# Uninsured Idiosyncratic Production Risk With Borrowing Constraints*

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**Abstract**

This paper analyzes a general-equilibrium model of a heterogeneous agents economy in which the agents are subject to borrowing constraints and uninsurable idiosyncratic production risk. In particular, it addresses the impact of these frictions for aggregate capital accumulation. In contrast to other studies, the results suggest that, when entrepreneurs are poorly diversified, the underaccumulation of capital in the entrepreneurial sector of the model economy is less likely to hold, because of a strong precautionary savings motive. Furthermore, the effect of these frictions on entrepreneurial investment exacerbates the overaccumulation of capital in the non-entrepreneurial sector of the economy that is reported in Bewley models with uninsurable labor income risk.

**Keywords:** Incomplete markets, Precautionary savings, Entrepreneurial investment, Borrowing constraints, Aggregate savings.

**JEL Classification:** E22, G11, M13

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# 1 Introduction

This paper investigates the importance of uninsured idiosyncratic production risk and borrowing constraints, by the firm owned by the entrepreneur, for aggregate capital accumulation. More specifically, I study the implications of the lack of diversification in entrepreneurs' earnings for aggregate capital accumulation. This is important, in this setting, for the following reasons. First, the volatility of entrepreneurial earnings is substantially larger than that of wages from paid employment (Hamilton, 2000). Second, the portfolio of entrepreneurial households is biased towards their business (Gentry and Hubbard, 2004), which makes their equity return highly correlated with their human capital return (Moskowitz and Vissing-Jørgensen, 2002). This paper shows that, when borrowing constraints exist, and in the presence of uninsurable investment risks with poorly diversified entrepreneurs, I obtain a higher steady-state level of capital in the entrepreneurial and non-entrepreneurial sectors of the economy. The driving force of this result is the lack of diversification in entrepreneurs' earnings, which generates strong precautionary savings, and leads to overaccumulation of capital in the entrepreneurial sector. Because entrepreneurs accumulate a buffer stock of wealth and are relatively wealthy households, they also exert a large influence on the accumulation of capital in the non-entrepreneurial sector. Understanding this mechanism is useful for analyzing important issues in macroeconomics, particularly the implications of incomplete markets for asset pricing and business cycles.

To illustrate this mechanism, I use a general-equilibrium model of entrepreneurial investment in which agents are subject to uninsurable production risk and borrowing constraints. In the model, a large number of entrepreneurs are able to pursue different investment portfolio choices. In particular, entrepreneurial wealth is allocated between a risky and a safe investment. The return on the risky investment is subject to uninsurable idiosyncratic productivity shocks. The return on the safe investment is the equilibrium interest rate. I introduce borrowing constraints as a *short-sales* constraint on the safe investment. Also, the only source of non-stochastic earnings is the return on the safe investment.

In the presence of uninsurable production risk, risk-averse entrepreneurs prefer an investment portfolio biased towards the safe investment. More specifically, in the absence of binding borrowing constraints *and* when the return on the safe investment is not affected by the entrepreneurs' investment decisions, there is underaccumulation of capital in the risky investment. In general equilibrium, however, precautionary savings by all entrepreneurs increase the demand for the safe investment, which lowers its equilibrium return. This, in turn, increases the attractiveness of the risky investment relative to the safe investment, and weakens the underaccumulation result.

Borrowing constraints play a key role in the model: they make it more difficult for the entrepreneur to smooth consumption, which increases the desire to save more in general (precautionary savings). An increase in demand for the safe asset leads to a decrease in the equilibrium interest rate, because the safe asset is in zero net supply. Consequently, the risky investment becomes even more attractive and, in equilibrium, if the precautionary savings effect is strong enough, entrepreneurs accumulate excess capital in the steady state. In almost all cases considered, these two frictions yield a higher steady-state level of entrepreneurial capital than in the complete markets case. Particularly, for plausible parameterizations, aggregate entrepreneurial capital could be higher by as much as 50 percentage points.

I extend the model to include non-entrepreneurial agents who supply labor to the non-entrepreneurial (corporate) sector, and are subject to uninsurable labor income risk. These extensions are important for two reasons: (i) the zero net-supply assumption on the safe investment is relaxed, and (ii) they allow a comparison between my results and those in standard Bewley (1977) models with uninsurable labor income risk (e.g., Aiyagari, 1994). In this environment, I predict an overaccumulation of capital in the entrepreneurial sector by 25 per cent in an economy that is able to replicate the wealth concentration in the U.S. economy. I also find an overaccumulation of capital in the corporate sector of 35 per cent. This is considerably higher than the 11 per cent that is obtained from a standard Bewley (1977)-style model with uninsurable labor income risk, imposing the same parameterization.

Several theoretical studies exist on the macroeconomic implications of uninsurable production risks.[1] Angeletos (2005) analyzes the implications of uninsurable idiosyncratic production risk for aggregate investment and the macroeconomy. In particular, Angeletos (2005) assumes borrowing constraints never bind and entrepreneurs have two sources of non-stochastic income: labor income and the return on a bond. In my model, borrowing constraints occasionally bind, and entrepreneurs do not receive a *riskless* wage income. For example, Moskowitz and Vissing-Jørgensen (2002) suggest business income is highly correlated with entrepreneurs' human capital return. For empirically plausible parameterizations, I find that uninsurable production risks lead to an increase in investment demand relative to complete markets, and my model economy displays overaccumulation in both the entrepreneurial and corporate sectors of the economy. This contrasts with Angeletos' (2005) findings, in which uninsurable investment risks lead to underaccumulation of capital.

Meh and Quadrini (2005) consider a model that includes both uninsurable investment risks and borrowing constraints, and they use it to study welfare changes under different risk-sharing environments. Similar to Angeletos' (2005) model, Meh and Quadrini's (2005) model economy experiences underaccumulation of capital. There are two main differences between their model and my model. First, entrepreneurs are more diversified in Meh and Quadrini's model, because they have two different sources of income, in addition to business profits. Second, investment risk in their model appears both in the form of production and depreciation risks. Because of these two main differences, the entrepreneurs in my model are less diversified in terms of income. My results also suggest that the introduction of a corporate sector in this analysis is important because wealthy entrepreneurial households have a large influence on the accumulation of capital in the corporate sector.

[1] Namely, on the implications of uninsurable production/investment risks for growth (Khan and Ravikumar, 2001; Angeletos and Calvet, 2005), business cycle dynamics (Angeletos, 2005), welfare changes under different institutional environments (Meh and Quadrini, 2005), and policy experiments regarding corporate taxation (Meh, 2003).

This paper is organized as follows. Section 2 describes the entrepreneurial economy model in which all agents are subject to uninsurable production risk, and it derives the equilibrium conditions that are obtained from the individual decision problem. Section 3 introduces functional forms and the benchmark parameterization. Section 4 analyzes the deviation between the capital stock in the economy with frictions and the capital stock that would be chosen in an economy without production risk. Section 5 analyzes the overall efficiency of capital accumulation in an economy that is able to replicate the wealth concentration observed in the data. Finally, section 6 offers some conclusions.

# 2 Entrepreneurial Economy

This section introduces the benchmark economy. Consider an economy with a measure one of infinitely lived entrepreneurial households. Each household has the ability to operate its own technology. This technology represents the risky investment of the agent, because it is subject to uninsurable idiosyncratic productivity shocks. The model presented in this section is similar to the one used in Angeletos (2005), with three important exceptions: (i) entrepreneurs face occasionally binding borrowing constraints and, as in Aiyagari (1994), entrepreneurial wealth heterogeneity plays an important role in determining equilibrium prices; (ii) the return on a safe investment is the only source of non-stochastic earnings; and (iii) the idiosyncratic production process can exhibit positive serial correlation.[2]

## 2.1 Environment

For simplicity, there is only one consumption good. The utility function of each entrepreneur, $U(\cdot)$, is strictly increasing, strictly concave, obeys the Inada conditions, and is twice continuously differentiable in consumption, $c_t$. Since there are idiosyncratic shocks, $c_t$ will differ across agents. To simplify the notation, I do not index the variables to indicate this cross-sectional variation. The entrepreneur's problem is to maximize the expected lifetime utility derived from consumption:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t), \tag{1}$$

where $0 < \beta < 1$ is the discount factor.

Each period, the entrepreneur can invest both in the individual-specific technology, which represents the risky investment, and also in a safe investment that yields a sure return. In addition, the entrepreneur is allowed to borrow to finance both consumption and the risky investment; that is, the entrepreneur may choose to invest a negative amount of funds in the safe investment. Borrowing is constrained for reasons of moral hazard and

[2] Angeletos (2005) assumes that there are no binding borrowing constraints, productivity is serially uncorrelated, and entrepreneurs have two sources of non-stochastic earnings.

adverse selection not explicitly modelled, and the limit is fixed exogenously for simplicity. The risky technology available to the entrepreneur is represented by

$$y_t = z_t f(k_t), \tag{2}$$

where $z_t$ denotes productivity and $k_t$ is the capital stock in the risky investment. This investment is risky because the stock of capital, $k_t$, is chosen one period in advance; that is, before observing the level of productivity, $z_t$. The idiosyncratic productivity process, $z_t$, follows a first-order Markov process. Further, assume that $f(\cdot)$ is continuously differentiable, strictly increasing, strictly concave with $f(0) = 0$, and satisfying the Inada conditions. Also, capital depreciates at a fixed rate, $\delta$, and the gross risky investment is given by

$$i_t = k_{t+1} - (1-\delta)k_t. \tag{3}$$

Let $b_{t+1}$ denote the resources of the entrepreneur allocated to the safe investment. This investment pays a sure return, $r$, in each period. This rate of return is determined in equilibrium such that the bond market clears in each period. In this environment, the entrepreneur's budget constraint is as follows:

$$c_t + k_{t+1} + b_{t+1} = x_t, \tag{4}$$

$$x_{t+1} = z_{t+1} f(k_{t+1}) + (1-\delta)k_{t+1} + (1+r)b_{t+1}, \tag{5}$$

where $x_t$ denotes the entrepreneur's period $t$ wealth.

Let $v(z, x)$ be the optimal value function for an entrepreneur with productivity $z$ and wealth $x$.[3] The entrepreneur's optimization problem can be specified in terms of the following dynamic programming problem:

$$\begin{aligned} v(z,x) = \max_{k',b'} \; & U(x - k' - b') + \beta E[v(z',x')|z], \\ \text{s.t.} \quad & x' = z' f(k') + (1-\delta)k' + (1+r)b', \\ & k' \geqslant 0 \quad \text{and} \quad b' \geqslant \underline{b}, \end{aligned} \tag{6}$$

where $\underline{b}$ represents the exogenous borrowing constraint faced by the entrepreneur. From the properties of the utility and production functions of the entrepreneur, the optimal levels of consumption and the risky investment are always strictly positive. The only constraint that may be binding is the choice of $b'$. Taking first-order conditions of problem (6) and using the envelope condition, the first-order conditions of the problem are as follows:

$$U_c(c) = \beta(1+r)E[U_c(c')|z] + \lambda, \tag{7}$$

$$U_c(c) = \beta E[(z' f_k(k') + 1 - \delta)U_c(c')|z], \tag{8}$$

[3] Because the entrepreneurs' problem is recursive, the subscript $t$ is omitted for all variables in the current period, and I let the prime denote the value of variables one period ahead.

where $\lambda$ is the Lagrange multiplier associated with the entrepreneur's borrowing constraint, $b' \geqslant \underline{b}$. The Lagrange multiplier is positive if the constraint is binding, and zero otherwise. Definition 1 summarizes the steady-state equilibrium in this economy.

**Definition 1** *The steady-state equilibrium in this economy is: a value function for the entrepreneur, $v(z, x)$; the policy functions of the entrepreneur $\{k(z, x), b(z, x), c(z, x)\}$; a value for the interest rate, $r$; entrepreneurial bond demand, $B$; and a probability measure of entrepreneurs, $\Gamma(z, x)$, such that:*

*(i) Given $r$, the entrepreneur's policy functions solve the entrepreneur's decision problem (6).*

*(ii) The bond market clears:*

$$\int b(z, x) d\,\Gamma(z, x) = 0. \tag{9}$$

*(iii) Given the policy functions of the entrepreneur, the probability measure of entrepreneurs, $\Gamma(z, x)$, is invariant.*

Finally, aggregate entrepreneurial capital in this economy is defined as:

$$K = \int k(z, x) d\,\Gamma(z, x). \tag{10}$$

# 3 Parameterization

The properties of the model can be evaluated only numerically. Therefore, I need to assign functional forms and parameter values to find the numerical solution of the model. For this purpose I chose standard functional forms and parameter values. The period is one year, and so the discount factor, $\beta$, is set equal to 0.96. For the utility function, a constant relative risk-aversion (CRRA) specification is assumed:

$$U(c) = \frac{c^{1-\gamma}}{1-\gamma}, \tag{11}$$

where $\gamma$ is the risk-aversion parameter. In the benchmark model, this parameter is chosen to be 2. The entrepreneur's risky technology is given by $k^{\alpha}$, with the curvature parameter, $\alpha$, equal to 0.36. The depreciation rate, $\delta$, equals 0.08. The idiosyncratic productivity process is first-order Markov:

$$\ln(z') = \rho_z \ln(z) + \sigma_z (1 - \rho_z)^{1/2} \epsilon', \tag{12}$$

where $\epsilon \sim N(0, 1)$; the serial correlation parameter, $\rho_z$, is set at 0.90; and the unconditional standard deviation of productivity, $\sigma_z$, is set at 0.4. The specification presented in (12) is convenient; in section 4, I will show the sensitivity of the model's steady state to changes in the level of serial correlation of productivity risk, keeping the unconditional standard deviation constant. As part of the numerical algorithm, I use the procedure suggested by Tauchen and Hussey (1991) to approximate the first-order autoregressive process with a discrete-state stochastic process that has seven states.[4] I set the exogenous borrowing constraint at $\underline{b} = -4$, which corresponds to roughly twice the net income generated by the average entrepreneur in one year.[5] Table 1 summarizes the parameter values adopted in the benchmark entrepreneurial model.

# 4 Capital Accumulation in the Entrepreneurial Economy

In this section, I analyze the deviation between the capital stock in the economy with frictions and the capital stock that would be chosen in an economy without production risk. Although the model is a general-equilibrium model, I will consider a simple partial-equilibrium version to gain intuition regarding the importance of the different aspects of the model.

## 4.1 Numerical solution

The solution method used to solve the dynamic programming problem works directly on the first-order conditions of the problem, defined in equations (7) and (8). The appendix describes the numerical procedures used to find the solution to the entrepreneur's problem. Table 1 provides details concerning the discretization of the state space.

Figure 1 plots the entrepreneur's decision rules for consumption, the risky and safe investments, and the safe-to-risky investment ratio. The decision rules are a function of the two state variables of the model: wealth and the idiosyncratic productivity shock. For clarity, the figure describes only the policy rules for three different levels of idiosyncratic productivity: the lower and the upper bound, and the mean value. The two top plots depict the entrepreneur's investment-portfolio decision rules for the risky and safe investments. Poor entrepreneurs invest all their wealth in the risky investment and are at the short-sales constraint in the safe investment. As wealth levels increase, investment shifts towards the safe investment, but the rate at which entrepreneurs do so depends on their productivity level.

## 4.2 The mechanism

The uninsurable production risk and borrowing constraints give rise to two opposing effects on capital accumulation. First, an increase in the amount of production risk means that the investment itself becomes more risky, which implies that the agents would like to invest less in the risky investment. Second, an increase in the

[4] The results are robust to finer discretizations; namely, I solved the model with 15 and 21 states for the exogenous stochastic process and found minor changes in the results.

[5] Huggett (1993) suggests that a credit limit of one year's average endowment is a reasonable one. I loosen up his suggestion by one year. In the data, however, individuals can often borrow much more than that. For this reason, in section 4.3.1, I show the robustness of the results to this assumption.

amount of production risk also means that it becomes harder to smooth consumption, which implies that the agents would like to save more in general (precautionary savings). Since the agents can also invest in a safe asset, one might think that in this economy they would respond to an increase in production risk by substituting out of the risky investment and into the safe asset. In general equilibrium, however, this is not possible, since the safe asset is in zero net supply. Consequently, the increased demand for the safe asset leads to a decrease in the interest rate. This, in turn, makes the risky asset more attractive. If the precautionary savings motive is very strong, then the interest rate would decrease considerably and the investment in the risky asset might very well increase. To understand the quantitative importance of the different channels, I solve a partial-equilibrium version of the model, in which the interest rate is set equal to a value just below $1/\beta - 1$. Note that $1/\beta - 1$ corresponds to the interest rate under the assumption of complete markets.[6]

The left panel in Figure 2 plots the aggregate level of capital allocated to the risky technology, the safe investment, and aggregate consumption as a function of the standard deviation of production risk, $\sigma_z$.[7] First, the aggregate level of capital is below the complete markets case and declines as production risk increases. The underaccumulation of capital provides a measure of the importance of uninsurable production risk. Second, the supply of the safe investment is above zero (the equilibrium value) and increases sharply with increases in the amount of production risk.

Because the aggregate supply of the safe investment is greater than zero, the outcomes described above are not an equilibrium. To generate an equilibrium, the price of the safe investment must go up; that is, the interest rate must adjust downwards. The large drop in the interest rate that is needed to get back to equilibrium indicates that the precautionary motive is very strong for this model. The right panel in Figure 2 plots the equilibrium response of capital, interest rates, and consumption as a function of the standard deviation of productivity risk. For the specification of productivity risk used here, the price of the safe investment must go up significantly to generate equilibrium in the bond market. Consequently, the lower return on the safe investment increases the attractiveness of the risky investment, resulting in a higher steady-state level of entrepreneurial capital, despite the presence of uninsurable production risk.

## 4.3 Sensitivity analysis

I repeat this exercise for different parameterizations to verify the robustness of the results. I find that, for the specification of production risk considered here,[8] the results are robust, except for low values of the coefficient of risk aversion. Moreover, I find that the amount of overaccumulation of capital increases when the coefficient

[6] Chamberlain and Wilson (2000) show that, with $r = 1/\beta - 1$, consumption can grow without bound as $t$ goes to infinity. I set $r$ equal to 0.04166, whereas $r = 1/\beta - 1$ is equal to 0.041667.

[7] For simplicity, I assume that the process is serially uncorrelated.

[8] In section 4.4, I will consider another specification to describe the risk of the return on capital investment and show that the form of risk is crucial for the results reported here.

of relative risk aversion increases, the persistence of productivity risk increases, and the maximum amount by which agents can go short in the safe asset decreases.

#### 4.3.1 Borrowing constraints

Figure 3 plots the sensitivity of aggregate capital stock to changes in the short-sales constraint of the safe investment. The figure shows that tighter short-sales constraints exacerbate the overaccumulation result. This is not surprising, because under more restrictive borrowing constraints it is more difficult for entrepreneurs to smooth consumption in the presence of adverse shocks to productivity. Hence, entrepreneurs have a stronger incentive to self-insure and respond by increasing their demand for the safe investment. This, in turn, generates an even stronger response of the equilibrium interest rate, resulting in a higher steady-state level of entrepreneurial capital.

In addition, Figure 3 shows that the economy displays overaccumulation even with very loose borrowing constraints. For example, even when the credit limit is equivalent to four times the net income of the entrepreneur, there is still some overaccumulation. Hence, I do not need to resort to unreasonably tight borrowing constraints to generate a higher steady-state level of capital relative to the complete markets case. Even with the "natural" borrowing limit, the economy displays overaccumulation of capital of 0.15 per cent when $\sigma_z = 0.40$.[9]

#### 4.3.2 Risk aversion

Figure 4 plots the sensitivity of aggregate capital to different levels of the risk-aversion coefficient. The figure shows that, if the value of the coefficient of risk aversion is sufficiently low, then the economy displays underaccumulation of capital. Note that, if $\alpha = 1$ and $\delta = 1$, one would get the standard result: overaccumulation when $\gamma > 1$ and underaccumulation when $\gamma < 1$. For the values of $\alpha$ and $\delta$ considered here, there is still a small amount of underaccumulation when $\gamma = 1$, but the amount is small. According to Angeletos (2005), if the elasticity of intertemporal substitution is higher than the capital share, then that is a sufficient condition to give rise to underaccumulation of capital. Under CRRA preferences, this condition is equivalent to stating that the solution displays underaccumulation when $\frac{1}{\gamma} > \alpha$. Table 2 shows that, in the presence of borrowing constraints and when the entrepreneur cannot supply labor to a competitive labor market, I get overaccumulation even in cases where $\frac{1}{\gamma} > \alpha$. For example, with $\underline{b} = -2$, the solution displays underaccumulation when the coefficient of relative risk aversion is equal to 0.5. When $\gamma = 1$, and according to Angeletos' (2005) sufficient condition, the solution should display underaccumulation for $\alpha \in \{0.3, 0.5, 0.7\}$; however, the solution always

[9] In this model, the "natural" borrowing constraint is given by

$$b' \geqslant -\frac{z_1 f(k_1) - \delta k_1}{r},$$

where $z_1 = \min z$ and $k_1 = (\frac{1/\beta - 1 + \delta}{\alpha z_1})^{1/(\alpha-1)}$. In particular, for $\sigma_z = 0.40$ the "natural" borrowing limit is $\underline{b} \approx -11.9$.

displays overaccumulation for these values of $\alpha$. When $\underline{b} = -4$ and $\gamma = 1$, the solution displays overaccumulation for $\alpha \in \{0.5, 0.7\}$, and for $\underline{b} = -8$ it displays overaccumulation only for $\alpha = 0.7$. These results suggest that loosening the credit limit increases the number of examples where there is underaccumulation. Still, the experiment with the "natural" borrowing limit reported in Table 2 suggests that borrowing constraints are not sufficient to explain the overturning of the underaccumulation of capital reported by Angeletos (2005). Otherwise, the economy would display underaccumulation of capital at the "natural" borrowing constraint. There is another important feature of the model, which is the fact that entrepreneurs cannot supply labor in a competitive labor market. Under this assumption, entrepreneurial income is relatively more concentrated in the entrepreneurial risky investment. The presence of occasionally binding borrowing constraints and the lack of income diversification of the entrepreneur give powerful incentives to entrepreneurs to increase savings for consumption-smoothing (self-insurance) reasons, which further depresses the interest rate and increases the accumulation of capital. Empirical evidence suggests that "the average entrepreneur holds most of his investment in the same private firm in which he works, making his equity return highly correlated with his human capital return" (Moskowitz and Vissing-Jørgensen, 2002, p. 746); hence, the case where the entrepreneur is poorly diversified is highly relevant to this study.

More generally, these results are *qualitatively* consistent with the intuition provided by Angeletos (2005). In particular, $\alpha$ and $\gamma$ are also important parameters in my environment, and the qualitative effect of changing these two parameters is as in Angeletos (2005). I introduce other important dimensions that should be taken into account in these studies. Namely, under the presence of occasionally binding borrowing constraints, it is important to also consider the level of diversification of the entrepreneur. Finally, note that these results consider only the case where productivity is independent and identically distributed (i.i.d.). Relaxing this assumption increases the number of cases where the solution displays overaccumulation, as I show in section 4.3.3. Given all these dimensions in which results may change, it is more difficult to find a sufficient condition relevant to the assumptions advanced in this study.

#### 4.3.3 Persistence in productivity

This section analyzes the sensitivity of the model's solution to changes in the serial correlation of the risky technology. Figure 5 plots the difference between the aggregate capital stock under incomplete and complete markets, for different levels of serial correlation in productivity. Focusing on the case with $\sigma_z = 0.4$, I find that, as persistence is increased, the equilibrium interest rate drops considerably. In particular, when there is no serial correlation, the interest rate is equal to 3.5 per cent, whereas it is equal to 1.2 per cent in the case with $\rho = 0.9$ (Table 3). Not surprisingly, the large drop in the interest rate when the amount of persistence increases leads to a large increase in risky capital. In particular, when $\rho = 0.9$, the aggregate level of capital stock is 24 per cent higher than would be observed under complete markets. Note that investment is not irreversible, and so an increase in persistence does not increase the risk of the return on the risky investment. Persistence in the productivity process does mean, however, that if bad times occur they last longer, which increases the importance of the precautionary savings motive.

Table 3 also reports the Gini coefficient of wealth. Interestingly, there is an increase in wealth dispersion in the economy, even for very high levels of persistence. This contradicts the findings of Krusell and Smith (1997); in their environment, very high degrees of persistence tend to reduce wealth dispersion. The fact that agents can operate their own technology and the presence of borrowing constraints are key to generating this result: they induce a strong precautionary savings motive. This implies that the increase in the Gini index is being driven by the agents at the upper tail of the wealth distribution. The intuition is that, following an increase in persistence: (i) poorer agents increase the amount borrowed, and more agents exhibit binding borrowing constraints, and (ii) in response to the increase in the interest rate, rich agents do not invest as much in capital.

Table 4 compares, for different values of $z$, the average capital stock observed in this economy for agents that receive a productivity shock equal to $z$, and the aggregate capital stock in an economy with complete markets in which $z$ would represent the aggregate productivity shock.[10] Consistent with Figure 5, Table 4 shows that the average overaccumulation of capital is much smaller when there is less persistence. When there is no serial correlation, agents in the complete markets economy choose a constant capital stock. In contrast, in the economy with incomplete markets, agents choose a lower capital stock when productivity is low, to smooth consumption. Since the average amount of overaccumulation is small, the agents choose for low values of $z$ a value for capital that is *smaller* than the one chosen in the complete markets economy. If there is persistence, then agents in the complete markets economy decrease the choice of capital as $z$ decreases, because lower values of $z$ indicate lower future values of $z$. Agents in the economy with incomplete markets also choose lower capital stocks, but the capital stock decreases less if $z$ decreases. The intuition is that agents in an economy with incomplete markets are more careful in reducing their capital stock as $z$ decreases, since the capital stock helps them insure against more bad shocks. Consistent with this reasoning is the observation that, relative to the capital stock in the economy with complete markets, the capital stock in the economy with incomplete markets is now highest for low values of $z$.

## 4.4 Depreciation risk

I showed in section 4.3 that the overaccumulation of entrepreneurial capital, in the presence of uninsurable production risk, is robust to most changes in the parameters considered. In this section, I investigate whether the result is robust to a change in the specification of entrepreneurial risk. An important alternative considered in the literature is the specification for which the amount of depreciation is stochastic. Moreover, this alternative

[10] This is equivalent to weighting the capital choice under complete markets, conditional on current productivity, with the frequency at which the different states occur.

has been shown to be helpful in explaining the equity premium puzzle (Storesletten, Telmer, and Yaron, 2001). In particular, assume that the depreciation rate follows the law of motion given by:

$$\tilde{\delta}_t = \delta + s\eta_t, \tag{13}$$

where $\eta_t \sim N(0, \sigma_\eta^2)$. To ensure that the condition $\delta_t \in (0, 1]$ is not violated in the numerical example, $\eta_t$ is discretized into seven grid points and the value of $s$ is set equal to 0.08.[11] In the recursive formulation given in equation (6), the entrepreneur's resources become

$$x' = f(k') + (1 - \tilde{\delta}')k' + (1 + r)b', \tag{14}$$

where $\tilde{\delta}' = \delta + s\eta'$. Figure 6 plots aggregate entrepreneurial capital and aggregate bonds as a function of the level of the standard deviation of $\eta$. The left panel shows the results for the partial-equilibrium case and the right panel shows the results for the general-equilibrium case. Clearly, in the partial-equilibrium case, as uncertainty increases, the increase in bonds relative to the reduction in capital is considerably lower in the presence of depreciation risk.[12] In general equilibrium, the uninsurable risk effect dominates the precautionary savings effect and, as a result, there is underaccumulation of capital in the technology subject to depreciation risk. The presence of depreciation risk does not generate a substantial amount of volatility in consumption. Because output is not subject to shocks, the entrepreneur's flow of income is approximately constant. Only the stock of capital changes, owing to depreciation risk, but this generates a small precautionary savings effect. Consequently, the reduction in the interest rate that is needed to keep the bond market in equilibrium is small, and the steady-state level of capital is lower than the level observed in complete markets.

I next analyze the results when production and depreciation risk are both present. For simplicity, I focus on two special cases. In the first case, the two shocks are positively correlated; namely, periods of high productivity are accompanied by high depreciation. In particular, I set $s > 0$ and $\eta = \ln(z)$. In the second case, I assume that the two shocks are negatively correlated; that is, periods of high productivity coincide with periods of low depreciation. In particular, I assume that $\eta = \ln(z)$ and change the sign of $s$ (so that $s < 0$), but keep the absolute value of $s$ unchanged.

Figure 7 plots aggregate entrepreneurial capital and aggregate bonds as a function of the standard deviation of production and depreciation risk. With negative correlated shocks, the uninsurable risk effect is quite strong. This is not surprising, because periods of low productivity are accompanied by a large write-off, which exacerbates the risk associated with the investment in entrepreneurial capital. In fact, for lower levels of production risk, the uninsurable risk and the precautionary savings effect roughly offset each other, and

[11] For example, with $\sigma_\eta = 0.6$, $\bar{\delta} = [0.01, 0.03, 0.04, 0.08, 0.14, 0.25, 0.45]$.

[12] Although I examine the exact same values of the standard deviations, these are not comparable, owing to the different specifications. Still, the volatility of net profits generated by the stochastic depreciation case with $\sigma_\eta = 0.4$ is roughly the same as for the one generated with $\sigma_z = 0.10$.

there is no under- or overaccumulation of entrepreneurial capital. The effects are clearly non-linear in the amount of production risk, however, and when the standard deviation of productivity and depreciation risks exceeds 0.4, the model generates overaccumulation of capital. In the case where the two shocks are positively correlated, the uninsurable risk effect is clearly dominated by the precautionary savings effect. There is some diversification, because periods of low production are "compensated" with a lower depreciation rate. Under the positive correlation assumption, the risky investment is a relatively attractive asset with which to accumulate wealth and smooth consumption over time.

# 5 Extension to a Stylized Entrepreneurial Economy

In the previous section, I analyzed the effect of production risk on entrepreneurial capital in an economy that consisted only of entrepreneurs. I showed that the quantitative effect clearly depended on the level of the interest rate needed to keep entrepreneurs' aggregate demand for bonds equal to zero. The question arises as to how the results would change if entrepreneurs' aggregate demand for bonds could be non-zero; for example, because there are other types of agents in the economy. In this section, I modify the environment introduced in section 2 as follows: (i) a fixed fraction of agents do not have access to the risky technology (workers); (ii) a corporate sector replaces the investment in bonds as the safe investment; and (iii) workers face uninsurable wage risk, as in Aiyagari (1994). Several key parameters are chosen, so that the concentration of wealth generated by the model corresponds to the one observed in the data.

## 5.1 Environment

Assume that only a fixed fraction of agents has access to the risky technology. The agents that do not have access to this technology are denoted by workers in the model. These agents are heterogeneous with respect to wealth holdings and earnings ability. They choose consumption to maximize their expected lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(\tilde{c}_t), \tag{15}$$

subject to the following budget constraint:

$$\tilde{c}_t + a_{t+1} = w l_t + (1+r) a_t, \tag{16}$$

where $\tilde{c}_t$ denotes the worker's consumption in period $t$, $a_t$ denotes the worker's savings in period $t$, $w$ is the worker's wage rate, and $l_t$ represents a labor-efficiency process. I assume that workers are also subject to a borrowing constraint; that is, $a_{t+1} \geqslant \underline{a}$, where $\underline{a} \leqslant 0$.

Let $\tilde{v}(l, a)$ be the optimal value function for a worker with labor efficiency, $l$, and savings, $a$. The worker's optimization problem can be specified in terms of the following dynamic programming problem:

$$
\begin{aligned}
& \tilde{v}(l,a) = \max_{\tilde{c},a'} \; U(\tilde{c}) + \beta E[\tilde{v}(l',a')|l], \\
\text{s.t.} \quad & \tilde{c} + a' = wl + (1+r)a, \\
& \quad a' \geqslant \underline{a}.
\end{aligned}
\tag{17}
$$

The stochastic process for the labor-efficiency process is first-order autoregressive:

$$
\ln(l') = \rho_l \ln(l) + \sigma_l (1-\rho_l)^{1/2} \varepsilon', \tag{18}
$$

where $\varepsilon \sim N(0,1)$. The solution to this problem yields the agent's optimal decision rule with respect to consumption, $\tilde{c}(l,a)$, and the next period's asset-demand function, $g(l,a)$. In this economy, the consumption good is produced by two sectors: (i) the entrepreneurial sector and (ii) a corporate sector that uses a constant-returns-to-scale Cobb-Douglas production function, which uses capital and non-entrepreneurial labor as inputs. The aggregate technology is represented by:

$$
Y = F(K_c, L). \tag{19}
$$

The problem of the entrepreneur is exactly the same as that described in section 2 and will not be discussed here. Note that the safe investment of the entrepreneur corresponds to lending (borrowing) funds to (from) the corporate sector, and the equilibrium interest rate equals the marginal productivity of capital in this sector.

To complete the description of the model, I describe the market-clearing conditions for labor and capital in the new environment. With respect to labor, only the corporate sector technology demands labor input, so the wage rate equals the marginal productivity of labor in the corporate sector. With respect to capital, the equilibrium interest rate is also equal to the marginal productivity of capital in equilibrium. The supply of capital to the corporate sector equals non-entrepreneurs' savings in addition to entrepreneurs' portfolio holdings in the safe investment. Definition 2 summarizes the steady-state equilibrium in the extended economy.

**Definition 2** *The steady-state equilibrium in this economy is: a value function for the entrepreneur, $v(z,x)$, and for the worker, $\tilde{v}(l,a)$; the entrepreneur's policy functions $\{k(z,x), b(z,x), c(z,x)\}$; the worker's policy functions $\{g(l,a), \tilde{c}(l,a)\}$; factor prices, $(r,w)$; capital and labor demand from the corporate sector, $K_c$ and $L$; a constant cross-sectional distribution of entrepreneurs' characteristics, $\Gamma_e(z,x)$, with mass $\chi$; a constant cross-sectional distribution of workers' characteristics, $\Gamma_w(z,x)$, with mass $(1-\chi)$, such that:*

*(i) Given $r$, the entrepreneur's policy functions solve the entrepreneur's decision problem (6).*

*(ii) Given $r$, and $w$, the worker's policy functions solve the worker's decision problem (17).*

*(iii) Capital and labor markets clear:*

$$K_c = \int g(l,a)d\Gamma_w(l,a) + \int b(z,x)d\Gamma_e(z,x), \tag{20}$$

$$L = \int d\Gamma_w(l,a), \tag{21}$$

*where the household sector integrals are defined over the state space $L \times \mathcal{A}$, and the entrepreneurial sector integrals are defined over the state space $Z \times \mathcal{X}$.*

*(iv) The factor prices are equal the marginal productivity of capital (net of depreciation) and labor:*

$$r = F_K(K_c, L) - \delta, \tag{22}$$

$$w = F_L(K_c, L). \tag{23}$$

*(v) Given the policy functions of entrepreneurs and workers, the probability measures of entrepreneurs, $\Gamma_e$, and workers, $\Gamma_w$, are invariant.*

Finally, aggregate entrepreneurial capital in this economy is defined as:

$$K_e = \int k(z,x)d\Gamma_e(z,x). \tag{24}$$

## 5.2 Aggregate capital accumulation and the wealth distribution

So far, I have studied capital accumulation in an economy where all agents are subject to uninsurable production risks and the safe investment is in zero net supply. With the introduction of a corporate sector, I analyze the efficiency of capital accumulation in the entrepreneurial and corporate sectors, respectively. For the results, the target is an artificial economy that is able to replicate the wealth concentration in the U.S. economy, as measured by the Gini index reported in Quadrini (2000).

The choice of parameter values is as follows. The fraction of entrepreneurs is set at 10 per cent, following Quadrini (1999). Both groups have the same preference parameter; that is, the coefficient of relative risk aversion is equal to 2. For workers, the law of motion for labor earnings follows the estimates reported in Storesletten, Telmer, and Yaron (2004); namely, it is very persistent, with an autocorrelation coefficient of 0.95 and an unconditional standard deviation of 0.3. For entrepreneurs, the productivity process is also assumed to be very persistent, with an autocorrelation coefficient of 0.95 and a standard deviation of 0.8. Evidence reported by Hamilton (2000) suggests that the standard deviation of self-employment earnings is at least 2 to 4 times larger than wages received from paid employment. This parameterization is consistent with this empirical evidence, but I do not have a direct estimate of the persistence of entrepreneurial productivity shocks. To match the Gini index of wealth observed in the United States, 0.76, the curvature of the entrepreneurial technology, $\alpha$, is chosen to be 0.45. In addition, the short-sales constraint on bonds is $\underline{b} = -4$, which is about the net income of the average entrepreneur for one year. This constraint is not very tight. In equilibrium, only 8 per cent of the entrepreneurial population are borrowing the maximum amount. Finally, the remaining parameters $(\beta, \delta, \tilde{\alpha}, \underline{a})$ are taken from Aiyagari (1994). Table 5 summarizes the parameter values assumed for this exercise.

Suppose that the fraction of entrepreneurs in this economy is zero; that is, all agents are workers, $\chi = 0$. The equilibrium return on capital is 3.4 per cent. The economy displays an aggregate overaccumulation of capital of 11 per cent. This exercise replicates Aiyagari's result. Next, I assume that 10 per cent of the workers are entrepreneurs. I find an overaccumulation of capital in the entrepreneurial as well as in the corporate sectors. In this case, the equilibrium interest rate is 2 per cent. The entrepreneurial sector overaccumulates capital by about 25 per cent. The corporate sector overaccumulates capital by 36 per cent.

The intuition for this result is straightforward. In this economy, entrepreneurs face large uninsurable risks in production, which induces them to accumulate a buffer stock of wealth. Because entrepreneurs are so wealthy relative to workers, they have a large influence on the accumulation of capital in the corporate sector. Thus, uninsurable production risks introduce sizable distortions in both the entrepreneurial and corporate sectors of the economy. Finally, the results of the model are consistent with empirical evidence found in Heaton and Lucas (2000) and Guvenen (2003), who argue that uninsurable idiosyncratic risks and incomplete markets are more important for entrepreneurial households.

# 6 Conclusion

I have analyzed a general-equilibrium model of a heterogeneous agents economy with incomplete markets, to understand the size and magnitude of the distortions to entrepreneurial investment introduced by production risk *and* borrowing constraints. This economy displays overaccumulation of capital under a wide range of plausible parameterizations. The strong overaccumulation result is due to the interaction of uninsurable risks and credit market frictions, which generate a strong precautionary savings effect. I have also shown that combining models with entrepreneurial and non-entrepreneurial uninsurable risks lowers the net return to capital substantially more. On the one hand, this implies that the distortions found in Bewley-type economies are exacerbated. On the other hand, the decrease in the interest rate has potentially large implications for the welfare of workers in models with incomplete markets.

Given the importance of credit market frictions on the results of this study, other types of financial contracts should be explored that mimic more closely the contracts offered by financial intermediaries. An interesting feature of the data that is abstracted in most studies is the possibility of defaults arising in equilibrium. This is important, because the possibility of default increases risk-sharing as long as filing for bankruptcy allows entrepreneurs to keep a fraction of their assets together with current and future earnings. Thus, extending the current model along the lines of Chatterjee, Corbae, Nakajima, and Ríos-Rull (2002) would be a fruitful avenue of research. Meh and Terajima (2005) take interesting steps in this direction.

An important limitation of my analysis is the absence of aggregate risk. It would be interesting to extend the entrepreneurial economy along the lines of den Haan (1996, 1997) and Krusell and Smith (1997), to measure the size of the precautionary savings effect in this environment and link it to business cycles fluctuations. That is left for future research.

## Appendix: Numerical Methods

In this appendix, I describe the numerical procedures used to compute equilibrium in the entrepreneurial economy. This can be done in two steps. In the first step, the numerical procedure solves the individual's problem taking the interest rate as given. In the second step, the equilibrium interest rate is determined.

The algorithm can be started by guessing bounds on the interest rate, $r$; that is, by assuming that the equilibrium interest rate lies in the interval $[r_l, r_u]$. Given this interval, let the equilibrium interest rate equal $\frac{1}{2}[r_l + r_u]$ and solve the entrepreneur's problem, for example, using Coleman's (1990) algorithm, which consists in solving for a fixed point in the consumption function. The policy function $c(z, x)$ is approximated with a piecewise linear interpolant of the state variable, $x$. This variable is discretized in non-uniformly spaced grid points. In particular, there are more grid points to lower values of wealth, because there is more curvature in the consumption function owing to the presence of borrowing constraints. The productivity process, $z$, is assumed to follow a Markov chain with $E(z) = 1$, and the probability of transiting from state $i$ to state $j$ is given by $\pi_{i,j}$, $i, j = 1, \ldots, n_z$. The discretization of the exogenous stochastic process follows the numerical method proposed by Tauchen and Hussey (1991).

Given an initial guess, $c_0$, use expressions (4), (7), and (8) with $\lambda = 0$ to find $(c_1, k_1', b_1')$ at each grid point. After computing the solution at each grid point, I check whether the choice of bond holdings violates the short-sales constraint. In cases where the short-sales constraint is violated—that is, $b_1' < \underline{b}$—let $b_1' = \underline{b}$ and use (4) and (8) to determine $(c_1, k_1')$ at those grid points. Use $c_1$ as the new initial guess and iterate on this procedure until $\sup |\ln c_1 - \ln c_0|$ over all grid points is less than some convergence parameter, $\epsilon = 0.0000001$.

After obtaining the decision rule $c(z, x)$, it is necessary to compute the mean bond holdings $E(b')$ in order to check whether the interest rate clears the bond market. One easy way to evaluate this expectation is to generate a long time series for bond holdings and approximate it with its sample average. Before simulating this time series, solve for the equilibrium $(c, k', b')$ in a fine set of grid points for $x$ at each productivity state, to speed-up the simulation step. Then generate a long time series of the exogenous productivity state conditional on the initial state, $z_0$. Given an initial wealth level, $x_0$, generate a time series for consumption and capital using piecewise linear interpolants, and use expression (4) to find the time series of bondholdings. The size of the simulation is 10,000 and the first 1,000 observations are discarded. Finally, compute the sample average of bondholdings. If it is negative, then set $r_l = r$ and repeat the above steps. Otherwise, set $r_u = r$ and repeat until $|E(b')| < \epsilon$.

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**Figure 1: Entrepreneur's Policy Rules**

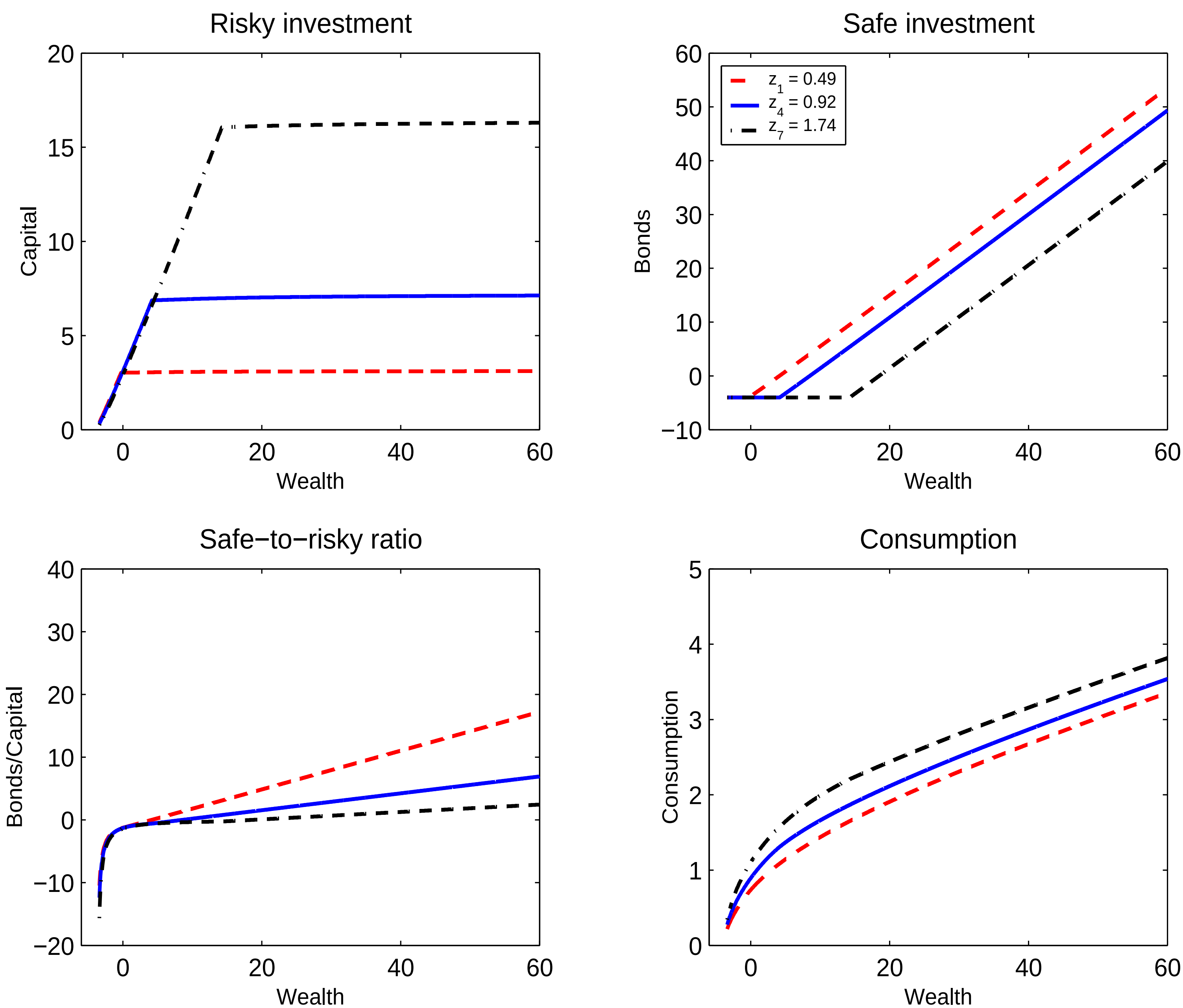
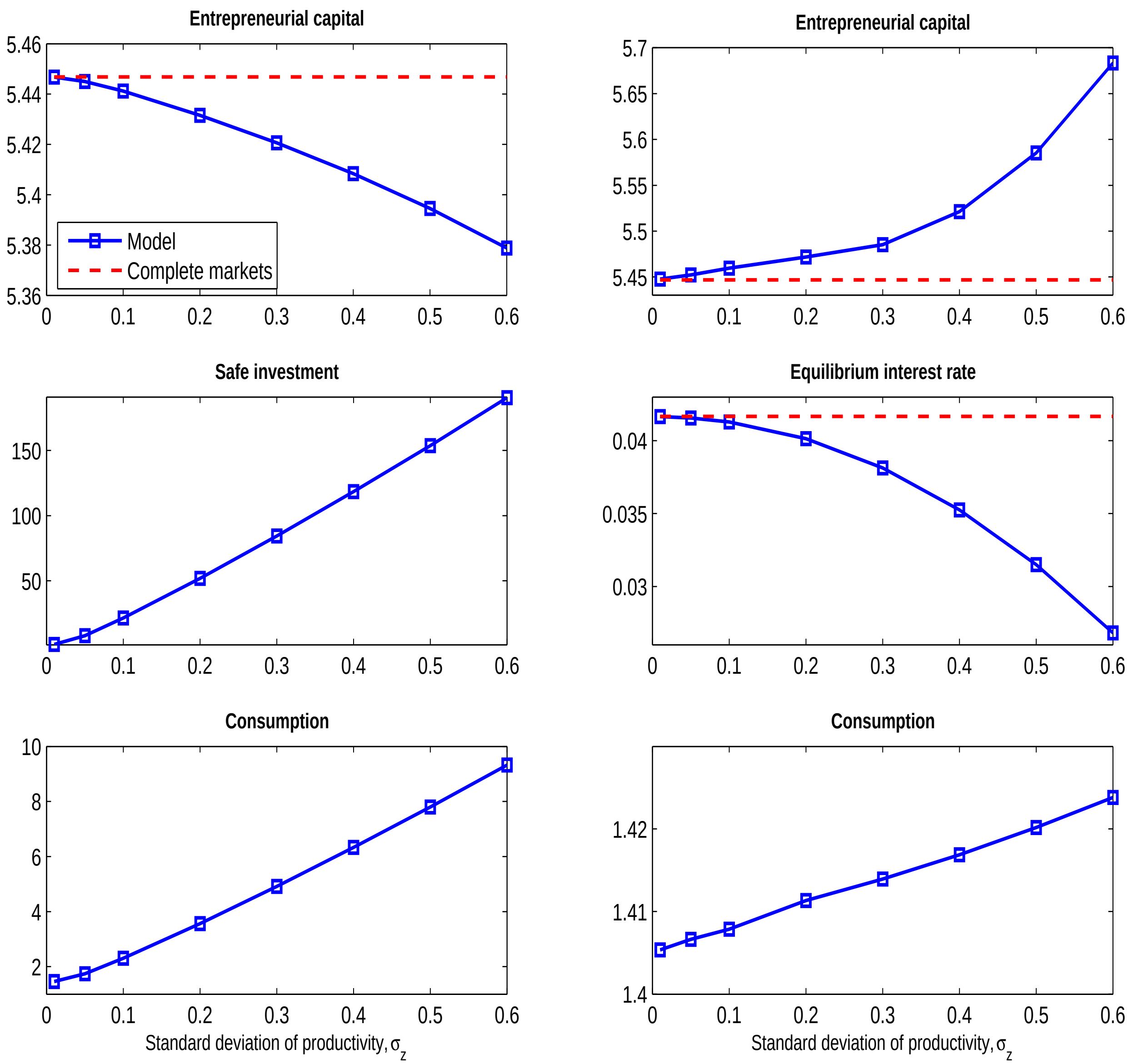


**Figure 2: Volatility of Productivity Risk, Aggregate Values, and Interest Rates**




**Left panel:** Aggregate values as a function of the standard deviation of idiosyncratic productivity with $r = 0.04166$. **Right panel:** Equilibrium values as a function of the standard deviation of productivity risk.

**Figure 3: Short-Sales Constraints and Aggregate Capital Stock**

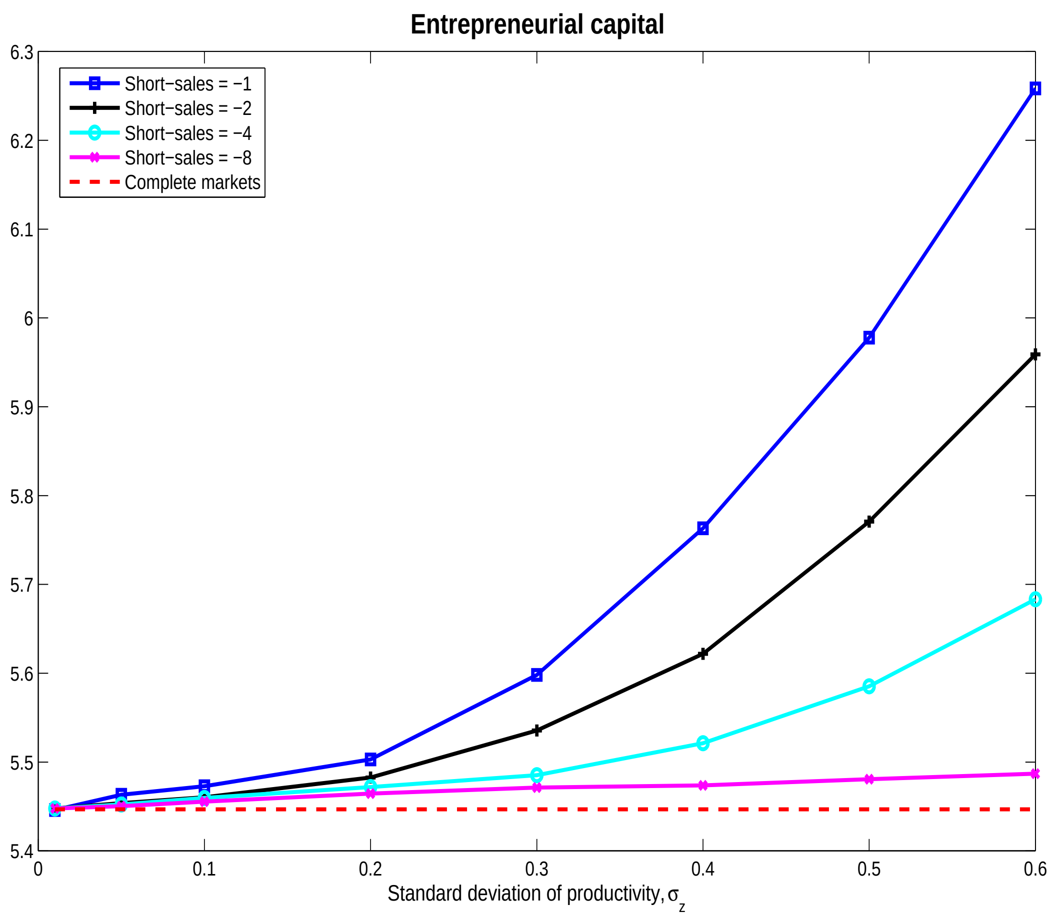


**Figure 4: Relative Risk Aversion and Aggregate Capital Stock**

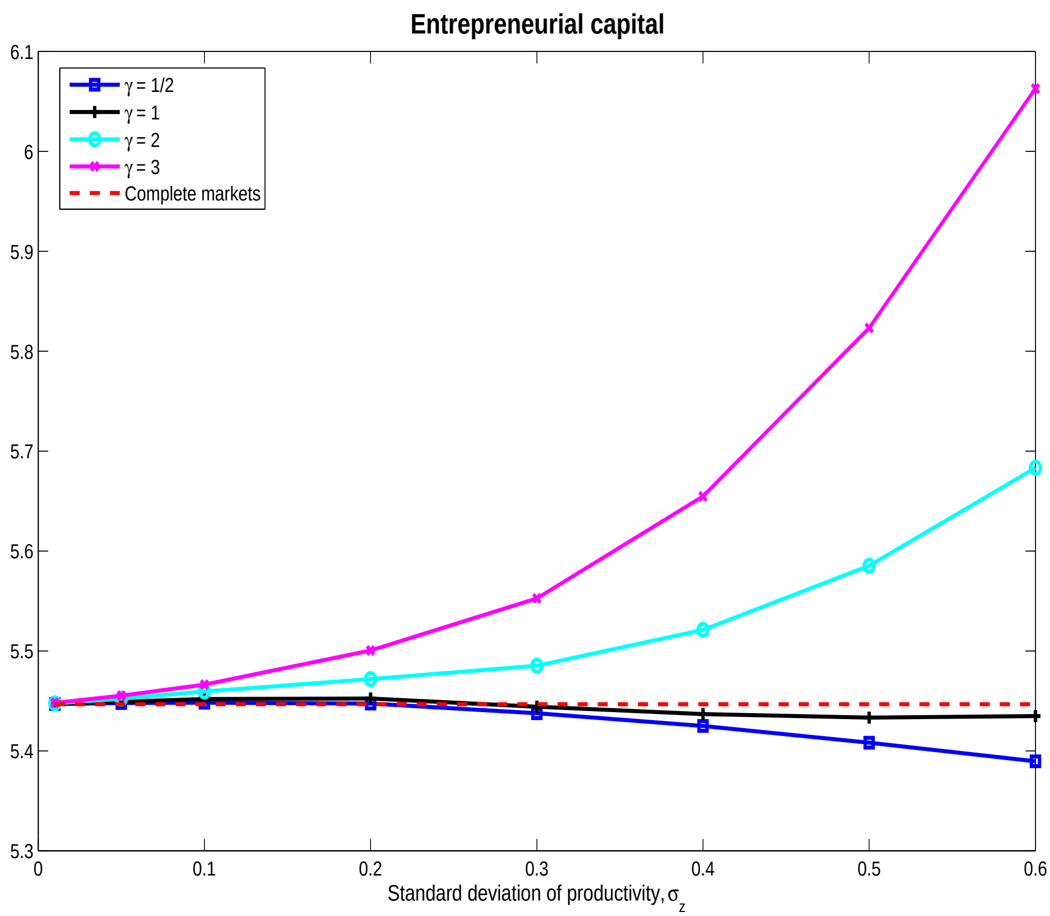


**Figure 5: Serial Correlation of Productivity Risk and Aggregate Capital Stock**

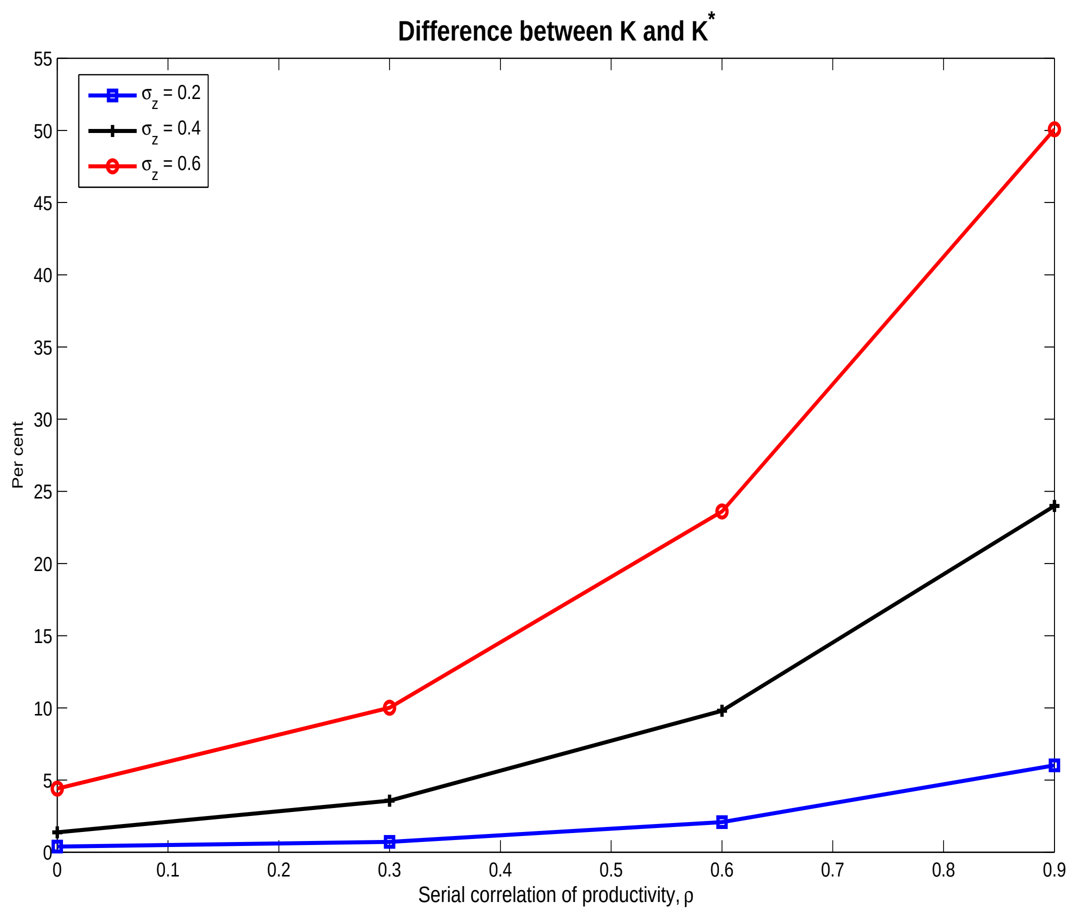
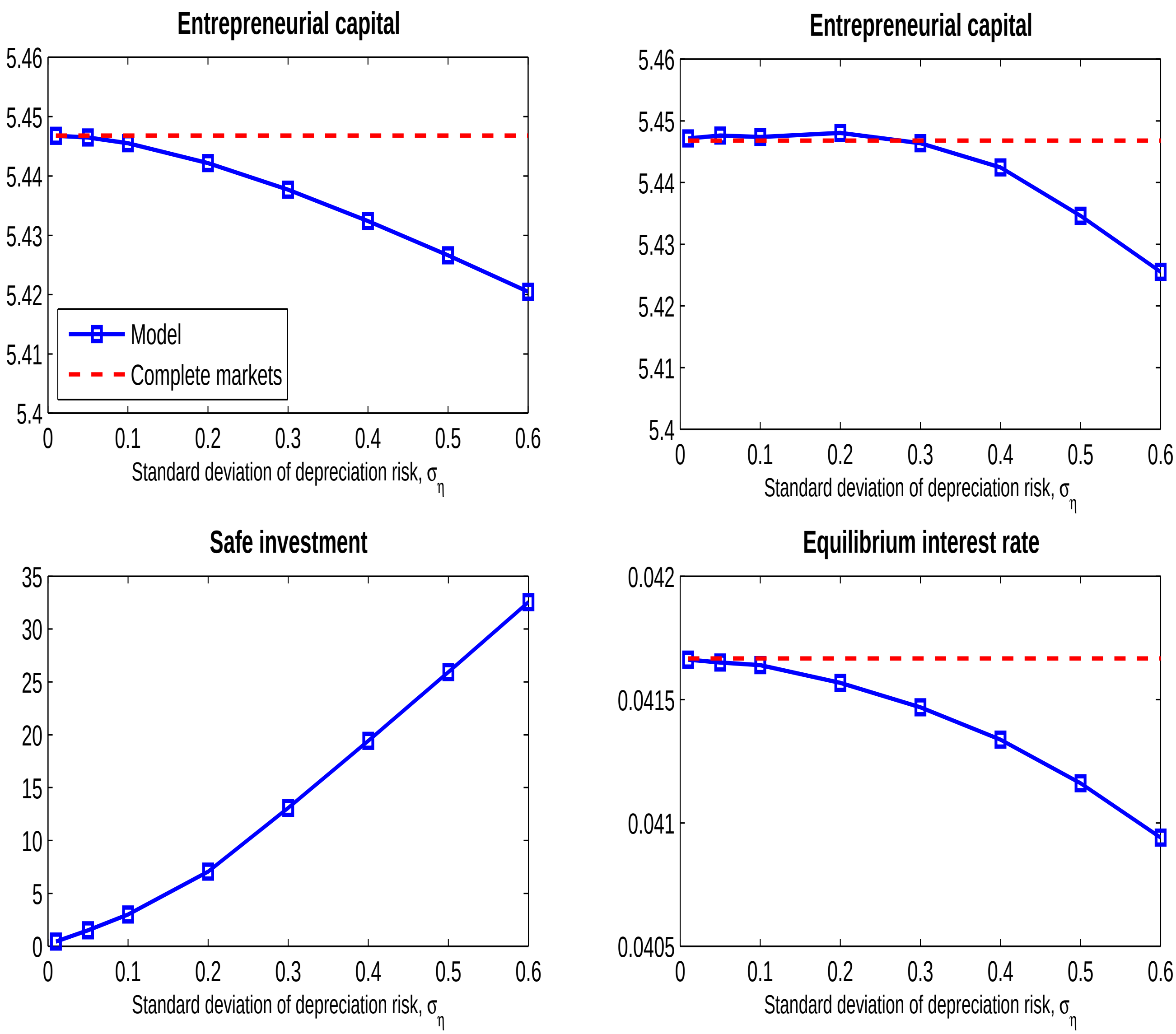
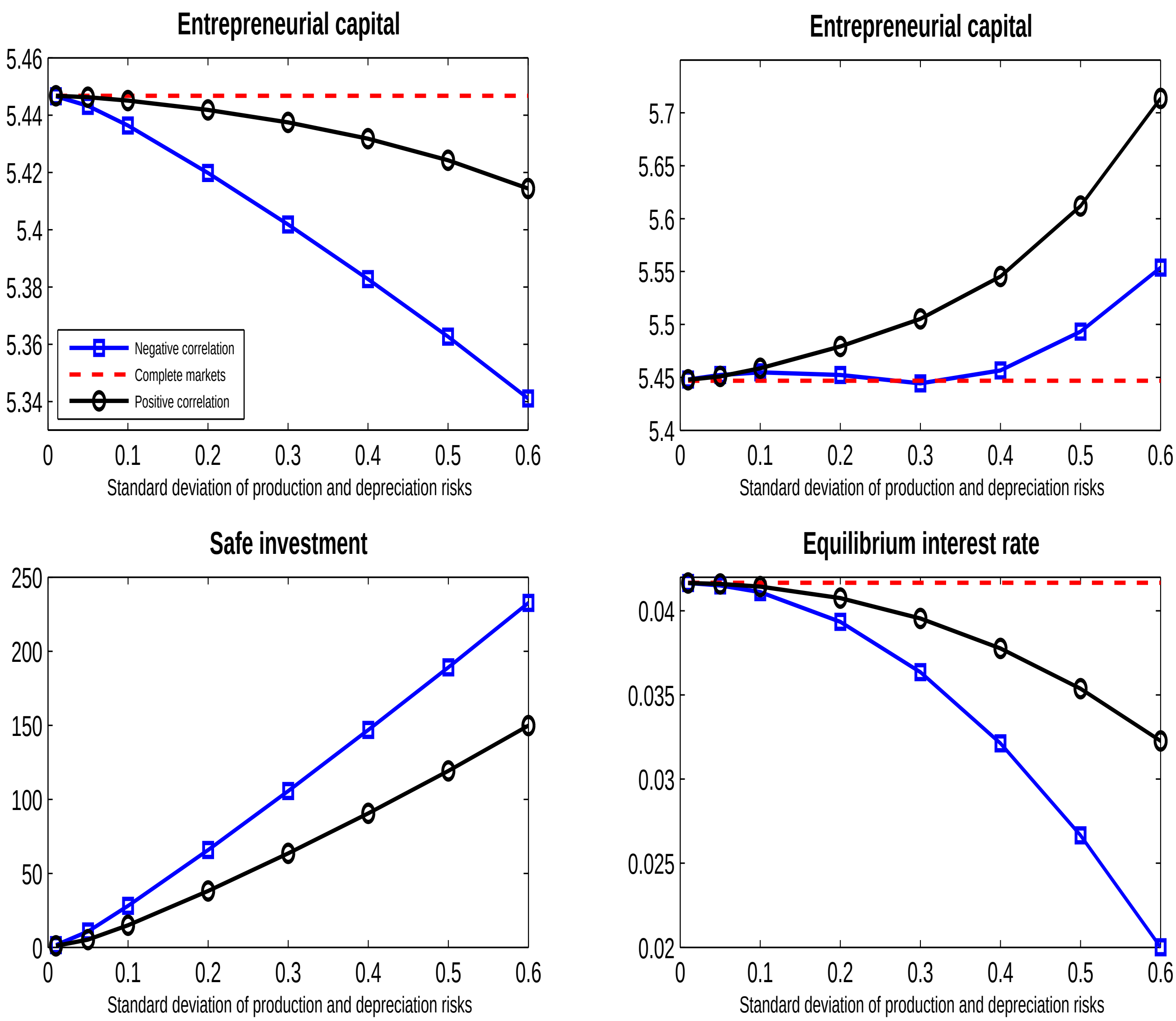


**Figure 6: Volatility of Depreciation Risk, Aggregate Values, and Interest Rates**




**Left panel:** Aggregate entrepreneurial capital and aggregate bonds as a function of the standard deviation of idiosyncratic productivity with $r = 0.04166$. **Right panel:** Equilibrium values as a function of the standard deviation of productivity risk.

**Figure 7: The Volatility of Depreciation and Productivity Risk, Aggregate Values, and Interest Rates**




**Left panel:** Aggregate entrepreneurial capital and aggregate bonds as a function of the standard deviation of idiosyncratic productivity with $r = 0.04166$. **Right panel:** Equilibrium values as a function of the standard deviation of productivity risk.

**Table 1: Parameter Values for the Benchmark Entrepreneurial Economy**

| | | |
|---|---|---|
| Discount factor | $\beta$ | 0.96 |
| Risk aversion | $\gamma$ | 2 |
| Curvature of production | $\alpha$ | 0.36 |
| Depreciation rate | $\delta$ | 0.08 |
| Serial correlation of productivity risk | $\rho_z$ | 0.90 |
| Unconditional standard deviation of productivity risk | $\sigma_z$ | 0.40 |
| Short-sales constraint on bonds | $\underline{b}$ | -4 |
| Discretization of the state space | | |
| Productivity risk | | |
| Number of states | $n_z$ | 7 |
| Discrete states: | | |
| $\bar{z} = [0.49; 0.67; 0.80; 0.92; 1.07; 1.27; 1.74]$ | | |
| Transition matrix: | | |
| $\Pi = \begin{bmatrix} 0.7186 & 0.2223 & 0.0499 & 0.0083 & 0.0009 & 0.0000 & 0.0000 \\ 0.2223 & 0.4099 & 0.2502 & 0.0938 & 0.0215 & 0.0022 & 0.0000 \\ 0.0499 & 0.2502 & 0.3324 & 0.2411 & 0.1040 & 0.0215 & 0.0009 \\ 0.0083 & 0.0938 & 0.2411 & 0.3136 & 0.2411 & 0.0938 & 0.0083 \\ 0.0009 & 0.0215 & 0.1040 & 0.2411 & 0.3324 & 0.2502 & 0.0499 \\ 0.0000 & 0.0022 & 0.0215 & 0.0938 & 0.2502 & 0.4099 & 0.2223 \\ 0.0000 & 0.0000 & 0.0009 & 0.0083 & 0.0499 & 0.2223 & 0.7186 \end{bmatrix}$ | | |
| Wealth | | |
| Number of grid points* | $n_x$ | 40 |
| Lower and upper bound: | | |
| $x \in [-3.4, \ldots, 60.0]$ | | |

* $c(z, x)$ is defined over a continuum of wealth levels. Points outside the grid are found by piecewise linear interpolation.

**Table 2: Difference in Capital Accumulation Relative to Complete Markets**

| | | $\underline{b} = -2$ | | | $\underline{b} = -4$ | | | $\underline{b} = -8$ | | | natural borrowing limit | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $\alpha$ | | | $\alpha$ | | | $\alpha$ | | | $\alpha$ | | |
| | | 0.3 | 0.5 | 0.7 | 0.3 | 0.5 | 0.7 | 0.3 | 0.5 | 0.7 | 0.3 | 0.5 | 0.7 |
| $\gamma$ | 0.5 | -0.11 | -0.23 | -0.07 | -0.26 | -0.43 | -0.20 | -0.24 | -0.49 | -0.41 | -0.20 | -0.36 | -1.00 |
| | 1 | 0.40 | 0.69 | 1.70 | -0.12 | 0.08 | 1.45 | -0.25 | -0.22 | 1.12 | -0.24 | -0.30 | -0.53 |
| | 2 | 3.54 | 4.24 | 7.60 | 1.55 | 2.75 | 7.43 | 0.34 | 1.40 | 6.64 | 0.03 | 0.65 | 2.85 |
| | 3 | 5.46 | 8.98 | 16.17 | 2.66 | 6.56 | 15.72 | 1.07 | 4.18 | 14.96 | 0.73 | 2.48 | 8.70 |

**Table 3: Effect of Increased Idiosyncratic Persistence**

| | $\rho = 0.0$ | $\rho = 0.30$ | $\rho = 0.60$ | $\rho = 0.90$ | $\rho = 0.98$ |
|---|---|---|---|---|---|
| Equilibrium interest rate | 0.035 | 0.032 | 0.024 | 0.012 | 0.016 |
| Risk premium | 0.006 | 0.009 | 0.013 | 0.018 | 0.014 |
| $K^*$ | 5.446 | 5.494 | 5.634 | 5.899 | 6.317 |
| $K$ | 5.521 | 5.690 | 6.186 | 7.314 | 7.667 |
| Difference, % | 1.38 | 3.57 | 9.80 | 23.99 | 21.37 |
| Safe-to-risky ratio | -0.044 | -0.064 | -0.080 | -0.232 | -0.479 |
| Fraction constrained | 0.158 | 0.217 | 0.312 | 0.453 | 0.518 |
| Gini coefficient of wealth | 0.337 | 0.387 | 0.453 | 0.573 | 0.676 |

**Table 4: Aggregate Capital Conditional on Idiosyncratic Productivity**

| Persistence | $\rho = 0.0$ | | | $\rho = 0.30$ | | | $\rho = 0.60$ | | | $\rho = 0.90$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $K^*$ | $K$ | % | $K^*$ | $K$ | % | $K^*$ | $K$ | % | $K^*$ | $K$ | % |
| State 1 | 5.446 | 5.249 | -3.6 | 3.144 | 3.340 | 6.2 | 1.812 | 2.164 | 19.5 | 1.178 | 1.647 | 39.8 |
| State 2 | 5.446 | 5.337 | -2.0 | 3.762 | 3.920 | 4.2 | 2.569 | 2.981 | 16.0 | 1.878 | 2.530 | 34.7 |
| State 3 | 5.446 | 5.412 | -0.6 | 4.501 | 4.679 | 3.9 | 3.657 | 4.199 | 14.8 | 3.037 | 3.887 | 28.0 |
| State 4 | 5.446 | 5.518 | 1.3 | 5.387 | 5.580 | 3.6 | 5.209 | 5.804 | 11.4 | 4.919 | 6.111 | 24.2 |
| State 5 | 5.446 | 5.626 | 3.3 | 6.444 | 6.632 | 2.9 | 7.419 | 7.991 | 7.7 | 7.973 | 9.861 | 23.7 |
| State 6 | 5.446 | 5.722 | 5.1 | 7.703 | 8.023 | 4.1 | 10.541 | 10.985 | 4.2 | 12.918 | 15.882 | 22.9 |
| State 7 | 5.446 | 5.759 | 5.7 | 9.193 | 9.653 | 5.0 | 14.778 | 15.238 | 3.1 | 20.431 | 23.427 | 14.7 |

**Table 5: Parameter Values for the Extended Economy**

| | | |
|---|---|---|
| Fraction of entrepreneurs | $\chi$ | 0.10 |
| Discount factor | $\beta$ | 0.96 |
| Risk aversion | $\gamma$ | 2 |
| Capital share in corporate sector | $\tilde{\alpha}$ | 0.36 |
| Labor share in corporate sector | $(1-\tilde{\alpha})$ | 0.64 |
| Depreciation rate | $\delta$ | 0.08 |
| Entrepreneurs | | |
| Curvature of production | $\alpha$ | 0.45 |
| Serial correlation of productivity risk | $\rho_z$ | 0.95 |
| Unconditional standard deviation of productivity risk | $\sigma_z$ | 0.80 |
| Short-sales constraint on bonds | $\underline{b}$ | -4 |
| Workers | | |
| Serial correlation of earnings risk | $\rho_l$ | 0.95 |
| Unconditional standard deviation of earnings risk | $\sigma_l$ | 0.30 |
| Liquidity constraint | $\underline{a}$ | 0 |