# Optimal Capital Structure and the Term Structure of Interest Rates

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*Key words:* Capital Structure; Term Structure; Credit Spreads

*Classification:* G12; G32; G33

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# Optimal Capital Structure and the Term Structure of Interest Rates


**Abstract**

In this paper we study corporate debt values, capital structure, and the term structure of interest rates in a unified framework. We employ numerical techniques to compute the firm's optimal capital structure and the value of its long-term risky debt and yield spreads when the value of the firm's unleveraged assets and the instantaneous default-free interest rate are risk factors. Debt and leveraged firm value are thus explicitly linked to properties of the firm's unleveraged assets, the term structure of default-free interest rates, taxes, bankruptcy costs, payout rates, and bond covenants. The results clarify the relationship between a firm's capital structure and movements in the term structure and other important aspects of the capital structure decision.

# 1 Introduction

This paper considers the optimal capital structure of a firm when the firm chooses both the amount and maturity of its debt. The model extends that of Leland and Toft (1996) to the case where the default-free interest rate follows the square-root diffusion of Cox, Ingersoll and Ross (1985) and the firm can choose arbitrary debt structures. The incorporation of the interest rate as a priced risk factor allows us to study how the shape of the term structure and the correlation of a firm's asset returns with movements in interst rates interact to determine a firm's optimal capital structure. Allowing firms to choose arbitrary debt structures enhances the realism of the model and allows us to analyze how firms adjust their capital structures in response to changes in the term structure.

Our model is also related to that of Longstaff and Schwartz (1995), who analyze the valuation of risky fixed and floating-rate debt in a model with stochastic firm value and interest rates.[1] While similar, our modeling framework differs significantly from theirs: (i) we explicitly consider firm payouts; (ii) the interest rate process that we utilize does not allow negative interest rates and is estimated against Treasury data (producing an estimate of the price of interest rate risk, which is apparently set to zero in Longstaff and Schwartz (1995)); and (iii) we do not assume that firm value is independent of the capital structure of the firm. These additional features come at a cost: we must employ numeric techniques in order to solve our model.

Our key result is that the level of the short rate has a significant impact on the optimal leverage ratio of the firm. For example, if the short rate is three percent, 20 year debt is optimal and the optimal leverage ratio (value of debt divided by unleveraged asset value) is approximately 30 percent. When the short rate rises to 15 percent, 20 year debt is again optimal and the leverage ratio is 60 percent. The logic for why the leverage ratio rises with the risk-free rate is analogous to the logic offered by Longstaff and Schwartz (1995) for how

[1]Huang, Ju and Ou-Yang (2003) examine a firm's optimal capital structure under the Longstaff and Schwartz (1995) model.

the risk-free rate affects credit spreads: in the risk-neutral setting, a higher risk-free rate means all assets are expected to have higher returns. Hence the unleveraged assets of the firm are expected to appreciate at a higher rate and the firm's debt capacity increases.

We also find that the correlation of a firm's asset returns with the risk-free rate potentially plays an important role in determining the risk spread on its debt. Firm's with assets that are negatively correlated with interest rates face lower risk spreads than firms with assets that have zero or positive correlation with interest rates. However, optimal leverage ratios show very little variation across different settings of the correlation parameter. In effect, the firm is a portfolio of a long position in an asset and a short position in bonds. While the mix of assets and bonds that maximizes the value of this portfolio does not change, firms with positive asset correlations are unable to diversify as well and hence face higher risk spreads. An implication of this is that the model will do a better job at explaining cross-sectional variation in risk spreads than capital ratios, though in this version of the paper we do not test any of the empirical implications of our model.

Higher interest-rate volatility boosts risk spreads, principally at longer maturities. However, under plausible settings of this parameter, interest rate volatility plays a minor role in determining the optimal leverage ratio. We also examine how the rate of mean reversion in interest rates affects optimal leverage. While our interest rate model is estimated against a long time span of observed data, and these data indicate only weak mean-reversion, we consider cases where mean reversion is stronger as a way of better understanding the operation of the model, and as a way of gaining insight into a more complicated model that might allow the rate of mean reversion to change over time. Naturally, we find that when mean reversion is very strong, there is far less sensitivity of the optimal capital structure to the initial level of the short rate—the optimal capital structure is that prevailing at the mean interest rate, since firms expect rates to revert back to the mean rapidly. The price of interest rate risk is also an important determinant of leverage. A higher price of interest rate risk is associated with a higher term premium and hence a higher value of the tax shield associated with longer-term debt.

Finally, we examine how optimal capital structure varies under the parameters that define the process for the firm's unleveraged asset value. There is little news here; our results are in line with previous models that considered capital structure where the firm's assets follow the log-normal diffusion that we use here.

This paper is organized as follows. Section 2 lays out our valuation framework and our key assumptions. Section 3 discusses our comparative statics exercises designed to elucidate the relationship between optimal leverage, risk spreads, and interest rates. Section 4 concludes with an idea of what future revisions of this paper will contain.

# 2 Valuation Framework

In this section, we extend the Leland and Toft (1996) model to incorporate a stochastic interest rate and arbitrary debt structures. We then use this framework to analyze the capital structure decision of the firm. Our basic assumptions parallel those of Merton (1974), Black and Cox (1976), Longstaff and Schwartz (1995), and Leland and Toft (1996); we outline our assumptions below.

Assumption 1: *Let $V$ denote the value of a firm's unleveraged assets. We assume that $V$ evolves according to:*

$$dV = (\mu(V,t) - \alpha)V\,dt + \sigma_V V\,dW_V, \tag{1}$$

*where $\mu(V,t)$ is the expected rate of return on the firm's assets, $\alpha$ is the fraction of the firm paid out to security holders, and $dW_V$ is a standard Wiener process.*

This assumption is identical to that in Leland and Toft (1996), and differs from that in Longstaff and Schwartz (1995) only by the inclusion of $\alpha$, which explicitly parameterizes the payout policy of the firm. As in previous studies, we do not allow $\alpha$ to vary with the decisions of the firm, thus abstracting away from broader issues associated with how the investment decisions of the firm might change with capital structure. The implications of the firm's investment policy for its value are proxied by the term $\mu(V, t) - \alpha$.

ASSUMPTION 2: *Let $r$ denote the short-term riskless (default-free) interest rate. We assume that $r$ evolves according to:*

$$dr = \gamma(\theta - r)dt + \sigma_r \sqrt{r} dW_r, \tag{2}$$

*where $\gamma$ is the rate of mean-reversion to $\theta$, the long-term average interest rate, $\sigma_r$ is the volatility of the risk-free rate, and $dW_r$ is a standard Wiener process. The instantaneous correlation between $dW_V$ and $dW_r$ is $\rho dt$. The price of interest-rate risk is given by $\lambda$.*

This assumption about the term structure dynamics is drawn from Cox et al. (1985). This process differs in important respects from that employed in Longstaff and Schwartz (1995). Negative interest rates are ruled out, and the volatility of interest rates is proportional to the level of interest rates. Moreover, we will explicitly consider the price of interest rate risk; incorporating a stochastic interest rate process but then setting the price of interest rate risk to zero understates the importance of the interest rate factor in all respects.

We estimate the coefficients of equation (2) using the methodology of Pearson and Sun (1989) and 3-month and 10-year constant-maturity Treasury rates for the period 1965-2003. The estimation results are displayed in Table 1. As can be seen, the data are consistent with weak mean-reversion to a long-term average short rate of approximately 5.7 percent. Short rate volatility is about six percent per annum, and $\lambda = -7.6$, indicating a positive term premium.

ASSUMPTION 3: *Following Black and Cox (1976) and Longstaff and Schwartz (1995), we assume that the firm defaults on its obligations when $V \leq V_B$, where $V_B$ is a constant lower threshold. We assume that $V_B = P$, the face value of the firm's debt.*

In this version of the paper, we hold the bankruptcy boundary fixed purely for technical convenience. As shown in Leland and Toft (1996), the bankruptcy-triggering value $V_B$ can be endogenized by invoking a smooth-pasting condition. In their single-factor model (interest rates fixed), $V_B$ is found by solving the equation:

$$\frac{\partial E(V; V_B, T)}{\partial V} = 0 \text{ at } V = V_B, \tag{3}$$

where $T$ is the maturity of the debt. In future revisions of the paper, we plan to implement an analagous condition for our model.

ASSUMPTION 4: *The corporate tax rate is $\tau$ and the interest paid on debt is tax deductible.*

ASSUMPTION 5: *Following Longstaff and Schwartz (1995) and Leland and Toft (1996), if the firm declares bankruptcy, debt-holders receive $1 - w$ times the face value of the debt, and equity holders receive nothing.*

As is well known in the finance literature, in the absence of taxes and bankruptcy costs the value of a firm is independent of the capital structure of the firm (Modigliani and Miller (1958)). In our model the key element driving the capital structure decision the tradeoff between the value of the tax shield offered by debt and the potential costs of bankruptcy. Like Leland and Toft (1996), we assume that the tax shield is lost when firm value falls to the point $V_\tau$ such that $\alpha V_\tau < C$, that is, the firm's cash flow is insufficient to pay the coupon in its debt.

ASSUMPTION 6: *We assume there are no arbitrage opportunities and that securities trade in continuous time.*

Under these assumptions, we invoke the usual arguments to derive the partial differential equation that defines the price $F(V, r, t; T)$ of any security with maturity $T$ and payoffs contingent on the values of $V$ and $r$:

$$\frac{1}{2}\sigma_V^2 V^2 F_{VV} + \rho\sigma_V\sigma_r V\sqrt{r}F_{Vr} + \frac{1}{2}\sigma_r^2 r F_{rr} + (r-\alpha)VF_V + (\gamma(\theta - r) - \lambda r)F_r - rF + c = F_t, \tag{4}$$

where $c \geq 0$ is the cash flow, if any, paid by the security, and subscripts on $F$ denote partial derivatives. The value of any derivative security is found by solving (4) subject to appropriate boundary conditions.

We are principally concerned with the valuation of two linked cash flows: the payments on the firm's defaultable debt securities and the value of the tax shield provided by the interest paid on these debt securities. We treat the tax shield as a security with cash flows contingent on the firm not defaulting on its debt; hence the values of the debt and tax shield are linked and must be solved for simultaneously.

The firm is assumed to issue standard straight coupon debt promising to pay a coupon $C$ each period and to return the principal $P$ at maturity. In contrast to Leland and Toft (1996), we do not consider a stationary debt structure where the firm makes a constant payment of principal and interest each period. Rather, we consider the more realistic case where firms issue debt in discrete tranches. This has a number of implications for the analysis. First, the placement of the principal payment in time has a significant impact on the risk of bankrupty and hence the expected present value of bankruptcy costs. In general, a firm is always better off to push the principal payment far into the future. Second, the consideration of multiple tranches of debt requires the simultaneous solution of the valuation equation for each tranche of debt. However, the case of multiple debt tranches will allow us to analyze how firms dynamically adjust their capital structure. We will consider this issue in future revisions of the paper.

The value of the firm's debt, $D(V, r, t; V_B, T)$ is the solution to equation (4) subject to the following boundary conditions:

$$D(0, r, t; V_B, T) = 0; \tag{5}$$

$$\lim_{r\to\infty} D(V, r, t; V_B, T) \rightarrow 0; \tag{6}$$

$$D(V, r, t; V_B, T) = (1 - w)V \text{ for } V \leq V_B; \tag{7}$$

$$D(V, r, T; V_B, T) = P \text{ for } V_T > V_B. \tag{8}$$

Boundary condition (5) states that the debt is worthless if the value of the firm's assets falls to zero; under our model, zero is an absorbing boundary on firm value. Condition

(6) captures the notion that at very high interest rates future cash flows are worthless.[2] Condition (7) is the bankruptcy condition, and the boundary condition (8) states that the firm will return the principal amount $P$ at maturity as long as its assets are worth more than $V_B$; otherwise, condition (7) applies.

The value of the tax shield, $S(V, r, t; V_B, T)$ is the solution to equation (4) subject to the following boundary conditions:

$$S(0, r, t; V_B, T) = 0; \tag{9}$$

$$\lim_{r\to\infty} S(V, r, t; V_B, T) \rightarrow 0; \tag{10}$$

$$S(V, r, t; V_B, T) = 0 \text{ for } V \leq V_B; \tag{11}$$

$$S(V, r, T; V_B, T) = \tau C \text{ for } V_T > V_B.. \tag{12}$$

Boundary conditions (9) and (10) are completely analagous to (5) and (6). Condition (11) says that the value of the tax shield falls to zero in bankruptcy. Condition (12) captures an important aspect of the debt structures we study: when the debt matures, the firm is *not* assumed to issue more debt. Hence on the maturity date of the firm's debt its tax shield is only worth the tax rate on the final coupon payment.[3]

Under this framework, the value of the firm, $F(V, r, t; V_B, T)$ is given by:

$$F = V + D + S, \tag{13}$$

where we have dropped function arguments for brevity. The value of the firm is equal to its unleveraged asset value, $V$, plus the value of its defaultable debt, $D$, plus the value of the tax shield, $S$. Noting that the costs of bankruptcy are incorporated into the value of debt, equation (13) is completely analagous to the standard decomposition of leveraged firm value

[2]Technically, there is a boundary condition for zero interest rates. Zero is an inaccessible bound for our interest rate process; we handle this numerically by assuming that at zero interest rates, interest rates move up by the drift rate of the interest rate process. We note that this has an undetectable effect on the security prices for realistic values of $r$ and $V$.

[3]In future revisions of the paper we will consider the case of debt rollovers.

into unleveraged assets, debt, tax shield, and bankruptcy costs. The equity value of the firm is given by $E = F - D$.

We solve for the values of the debt and the tax shield using standard numerical techniques. This involves discretizing the PDE and applying a finite difference algorithm; see Gourlay and McKee (1977) and Downing, Stanton and Wallace (2005) for details.

# 3 Optimal Capital Structure

In this section we conduct comparative statics experiments on our model. We compute the optimal leverage ratio (hereafter, $\frac{D}{V}$, the value of the firm's debt divided by the unlevered asset value) for different combinations of the key variables of the model.[4] For each combination of input parameters, we report the par coupon ($C$), leveraged firm value ($F$), and the risk spread at the optimal leverage ratio. We measure the risk spread as the difference between the par coupon and the par coupon on equal-maturity risk-free bonds.[5] To facilitate these comparisons, we consider a "base case" scenario in Table 2 with the following parameter settings: the bankruptcy cost fraction $w = 0.50$, the corporate tax rate $\tau = 0.35$, and the firm payout ratio $\alpha = 0.07$. We assume the bankruptcy trigger $V_B$ is equal to the face value of the debt.

The initial value of the firm's underlying assets is normalized to one ($V = 1$ at $t = 0$); this is without loss of generality since the implications of the debt load only depend on the par value of the debt relative to the value of the firm's assets. Under the base case scenario, the volatility of unlevered firm value $\sigma_v = 0.2$. As noted earlier, the interest rate process is parameterized at the values shown in Table 1. Finally, in the base case we set the correlation between risk-free interest rate and unlevered firm value $\rho = 0$.

[4] We will always work with par-valued debt since this eases comparisions and because firms typically issue debt at par; hence at the time point we consider—when the debt is first issued—there is no difference between $D$ and $P$ and we use the two interchangeably when no confusion will arise.

[5] In future revisions of the paper we will duration-match the securities; here we use maturity because it is simpler, but this likely matches the firm's risky debt to a risk-free bond that is "too long" in duration terms and hence the risk-spreads will be under-stated when the term structure is upward sloping, and vice-versa.

Table 2 reports the optimal leverage ratio, the firm's leveraged value at this point, and the par coupon and risk spread on the firm's debt. The initial risk-free interest rate, $r_0$, is alternately set to three percent, six percent and 15 percent. At each level of the risk-free rate, we report results for debt maturities, $T$, equal to one year, five years, 10 years and 20 years. Observe that for any given initial default-free interest rate, the leverage ratio which maximizes the value of the firm falls as the maturity of the debt increases. This result differs from Leland and Toft (1996), where the optimal leverage ratio rises with maturity. Under their model, the bankruptcy trigger, $V_B$, falls as the maturity of the bond increases, which implies lower bankruptcy risk for longer maturity debt. In contrast, we assume that bondholders have the right to force the firm into bankruptcy when the firm's value falls below the face value of its debt—the bankruptcy trigger $V_B$ is constant in our model. All else equal, as the maturity of the debt lengthens, the likelihood of ending up in default rises, reducing the optimal debt load.

It is obvious from Table 2 that for any given maturity, a higher initial value of the risk-free rate corresponds to higher leverage. The rate of mean-reversion in interest rates is weak, so a higher initial risk-free rate implies a fairly long period of greater expected appreciation in asset values and thus a higher debt capacity. We also see the reflection of the shape of the term structure in the firm's par coupon term structure. For low $r_0$, the par coupon rises with maturity $T$, since the risk-free term structure is upward-sloping. For moderate-to-high $r_0$, the risk-free term structure is "hump" shaped, as is the firm's par-coupon term structure. Finally, the risk-free and par-coupon term structures are downward sloping for very high $r_0 = 15$ percent, reflecting the assumed mean-reversion in interest rates.

Figure 1 examines risk spreads as a function of maturity, $T$, for firms with leverage ratios of 40 percent (solid line), 50 percent (short dashed line), 60 percent (medium dashed line), and 70 percent (long dashed line). We use the parameters from the base case. The initial risk-free interest rate $r_0$ is set to 3 percent (Panel A), 6 percent (Panel B), and 15 percent (Panel C). The panels exhibit similar patterns of risk spreads: for high or moderate leverage, risk spreads are high with a distinct hump shape; for firms with low leverage levels, risk spreads are low and rise nearly monotonically with maturity. At a given leverage ratio, risk spreads decrease as the initial risk-free rate rises. This negative correlation between spreads and interest rates has been verified empirically by Longstaff and Schwartz (1995) using Moody's corporate bond yield data. We should note that the risk spreads here are significantly larger than those in Longstaff and Schwartz (1995) and Leland and Toft (1996) for similar parameter settings, reflecting the fact that we are treating interest rates as a *priced* risk factor.

## 3.1 Capital Structure and The Term Structure

We now turn to the behavior of the optimal leverage ratio as we vary the coefficients of the assumed term structure process. The correlation between the stochastic processes for firm value and interest rates is characterized by the parameter $\rho$. Longstaff and Schwartz (1995) suggest that differences in credit spreads across industries and sectors might be related to cross-sectional variation in the correlation of asset returns and interest rates. To examine how such differences might explain the cross-sectional variation in capital structures and risk spreads, we alternately set the correlation coefficient $\rho$ to -0.75, 0.00, and 0.75, and hold all of the other parameters at their base-case values.

Table 3 reports calculations of optimal leverage ratios, firm values, par coupons, and risk spreads for the three different correlation settings. Holding the initial risk-free rate, $r_0$, and maturity, $T$, fixed, changes in the correlation coefficient $\rho$ have little effect on the optimal leverage ratio. As shown in Figure 2 for the case of 20 year debt ($T = 20$) and an initial risk-free interest rate of 15 percent ($r = 15\%$), this is because changes in correlation produce greater convexity in the relationship between firm value and leverage, but do not move the optimal leverage point much. However, the correlation setting *does* affect risk spreads, particularly at longer maturities. Thinking of a firm as a portfolio long in assets and the tax shield (and short defaultable bonds), this result says that the optimal portfolio weights between assets and bonds do not change much, but the diversification benefit of adding debt is not as large when the correlation between assets and bonds is positive; hence risk spreads are higher relative to the case where the correlation coefficient is zero or negative.

The volatility of interest rates is governed by the coefficient $\sigma_r$. Table 4 reports the relationship between interest rate volatility and optimal leverage. We use the parameters of the base case and alternately set $\sigma_r$ to 0.03, 0.06 and 0.09. Table 4 shows that, holding $r$ fixed, higher interest-rate volatility has little effect on leverage or risk spreads at short maturities, but at longer maturities risk spreads widen substantially as interest-rate volatility increases. The optimal leverage ratio is largely unaffected by changes in interest-rate volatility, though firm values do decline somewhat as $\sigma_r$ increases.

The rate at which interest rates revert to the mean is given by the parameter $\gamma$. Our estimates of this parameter suggest a relatively weak rate of mean reversion, and hence the term structure model at the estimated parameter values cannot produce steeply sloped term structures. This may in part explain why the model is rejected, and may also indicate that the rate of mean reversion moves through time—that is, that investors' convictions regarding the likelihood of rising or falling short rates may change over time, motivated, for example, by the effect of monetary policy on interest rate expectations. To better understand the operation of our model as well as to gain some insight into how a more complicated model with time-varying mean-reversion might behave, we consider some alternative settings of the mean-reversion coefficient.

We use the parameters of the base case and set the rate of mean reversion $\gamma$ to 0.13131, 0.50 and 1.0. Table 5 reports the relationship between the optimal leverage ratio and the rate of mean reversion $\gamma$. For any given maturity, the increase of the rate of mean reversion has more effect on the optimal leverage decision when the initial risk-free interest rate is high, since obviously it is for these values that mean reversion is an issue. Naturally enough, high rates of mean reversion produce a situation where the current setting of the risk-free rate, $r$, has little effect on capital structure. Firms expect interest rates to move quickly back to the mean, and hence the optimal capital structures resemble those at the mean interest rate of $r = 6$ percent.

The price of interest rate risk is captured by $\lambda$, which we estimate to be equal to -0.0758. As noted earlier, other studies have typically set $\lambda = 0$. In order to understand how that restriction affects the results, as well as to better understand how our model operates, we consider alternative settings for $\lambda$ equal to 0.0, -0.0758 and -0.16. Observe from Table 6 that, at any given initial risk-free interest rate and maturity combination, both firm value at the optimal leverage ratio and the par coupon rise with the price of interest rate risk. The reason we find this result is that the price of interest rate risk does not affect the probability of bankruptcy, but it does boost longer-term interest rates and hence par coupons. Hence the value of the tax shield is higher but the expected present value of bankruptcy costs is unchanged, leading firms to choose higher levels of leverage. This effect is most pronounced when the initial risk-free interest rate, $r$, is relatively high and the issuance maturity, $T$, is greater than five years.

## 3.2 Capital Structure and Firm Characteristics

For completeness, we consider how capital structure is related to firm characteristics. Since we employ the same process for firm value as Leland and Toft (1996), there are few surprises here; the point of this brief section is to establish that, in these dimensions, our model is consistent with previous studies.

The volatility of the firm's underlying assets is given by $\sigma_v$. Table 7 reports the relationship between optimal leverage ratio and firm risk $\sigma_v$. We use the parameters of the base case and set the volatility of unlevered firm value to 0.1, 0.2 and 0.4. Observe that the optimal leverage ratio, as well as firm value, falls as volatility of the firm's assets rises, as expected.

Because bankruptcy risk is cumulative over time, firms with higher asset volatilities see less benefit to long-term debt than firms with safer assets. For example, when initial risk-free interest rate $r = 3$ percent, firms with the asset volatility $\sigma_v = 0.4$ increase their market value from \$1.0031 with one-year debt to \$1.0134 with 20-year debt, an increase of about one percent, whereas a firm with asset volatility $\sigma_v = 0.1$ sees its value jump from \$1.0074 with one-year debt to \$1.0859 dollars with 20-year debt, an increase of about eight percent. This is consistent with the empirical evidence of Stohs and Mauer (1996). They report that larger, less risky firms use relatively longer-term debt than smaller, riskier firms.

In our model, we assume that the bankruptcy cost, $w$, is the fraction of the market value of the firm that will be lost in bankruptcy. As noted earlier, the magnitude of bankruptcy costs is a key determinant of the optimal leverage ratio, since the trade-off between tax savings and bankuptcy costs is central to the capital structure decision. Moreover, it is well known that bankruptcy costs (conversely, recovery rates) exhibit substantial variation through time and across industries. In order to shed more light on the relationship between the magnitude of $w$ and the leverage ratio and risk spreads in our model, Table 8 reports the usual set of results for $w$ set to 0.0, 0.50, and 1.00. As expected, at any given initial risk-free interest rate and maturity, optimal leverage falls as bankruptcy costs rise. The effect of higher bankruptcy costs is most pronounced for combinations of $r$ and $T$ that produce the highest risk of bankruptcy. Hence we see substantial variation in the optimal leverage ratio and risk spreads with $w$ for low interest rates and long maturities.

Table 9 examines the effects of varying a firm's payout ratio, $\alpha$. We set the firm payout ratio $\alpha$ to 0.02, 0.07, and 0.10. At any given initial risk-free interest rate and maturity combination, an increase in the payout ratio causes the optimal leverage ratio as well as firm value to fall. The reason for this result is that, if firms pay out at a faster rate, their assets grow at a slower rate—a pecking-order like result that firms will maximize their growth and hence value by growing through retained earnings first and then raising external capital.

# 4 Conclusion

In this paper, we made some first steps in understanding the relationship between a firm's capital structure and the term structure of interest rates when we consider interest rates as a priced risk-factor and where the firm issues a single tranche of straight debt. We found that the level of the risk-free rate has a large impact on the firm's optimal leverage level, with firm's leveraging more aggressively when interest rates are high. Firms with assets negatively correlated with interest rates face significantly lower risk-spreads; we offered a diversification-like argument to support these results. In regimes marked by faster interest-rate mean-reversion, our model predicts less cross-sectional dispersion in capital structures due to interest rate variation: firms expect a return to the mean in short order, hence all capital structures are "at-the-mean" capital structures. Interest rate volatility has a modest effect on capital structure and risk spreads, principally exerting an upward influence at longer maturities.

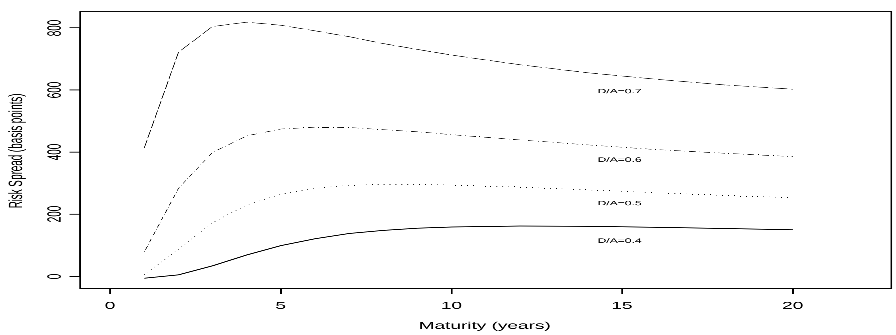
This model has a number of shortcomings that we plan to address in future revisions of the paper. First, the model predicts that firms will always choose the longest-maturity debt, a shortcoming our model shares with closely related models such as Longstaff and Schwartz (1995) and Leland and Toft (1996). The principal reason for this prediction is that, at longer maturities both the value of the tax shield is higher (there are more payments) and the expected present discounted value of bankruptcy costs is lower (costs are pushed further into the future). There are at least two interesting ways to try and break this relationship. First, we plan to consider dynamic debt policy where a firm issuing short-term debt is assumed to roll the debt over at its maturity date. The aim is to examine how matching the time period over which the tax shield is earned might under certain circumstances swing optimal leverage toward short-term debt. Second, we plan to explicitly consider agency costs. As noted in Leland and Toft (1996), agency costs could explain why some firms issue short-term debt: short-term debt induces market-discipline on firms by forcing them to re-contract with debtholders at a higher frequency. The challenge is to make this connection explicit in the model.

Another important extension that we are currently working on is to consider the case of multiple tranches of debt. Obviously, when firms issue debt they do so in a way that is perceived to be optimal, but then the world changes and the firms are forced into sub-optimal capital structures. A model that can handle several tranches of debt is capable of examining how firms dynamically adjust their capital structures as state variables change. Finally, we plan to examine the case for callable debt under different term structures.

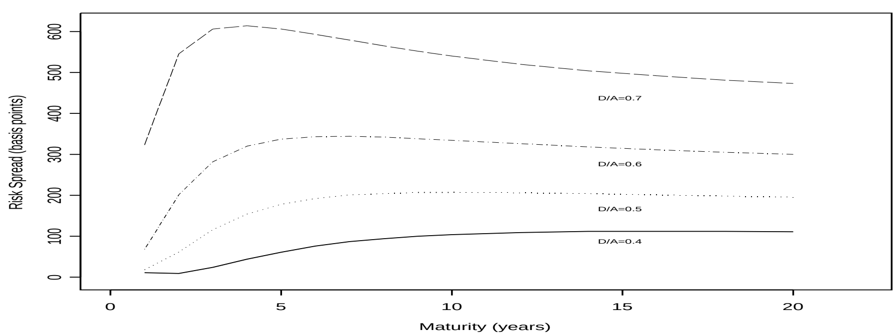
## Figure 1: The Term Structure of Risk Spreads

The figure shows risk spreads, measured as the basis point difference between the par coupon on the firm's debt and the par coupon yield on an equal-maturity risk-free bond, as a function of issuance maturity $T$ (in years) for firms with leverage ratios of 40 percent (solid line), 50 percent (short dashed line), 60 percent (medium dashed line), and 70 percent (long dashed line). Panel A displays risk spreads for an initial risk-free rate of three percent; Panels B and C show risk spreads for initial risk-free rates of six and 15 percent, respectively. The other parameters are held at their base case values.

### Panel A: $r = 3$ percent




### Panel B: $r = 6$ percent




### Panel C: $r = 15$ percent

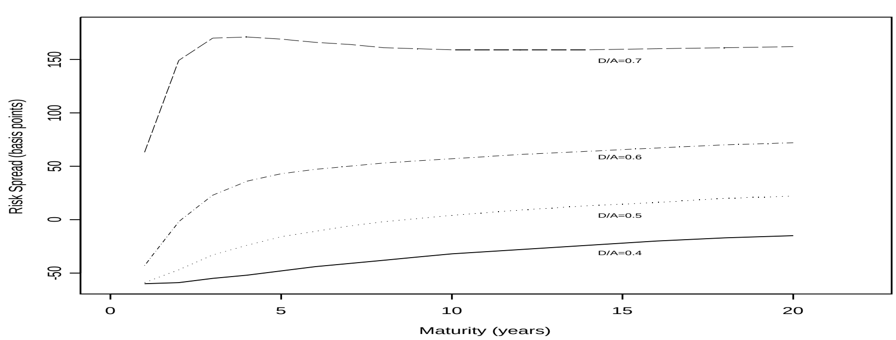


Figure 2: Firm value as a function of leverage ratios.

The lines plot levered firm value (in dollars) against leverage ratios $D/V$. The solid line shows how firm value varies with leverage under the base case with a setting of $\rho = -0.75$, the short dashed line shows $\rho = 0.0$, and the medium dashed line shows $\rho = 0.75$. The maturity of the firm's debt is held constant at 20 years.

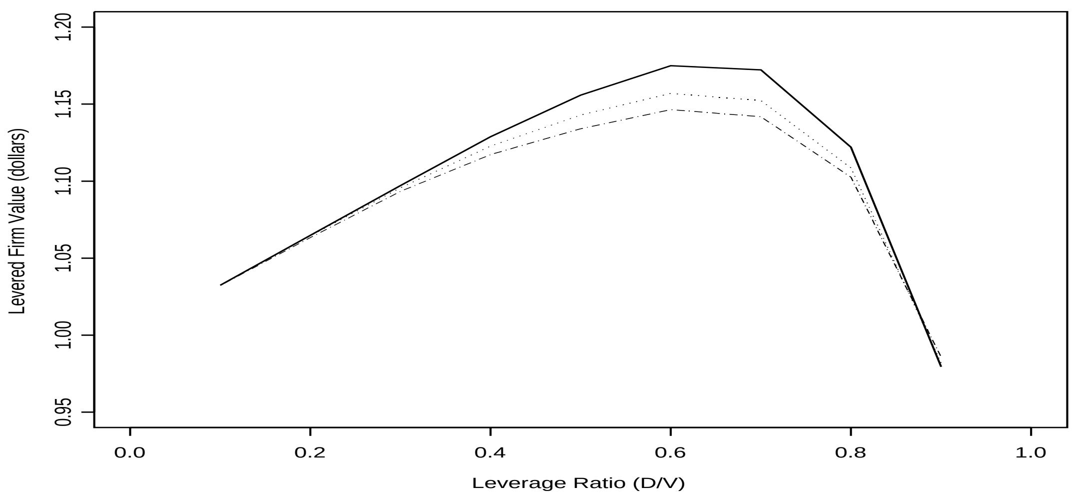


Table 1: Maximum-Likelihood Estimates of Interest-Rate Process Coefficients

The table displays maximum-likelihood estimates of the coefficients of the interest rate process

$$dr = (\gamma(\theta - r) - \lambda r)dt + \sigma_r \sqrt{r} dW_r,$$

where $\gamma$ is the rate of reversion to the long-term mean $\theta$, $\lambda$ is the price of interest-rate risk, and $\sigma_r$ is the volatility of interest rates. The data are daily 3-month and 10-year constant-maturity Treasury rates for the period January 4, 1965 through December 31, 2003, for a total of 9,734 daily observations. The estimates are made using the methodology of Pearson and Sun (1989).

| Coefficient | Estimate | Std. Err. |
|---|---|---|
| $\gamma$ | 0.13131 | |
| $\theta$ | 0.05740 | |
| $\lambda$ | -0.07577 | |
| $\sigma_r$ | 0.06035 | |
| N=9,734 | | |

Table 2: Optimal Capital Structure under the Base Case

This table computes the optimal leverage ratio ($D/V$), the par coupon on the firm's debt, and firm value at the optimal leverage ratio, given initial risk-free interest rates $r$ and alternative choices of maturities $T$ ranging from one year to 20 years. The par coupon and the initial value of risk-free interest rate $r$ are expressed in percent; the risk spreads, measured as the difference between the par coupon on the firm's bond and the par coupon yield on an equal-maturity risk-free bond, is displayed in basis points; the firm value is expressed in dollars. The bankruptcy cost fraction $w = 0.50$, the corporate tax rate $\tau = 0.35$, and the firm payout ratio $\alpha = 0.07$. The initial value of the firm's underlying assets $V_0 = \$1$. The volatility of unlevered firm value $\sigma_v = 0.20$. The rate of mean reversion in interest rate $\gamma = 0.1313$, the long-term interest rate mean $\theta = 0.0574$, risk-adjustment to interest rate $\lambda = -0.0758$, and the volatility of short rate $\sigma_r = 0.0603$. The correlation between riskfree interest rate and unlevered firm value $\rho = 0.00$. It is assumed that the bankruptcy trigger $V_B$ is equal to $P$, the face value of the firm's debt.

| $r$ | $T$ | $D/V$ | Firm Value | Par Coupon | Risk-free Par Coupon | Risk Spread |
|---|---|---|---|---|---|---|
| 3 | 1 | 0.5 | 1.0052 | 3.40 | 3.34 | 6 |
| | 5 | 0.3 | 1.0188 | 4.46 | 4.35 | 11 |
| | 10 | 0.3 | 1.0332 | 5.83 | 5.25 | 58 |
| | 20 | 0.3 | 1.0514 | 6.98 | 6.23 | 75 |
| 6 | 1 | 0.6 | 1.0109 | 7.01 | 6.33 | 68 |
| | 5 | 0.4 | 1.0343 | 7.69 | 7.08 | 61 |
| | 10 | 0.4 | 1.0518 | 8.66 | 7.62 | 104 |
| | 20 | 0.3 | 1.0716 | 8.64 | 8.12 | 52 |
| 9 | 1 | 0.6 | 1.0173 | 9.89 | 9.55 | 34 |
| | 5 | 0.5 | 1.0507 | 10.95 | 9.90 | 105 |
| | 10 | 0.4 | 1.0749 | 10.65 | 10.09 | 56 |
| | 20 | 0.4 | 1.0964 | 10.90 | 10.19 | 71 |
| 12 | 1 | 0.7 | 1.0214 | 14.41 | 12.79 | 162 |
| | 5 | 0.6 | 1.0719 | 14.04 | 12.81 | 123 |
| | 10 | 0.6 | 1.1040 | 14.05 | 12.69 | 136 |
| | 20 | 0.6 | 1.1282 | 13.86 | 12.44 | 142 |
| 15 | 1 | 0.7 | 1.0293 | 16.76 | 16.13 | 63 |
| | 5 | 0.6 | 1.0941 | 16.25 | 15.82 | 43 |
| | 10 | 0.6 | 1.1321 | 15.98 | 15.41 | 57 |
| | 20 | 0.6 | 1.1569 | 15.59 | 14.87 | 72 |

Table 3: Asset Value-Interest Rate Correlation and Optimal Capital Structure, Firm Value, and Risk Spreads

This table examines the effect of correlation between firm value and default-free interest rate on firm's leverage decision. For each setting of the intitial risk-free rate, $r$, and maturity, $T$, we compute the optimal leverage ratio $D/V$, firm value, par coupon, and risk spread for three settings of $\rho$, the correlation between movements in the unleveraged value of the firm's assets and interest rates. The other parameters are as in Table 2 above.

| $r$ | $T$ | $\rho$ | $D/V$ | Firm Value | Par Coupon | Risk-free Par Coupon | Risk Spread |
|---|---|---|---|---|---|---|---|
| 3.0 | 1 | -0.75 | 0.5 | 1.0053 | 3.37 | 3.34 | 3 |
| | | 0.00 | 0.5 | 1.0052 | 3.40 | 3.34 | 6 |
| | | 0.75 | 0.5 | 1.0052 | 3.42 | 3.34 | 8 |
| | 5 | -0.75 | 0.3 | 1.0194 | 4.35 | 4.35 | 0 |
| | | 0.00 | 0.3 | 1.0188 | 4.46 | 4.35 | 11 |
| | | 0.75 | 0.3 | 1.0181 | 4.57 | 4.35 | 22 |
| | 10 | -0.75 | 0.3 | 1.0370 | 5.44 | 5.25 | 19 |
| | | 0.00 | 0.3 | 1.0332 | 5.83 | 5.25 | 58 |
| | | 0.75 | 0.3 | 1.0305 | 6.20 | 5.25 | 95 |
| | 20 | -0.75 | 0.3 | 1.0596 | 6.41 | 6.23 | 18 |
| | | 0.00 | 0.3 | 1.0514 | 6.98 | 6.23 | 75 |
| | | 0.75 | 0.3 | 1.0472 | 7.49 | 6.23 | 126 |
| 6.0 | 1 | -0.75 | 0.6 | 1.0112 | 6.91 | 6.33 | 58 |
| | | 0.00 | 0.6 | 1.0109 | 7.01 | 6.33 | 68 |
| | | 0.75 | 0.6 | 1.0106 | 7.11 | 6.33 | 78 |
| | 5 | -0.75 | 0.4 | 1.0371 | 7.38 | 7.08 | 30 |
| | | 0.00 | 0.4 | 1.0343 | 7.69 | 7.08 | 61 |
| | | 0.75 | 0.4 | 1.0320 | 8.00 | 7.08 | 92 |
| | 10 | -0.75 | 0.4 | 1.0591 | 8.04 | 7.62 | 42 |
| | | 0.00 | 0.4 | 1.0518 | 8.66 | 7.62 | 104 |
| | | 0.75 | 0.3 | 1.0485 | 8.29 | 7.62 | 67 |
| | 20 | -0.75 | 0.4 | 1.0832 | 8.46 | 8.12 | 34 |
| | | 0.00 | 0.3 | 1.0716 | 8.64 | 8.12 | 52 |
| | | 0.75 | 0.3 | 1.0668 | 9.15 | 8.12 | 103 |
| 15.0 | 1 | -0.75 | 0.7 | 1.0302 | 16.49 | 16.13 | 36 |
| | | 0.00 | 0.7 | 1.0293 | 16.76 | 16.13 | 63 |
| | | 0.75 | 0.7 | 1.0285 | 17.02 | 16.13 | 89 |
| | 5 | -0.75 | 0.7 | 1.0967 | 16.58 | 15.82 | 76 |
| | | 0.00 | 0.6 | 1.0941 | 16.25 | 15.82 | 43 |
| | | 0.75 | 0.6 | 1.0895 | 16.85 | 15.82 | 103 |
| | 10 | -0.75 | 0.7 | 1.1403 | 15.84 | 15.41 | 43 |
| | | 0.00 | 0.6 | 1.1321 | 15.98 | 15.41 | 57 |
| | | 0.75 | 0.6 | 1.1247 | 16.81 | 15.41 | 140 |
| | 20 | -0.75 | 0.7 | 1.1723 | 15.15 | 14.87 | 28 |
| | | 0.00 | 0.6 | 1.1569 | 15.59 | 14.87 | 72 |
| | | 0.75 | 0.6 | 1.1463 | 16.59 | 14.87 | 172 |

Table 4: Interest Rate Volatility and Optimal Capital Structure, Firm Value, and Risk Spreads

This table examines the optimal leverage ratio under different settings of the level of interest rate volatility $\sigma_r$ for given initial risk-free interest rates, $r$, and maturities, $T$. The other parameters are as in Table 2.

| $r$ | $T$ | $\sigma_r$ | $D/V$ | Firm Value | Par Coupon | Risk-free Par Coupon | Risk Spread |
|---|---|---|---|---|---|---|---|
| 3.0 | 1 | 0.03 | 0.5 | 1.0052 | 3.40 | 3.34 | 6 |
| | | 0.06 | 0.5 | 1.0052 | 3.40 | 3.34 | 6 |
| | | 0.09 | 0.5 | 1.0052 | 3.39 | 3.34 | 5 |
| | 5 | 0.03 | 0.3 | 1.0190 | 4.48 | 4.38 | 10 |
| | | 0.06 | 0.3 | 1.0188 | 4.46 | 4.35 | 11 |
| | | 0.09 | 0.3 | 1.0185 | 4.43 | 4.29 | 14 |
| | 10 | 0.03 | 0.3 | 1.0345 | 5.87 | 5.36 | 51 |
| | | 0.06 | 0.3 | 1.0332 | 5.83 | 5.25 | 58 |
| | | 0.09 | 0.3 | 1.0311 | 5.77 | 5.08 | 69 |
| | 20 | 0.03 | 0.3 | 1.0557 | 7.05 | 6.51 | 54 |
| | | 0.06 | 0.3 | 1.0514 | 6.98 | 6.23 | 75 |
| | | 0.09 | 0.3 | 1.0456 | 6.86 | 5.84 | 102 |
| 6.0 | 1 | 0.03 | 0.6 | 1.0109 | 7.01 | 6.33 | 68 |
| | | 0.06 | 0.6 | 1.0109 | 7.01 | 6.33 | 68 |
| | | 0.09 | 0.6 | 1.0109 | 7.01 | 6.33 | 68 |
| | 5 | 0.03 | 0.4 | 1.0349 | 7.71 | 7.13 | 58 |
| | | 0.06 | 0.4 | 1.0343 | 7.69 | 7.08 | 61 |
| | | 0.09 | 0.4 | 1.0334 | 7.66 | 6.98 | 68 |
| | 10 | 0.03 | 0.4 | 1.0545 | 8.68 | 7.79 | 89 |
| | | 0.06 | 0.4 | 1.0518 | 8.66 | 7.62 | 104 |
| | | 0.09 | 0.3 | 1.0486 | 7.82 | 7.35 | 47 |
| | 20 | 0.03 | 0.4 | 1.0775 | 9.31 | 8.51 | 80 |
| | | 0.06 | 0.3 | 1.0716 | 8.64 | 8.12 | 52 |
| | | 0.09 | 0.3 | 1.0653 | 8.41 | 7.59 | 82 |
| 15.0 | 1 | 0.03 | 0.7 | 1.0294 | 16.75 | 16.14 | 61 |
| | | 0.06 | 0.7 | 1.0293 | 16.76 | 16.13 | 63 |
| | | 0.09 | 0.7 | 1.0293 | 16.76 | 16.14 | 62 |
| | 5 | 0.03 | 0.6 | 1.0957 | 16.26 | 15.95 | 31 |
| | | 0.06 | 0.6 | 1.0941 | 16.25 | 15.82 | 43 |
| | | 0.09 | 0.6 | 1.0918 | 16.22 | 15.61 | 61 |
| | 10 | 0.03 | 0.6 | 1.1366 | 16.07 | 15.76 | 31 |
| | | 0.06 | 0.6 | 1.1321 | 15.98 | 15.41 | 57 |
| | | 0.09 | 0.6 | 1.1253 | 15.87 | 14.88 | 99 |
| | 20 | 0.03 | 0.6 | 1.1650 | 15.80 | 15.54 | 26 |
| | | 0.06 | 0.6 | 1.1569 | 15.59 | 14.87 | 72 |
| | | 0.09 | 0.6 | 1.1453 | 15.33 | 13.91 | 142 |

Table 5: The Rate of Mean Reversion of Interest Rates and Optimal Capital Structure, Firm Value, and Risk Spreads

This table reports the relationship between optimal leverage ratio and rate of mean reversion $\gamma$ for different initial risk-free interest rates and debt maturities. The other parameters are as in Table 2 above.

| $r$ | $T$ | $\gamma$ | $D/V$ | Firm Value | Par Coupon | Risk-free Par Coupon | Risk Spread |
|---|---|---|---|---|---|---|---|
| 3.0 | 1 | 0.1 | 0.5 | 1.0052 | 3.40 | 3.34 | 6 |
| | | 0.5 | 0.5 | 1.0059 | 3.80 | 3.71 | 9 |
| | | 1.0 | 0.5 | 1.0067 | 4.22 | 4.12 | 10 |
| | 5 | 0.1 | 0.3 | 1.0188 | 4.46 | 4.35 | 11 |
| | | 0.5 | 0.3 | 1.0229 | 5.29 | 5.25 | 4 |
| | | 1.0 | 0.3 | 1.0244 | 5.62 | 5.61 | 1 |
| | 10 | 0.1 | 0.3 | 1.0332 | 5.83 | 5.25 | 58 |
| | | 0.5 | 0.3 | 1.0390 | 6.29 | 5.88 | 41 |
| | | 1.0 | 0.3 | 1.0394 | 6.34 | 5.94 | 40 |
| | 20 | 0.1 | 0.3 | 1.0514 | 6.98 | 6.23 | 75 |
| | | 0.5 | 0.3 | 1.0539 | 6.92 | 6.23 | 69 |
| | | 1.0 | 0.3 | 1.0525 | 6.84 | 6.10 | 74 |
| 6.0 | 1 | 0.1 | 0.6 | 1.0109 | 7.01 | 6.33 | 68 |
| | | 0.5 | 0.6 | 1.0107 | 6.93 | 6.33 | 60 |
| | | 1.0 | 0.6 | 1.0105 | 6.85 | 6.33 | 52 |
| | 5 | 0.1 | 0.4 | 1.0343 | 7.69 | 7.08 | 61 |
| | | 0.5 | 0.4 | 1.0312 | 7.24 | 6.62 | 62 |
| | | 1.0 | 0.4 | 1.0294 | 6.99 | 6.35 | 64 |
| | 10 | 0.1 | 0.4 | 1.0518 | 8.66 | 7.62 | 104 |
| | | 0.5 | 0.3 | 1.0450 | 7.12 | 6.73 | 39 |
| | | 1.0 | 0.3 | 1.0424 | 6.79 | 6.37 | 42 |
| | 20 | 0.1 | 0.3 | 1.0716 | 8.64 | 8.12 | 52 |
| | | 0.5 | 0.3 | 1.0601 | 7.48 | 6.80 | 68 |
| | | 1.0 | 0.3 | 1.0555 | 7.15 | 6.38 | 77 |
| 15.0 | 1 | 0.1 | 0.7 | 1.0293 | 16.76 | 16.13 | 63 |
| | | 0.5 | 0.7 | 1.0255 | 15.51 | 14.42 | 109 |
| | | 1.0 | 0.6 | 1.0227 | 12.82 | 12.69 | 13 |
| | 5 | 0.1 | 0.6 | 1.0941 | 16.25 | 15.82 | 43 |
| | | 0.5 | 0.5 | 1.0578 | 11.91 | 10.98 | 93 |
| | | 1.0 | 0.4 | 1.0420 | 9.30 | 8.65 | 65 |
| | 10 | 0.1 | 0.6 | 1.1321 | 15.98 | 15.41 | 57 |
| | | 0.5 | 0.5 | 1.0693 | 11.15 | 9.53 | 162 |
| | | 1.0 | 0.4 | 1.0525 | 9.00 | 7.71 | 129 |
| | 20 | 0.1 | 0.6 | 1.1569 | 15.59 | 14.87 | 72 |
| | | 0.5 | 0.4 | 1.0812 | 9.92 | 8.69 | 123 |
| | | 1.0 | 0.3 | 1.0630 | 8.21 | 7.25 | 96 |

Table 6: The Price of Interest Rate Risk and Optimal Capital Structure, Firm Value, and Risk Spreads

This table reports the relationship between the firm's optimal leverage ratio and the price of interest rate risk, $\lambda$, for different initial risk-free interest rates and debt maturities. The other parameters are as in Table 2 above.

| $r$ | $T$ | $\lambda$ | $D/V$ | Firm Value | Par Coupon | Risk-free Par Coupon | Risk Spread |
|---|---|---|---|---|---|---|---|
| 3.0 | 1 | 0.0000 | 0.5 | 1.0050 | 3.28 | 3.22 | 6 |
| | | -0.0758 | 0.5 | 1.0052 | 3.40 | 3.34 | 5 |
| | | -0.1600 | 0.5 | 1.0054 | 3.53 | 3.48 | 5 |
| | 5 | 0.0000 | 0.3 | 1.0160 | 3.92 | 3.75 | 17 |
| | | -0.0758 | 0.3 | 1.0188 | 4.46 | 4.35 | 11 |
| | | -0.1600 | 0.3 | 1.0225 | 5.21 | 5.18 | 3 |
| | 10 | 0.0000 | 0.3 | 1.0232 | 4.98 | 4.15 | 84 |
| | | -0.0758 | 0.3 | 1.0332 | 5.83 | 5.25 | 58 |
| | | -0.1600 | 0.3 | 1.0470 | 7.25 | 7.01 | 24 |
| | 20 | 0.0000 | 0.3 | 1.0286 | 5.84 | 4.52 | 131 |
| | | -0.0758 | 0.3 | 1.0514 | 6.98 | 6.23 | 75 |
| | | -0.1600 | 0.3 | 1.0781 | 9.14 | 9.07 | 7 |
| 6.0 | 1 | 0.0000 | 0.6 | 1.0104 | 6.80 | 6.16 | 64 |
| | | -0.0758 | 0.6 | 1.0109 | 7.01 | 6.40 | 61 |
| | | -0.1600 | 0.6 | 1.0115 | 7.26 | 6.68 | 58 |
| | 5 | 0.0000 | 0.4 | 1.0277 | 6.85 | 6.06 | 79 |
| | | -0.0758 | 0.4 | 1.0343 | 7.69 | 7.08 | 61 |
| | | -0.1600 | 0.4 | 1.0428 | 8.88 | 8.50 | 38 |
| | 10 | 0.0000 | 0.3 | 1.0392 | 6.56 | 5.94 | 62 |
| | | -0.0758 | 0.4 | 1.0518 | 8.66 | 7.62 | 104 |
| | | -0.1600 | 0.4 | 1.0765 | 10.70 | 10.28 | 42 |
| | 20 | 0.0000 | 0.3 | 1.0483 | 6.91 | 5.79 | 112 |
| | | -0.0758 | 0.3 | 1.0716 | 8.64 | 8.12 | 51 |
| | | -0.1600 | 0.4 | 1.1075 | 12.08 | 11.94 | 15 |
| 15.0 | 1 | 0.0000 | 0.7 | 1.0279 | 16.30 | 15.50 | 80 |
| | | -0.0758 | 0.7 | 1.0293 | 16.76 | 16.13 | 63 |
| | | -0.1600 | 0.7 | 1.0310 | 17.30 | 16.87 | 43 |
| | 5 | 0.0000 | 0.6 | 1.0777 | 14.58 | 13.47 | 111 |
| | | -0.0758 | 0.6 | 1.0941 | 16.25 | 15.82 | 43 |
| | | -0.1600 | 0.7 | 1.1137 | 19.66 | 19.06 | 60 |
| | 10 | 0.0000 | 0.6 | 1.0985 | 13.60 | 11.90 | 170 |
| | | -0.0758 | 0.6 | 1.1321 | 15.98 | 15.41 | 57 |
| | | -0.1600 | 0.8 | 1.1328 | 22.61 | 20.64 | 197 |
| | 20 | 0.0000 | 0.6 | 1.1097 | 12.65 | 10.34 | 232 |
| | | -0.0758 | 0.6 | 1.1569 | 15.59 | 14.87 | 72 |
| | | -0.1600 | 0.8 | 1.1548 | 22.95 | 21.45 | 150 |

Table 7: Asset Volatility and Optimal Capital Structure, Firm Value, and Risk Spreads

This table computes the optimal leverage ratio, firm value, par coupon, and risk spread for different levels of asset volatility, $\sigma_v$. The other parameters are as in Table 2 above.

| $r$ | $T$ | $\sigma_v$ | $D/V$ | Firm Value | Par Coupon | Risk-free Par Coupon | Risk Spread |
|---|---|---|---|---|---|---|---|
| 3.0 | 1 | 0.1 | 0.7 | 1.0074 | 3.38 | 3.34 | 4 |
| | | 0.2 | 0.5 | 1.0052 | 3.40 | 3.34 | 6 |
| | | 0.4 | 0.3 | 1.0031 | 3.48 | 3.34 | 14 |
| | 5 | 0.1 | 0.5 | 1.0303 | 4.48 | 4.35 | 13 |
| | | 0.2 | 0.3 | 1.0188 | 4.46 | 4.35 | 11 |
| | | 0.4 | 0.1 | 1.0059 | 4.61 | 4.35 | 26 |
| | 10 | 0.1 | 0.4 | 1.0532 | 5.34 | 5.25 | 9 |
| | | 0.2 | 0.3 | 1.0332 | 5.83 | 5.25 | 58 |
| | | 0.4 | 0.1 | 1.0094 | 6.19 | 5.25 | 94 |
| | 20 | 0.1 | 0.4 | 1.0859 | 6.51 | 6.23 | 28 |
| | | 0.2 | 0.3 | 1.0514 | 6.98 | 6.23 | 75 |
| | | 0.4 | 0.1 | 1.0134 | 7.37 | 6.23 | 114 |
| 6.0 | 1 | 0.1 | 0.7 | 1.0149 | 6.47 | 6.33 | 14 |
| | | 0.2 | 0.6 | 1.0109 | 7.01 | 6.33 | 68 |
| | | 0.4 | 0.3 | 1.0062 | 6.60 | 6.33 | 27 |
| | 5 | 0.1 | 0.6 | 1.0575 | 7.36 | 7.08 | 28 |
| | | 0.2 | 0.4 | 1.0343 | 7.69 | 7.08 | 61 |
| | | 0.4 | 0.2 | 1.0120 | 8.55 | 7.08 | 147 |
| | 10 | 0.1 | 0.6 | 1.0931 | 8.25 | 7.62 | 63 |
| | | 0.2 | 0.4 | 1.0518 | 8.66 | 7.62 | 104 |
| | | 0.4 | 0.1 | 1.0153 | 8.36 | 7.62 | 74 |
| | 20 | 0.1 | 0.6 | 1.1313 | 8.88 | 8.12 | 76 |
| | | 0.2 | 0.3 | 1.0716 | 8.64 | 8.12 | 52 |
| | | 0.4 | 0.1 | 1.0200 | 9.13 | 8.12 | 101 |
| 15.0 | 1 | 0.1 | 0.9 | 1.0308 | 18.01 | 16.13 | 188 |
| | | 0.2 | 0.7 | 1.0293 | 16.76 | 16.13 | 63 |
| | | 0.4 | 0.4 | 1.0170 | 16.56 | 16.13 | 43 |
| | 5 | 0.1 | 0.9 | 1.1233 | 16.97 | 15.82 | 115 |
| | | 0.2 | 0.6 | 1.0941 | 16.25 | 15.82 | 43 |
| | | 0.4 | 0.3 | 1.0441 | 16.86 | 15.82 | 104 |
| | 10 | 0.1 | 0.9 | 1.1837 | 16.32 | 15.41 | 91 |
| | | 0.2 | 0.6 | 1.1321 | 15.98 | 15.41 | 57 |
| | | 0.4 | 0.3 | 1.0580 | 16.89 | 15.41 | 148 |
| | 20 | 0.1 | 0.9 | 1.2247 | 15.76 | 14.87 | 89 |
| | | 0.2 | 0.6 | 1.1569 | 15.59 | 14.87 | 72 |
| | | 0.4 | 0.3 | 1.0658 | 16.64 | 14.87 | 177 |

Table 8: Bankruptcy Costs and Optimal Capital Structure, Firm Value, and Risk Spreads

This table computes the optimal leverage ratio for different levels of bankruptcy costs, $w$. The other parameters are set as in Table 2 above.

| $r$ | $T$ | $w$ | $D/V$ | Firm Value | Par Coupon | Risk-free Par Coupon | Risk Spread |
|---|---|---|---|---|---|---|---|
| 3.0 | 1 | 0.0 | 0.7 | 1.0086 | 3.66 | 3.34 | 32 |
| | | 0.5 | 0.5 | 1.0052 | 3.40 | 3.34 | 6 |
| | | 1.0 | 0.5 | 1.0048 | 3.51 | 3.34 | 17 |
| | 5 | 0.0 | 0.7 | 1.0385 | 4.89 | 4.35 | 54 |
| | | 0.5 | 0.3 | 1.0188 | 4.46 | 4.35 | 11 |
| | | 1.0 | 0.3 | 1.0173 | 4.63 | 4.35 | 28 |
| | 10 | 0.0 | 0.7 | 1.0585 | 5.47 | 5.25 | 22 |
| | | 0.5 | 0.3 | 1.0332 | 5.83 | 5.25 | 58 |
| | | 1.0 | 0.3 | 1.0242 | 6.43 | 5.25 | 118 |
| | 20 | 0.0 | 0.9 | 1.0367 | 6.25 | 6.23 | 2 |
| | | 0.5 | 0.3 | 1.0514 | 6.98 | 6.23 | 75 |
| | | 1.0 | 0.2 | 1.0342 | 7.01 | 6.23 | 78 |
| 6.0 | 1 | 0.0 | 0.8 | 1.0183 | 7.48 | 6.33 | 115 |
| | | 0.5 | 0.6 | 1.0109 | 7.01 | 6.33 | 68 |
| | | 1.0 | 0.5 | 1.0103 | 6.58 | 6.33 | 25 |
| | 5 | 0.0 | 0.7 | 1.0608 | 7.61 | 7.08 | 53 |
| | | 0.5 | 0.4 | 1.0343 | 7.69 | 7.08 | 61 |
| | | 1.0 | 0.3 | 1.0297 | 7.24 | 7.08 | 16 |
| | 10 | 0.0 | 0.7 | 1.0893 | 8.02 | 7.62 | 40 |
| | | 0.5 | 0.4 | 1.0518 | 8.66 | 7.62 | 104 |
| | | 1.0 | 0.3 | 1.0467 | 8.27 | 7.62 | 65 |
| | 20 | 0.0 | 0.6 | 1.1141 | 8.20 | 8.12 | 8 |
| | | 0.5 | 0.3 | 1.0716 | 8.64 | 8.12 | 52 |
| | | 1.0 | 0.3 | 1.0625 | 9.12 | 8.12 | 100 |
| 15.0 | 1 | 0.0 | 0.9 | 1.0366 | 17.09 | 16.13 | 96 |
| | | 0.5 | 0.7 | 1.0293 | 16.76 | 16.13 | 63 |
| | | 1.0 | 0.7 | 1.0247 | 17.87 | 16.13 | 174 |
| | 5 | 0.0 | 0.9 | 1.0946 | 16.21 | 15.82 | 39 |
| | | 0.5 | 0.6 | 1.0941 | 16.25 | 15.82 | 43 |
| | | 1.0 | 0.6 | 1.0822 | 17.15 | 15.82 | 133 |
| | 10 | 0.0 | 0.8 | 1.1617 | 15.49 | 15.41 | 8 |
| | | 0.5 | 0.6 | 1.1321 | 15.98 | 15.41 | 57 |
| | | 1.0 | 0.6 | 1.1145 | 16.90 | 15.41 | 149 |
| | 20 | 0.0 | 0.7 | 1.1920 | 14.89 | 14.87 | 2 |
| | | 0.5 | 0.6 | 1.1569 | 15.59 | 14.87 | 72 |
| | | 1.0 | 0.6 | 1.1363 | 16.48 | 14.87 | 161 |

Table 9: The Firm's Payout Policy and Optimal Capital Structure, Firm Value, and Risk Spreads

This table computes the optimal leverage ratio for different firm payout rates, $\alpha$. The other parameters are set as in Table 2 above.

| $r$ | $T$ | $\alpha$ | $D/V$ | Firm Value | Par Coupon | Risk-free Par Coupon | Risk Spread |
|---|---|---|---|---|---|---|---|
| 3.0 | 1 | 0.02 | 0.6 | 1.0052 | 3.73 | 3.34 | 39 |
| | | 0.07 | 0.5 | 1.0052 | 3.40 | 3.34 | 6 |
| | | 0.10 | 0.5 | 1.0050 | 3.46 | 3.34 | 12 |
| | 5 | 0.02 | 0.4 | 1.0223 | 4.64 | 4.35 | 29 |
| | | 0.07 | 0.3 | 1.0188 | 4.46 | 4.35 | 11 |
| | | 0.10 | 0.3 | 1.0172 | 4.67 | 4.35 | 32 |
| | 10 | 0.02 | 0.4 | 1.0428 | 5.75 | 5.25 | 50 |
| | | 0.07 | 0.3 | 1.0332 | 5.83 | 5.25 | 58 |
| | | 0.10 | 0.3 | 1.0245 | 6.46 | 5.25 | 121 |
| | 20 | 0.02 | 0.4 | 1.0756 | 6.66 | 6.23 | 43 |
| | | 0.07 | 0.3 | 1.0514 | 6.98 | 6.23 | 75 |
| | | 0.10 | 0.3 | 1.0331 | 7.68 | 6.23 | 145 |
| 6.0 | 1 | 0.02 | 0.6 | 1.0119 | 6.73 | 6.33 | 40 |
| | | 0.07 | 0.6 | 1.0109 | 7.01 | 6.33 | 68 |
| | | 0.10 | 0.5 | 1.0104 | 6.55 | 6.33 | 22 |
| | 5 | 0.02 | 0.5 | 1.0407 | 7.80 | 7.08 | 72 |
| | | 0.07 | 0.4 | 1.0343 | 7.69 | 7.08 | 61 |
| | | 0.10 | 0.3 | 1.0296 | 7.28 | 7.08 | 20 |
| | 10 | 0.02 | 0.5 | 1.0697 | 8.40 | 7.62 | 78 |
| | | 0.07 | 0.4 | 1.0518 | 8.66 | 7.62 | 104 |
| | | 0.10 | 0.3 | 1.0458 | 8.37 | 7.62 | 75 |
| | 20 | 0.02 | 0.5 | 1.1050 | 8.80 | 8.12 | 68 |
| | | 0.07 | 0.3 | 1.0716 | 8.64 | 8.12 | 52 |
| | | 0.10 | 0.3 | 1.0592 | 9.20 | 8.12 | 108 |
| 15.0 | 1 | 0.02 | 0.7 | 1.0315 | 16.19 | 16.13 | 6 |
| | | 0.07 | 0.7 | 1.0293 | 16.76 | 16.13 | 63 |
| | | 0.10 | 0.7 | 1.0275 | 17.25 | 16.13 | 112 |
| | 5 | 0.02 | 0.7 | 1.1088 | 16.29 | 15.82 | 47 |
| | | 0.07 | 0.6 | 1.0941 | 16.25 | 15.82 | 43 |
| | | 0.10 | 0.6 | 1.0849 | 16.90 | 15.82 | 108 |
| | 10 | 0.02 | 0.7 | 1.1584 | 15.78 | 15.41 | 37 |
| | | 0.07 | 0.6 | 1.1321 | 15.98 | 15.41 | 57 |
| | | 0.10 | 0.6 | 1.1151 | 16.77 | 15.41 | 136 |
| | 20 | 0.02 | 0.7 | 1.1933 | 15.21 | 14.87 | 34 |
| | | 0.07 | 0.6 | 1.1569 | 15.59 | 14.87 | 72 |
| | | 0.10 | 0.6 | 1.1331 | 16.48 | 14.87 | 161 |

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