \frac{p^S}{N} \int_{\underline{a}^m}^{\overline{a}} \int_{\underline{w}}^{\overline{w}} \left[\frac{1}{V} \int_{a^i}^{\overline{a}} g(a_j) da_j \right] I_i f(a_i, w_i) dw_i da_i. \tag{16}$$

Workers pool their incomes each period and choose consumption in period t to maximize the expected discounted sum of the utility of a representative worker as

$$E\left[\sum_{t=0}^{\infty}\beta^t u(C)\right] = E\left[\sum_{t=0}^{\infty}\beta^t \frac{C^{1-\gamma}-1}{1-\gamma}\right] \tag{17}$$

subject to the budget constraint

$$C + K' = H + bU + (r+1+\delta)K, \tag{18}$$

where β is the discount factor, C is aggregate consumption, K is the aggregate capital stock at the beginning of the current period, bU is the nontradeable units of consumption produced at home by unemployed workers, and δ is the depreciation rate. Aggregate income less search costs is

$$H = \int_{\underline{a}^m}^{\overline{a}}\int_{\underline{w}}^{\overline{w}}[za_i(k_i^*)^\alpha - rk_i^*]f(a_i,w_i)dw_i da_i - \nu V - \chi L. \tag{19}$$

2.2 Wage Setting

Wage setting occurs by a bargaining process similar to that modeled by Cahuc, Postel-Vinay, and Robin (2004) and Dey and Flinn (2003). Wages are established by long-term contracts that can be renegotiated only by mutual agreement of the firm and worker. As in those papers, renegotiation can be optimal when a worker searching on the job receives a job offer from a competing firm. Because of the stochastic environment in this paper, it is also sometimes mutually advantageous for a firm and worker to renegotiate the wage when there is a shock to the job's productivity.

I first consider wage adjustments due to changes in productivity. Long-term wage contracts imply $w_i' = \widehat{w}_i$ unless the observations of a_i' and z' at the beginning of the subsequent period make it mutually advantageous to the firm and worker to adjust the wage. Wage adjustment would be optimal under three scenarios. First, if at $w_i' = \widehat{w}_i$ the firm's payoff to staying in the match (eq. 4) is less than its payoff outside the match (eq. 6), but there exists a lower wage for which both the firm and the worker would choose not to separate the match, then they will lower w_i' to the firm's reservation wage, satisfying

$$z'a_i'(k_i^{*\prime})^\alpha - r'k_i^{*\prime} - w_i' + W^{fm}(a_i',w_i',I_i',z',X') = \max\left\{W^{fu}(a_i',z',X') - \nu, 0\right\}. \tag{20}$$

Second, if at $w_i' = \widehat{w}_i$ the worker's payoff to staying in the match (eq. 2) is less than her payoff outside the match (eq. 5), but there exists a higher wage for which both the firm and the worker would choose not to separate the match, then they will raise w_i' to the worker's reservation wage, determined by[2]

$$w_i' - I_i'\kappa + W^{wm}(a_i',w_i',I_i',z',X') = b + W^{wu}(z',X'). \tag{21}$$

[2] This situation can emerge because the bargaining process described below implies that a worker may be willing to accept a lower wage when a job has a higher productivity since the asset value of the job at the same wage would be greater. Thus, when a negative idiosyncratic productivity shock occurs, the worker's reservation wage at the new productivity level may be higher than it was last period.

This process of moving the wage from outside the set of mutually acceptable wages to the boundary is similar to the wage setting mechanism in Hall (2003). Finally, if at $w_i' = \widehat{w}_i$ the worker would choose to search on the job, but there exists a higher wage such that the worker would not search on the job and the firm's payoff would be higher, they will agree to raise the wage to the minimum value needed to induce the worker not to search. That wage would satisfy

$$W^{wm}(a_i', w_i', 0, z', X') > W^{wm}(a_i', \widehat{w}_i, 1, z', X') - \kappa. \tag{22}$$

After observing the shocks to productivity and adjusting wages, the matching process begins. When a searching worker receives a job offer, the wage $\widehat{w}_i$ is negotiated by the mechanism described in Cahuc, Postel-Vinay, and Robin (2004). Those authors use a Rubenstein (1982) game of alternating offers for wage negotiation when an worker meets a firm. They show that the outcome of the game is a generalized Nash bargaining solution in which the worker receives a constant share π of the match rent. Let $\widehat{w}_i^r$ be the maximum wage a firm would be willing to offer, the wage at which the firm would receive none of the match rent. Since matches formed this period come into existence as producing firms at the beginning of the next period, $\widehat{w}_i^r$ satisfies

$$\widehat{W}^{fm}(a_i, \widehat{w}_i^r, z, X) - \widehat{W}^{fu}(a_i, z, X) = 0, \tag{23}$$

where the $\widehat{W}$'s are expected present values *after* matching has occurred. When an *unemployed* worker gets matched, she accepts the offer and observes the firm's productivity level a_i. Wage negotiation results in $\widehat{w}_i$ such that the worker's payoff includes a fraction π of the match rent in excess of her outside option

$$\widehat{W}^{wm}(a_i, \widehat{w}_i, z, X) = \widehat{W}^{wu}(z, X) + \pi \left[\widehat{W}^{wm}(a_i, \widehat{w}_i^r, z, X) - \widehat{W}^{wu}(z, X) \right]. \tag{24}$$

For a worker employed in the previous period, $\widehat{w}_i = w_i$ if she either does not search on the job or searches but does not get matched to a new firm. If the worker searches and gets matched, the two firms enter into a Bertrand kind of competition for the worker. The worker and the two firms are assumed to observe the productivity levels of both firms. If the worker meets a firm with productivity $a_j > a_i$, the new firm will be able to offer a wage than gives the worker a payoff in excess of the maximum that the current firm is able to offer, $\widehat{W}^{wm}(a_i, \widehat{w}_i^r, z, X)$. Thus, the worker will switch firms. The negotiated wage $\widehat{w}_j$ at the new firm will give the worker a payoff with a fraction π of the match rent in excess of her outside option, which would be staying with the current firm and receiving the maximum wage it would pay

$$\widehat{W}^{wm}(a_j, \widehat{w}_j, z, X) = \widehat{W}^{fm}(a_i, \widehat{w}_i^r, z, X) + \pi \left[\widehat{W}^{wm}(a_j, \widehat{w}_j^r, z, X) - \widehat{W}^{fm}(a_i, \widehat{w}_i^r, z, X) \right]. \tag{25}$$

If the worker meets a firm with productivity $a_j \leq a_i$, the current firm will be able to offer a wage than gives the worker a payoff in excess of the maximum that the new firm is able to offer, $\widehat{W}^{wm}(a_j, \widehat{w}_j^r, z, X)$. Thus, the worker will not switch firms. However, bargaining between the two firms and worker may result in a wage increase for the worker at her current firm. The new negotiated wage $\widehat{w}_i$ will give the worker a

payoff with a fraction π of the match rent in excess of receiving the maximum wage the new firm would pay, as long as that payoff is greater than that associated with the worker's current wage

$$\widehat{W}^{wm}(a_i, \widehat{w}_i, z, X) = \max \left\{ \widehat{W}^{wm}(a_i, w_i, z, X), \right. \tag{26}$$
$$\left. \widehat{W}^{fm}(a_j, \widehat{w}_j^r, z, X) + \pi \left[\widehat{W}^{wm}(a_i, \widehat{w}_i^r, z, X) - \widehat{W}^{fm}(a_j, \widehat{w}_j^r, z, X) \right] . \right\}$$

2.3   Equilibrium

Equilibrium in the model consist of

1. choices of k_i^*, I_i, $\underline{a}^m$, and $\underline{a}^u$ that maximize payoffs conditional on expected future payoffs $W^{fm}(\cdot)$, $W^{fu}(\cdot)$, $W^{wm}(\cdot)$, $W^{wu}(\cdot)$, the rental rate r_t and the expected future distribution of aggregate endogenous state variable X,

2. the interest rate r_t, the expected evolution of X, and equilibrium determination of the expected future payoffs condition on the choices in 1.,

3. equilibrium in the capital market, and

4. updating of X conditional on the choices in 1.

The optimal choices of k_i^*, I_i, $\underline{a}^m$, and $\underline{a}^u$ are given in (4), (3), (8), and (9), respectively. In equilibrium, the expected present value of future payoffs *after* matching must satisfy the following. An unmatched worker will be unemployed and searching next period, so her expected future payoff satisfies

$$\widehat{W}^{wu}(z, X) = E_t \left[\beta \frac{mu(C')}{mu(C)} \left(b + W^{wu}(z', X') \right) \right], \tag{27}$$

where $E_t[\cdot]$ is the expectations operator conditional on information in period t (the current period) and $mu(C)$ is the marginal utility of consumption in the current period. An unmatched firm will search next period if its productivity level a_i' is above the threshold value $\underline{a}^{u'}$ or will cease to exist otherwise, so its expected future payoff satisfies

$$\widehat{W}^{fu}(a_i, z, X) = E_t \left[\beta \frac{mu(C')}{mu(C)} \int_{\underline{a}^{u'}}^{\overline{a}} \left(-\nu + W^{fu}(a_i', z', X') \right) \tau(a_i'|a) da_i' \right]. \tag{28}$$

For a matched worker, her expected future payoff depends on whether her match is separated exogenously or endogenously at the beginning of next period

$$\widehat{W}^{wm}(a_i, \widehat{w}_i, z, X) = E_t \left[\beta \frac{mu(C')}{mu(C)} \left\{ \left(p + (1-p) \int_{\underline{a}}^{\underline{a}^{m'}} \tau(a_i'|a_i) da_i' \right) \left(b + W^{wu}(z', X') \right) \right. \right.$$
$$\left. \left. + (1-p) \int_{\underline{a}^{m'}}^{\overline{a}} \left(w_i' + W^{wm}(a_i', w_i', I_i', z', X') \right) \tau(a_i'|a_i) da_i' \right\} \right]. \tag{29}$$

A matched firm's expected future payoff also depends on whether separation occurs at the beginning of next period. If the match is separated, the firm chooses whether to continue to exist and search

$$
\begin{aligned}
\widehat{W}^{fm}(a_i, \widehat{w}_i, z, X) \;=\; & E_t\left[\beta\frac{mu(C')}{mu(C)}\left\{p\int_{\underline{a}^{u'}}^{\overline{a}}\left(-\nu + W^{fu}(a_i', z', X')\right)\tau(a_i'|a_i)da_i'\right.\right. \\
& \left.\left. +(1-p)\int_{\underline{a}^{m'}}^{\overline{a}}\left(z'a_i'\left(k_i^{*'}\right)^{\alpha} - r'k_i^{*'} - w_i' + W^{fm}(a_i', w_i', I_i', z', X')\right)\tau(a_i'|a_i)da_i\right\}\right].
\end{aligned}
\tag{30}
$$

Equilibrium future value functions relevant *prior* to matching in the current period can be expressed as functions of the above equations for expected future values *after* matching. For an unmatched worker, the expected future value of her payoff depends on whether she finds a match this period

$$
W^{wu}(z, X) = p^S \int_{\underline{a}^u}^{\overline{a}} \widehat{W}^{wm}(a_i, \widehat{w}_i, z, X)g(a_i)da_i + (1-p^S)\widehat{W}^{wu}(z, X).
\tag{31}
$$

An unmatched firm has expected future payoff

$$
\begin{aligned}
W^{fu}(a_i, z, X) \;=\; & p^V\frac{Q}{S}\int_{\underline{a}^m}^{a_i}\int_{\underline{w}}^{\overline{w}}\left[\widehat{W}^{fm}(a_i, \widehat{w}_i, z, X) - \widehat{W}^{fu}(a_i, z, X)\right]I_j f(a_j, w_j)dw_j da_j \\
& + p^V\frac{U}{S}\left[\widehat{W}^{fm}(a_i, \widehat{w}_i, z, X) - \widehat{W}^{fu}(a_i, z, X)\right] + \widehat{W}^{fu}(a_i, z, X),
\end{aligned}
\tag{32}
$$

where the first term in the equation is relevant if the firm gets matched and the match is to a worker searching on the job and the second term applies if the firm gets matched and the match is to an unmatched worker. For matched workers and firms, the value functions depends on whether the worker searches on the job, and if so, whether she gets matched and accepts the match

$$
\begin{aligned}
W^{wm}(a_i, w_i, I_i, z, X) \;=\; & I_i\left\{p^S\int_{a_i}^{\overline{a}}\widehat{W}^{wm}(a_j, \widehat{w}_j, z, X)g(a_j)da_j\right. \\
& \left. +p^S\int_{\underline{a}^u}^{a_i}\widehat{W}^{wm}(a_i, \widehat{w}_i, z, X)g(a_j)da_j - \kappa\right\} \\
& +(1 - I_i p^S)\widehat{W}^{wm}(a_i, w_i, z, X),
\end{aligned}
\tag{33}
$$

$$
\begin{aligned}
W^{fm}(a_i, w_i, I_i, z, X) \;=\; & I_i p^S\left\{\int_{a_i}^{\overline{a}}\widehat{W}^{fu}(a_i, z, X)g(a_j)da_j + \int_{\underline{a}^u}^{a_i}\widehat{W}^{fm}(a_i, \widehat{w}_i, z, X)g(a_j)da_j\right\} \\
& +(1 - I_i p^S)\widehat{W}^{fm}(a_i, w_i, z, X).
\end{aligned}
\tag{34}
$$

Equilibrium in the capital market is characterized as follows. The market clears when demand for capital is equal to the supply

$$
\int_{\underline{a}^m}^{\overline{a}}\int_{\underline{w}}^{\overline{w}} k_i^* f(a_i, w_i)dw_i da_i = K.
\tag{35}
$$

The aggregate capital stock in the subsequent period K' is determined by maximizing (17) subject to (18) for which the following condition is sufficient

$$
mu(C) = \beta E_t\left[mu(C')(r' + 1 - \delta)\right].
\tag{36}
$$

The endogenous aggregate state variables after matching has occurred are K' and the post-match distributions of firm-worker matches $\widehat{f}(a_i, \widehat{w}_i)$ and existing unmatched firms $\widehat{g}(a_i)$. These variables form X', the vector of endogenous aggregate state variables at the beginning of the subsequent period before shocks are observed. The distribution of matches evolves as

$$f(a_i', w_i') = (1-p) \int_{\underline{a}^{m\prime}}^{\overline{a}} \tau(a_i'|a_i) \widehat{f}(a_i, \widehat{w}_i) da_i, \tag{37}$$

where w_i' is determined from $\widehat{w}_i$ according to the wage-setting mechanism described above. The distribution of existing unmatched firms after observing the shocks at the beginning of the subsequent period is determined as

$$g(a_i') = p \int_{\underline{a}^{u\prime}}^{\overline{a}} \int_{\underline{w}}^{\overline{w}} \tau(a_i'|a_i) \widehat{f}(a_i, \widehat{w}_i) d\widehat{w}_i da_i + \int_{\underline{a}^{u\prime}}^{\overline{a}} \tau(a_i'|a_i) \widehat{g}(a_i) da_i + Lh(a_i|a_i > \underline{a}_u). \tag{38}$$

The results of the matching process dictate the evolution from pre- to post-matching distributions. The post-match distribution of firm-worker matches with productivity a_i comes from matches with a_i whose worker did not search or did not receive an acceptable job offer from another firm and from unmatched firms with productivity a_i that got matched to a worker who accepted the job offer

$$
\begin{aligned}
\widehat{f}(a_i, \widehat{w}_i) &= \left[(1 - I_i) + I_i(1 - p^S) + I_i p^S \int_{\underline{a}^u}^{a_i} g(a_j) da_j \right] f(a_i, w_i) \\
&\quad + \left[p^V \frac{U}{S} + p^V \frac{Q}{S} \int_{\underline{a}^u}^{a_i} I_j f(a_j, w_j) da_j \right] g(a_i),
\end{aligned}
\tag{39}
$$

where $\widehat{w}_i$ is determined according to the wage-setting mechanism. The post-match distribution of unmatched firms with productivity a_i comes from unmatched firms with a_i that did not get matched or got matched to a worker who did not accept that match and from firm-worker matches whose worker searched on the job and accepted a new job offer

$$\widehat{g}(a_i) = \left[(1 - p^V) + p^V \frac{Q}{S} \int_{a_i}^{\overline{a}} I_j f(a_j, w_j) da_j \right] g(a_i) + \int_{\underline{w}}^{\overline{w}} \left[p^S \int_{a_i}^{\overline{a}} g(a_j) da_j \right] I_i f(a_i, w_i) dw_i. \tag{40}$$

3 Calibrating and Solving the Model

3.1 Specification

Because many unemployment spells in the data are short, the model is specified such that a period corresponds to a month. However, the model imbeds a labor search model in a real business cycle model, and real business cycle models are usually calibrated with quarterly aggregate shocks. Also, the fit of the model will be evaluated on the basis of properties of quarterly averages of monthly data, as in Shimer (2004a, 2004b). To make the monthly model similar to a model responding to quarterly shocks, the driving process for the monthly aggregate shock is specified such that an innovation to the persistent component of aggregate productivity is realized gradually over the months of a quarter. Let z_t be composed of two shocks, one highly

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persistent z_t^P and the other with little persistence z_t^S. The laws of motion are specified as

$$
\begin{aligned}
\ln z_t &= \ln z_t^P + \ln z_t^S \\
\ln z_t^P &= \rho_P \ln z_{t-1}^P + \varepsilon_t^P & \varepsilon_t^P &\sim \text{iid} N(0, \sigma_P^2) \\
\ln z_t^S &= \rho_S \ln z_{t-1}^S + \varepsilon_t^S & \varepsilon_t^S &\sim \text{iid} N(0, \sigma_S^2).
\end{aligned}
\tag{41}
$$

Agents in the model observe z_t but not its two components, and they use the Kalman filter to forecast future values of the shocks. The expectation of z_t^P is more important than z_t^S for decision making in the model because it has a larger effect on the present discounted value of future payoffs specified above due to the high persistence of z_t^P.

The calibrated parameter values of the driving processes are given in the top panel of Table 1. The parameters are chosen such that a AR(1) on quarterly averages of z_t^P has an autoregressive coefficient of 0.95 and the standard deviation of the error is 0.007, common values used for the Solow residual in the RBC literature. The calibration also implies that an innovation to z_t^P causes $E_t[z_t^P]$ to approach z_t^P gradually over approximately three months. Figure 1 illustrates this pattern.

The equilibrium distributions of firm specific productivity a_{it} for matched and unmatched firms ultimately derive from the distribution of idiosyncratic productivity levels for potential new firms $h(a)$ and the law of motion for a_i, $\tau(a_i'|a_i)$. The distribution $h(a)$ is taken from a normal distribution $N(1, \sigma_h^2)$. The law of motion for a_i is specified as

$$
a_i' = \rho_a a_i + e_i \qquad e_i \sim \text{iid} N(0, \sigma_e^2).
\tag{42}
$$

This specification implies that $\tau(a_i'|a_i) = N(\rho_a a_i, \sigma_e^2)$. The range of a_i is restricted to the interval $[1 - 4\sigma_h^2, 1 + 4\sigma_h^2]$, with $h(a)$ normalized to integrate to one over that interval.

3.2 Calibration and Data

For the calibration, some parameters are fixed in advance and others are adjusted to make the data from the model match certain moments of the actual data. In the first category are the parameters α, β, δ, γ, and π. The parameter α in the production function is set to 0.36. The value of β is set such that the effective annual discount factor is six percent. The value of δ is set such that the effective annual depreciation rate is five percent. For the utility function, $\gamma = 1$ implying that $mu(c) = 1/c$. The value of the bargaining share parameter π is based on the results of Cahuc, Postel-Vinay, and Robin (2004). They estimate a model with on-the-job search and the wage bargaining mechanism described above on matched French employer-employee data. Their results indicate that competition between firms resulting from workers searching on the job has a substantial positive effect on wages allowing workers to obtain a larger share of match surplus than the bargaining share in isolation would imply. They estimate the bargaining share to be between 0 and 0.2 for unskilled workers and between 0.2 and 0.4 for skilled workers. For this paper, π is set to 0.3.

The remaining parameters to be calibrated are b, χ, ν, κ, ℓ, ψ, p, ρ_a, σ_a, and σ_h. Data on employment, unemployment, and the rate of labor turnover provide benchmarks that simulated data from the model should match. Parameter values are selected to match these benchmarks. Using Dutch survey data, van Ours and Ridder (1992) found that 71 percent of vacancies reported in an initial survey were found to be filled in a subsequent survey approximately one quarter later. Den Haan, Ramey, and Watson (2000) found that in the context of their labor search model, data on job creation and job destruction in U.S. manufacturing in Davis, Haltiwanger, and Schuh (1992) also implied a quarterly job finding rate of 0.71 in the steady state. For the monthly model in this paper, the probability that a vacancy gets filled within a quarter is approximately $q^V + q^V(1 - q^V) + q^V(1 - q^V(1 - q^V))$ where q^V is the monthly probability that a vacancy is filled. A value of .71 for the average quarterly fill probability implies that the model need to be calibrated to generate an average value of q^V of approximately 0.28.

Since separation and matching in the world actually occurs simultaneously and spread throughout a month, questions arise of how to use observed data to calibrate a discrete-time model. For example, should observed unemployment correspond to unemployment in the model prior to matching or after matching in the period? Shimer (2004b) describes estimating job finding and separation rates in a continuous-time environment using employment and unemployment data observed at discrete points in time. Workers are assumed to be either employed or unemployed, data are observed at discrete dates $\tau \in \{0, 1, 2, ...\}$, and the instantaneous rates at which unemployed workers find a job, f_τ, and employed workers lose a job, x_τ, are assumed to be constant over the interval $[\tau, \tau + 1)$ for all workers. Shimer shows that if u_τ is unemployment at date τ and n_τ is the size of the labor force, then

$$u_{\tau+1} = \frac{(1 - e^{-f_\tau - x_\tau})(n_\tau + u_\tau)x_\tau}{f_\tau + x_\tau} + e^{-f_\tau - x_\tau}u_\tau. \tag{43}$$

Likewise, if u_τ^s is short-term unemployment, consisting of workers unemployed at date τ for less than one period, then

$$u_{\tau+1} = e^{-f_\tau}u_\tau + u_{\tau+1}^s. \tag{44}$$

Given data on employment, unemployment and short-term unemployment, one can estimate f_τ and x_τ. These estimates can then be used in approximating data relevant for calibrating the discrete-time search model in this paper. Let period t in the discrete-time environment correspond to the interval $[\tau, \tau + 1)$ in the continuous-time environment. The total flow into unemployment during the interval can be approximated as $x_\tau((n_\tau + n_{\tau+1})/2)$, so the number of workers who were unemployed and searching for a job at some point during the interval would be $u_\tau + x_\tau((n_\tau + n_{\tau+1})/2)$. These values imply that number of searching workers relative to the size of the labor force in period t, which corresponds to the variable U_t in the model, can be approximated as

$$U_t = \frac{u_\tau + x_\tau(n_\tau + n_{\tau+1})/2}{(n_\tau + u_\tau + n_{\tau+1} + u_{\tau+1})/2}. \tag{45}$$

The separation rate into unemployment is approximated as

$$s_t^u = \frac{x_\tau (n_\tau + n_{\tau+1})/2}{(n_\tau + u_\tau + n_{\tau+1} + u_{\tau+1})/2}/N_t. \tag{46}$$

This separation rate is distinct from the total separation rate in the model because it does not include separation associated with workers moving from one job to another without passing through a phase of unemployment, as happens when a worker searching on the job accepts a new match. The total flow into employment during the interval can be approximated as $f_\tau (u_\tau + u_{\tau+1})/2$. Thus, the average job finding rate for workers unemployed during period t, which corresponds to the variable p_t^S in the model, can be estimated as

$$p_t^S = \frac{f_\tau (u_\tau + u_{\tau+1})/2}{(n_\tau + u_\tau + n_{\tau+1} + u_{\tau+1})/2}/U_t. \tag{47}$$

Since in the model unemployed workers accept all matches, the job finding rate for unemployed workers is equal to the probability of an unemployed worker being matched.

Data on n_t, u_t, and u_t^s are used to estimated time series on U_t, s_t^u, and p_t^S. The data are monthly observations on employment and unemployment, 16 years and older, seasonally adjusted for the U.S, from January 1951 through September 2004. Persons unemployed for less than five weeks is used as a measure of u_t^s, with the adjustment to the series after January 1994 suggested by Shimer (2004b) to accommodate changes in data collection. Mean values of U_t, s_t^u, and p_t^S are reported in Table 2. These values serve as benchmarks for calibrating the model. Though the model parameters interact in their effects on the simulated data, b and χ are closely tied to matching the mean of U_t, and ρ_a is used to match the mean of s_t^u. The parameters of the matching function, ℓ and ψ, and the cost of on the job search, κ, are important for matching the means of q_t^V and p_t^S.

Nagyapál (2004a) uses data from CPS dependent interviewing to estimate the frequency of job-to-job transitions. She estimates that the probability of employed workers being at a different employer in the following month is 2.73%. Though this value as a measure of direct job-to-job flows may be biased upward because the measurement does not preclude very short unemployment spells, Nagyapál provides arguments that this bias is small. The model is calibrated with a benchmark value of the mean of T_t of 0.022.

The remaining parameters must be set somewhat arbitrarily. I set the exogenous separation rate p to 0.01. With σ_h set to 0.075, $a_i \in [0.7, 1.3]$. The standard deviation of innovations to a_i, σ_e, is set 0.0076, a value slightly larger than $(1 - \rho_a)$.

A full set of statistics on the data is reported in Table 3. In addition to the data described above, real U.S. GDP seasonally adjusted is used to measure aggregate output Y_t Vacancies are measured by the Conference Board help-wanted advertising index. Quarterly averages are taken of monthly data, then the data is HP-filtered with a smoothing parameter of 10^5. This is the data adjustment procedure used by Shimer (2004a, 2004b). The statistics show the procyclicity of vacancies mentioned in the introduction.

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3.3 Solution Procedure

The model is solved by a procedure similar to that in Hussey (2004b). That paper shows how to solve a model in which distributions of a variable such as a_i across agents is a state variable and the truncation points of those distributions vary endogenously. The additional complication in the model of this paper is that one of the distributions, $f(a_i, w_i)$, is a function of two firm-specific variables. However, the methodology can be extended by specifying the distribution over a grid of both variables. The grid varies in the a_i dimension as the threshold value $\underline{a}_t^m$ changes from period to period, but the w_i grid can remain fixed. For the results presented below, the model is solved for a grid of 80 points on a_i and 200 points on w_i.

The solution procedure uses the method of parameterized expectations (den Haan and Marcet, 1990; Wright and Williams, 1984). The expectations terms that are parameterized are a function of the aggregate endogenous state variable X_t, which potentially has a very large dimension because it can include the distributions of productivity levels and wages. To make the solution method tractable, I assume that individual firms and workers observe only a limited set of aggregate state variables. This set can include functions of integrals of the distribution, which would include variables such as aggregate employment and the average wage in the economy. This simplification is similar to the approach of Krusell and Smith (1998) who found that equilibrium in a model with heterogeneous agents could be closely approximated as a function of a limited set of statistics on the distribution of wealth across agents in the economy. For the solutions reported below, the set of aggregate exogenous and endogenous state variables on which agents base their decisions is $\{z_t, E_t[z_t^p], K_t, \widehat{N}_{t-1}, \widehat{V}_{t-1}, W_{t-1}\}$, where $\widehat{N}_{t-1}$ is the number of matched workers at the end of period $t-1$ after matching has occurred, $\widehat{V}_{t-1}$ is the number of firms remaining unmatched after matching, and W_{t-1} is the average wage earned in the previous period

$$W = \int_{\underline{a}^m}^{\overline{a}} \int_{\underline{w}}^{\overline{w}} w_i f(a_i, w_i) dw_i da_i. \tag{48}$$

To evaluate the expectations terms with respect to future values of the *exogenous* aggregate state variables, Gaussian quadrature as in Tauchen and Hussey (1991) is used with five quadrature points for z_{t+1}.

4 Results

Current results are preliminary as the calibration is fine tuned and the model solution procedure is allowed to reach tighter convergence. The model is solved for the parameterization given in Table 1, and data are simulated to characterize the properties of the solution. One hundred realizations of length 660 are simulated with additional initial observations dropped to induce randomness. The length of the simulations is the same as the number of months in the data. Means across realizations of statistics on the simulated data are reported in Tables 2 and 4.

The calibration matches closely the benchmark unconditional moments reported in Table 2. Standard

deviations and correlations for the simulated data are reported in Table 4. The model is able to generate highly procyclical vacancies as seen by the large positive correlation of vacancies with output and negative correlation with unemployment. The vacancy-unemployment ratio is even more variable than in the data. Overall, measures of labor turnover are more volatile in the simulated data than in the U.S. data. The one exception is the lower volatility of the rate of separation to unemployment.

Impulse responses to a positive aggregate technology shock are graphed in Figures 2-4. The waviness of the lines indicates that the solution algorithm has not fully converged. The unevenness in the functions may also be due to issues associated with the coarseness of the grid of wages, especially near the truncation points. Nonetheless, the overall patterns in the impulse response functions are indicative of the dynamics implied by the model. Figure 2 shows the positive response of output and employment to the technology shock. Though it is not clear that all the small waves in the lines are revealing underlying economic behavior, the graph does suggest that on-the-job search and wage rigidity may serve as a significant internal propagation mechanism, similar to the result of Krause and Lubik (2004).

Figure 3 shows the clear negative correlation between vacancies and unemployment. During a boom there is a surge in vacancies and unemployment falls leading to substantial movement in the vacancy-unemployment ratio. Figure 4 also shows that a positive aggregate shock leads to an increase in the job-to-job transition rate, and this value remains high for multiple quarters. The separation rate to unemployment appears less sensitive to the shock, which may be consistent with the finding of Shimer (2004b) that fluctuations in the job finding rate are a more important component of cyclical variations in unemployment than are changes in the job separation rate. As reported above, the model generates variation in the rate of separation to unemployment lower than in the data, but it is still and interesting result that even if a model has endogenous job destruction, it does not have to generate large variation in job destruction in response to shocks.

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Table 1. Parameter Values

Aggregate Productivity

ρ_P	0.975	σ_P	0.005
ρ_S	-0.5	σ_S	0.007

Firm-Specific Productivity

ρ_a	0.9973	σ_a	.0076
		σ_h	0.075

Other Parameters

α	0.36	b	5.15
β	$1/1.06^{1/12}$	χ	1.98
δ	$1-(1-0.05)^{1/12}$	ν	0.20
γ	1.0	κ	0.31
π	0.3	ℓ	0.36
		ψ	0.70
		p	0.01

Table 2. Calibration Benchmarks

	Benchmark	Mean of Simulation	
U	0.089	0.085	unemployment
s^u	0.036	0.032	separation rate to unemployment
p^S	0.37	0.28	job finding rate for unemployed workers
q^V	0.28	0.32	probability a vacancy is filled
T	0.022	0.023	job-to-job transition rate

Table 3. Statistics on U.S. Data

	Y	N	U	s^u	p^S	V	$\frac{V}{U}$
			Standard Deviations				
		0.53	5.36	5.4	7.7	8.38	0.23
			Correlation Matrix				
Y	1.00	0.90	-0.89	-0.58	0.88	0.84	0.78
N		1.00	-0.98	-0.70	0.91	0.91	0.82
U			1.00	0.73	-0.92	-0.91	-0.83
s^u				1.00	-0.51	-0.62	-0.52
p^S					1.00	0.90	0.80
V						1.00	0.81
$\frac{V}{U}$							1.00

Data except for Y are quarterly averages of monthly data. All series are logged and HP-filtered with a smoothing parameter of 10^5 as in Shimer (2004a, 2004b). Standard deviations of N, U, and V are expressed relative to the standard deviation of Y. Standard deviations of s^u and p^S are expressed relative to the standard deviation of N.

Table 4. Statistics on Simulated Data

	Y	N	U	s^u	p^S	V	$\frac{V}{U}$
				Standard Deviations			
		0.76	9.21	12.7	2.30	9.13	0.632
				Correlation Matrix			
Y	1.00	0.99	-0.97	-0.91	0.82	0.89	0.97
N		1.00	-0.96	-0.89	0.82	0.85	0.94
U			1.00	0.92	-0.78	-0.84	-0.96
s^u				1.00	-0.68	-0.77	-0.88
p^S					1.00	0.81	0.83
V						1.00	0..96
$\frac{V}{U}$							1.00

Data except for Y are quarterly averages of monthly data. All series are logged and HP-filtered with a smoothing parameter of 10^5 as in Shimer (2004a, 2004b).. Standard deviations of N, U, and V are expressed relative to the standard deviation of Y. Standard deviations of s^u and p^S are expressed relative to the standard deviation of N.
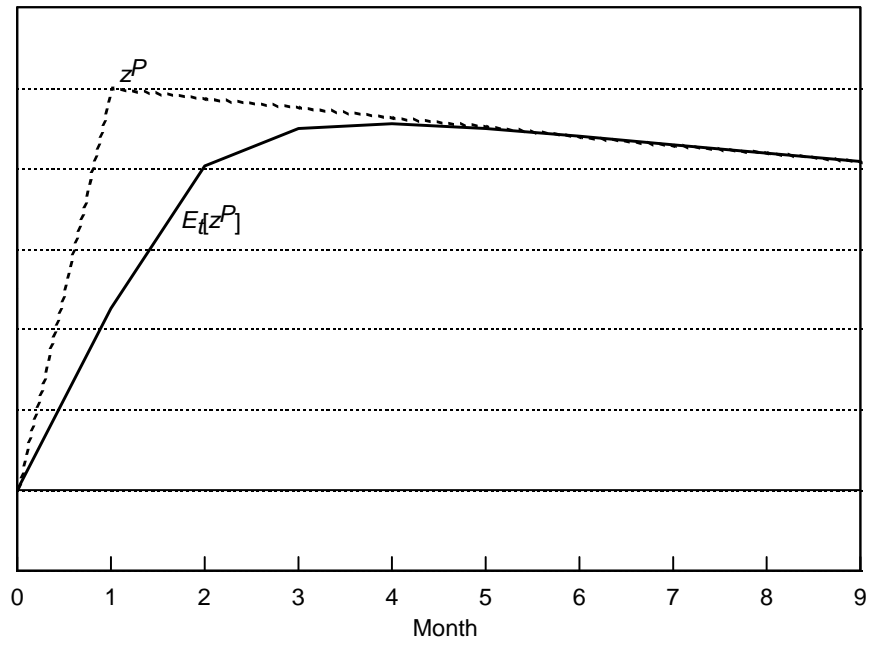
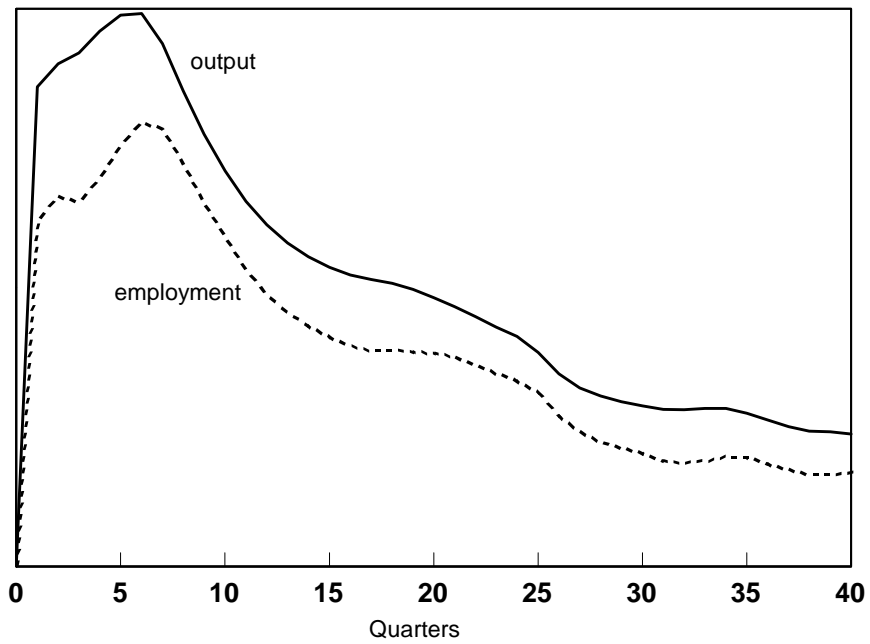Figure 1: Response of $E_t[z_t^P]$ to an innovation in z_t^P.

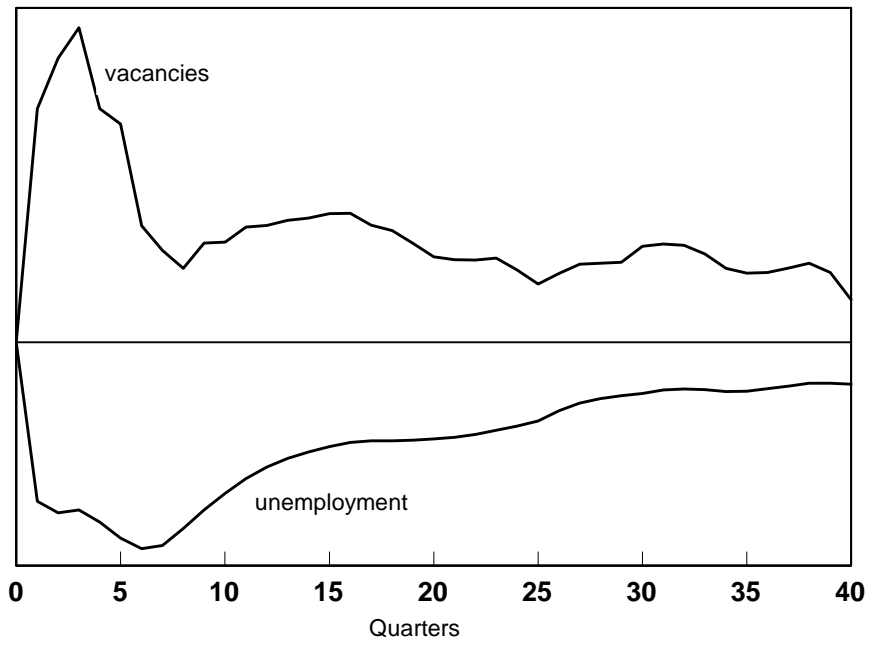Figure 2: Response of Y and N to a Positive Productivity Shock

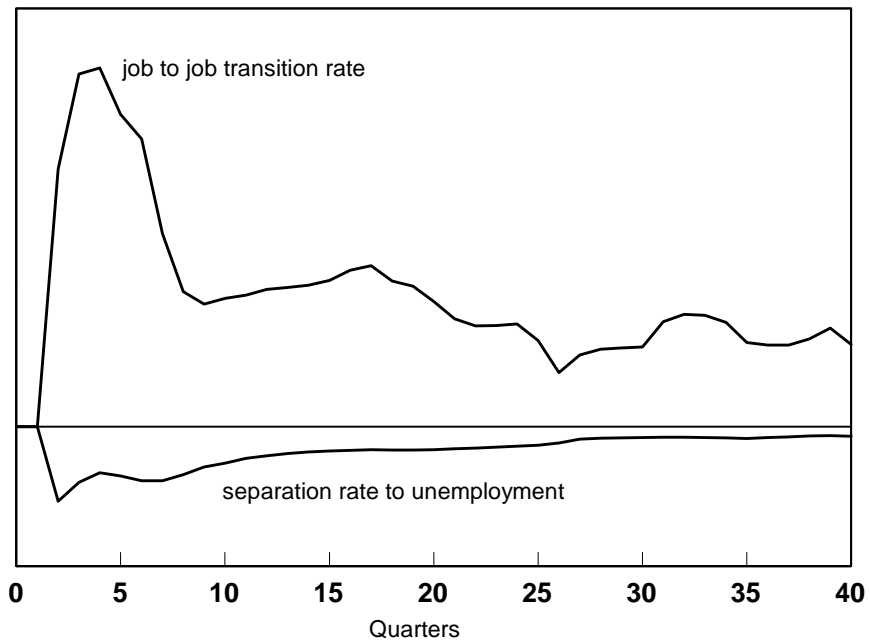Figure 3: Response of V and U to a Positive Productivity Shock

Figure 4: Reponse of s^{U} and T to a Positive Productivity Shock