Employment, New Equipment, Skill, and Growth

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Employment, New Equipment, Skill, and Growth

Abstract

This paper studies the conditions under which new equipment may endogenously occur. To this end, we construct an endogenous growth multisectoral model with a preeminent new equipment sector. Technological progress is embodied: New machines can only be run on the most recent generations of hardware. While the new equipments are copyrighted during a fixed period of time, they become public knowledge at a certain point in time, which generates positive externalities in the rest of the economy. First, we find that our model can give rise to multiple steady states due to strategic complementarities. Substitution effects are shown to arise: The labor resources are diverted from the final goods sector to sustain the creation and production of new softwares. During the new equipments (for example during IT boom), labor productivity is growth slowdowns, the skill premium rises as well as the value of firms undertaking research. However, the registered new equipments is always transitory and nothing can be said about the long run sustainability of a new equipment-driven growth regime. We analyze consequences of the introduction the new equipments on the job and more in particularity on that the unskilled. We study the analysis of parameters and the economic policies.

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Introduction

The sector of new equipments has been recently considered of fundamental importance in the explanation of the economic performance of several countries. The huge productivity growth figures registered for the durable goods sector, and in particular for the computer sector makes it difficult to argue against such a view. However, some issues are still debated and will be debated until a more substantial historical experience is available. The main debated issue concerns the status of this new equipments age from an historical perspective. Some authors like Greenwood, Yorukoglu or Jovanovic (see Greenwood and Yorukoglu, 1997, and Greenwood and Jovanovic, 1999a) have argued that we are witnessing the Third Industrial Revolution: After an adoption period along which the productivity slowdown takes place due to learning costs and slow diffusion, the new equipments are now driving the rest of the sectors. The productivity gains should accordingly spread over the economy exactly as the major discoveries affected the pace of economic activity during the nineteenth century's Industrial Revolution.

For all these reasons, a great attention has been devoted to the study of what has been called the new equipments Revolution and of its effects on the economy, and the debate is largely open, both from an empirical and from a theoretical point of view. On the empirical side, the main studies (Gordon(1999, 2000), Jorgenson and Stiroh (2000), Oliner and Sichel (2000), Whelan (2000)) outline the strong productivity growth in the computer sector(particularly in the years 1995-1999), but at the same time evidence also problems of measurement of the real contribution of new equipments to the growth and productivity of the economy, together with the fact that the productivity growth in the computer sector has not been accompanied by spillovers from this sector to the rest of the economy. The fact that after 1974 there has been acceleration in the rate of embodied technological progress is indeed reflected by the observed acceleration in the rate of decline of the relative price of equipment as reported by Gordon (1990) for example. This is even more striking for new equipments as emphasize Jorgenson and Stiroh (2000): The price of computer investment fell around 17% per year from 1990 to 1996 while the price of new equipment to households fell 24% annually. Within a computable general equilibrium set-up, Greenwood and Yorukoglu introduce these features by assuming that the rate of embodied technological change has exogenously accelerated suddenly and permanently from 1974. As the pre-existing firms are unable to immediately use the new techniques at their full potential, a relatively long adoption period takes place which duration depends upon different endogenous costs. This paper is intended to remedy this shortcoming measured. Clearly a considerable research effort should be done to appraise and as-less the real contribution of new equipments to growth and productivity so as to conclude more safely about the long run sustainability of the current growth regime.

The model presented here takes a different view and tries to explain some of the essential features of the new equipments considering the framework of endogenous growth theories. In particular, it is a Romer-like model (1990) in order to capture the R&D effort of therms operating in the new equipments sector, and in addition it considers embodied technological progress. More precisely, it is based on the original contribution developed by Boucekkine and de la Croix (2003), with the main difference represented by the labor market (see Manning 1993 as well as Cahuc and Zylberberg 1996).

The article is organized as follows: in the firsthand, it presents the model and provide a characterization of its balanced growth path and derives the corresponding steady state system. From this system it is possible to find some analytical results concerning the effects on employment of different shocks that can interest the economy. The next section analyses the results. Finally, in last section some concluding remarks are made.

The model

Time is discrete and goes from 0 to infinity. We first describe the final good sector, then the intermediate good sector and the research activity. Second, labor market, households behavior and equilibrium conditions are introduced.

The final good sector

The final good produces a composite good that is used either to consume or to invest in physical capital. It uses physical capital, immaterial capital and two types of labor (unskilled labor and skilled labor). Each vintage of physical has its own embodied productivity.

The problem of the firm

Let $M_{m,t}$ represent the number of machines or capital units produced at time t (e.g. the vintage m) and still in use at time $t \geq m$. The quantity $I_m = E_{m,m}$ stands for gross investment, e.g., capital goods production at time m. We assume that the physical depreciation rate δ, is constant so

$$M_{m,t} = I_t(1-\delta)^{t-m}, \tag{1}$$

At time $t \geq m$, the vintage m is operated by certain amount of unskilled labor, say $L_{nq,m,t}$, and skilled labor, say $L_{q,m,t}$. Let $Y_{m,t}$ be the output produced at time t with vintage m. Under the following Cobb-Douglass we have

$$Y_{m,t} = A_t(q_{m,t}M_{m,t})^{\gamma} L_{nq,m,t}^{\alpha} L_{q,m,t}^{\beta}; \text{ with } \alpha, \beta \in [0,1] \text{ and } \gamma = 1-\alpha-\beta, \tag{2}$$

The variables A_t and q_t represent the state of knowledge at time t. An increase in A_t rises marginal productivity of all the capital stock, independently of its age structure. Hence, A_t represents disembodied technological progress. In sharp contrast, q_m is specific to the equipment of vintage m and represents embodied technological progress.

We relate q_m to the immaterial capital embodied in the in the vintage $M_{m,t}$. This immaterial capital is build from a series of specialized intermediate goods; following a Dixit-Stiglitz (1977) CES functions:

$$q_m = (\int_0^{n_m} x_{i,m}^{\frac{\sigma-1}{\sigma}} d_i)^{\frac{\sigma}{\sigma-1}} \text{ where } \sigma > 1, \tag{3}$$

Where n_m is the number of variety available in m, x_{im} is the quantity of input used in m of variety i and σ the elasticity of substitution between two varieties ($\sigma > 1$).

$$Y_{m,t} = A_t \sum_{m=-\infty}^{t} (q_{m,t} M_{m,t})^{\gamma} L_{nq,m,t}^{\alpha} L_{q,m,t}^{\beta}, \tag{4}$$

The discounted profits of investing I_t in physical equipment of vintage m, and in $x_{i,m}$ input of immaterial capital of variety i are given by:

$$\begin{aligned}\Pi_t = \sum_{m=t}^{\infty} [(1-\tau_1) Y_{t,m} - w_{nq}(1+\tau_2) L_{nq,t,m} - w_q (1+\tau_3) L_{q,t,m}] R_t^m \\ -(1+\tau_4) I_t - (1+\tau_5) \left[\int_0^{nt} p_{i,t} x_{i,t} d_i \right],\end{aligned} \tag{5}$$

where

$$R_t^t = 1, \text{ and } R_t^m = \prod_{T=t+1}^{m} \left[\frac{1}{1+r_T} \right],$$

is the discounted factor at time m et r_T is the interest rate at time T. w_{nq} and w_q are respectively the wages of unskilled and skilled labor input at time T. $p_{i,t}$ is the price of variety i. τ_1, τ_2, τ_3, τ_4, τ_5 are respectively production tax, employer contributions for unskilled labor and skilled labor, investment tax and immaterial capital tax.

The representative firm chooses physical and immaterial investment and the labor allocation across vintages in order maximize its discounted profits taking, prices as given a subject to its technological constraint:

$$\underset{I_t, \{x_{i,t}\}_{i=0}^{nt}; \{L_{nq,t,m}\}_{m=t}^{\infty}; \{L_{q,t,m}\}_{m=t}^{\infty};}{Max} \Pi_t$$

The first order conditions characterizing an interior maximum for Π_t are

$$\left(\frac{1-\tau_1}{1+\tau_4}\right)\left[\gamma q_t^{\gamma} I_t^{\gamma-1}\sum_{m=t}^{\infty} R_t^m A_m (1-\delta)^{\gamma(m-t)} L_{nq,t,m}^{\alpha} L_{q,t,m}^{\beta}\right]=1;\ \forall m \geq t \quad (6)$$

$\forall m \geq t,$

$$\left(\frac{1-\tau_1}{1+\tau_2}\right)\left[\alpha A_m \left(q_t M_{t,m}\right)^{\gamma} L_{nq,t,m}^{\alpha-1} L_{q,t,m}^{\beta}\right]= w_{nq}, \quad (7)$$

$$\left(\frac{1-\tau_1}{1+\tau_3}\right)\left[\beta A_m \left(q_t M_{t,m}\right)^{\gamma} L_{nq,t,m}^{\alpha} L_{q,t,m}^{\beta-1}\right]= w_{q}, \quad (8)$$

$\forall j \in [0, n_t],$

$$\left(\frac{1-\tau_1}{1+\tau_5}\right)\left[\gamma q_t^{\gamma-1} I_t^{\gamma}\left[\sum_{m=t}^{\infty} R_t^m A_m (1-\delta)^{\gamma(m-t)} L_{nq,t,m}^{\alpha} L_{q,t,m}^{\beta}\right]\left(\frac{q_t}{x_{j,t}}\right)^{\frac{1}{\sigma}}\right]= p_{j,t}, \quad (9)$$

equation (6) determines investment at time t by equalizing marginal returns to marginal costs. Equation (7) and (8) determine the labor allocation at time m to vintage t. Equation (9) gives is the demand function for the intermediate input of type i.

Aggregation properties

We define the total stock of efficient capital, which includes both material and immaterial aspects, as

$$k_t = \sum_{m=-\infty}^{t} q_m M_{m,t} = \sum_{m=-\infty}^{t} q_t I_m (1-\delta)^{t-m}\, dm, \quad (10)$$

It is thus the sum of surviving machines weighted by their respective productivity. The productivity of each machine depends itself on the embedded immaterial capital. The corresponding law of motion of capital is

$$k_t = (1-\delta)k_{t-1} + q_t I_t, \quad (11)$$

Note that the embodied technological progress variable q_t can be seen as a measure of marginal productivity or efficiency of new equipment, it is endogenous in our model in contrast to the canonical model of Greenwood, Hercowitz and Krusell (1997) and Boucekkine and de la Croix (2000), and we study the consequences in labor market. A previous theoretical attempt at endogenizing q_t is in Krusell (1998). However the research sector in this contribution is extremely ad-hoc as one can see. Our specifications are much more in line with the vintage capital models of Boucekkine, del Rio and Licandro (2000) and Hsieh (2000). However we rely on a much more complete setting in order to meet the basic characteristics of the new equipment sector as stated in the introduction, and this clearly differentiates our approach. We next define aggregate skilled and unskilled labor demand and relate them to capital. Combining equations (7) and (8) one obtains

$$(1+\tau_2)\beta L_{nq,t,m} w_{nq,t,m} = (1+\tau_3)\alpha L_{q,t,m} w_{q,t,m}, \tag{12}$$

We next transform equations (7) and (8) into

$$\frac{(1+\tau_2)^{\beta-1}}{(1+\tau_3)^{\beta}}(1-\tau_1)\left[\beta^{\beta}\alpha^{1-\beta}A_m(q_t M_{t,m})^{\gamma} L_{nq,m}^{-\gamma}\right] = w_{nq,m}^{1-\beta} w_{q,m}^{\beta}, \tag{13}$$

$$\frac{(1+\tau_3)^{\alpha-1}}{(1+\tau_2)^{\alpha}}(1-\tau_1)\left[\alpha^{\alpha}\beta^{1-\alpha}A_m(q_t M_{t,m})^{\gamma} L_{q,m}^{-\gamma}\right] = w_{q,m}^{1-\alpha} w_{nq,m}^{\alpha}, \tag{14}$$

Using (13), the aggregate unskilled employment level at time t is:

$$L_{nq,t} = \sum_{m=-\infty}^{t} L_{nq,m,t} = \left[\frac{(1+\tau_2)^{\beta-1}}{(1+\tau_3)^{\beta}}(1-\tau_1)\left[\frac{\beta^{\beta}\alpha^{1-\beta}A_t}{w_{nq,t}^{1-\beta} w_{q,t}^{\beta}}\right]\right]^{\frac{1}{\gamma}} \sum_{m=-\infty}^{t} q_m M_{m,t},$$

Hence, the demand for unskilled employment can be written

$$L_{nq,t} = \left[\frac{(1+\tau_2)^{\beta-1}}{(1+\tau_3)^{\beta}}(1-\tau_1)\left[\frac{\beta^{\beta}\alpha^{1-\beta}A_t}{w_{nq,t}^{1-\beta} w_{q,t}^{\beta}}\right]\right]^{\frac{1}{\gamma}} k_t, \tag{15}$$

and, using (12) the aggregate skilled employment level is

$$L_{q,t} = \left[\frac{(1+\tau_3)^{\alpha-1}}{(1+\tau_2)^{\alpha}}(1-\tau_1)\left[\frac{\alpha^{\alpha}\beta^{1-\alpha}A_t}{w_{q,t}^{1-\alpha} w_{nq,t}^{\alpha}}\right]\right]^{\frac{1}{\gamma}} k_t, \tag{16}$$

Replacing now Replacing now $L_{q,t}$ and $L_{nq,t}$ in (4) by their value taken from equations (13) and (14), one obtains:

$$Y_t = A_t\left[\frac{(1-\tau_1)(1+\tau_2)^{\beta-1}}{(1+\tau_3)^{\beta}}\left[\frac{\alpha^{1-\beta}\beta^{\beta}A_t}{w_{nq,t}^{1-\beta}w_{q,t}^{\beta}}\right]\right]^{\frac{\alpha}{\gamma}}\left[\frac{(1-\tau_2)(1+\tau_3)^{\alpha-1}}{(1+\tau_2)^{\alpha}}\left[\frac{\alpha^{\alpha}\beta^{1-\alpha}A_t}{w_{nq,t}^{\alpha}w_{q,t}^{1-\alpha}}\right]\right]^{\frac{\beta}{\gamma}}\sum_{m=-\infty}^{t} q_m M_{m,t}, \tag{17}$$

Equation (10), (13), (14), and (17) jointly imply that

$$Y_t = A_t k_t^{\gamma} L_{nq,t}^{\alpha} L_{q,t}^{\beta}, \quad (18)$$

Hence, if one redefines the capital stock as we did in equation (10), we retrieve a Cobb-Douglas production function as in Solow (1960).

The demand for intermediate inputs

Using equations (6) and (9) the demand for intermediate input j by the firms of the final good sector can be rewritten

$$\frac{x_{j,t}}{q_t} = \left(\frac{I_t}{q_t}\right)^{\sigma} p_{j,t}^{-\sigma} \left(\frac{1+\tau_5}{1+\tau_4}\right)^{\sigma}, \qquad (19)$$

The price elasticity of demand is thus $-\sigma$.

The intermediate good sector

The intermediate good sector produces a number of immaterial products that are sold to the final good sector. It uses unskilled labor to produce the goods and skilled labor to research for new varieties.

The production activity

The sector $[0, s_t]$ producing the intermediate goods is divided into a competitive sector $[0, s_t^c]$ and a monopolistic sector $[s_t^c, s_t]$. The market power is given by the presence of copyrights which have a lifetime of T. Hence, after a span of time T, monopolistic firms become competitive and we have

$$s_t^c = s_{t-T}, \qquad (20)$$

The intermediate good of type $i \in [0, s_t]$ is produced with a constant return to scale technology involving unskilled labor as the only input:

$$x_{i,t} = v\widetilde{L}_{nq,it}, \qquad (21)$$

where $\widetilde{L}_{nq,it}$ denotes unskilled labor employed in the intermediate sector and v measures labor productivity. In the side of the sector that behaves competitively, the output price is equal to the marginal cost:

$$p_{i,t} = \frac{w_{nq,t}}{v}\left(\frac{1+\tau_2}{1-\tau_1}\right) \text{ with } \forall i \in \left[0, s_t^c\right], \tag{22}$$

In the side of the sector that behaves monopolistically, the output price is chosen so as to maximize profits subject to the demand formulated by the final good sector:

$$\max\left(p_{i,t} - \frac{w_{nq,t}}{v}\left(\frac{1+\tau_2}{1-\tau_1}\right)\right)x_{i,t},$$

This leads to

$$p_{i,t} = \left[\left(1-\frac{1}{\sigma}\right)^{-1}\frac{w_{nq,t}}{v}\left(\frac{1+\tau_2}{1-\tau_1}\right)\right] \text{ with } \forall i \in \left]s_t^c, s_t\right], \tag{23}$$

and the price is a mark-up over unit labor costs, whose mark-up rate depend on the price elasticity of demand.

The research activity

Following Grossman and Helpman (1991) and Michel and Nyssen (1998), the research activity requires labor and public knowledge. The stock of public knowledge f_t that is used in the production of new types of input consists in the inputs being in the public domain $\left[0, s_t^c\right]$ but is also influenced by the inputs covered by copyrights. This latter influence is moderated by the parameter $\theta < 1$.

$$f_t = s_t^c + \theta(s_t - s_t^c), \tag{24}$$

The parameter θ is called the diffusion coefficient in the literature. It is equal to one when knowledge is non excludable despite the existence of copyrights. On the contrary it is equal to zero, as in Judd (1985), when copyrights prevent any positive externality from protected software to public knowledge. In this latter case, endogenous growth is made impossible.

The production of new inputs is made with skilled labor, according to the following constant return to scale technology:

$$\Delta s_t = s_t - s_{t-1} = af_t \widetilde{L}_{q,t}, \tag{25}$$

and the unit cost of research z_t is given by

$$z_t = \frac{w_{q,t}}{af_t}, \tag{26}$$

The unit cost increases with the skilled wage and decreases with the level of public knowledge. There will be entry of new firms until this cost is equal to the discounted flow of profits linked to one invention. This equilibrium condition that determines the number of new firms s_t can be written:

$$z_t(1+\tau_3) = \sum_{\mathbf{A}=t}^{t+T-1} R_t^{\mathbf{A}} \frac{1}{\sigma - 1} \frac{w_{nq,\mathbf{A}}}{v} x_{i,\mathbf{A}} (1-\tau_1), \tag{27}$$

Note that by (19) the discounted flow of profits depends on the investment made by the firms in the final goods sector. This is the main consequence of embodiment in our model: The return to research is related to investment in the final goods sector. Such a property does not arise in research-based growth models if technological progress is fully disembodied as one can infer from the models built up by Howitt and Aghion (1998) and by Boucekkine, del Rio and Licandro (2000). We will see later that this characteristic of the model, featuring a kind of strategic complementarily between investment and R&D, is responsible for multiple steady states to occur.

Finally, the demand for skilled labor by the research sector is given by

$$\widetilde{L}_{q,t} = \frac{\Delta s_t}{af_t} \frac{(1-\tau_1)}{(1+\tau_3)}, \tag{28}$$

Labor Market

The salary-making process is subject to a salary negotiation model, such as the one developed by Manning (1993) as well as Cahuc and Zylberberg (1996). The choice of this method is explained by empirical reasons. In fact, it has been observed that union activity is high in the United States (20%); due to legal constraints, salary negotiation is required only if the majority of wage-earners in a firm vote to be represented by a trade union (Hartog and Thoeuwes 1993). This probably explains why collective negotiations are so weakly covered in the United States (15%). In short, the cover rate is certainly a weaker indicator of trade unions power than the rate of trade unionism.

Determination of employment rate and wages

As far as the working population is concerned, we estimate that it is made up of two categories of workers: unskilled workers (L_{nq}) and skilled workers (L_q), knowing that only the skilled workers are fully employed. We assume that unskilled workers are paid according to the w_{nq} index which is based on the consumption price index. Furthermore, at equal qualification, work circulates between the different economic sectors without any cost. We also assume that unskilled workers are not subject to taxes. Finally, we suppose that, at equal qualification and in all sectors of activity, workers receive the same wage (with ω = net wage). We respectively obtain the net wage of unskilled workers and of skilled workers using equations (29) and (30):

$$\omega_{nq,t} = w_{nq,t}\left(1-\tau_{6,t}\right), \qquad (29)$$
$$\omega_{q,t} = w_{q,t}\left(1-\tau_{7,t}\right)\left(1-\tau_{8,t}\right), \qquad (30)$$

Variables t_6 and t_7 represent the rate unskilled and skilled employees' contribution, and the variable t_8 indicates the taxation rate of skilled workers.

Case of the trade union of firm i

Concerning wage negotiation, skilled workers benefit from full employment, and only unskilled workers are represented by an union. At each t period, the unskilled worker receives a net wage ($\omega_{nq,t}$,). At the end of that period, he leaves the firm i with a exogenous probability (P^x). At the beginning of the period $(t+1)$ he may find a job with a endogenous (P^e_{t+1}) probability depending negatively on the unemployment rate, or he may become unemployed with a $(1-P^e_{t+1})$ probability. Thus, intertemporal utility of a representative agent employed by the firm i [1].

$$V^e_{i,t} = \omega_{nq,t} + TR_{i,t} + \tilde{\lambda}\left[P^x[P^e_{t+1}V^e_{t+1} + (1-P^e_{t+1})V^u_{t+1}] + (1-P^x)V^e_{i,t+1}\right], \qquad (31a)$$

where $TR_{i,t}$ is the amount of State's inclusive transfers to representative employees, $\tilde{\lambda}$ $\left(0 < \tilde{\lambda} < 1\right)$ is the trade union's rate of discount, V^e_t is the average utility of an employee and V^u_t is the average utility of an unemployed over the $t+1$ period. Over the t period, an unemployed receives an unemployment allowance that is proportional to the average wage. At the beginning of the $t+1$ period, he can be hired with a probability P^e_{t+1} or remain unemployed[2]. Thus, the intertemporal utility of representative unemployed is:

$$V^u_t = tr\omega + TR_t + \tilde{\lambda}[P^e_{t+1}V^e_{t+1} + (1-P^e_{t+1})V^u_{t+1}], \qquad (31b)$$

[1]We estimate that unskilled workers had not access to the stock market and consumed entirely their income. We also suppose that their utility over a period is simply given by their consumption.

[2]We estimate that the probability to find a job is the same for an unemployed as for a worker that loses his job.

Collective negotiation

If the negotiation relating to the period t fails, workers leave the firm. Workers can find a job in another firm with a P_t^e probability or be unemployed during this period. In case of failure, the union's utility is:

$$V_t^{failure} = P_t^e V_t^e + \left(1 - P_t^e\right) V_t^u, \qquad (32)$$

Collective negotiation is formalized by a sequential cooperative set in complete information such as Nash's (1953)[3]. Nash's criterion is given by the maximization of the weighted profits product of both parts to the negotiation.

$$\underset{\omega_{nq,t}}{Max}\left[V_{i,t}^e - V_t^{failure}\right]^{\ell}\left[\pi_{i,t}\right]^{(1-\ell)}, \qquad (33)$$

with

$$\pi_t = \sum_{t=0}^{\infty}(1-\tau_1)Y_t - w_{nq,t}(1+\tau_2)L_{nq,t} - w_{q,t}(1+\tau_3)L_{q,t},$$

Where ℓ $\left(0 \le \ell \le 1\right)$ is a parameter indicating the unions's power of negotiation. This maximization program is solved as follows:

$$V_{i,t}^e - V_t^{failure} = \frac{\tilde{\ell}}{W_t}\frac{\pi_{i,t}}{L_{nq,t}}, \qquad (34)$$

With $\tilde{\ell}$ $\left(\tilde{\ell} = \dfrac{\ell}{1-\ell}\right)$ and $W_t = \left(\frac{1}{1-\tau_2-\tau_3}\right)$ respectively the relative power of negotiation of unions and the "*wage corner*"[4].

As in the symmetric $V_{i,t}^e = V_t^e$ case and taking (32) into account, the preceding equation is:

[3]It is for this reason that negotiations never effectively fail. Indeed, protagonists always reach an agreement, their gain in case of agreement is by definition superior than in case of failure. Strike is just a threat that wage earners never put into execution.

[4]which is the ratio between the salary paid by firms and that received by wage-earners

$$w_{nq,t} = \frac{(1-\tau_L)\tilde{\ell}}{(1-tr)}\left[\frac{\pi_t}{(1-P_t^e)L_{nq,t}} - \tilde{\lambda}(1-P_t^x)\frac{\pi_{t+1}}{L_{nq,t}}\right], \qquad (35)$$

Thus the negotiated gross wage is a relatively positive negotiation function for trade unions, of the replacement rate, the profit of the period t, the probability of entry in the unemployment and the volume of unskilled labor of the period $t+1$ and a function negative of the W_t, the volume of unskilled labor of the period t, the rate of union discount and the profit of the period $t+1$. To put in relationship the negotiated wage and the unemployment rate, it is well-off to write the number of unemployed in the t period $(\overline{L}_{nq,t} - L_{nq,t})$ according to $(1-P_t^e)$.

Indeed, unemployed of the period t w were, during $t-1$, unemployed or again on the job market or is again the employed and that come to lose their job but do not find a new one. To the total, the number of unemployed to the t period is given by:

$$\overline{L}_{nq,t} - L_{nq,t} = (1-P_t^e)\left[(\overline{L}_{nq,t-1} - L_{nq,t-1}) + (P_t^x L_{nq,t-1})\right], \qquad (36)$$

By dividing this equation by the active population in the t period $(\overline{L}_{nq,t})$ and $u_t = \frac{\overline{L}_{nq,t} - L_{nq,t}}{\overline{L}_{nq,t}}$, we have:

$$u_t = (1-P_t^e)\left[(1-P_t^x)u_{t-1} + P_t^x\right], \qquad (37)$$

The relationship between the wage and the unemployment rate is obtained by replacing $(1-P_t^e)$ pulls (37) and (35).Eventually, the equation will be the form:

$$u_t = \frac{\tilde{\ell}\pi_t\left[(1-P_t^x)u_{t-1} + P_t^x\right]}{L_{nq,t}w_{nq}W(1-tr) + \tilde{\lambda}\tilde{\ell}(1-P_t^x)(\pi_{t+1})}, \qquad (38)$$

Thus, the unemployment balance rate is a positive function of π , negative ℓ .

Household behavior

There are two types of households, skilled and unskilled. The household unskilled consume all income (wage and jobless benefits). The household skilled both consume, save for future consumption and supply labor inelastically. The household skilled savings are invested either in physical capital or in the research activity.

$$\begin{bmatrix} C_{nq,t} = R_{nq,t}(1+\tau_9) \\ R_{nq,t} = \omega_{nq,t}L_{nq,t} + TR_t \end{bmatrix} \tag{39}$$

$$\sum_{t=0}^{\infty} \rho^t \ln C_{q,t}$$

where ρ is the psychological discount factor and the utility function is logarithmic. Their budget constraints is

$$P_{q,t+1} = (1+r_{t+1})P_{q,t} + (1-\tau_8)\left[\omega_q L_q + rI(1-\tau_{10})\right] - (1+\tau_9)(C_{q,t}),$$

And first-order necessary condition for this problem is:

$$\frac{C_{q,t+1}}{C_{q,t}} = (1+r_{t+1})\rho, \tag{40}$$

Their income and consumption are:

$$R_q = (1-\tau_8)\left[\omega_q L_q + rI(1-\tau_{10})\right], \tag{41}$$

$$(1+\tau_9)C_{q,t},$$

which, together with the usual transversality condition, is sufficient for an optimum. And τ_9, τ_{10} represent consumption tax and capital income tax.

The government

The government impose the different taxes and gives in the form allocation to households unskilled.

Thus we have:

The employer contributions are:

$$CS_1 = \tau_2 L_{nq} w_{nq} + \tau_3 L_q w_q,$$

The employee contributions are:

$$CS_2 = \tau_6 L_{nq} w_{nq} + \tau_7 L_q w_q,$$

The income tax of households skilled is:

$$IR_q = \tau_8 \lfloor \omega_q L_q + rI(1-\tau_{10}) \rfloor,$$

The income capital tax is:

$$Ir = \tau_{10} rI,$$

The investment tax is

$$Ii = \tau_4 I,$$

The consumption tax is:

$$Ic = \tau_9 (C_{nq} + C_q),$$

The production tax is:

$$Iy = \tau_1 Y,$$

The intermediate good tax is:

$$Ix = \tau_5 p.x,$$

The government allocations is for households unskilled

$$TR_t = \sum \left(CS_{1,t} + CS_{2,t} + IR_t + Ir_t + Ii_t + Ic_t + Iy_t + Ix_t \right).$$

Market equilibrium

Equilibrium on the skilled labor market implies the skilled labor force is employed in the final good sector or in the research sector:

$$L_{q,t} = L_{q,t} + \widetilde{L}_{q,t}, \tag{42}$$

Equilibrium on the unskilled labor market implies the unskilled labor force is employed in the final good sector or in the intermediate good sector:

$$L_{nq} = L_{nq,t} + \int_0^{nt} \widetilde{L}_{nq,i,t} di, \tag{43}$$

Equilibrium on the final good market implies

$$Y_t = C_t + I_t, \tag{44}$$

which, after using the budget constraints of the agents, is equivalent to

$$\Delta P_{q,t+1} = I_t + z_t \Delta v_t,$$

g.e saving finance either investment in physical capital or in research.

The equilibrium

In this section we characterize the equilibrium and give some analytical characterization of a balanced growth path.

Characteristics

We begin by stating a proposition summarizing the equilibrium and optimality conditions of the model. The proof is reported in Appendix A.

Proposition 1

Given the initial conditions k_{-1} and $\{s_t\}_{t=T....-1}$ an equilibrium is a path :

$$w_{nq,t}, w_{q,t}, q_t, I_t, k_t, r_{t+1}, s_t, C_{nq}, C_{q,t}, f_t, u_t$$

that satisfies the following conditions:

$$L_{nq,t} = \left[\left[\frac{(1-\tau_1)(1+\tau_2)^{\beta-1}}{(1+\tau_3)^{\beta}}\right]\left[\frac{\beta^{\beta}\alpha^{1-\beta}A_t}{w_{nq,t}^{1-\beta}w_{q,t}^{\beta}}\right]\right]^{\frac{1}{\gamma}} k_t + (1+\tau_4)^{\sigma} \left[s_{t-T} + (1-\frac{1}{\sigma})^{\sigma}(s_t - s_{t-T})\right]\left(\frac{vI_t}{w_{nq,t}}\right)^{\sigma} q_t^{1-\sigma}, \tag{45}$$

$$L_{q,t} = \left[\left[\frac{(1-\tau_1)(1+\tau_3)^{\alpha-1}}{(1+\tau_2)^{\alpha}}\right]\left[\frac{\alpha^{\alpha}\beta^{1-\alpha}A_t}{w_q^{1-\alpha}w_{nq}^{\alpha}}\right]\right]^{\frac{1}{\gamma}} k_t + \frac{(1-\tau_1)}{(1+\tau_3)}\frac{\Delta s_t}{af_t}, \tag{46}$$

$$A_t^{\frac{1}{\gamma}} k_t \left(\frac{\alpha}{w_{nq,t}}\right)^{\frac{\alpha}{\gamma}}\left(\frac{\beta}{w_{q,t}}\right)^{\frac{\beta}{\gamma}} = C_t + I_t, \tag{47}$$

$$\left(\frac{1-\tau_1}{1+\tau_4}\right)\left[\frac{(1+\tau_3)^{\frac{\alpha}{\gamma}}}{(1+\tau_2)^{\frac{\beta}{\gamma}}}\right]\gamma q_t A_t^{\frac{1}{\gamma}}\left(\frac{\alpha}{w_{nq,t}}\right)^{\frac{\alpha}{\gamma}}\left(\frac{\beta}{w_{q,t}}\right)^{\frac{\beta}{\gamma}} = 1 - \frac{(1-\delta)q_t}{(1+r_{t+1})q_{t+1}}, \tag{48}$$

$$\frac{C_{q,t+1}}{C_{q,t}} = (1+r_{t+1})\rho, \tag{49}$$

with

$$R_{nq,t} = L_{nq,t}w_{nq,t} + TR_t = C_{nq,t}(1-\tau_{9)} \text{ and}$$
$$TR_t = \left(CS_{1,t} + CS_{2,t} + IR_t + Ir_t + Ii_t + Ic_t + Iy_t + Ix_t\right) \tag{50}$$

$$k_t = (1-\delta)k_{t-1} + q_t I_t, \tag{51}$$

$$\left(\frac{\left(\frac{1+\tau_2}{1-\tau_1}\right) w_{nq,t} q_t}{\left(\frac{1+\tau_5}{1+\tau_4}\right) vI_t}\right) = \left[s_t^c + \left(s_t - s_t^c\right)\left(1-\frac{1}{\sigma}\right)^{\sigma-1}\right]^{\frac{\sigma}{\sigma-1}}, \tag{52}$$

$$\begin{aligned}&\left[v^{1-\sigma}\left(\frac{(\sigma-1)^{1-\sigma}\sigma^{\sigma}}{a}\right)\left(\frac{1+\tau_4}{1+\tau_5}\right)^{\sigma}\left(\frac{1+\tau_3}{1-\tau_1}\right)\left(\frac{w_{q,t}}{f_t} - \frac{R_t^{t+1} w_{q,t+1}}{f_{t+1}}\right)\right] \\ &= \left(w_{nq,t}^{1-\sigma} - q_t^{1-\sigma} I_t^{\sigma} - R_t^{t+T} w_{nq,t+T}^{1-\sigma} q_{t+T}^{1-\sigma} I_{t+T}^{1-\sigma}\right),\end{aligned} \tag{53}$$

$$f_t = (1-\theta) s_{t-T} + \theta s_t, \tag{54}$$

$$u_t = \frac{\tilde{\ell}\,\pi_t\left[\left(1-P_t^x\right)u_{t-1} + P_t^x\right]}{L_{nq,t} w_{nq,t} W(1-tr) + \tilde{\lambda}\,\tilde{\ell}(1-P_t^x)(\pi_{t+1})}, \tag{55}$$

Equations (45), (46) and (47) describe the equilibrium on the unskilled labor, skilled labor and final goods markets respectively. The equilibrium interest rate obtains from (48). Optimal consumption of households skilled is given in equation (49) and equation (50) is unskilled income. Equation (51) is the income accumulation rule of capital. Equation (53) links the embodied technological progress to the expansion in the varieties of intermediate products. Equation (54) is derived from the free entry condition. And equation (55) is unemployment unskilled.

The balanced growth path

We assume that labor supplies w_{nq} and w_q are constant. The disembodied technological progress A_t is also assumed constant in the long-term. Along a balanced growth path, each variable grows at a constant rate. For output we have

$$Y_t = \overline{Y} g_Y^t,$$

where g_Y is the growth factor and $\overline{Y}$ the initial level of output. $s_t, C_t,$ $I_t, q_t,$ $w_{nq},$ $w_q,$ and k_t grows respectively with factors g_s, g_C, g_I, g_q, $g_{w_{nq}}$, g_{w_q}, and g_k. The interest rate r_t and unemployment rate u_t are constant.

Proposition 2

If q_t grows at a rate g $g_q > 1$, then all the other variables grow at strictly positive rates with

$$g_s = g_q^{\sigma-1}, \tag{56}$$

$$g_Y = g_C = g_I = g_{w_{nq}} = g_{w_q} = g_q^{\frac{\gamma}{1-\gamma}}, \tag{57}$$

$$g_k = g_q^{\frac{1}{1-\gamma}}, \tag{58a}$$

Proof: If a balanced growth path should satisfy the nine equations (39)-(49), then one should have the following eight restrictions among the various growth rates:

$$\left(g_{w_{nq}}\right)^{\frac{-(1-\beta)}{\gamma}}\left(g_{w_q}\right)^{\frac{-\beta}{\gamma}} g_k = 1 = \frac{g_N}{g_q} = \left(\frac{g_f}{g_{w_{nq}}}\right)^{\sigma}, \tag{58b}$$

$$g_k = \left(g_{w_{nq}}\right)^{\frac{\alpha}{\gamma}}\left(g_{w_q}\right)^{\frac{1-\alpha}{\gamma}}, \tag{59}$$

$$g_k = g_Y\left(g_{w_{nq}}\right)^{\frac{\alpha}{\gamma}}\left(g_{w_q}\right)^{\frac{\beta}{\gamma}}, \tag{60}$$

$$g_q = \left(g_{w_{nq}}\right)^{\frac{\alpha}{\gamma}}\left(g_{w_q}\right)^{\frac{\beta}{\gamma}}, \tag{61}$$

$$g_N^{\frac{\sigma}{1-\sigma}} = \left(\frac{g_Y}{g_{w_{nq}} g_{w_q}}\right)^{\sigma}\rho, \tag{62}$$

$$g_Y = (1+r)\rho, \tag{63}$$

$$g_k = g_g g_Y, \tag{64}$$

$$\frac{g_{w_q}}{g_N} = g_{w_{nq}}^{1-\sigma} g_q^{1-\sigma} g_Y^{\sigma}, \tag{65}$$

$$g_f = g_N, \tag{66}$$

$$u_t = \overline{L}_{nq} - L_{nq}, \tag{67}$$

We use implicitly the condition $g_Y = g_C = g_I$ in (52)-(61), a condition implied by the good market equilibrium and by the fact that the share of consumption in production cannot tend to zero or to infinity along a balanced growth path. Using (52) and (53) to eliminate g_k we have $g_{w_{nq}} = g_{w_q}$. The (56) gives:

$$g_{w_{nq}} = g_{w_q} = g_q^{\frac{\gamma}{1-\gamma}},$$

and by (60), $g_k = g_q$. Equation (61) yields $g_Y = g_q$. It turns out that (64) is redundant with (60). Now, by using (62) we get

$$g_N = \left(\frac{g_Y}{g_{w_{nq}} g_q} \right)^{1-\sigma},$$

This result is the same obtained from equation (65)

$$\frac{g_{w_q}}{g_N} = g_{w_{nq}}^{1-\sigma} g_q^{1-\sigma} g_Y^{\sigma},$$

Hence, the two latter equations are redundant with (62). At the end, the eight unknown of the problem ($g_{w_{nq}}$, g_{w_q}, g_q, g_Y, g_N, g_k, $\bar{r}$, $\bar{u}$,) are shown to be truly related by a system of seven equations (out of the nine initial restrictions since three redundant equations have been identified). For given g_q, all the other unknowns can be found. They are thus parameterized by g_q, including $\bar{r}$ since by (63), we have :

$$\bar{r} = g_q^{\frac{\gamma}{1-\gamma}} / \rho - 1.$$

Hence, along a balanced growth path, output, consumption, investment and wages grow at the same rate. The stock of capital grows faster as it includes improvement in the embodied productivity. To determine g_q, we need additional information, which is provided by the restrictions on the long-run levels. Computing these restrictions from the dynamic system (46)-(54), we end with 9 equations for 10 unknowns ($\overline{w}_{nq}$, $\overline{w}_q$, $\bar{s}$, $\bar{q}$, $\bar{I}$, $\overline{C}$, $\bar{k}$, $\bar{r}$, $\bar{u}$, $\bar{g}_q$) since all the other growth rates can be expressed in terms of g_q. The system in terms of levels is therefore undetermined, which is a usual property of endogenous growth models. Fortunately, it is always possible to rewrite this system in such way that we get rid of this indeterminacy. As usual, this is done by stationarizing the equations by the means of some auxiliary variables. Indeed, the dynamic system (45)-(55) can be rewritten as a function of eight stationary variables, which are: $\hat{w}_t = \frac{w_{q,t}}{w_{nq,t}}$, $\hat{k} = \frac{k_t}{s_t^{\frac{1}{(1-\gamma)(\sigma-1)}}}$, $\hat{C}_t = \frac{C_t}{w_{nq,t}}$, $\hat{I} = \frac{I_t}{w_{nq,t}}$, $\hat{q} = \frac{q_t}{w_{nq}^{\frac{1-\gamma}{\gamma}}}$, $\hat{g}_t = \frac{s_t}{s-1}$, $\hat{s}_t = \frac{s_t}{w_{nq,t}^{\frac{(1-\gamma)(\sigma-1)}{\gamma}}}$, $\hat{f} = \frac{f_t}{s_t}$ and $\bar{r}_t$, $\bar{u}_t$.

The stationarized dynamic system is given in appendix B. Note that as for the original system, we have two pre-determined variables $\hat{k}_t$ and $\hat{g}_t$. Hence our stationarization does not alter the dynamic order of the original system. The corresponding steady state system is summarized in the following

proposition 3

Denote $g_s = g$ and $\alpha_1 = (\sigma-1)(1-\gamma)$. Considering that lim $A_t = A$ and defining the following stationary variables, $\hat{w}_t = \frac{\bar{w}_q}{\bar{w}_{nq}}$, $\hat{k} = \frac{k_t}{\bar{s}^{\frac{1}{\alpha_1}}}$, $\hat{I} = \frac{\bar{I}}{\bar{w}_{nq}}$, $\hat{C} = \frac{\bar{C}}{\bar{w}_{nq}}$, $\hat{q} = \frac{\bar{q}}{\bar{w}_{nq}^{\frac{-1-\gamma}{\gamma}}}$, $\hat{s} = \frac{\bar{s}}{\bar{w}_{nq}^{\frac{-(1-\gamma)(\sigma-1)}{\gamma}}}$,

the restrictions on the levels can be rewritten as

$$L_{nq} = \left[\left[\frac{(1-\tau_1)(1+\tau_2)^{\beta-1}}{(1+\tau_3)^{\beta}}\right]\left[\frac{\beta^{\beta}\alpha^{1-\beta}A}{\hat{w}_q^{\beta}}\right]\right]^{\frac{1}{\gamma}}$$

$$\hat{k}_t\hat{s}_t^{\frac{1}{\alpha_1}} + (1+\tau_4)^{\sigma}\left[\prod_{i=0}^{T-1}\frac{1}{g_{t-i}} + (1-\frac{1}{\sigma})^{\sigma}(1-\prod_{i=0}^{T-1}\frac{1}{g_{t-i}})\right](v\hat{I})^{\sigma}\hat{q}_t^{1-\sigma}\hat{s}_t,$$

$$L_q = \left[\left[\frac{(1-\tau_1)(1+\tau_3)^{\alpha-1}}{(1+\tau_2)^{\alpha}}\right]\left[\frac{\alpha^{\alpha}\beta^{1-\alpha}A}{\hat{w}_{q,t}^{1-\alpha}}\right]\right]^{\frac{1}{\gamma}}\hat{k}\hat{s}^{\frac{1}{\alpha_1}} + \frac{(1-\tau_1)}{(1+\tau_3)}\frac{(g_t-1)}{a[g_t\hat{m}_t]},$$

$$A_t^{\frac{1}{\gamma}}\hat{k}_t\hat{s}_t^{\frac{1}{\alpha_1}}\alpha^{\frac{\alpha}{\gamma}}\left(\frac{\beta}{\hat{w}_{q,t}}\right)^{\frac{\beta}{\gamma}} = \hat{C}_t + \hat{I}_t,$$

$$\left(\frac{1-\tau_1}{1+\tau_4}\right)\left[\frac{(1+\tau_3)^{\frac{\alpha}{\gamma}}}{(1+\tau_2)^{\frac{\beta}{\gamma}}}\right]\gamma\hat{q}_t A_t^{\frac{1}{\gamma}}\alpha^{\frac{\alpha}{\gamma}}\left(\frac{\beta}{\hat{w}_{q,t}}\right)^{\frac{\beta}{\gamma}} + \frac{(1-\delta)}{(1+r)}g_{t+1}^{\frac{-1}{\sigma-1}}\left(\frac{\hat{s}_t}{\hat{s}_{t+1}}\right)^{\frac{-1}{\sigma-1}}\frac{\hat{q}_t}{\hat{q}_{t+1}} = 1,$$

$$g_t^{\frac{\gamma}{\alpha_1}} = (1+r_{t+2})\rho,$$

$$\hat{k}_t((1-\delta)\hat{k}_{t-1}g_t^{\frac{-1}{\alpha_1}}) = \hat{q}_t\hat{I}_t\hat{s}_t^{\frac{-1}{\alpha_1}},$$

$$\left(\frac{\left(\frac{1+\tau_2}{1-\tau_1}\right)}{\left(\frac{1+\tau_5}{1+\tau_4}\right)}\frac{\hat{q}_t}{v\hat{I}_t}\right)^{\sigma-1}\frac{1}{\hat{s}} = \left[g^{-T} + (1-g^{-T})\left(1-\frac{1}{\sigma}\right)^{\sigma-1}\right]$$

$$\left[\left[\left(\frac{1+\tau_4}{1+\tau_5}\right)^{\sigma}\left(\frac{1+\tau_3}{1-\tau_1}\right)\right]\frac{v^{1-\sigma}(\sigma-1)^{1-\sigma}\sigma^{\sigma}\hat{w}}{a((1-\theta)g^{-T}+\theta)}\left(1-\frac{g^{\frac{\gamma}{\alpha_1}-1}}{1+r}\right)-\hat{I}^{\sigma}\hat{q}^{1-\sigma}\hat{s}\left(1-\left(\frac{g^{\frac{\gamma}{\alpha_1}-1}}{1+r}\right)^{T}\right)\right]=0,$$

$$u=\frac{\tilde{\ell}\,\pi\left[\left(1-P^{x}\right)\bar{u}_{t-1}+P^{x}\right]}{L_{nq,t}\hat{w}_{nq,t}W\left(1-tr\right)+\tilde{\lambda}\,\tilde{\ell}(1-P^{x})\left(\pi_{t+1}\right)},$$

with

$$\pi_t=\sum_{t=0}^{\infty}(1-\tau_1)Y_t-\hat{w}_{nq,t}(1+\tau_2)L_{nq,t}-\hat{w}_{q,t}(1+\tau_3)L_{q,t},$$

Note that since the other growth rates g_q, g_k, g_w, and g_Y depend on $g_s=g$ through (56)-(58), this system determines all the growth rates of the variables of the model, together with eight other ratios, namely $\hat{w}$, $\hat{s}$, $\hat{q}$, $\acute{I}$, $\hat{C}$, $\hat{k}$, $\bar{r}$, et $\bar{u}$. Our choice of stationarization is indeed the simplest algebraically speaking given the long run relationships described in Proposition 2. Obviously, we can recover any relevant stationary ratio from the seven previous one. For example, the ratio consumption to output can be simply computed as $\frac{\hat{C}}{\hat{C}+\hat{I}}$. Given the complexity of the long run steady state described above, it is impossible to derive an analytical solution. However, though the corresponding system of equations is indeed extremely heavy to manipulate, it is possible to bring out some interesting intermediate results which turn out to be crucial to understand the issues related to the existence and uniqueness of steady state growth paths in our model. In particular, the following proposition reveals most useful.

Proposition 4

At any growth rate value g, there exist explicit functions expressing the long run level $\hat{k}$, $\acute{I}$, $\hat{r}$, $\hat{s}$ $\hat{C}$, $\hat{q}$, $\hat{u}$, and $\hat{w}$ exclusively in terms of $\hat{k}=\Psi_k(g)$, $\hat{I}=\Psi_I(g)$, $\hat{s}=\Psi_s(g)$, $\hat{r}=\Psi_r(g)$, $\hat{C}=\Psi c(g)$, $\hat{q}=\Psi_q(g)$, $\hat{u}=\Psi_s(g)$ and $\hat{w}=\Psi_w(g)$. It follows the obvious corollary:

Corollary 1

There exists an explicit function $\Psi(g)$ such that the long run equilibrium growth rate value solves the equation $\Psi(g)=0$.

Clearly if Proposition 4 holds, then we can obtain an explicit equation involving only g by using the g-functional expressions of the long run levels in any equation of the steady state system. Therefore we can reduce our 9-dimensional system to an explicit scalar equation involving the growth rate g. Once this equation is solved, the remaining long run levels can be recovered using the explicit g-functions of Proposition 4. A proof of Proposition 4 can be found in Appendix C. The proof of the following useful property can be also found in the same appendix.

Proposition 5
Assuming that a solution for the steady state system exists, the long value of A only affects the stationary values $\hat{s}$, $\hat{q}$ and $\hat{k}$.

As argued in the introduction, our model can generate steady state as it entails a clear strategic complementarily due to the embodiment hypothesis: Investment in physical capital and R&D efforts are complementary. Although we can find by Corollary 1 an explicit equation $\Psi(g) = 0$ giving the eventual steady state growth rate(s), this equation is unsurprisingly so complicated- as it summarizes the algebra of 9 non-redundant equations- that no exact solution(s) can be found out. So we resort to numerical resolution using various parameterizations.

Consider the following calibration or the model. A first set of parameters is fixed a priori to what we view as reasonable values given the empirical evidence available. The skilled population is 20 % of total population (roughly the share of workers with higher education is developed countries). The length of copyrights is set at 5 years. It means that the profits made on software invented 5 years ago falls to zero. The total factor productivity in the final sector is normalized to 1. The rate of depreciation of physical capital is 4% and the psychological discount factor is 97%.

Parameters fixed *a priori*

Unskilled labor supply	L_{nq}	8
Skilled labor supply	L_q	2
Copyrights length	T	5
Total factor productivity in the final sector	A	1
Rate of depreciation of capital	δ	0.04
Psychological discount factor	ρ	0.97

A second set of parameters is fixed in order to match a series of moments of the steady state we consider. The parameters α and β are such that the share of labor in the final sector is 70% and the ratio of the two wages about 3.7. The total factor productivity in the research sector is set in order to obtain a growth rate of embodied technological progress around 2%. We select the elasticity of substitution between varieties of software to obtain a mark-up rate of 1.5. Finally the unskilled labor productivity in the intermediate sector is such that the share of unskilled workers in this sector is about 4%; and the psychological discount factor is 97/100.

Other parameters

Unskilled labor share in the final sector	α	0.4
Skilled labor share in the final sector	β	0.3
diffusion rate	θ	0.8
Elasticity of substitution between varieties of softwares	σ	3
Unskilled labor productivity in the intermediate sector	v	0.1
Total factor productivity in the research sector	a	1.2

Calibration

To realize that, we have chosen 1994 as reference's year. This choice is based in function of the information's new technologies' growth and maturity's period in the American's economic cycle[5].

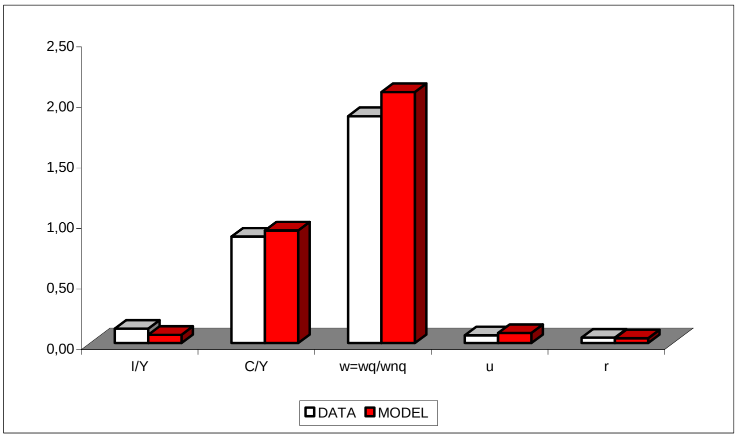
[5] For calibration, the choice of the referenced year consist in realize averages on three to five years; See also Pyatt et Round [1985].

In general, the economic indicators the more significant are: relations of I/Y, C/Y, $\omega = w_q / w_{nq}$ (salaries relations), u (unemployment rate), r (interest rate).

We observe that given values for the simulation are close to the data.

Reference Values

T=5	I/Y	C/Y	$\omega = w_q / w_{nq}$	u	r
Data	0.12	0.88	1.88	0.06	0.045
Model	0.7	0.93	2.08	0.068	0.040



Analysis of parameters and Economic Policies

In section, first time, we analysis the parameters, and second hand, we study the implication

The parameters' analysis

Analysis of parameters defines as the measure of incertitude of the resultants of simulations, linked has incertitude the parameters free. To models this, we have realized an analysis of sensibility on the different parameters and we have given that two resultants regrouping all analyses:

- The first simulation concerns during of copyright (*T*), the second focuses on the variation simulate of elasticity substitution between the varieties of software (σ) and the copyright (*T*) and the 3 the variation simulate of parameter of diffusion (θ) and during of copyright (T). To make this, we have taken in consideration three scripts:

(i) the first calls "*ref*", indicates the simulation of reference: alone the value of the parameter T varies,

(ii) the second appoints simulation 1, corresponds has a variation simulate the parameters σ and *T*, allowing thus observer evolution economic indicators,

(iii) in the third scripts titles simulation 2, we examine implications of the variation simulate of θ and T on economic indicators.

The obtained results describe parameters' variation effect chosen on the economic large aggregates like this :

If the length copyright (T) reduces, it encourages to invest more. But it has as consequence of reducing innovations. Conversely, when the copyright length increases, the capacity to reproduce the innovations reduces (innovations are more protected)[6].Which has like consequence an investments decline, generating an indeterminate situation!

Concerning unemployment, we can conclude that it owns a similarly movement to the parameter T: thus meaning when the copyright's length parameter increases, unemployment raises too (due to an investment's reducing). Conversely, the decline of value of T leads to an unemployment reducing (growth in investments).

The value reducing $\sigma = 2$ with a variation T accentuates more situations described previously. This means that a decline of σ associated to a value reducing of T increases more the investment

[6] For same results see Jones [1995].

and reduces unemployment and innovative capacity too. If $\sigma = 4$ with a reducing of the value T, results in an investment decline and a growth of innovative capacity. The lengthening of the length's copyright leads to an unemployment and investment convergence towards the reference situation (whatever's value).

Concerning the parameters' simultaneous variation θ (diffusion ratio) and T (copyright length), we could be waiting for a certain sensibility from the model: and yet the different simulations show that it's not the case concerning parameter's θ variation.

The resultants of the different simulations realizes are synthesizes in figures of annex n 2.

Economic policy analysis

It consists in noting economic and fiscal measurement effects on model's variables, in order to watch the evolution of different economic aggregates. We have chosen six fiscal measures which have a variation of 10 points (the results group is settled in annex 2).The different taxations chosen are the following :

(i)Employer's social contributions' rate diminution for unskilled workers of 10 points:
An obvious effect; this rate's diminution generates a diminution of the unemployment rate, with a growth of Y and C. This explains the production factor's growth (unskilled work).

(ii) Employer's social contributions' rate augmentation for skilled workers of 10 points:

The contribution's growth of this category socio-professional encourages the job situation for people without qualifications, producer using more it. Although this second measure is favorable to the situation of people's job without qualifications, it's not without consequences on Y and C.

(iii) Investment taxation's rate diminution (material and immaterial) of 10 points:

Decreasing taxes generates an investment's increase as Y and C's decreases. But under no circumstances does this make it an advantage for the situation of people without qualifications.

(iv) Employees' contribution reducing (unskilled and skilled) of 10 points:

This case has not impact on unemployment rate development.

In fact, it can be concluded that a taxes' general reducing advantages admittedly the investment's reflation (capital less costly) and increases the production, but it can present a negative effect on job of unskilled people. Thus, fiscal measurement isn't favourable for employment.

It seems that in fiscal politic the reducing of different contributions and taxes, presents some borders concerning reflation job unskilled. Thus, a strong diminution of taxes or their quasi-complete disappearance doesn't curb the problem of unskilled unemployment. Therefore, it exists an unemployment rate (about 5%) below this one, none fiscal politic is effective (because this rate is close to the natural unemployment's rate of 4%).

Conclusion

This paper developed a calculable general balance's model with the incorporated technical progress. This model is built from the Solow's endogenous growth model (1960), within it coexists two categories of employees (skilled and unskilled). Moreover, we considered a predominated economy by the new equipments' sector, where new software's equipments just work with the latest hardware's generations.

Our calibration on the American's economy permitted us to analyse the unemployment evolution of workers unskilled thanks to an analysis of sensibility and fiscal policies. The following measures: sensibility analysis puts in obvious that when the copyright length reduces, it advantages investment so employment, but it generates a decline of the innovative capacity. And conversely, the length advantage in innovation slows down investment and aggravate unemployment rate. In economic politic, different measures permitted to notice the unskilled unemployment rate's evolution. Thus we could observe that the measure, the most favourable for this workers' category is the employees contributions' reducing.

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Annex 1

The equilibrium

The demand of unskilled labor from the intermediate goods sector is obtained using equations (21), (19), (22) and (23):

$$\int_0^{s_t} \widetilde{L}_{nq,i,t} di = \int_0^{s_t^c} \left[\frac{I_t(1+\tau_4)}{w_{nq,t}}\right]^{\sigma} q_t^{1-\sigma} di + \left(1-\frac{1}{\sigma}\right)^{\sigma} \int_{s_t^c}^{s_t} \left[\frac{vI_t(1+\tau_4)}{w_{nq,t}}\right]^{\sigma} q_t^{1-\sigma} di =$$

$$(1+\tau_4)^{\sigma}\left[s_t^c + \left(1-\frac{1}{\sigma}\right)^{\sigma}\left(s_t - s_t^c\right)\right]\left(\frac{I_t}{w_{nq,t}}\right)^{\sigma} q_t^{1-\sigma},$$

Using this result and equations (15) and (31), the equilibrium on the unskilled labor market can be rewritten

$$L_{nq,t} = \left[\left[\frac{(1-\tau_1)(1+\tau_2)^{\beta-1}}{(1+\tau_3)^{\beta}}\right]\left[\frac{\beta^{\beta}\alpha^{1-\beta}A_t}{w_{nq,t}^{1-\beta}w_{q,t}^{\beta}}\right]\right]^{\frac{1}{\gamma}} k_t$$

$$+(1+\tau_4)^{\sigma}\left[s_t^c + (1-\frac{1}{\sigma})^{\sigma}(s_t - s_t^c)\right]\left(\frac{vI_t}{w_{nq,t}}\right)^{\sigma} q_t^{1-\sigma},$$

which is equation (33) of the main text.

From equations (16), (30), and (28) we have

$$L_{q,t} = \left[\left[\frac{(1-\tau_1)(1+\tau_3)^{\alpha-1}}{(1+\tau_2)^{\alpha}}\right]\left[\frac{\alpha^{\alpha}\beta^{1-\alpha}A_t}{w_q^{1-\alpha}w_{nq}^{\alpha}}\right]\right]^{\frac{1}{\gamma}} k_t + \frac{(1-\tau_1)}{(1+\tau_3)}\frac{\Delta s_t}{af_t},$$

which is equation (34) of the main text.

The equilibrium on the final good market, using (15), (16), (32), (18), is

$$A_t^{\frac{1}{\gamma}} k_t \left(\frac{\alpha}{w_{nq,t}}\right)^{\frac{\alpha}{\gamma}}\left(\frac{\beta}{w_{q,t}}\right)^{\frac{\beta}{\gamma}} = C_t + I_t,$$

with

$$\frac{C_{q,t+1}}{C_{q,t}} = (1+r_{t+2})\rho, \text{ and } w_{nq,t} + TR_t = C_{nq,t},$$

which is equation (35) of the main text.

Solving (13) for $L_{nq,t,s}$ and (14) for $L_{q,t,s}$,

$$\left[\frac{(1+\tau_2)^{\frac{\alpha(\beta-1)}{\gamma}}}{(1+\tau_3)^{\frac{\alpha\beta}{\gamma}}}\right]\left(\frac{\beta^{\beta}\alpha^{1-\beta}A_m}{w_{nq,m}^{1-\beta}w_{q,m}^{\beta}}\right)^{\frac{\alpha}{\gamma}}\left(q_t M_{t,s}\right)^{\alpha} = L_{nq,t,s}^{\alpha},$$

$$\left[\frac{\left(1+\tau_3\right)^{\frac{\beta(\alpha-1)}{\gamma}}}{\left(1+\tau_2\right)^{\frac{\alpha\beta}{\gamma}}}\right]\left(\frac{\alpha^{\alpha}\beta^{1-\alpha}A_m}{w_{q,m}^{1-\alpha}w_{nq,m}^{\alpha}}\right)^{\frac{\beta}{\gamma}}\left(q_t M_{t,s}\right)^{\beta}=L_{q,t,s}^{\beta},$$

replacing them in (6), using the definition of $M_{t,s}$ given in (1), and simplifying yields

$$\left(\frac{1-\tau_1}{1+\tau_4}\right)\left[\frac{\left(1+\tau_3\right)^{\frac{\alpha}{\gamma}}}{\left(1+\tau_2\right)^{\frac{\beta}{\gamma}}}\right]\gamma q_t A_t^{\frac{1}{\gamma}}\left(\frac{\alpha}{w_{nq,t}}\right)^{\frac{\alpha}{\gamma}}\left(\frac{\beta}{w_{q,t}}\right)^{\frac{\beta}{\gamma}}=1-\frac{\left(1-\delta\right)q_t}{\left(1+r_{t+1}\right)q_{t+1}},$$

Which gives the following law of motion for q_t:

Capital accumulation is given by (11):

$$k_t=\left(1-\delta\right)k_{t-1}+q_t I_t,$$

q_t can be determined in using equations (3), (19), (22) and (23):

$$\left(\frac{\left(\frac{1+\tau_2}{1-\tau_1}\right)}{\left(\frac{1+\tau_5}{1+\tau_4}\right)}\frac{w_{nq,t}q_t}{vI_t}\right)^{\sigma}=\left[s_t^c+\left(s_t-s_t^c\right)\left(1-\frac{1}{\sigma}\right)^{\sigma-1}\right]^{\frac{\sigma}{\sigma-1}},$$

Using (26) and (27), the free entry condition becomes:

$$\left[v^{1-\sigma}\left(\frac{\sigma-1}{a}\right)\left(\frac{1+\tau_4}{1+\tau_5}\right)^{\sigma}\left(\frac{1+\tau_3}{1-\tau_1}\right)\left(\frac{w_{q,t}}{f_t}-\frac{R_t^{T+1}w_{q,t+1}}{f_{t+1}}\right)\right]$$

$$=\left(1-\frac{1}{\sigma}\right)^{\sigma}\left(w_{nq,t}^{1-\sigma}-q_t^{1-\sigma}I_t^{\sigma}-R_t^{t+T+1}w_{nq,t+T+1}^{1-\sigma}q_{t+T+1}^{1-\sigma}I_{t+T+1}^{1-\sigma}\right),$$

using equation (24) gives:

$$f_t=\left(1-\theta\right)s_{t-T}+\theta s_t,$$

Finally Equation (32) gives the free entry condition in unemployment for unskilled labor:

$$u_t=\frac{\tilde{\ell}\pi_t\left[\left(1-P_t^x\right)u_{t-1}+P_t^x\right]}{L_{nq,t}w_{nq,t}W\left(1-tr\right)+\tilde{\lambda}\tilde{\ell}(1-P_t^x)\left(\pi_{t+1}\right)}$$

with

$$u_t=\frac{1}{\overline{L}_{nq}}-\frac{L_{nq}}{\overline{L}_{nq}},$$

$$\pi_t=\sum_{t=0}^{\infty}(1-\tau_1)Y_t-w_{nq,t}(1+\tau_2)L_{nq,t}-w_{q,t}(1+\tau_3)L_{q,t},$$

The stationarized dynamic system

The dynamic system (39)-(47) can be rewritten as:

$$L_{nq}=\left[\left[\frac{(1-\tau_1)(1+\tau_2)^{\beta-1}}{(1+\tau_3)^{\beta}}\right]\left[\frac{\beta^{\beta}\alpha^{1-\beta}A}{\hat{w}_q^{\beta}}\right]\right]^{\frac{1}{\gamma}}$$

$$\hat{k}_t\hat{s}_t^{\frac{1}{\alpha_1}}+(1+\tau_4)^{\sigma}\left[\prod_{i=0}^{T-1}\frac{1}{g_{t-i}}+(1-\frac{1}{\sigma})^{\sigma}(1-\prod_{i=0}^{T-1}\frac{1}{g_{t-i}})\right](v\hat{I})^{\sigma}\hat{q}_t^{1-\sigma}\hat{s}_t,$$

$$L_q=\left[\left[\frac{(1-\tau_1)(1+\tau_3)^{\alpha-1}}{(1+\tau_2)^{\alpha}}\right]\left[\frac{\alpha^{\alpha}\beta^{1-\alpha}A}{\hat{w}_{q,t}^{1-\alpha}}\right]\right]^{\frac{1}{\gamma}}\hat{k}\hat{s}^{\frac{1}{\alpha_1}}+\frac{(1-\tau_1)}{(1+\tau_3)}\frac{(g_t-1)}{a[g_t\hat{m}_t]},$$

$$A_t^{\frac{1}{\gamma}}\hat{k}_t\hat{s}_t^{\frac{1}{\alpha_1}}\alpha^{\frac{\alpha}{\gamma}}\left(\frac{\beta}{\hat{w}_{q,t}}\right)^{\frac{\beta}{\gamma}}=\hat{C}_t+\hat{I}_t,$$

$$\left(\frac{1-\tau_1}{1+\tau_4}\right)\left[\frac{(1+\tau_3)^{\frac{\alpha}{\gamma}}}{(1+\tau_2)^{\frac{\beta}{\gamma}}}\right]\gamma\hat{q}_tA_t^{\frac{1}{\gamma}}\alpha^{\frac{\alpha}{\gamma}}\left(\frac{\beta}{\hat{w}_{q,t}}\right)^{\frac{\beta}{\gamma}}+\frac{(1-\delta)}{(1+r)}g_{t+1}^{\frac{-1}{\sigma-1}}\left(\frac{\hat{s}_t}{\hat{s}_{t+1}}\right)^{\frac{-1}{\sigma-1}}\frac{\hat{q}_t}{\hat{q}_{t+1}}=1,$$

$$g_t^{\frac{\gamma}{\alpha_1}}\left(\frac{\hat{s}_t}{\hat{s}_{t+1}}\right)^{\frac{\gamma}{\alpha_1}}\frac{\hat{C}_t}{\hat{C}_{t+1}}=(1+r_{t+2})\rho,$$

$$TR_t=\hat{C}_{nq,t}(1-\tau_9),$$

$$\hat{k}_t-(1-\delta)\hat{k}_{t-1}g_t^{\frac{-1}{\alpha_1}}=\hat{q}_t\hat{I}_t\hat{s}_t^{\frac{-1}{\alpha_1}},$$

$$\left(\frac{\left(\frac{1+\tau_2}{1-\tau_1}\right)\hat{q}_t}{\left(\frac{1+\tau_5}{1+\tau_4}\right)v\hat{I}_t}\right)^{\sigma-1}\frac{1}{\hat{s}}=\left[\prod_{i=0}^{T-1}\frac{1}{g_{t-i}}+\left(1-\prod_{i=0}^{T-1}\frac{1}{g_{t-i}}\right)\left(1-\frac{1}{\sigma}\right)^{\sigma-1}\right]$$

$$\left[\frac{\upsilon^{1-\sigma}(\sigma-1)^{1-\sigma}\sigma^{\sigma}}{a}\left(\frac{1+\tau_4}{1+\tau_5}\right)^{\sigma}\left(\frac{1+\tau_3}{1-\tau_1}\right)\left(\frac{\hat{w}}{\hat{f}_t}-\frac{\hat{w}_{t+1}g_{t+1}^{\frac{\gamma}{(1-\gamma)(\sigma-1)}-1}\left(\frac{\hat{s}_t}{\hat{s}_{t+1}}\right)^{\frac{\gamma}{(1-\gamma)(\sigma-1)}}}{(1+r_{t+1})\hat{f}_{t+1}}\right)\right]$$

$$+(\prod_{i=0}^{T-1}\frac{1}{1+r_{t+i}})I_{t+T}^{\sigma}q_{t+T}^{1-\sigma}+n_{t+T}\left(\prod_{i=0}^{T-1}g_{t+i+1}^{\frac{\gamma}{(1-\gamma)(\sigma-1)}-1}\left(\frac{\hat{s}_{t+i}}{\hat{s}_{t+1+i}}\right)^{\frac{\gamma}{(1-\gamma)(\sigma-1)}}\right)$$

$$=I_t^{\sigma}q_t^{1-\sigma}s_t$$

$$\hat{m}_t=\theta+(1-\theta)\prod_{i=0}^{T-1}\frac{1}{g_t},$$

$$\hat{u}_t=\frac{\tilde{\ell}\pi_t[(1-P_t^x)u_{t-1}+P_t^x]}{L_{nq,t}W(1-tr)+\tilde{\lambda}\tilde{\ell}(1-P_t^x)(\pi_{t+1})},\ \text{where}\ W=\left(\frac{1}{1-\tau_2-\tau_3}\right)$$

with:

$$\pi_t=\sum_{t=0}^{\infty}(1-\tau_1)Y_t-(1+\tau_2)L_{nq,t}-\hat{w}_{q,t}(1+\tau_3)L_{q,t},$$

$$\alpha_1=(\sigma-1)(1-\gamma)$$

The stationarized long run system

Let us denote by (SL) the steady state system as formulated in Proposition 3. We write $\hat{x} = x$ for any variable x to unburden the notations in this appendix. Using the fifth equation of (SL), we get directly

$$r = \Psi_r(g) = \frac{g^{\frac{\gamma}{\alpha_1}}}{\rho} - 1.$$

Using the last equation of (SL), we can derive the following important relation

$$w_q = \Psi_1(g) I^{\sigma} q^{1-\sigma} s,$$

with $\Psi_1(g)$ given by the following expression provided $\Psi_r(g)$:

$$\Psi_1(g) = \frac{1-(\frac{\rho}{g})^T}{\Omega_1 \Phi_1 (1-\frac{\rho}{g})} \left((1-\theta)g^{-T} + \theta\right),$$

with $\Omega_1 = v^{1-\sigma} \frac{(\sigma-1)^{1-\sigma}\sigma^{\sigma}}{a}$ et $\Phi_1 = \left[\left(\frac{1+\tau_4}{1+\tau_5}\right)^{\sigma}\left(\frac{1+\tau_3}{1-\tau_1}\right)\right]$ On the other hand, the second equation of (SL) yields

$$Ks^{\frac{1}{\alpha_1}} = \Psi_2(g) w_q^{\frac{1-\alpha}{\gamma}},$$

with

$$\Psi_2(g) = \frac{L_q - \frac{g-1}{ag((1-\theta)g^{-T}+\theta)}}{\Omega_2 \Phi_2 A^{\frac{1}{\gamma}}},$$

where $\Omega_2 = (\alpha^{\alpha}\beta^{1-\alpha})^{\frac{1}{\gamma}}$ et $\Phi_2 = \left[\frac{(1-\tau_1)(1+\tau_3)^{\alpha-1}}{(1+\tau_2)^{\alpha}}\right]^{\frac{1}{\gamma}}$. Putting SL1 and SL2 into the first equation of (SL) we get the intended functional relation $w_q = \Psi_{w_q}(g)$ with:

$$\Psi_w(g) = \frac{\beta L_{nq}}{\alpha\Phi_3\left(L_{q-}\Phi_4 \frac{(g-1)}{ag\left[(1-\theta)g^{-T}+\theta\right]}\right) + \beta \frac{\Phi_5\left[g^{-T}+(1-\frac{1}{\sigma})^{\sigma}(1-g^{-T})\right]v^{\sigma}}{\Psi_1(g)}}$$

and $\Phi_3 = \frac{(1+\tau_3)}{(1+\tau_2)}$, $\Phi_4 = \frac{(1-\tau_1)}{(1+\tau_3)}$, $\Phi_5 = (1+\tau_4)^{\sigma}$.

Now using the seventh equation of (SL) and provided (SL1), we can derive immediately a g-function for I given $\Psi_I(g)$:

$$I = \Psi_I(g) = \frac{\Psi_{w_q}(g)}{\Psi_1(g)}\left(g^{-T} + (1-\frac{1}{\sigma})^{\sigma-1}(1-g^{-T})\right)\left(\frac{v}{\Phi_6}\right)^{\sigma-1},$$

with $\Phi_6 = \left(\frac{\left(\frac{1+\tau_2}{1-\tau_1}\right)}{\left(\frac{1+\tau_5}{1+\tau_4}\right)}\right)$.

The g-functional expression for C_q is then computed from the third equation of (SL)

$$C_q = \Psi_C(g) = \frac{1}{\beta}\left(L_q - \frac{g-1}{ag((1-\theta)g^{-T}+\theta)} \right)\Psi_w(g) - \Psi_I(g).$$

The g-functional expression for q is:

$$q = \Psi_q(g) = (1-\rho(1-\delta)g^{\frac{1}{\alpha_1}})\frac{(\Psi_w(g))^{\frac{\beta}{\gamma}}}{\gamma\Omega_3 A^{\frac{1}{\gamma}}},$$

With $\Omega_3 = (\alpha^{\alpha}\beta^{\beta})^{\frac{1}{\gamma}}$.

The g-functional expressions of the last variables are:

$$s = \Psi s(g) = \left(\frac{\Psi_{wq}(g)}{\Psi_1(g)} \right) (\Psi_I(g))^{-\sigma} (\Psi_q(g))^{\sigma-1},$$

$$K = \Psi_K(g) = \Psi_2(g)(\Psi_{w_q}(g))^{\frac{1-\alpha}{\gamma}} (\Psi_s(g))^{\frac{-1}{\alpha}},$$

$$\begin{aligned} TR = \Psi_{TR}(g) = \Psi_{C_{nq}}(1-\tau_9) &= \tau_2 L_{nq} + \tau_3 L_q \Psi_w(g) + \tau_6 L_{nq} + \tau_7 L_q \Psi_w(g) + \\ &\tau_8\left[\Psi_w(g)L_q + \Psi_r(g)\Psi_I(g)(1-\tau_{10})\right] + \tau_{10}\Psi_r(g)\Psi_I(g) + \tau_4\Psi_I(g) + \tau_9\Psi_{C_q}(g) + \\ &\tau_1(\Psi_{C_q}(g) + \Psi_I(g)) + \tau_5 Px, \end{aligned}$$

$$\Psi_{C_{nq}} = \frac{\Psi_{TR}(g)}{(1-\tau_9)},$$

$$\pi = \Psi_\pi(g) = (1-\tau_1)(\Psi_{C_q}(g) + \Psi_I(g)) - (1+\tau_2)L_{nq} - w_q(g)(1+\tau_3)L_q,$$

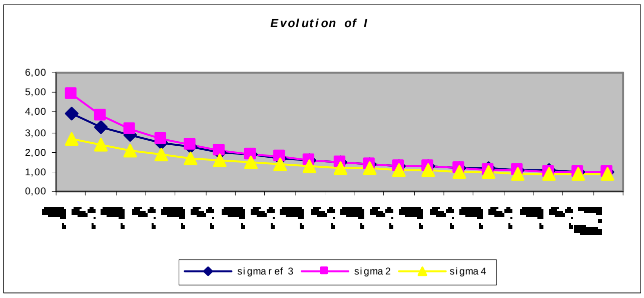
$$u = \Psi_u(g) = \frac{\ell\Psi_\pi(g)\left[(1-P^x)u + P^x\right]}{L_{nq}W(1-tr) + \lambda\ell(1-P^x)(\Psi_\pi(g))}$$

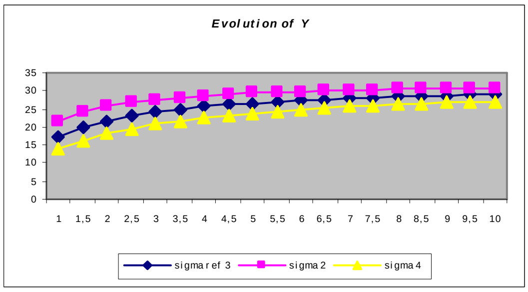
Annex 2 :

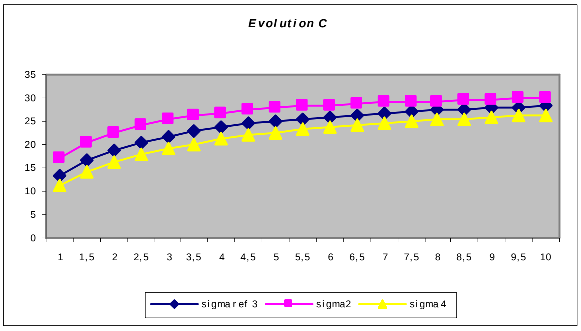
The parameters' analysis

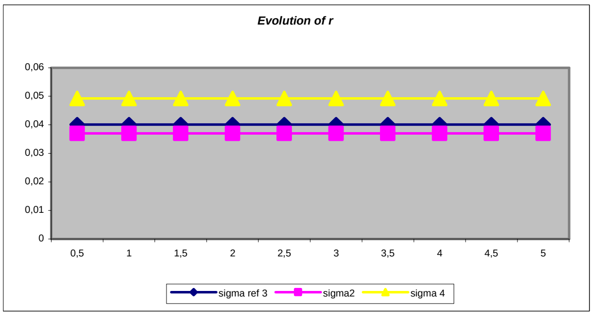
Variation of Sigma

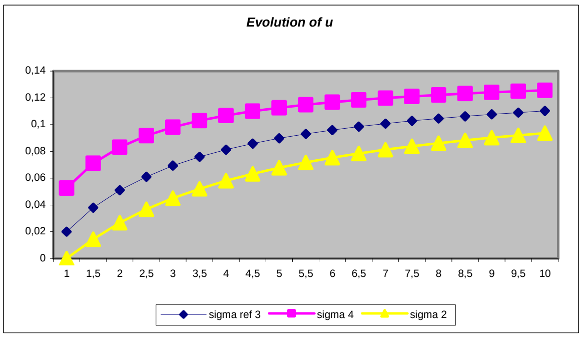
Figures

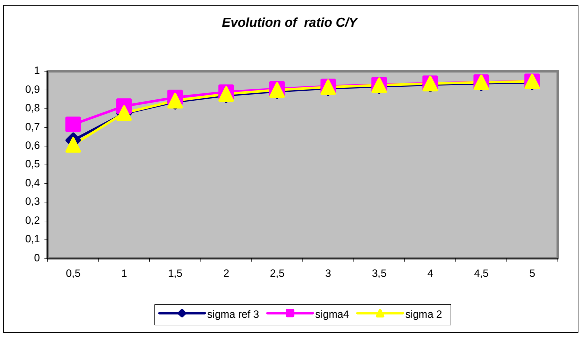


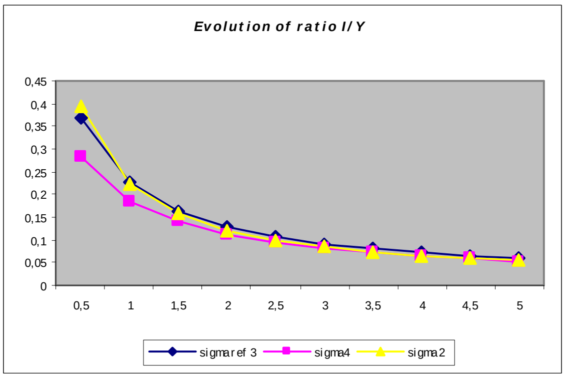


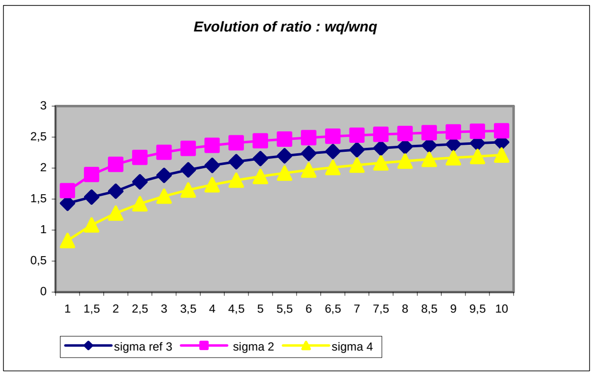






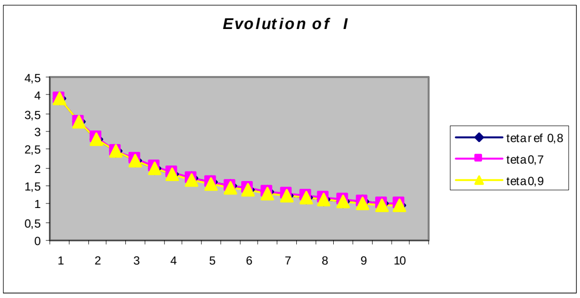


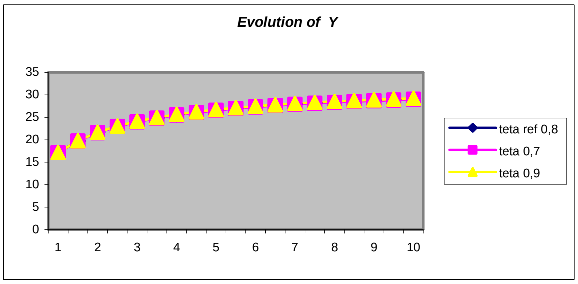


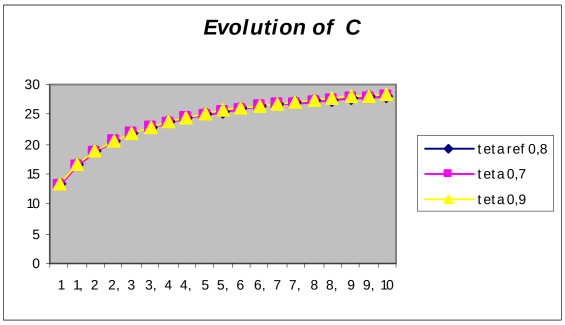


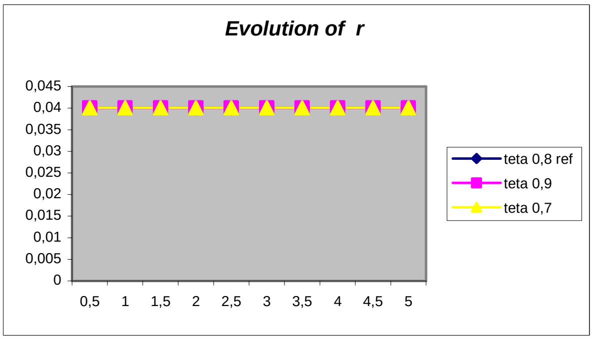
Variation of Teta

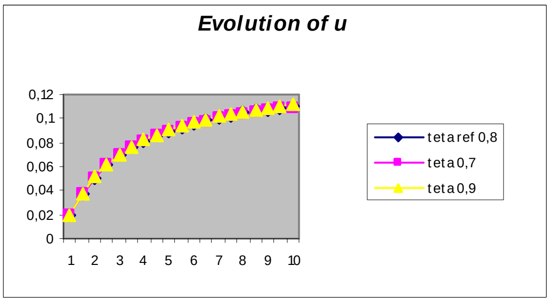
Figures

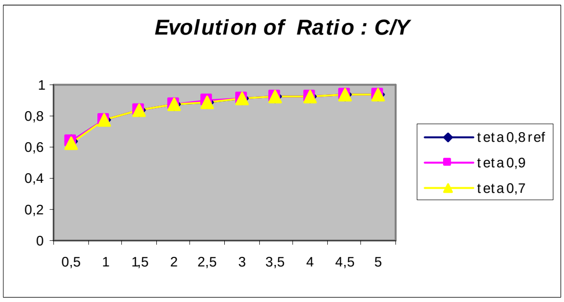


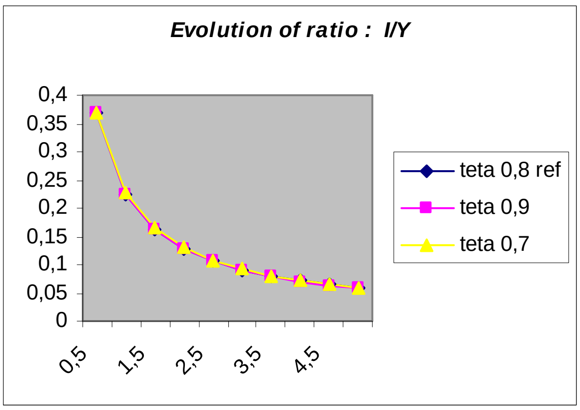


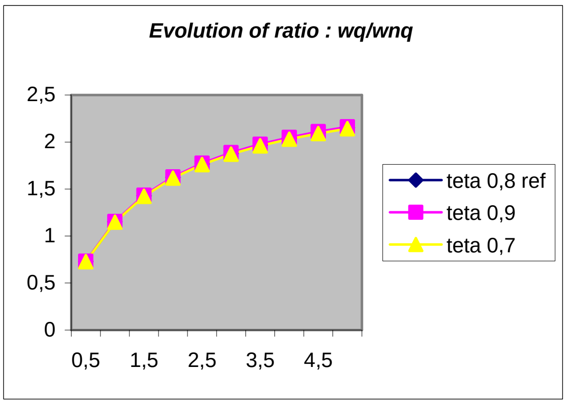




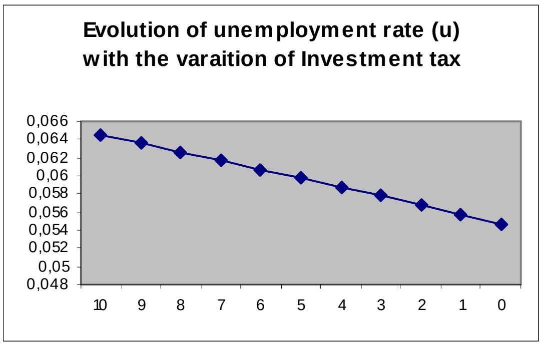


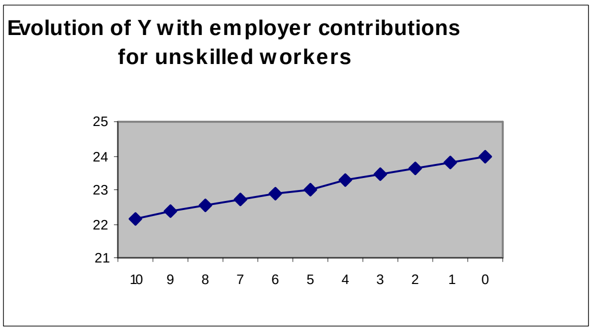


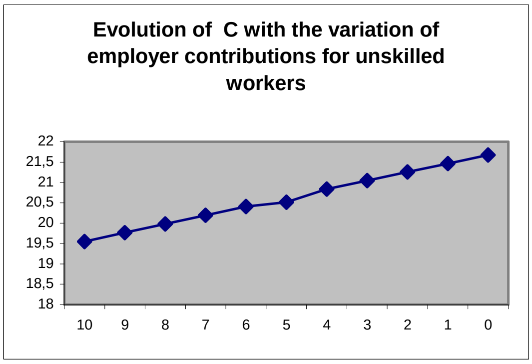


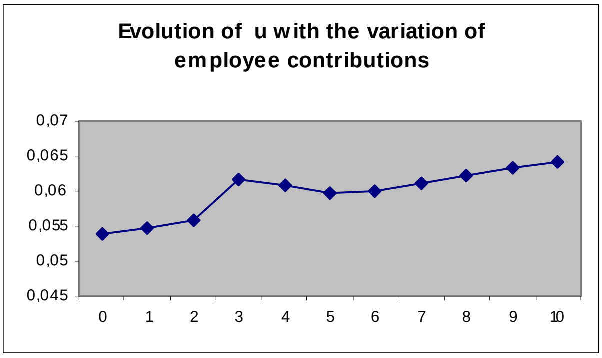


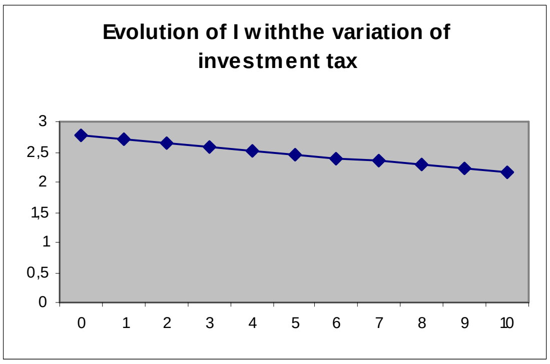
Economic policy analysis

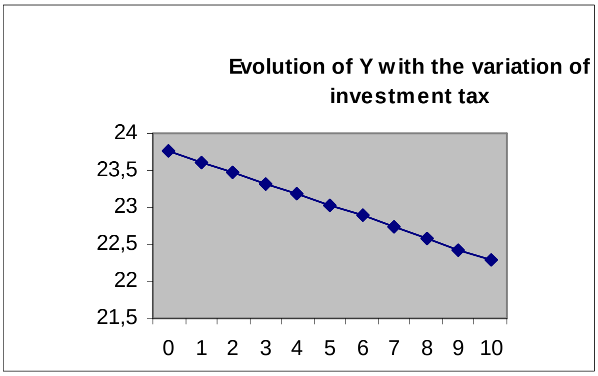


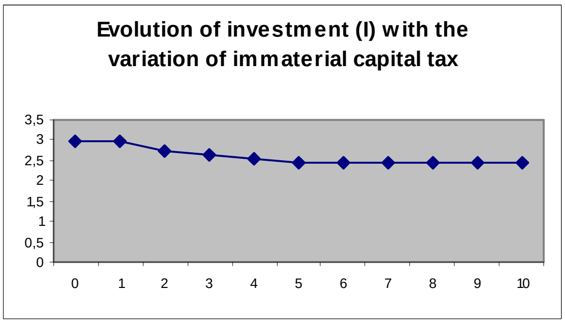


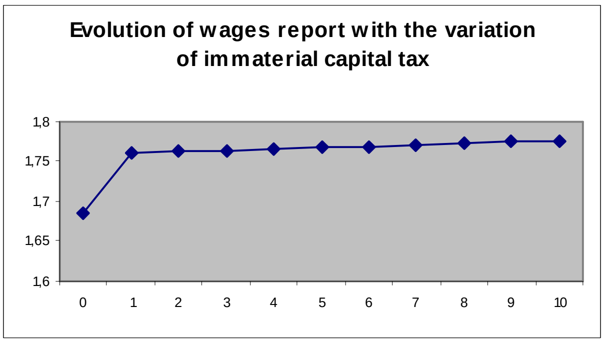


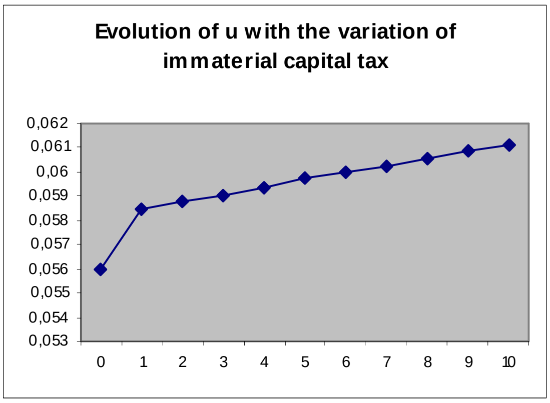


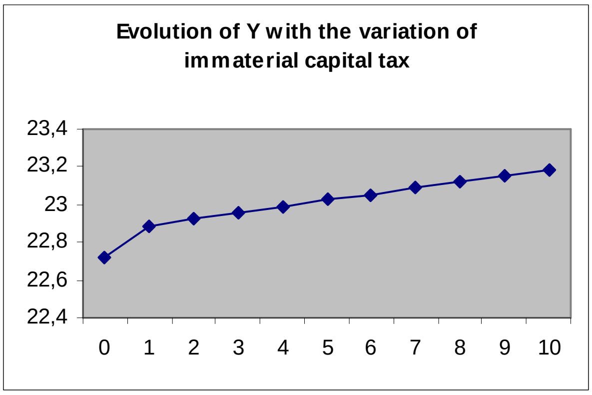


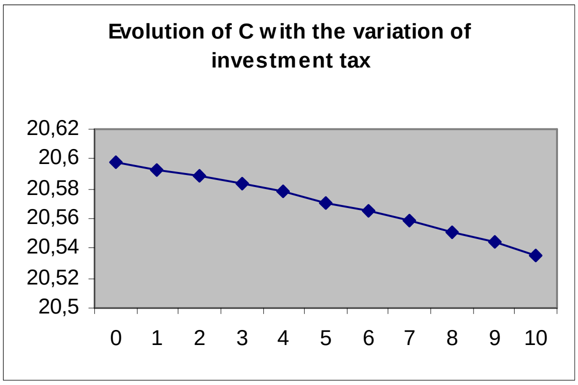


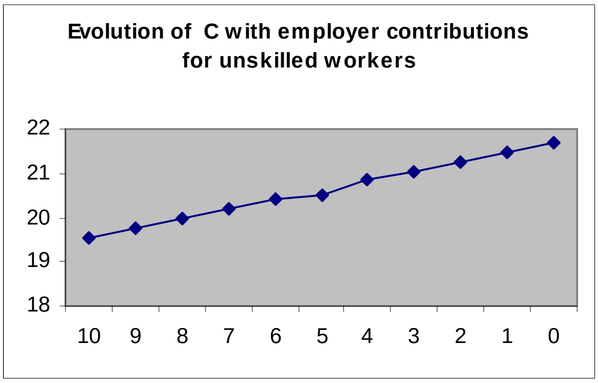


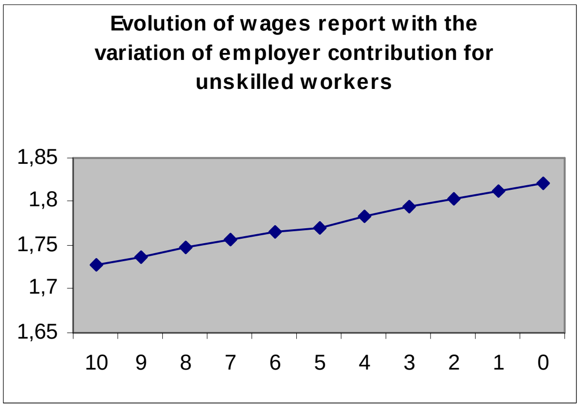


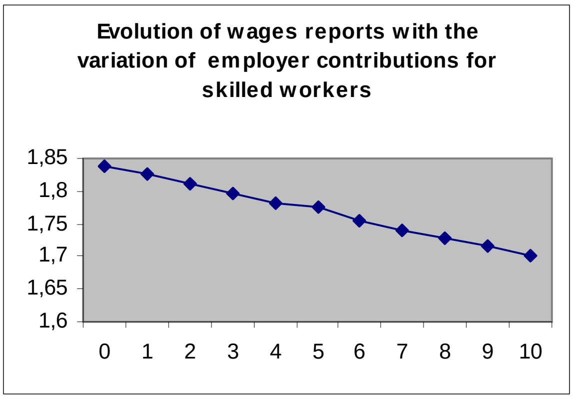

Evolution of u with the variation of immaterial capital tax
0,062
0,061
0,06
0,059
0,058
0,057
0,056
0,055
0,054
0,053
0 1 2 3 4 5 6 7 8 9 10

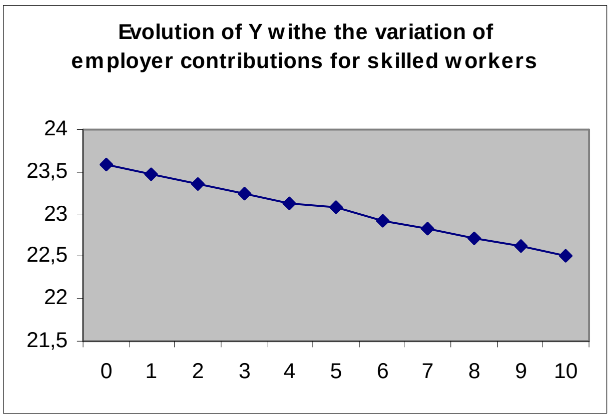

Evolution of Y with the variation of immaterial capital tax
23,4
23,2
23
22,8
22,6
22,4
0 1 2 3 4 5 6 7 8 9 10

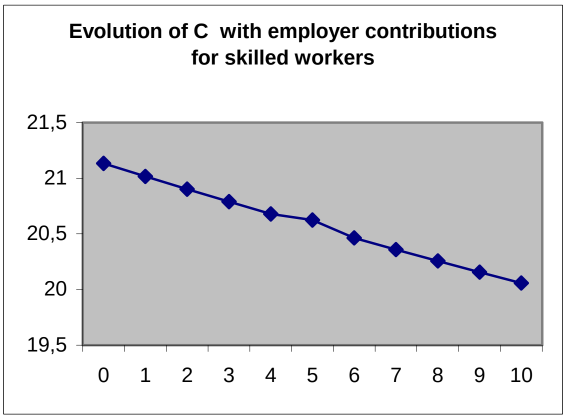

Evolution of C with the variation of investment tax
20,62
20,6
20,58
20,56
20,54
20,52
20,5
0 1 2 3 4 5 6 7 8 9 10


Evolution of C with employer contributions for unskilled workers
22
21
20
19
18
10 9 8 7 6 5 4 3 2 1 0


Evolution of wages report with the variation of employer contribution for unskilled workers
1,85
1,8
1,75
1,7
1,65
10 9 8 7 6 5 4 3 2 1 0


Evolution of wages reports with the variation of employer contributions for skilled workers
1,85
1,8
1,75
1,7
1,65
1,6
0 1 2 3 4 5 6 7 8 9 10


Evolution of Y withe the variation of employer contributions for skilled workers
24
23,5
23
22,5
22
21,5
0 1 2 3 4 5 6 7 8 9 10


Evolution of C with employer contributions for skilled workers
21,5
21
20,5
20
19,5
0 1 2 3 4 5 6 7 8 9 10