Commercial Mortgage Backed Securities: How Much Subordination is Enough?

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January 4, 2005, 12:45 A.M.

Abstract

The commercial mortgage-backed security market has experienced rapid growth in recent years, but relatively little academic research has questioned the apparent success of the CMBS capital structure. In this paper, we study whether the recent growth in the CMBS market actually reflects the mitigation of a market imperfection, as some have suggested, or reflects ever thinner subordination levels, perhaps backed up by overly aggressive assumptions about future commercial real estate performance. Preliminary results from our analysis based on a multi-factor structural model of expected commercial mortage defaults indicate that, somewhat surprisingly, the optimal levels of subordination are lower than those observed in recent CMBS deals. One interpretation of these results is that, in the absence of a performance track record for securitized commercial mortgages, the subordination levels of early deals were set too high. Under this interpretation, our results imply that in the future the CMBS market will likely see further reductions in subordination and continued rapid growth. An indirect implication of this conclusion is that the CMBS capital structure does create value.

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1 Introduction

The commercial mortgage-backed security (CMBS) market has experienced rapid growth in recent years, registering an average annual growth rate of about 18 percent since 1997. CMBS now surpass life insurance companies, and stand second only to commercial banks, as a primary channel of credit provision to the commercial real estate sector. Indeed, by the end of the third quarter of 2004, outstanding CMBS funded $322 billion, commercial banks $809 billion, and insurance companies $210 billion of the total $1.6 trillion of outstanding commercial mortgages.[1] Despite the rapidly growing importance of CMBS in commercial real estate finance, relatively little academic research has questioned the apparent success of the CMBS conduit capital structure.

CMBS are issued by legal entities ("conduits") that exist solely to issue CMBS so as to buy commercial mortgages.[2] CMBS conduits rely on subordinated bond structures in order to parse the credit risk inherent in commercial mortgages into bonds exhibiting different risk and return profiles. Bond tranches at the bottom of the structure are subordinated to the upper tranches, and thus stand first in line in the event that defaults in the mortgage pool translate into the capital losses. Typically, the topmost tranche of a CMBS structure will carry a 'AAA' (or equivalent) rating from at least one of the major bond rating agencies, indicating that, in the agency's opinion, the chance that these bondholders will experience a default is exceedingly remote.

In an efficient capital market, a CMBS conduit would have no economic value over and above the value of the mortgage pool since it simply repackages an existing set of securities. Yet the CMBS market has grown rapidly, as have all other asset-backed security classes (for example, asset-backed commercial paper), suggesting that the effects of important market imperfections are mitigated in these capital structures.[3] One plausible explanation that has been advanced in the theoretical literature is that the uppermost tranches of a securitization enjoy greater liquidity than the underlying mortgage collateral; hence a liquidity premium on these securities pulls the cost of raising capital through a CMBS structure below the cost of acquiring the mortgage pool (DeMarzo and Duffie (1998) and DeMarzo (2004)).

On the other hand, a key determinant of the cost of capital for CMBS conduits is the amount of subordination that must be applied to produce the relatively cheap highly rated tranches in the security structures. In order to simplify this assessment by reducing the im-

[1]Federal Reserve Z.1 Release (Flow of Funds), September 24, 2004.

[2]Accounting rules require CMBS conduits to be unmanaged following their inception if they are to be treated as separate (off balance-sheet) from the entities that sell the mortgages into the conduits.

[3]Residential mortgage-backed securities (MBS) require a somewhat different economic analysis because the MBS market is dominated by Fannie Mae and Freddie Mac, entities that enjoy at least implicit government subsidies.

portance of idiosyncratic risks, the conduits typically assemble large pools of mortgages that are diversified geographically, across property types (retail, office, multifamily, hotel, etc.), and in many other dimensions. As we show below, subordination levels have experienced a secular decline since 1997—falling in half by 2003. This stylized fact motivates the focus of our paper: What is the optimal level of subordination? Does the recent growth in the CMBS market actually reflect the mitigation of a market imperfection, or is growth being boosted by ever thinner subordination levels, perhaps backed up by overly aggressive assumptions about future commercial real estate performance?

Calculating the optimal level of subordination is essentially a calculation of expected default risk. For a given commercial mortgage, the idiosyncratic movements in the underlying property collateral would dominate the analysis of credit risk for the mortgage. However, as noted above, CMBS mortgage pools contain a large number of mortgages that presumably diversify away at least some idiosyncratic risk. In this paper, we leave aside the issue of diversification across mortgages and instead focus on the issue of systematic risk, taking the short rate of interest on default-free bonds as an indicator of the state of the economy.

Our empirical analysis indicates that, over a long horizon, flat and inverted yield curves have been contemporaneously correlated with higher real estate returns, but precede periods of declining real estate returns. Moving from these basic stylized facts to specific predictions of the optimal subordination levels for particular CMBS structures requires a model of the default behavior of commercial mortgages. We develop and empirically estimate a two-factor structural model for CMBS prices that values a mortgage pool as a function of property prices (the aggregate value of the properties underlying the mortgage pool) and the default-free short rate. This framework allows us to analyze how correlation between the level of commercial property values and interest rates affects commercial mortgage default risk and hence subordination levels. We first estimate the model on a comprehensive dataset of CMBS default rates and security prices, and then employ Monte Carlo simulations to estimate the optimal levels of subordination required to produce default rates on CMBS consistent with the ratings.

The preliminary results of our Monte Carlo experiments indicate that, somewhat surprisingly, the optimal level of subordination for a 'BBB' rated CMBS (lowest investment-grade tranche) is 5 percent— *lower* than the subordination levels observed in recent CMBS deals. There are two possible interpretations of this result. First, our model of commercial mortgage defaults might be wrong, and indeed more work needs to be done to assess the robustness of our results. The second more substantive interpretation is that, in the absence of a track record of the performance of securitized commercial mortgages, subordination levels were initially set too high. Under this interpretation, our results imply that the CMBS market will likely see further reductions in subordination and further rapid growth. An indirect implication of this conclusion is that the CMBS capital structure does create value, perhaps by exploiting liquidity differences as suggested earlier.

The paper is organized as follows. In Section 2 we analyze real estate returns and interest rates. Section 3 discusses our CMBS dataset and Section 4 outlines our modelling framework for calculating optimal subordination levels. Section 5 presents our estimation results and Monte Carlo results and Section 6 concludes.

2 Real Estate Returns and Interest Rates

In this section, we study the empirical correlation between real estate returns and the term structure of interest rates. In two recent papers, Torous, Valkanov, and Plazzi (2004) find evidence that variations in commercial real estate prices are primarily driven by fluctuations in discount rates rather than cash flows and Plazzi, Torous, and Valkanov (2004) find that the prices of apartment, industrial, retail, and office properties trend with the term spread and short rates. Other recent research has considered the relationship between the yield curve and macro-fundamentals such as GDP growth, consumption growth, and inflation (See for example, Ang, Piazzesi, and Wei (2004), Berardi and Torous (2002), and Ang and Bekaert (2003)). Since term structure factors summarize expectations about future interest rates, in a world with forward looking agents, these expectations should be important determinants of current and future macroeconomic variables. Ang and Bekaert (2003) find that inflation is the primary determinant of nominal interest rate spreads and Ang, Piazzesi, and Wei (2004) find that two yield curve factors, the short rate and term spread, are sufficient to model the dynamics of yield and GDP growth.[4]

Analysis of the relationship between real estate yields and term structure is hampered by shortcomings in available real estate returns indices. Measurements of real estate returns must contend with numerous impediments such as infrequent trading and heterogeneity of the assets (Geltner and Goetzmann (1998) and Tu, Yu, and Sun (2004)), potential bias in transaction based indices induced by selection problems and liquidity variability (Fisher, Gatzlaff, Geltner, and Haurin (2003), Fisher, Geltner, and Webb (1994), and Gatzlaff and Geltner (1998)) and problems with seasonality and appraisal smoothing in appraisal based indices (Clayton, Geltner, and Hamilton (2001)). Despite its well-known deficiencies, the National Council of Real Estate Investment Fiduciaries (NCREIF) Property Index (NPI) is

[4]Knez, Litterman, and Scheinkman (1994) find a third principal component related to heteroskedasticity in high frequency data accounts for 2% of movements of yields, however, Ang, Piazzesi, and Wei (2004) find that it accounts for less than 0.3% in lower frequency data

the most widely used benchmark of property-level commercial real estate investment performance in the United States. The NPI is a total returns index and it is compiled from both the capital and the income performance components of NCREIF monitored properties. Its primary shortcoming is that it includes appraisals of property values in addition to transaction prices. The properties in the NCREIF database that do not transact are typically not re-appraised every quarter, and more properties are re-appraised in the fourth calendar quarter than in any other quarter. As noted by Geltner and Goetzmann (1998), the shortcomings of the NPI induce smoothing, lags, and artificial seasonality in the indexes. The primary advantage of the NPI is that it is available over a long time series and it monitors a diverse base of property types across a wide range of geographic locations. Since our interest is in the long time-series properties of the indices and their correlation with term structure, we base our analysis and modelling on the NPI, as well.

In Figure 1, we plot the aggregate NPI index against the three-month treasury yield series obtained from the CRSP Fama risk-free rate file (Fama (1990)) and the term spread computed as the difference between the three-month zero-coupon yield and the five-year zero-coupon yield obtained from the CRSP Fama-Bliss discount bond file (Fama and Bliss (1987)) from March, 1978 through December, 2003. Typically the yield curve has been upward sloping over the period; however, as seen in Figure 1, there have been four important periods of yield curve inversions. Each of these episodes preceded recessions as defined by NBER and summarized in Table 1.[5]

As shown in Figure 1, flatter term structures and episodes of yield curve inversions appear contemporaneously associated with higher real estate returns. Inversions, however, also precede periods of recession induced declines in real estate returns when premia on long-bonds tend to be high and yields on short bonds are low due to Federal Reserve credit policy. Three month treasury yields trend positively with the aggregate NPI; however, there are several periods when higher treasury yields and a more steeply sloped yield curve are associated with lower aggregate real estate returns. Figures 2 and 3 plot the property-type specific NPI against both the three-month yields and the term spread. The trends plotted in these Figures are strikingly similar to aggregate results in Figure 1. Together these Figures suggest that correlations between the NPI real estate return indexes, the term spread, and short-term yields may be time varying, perhaps because they alternate between effects through the credit channel that decrease real estate returns, and effects on real estate cash flows due to inflationary expectations that increase returns.

[5] There is one inversion period from 1966:Q3 through 1996:Q4 that was not followed by an NBER recession, but generally since the 1950's inverted yield curves have precede recessions although lead times have varied Ang, Piazzesi, and Wei (2004).

Table 2 presents the Pearson's correlation coefficients between the aggregate NPI, the NPI's by property-type, the one-month and three-month yields from the CRSP Fama risk-free rates, and the one-year, three-year, and five-year yields from the CRSP Fama-Bliss discount bonds. The high cross-correlations of yields of different maturities suggest that a parsimonious representation of the term structure should be sufficient to uncover the broad effects of interest rate movements on real estate asset returns. Of course, these simple correlations cannot distinguish between the effects of risk premia and expectations of future interest rates on real estate return dynamics. The correlation coefficients do, however, indicate that treasury yields of all maturities are positively correlated with the aggregate and property-type NPI's and that the property-type NPIs are highly correlated with the aggregate diversified index.

Translating these broad results on real estate returns and interest rates to specific statements about the credit risk in CMBS mortgage pools requires a model that can link property returns and interest rates to mortgage defaults. The first characteristic of these market dynamics that we will exploit in our default modelling framework is the relatively high historic correlations in returns between the aggregate diversified NPI and the indexes for diversified portfolios by property-type. The co-movement of these indexes suggests that the aggregate diversified NPI is a reasonable proxy for the real estate return dynamics of diversified commercial mortgage pools. The positive co-movement of yields of various maturities is a second key characteristic of recent historic interest rate dynamics. The evident collinearity of yields of treasuries of various maturities suggests that a suitable default modelling framework should impose estimation restrictions to assure "no-arbitrage" conditions across bonds of various maturities. The "no-arbitrage" assumption is reasonable in this context because it would be expected that traders in treasury and CMBS markets would act to eliminate arbitrage opportunities arising from bond prices that are inconsistent either in the time series or the contemporaneous cross section. Additionally, the collinearity of yields suggests that one factor interest rate models may be a reasonable first approximation to the dynamics of different parts of the yield curve in a model where the second factor is asset yields. In the following sections, we discuss how our estimating model builds upon these linkages, the data we use to estimate the model on CMBS data, and the optimal subordination results that we uncover.

3 CMBS Data

The data for this analysis include two hundred and twenty commercial mortgage backed security pools originated between 1995 and 2002. The data were obtained from Commercial

Mortgage Alert and Trepp and include detailed information on the mortgage collateral at origination and performance data on the pools through October of 2004. As shown in Figure 4, our sample of pools represents about fifty percent of all CMBS issued over the period. Data availability forces us to exclude Resolution Trust Corporation and government sponsored enterprise multi-family deals in 1995 and 1996 and single loan pools over the entire sampling period. Overall, our sample represents the securitization of about 40,000 commercial mortgages and about $223 billion of capitalization.

Table 3 provides summary statistics for the mortgage collateral at origination. The weighted average coupons (WAC) on the pools range from a high of 9.26% in 1995 to a low of 7.02% in 2002 and the standard deviations range between seventeen and sixty two basis points. Most of the underlying mortgage collateral are balloon or bullet mortgages that amortize over a fifteen to twenty year horizon and are due in five to ten years. The weighted average maturity (WAM) of the pools reflects the average horizon for the balloon payments and these range from a high of 134 months in 1997 to a low of about 113 in 2001 and 2002. As is clear from the standard deviations, there is more heterogeneity in the balloon maturities in the earlier pools than in the later pools. The quality of the property cash flows improved over the sample period with average debt service coverage ratios (DSCR) rising to a high of 1.57 in 2002. However, average loan-to-value (LTV) ratios also increased over the period as borrowers increased leverage in the low interest rate environment from 1998 through 2002. The average number of loans in the pools grew to a high of about 250 loans in 1998 and 1999 and then fell in 2000 through 2002. Since the average size of the pools in 2000 through 2002 remained at about one billion dollars of mortgage principal at origination, this trend suggests that loan sizes have increased.

Prepayment options are severely curtailed or extinguished in nearly all the pool collateral in the sample. By 1998, most of the CMBS pools required prepayment lockout for a minimum of sixty months and by 2001 nearly all securitized mortgages had lockout again usually for sixty months. In conjunction with lockout, by the first quarter of 1998 more than 60 percent of the pools required defeasance to prepay and from 1999 through the end of the sample period more that 90 percent of the pools required defeasance. After, the lockout period most pools require some form of prepayment penality until a six month “free period” just prior to the balloon payment due date at the end of the tenth year. Yield maintenance was used to curtail prepayments in the earlier pools, until lockout became the preferred mechanism in 1998, and staggered prepayment penalities were heavily used in the remaining pools. Based on these results, in our estimation and simulation exercises we adopt the simplifying assumption that all of the mortgages in the mortgage pools are not prepayable.

Table 4 reports summary statistics for the property-type and geographic diversification of the CMBS pools in the sample. As shown, California has consistently accounted for the largest, or nearly the largest, average share of CMBS pool collateral over the sample period. The next largest state concentration is Texas, with Florida and New York accounting for an average of about six percent of CMBS pool principal at origination. The property-type distribution of the pool collateral is nearly evenly allocated to office, apartment, and retail properties with retail consistently representing about 29 percent of the sample pool collateral. Apartments collateral had high average allocations in 1995 and 1996 pools and then fell to about 22 percent of collateral at the end of the sample period. Average office allocations were at a low of 14 percent in 1995 and rose to a high of about 29 percent in 2001. Industrial property average allocations rarely exceed 10 percent over the period. Hotel allocations are at very high average allocations in the earlier pools, with a 16 percent allocation in 1995, and then fell to a little more than three percent of pool collateral.

Figure 5 summarizes the evolution of average loan-to-value ratios for the mortgages in the sample pools. As is clear from the figure there has been a rightward shift in the loan-to-value distributions over all origination vintages. In particular, the 1995 through 1997 vintages now have as much as five percent of their principal balances with loan-to-value ratios in excess of 100 percent.

Figures 6 and 7 present the evolution of the tranching and subordination structure of the CMBS pools in the sample. On average, the pool structures have become more complicated over the sampling period and the average number of bonds created from the underlying collateral has nearly doubled. Since the size of the collateral pools has not increased as rapidly, the face value of each bond class other than the AAA bonds has decreased over time. In addition, as noted at the outset, subordination levels for the 'BBB' rated tranches have decreased from a high of about 15 percent to a low of about seven percent in the 2002 pools. These trends suggests that the assumptions about the underlying credit risk in the commercial mortgage pool have been changing over time. In the next section, we develop a model that can reveal some of the economic factors underlying these trends.

4 Model

Two quantitative modelling strategies have emerged from the extensive literature focusing on the prices of defaultable bonds: structural and reduced-form models. The structural models make a direct causal link between the dynamics of the underlying collateral value (the unleveraged value of the firm or of the commercial property) and the price of the bond. Default is triggered by random movements in the collateral value relative to a threshold that is the exercise price on the default option. Recent examples of structural models of defaultable corporate debt include Longstaff and Schwartz (1995), Jarrow and Turnbull (1995), Collin-Dufresne and Goldstein (2001), and Huang and Huang (2002). Examples of structural valuation models of commercial mortgage and commercial mortgage backed securities include Dierker, Quann, and Torous (2005), Kau (1990), and Titman and Torous (1989).

In the reduced-form approach, the value of the borrower's assets and capital structure are not modelled explicitly. Rather, these models typically estimate exogenously specified competing risk hazard specifications of default behavior. Recent examples of the reduced-form approaches to the valuation of defaultable corporate debt include Jarrow and Turnbull (1995), Duffee (1998), Duffie and Singleton (1999), Collin-Dufresne and Solnik (2001), and Duffie and Lando (2001). Reduced form approaches have also been applied to the valuation of commercial mortgages and commercial mortgage backed securities (Ciochetti, Deng, Lee, Shilling, and Yao (2003) and Ambrose and Sanders (2004)). There are two principal drawbacks to the reduced-form approach in our application. First, these models do not incorporate term structure models in which bonds of different maturities are arbitrage-free. Related to this, these models do not make an internally-consistent causal link between property prices, interest rates, and default behavior, and thus are of limited use for calculating ex-ante expectations for default.

Hence in this section our objective is to develop a structural model that incorporates both interest-rate and property-price dependence, and that produces default behavior close to that seen in practice. The model that we develop incorporates frictions and transaction costs along the lines of Stanton (1995) and Downing, Stanton, and Wallace (2005) into the basic commercial mortgage valuation model of Titman and Torous (1989).

We begin with a basic two-factor structural model in which the value of a mortgage, M, is a function of interest rates, r, property prices, p, and time t. As discussed earlier in connection with Table 2, the short rate seems a reasonable representation of term structure, treasury yields of all maturities are positively correlated with the aggregate and property-type NPI's, and the property-type NPIs are highly correlated with the aggregate index. With these considerations in mind, we assume interest rates are governed by the Cox, Ingersoll, and Ross (1985) model:[6]

$$dr_t = (\kappa(\theta_r - r_t) - \eta r_t)dt + \phi_r \sqrt{r_t} dW_{r,t}, \tag{1}$$

where κ is the rate of reversion to the long-term mean of θ_r, η is the price of interest rate risk, and ϕ_r is the proportional volatility in interest rates. The process $W_{r,t}$ is a standard

[6] This model is widely used in the mortgage pricing literature. See, for example, Kau, Keenan, Muller, and Epperson (1995), Stanton (1995), and Downing, Stanton, and Wallace (2005).

Wiener process.

We estimated the following parameters for the interest rate process using the methodology of Pearson and Sun (1989) and daily data on constant maturity 3-month and 10-year Treasury rates for the period 1968-1998:

$$\begin{aligned} \kappa &= 0.13131 \\ \theta_r &= 0.05740 \\ \phi_r &= 0.06035 \\ \eta &= -0.07577 \end{aligned}$$

The coefficient estimate $\kappa = 0.13$ indicates that the mean reversion in interest rates is relatively weak. The long-term average short rate is 5.74 percent, as indicated by the estimate θ_r, and the annualized volatility of interest rates is estimated to be approximately 6 percent. The risk adjustment factor η is estimated to be approximately -0.076.

Property prices are assumed to evolve according to a geometric Brownian motion:

$$dp_t = \theta_p p_t dt + \phi_p p_t dW_{p,t}, \tag{2}$$

where θ_p is the expected appreciation in property prices, and ϕ_p is the volatility of property prices. Denoting the net income (on an unleveraged basis) accruing to the property owner by q_p, after risk-adjustment property prices evolve according to:

$$dp_t = (r_t - q_p)p_t dt + \phi_p p_t dW_{p,t}, \tag{3}$$

where $W_{p,t}$ is a Wiener process and $E\left[dW_r dW_p\right] = \rho$. We calibrate equation (3) as follows:

$$\begin{aligned} q_p &= 0.08 \\ \phi_p &= 0.06. \end{aligned}$$

The value of q_p is equal to the average income return to commercial properties as measured by the NPI, disucssed in section 2 above. The value of ϕ_p is equal to the average annual volatility of total commercial real estate returns, again as measured by the NPI. Finally, the contemporaneous correlation between the interest-rate and property-price shocks, ρ, is set to 0.5 based on the correlation between the NPI and treasury yields, discussed earlier.

Given these processes for interest rates and property prices, standard arguments show that, in the absence of arbitrage, the value of a commercial mortgage $M(p_t, r_t, t)$ with maturity date $T > t$ paying coupon C must satisfy the partial differential equation:

$$\frac{1}{2}\phi_r^2 r M_{rr} + \frac{1}{2}\phi_p^2 p^2 M_{pp} + \frac{1}{2}\rho\phi_r\phi_p p\sqrt{r} M_{rp} + \left(\kappa(\theta_r - r) - \eta r\right) M_r + \left((r - q_p)p_t\right) M_p + M_t - rM + C = 0, \tag{4}$$

subject to the boundary conditions:

$$\begin{aligned} M(0, r, t) &= 0, \qquad (5) \\ \lim_{r \to \infty} M(p, r, t) &= 0, \qquad (6) \\ M(p, r, T) &= P_T, \qquad (7) \end{aligned}$$

Boundary condition (5) requires that, when property prices are zero, the mortgage holder always defaults; because zero is an absorbing barrier for the property price process, the mortgage is therefore worthless. Boundary condition (6) arises because all future payments are worthless when interest rates approach infinity. Equation (7) reflects the amortization of the mortgage, where P_T is the balance reamining at time T.[7]

Consistent with standard practice in the commercial mortgage securitization industry, we assume that prepayment is not an option for the commercial mortgage borrower.[8] However, the borrower does retain an option to default. We value this option as a call on the underlying mortgage with a strike price equal to the value of the property, p_t. In other words, the borrower can turn over the property in return for the mortgage. Hence default is optimal when

$$M_t \geq p_t. \qquad (8)$$

The default option is embedded in the commercial mortgage and does not trade separately; hence we solve for its value at the same time that we solve for the mortgage value function that satisfies equations (4)-(7) above. As part of this solution, we identify the time-varying boundary in interest rate and property price space that separates the space into a region where continuation of the mortgage is optimal for the borrower and a region where default is optimal.

The basic model is solved numerically by replacing the partial derivatives in equation (4) with finite difference approximations.[9] This produces an algebraic expression for the value of the mortgage on a discrete grid of interest rates and property prices, where the value of the mortgage at each point (r_i, p_j, t) for $i = 1, \ldots, N_r$ and $j = 1, 2, \ldots, N_p$ is a function of

[7] While commercial mortgages take a variety of structures, it is fairly common for these mortgages to amortize on a long horizon, often 20-30 years, but to mature on a shorter horizon, often 10 years, leaving a balloon payment at maturity. This balloon payment is represented by P_T.

[8] As noted by Dierker, Quann, and Torous (2005) defeasance is an important option held by borrowers. Because defeasance does not affect the outstanding principal balances on the pool, it does not affect the mortgage termination factors. It does affect the lower tail of the underlying loan-to-value distribution in the pool as treasury securities are substituted for the mortgage collateral. The default boundary conditions of our model focus on the upper tail of the distribution (high loan-to-value ratios) which would be unaffected by defeasance.

[9] For further details of the finite difference method we use here, see Gourlay and McKee (1977) and Downing, Stanton, and Wallace (2005).

(known) values at various points on the grid at time $t+1$. The algorithm is initialized for time $t = T$ using the boundary conditions (5)-(7). Noting that these boundary conditions completely specify the mortgage value at time T, we make a step back to time $T-1$ using the finite difference expression. The default condition (8) is then checked at each point on the grid, and the resulting values are used as the starting point for the next step back in time to $T-2$, and so on back to $t = 0$.[10]

The algorithm for finding the value of the mortgage and solving for the borrower's optimal default policy illustrates the "ruthless" nature of default under the basic model. As soon as $M_t \geq p_t$, the basic model predicts that *all* mortgages with maturity T and coupon C will default. Of course, this is highly unrealistic—default is a costly event for all involved, and the incidence of default in any given pool of mortgages, even mortgages sharing nearly identical contract features, is very low. Clearly there are unobservable differences across borrowers that lead to fractional default rates in a pool of commercial mortgages.

4.1 Transaction Costs and Borrower Heterogeneity

To better capture the observed default behavior of commercial mortgage borrowers, we suppose that borrowers face significant transaction costs whenever they default on a mortgage and that these costs vary across borrowers. These costs include both direct monetary costs, such as legal fees, as well as implicit costs, such as the effects that the loss of one's credit rating might have on the cost of future credit.

Transaction costs of this nature will introduce a "wedge" into the basic default condition in equation (8) above. In the presence of such default costs, the borrower will wait to default until the property value sinks below the mortgage value by an amount equal to the costs of default. If we think of these costs as a proportion of the property value, we can represent this modified default condition as

$$M_t \geq (1+x)p_t, \tag{9}$$

where $0 \leq x$ represents the cost of defaulting. The lower bound on x reflects our assumption that x represents costs of defaulting, and these costs make default less likely.

Different borrowers might face different transaction costs. To account for this possibility, we assume that the costs x are distributed according to a beta distribution with parameters β_1 and β_2. This distribution is chosen because it can take many possible shapes, and is

[10]It is perhaps helpful to note that this algorithm prices the mortgage from the perspective of the *lender*. Hence, when checking the default condition, at points on the grid where the default condition is *true*—that is, where default is optimal for the borrower—we replace $M_t(r_i, p_j)$ with p_j. It is at this point where a severity rule could also be applied.

bounded by zero and one. The mean and variance of this distribution are given by:

$$\mu = \frac{\beta_1}{\beta_1 + \beta_2}$$

$$\sigma^2 = \frac{\beta_1 \beta_2}{(\beta_1 + \beta_2)^2(\beta_1 + \beta_2 + 1)}$$

It is straightforward to modify our solution algorithm to take account of the assumed structure of transaction costs. First, we form sub-pools of mortgages differentiated by their transaction cost levels. We choose the transaction cost levels on a discrete grid of $N_x = 30$ points, and we set the principal amount for the sub-pool at each transaction cost level so as to mimic a beta distribution defined by parameters β_1 and β_2.[11] We solve separately for the price of each of the sub-pools using the basic pricing algorithm, but where we replace the default condition with the condition specific to each sub-pool:

$$M_{i,j,k,t} \geq (1 + x_k)p_{j,t} \tag{10}$$

for $k = 1, 2, \ldots, N_x$ and i, j, t as before. It should be clear from this discussion how the transaction cost distribution produces fractional default rates in a pool of mortgages: When property values decline, the relatively low transaction cost sub-pools will default first, but these are only a (potentially small) fraction of the overall pool of mortgages.

A second source of heterogeneity that we consider is the loan-to-value (LTV) ratio. For each pool, we observe the initial weighted-average LTV. We also observe the distributions of the loan-to-value ratios within each pool as of October 15, 2004. We make use of this snapshot of distributions to impose a distribution of LTVs, centered on the weighted-average LTV, at origination. Specifically, for each pool we impose the following distribution of LTVs:

LTV	Share of Pool Balance
0.3	0.05
0.5	0.15
0.7	0.60
0.9	0.15
1.1	0.05

Because LTV is a transformation of a state variable, property value, and the fixed loan amount, the imposition of an LTV distribution does not require additonal solutions of the pricing model. When computing predicted default rates, within each transaction cost sub-

[11] The total principal amount is constrained to \$1 since the mortgage value is homogeneous in the principal amount. Hence the distribution of principal amounts across the sub-pools traces out the beta distribution.

pool we form further sub-pools by LTV and keep track of the principal balances at this level of disaggregation.

4.2 Seasoning and Burnout

While the imposition of a transaction cost distribution produces fractional default rates, the model still suffers from some important shortcomings. Commercial mortgages tend to exhibit average default rates that are relatively low for 1-3 years following origination, followed by a 3-7 year period of "ramp-up" to a maximum default rate, and then a decline in defaults (Vandell, Barnes, Hartzell, Kraft, and Wendt (1993)). The ramp-up in defaults, often referred to as the "seasoning" effect, is something that the transaction cost model cannot produce. Moreover, the model cannot produce defaults for "exogenous" reasons—that is, for reasons unrelated to the movements of interest rates and asset prices.

In order to capture possible seasoning effects in the data, we assume that the probability of default in any time interval is governed by a state- and time-dependent hazard function, λ. The value of λ depends on whether it is currently optimal for the borrower to default, which in turn is determined as part of the valuation of the mortgage. When default is not optimal, we will apply a lower hazard rate than when default is optimal. The hazard function is chosen to be flexible enough to deliver a seasoning effect.

The overall hazard rate governing default is given by:

$$\lambda(t) = \beta_3 + \beta_4 \operatorname{atan}\left(\frac{t}{\beta_5}\right) D_t, \tag{11}$$

where β_3 denotes the "background hazard rate," and the indicator variable D_t is one when default is optimal and zero otherwise. The atan function is employed to capture seasoning effects. In the continuation region, the overall hazard function implies that defaults arrive at a constant rate governed by β_3. In the default region, the atan function produces default rates that rise over time at a rate governed by β_4 to a maximum given by β_5. Since we hold β_3, β_4, and β_5 constant across the sub-pools, the hazard function will hold down the overall default rate early on in the life of a mortgage pool. As the pool ages, the hazard rate rises, producing the ramp-up effects observed in commercial mortgage data.

We modify our solution algorithm to incorporate the hazard function as follows. As before, we check the sub-pool specific default condition $M_{i,j,k,t} \geq (1+x_k)p_{j,t}$. If this condition is true, then default is optimal and we set $D_t = 1$; if the condition is false, then we set $D_t = 0$. We next compute the hazard λ using equation (11). The rate of default in a given sub-pool is equal to $P_d = 1 - e^{-\lambda\delta t}$ where δt is the length of time (fraction of a year) between t and

$t+1$. The value of a mortgage sub-pool is then

$$M_{i,j,k,t} = P_d \, p_{j,t} + (1 - P_d)(M_{i,j,k,t+1} + c), \tag{12}$$

where c is the gross mortgage coupon. As can be seen, the value of the sub-pool is just a probability-weighted average of the payoff in default and the payoff from holding the mortgage an additional period. The seasoning effect is captured by the rise in λ over time in the region of the state space where $D_t = 1$. When $D_t = 0$, a constant rate of default given by $\lambda = \beta_3$ is predicted by the model. Finally, the value of the overall pool of mortgages is equal to the sum of the values of the sub-pools, where we sum over k.

5 Estimation Strategy and Results

We employ a non-linear least squares procedure to estimate the coefficients of the model, $\beta_1 - \beta_5$. For any given set of coefficients, the valuation procedure described above generates a predicted termination rate for each month. If we have the "true" set of coefficients, these predicted default rates ought to be close, on average, to those we actually observe. Our estimation strategy involves searching for the set of coefficients that minimizes the sum of squared differences between the default rates predicted by our model and those observed in the data.

Formally, let $\hat{\omega}_{it}(\Theta)$ denote the predicted proportion of the balance of pool i that defaults in month t, as a function of the vector of coefficients to be estimated, Θ. If ω_{it} denotes the observed proportion that defaults (the single month mortality rate), our objective function is:

$$\chi(\Theta) = \sum_{i=1}^{N} \sum_{t=1}^{T_i} \left(\omega_{it} - \hat{\omega}_{it}(\Theta)\right)^2 \tag{13}$$

where N is the number of mortgage pools, and T_i is the number of observations on pool i. We use the Nelder-Mead downhill simplex algorithm to find the vector of coefficients Θ that minimizes $\chi(\Theta)$.

5.1 Results

Table 5 displays our preliminary estimation results. The estimates for β_1 and β_2 indicate that the mean transaction cost is quite high—sufficient to prevent most of the pool from defaulting, consistent with the very low default rates observed in the data over the period. The coefficient estimate β_3 governs the background default rate, and implies that 0.7 percent of the pool defaults per quarter. Finally, the estimates for β_4 and β_5 indicate that, when the default options held by the borrowers in a particular sub-pool are in the money, approximately 9 percent of the sub-pool will default after 4 quarters, a share that rises to 25 percent by 20 quarters.[12] Figure 8 illustrates that the model somewhat overpredicts default rates early on in the life of a pool, and under-predicts default rates later on.

5.2 Subordination Levels

It is straightforward to compute the optimal subordination levels for the tranched and subordinated securities that are used to finance the commercial mortgage pool. For a given tranche at date t, a principal loss will be realized if the cumulative defaulted principal in the pool surpasses the face value of the notes subordinated to the given tranche. For a given level of subordination, we can use Monte Carlo simulations of our model to estimate the ex-ante likelihood of experiencing a loss over a given time horizon. In this exercise, we assume that the rating agencies use the default experience of senior unsecured corporate bonds to set their required subordination levels.[13]

The Monte Carlo simulations are carried out as follows. The basic unit of analysis is a mortgage. We hold the maturity of all of the mortgages in our simulated pools fixed at 10 years, and we assume a 30 year amortization schedule (hence there is a fairly sizable balloon payment at 10 years). First, we solve the model for a particular coupon level (in our simulations, we compute simulations for coupons of 7, 8, and 9 percent); this solution delivers a mapping from interest rates and property prices to the fraction of the pool that defaults in each time period. Second, for each coupon group, we use a first-order Euler method to simulate 500 draws from the vector process for interest rates and property prices, starting from an interest rate of 5.7 percent and a property price level of 1.4 (which is equivalent to a 70% ltv, since the loan balance is always constrained to $1). These draws are fed through the solution of the model to generate predictions of default in each simulated pool. The output from each Monte Carlo simulation is a 10-year sequence of quarterly default rates. Third, we compute the cumulative amount of defaulted principal in each simulated pool by adding up the quarterly default rates. It is these cumulative default rates that are the main object of interest in this section.

Figure 9 displays the average cumulative default rate across all of the coupon groups in our simulation, and Figure 10 provides some information about the variance in cumulative

[12]Future revisions of the paper will discuss standard error calculations.

[13]The rating agencies from time to time publish guidance on how they arrive at the ratings and subordination levels for CMBS (specific document cite here). It is clear from these statements that they do in fact attempt to make the default risk on the 'BBB' rated CMBS, for example, equivalent to that on a 'BBB' rated corporate bond. One reason for doing this is that there is not enough history on the performance of CMBS to support the calibration of subordination levels.

default rates across our Monte Carlo simulations. Here we compute the cumulative default rates for $\rho = 0.5$, roughly the long-term correlation between interest rates and property values as discussed in previous sections. As can be seen, the median cumulative default rate takes a maximum of approximately 0.6 percent after about 15 quarters. The 25^{th} percentile takes a maximum of about 0.2 percent, while the 75^{th} percentile takes a maximum of 1.3 percent.[14]

A particular quantile of the sample of simulated cumulative default rates tells us the probability that principal equal to the quantile value will be wiped out due to defaults. For example, the 98^{th} quantile attains a maximum equal to 5 percent of origination balance after approximately 20 quarters. This means that, 98 percent of the time, the cumulative defaults in the pool will not exceed 5 percent over a five year horizon. Hence notes with 5 percent or greater subordination would experience a loss 2 percent of the time 5 years from origination. According to Moody's, from 1970-2001 1.63 percent of corporate bonds originally rated Baa transitioned into default over a five year horizon.[15] Over a ten year horizon, 3.77 percent of corporate bonds originally rated Baa transitioned into default. Since CMBS notes tend to be between five and ten years to maturity, this suggests that the 2 percent default rate above is a reasonable benchmark for 'Baa' rated bonds.

Figure 11 explores how the shape of the term structure affects optimal subordination levels. In this Figure, we show the expected cumulative default rates for two different starting values of the short rate, $r_0 = 0.01$ and $r_0 = 0.20$, holding ρ fixed at 0.5.[16] When $r_0 = 0.01$, the term structure is upward sloping because future short rates are expected to be higher as the interest rate reverts to its mean; the opposite is the case when $r_0 = 0.2$. As can be seen, under the upward sloping term structure default rates are significantly higher than under the downward sloping scenario, reflecting the lower expected returns on all assets in the low interest-rate environment.

As discussed earlier, it is generally the case that short rates have been falling over the sample period. Our results here suggest that, if anything, falling short rates imply that subordination levels ought to have been *rising*, unless they were set too high to begin with and the CMBS market is moving toward equilibrium. Further work is needed to assess the robustness of this tentative conclusion, in particular to examine additional processes for

[14] We are only making 500 draws from the system of interest rates and property prices. It is likely at this number of draws the tail quantiles are being estimated with high variance. In future versions of the paper we will use many more simulation draws so as to pin down the quantiles more precisely.

[15] Moody's Investors Service, Global Credit Research, Special Comment. February 2002. "Default and Recovery Rates of Corporate Bonds."

[16] In these calculations, we are holding the coupon levels fixed at 7, 8, and 9 percent, as before. Hence the coupon effect is influencing the results; in future revisions of the paper, we will hold the coupons fixed at the relevant par-coupon level under each term structure scenario so that the coupon effect is eliminated.

property price movements.

6 Conclusions

In this paper we have tried to shed some light on the rapid growth in the market for commercial mortgage-backed securities. While relatively little research has been devoted to understanding the CMBS market, some theoretical work has suggested that CMBS conduits exist because they capitalize on liquidity differences between CMBS and the underlying mortgage collateral. On the other hand, more recent CMBS structures embed far less subordination than deals done a decade ago, raising the possibility that growth is being fueled by overly aggressive assumptions regarding commercial mortgage credit risk.

We construct a two-factor model of defaultable mortgage prices that links default behavior to expectations for property prices and default-free short-term interest rates. The model employs transaction costs and frictions that we estimate on CMBS data. Monte Carlo simulations of expected defaults under the model indicate that the optimal subordination levels lie *below* those currently seen in practice. There are two possible interpretations of this result. First, our model of commercial mortgage defaults might be wrong. Second, in the absence of any track record for the performance of securitized commercial mortgages, subordination levels were too high early in the history of the CMBS market. Under this interpretation, our results imply that the market might undergo further reductions in subordination and hence further growth. An indirect implication of this conclusion is that the CMBS capital structure does create value, perhaps by exploiting liquidity differences as suggested earlier, though more research is needed to uncover the value proposition in CMBS conduits.

In future revisions of this paper, we plan to explore additional specifications for the term structure and property price processes. In particular, we plan to explore the effect a jump process in property prices might have on optimal subordination levels. We also plan to carry out more extensive sensitivity analysis of our basic results.

Table 1: NBER Recessions and Yield Curve Inversions

NBER Recessions	**Yield Curve Inversions**
January 1980 - July 1980	January 1979 – April 1980 January 1980 – March 1980
July 1981 - November 1982	November 1980 – August 1989
July 1990 - March 1991	March 1989–July 1989
March 2001 - November 2001	August 2000–December 2000 February 2001

Sources: NBER, the U.S. Department of Commerce, *Survey of Current Business*, October 1994, Table C-51.

Table 2: Pearson's Correlation Coefficients (Series from March, 1978 - December, 2003)

	Aggregate Real Estate	Apartment Yield	Industrial Yield	Office Yield	Retail Yield	Treasury. One Mo. Yield	Treasury. Three Mo. Yield	Treasury One Yr. Yield	Treasury Three Yr. Yield	Treasury Five Yr. Yield
Aggregate Real Estate	1									
Apartment Yield	0.742	1								
Industrial Yield	0.946	0.692	1							
Office Yield	0.937	0.610	0.883	1						
Retail Yield	0.690	0.415	0.594	0.482	1					
Treasury One Mo. Yield	0.519	0.446	0.491	0.509	0.210	1				
Treasury Three Mo. Yield	0.522	0.454	0.494	0.504	0.237	0.991	1			
Treasury One Yr. Yield	0.497	0.447	0.471	0.476	0.233	0.974	0.988	1		
Treasury Three Yr Yield	0.417	0.382	0.393	0.393	0.217	0.935	0.953	0.983	1	
Treasury Five Yr Yield	0.363	0.342	0.336	0.336	0.209	0.905	0.925	0.960	0.994	1

Source: NCREIF Total Return Indices, CRSP Fama risk-free rate files, and CRSP Fama/Bliss discount bond files
All correlation coefficients are statistically significant at better than the .001 level

Table 3: Summary Statistics for the Commercial Mortgage Collateral

Origination Year	Number of Pools		WAC Percent	DSCR	WAM Months	Number of Loans	LTV Ratio Percent	Orig. Prin. $ Billions	Lockout % of Pool	Yield Maint. % of Pool
1995	8	Mean	9.26	1.31	123.39	102.00	65.95	459.18	23.01	32.29
		Std. Deviation	0.63	0.54	69.60	114.39	3.37	345.55	39.56	34.21
1996	15	Mean	8.79	1.49	129.73	120.60	65.77	545.69	23.18	64.63
		Std. Deviation	0.37	0.16	50.72	64.84	5.77	267.01	40.18	39.71
1997	22	Mean	8.38	1.49	134.07	158.05	67.13	1035.67	44.21	45.56
		Std. Deviation	0.55	0.19	36.44	106.92	5.24	483.34	38.72	36.31
1998	37	Mean	7.40	1.50	127.87	256.41	68.80	1441.15	59.66	36.90
		Std. Deviation	0.34	0.17	10.25	145.26	3.86	752.96	35.02	32.13
1999	38	Mean	7.57	1.42	124.79	263.50	68.63	988.27	79.60	16.67
		Std. Deviation	0.37	0.11	19.80	420.52	4.36	355.69	25.38	21.55
2000	28	Mean	8.19	1.42	116.57	159.46	67.08	900.03	87.43	12.08
		Std. Deviation	0.17	0.14	8.23	46.51	4.00	187.99	19.57	19.56
2001	36	Mean	7.61	1.46	112.75	144.78	67.75	1011.52	90.05	9.67
		Std. Deviation	0.26	0.14	4.84	30.19	4.16	161.77	18.18	17.48
2002	36	Mean	7.02	1.57	112.94	123.56	68.06	993.85	94.74	4.20
		Std. Deviation	0.40	0.33	6.65	34.15	4.35	224.22	8.81	8.33

Source: Commercial Mortgage Alert and Trepp

Table 4: Summary Statistics for the Pool Collateral Large State and Property Type Diversification

Origination Year		California % of Pool	Florida % of Pool	New York % of Pool	Texas % of Pool	Office % of Pool	Hotel % of Pool	Apartment % of Pool	Retail % of Pool	Industrial % of Pool
1995	Mean	18.80	2.00	0.83	20.83	14.34	15.69	31.44	28.79	12.10
	Std. Deviation	30.05	3.74	2.33	19.02	11.59	13.71	17.07	20.36	8.59
1996	Mean	11.68	4.43	3.11	13.07	9.59	15.08	41.98	29.91	6.45
	Std. Deviation	9.32	4.55	5.56	6.50	7.53	13.60	17.85	9.57	4.68
1997	Mean	17.06	5.68	9.23	8.64	17.98	13.64	28.16	29.99	7.23
	Std. Deviation	8.31	5.47	7.64	6.70	12.19	8.79	11.50	7.35	5.16
1998	Mean	17.67	5.03	6.96	7.88	17.17	11.86	25.52	30.06	8.82
	Std. Deviation	10.09	5.31	6.97	6.52	6.97	5.84	8.37	12.77	5.89
1999	Mean	21.29	6.10	6.50	5.89	20.72	7.81	26.67	28.19	9.41
	Std. Deviation	13.23	4.64	6.45	6.35	7.56	4.06	9.00	10.06	6.61
2000	Mean	19.21	5.95	7.55	6.49	26.32	6.47	22.66	28.77	11.60
	Std. Deviation	7.24	5.13	8.11	4.64	9.76	4.13	7.10	8.74	7.48
2001	Mean	20.59	4.56	6.63	6.93	28.95	5.30	21.94	29.47	10.73
	Std. Deviation	6.79	4.28	6.78	5.54	7.98	2.68	9.67	7.25	6.06
2002	Mean	16.45	8.94	5.84	6.93	24.58	3.55	22.70	35.69	10.61
	Std. Deviation	8.68	6.67	7.67	5.66	7.38	3.96	9.07	7.31	5.43

Source: Commercial Mortgage Alert and Trepp

Table 5: Preliminary Estimation Results

Coefficient	Estimate
β_1	0.00057
β_2	1.383
β_3	0.029
β_4	1.059
β_5	10.547
χ =2.842	
N =3,566	

Figure 1: U.S. real estate returns, the three-month Treasury Bill yield, and the term structure spread (five year Treasury yield minus the three-month Treasury yield) on Real Estate Returns)

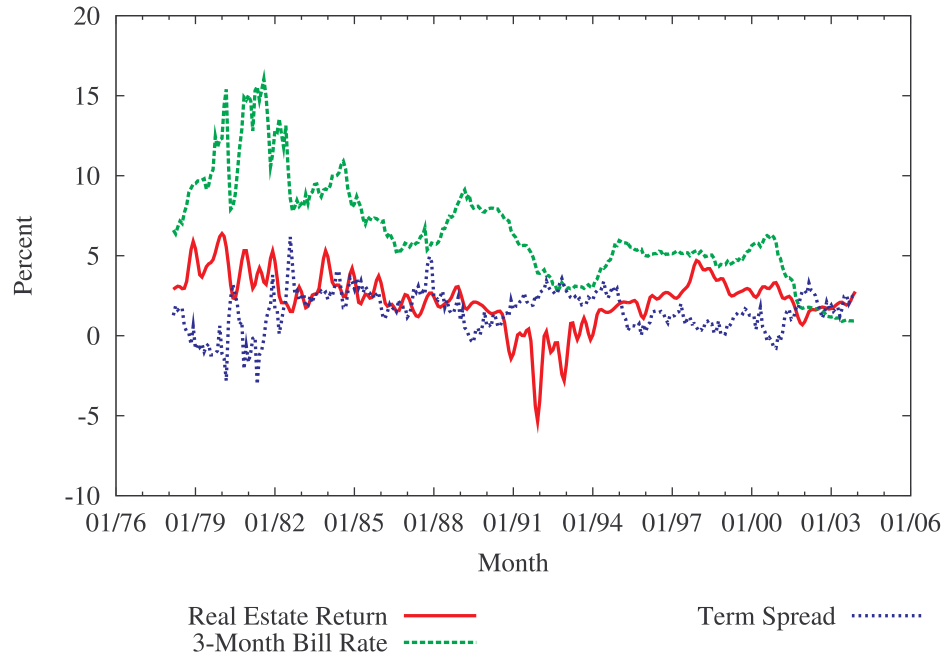


Figure 2: Real estate returns by property type and the term structure spread (five year treasury yield minus the three-month treasury yield) on real estate returns)

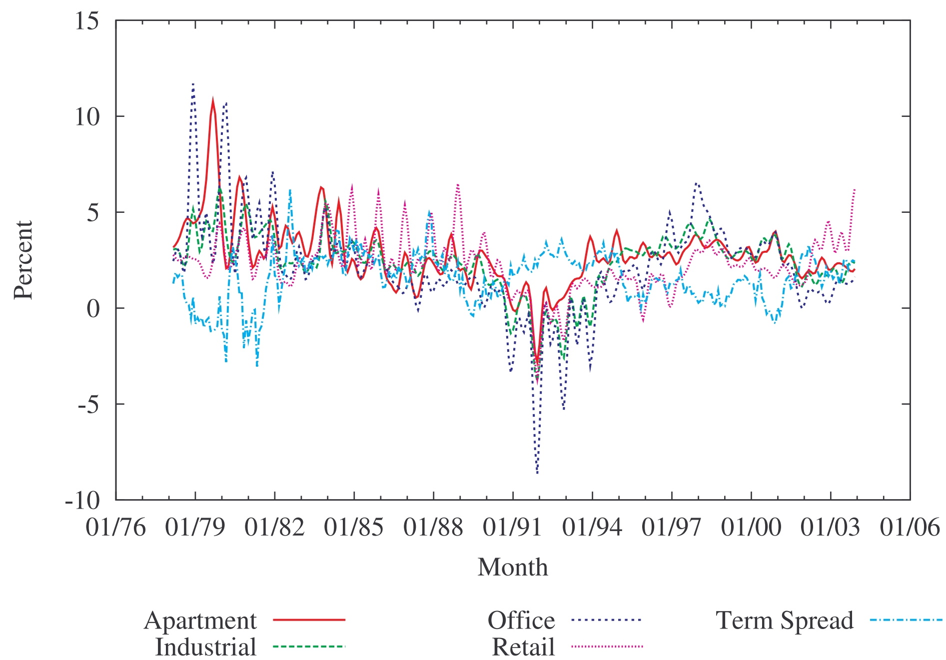


Figure 3: Real Estate Returns by property type and the three month Treasury Bill yield

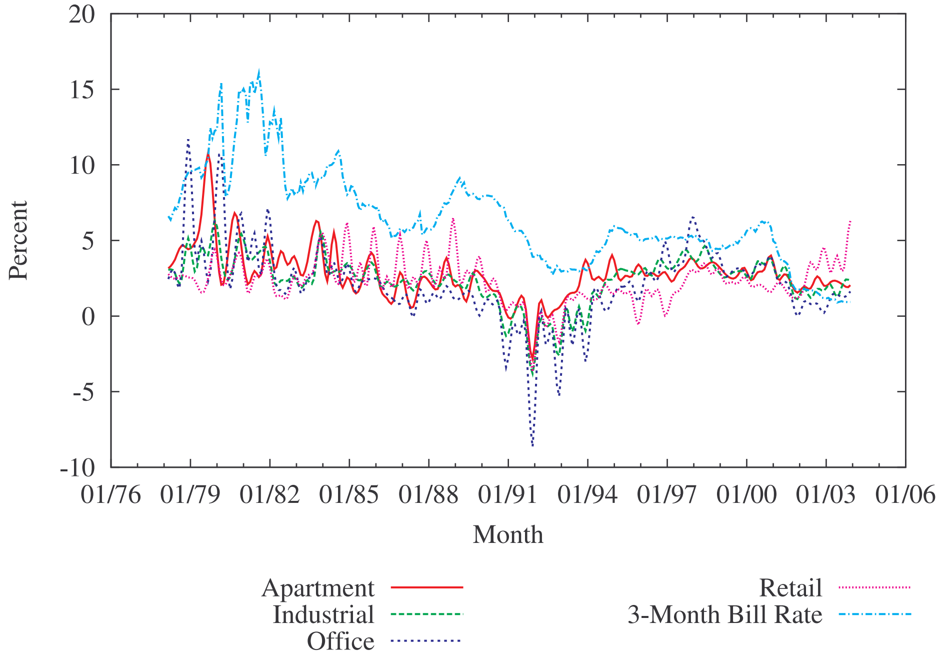


Figure 4: Comparison of Market and Sample Origination Levels

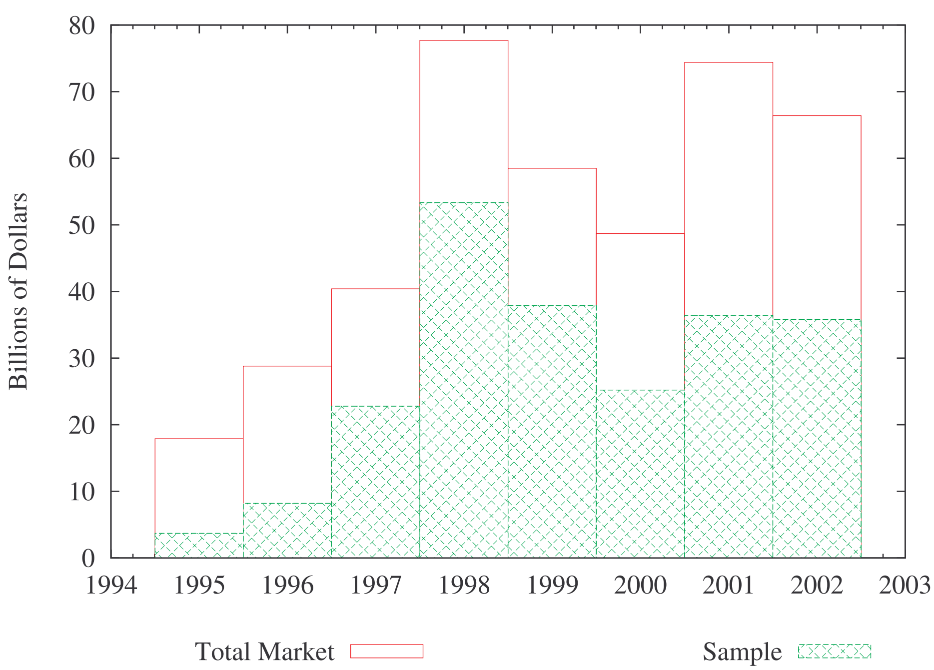


Figure 5: Distribution of Loan-to-Value Ratios on Remaining Loans by Year of Origination for Sample of 220 CMBS Pools (October 15, 2004)

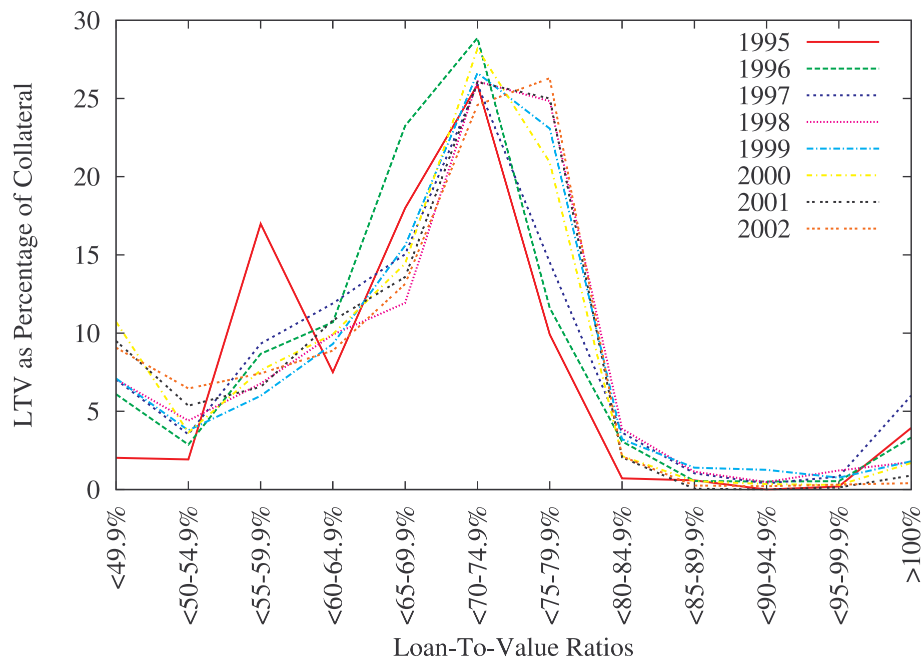


Figure 6: Average Number of Tranches for Sample of 220 CMBS Pools

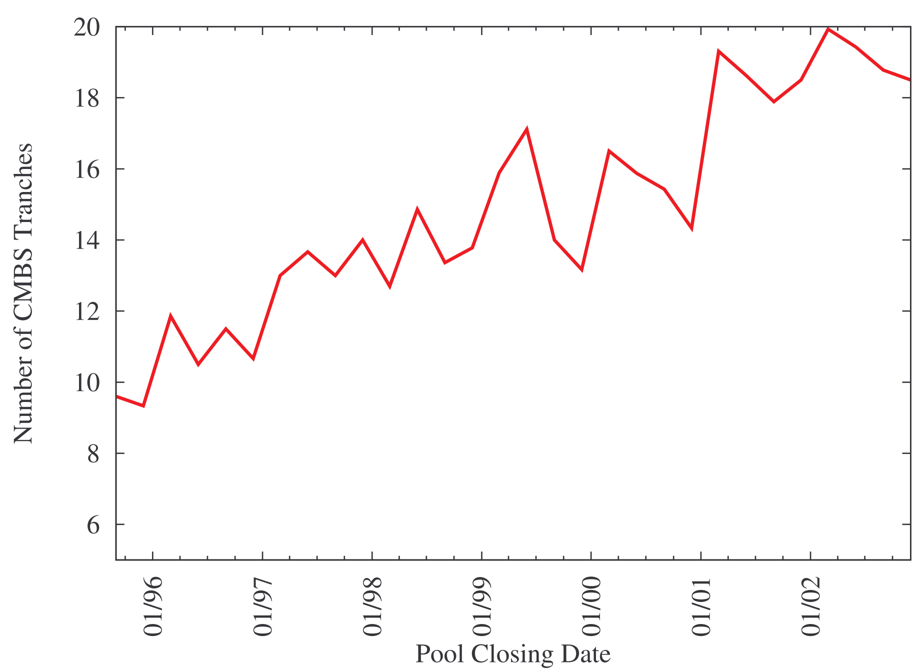


Figure 7: Subordination Levels at Origination for Sample of 220 CMBS Pools (Subordinate bonds are rated as below single B minus and Mezzanine bonds are rated between BBB and B)

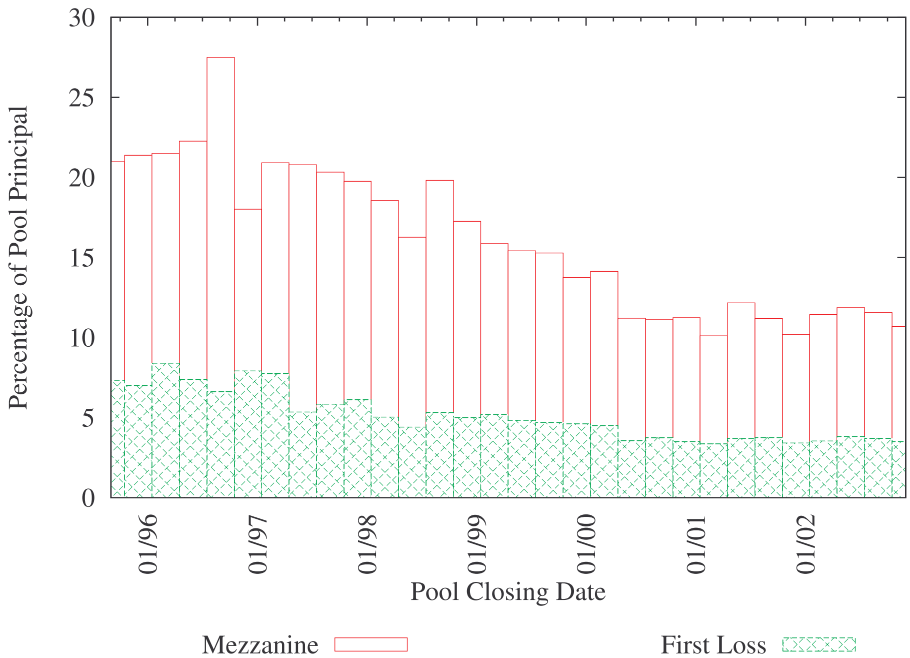


Figure 8: Observed and Predicted Default Rates

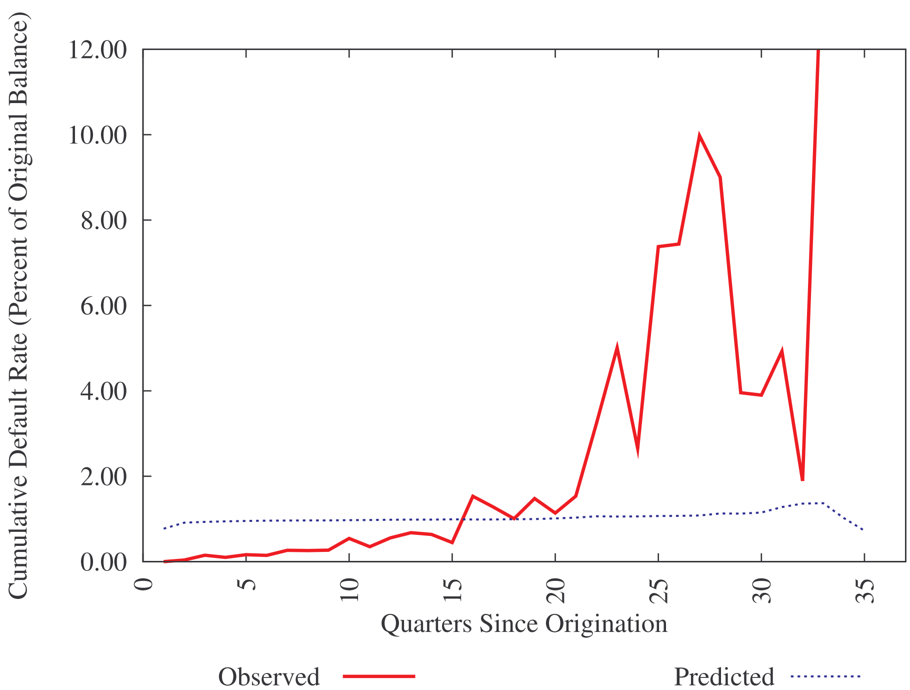


Figure 9: Simulated Cumulative Default Rates

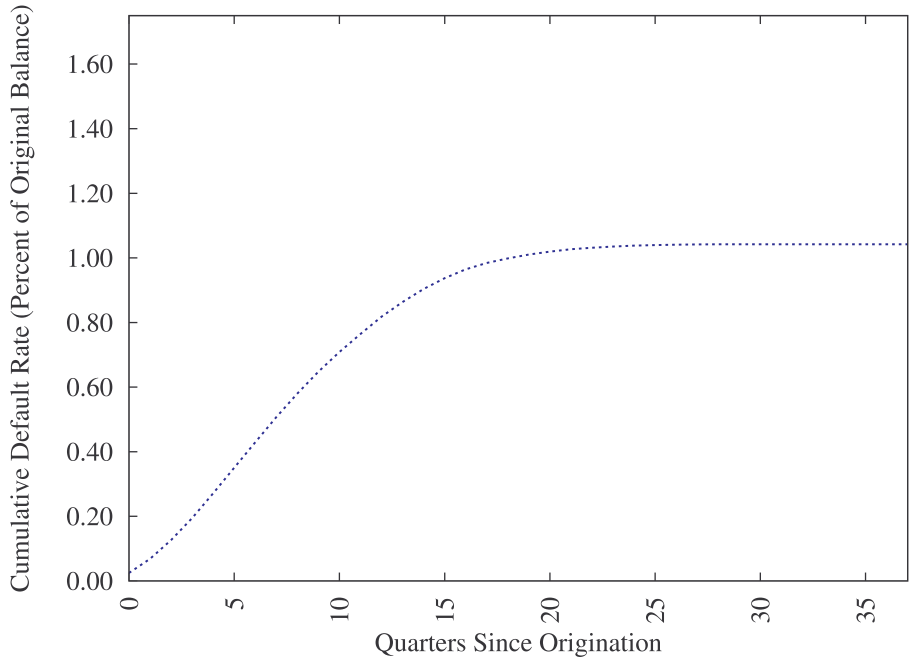


Figure 10: Distribution of Simulated Cumulative Default Rates

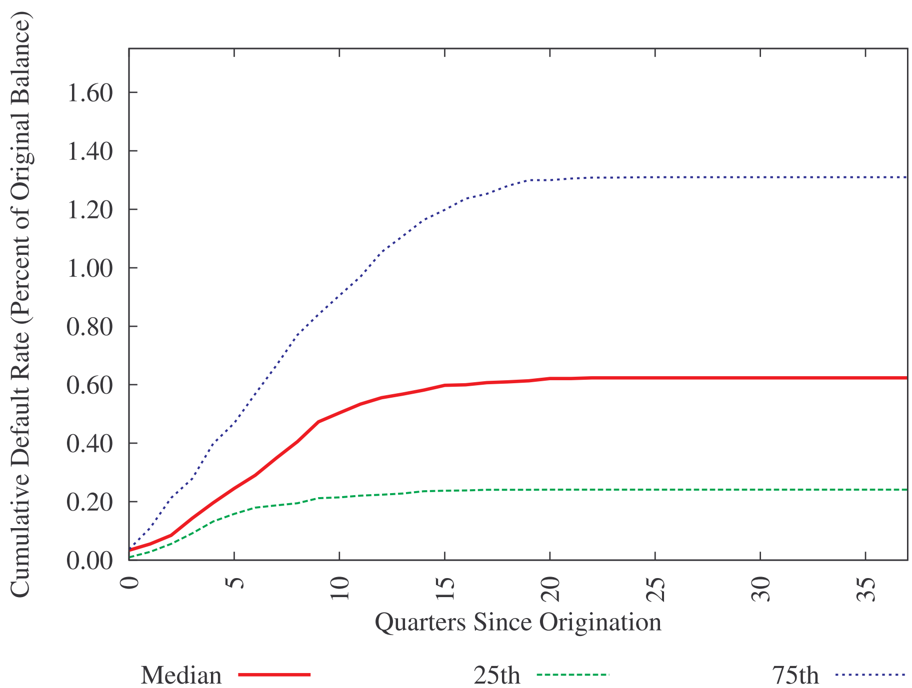
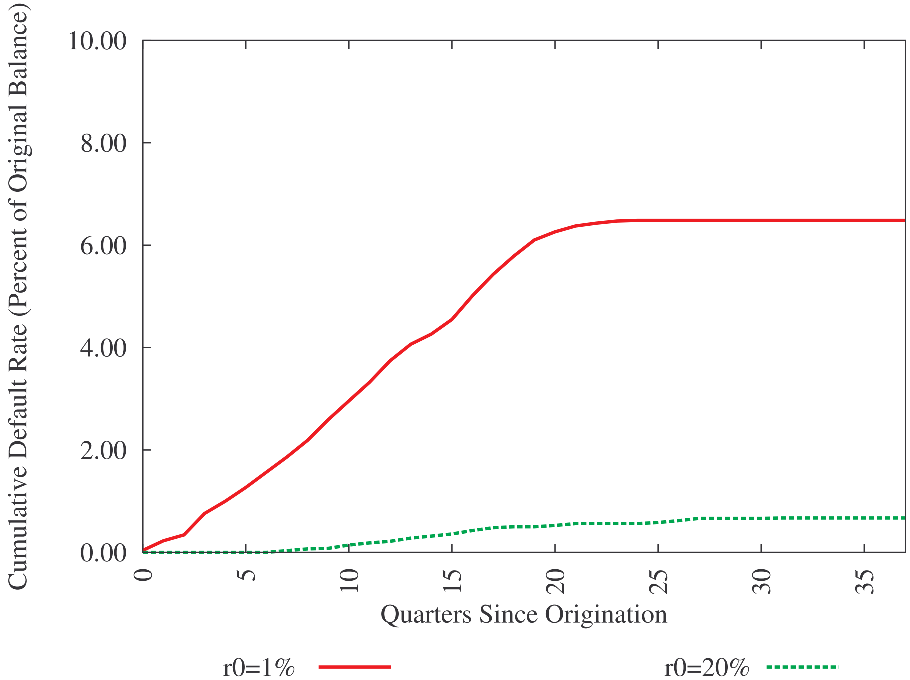


Figure 11: Simulated Default Rates for Different Short-Rate Starting Values



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