Monetary Policy Shifts, Indeterminacy and Inflation Dynamics

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First draft: March 2005
This draft: June 2005

Abstract

The New-Keynesian Phillips curve plays a central role in modern macroeconomic theory. A vast empirical literature has estimated this structural relationship over various postwar full-samples. While it is well know that in a New-Keynesian model a 'weak' central bank response to inflation generates sunspot fluctuations, the consequences of pooling observations from different monetary policy regimes for the estimates of the structural Phillips curve had not been investigated. Using Montecarlo simulations from a purely forward-looking model, this paper shows that indeterminacy can introduce a sizable persistence in the estimated process of inflation. This persistence however is not an intrinsic feature of the economy; rather it is *endogenous* to the policy regime and results from the self full-filling nature of inflation expectations. By neglecting indeterminacy the estimates of the forward-looking term of the structural Phillips curve are shown to be biased downward. The implications are in line with the empirical evidence for the U.K and U.S.

JEL Classification: E58, E31, E32.

Keywords: indeterminacy, New-Keynesian Phillips curve, Montecarlo, bias, persistence

1 Introduction

The New-Keynesian Phillips Curve (NKPC) has recently become the building block of many monetary policy models. This relation plays a central role in understanding aggregate fluctuations and quantifying the transmission mechanism of monetary policy. Most of the success of the NKPC hinges on the fact that it is derived from first principles, thereby implying that its estimates survive the Lucas (1976) critique.

As shown many times in the literature, a log-linearized version of the New-Keynesian model gives rise to self-fulfilling expectations if the central bank does not raise the nominal interest rate sufficiently in response to a deviation of inflation from the target. This implies that sunspot fluctuations can influence the properties of the inflation process even if the 'true' NKPC is a structurally invariant relation.

Using Montecarlo simulations from a monetary DSGE model, this paper shows that the estimates of an hybrid NKPC are severely biased downward when two conditions are met. First, the data are generated under indeterminacy. Second, the empirical analysis implicitly and arbitrarily limits the solution of the model to the determinacy region. Specifically, the null hypothesis of no backward-looking component is strongly rejected in spite of the fact that the data generating process does not exhibit any exogenous or endogenous persistence. The slope of the Phillips curve takes a value that is not statistically different from zero. Moreover, the sum of autoregressive coefficients in the reduced-form process of inflation is close to one and, most importantly, is significantly different from the value of zero that would emerge in the unique rational expectations equilibrium. As under determinacy the estimates match the 'true' coefficients used to simulate the data, we refer to the difference between the generating process parameters and the relative estimates as 'neglected indeterminacy bias'.

This paper cuts across two bodies of research. The first body is the literature on interest rate rules inspired by the works of Taylor (1993) and Clarida Galí and Gertler (2000) which documents a shift in the conduct of monetary policy around the beginning of the 1980s for a number of major industrialized economies. The second body includes Galí and Gertler (1999), Sbordone (2003), Eichenbaum and Fisher (2004), Lindé (2005) and Rudd and Whelan (2005) among many others, and uses different econometric techniques to estimate the NKPC over various postwar full-samples for several countries.

The results presented here suggest some caution is needed when interpreting the estimates of the structural NKPC obtained using a pool of observations that mixes different monetary policy regimes. The reason is that neglected indeterminacy can distort inference in an impor-

tant dimension. In particular, it is possible to introduce additional elements of persistence that are not present in the data generating process of inflation and thus are not an intrinsic, structural feature of the economy.

This paper also contributes to the literature on inflation persistence. Several authors including Cogley and Sargent (2005), and Benati (2005) show that inflation inertia has been an historically limited phenomenon in the U.S. and the U.K. In particular, inflation can be characterized as highly persistent only during those times that, in the empirical literature on monetary policy rules, are typically associated with a weak central bank response to inflation. Our simulations reveal that a *passive* monetary policy, in the form of a *less-than-proportional* response of the nominal interest rate to inflation, does actually produces inflation persistence. This result can thus provide a rationale for the empirical regularity in Cogley and Sargent (2005) and Benati (2005).

The paper is organized as follows. Section 2 presents the model that will serve as data generating process. Section 3 performs a set of montecarlo experiments and presents the estimates of the structural process and the reduced-form process of inflation based on the simulated data. The following Section reports sub-sample evidence on the U.K. and U.S. and shows that the data are consistent with the 'neglected indeterminacy bias' hypothesis. Conclusions are discussed in the last part while the Appendix describes a method to obtain a solution of the linear rational expectations model under indeterminacy and determinacy.

2   The model

This section describes a log-linearized New-Keynesian sticky price model of the business cycle. This model consists of the following three aggregate equations that King (2000) and Woodford (2003) derive from first principles:

$$\pi_t = \beta E_t \pi_{t+1} + k\left(x_t - z_t\right) \tag{1}$$

$$x_t = E_t x_{t+1} - \tau\left(i_t - E_t \pi_{t+1}\right) + g_t \tag{2}$$

$$i_t = \psi_\pi \pi_t + \psi_x\left(x_t - z_t\right) + u_t \tag{3}$$

where x_t is defined as the deviation of output from a long-run trend, π_t represents inflation, and i_t is the nominal interest rate. Inflation and the interest rate are expressed in percentage deviations from their steady state values.

Equation (1) captures the staggered feature of a Calvo-type world in which each firm adjusts its price with a constant probability in any given period, and independently from the

time elapsed from the last adjustment. The discrete nature of price setting creates an incentive to adjust prices more the higher is the future inflation expected at time t. The parameter $0 < \beta < 1$ is the agents' discount factor and k is the slope of the Phillips curve. The shock z_t is identically and independently distributed (iid) with standard deviation σ_z and it is meant to capture exogenous shifts in the marginal costs of production.

As there is no capital in the model, the second equation is a standard Euler equation for consumption combined with the relevant market clearing condition. It brings the notion of consumption smoothing into an aggregate demand formulation by making x_t a positive function of its future value and a negative function of the ex-ante real interest rate, $i_t - E_t\pi_{t+1}$. The parameter $\tau > 0$ can be interpreted as intertemporal elasticity of substitution. Preference shifts and government spending shocks are embodied in the iid process g_t which has standard deviation σ_g.

Equation (3) characterizes the behavior of the monetary authorities. As in Lubik and Schorfheide (2004), this is an interest rate rule according to which the central bank sets the policy rate in response to deviations of inflation and output from their respective targets.[1] Without loss of generality, the target for inflation is normalized to zero. The shock u_t represents an iid monetary policy disturbance with standard deviation σ_u. There is no correlation between innovations.

The specification (1) to (3) with iid shocks and no interest rate smoothing has been deliberately designed to maximize the power of the tests on the (in)significance of the backward-looking components of the Phillips curve. As the data generating process does exhibit no persistence, a rejection of the null hypothesis $(1 - \beta) = 0$ on the simulated data can only be interpreted as a spurious result from neglecting indeterminacy in the estimation procedure.

The linear rational expectations model described by equations (1) to (3) can be represented in the following canonical form:

$$\Gamma_0(\theta) s_t = \Gamma_1(\theta) s_{t-1} + \Psi(\theta) \varepsilon_t + \Pi(\theta) \eta_t \tag{4}$$

where

$$
\begin{aligned}
\theta &= [\psi_\pi, \psi_x, \beta, k, \tau] \\
s_t &= [x_t, \pi_t, i_t, E_t(x_{t+1}), E_t(\pi_{t+1})]' \\
\varepsilon_t &= [u_t, g_t, z_t]' \\
\eta_t &= [x_t - E_{t-1}(x_t), \pi_t - E_{t-1}(\pi_t)]'
\end{aligned}
$$

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[1] The results below are not affected by excluding z_t from the policy rule.

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The matrices Γ_0, Γ_1, Ψ and Π are given by the following expressions:

$$
\Gamma_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\tau & 1 & \tau \\ 0 & 0 & 0 & 0 & \beta \end{bmatrix}, \Gamma_1 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \psi_2 & \psi_1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -k & 1 \end{bmatrix}, \Psi = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & -\psi_2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \Pi = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \psi_2 & \psi_1 \\ 1 & 0 \\ -k & 1 \end{bmatrix}
$$

and they are conformable to the vectors of state variables s_t and s_{t-1}, to the vector of structural disturbances ε_t and to the vector of endogenous forecast errors η_t.

This log-linearized system gives rise to self-fulfilling expectations if the central bank does not raise the nominal interest rate enough in response to a deviation of inflation from the target. For the model used in this paper, Woodford (2003) show that the following condition must hold for the existence of a unique stable solution:

$$
\psi_\pi \geq 1 - \frac{(1-\beta)\,\psi_x}{k} \tag{5}
$$

In all other cases, a sunspot shock ζ_t will affect the dynamics of output and inflation through the endogenous forecast errors, thereby causing the existence of multiple stable solutions.

3 A Montecarlo experiment

The main experiment of the paper is now ready to be run. We apply the solution method outlined in the Appendix to the New-Keynesian model (1) to (3) and we generate artificial data under both determinacy and indeterminacy. To compute a solution under indeterminacy we follow Lubik and Schorfheide (2004) and present results for two different identifying assumptions. Under the first assumption, the sunspot shocks are orthogonal to the structural shocks and the solution is referred to as 'orthogonality'. Under the second scenario, we assume that the impulse responses $\frac{\partial s_t}{\partial \varepsilon_t'}$ associated with the system (4) are continuous at the boundary between the determinacy and the indeterminacy region, and the solution is labelled 'continuity'.

It is worth emphasizing that in the data generating process, inflation and output are purely forward-looking, errors are *iid* and there is no interest rate smoothing. In other words, the model deliberately lacks any source of either endogenous or exogenous persistence. We then use the simulated data to estimate the following hybrid version of the Phillips curve relation:

$$
\pi_t = \omega \pi_{t+1} + (1-\omega)\,\pi_{t-1} + k x_t + e_t \tag{6}
$$

where $e_t \equiv -kz_t - \omega(\pi_{t+1} - E_t \pi_{t+1})$ and $\beta = 0.99$. Using the alternative parametrization $\pi_t = \beta[\omega E_t \pi_{t+1} + (1-\omega)\,\pi_{t-1}] + k x_t + v_t$ does not affect the results.

Table 1 show the value of the parameters in the data generating process under indeterminacy and determinacy. These values are borrowed from Lubik and Schorfeide (2004) who use Bayesian techniques to estimate a version of the model (1) to (3) augmented with autoregressive error terms and interest rate smoothing on US data. To make the indeterminacy bias transparent, we eliminate the persistence in the shocks and in the nominal interest rate by setting the autoregressive coefficients of the processes for g_t, z_t and i_t to zero across all simulations.

The second columns corresponds to the pre-Volcker period estimates. The interest rate response to inflation is below unity and therefore violates the Taylor principle (5). We use these estimates to generate artificial series of inflation, output and interest rate under the orthogonality and the continuity identifications. The third column reports the values that parameterize the model under determinacy. For the sake of comparison, these coefficients are set to the same values used under indeterminacy, but with two important exceptions: both coefficients of the monetary policy rule do now generate a unique rational expectations solution and they correspond to the estimates in Lubik and Schorfeide (2004) over the post-Volcker sample.

We consider three sample lengths. The baseline case consists of 200 observations, which at quarterly frequencies correspond to fifty years. To explore the extent to which the estimates are sensitive to the sample length we also present results for periods of 80 and 400 observations. The former roughly matches the number of data points available to an econometrician from the beginning of the 1960s to the end of the 1970s.

3.1 Results

Figure 1 and 2 present the results based on 10,000 repetitions. The hybrid New-Keynesian Phillips curve (6) is estimated with the Generalized Method of Moments (GMM) and Two Stage Least Squares (TSLS) under the hypothesis of rational expectations. Starting from period $t - 1$ the instrument set includes past values of inflation, output and nominal interest rate. The selection of the number of lags is based on the Schwartz lag length criterion from an unrestricted Vector AutoRegression (VAR) in the three simulated series.

Figure 1 shows the probability distributions of the estimates of the forward-looking component of the Phillips Curve. The first two rows reveal that using the data generated under indeterminacy the estimates of ω from a conventional hybrid specification are significantly biased, with the median of the distribution around 0.64 (0.77) under orthogonality (conti-

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nuity) using GMM. The bias is slightly less pronounced using TSLS. Hence, by neglecting indeterminacy the null hypothesis of no backward-looking component in the Phillips curve is strongly rejected even if the data generating process is purely forward-looking.

The intuition for this result comes from the self-fulfilling nature of inflation expectations under indeterminacy. The private sector anticipates that in response to a positive shock to inflation the monetary authorities will not raise sufficiently the nominal interest rate, and therefore anticipates a negative real rate. The fall in the ex-ante real interest rate fuels a boom in real activity, and the boom in turn fuels further inflation. This implies not only that the expectations of high inflation are indeed confirmed but also that inflation remains persistently above target.

An aggressive monetary policy stance to deviations of inflation from target implies, in contrast, that the real interest rate is implicitly set such as to outweigh a rise in expected inflation. This means that a pick up in actual inflation is promptly followed by a reversal towards the target and in the case of a perfectly credible inflation targeting regime and a purely forward-looking model inflation is white noise.

The technical reason for the bias is that the solution of a linear rational expectations model requires that all unstable roots in the matrix of autoregressive coefficients Γ_1^* be suppressed. The New-Keynesian model is characterized by two roots, λ_j with $j = 1, 2$. When monetary policy conforms to the Taylor Principle the two roots are unstable, i.e. the system is determinate, and the solution generates no 'extra' persistence relative to the specification of the model. This means that if the data generating process is purely forward-looking, as it is here, the backward-looking term of the Phillips curve $(1 - \omega)$ should be zero statistically.

In contrast, indeterminacy is characterized by only one *unstable* root, thereby implying that the solution now generates 'extra' persistence through the *stable* root λ_1 - see equation (14) in the Appendix. This is confirmed by the third row of Figure 1. Under determinacy, the median estimates of ω are not statistically different from the true value of 0.99 at the 1% significance level, though they are somewhat smaller numerically. As shown below using simulations from a longer sample, this is likely to reflect a small sample problem.

Figure 2 shows the results for the slope of the Phillips curve. The data are generated under the assumption that the true parameter is 0.77 but only the estimates on the series simulated under determinacy are consistent with this value. In contrast, using the orthogonality or the continuity assumption the estimates of k are severely biased towards zero and largely below the 'true' value.

Indeterminacy may also have important implications for the reduced-form properties of the (simulated) data. To explore this possibility we estimate with OLS the following process for inflation

$$\pi_t = \mu + \phi_1 \pi_{t-1} + \phi_2 \pi_{t-2} + ... + \phi_p \pi_{t-p} + \xi_t \tag{7}$$

with $3 < p < 8$. Figure 3 reports the probability distributions of the sum of the autoregressive coefficients in equation (7). Indeterminacy generates a sizable persistence, though the reduced-form persistence of a purely forward-looking model solved for the unique rational expectations equilibrium is zero. In contrast, the estimates on the inflation series are centered in zero under determinacy. Empirical support for a persistent reduced-form process for inflation under indeterminacy can also be found in Benati (2004).

This finding also suggests that weak instruments are unlikely a concern under indeterminacy where inflation is quite a persistent process. Furthermore, while in principle it seems more reasonable to question the relevance of the instruments under determinacy, the third rows of Figure 1 and Figure 2 show that in practice the GMM estimates match the 'true' values of parameters under determinacy.

The results so far reveal the extent to which the estimates of the New-Keynesian Phillips curve are sensitive to a different monetary policy response to inflation. Figure 4 presents then the estimates and the confidence intervals of the parameters of the inflation process as a function of ψ_π. The estimates are computed for a grid of 20 points over the interval $[0, 2]$. The interesting result from this experiment is that -with the exception of the slope of the Phillips curve - the size of the bias is a negative function of the distance of ψ_π from the border between the indeterminacy and the determinacy region. As far as the forward-looking component of the Phillips curve is concerned, only a central bank response to inflation close to zero would deliver an unbiased estimate of ω within the indeterminacy region.

3.2   Robustness analysis

To investigate the relevance of the sample length for our findings Figure 5 presents results for 80 and 400 observations using the orthogonality solution. As the previous results were robust to running 1000 simulations, we set the number of repetitions to the latter value in an effort to make the computational burden lighter.

The bias is still sizable in both experiments, though the estimates over a longer period are, unsurprisingly, more accurate and precise. Moreover, the median estimates of the forward-looking component of the Phillips curve in the large sample is now close to 0.99 also numer-

ically. This suggests not only that the 'neglected indeterminacy bias' is more than simply a small sample bias, but also that it is not likely to be merely a peculiarity of instrumental variable estimators.

The results are also robust to using a 'mixed' sample of 160 observations in which the monetary policy rule switches from passive to active midway through the period. Specifically, the first 80 observations are generated under indeterminacy while the second half of observations are generated under determinacy. The estimate of the forward-looking component of the NKPC is 0.81 (0.84) using the orthogonality (continuity) identification, the slope takes a value of 0.06 (0.12) while the sum of the autoregressive coefficients of the reduced-form process is 0.56 (0.72).

Figure 6 presents an experiment where, conditional on the parameters for the orthogonality and continuity cases, the data are generated using different values of the standard deviation of the sunspot shocks, σ_ζ. The estimates are computed for a grid of 15 points in the interval $[0, 1.4]$. The first row shows that the bias of the estimates of the forward-looking component increases with σ_ζ for empirically plausible values of this standard deviation. For values larger than 0.3, which exceeds the estimates in Lubik and Schorfheide (2004), the bias of the forward-looking term appears stable.

The estimates of the slope of the Phillips curve seem virtually unchanged by the size of the standard deviation. As indeterminacy can influence aggregate fluctuations both by affecting directly the equilibrium dynamics through the sunspot shock and by affecting indirectly the transmission of the structural shocks to the endogenous variables, this result suggests that the bias in the slope is mostly due to the indirect effect. The last row shows that the sum of the autoregressive coefficients of the reduced-form process of inflation is a decreasing function of σ_ζ. This is probably due to the fact that a larger variance of the sunspots shocks translates into a larger variance of the endogenous state variables without implying an higher covariance between inflation and its own lags. The overall effect is therefore a reduction in the OLS estimates.

4   Empirical Evidence

The previous section showed that pooling observations from different monetary policy regimes can be highly misleading for the inference based on the full-sample estimates of the NKPC. In this section, we present some evidence on U.K. and U.S. quarterly data that appears consistent with the 'neglected indeterminacy bias' hypothesis.

As a preview of the results, the policy regimes that the empirical literature on monetary policy rules typically associates with a weak interest rate response to inflation are characterized by a higher degree of inertia in the structural process of inflation.

The NKPC is specified in the following hybrid version:

$$\pi_t = \omega_f E_t \pi_{t+1} + \omega_b \pi_{t-1} + k x_t + v_t \tag{8}$$

Inflation is measured as the annualized quarterly change in the GDP deflator. As far as excess demand is concerned, we present results using two alternative measures of the business cycle. The first measure is the output gap. For the U.S., this corresponds to the deviation of real GDP from the official estimates of real potential output provided by the Congressional Budget Office (CBO), whereas for the U.K. it is the residuals from a regression of real GDP on a quadratic trend. The second measure is the labor share calculated as the ratio of nominal compensation to employees to nominal GDP. The data have been obtained in January 2005 from the Bank of England and the Federal Reserve Bank of St. Louis.

For the U.K., we consider the period 1979:2 to 2003:4. The starting point corresponds to the date Thatcher government was first elected and moved towards a more explicit counter-inflationary monetary policy. Moreover, the data on the U.K. labor market, including unit labor costs, began to be systematically collected and published only in 1979 with the establishment of the Labour Force Survey. The full-sample is divided around the fourth quarter of 1992 when the Bank of England announced for the first time an explicit target for inflation. Given the short length of the later period, we compare the estimates of the pre-1992 regime with the full-sample estimates. Nelson (2003) shows that the pre- and post-1992 periods are characterized by a marked difference in the monetary policy stance in that the nominal interest rate has been raised more than proportionally in response to inflation movements only after 1992.[2]

For the U.S., we consider the period 1966:1 - 1997:4. The beginning of the sample corresponds to the date the Federal funds rate has been first traded consistently above the discount rate. The first sub-sample ends in 1979:2 when Paul Volcker was appointed Chairman of the Fed and fighting inflation became a clear policy objective. The later sub-sample begins in 1982:3 and therefore excludes the period in which Bernanke and Mihov (1998) document that the operating procedure of the Fed temporarily switched from federal funds rate to non-borrowed reserves targeting. The end of sample is chosen such as to make our results

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[2] As the paper focuses on monetary policy, we abstract from fiscal policy considerations which may have also contributed to the inflation outcome of the 1980s.

comparable to the available literature which typically uses observations until 1997:4 (see Galí and Gertler, 1999, and Lubik and Schorfeide, 2004). The results are not affected however by expanding the sample until 2003:4. Clarida, Galí and Gertler (2000) pioneered a vast empirical literature finding that the monetary policy stance of the Fed can be described as *passive* during the Pre-Volcker regime and *active* during the post-Volker regime.

4.1 The estimates

Equation (8) is estimated with GMM using an optimal weighting matrix that accounts for possible heteroskedasticity and serial correlation in the error terms. In practice, we employ a three lag Newey-West estimate of the covariance matrix where the number of lags is selected according to standard lag length criteria on a four-variate VAR in inflation, output gap, unit labor cost and nominal interest rate. Starting from date $t-1$, three lags of these four variables are included as instruments corresponding to 9 overidentifying restrictions that can be tested for. The null hypothesis of valid overidentifying restrictions is never rejected.

Table 2 reports the results for the U.K.. Regardless the measure of excess demand, the pre-1992 estimates of the forward-looking component of the Phillips curve is statistically smaller than its full-sample counterpart, consistently with the prediction of the 'neglected indeterminacy bias'. In particular, the hypothesis of no backward-looking in the NKPC can only be rejected in the earlier period. The estimates of the slope display a positive sign only when the labor share measure is used and they are larger in the full-sample, though they are not statistically different from zero.

Restricting $\omega_b = (1 - \omega_f)$ does not alter our conclusions. Furthermore, letting the later sub-sample begin in the first quarter of 1993 produces results, not reported but available upon request, which are very similar to the full-sample estimates. Given the limited number of observations available since the introduction of the inflation targeting regime however we prefer not to give much weight to the finding on the later sub-sample. Interestingly, these results are consistent with and complement the reduced-form evidence in Kuttner and Posen (1999), Batini and Nelson (2001) and Benati (2005) who shows that the persistence of inflation in the U.K. has dramatically declined since the announcement of an explicit target for inflation, moving from a value between 0.79 and 0.96 before 1992 to a value not statistically different from zero afterward.

The findings for the U.S. are displayed in Table 3 and appear to bear out the evidence for the U.K.. The estimates of the forward-looking component are larger over the most recent

monetary policy regime and they are significantly so using the labor share measure. Unlike the U.K., the later sample seems characterized by a significant, albeit smaller, backward-looking term. The slope of the Phillips curve takes a positive sign only using unit labor costs and, consistently with the simulations in the previous section, it is statistically different from zero only in the post-Volcker period. The full-sample estimates based on the labor share measure, not reported but available upon request, read a slope coefficient of 0.02 which is not statistically different from zero. The reduced-form analysis in Cogley and Sargent (2005 and 2002) reveal that the persistence of U.S. inflation has increased during the second half of the 1960s and during the 1970s and then has fallen in the 1980s and 1990s. Our results are compatible with the notion of a fall in inflation inertia.

Obviously, the results in this section are only suggestive and it is beyond the scope of this paper to discriminate whether the observed decline in inflation inertia represents a genuine structural break of an intrinsic feature of the economy or rather it is the effect of indeterminacy over the earlier samples. Nevertheless, it is intriguing to observe that the structural and the reduced-form inertia of the inflation process appear a peculiarity of the periods associated with a passive monetary policy reaction function. Along this line, Cogley and Sbordone (2005) show that a constant-parameter version of the NKPC can be consistent with a drifting-parameter VAR, thereby suggesting that a structural break in the Phillips curve does not seem to account for the changing persistence of U.S. inflation.

4.2 Weak instruments

Weak instruments is an important issue we must confront with to validate our estimates. Stock and Yogo (2003) tabulate critical values for the multiple endogenous regressor analog of the first-stage F-statistics and define weak instruments in terms of bias and in terms of size of the test. In particular, a set of instruments can be deemed strong if the analog of the F-statistics is sufficiently large that either the instrumental variable bias is no more than $x\%$ of the inconsistency of OLS or a 5% hypothesis test rejects no more than $y\%$ of the time. The first definition is useful for inference purpose whereas the second seems appropriate for hypothesis testing. Unfortunately, there is no particular guidance for the selection of x and y other than the researcher's tolerance.

In general, we find that our set of instruments can be deemed strong using $x = 10$ and $y = 15$ -even more ambitious tolerance level can be met in several cases - with two exceptions. Both of them correspond to the pre-1992 regime in the U.K.. We expand then the list of instrumental

variables in these two cases to include also wage inflation according to the reasoning that important reforms in the labor market took place under Thatcher government and it seems plausible to think they also had an impact on inflation. Moreover, we reduce in this estimation the number of lags of the instrumental variables from three to two in an effort to minimize the potential small-sample bias that may arise when too many over-identifying restrictions are imposed. The second and the third columns of Table 2 show that the expanded set of instruments can now be deemed strong also over the the pre-1992 period and the estimates reported in these columns refer to the expanded instrument set.

5   Conclusions

This paper begins to bridge the gap between two bodies of research on inflation dynamics. The first body uses a microfounded NKPC to estimate the structural relation between inflation and marginal costs. On the promise of identifying truly structural parameters, this literature mainly focuses on the full postwar period with a typical sample starting in 1960. The second body uses the New-Keynesian model to demonstrate that bad monetary policy in the form of a weak interest rate reaction to inflation generates sunspot fluctuations which can sizably influence the macroeconomic dynamics.

Using a purely forward-looking New-Keynesian model as the data generating process, this paper computes the solutions of the rational expectations model for two classes of parameterizations of the interest rate rule. These parameterizations roughly correspond to the shift in the conduct of monetary policy that occurred in a number of industrialized countries around the beginning of the 1980s. Specifically, one class of coefficients represents a passive monetary policy stance according to which the central bank can generate indeterminacy by moving the nominal interest rate insufficiently in response to inflation pressures. The second class of parameterizations describes an activist conduct that conforms to the Taylor principle and therefore produces a unique stable solution.

Montecarlo simulations demonstrate that the estimates of the forward-looking component and the slope of the NKPC can be severely biased downward whenever two conditions hold. First, the data are generated under a passive monetary policy rule. Second, the estimation procedure arbitrarily rule out the possibility of indeterminacy. Furthermore, this paper shows that the bias becomes larger the closer the interest rate response to inflation approaches the boundary between indeterminacy and determinacy. These results are robust to the number of observations in the simulated sample and to the selection of the instrumental variable estima-

tor. Finally, when the above two conditions are met the sum of autoregressive coefficients in the reduced-form representation of the inflation process is close to one, even though the data generating process exhibits no intrinsic persistence.

Empirical evidence on the NKPC using data for the U.K. and U.S. economies shows that inflation inertia is far more pronounced during the monetary policy regimes characterized by a less-than-proportional response of nominal interest rate to inflation. This result holds independently from whether the measure of excess demand is labor share or output gap, and is in line with the prediction of the 'neglected indeterminacy bias' hypothesis. Moreover, our structural estimates are consistent with and complement the reduced-form evidence in Benati (2005) for the U.K. and in Cogley and Sargent (2005) for the U.S. that the change in inflation persistence is concomitant with a policy regime shift.

Structural breaks in the monetary policy rule have therefore serious implications for the inference based on the NKPC. This finding indicates some caution is needed to interpret the results from full-sample analyses which pool observations from different monetary policy regimes. And, the neglected indeterminacy bias can arise even if the Phillips curve is a structurally invariant relation.

An interesting avenue for future research is to estimate a time-varying structural model that at each point in time contemplates the possibility of a switch between the indeterminacy and the determinacy solution. Furthermore, a richer model of the business cycle may relax the tight link between the degree of activism in the policy rule and indeterminacy, with consequences for the 'neglected indeterminacy bias' that are worth exploring.

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Appendix: Solution of the LRE Model

In order to transform the canonical form and solve the model, we follow Sims (2001) and exploit the QZ decomposition of the matrices Γ_0 and Γ_1. This corresponds to computing the matrices Q, Z, Λ and Ξ such that $QQ' = ZZ' = I_n$, Λ and Ξ are upper triangular, $\Gamma_0 = Q'\Lambda Z$ and $\Gamma_1 = Q'\Xi Z$. Moler and Stewart (1973) prove that the QZ decomposition always exists. Defining $w_t = Z's_t$ and pre-multiplying (4) by Q, we obtain:

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ 0 & \Lambda_{22} \end{bmatrix} \begin{bmatrix} w_{1,t} \\ w_{2,t} \end{bmatrix} = \begin{bmatrix} \Xi_{11} & \Xi_{12} \\ 0 & \Xi_{22} \end{bmatrix} \begin{bmatrix} w_{1,t-1} \\ w_{2,t-1} \end{bmatrix} + \begin{bmatrix} Q_{1.} \\ Q_{2.} \end{bmatrix} (\Psi \varepsilon_t + \Pi \eta_t) \tag{9}$$

where the vector of generalized eigenvalues λ, which is the ratio between the diagonal elements of Ξ and Λ, has been partitioned such that the lower block collects all the explosive eigenvalues. The matrices Ξ, Λ and Q have been partitioned accordingly, and therefore $Q_{j.}$ collects the blocks of rows that correspond to the stable ($j = 1$) and unstable ($j = 2$) eigenvalues respectively.

The explosive block of (9) can be rewritten as:

$$w_{2,t} = \Lambda_{22}^{-1} \Xi_{22} w_{2,t-1} + \Lambda_{22}^{-1} Q_{2.} (\Psi \varepsilon_t + \Pi \eta_t)$$

A non-explosive solution of the linear rational expectations model (4) for s_t requires $w_{2,t} = 0$ $\forall t \geq 0$. This can be obtained by setting $w_{2,0} = 0$ and choosing for every vector ε_t the endogenous forecast error η_t that satisfies the following condition:

$$\Psi^* \varepsilon_t + \Pi^* \eta_t = 0 \tag{10}$$

where $\Psi^* = Q_{2.} \Psi$ and $\Pi^* = Q_{2.} \Pi$.

In general, we can be confronted with three cases. If the number of endogenous forecast errors is equal to the number of unstable eigenvalues, the system is determined and the stability condition (10) uniquely determines η_t. If the number of endogenous forecast errors does exceed the number of unstable eigenvalues, the system is undetermined and sunspot fluctuations can arise. If the number of endogenous forecast errors is smaller than the number of unstable eigenvalues, the system has no solutions. This condition generalized Blanchard and Kahn's (1980) procedure of counting the number of unstable roots and predetermined variables.[3]

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[3] Sim's solution method has the advantage that it does not require the separation of predetermined variables from 'jump' variables. Rather, it recognizes that in equilibrium models expectational residuals are attached to equations and that the structure of the coefficient matrices in the canonical form implicitly selects the linear combination of variables that needs to be predetermined for a solution to exist.

A general solution for the endogenous forecast error can be computed through a singular value decomposition of $\Pi^* = UDV'$. Lubik and Schorfeide (2003) show that this solution takes the following form:

$$\eta_t = \left(-V_{.1}D_{11}^{-1}U_{.1}'\Psi^* + V_{.2}M_1\right)\varepsilon_t + V_{.2}M_2\zeta_t \tag{11}$$

where D_{11} is the upper-left diagonal block of D, U and V are orthonormal matrices, and M_s with $s = 1,2$ are the matrices that govern the influence of the sunspot shock on the model dynamics.

Solution (11) can be combined with (4) to yield the following law of motion for the state vector:

$$s_t = \Gamma_1^* s_{t-1} + \left[\Psi^* - \Pi^* V_{.1}D_{11}^{-1}U_{.1}'\Psi^*\right]\varepsilon_t + \Pi^* V_{.2}\left(M_1\varepsilon_t + M_2\zeta_t\right) \tag{12}$$

where for expositional convenience the notation (θ) is suppressed whenever we refer to a single vector of parameters equation-wide.

Equation (12) shows that indeterminacy has two consequences. First, sunspot fluctuations ζ_t can influence equilibrium dynamics as long as M_2 is a non-zero matrix. Second, the transmission of fundamental shocks ε_t to the endogenous variables is no longer uniquely identified as the elements of M_1 are not pinned down by the structure of the linear rational expectations model. Under determinacy $V_{.2} = 0$ and therefore the sunspot shock has no effect on aggregate fluctuations.

In order to compute the solutions of the model under indeterminacy, it is necessary to impose some additional restrictions on the endogenous forecast errors. In practice, we normalize $M_2 = 1$ such that ζ_t can be reinterpreted as a reduced-form sunspot shock. Moreover, we follow Lubik and Schorfeide (2003) and focus on two alternative identification schemes for M_1 which are labelled orthogonality and continuity. The first auxiliary assumption is that the effects of fundamental and sunspot shocks on the forecast error are *orthogonal* to each other. This correspond to assuming $M_1 = 0$.

The second identifying scheme corresponds to choosing M_1 such that the impulse responses $\partial s_t/\partial \varepsilon_t'$ are *continuous* at the boundary between determinacy and indeterminacy region. Let Θ^I and Θ^D be the sets of all possible vectors of parameters, $\theta's$, in the indeterminacy and determinacy region respectively. For every vector $\theta \in \Theta^I$ we identify a corresponding vector $\widetilde{\theta} \in \Theta^D$ that lies on the boundary of the two regions and choose M_1 such that the response of s_t to ε_t conditional on θ mimics the response conditional on $\widetilde{\theta}$. In practice, we minimize the

least squares deviations of the two impulse responses such that:

$$M_1 = \left[B_2'(\theta) B_2(\theta) \right]^{-1} B_2'(\theta) \left[B_1(\widetilde{\theta}) - B_1(\theta) \right] \tag{13}$$

where

$$B_1(\widetilde{\theta}) = \frac{\partial s_t}{\partial \varepsilon_t'}(\widetilde{\theta})$$

and

$$B_1(\theta) + B_2(\theta)M_1 = \left[\Psi^*(\theta) - \Pi^*(\theta)V_{.1}(\theta)D_{11}^{-1}(\theta)U_{.1}'(\theta)\Psi^*(\theta) \right] + \Pi^*(\theta)V_{.2}(\theta)M_1 = \frac{\partial s_t}{\partial \varepsilon_t'}(\theta, M_1)$$

The new vector $\widetilde{\theta}$ is obtained from θ by replacing ψ_1 with condition (5), which marks the boundary between the determinacy and indeterminacy region in the system (1) to (3).[4]

The solution of (9) is now fully characterized and for any given vector of parameters of the model it is possible to compute the evolution of the state variables under both determinacy and indeterminacy. In particular, the forecast error η_t and the law of motion for the latent state:

$$w_{1,t} = \Lambda_{11}^{-1}\Xi_{11}w_{1,t-1} + \Lambda_{11}^{-1}Q_{1.}\left(\Psi\varepsilon_t + \Pi\eta_t \right) \tag{14}$$

can be used to obtain $s_t = Zw_t$. The ratio $\Lambda_{11}^{-1}\Xi_{11} = \lambda_1(\theta)$ in (14) represents the generalized *stable* eigenvalue of $\Gamma_1^*(\theta)$ in the system (12) and it is the source of 'extra' persistence in the solution of the model (1) to (3) under indeterminacy.

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[4]Lubik and Schorfheide (2004) notice that this way of computing the vector M_1 relates to the search for the minimal-state-variable solution advocated by McCallum (1983), i.e. the most meaningful solution from an economic perspective among the n-possible ones under indeterminacy.

Table 1: Model Parameters

| Parameters	Indeterminacy	Determinacy
y_p	**0.77**	**2.19**
y_y	**0.17**	**0.30**
b	0.99	0.99
k	0.77	0.77
t^{-1}	1.45	1.45
s^R	0.23	0.23
s_g	0.27	0.27
s_z	1.13	1.13
s_z	0.20	-

Note: The parameterization of the data generating process under indeterminacy corresponds to the estimates in Lubik and Schorfheide (2004) over the pre-Volcker period. The solutions of the model under indeterminacy use the estimates in the second column. The solution of the model under determinacy uses the estimates in the third column.

Figure 1: Forward-looking component in the Phillips Curve
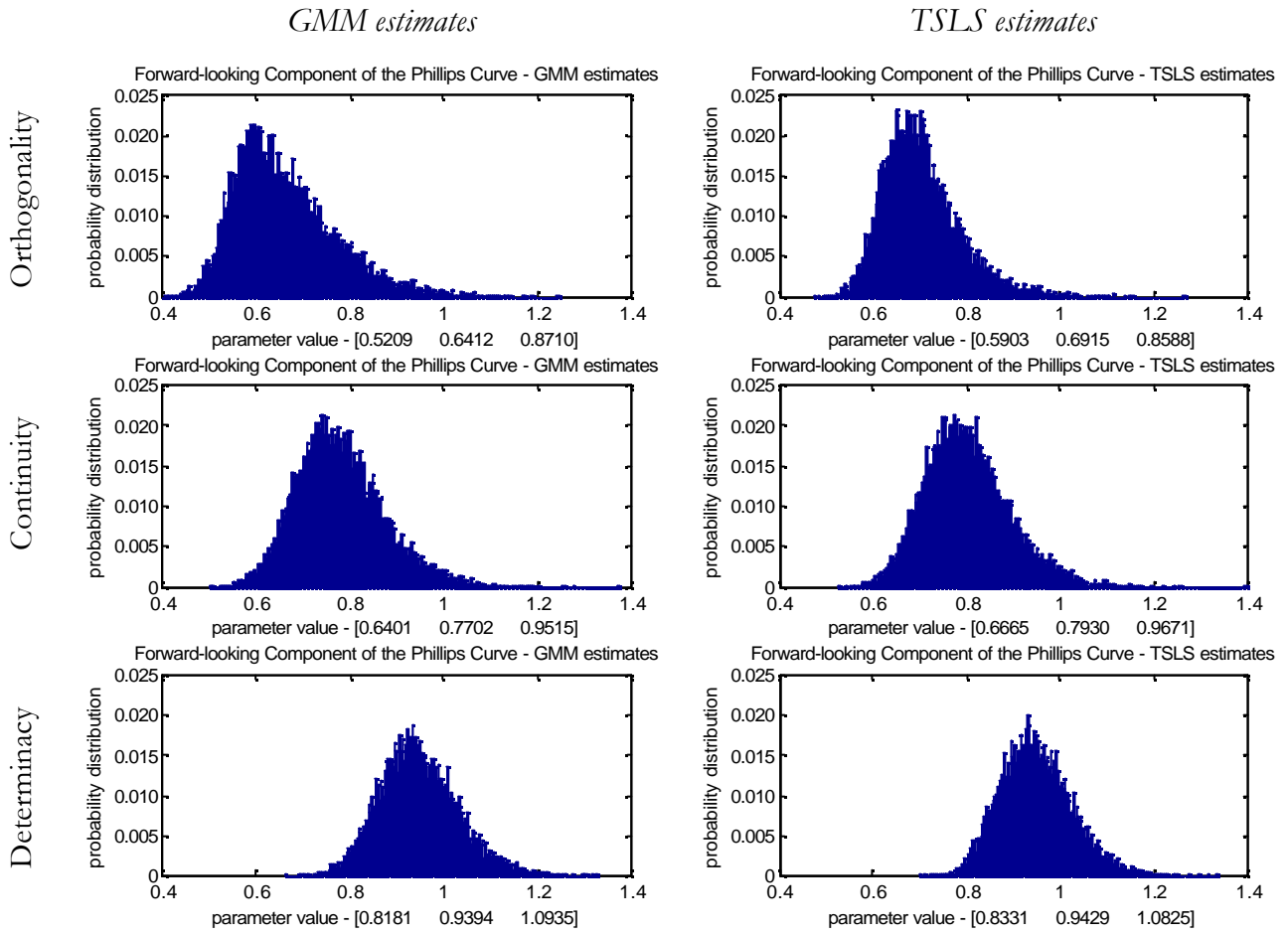
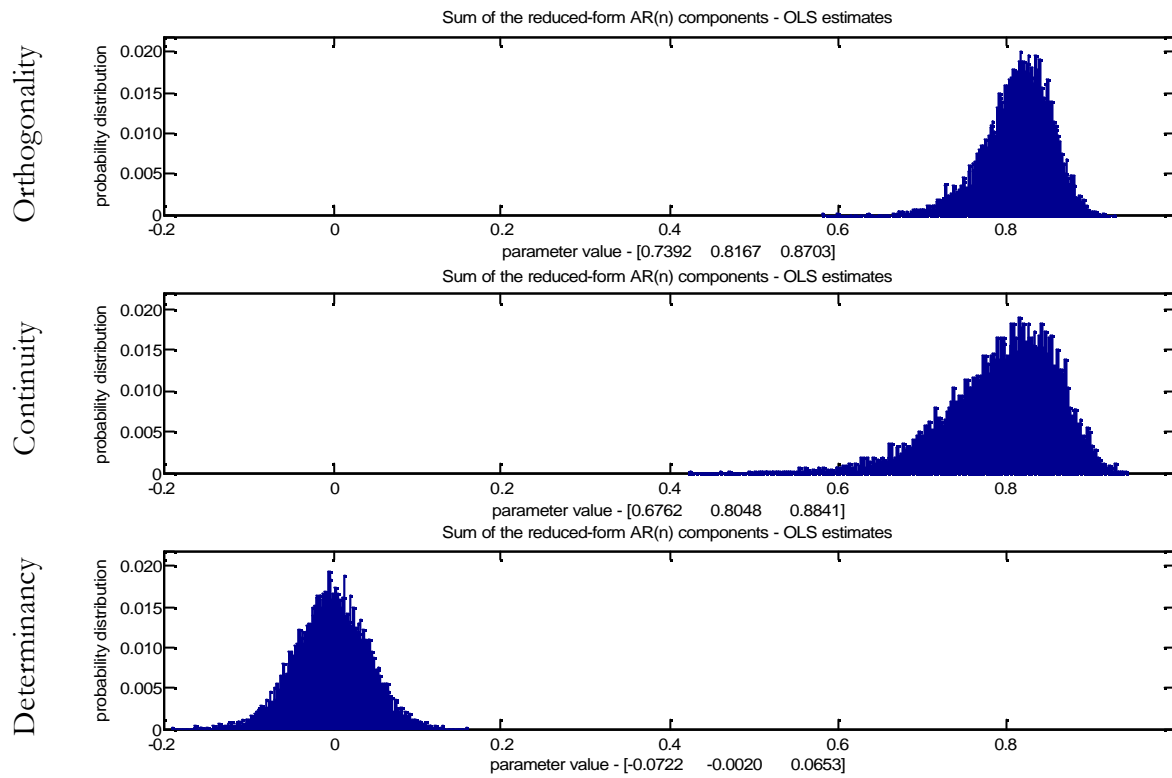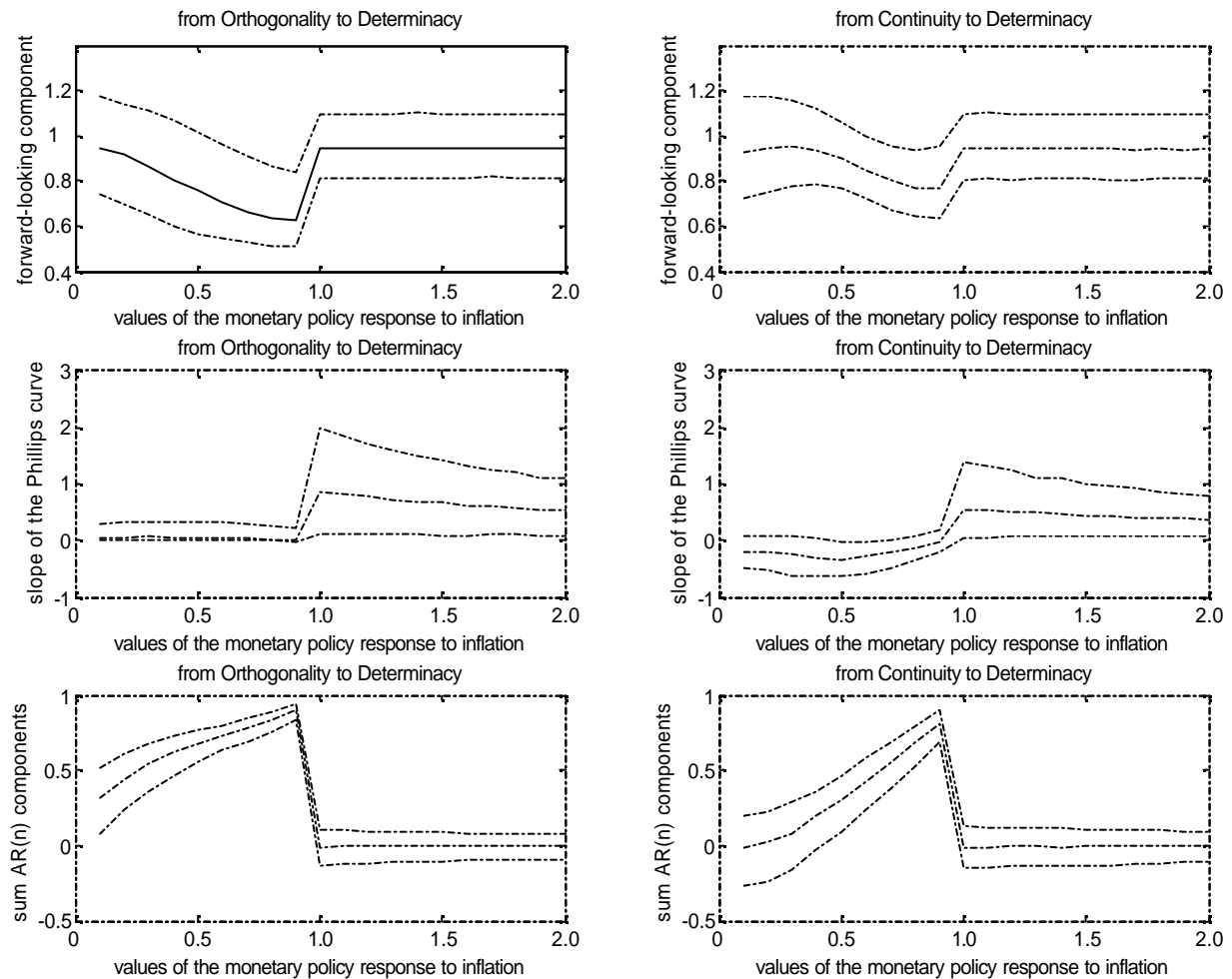*GMM estimates*   *TSLS estimates*

Note: The data generating process is a purely forward-looking model. The parameters are set to the values in Table 1. Estimates are based upon 10,000 repetitions of a sample of 200 observations. Each simulated sample is initiated with 100 extra observations to get a stochastic initial state, which are then discarded. Numbers in squared brackets represent the 5th, the 50th and the 95th percentile of the confidence interval, respectively.

Figure 2: Slope of the Phillips Curve
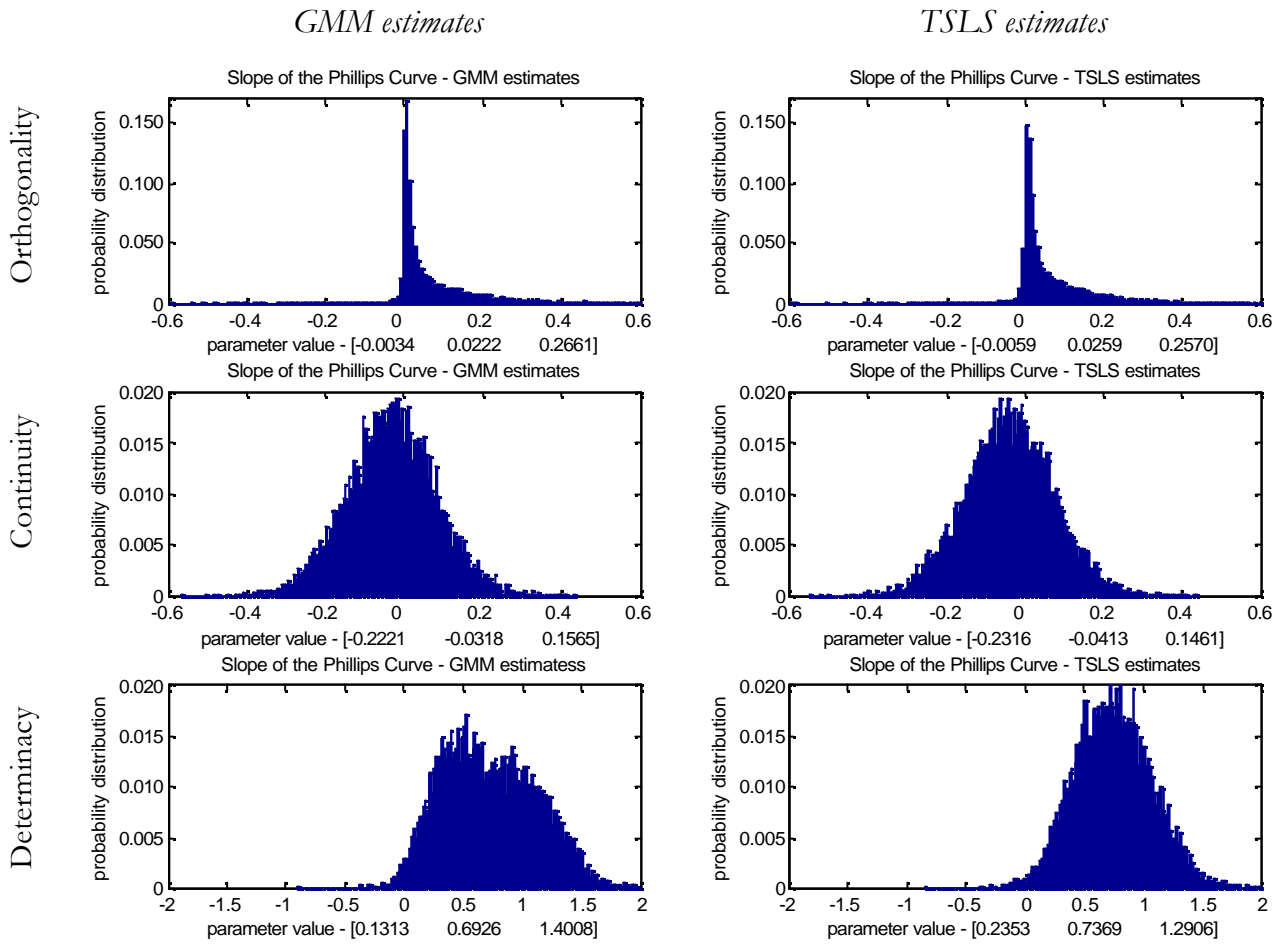
GMM estimates   *TSLS estimates*

Note: The data generating process is a purely forward-looking model. The parameters are set to the values in Table 1. Estimates are based upon 10,000 repetitions of a sample of 200 observations. Each simulated sample is initiated with 100 extra observations to get a stochastic initial state, which are then discarded. Numbers in squared brackets represent the 5th, the 50th and the 95th percentile of the confidence interval, respectively.

Figure 3: Sum of the Reduced-form AR(n) components – OLS estimates

Note: The data generating process is a purely forward-looking model. The parameters are set to the values in Table 1. Estimates are based upon 10,000 repetitions. Each simulated sample is initiated with 100 extra observations to get a stochastic initial state, which are then discarded. Numbers in squared brackets represent the 5th, the 50th and the 95th percentile of the confidence interval, respectively.
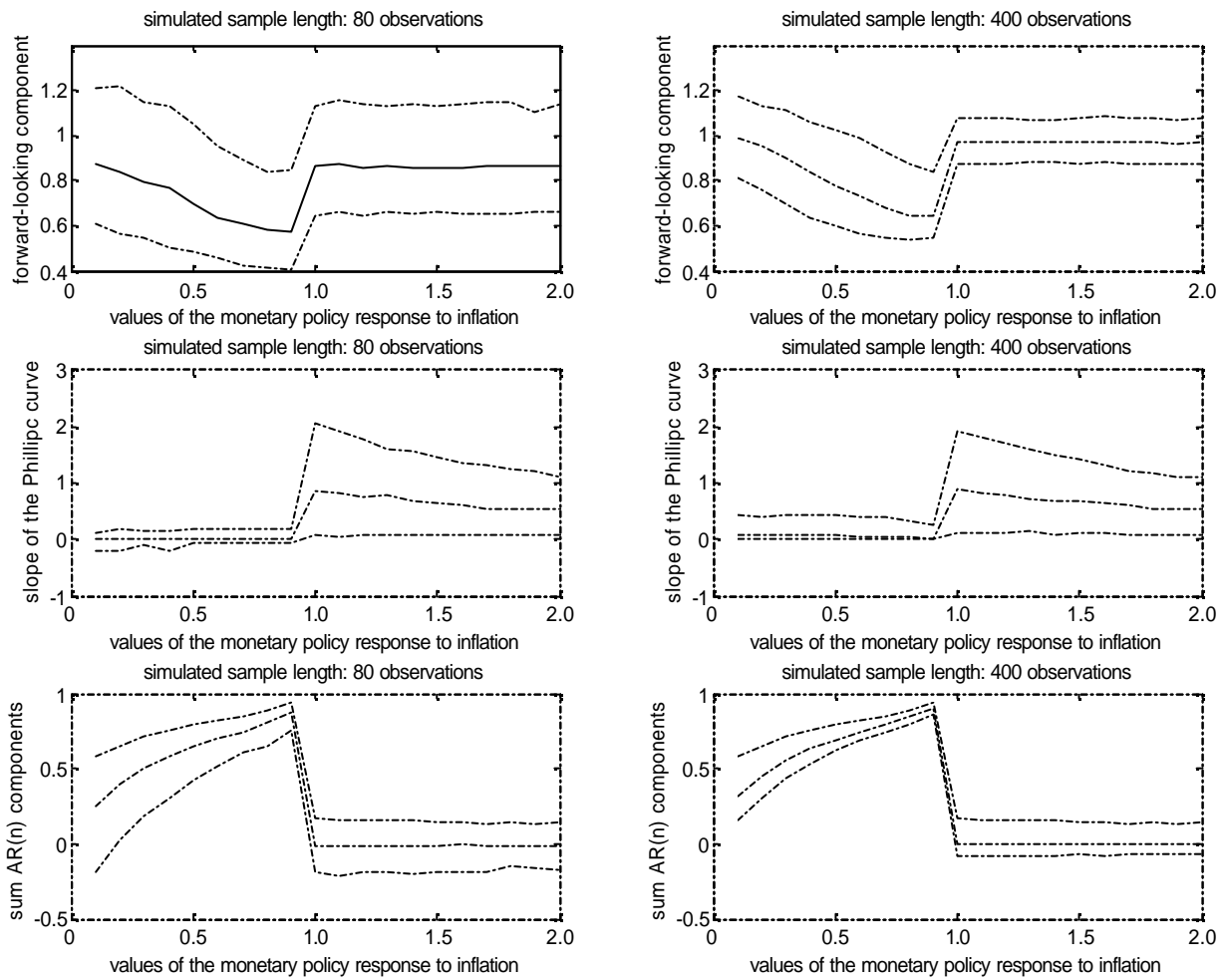
Figure 4: GMM estimates as a function of the monetary policy response to inflation
- from Indeterminacy to Determinacy –

Note: The data generating process is a purely forward-looking model. The parameters are set to the values in Table 1. Estimates are based upon 10,000 repetitions of a sample of 200 observations. Each simulated sample is initiated with 100 extra observations to get a stochastic initial state, which are then discarded. The dotted line corresponds to the point estimate whereas the dashed lines refer to the 5th and the 95th percentile of the confidence interval, respectively.

Figure 5: GMM estimates as a function of the monetary policy response to inflation
- from Indeterminacy to Determinacy with different number of observations –

Note: The data generating process is a purely forward-looking model. The parameters in the indeterminacy region are set to the values of Case 1 in Table 1. Estimates are based upon 1,000 repetitions of two samples of 80 and 400 observations respectively. Each simulated sample is initiated with 100 extra observations to get a stochastic initial state, which are then discarded. The dotted line corresponds to the point estimate whereas the dashed lines refer to the 5th and the 95th percentile of the confidence interval, respectively.

Figure 6: GMM estimates as a function of the standard deviation of the sunspot shock
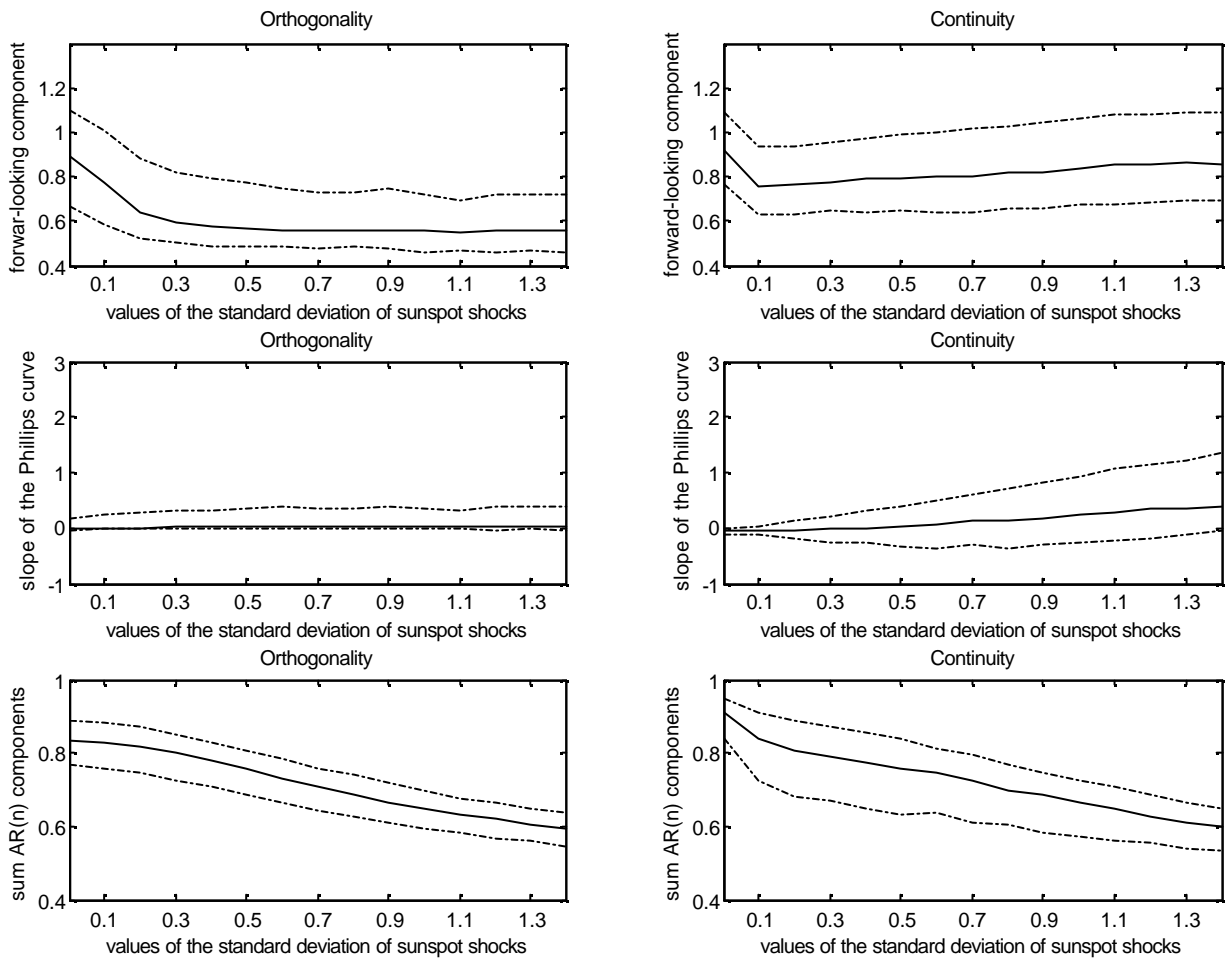
Note: The data generating process is a purely forward-looking model. The parameters are set to the values in Table 1. Estimates are based upon 1,000 repetitions of a sample of 200 observations. Each simulated sample is initiated with 100 extra observations to get a stochastic initial state, which are then discarded. The dotted line corresponds to the point estimate whereas the dashed lines refer to the 5th and the 95th percentile of the confidence interval, respectively.

Table 2: GMM estimate of the NKPC – United Kingdom

sample	1979:2 – 1992:4		1979:2 – 2003:4	
specification	*Labour share*	*Output gap*	*Labour share*	*Output gap*
w_f	0.594*** (0.126)	0.633*** (0.137)	1.002*** (0.134)	1.063*** (0.124)
w_b	0.396*** (0.119)	0.354*** (0.132)	0.016 (0.128)	-0.073 (0.126)
k	0.009 (0.072)	-0.023 (0.043)	0.037 (0.046)	-0.079* (0.042)
J-stat p-value	0.333	0.346	0.767	0.929
Analog F-stat	21.567[#]	17.858[#]	23.552[#]	25.472[#]

Notes: Standard errors using a three lag Newey-West estimate of the covariance matrix are reported in brackets. If not specified otherwise, the instrument set includes three lags of inflation, output gap, labour share and nominal interest rate. *J* refers to the statistics of Hansen's test for *m* over-identifying restrictions which is distributed as a $\chi^2(m)$ under the null hypothesis of valid over-identifying restrictions. *Analog F* refers to the minimum eigenvalue of the matrix analog of the first-stage F-statistics. The test rejects the null hypothesis of weak instruments in favour of the alternative of strong instruments if *Analog F* exceeds the critical value. The critical value is computed at the 5% significance level. The superscript ***, ** and * denote the rejection of the null hypothesis that the true coefficient is zero at the 1 percent, 5 percent and 10 percent significance levels, respectively. The superscript [#] denotes the rejection of the null hypothesis of weak instruments.

Table 3: GMM estimate of the NKPC – United States

sample	1966:1 – 1979:2		1982:3 – 1997:4	
specification	*Labour share*	*Output gap*	*Labour share*	*Output gap*
w_f	0.605*** (0.075)	0.721*** (0.080)	0.815*** (0.093)	0.802*** (0.068)
w_b	0.376*** (0.075)	0.274*** (0.083)	0.185** (0.084)	0.188*** (0.063)
k	0.120 (0.096)	-0.072 (0.062)	0.194** (0.089)	-0.034 (0.046)
J-stat p-value	0.606	0.471	0.515	0.325
Analog F-stat	21.661[#]	27.218[#]	33.143[#]	18.103[#]

See notes to Table 2 for details.