# Uncertainty and Judgment Aggregation in Monetary Policy Committees* 

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#### Abstract

Monetary policymakers face considerable uncertainty and have to use judgment. When the monetary policy committee (MPC) has to reach a decision based on different judgments among its members, various judgment aggregation problems may occur. Here, we consider an aggregation problem called the 'discursive dilemma', which is characterized by an inconsistency between the aggregate judgment on the premises for a conclusion and the aggregate judgment on the conclusion itself. If there is a discursive dilemma, the decision will depend on whether the MPC uses a premise-based or a conclusion-based decision procedure. We first show that the discursive dilemma is likely to be relevant for monetary policy decisions. Second, we analyze which decision procedure gives the better results. If there is only additive uncertainty, the two decision procedures are equally good. If there is multiplicative (Brainard) uncertainty, we find that a premise-based procedure gives better monetary policy decisions. Lastly, we discuss implications of our finding for optimal organization of interest rate decisions and for optimal characteristics of MPC members. We find that if the MPC votes directly on the interest rate, and not on the premises, the government should appoint overconfident MPC members, i.e., members who underestimate the true degree of uncertainty.


Keywords: Monetary policy committees, Uncertainty, Judgment aggregation, Discursive dilemma

JEL Classification: E52, E58, D71

[^0]
## 1 Introduction

Central banks conduct monetary policy under uncertainty about a wide range of issues. Consequently, a large part of the monetary policy literature considers policymaking under uncertainty. ${ }^{1}$ In most of this literature, as in the literature on monetary policy in general, it is assumed that monetary policy is conducted by a single policymaker. Only in the recent years, more attention has been directed to monetary policymaking in committees. Most of the monetary committee literature focuses on differences in preferences among the committee members. ${ }^{2}$ The focus of this paper, however, is differences in judgments among the committee members. Differences in judgments will typically arise when there is uncertainty and incomplete information.

The main reason for letting monetary policy in practice be decided by a committee and not a single policymaker is arguably to make better and more robust policy under uncertainty; "two heads are better than one". This motivation is also confirmed by experiments on economics students by Blinder and Morgan (2005) and Lombardelli et.al. (2005). Important analytical progress in understanding the gains from having committees is provided by Gerlach-Kristen (2003 and 2006). ${ }^{3}$ The topic for the present paper is related to this literature in dealing with MPC decisions under uncertainty, but we do not - as the abovementioned literature - consider gains from committees or the choice between voting or averaging. The topic of this paper is illustrated by the following example. Suppose the MPC sets the interest rate according to the following Taylor rule:

$$
\begin{equation*}
i_{t}=r_{t}^{*}+\pi^{*}+1.5\left(\pi_{t}-\pi^{*}\right)+0.5 y_{t}, \tag{1}
\end{equation*}
$$

where $i_{t}$ is the nominal interest rate, $r_{t}^{*}$ is the neutral real interest rate, $\pi^{*}$ is the desired rate of inflation (inflation target), $\pi_{t}$ is actual inflation, and $y_{t}$ is the output gap. The neutral real interest rate $r_{t}^{*}$ and the output gap $y_{t}$ are uncertain, particularly in real time, and the MPC members use judgment when quantifying them. We assume that $\pi_{t}$ can be perfectly observed and consider for simplicity a situation where inflation is on target, $\pi_{t}=\pi^{*}=2$. The MPC consists of three members, and the members' judgments on $r_{t}^{*}$ and $y_{t}$ are represented in Table 1.

[^1]Table 1:

|  | $r_{t}^{*}$ | $y_{t}$ | $i_{t}$ |
| :---: | :---: | :---: | :---: |
| Member 1 | 2.0 | 1.0 | 4.5 |
| Member 2 | 2.5 | 0.0 | 5.0 |
| Member 3 | 2.0 | 0.0 | 4.0 |
| Majority | 2.0 | 0.0 | 4.5 |

Suppose that the MPC aggregates judgments by majority voting and that the winner of the vote is the median judgment. Then, a direct vote on the interest rate would give $i_{t}=\operatorname{median}(4.5,5.0,4.0)=4.5$. We will call this decision-making procedure a 'conclusion-based procedure' (CBP). Suppose instead that the MPC uses what we will call a 'premise-based procedure' (PBP), where the MPC first votes on the premise-variables, which in this case are $r_{t}^{*}$ and $y_{t}$, and then let the interest rate decision be determined by the Taylor rule. We see that PBP would give $i_{t}=$ $\operatorname{median}(2.0,2.5,2.0)+2.0+(0.5 \times \operatorname{median}(1.0,0.0,0.0))=4.0 \neq 4.5$. There is thus an aggregation problem in the sense that the majority judgment on the conclusion is inconsistent with the majority judgments on the premise-variables. This aggregation inconsistency is known as the 'discursive dilemma ${ }^{4}$ and has received considerable attention in the recent years within social choice, philosophy, jurisprudence, and political science, see e.g. List (2005), Fallis (2005), List and pettit (2005). ${ }^{5}$

In this paper we analyze the relevance of the discursive dilemma for monetary policy decisions. The theoretical framework builds on Claussen and Røisland (2005) who developed a modelling framework that is suited for analyzing the discursive dilemma under judgments on continuous variables (like the output gap), as opposed to binary (true/false) judgments on propositions, which are considered in the social choice literature. ${ }^{6}$ Here, we extend the analysis in several ways. First, in terms of positive analysis of the discursive dilemma, we discuss how relevant the discursive dilemma is for monetary policy decisions, both measured by the probability of existence, the size of the difference between CBP and PBP, and potential policy biases. We find that the dilemma is indeed likely to exist in monetary policy decisions and that the size is likely to be non-negligible. Second, we analyze the normative question of which of the two decision-making procedures PBP versus CBP - that gives the better policy. We find that under addi-

[^2]tive uncertainty, CBP and PBP have on average equal performance. Under multiplicative (Brainard) uncertainty, PBP tends to perform better than CBP, because the latter gives too cautious policy decisions. These results have implications for how central banks (and other organizations) should organize the decision process. For example, if there is multiplicative uncertainty, there might be particularly important to have a core forecasting model, which reflects the MPC members aggregate views on the main economic mechanisms, and an inflation report, which reflects the MPC members aggregate judgments on the shocks. A core model and inflation reports may serve as institutional devices that support a premise-based decisionmaking procedure. Our results have also implications for central bank governance, such as delegation and optimal appointments. For example, the bias towards excessive cautiousness, in particular under CBP, may be removed if the government appoints "over-confident" MPC members, i.e., members who tend to underestimate the actual degree of uncertainty.

The relevance of the discursive dilemma for monetary policy has not, to our knowledge, been analyzed before. Faust and Henderson (2004) have a brief discussion of multi-stage decisionmaking, which is equivalent to our definition of PBP, and writes that "There is no theorem on public decisionmaking stating that the multistage decisionmaking approach is good for society." While it is probably not possible to derive a general normative theorem for this, we are able to reach some results that favours PBP under reasonable assumptions. ${ }^{7}$

The paper is organized as follows: In section 2, we define the issue and present the terminology to be used. In section 3, we study the relevance of the dilemma when there is only additive uncertainty. In section 4 we study the relevance of the dilemma when there is both additive and multiplicative uncertainty. The analysis suggests that the discursive dilemma is highly relevant for monetary policy decisions in MPCs. Furthermore, if there is multiplicative uncertainty, the premise-based decision procedure gives better policy decisions. In section 5 we discuss the implications of our findings for institutional design and central bank governance and suggests an institutional set-up that will support premise-based decisions. Section 6 summarizes the results and points out issues for future research.

## 2 Definitions and terminology

There are two popular ways of specifying monetary policy; either as a simple instrument rule, such as the Taylor rule, or as the result of minimizing a loss function. In the latter case, it is possible to derive an implicit instrument rule, or reaction function. Irrespective of which approach we take, monetary

[^3]policy can be represented as a relationship between the interest rate and a set of variables and parameters. These variables and parameters can be grouped into two types: (i) variables and parameters that are known by the MPC members or to which the MPC members never disagree, and (ii) variables and parameters that are unknown and where the MPC members may have different judgments. We will denote variables and parameters of type (ii) premise-variables, i.e.

Definition 1 A 'premise-variable' is a variable or parameter on which the MPC members may have different judgments and which is relevant for the interest rate decision

Let the vector of premise-variables be $p=\left(p_{1}, p_{2}, \ldots, p_{k}\right)$. Each MPC member $j \in M P C$ has a individual judgment $p_{j h}$ on each premise variable $p_{h}$ where $h \in H, H=(1,2, \ldots, k)$. The set of individual judgments on premise variable $h$ is denoted $P_{h}=\left\{p_{j h}\right\}_{j \in H}$. The set of possible judgments on the premise-variables is given by the Cartesian product $Q=\prod_{h \in H}\left[p_{h}^{-}, p_{h}^{+}\right]$ where $H=(1,2, \ldots, k)$ and $p_{h}^{-}<p_{h}^{+}$for all $h \in H, p_{h}^{-}, p_{h}^{+} \in \mathbb{R}$. The relationship between the conclusion and the premise-variables is called the decision function and is defined as follows:

Definition 2 'decision function' $f(p)$ is a continuously differentiable function that for each vector of judgments $\mathbf{p}=\left(p_{1}, p_{2}, \ldots, p_{k}\right) \in Q$ specifies a level of the interest rate:

$$
i=f(\mathbf{p}): Q \rightarrow \mathbb{R}
$$

Since only variables and parameters on which the committee members may disagree are arguments in the policy rule, the other variables and parameters are represented by the functional form $f(\cdot)$. To fix ideas and clarify the difference between a 'decision function' and more conventional interest rate rules or reaction functions, suppose that the central bank sets the interest rate according to the following simple rule: $i_{t}=i_{t-1}+\alpha\left(\pi_{t}^{\text {core }}-\pi^{*}\right)$, where $i_{t}$ is the interest rate at period $t, \pi_{t}^{\text {core }}$ is core (underlying) inflation, $\pi^{*}$ is the inflation target, and $\alpha$ is a parameter that says how much the interest rate should be adjusted if core inflation deviates from the target. If the committee members have different judgments on $\pi_{t}^{\text {core }}$ but agree on $\alpha$ and $\pi^{*}, f(p)=\left(i_{t-1}-\alpha \pi^{*}\right)+\alpha p$ where $\left(i_{t-1}-\alpha \pi^{*}\right)$ is a constant term and $\alpha$ is the coefficient on the premise-variable $p=\pi_{t}^{\text {core }}$. Alternatively, the members may agree on $\pi_{t}^{\text {core }}$, but disagree on $\alpha$. In this case, $\alpha$ is the premise-variable, $i_{t-1}$ is the constant term and $\left(\pi_{t}^{\text {core }}-\pi^{*}\right)$ is the (time-varying) coefficient on the premise-variable. If the members have different judgments on both $\pi_{t}^{\text {core }}$ and $\alpha$, the decision function is $i=f\left(p_{1,} p_{2}\right)=i_{t-1}-\pi^{*} p_{1}+p_{1} p_{2}$ where $p_{1}=\alpha$ and $p_{2}=\pi_{t}^{\text {core }}$. Thus, even if the interest rate rule $i_{t}=i_{t-1}+\alpha\left(\pi_{t}-\pi^{*}\right)$ is linear, the decision function, as defined above, may be non-linear depending
on which variables and parameters the committee members have different judgments on. The decision function can thus be derived from the underlying policy rule, but not the other way around. Note also that the dimension of the premise-variables is "cross-sectional", e.g., statically distributed among MPC members, while the underlying policy rule is a "time-series" function. The time-subscripts are therefore dropped in the decision function, but since the variables on which the MPC members have the same judgments vary over time, the $f(\cdot)$-function may also vary from one MPC meeting to the other. The reason for focussing on the decision function and not on the underlying policy rule is that they may have different functional forms, as shown above, and it is the functional form of the decision function that matters for the existence of the discursive dilemma.

We find it useful to distinguish between 'aggregation method' and 'aggregation procedure'. By aggregation method we mean the mechanism by which individual judgments on one variable is aggregated into a collective judgment on this variable. ${ }^{8}$ Blinder and Morgan (2005) list three general aggregation methods; (a) voting, (b) averaging, and (c) let the most skilled member decide. Aggregation methods can be represented by an aggregation function $g(\cdot)$ which for each possible set of judgments assigns an aggregate judgment. Aggregation functions for aggregation method (a)-(c) are (a) the median of the set of judgments, (b) the mean of the set of judgments and (c), the judgment of individual $j$. For any aggregation method there are two types of decision procedures. With a conclusion-based decision procedure (CBP) the MPC aggregates judgments on the interest rate directly. If, for example, the aggregation method is majority voting, the MPC votes directly on the interest rate. With a premise-based decision procedure (PBP) the MPC first aggregates the judgments on each premise-variable, and then let the decision function generate the interest rate based on these aggregate premise-variable judgments. Generally, for any given aggregation function the two decision procedures are defined as follows:

Definition $3 C B P: \quad i^{c b}=g\left(i_{1}, i_{2}, \ldots, i_{n}\right)=g\left(f\left(\mathbf{p}_{1}\right), f\left(\mathbf{p}_{2}\right), \ldots, f\left(\mathbf{p}_{n}\right)\right)^{9}$ PBP: $\quad i^{p b}=i^{p b}=f\left(p_{1}^{A}, p_{2}^{A}, \ldots, p_{k}^{A}\right)=f\left(g\left(P_{1}\right), g\left(P_{2}\right), \ldots, g\left(P_{k}\right)\right)$

Note that the MPC members agree on the decision function by construction since the part of the decision function they disagree on would otherwise be represented in the set of premise-variables.

We can now define the discursive dilemma the following way: ${ }^{10}$

[^4]Definition 4 There is a discursive dilemma if $i^{c b} \neq i^{p b}$, i.e., $g\left(f\left(\mathbf{p}_{1}\right), f\left(\mathbf{p}_{2}\right), \ldots, f\left(\mathbf{p}_{n}\right)\right) \neq f\left(g\left(P_{1}\right), g\left(P_{2}\right), \ldots, g\left(P_{k}\right)\right)$

It follows from definition 4 that both the decision function, $f(\cdot)$, and the aggregation function, $g(\cdot)$, matter for the existence of a discursive dilemma. Since most MPCs use majority voting when aggregating judgments on the interest rate, we will focus on majority voting. ${ }^{11}$ We assume that the MPC members' ordering on the judgments on each preference ordering on the judgments on each premise variable and on the interest rate are single peaked. Consequently, the median voter theorem applies, and we can use the median of the judgments as the aggregate judgment. The median judgment of individual judgments on premise-variable $h$ is denoted $p_{h}^{m}$ for the premise-variables and, equivalently, $i^{m}$ for the interest rate.

Since the decision function is assumed to be continuous, it follows that we treat the decision variable - the interest rate - as a continuous variable. It can be argued against this that most central banks change the signal rate in steps of 25,50 , or 75 basis points, which suggests that the decision-variable is discrete rather than continuous. However, the short-term interest rate that central banks set has in itself little impact on economic choices among households and firms. Svensson (2005) argues that " [...] the current instrument rate and the central-bank announcement matter and have an effect on the economy essentially only through the private-sector expectations about future instrument rates and about aggregate future inflation and output that they give rise to". It is increasingly accepted that it is the central bank's communication about the whole interest rate path that is the de facto monetary policy instrument. Therefore, market participants are often more interested in how an interest rate decision is explained and the signals about future policy than in the interest rate decision itself.

Central banks differ, however, somewhat in how directly they communicate. Most central banks explain the interest rate decision in a press release, often combined with a press conference. For example, the Federal Reserve uses statements of the sort; "policy accommodation can be maintained for a considerable period", and "the Committee believes that policy accommodation can be removed at a pace that is likely to be measured". By fine-tuning the exact wordings, central banks are to a large degree able to fine-tune the response of market rates. A more direct way to communicate monetary policy is to publish the bank's own forecast of future interest rates, which is currently done by the Norges Bank and the Reserve Bank of New Zealand. Since the modern view on monetary policy is that monetary policy is (almost) all about communication, it is too narrow only to consider the central bank's own short-term signal rate as the monetary policy instrument. Although the central bank does not control market interest rates

[^5]directly, it is more relevant to consider the short-term market interest rate (e.g., the three-month money market rate), which is a continuous variable, as the central bank's instrument. The monetary policy decision is to choose a communication strategy, where the setting of the short-term signal rate is one element in its communication, such that the central bank achieves a desired response on the market interest rate(s). Thus, we will argue that the decision-variable in monetary policy is a continuous variable. Most empirical studies also use the money market rate as the policy instrument based on similar arguments.

To study the relevance of the discursive dilemma for monetary policy decisions in MPCs, we ask three questions:

- How likely is the discursive dilemma?
- Is the difference between the interest rates following the CBP and the PBP likely to be non-negligible?
- Does one of the decision procedures give a better policy than the other?

We measure the likelihood of a dilemma as the probability that the two procedures produce a different interest rate, i.e. $\operatorname{Pr}\left(i^{c b} \neq i^{p b}\right)$. We measure the difference between the interest rates following the CBP and the PBP by the standard deviation of the difference between the interest rates following the two procedures, i.e. $\sqrt{E\left(i^{c b}-i^{p b}\right)^{2}}$. We denote this measure the 'size of the dilemma'. To investigate the third question, we look at the truthtracking capacity of the two decision methods, $E\left(i^{c b}-i^{*}\right)^{2}$ and $E\left(i^{p b}-i^{*}\right)^{2}$ where $i^{*}$ is the optimal full information interest rate. Note that in most cases this measure is a good approximate for the loss. ${ }^{12}$ Note also that the truth-tracking measure is the sum of the policy bias $\left(E\left(i^{c b}-i^{*}\right)\right)^{2}$ and the variance of the interest rate $\operatorname{var}\left(i^{c b}\right)$.

A starting point for answering the first question is the impossibility theorems on judgment aggregation in interconnected propositions (see e.g. Nehring and Puppe (2005) and Dietrich and List (2005b)). These theorems suggests that in genereal one cannot rule out a discursive dilemma for monetary policy decisions in committees. Claussen and Røisland (2005) derive an impossibility result for continuously differentiable decision functions and show how the existence of a dilemma depends on the functional form of the decision function:

Proposition 1 (A) If the MPC aggregates judgments by a simple majority rule and $k=1$, then
(i) $i^{c p}=i^{p b}$ for all $\mathbf{p} \subset Q$ if $f(p)$ is monotonic for $p \in Q$, and

[^6](ii) there exists a $\mathbf{p} \subset Q$ such that $i^{c p} \neq i^{p b}$ if $f(p)$ is non-monotonic for $p \in Q$.
(B) If the MPC aggregates judgments by a simple majority rule, and $k>1$, then there exists a $\mathbf{p} \subset Q$ such that $i^{c b} \neq i^{p b}$.

Proof. Claussen and Røisland (2005).
Part (A) of the proposition says that if the MPC aggregates judgments by majority voting, a discursive dilemma cannot be ruled out if the decision function is non-monotonic on its domain. It can only be ruled out if the decision function is monotonic in its domain. Part (B) of the proposition says that if the MPC aggregates judgments by majority voting and there is more than one premise-variable, then a discursive dilemma cannot be ruled out regardless of the functional form of the decision function. Thus, $a$ priory we can only rule out the discursive dilemma in the cases when there is only one premise-variable and the decision function is monotonic. This is typically not the case for monetary policy decisions.

A general proposition for when there will be a dilemma does not exist, since the existence of a dilemma depends both on the functional form of the rule of inference and the particular set of judgments. Therefore, to investigate the relevance and normative implications of the dilemma further, we have to consider specific functional forms. In section 3 we study the relevance of the dilemma when the decision function is linear. In section 4 we study the relevance of the dilemma when the decision function is nonlinear. The distinction between linear and non-linear decision functions has its counterpart in the distinction between additive and mulitiplicative uncertainty. Additive uncertainty gives rise to linear decision functions, while multiplicative uncertainty gives rise to non-linear decision functions.

## 3 Linear decision functions

Many of the policy rules considered in the monetary policy literature are linear (or linear approximations). However, as illustrated by the example in section 2, linear policy rules may give rise to non-linear decision functions if the MPC members may have different judgments on both the variables and the coefficients in the policy rule. Linear decision functions will only exist if the disagreement enters additively in the policy rule. Since judgment differences is a result of uncertain variables and parameters, linear decision rules is a result of additive uncertainty. For linear decision functions we can find quite general results. It is therefore useful first to consider the relevance of the discursive dilemma for this class of decision functions separately.

Let the linear decision function be

$$
\begin{equation*}
i=\alpha_{0}+A^{\prime} p \tag{2}
\end{equation*}
$$

where $A=\left(\alpha_{1}, \ldots, \alpha_{k}\right)$ is the vector of coefficients and $p$ is a vector of premise-variables as defined in section 2. Let $\mathbf{p}^{*}=\left(p_{1}^{*}, p_{2}^{*}, \ldots, p_{K}^{*}\right)$ be the vector of the true values of the premise-variables, and $\Sigma=\left(\sigma_{1}^{2}, \sigma_{2}^{2}, \ldots, \sigma_{k}^{2}\right)$ be a vector of variances for judgment errors. We will assume that the MPC members are equally good at making judgments, i.e. $E\left(p_{j h}\right)=p_{h}^{*}$ and $\sigma_{j h}^{2}=$ $\sigma_{h}^{2} \forall h \in H, j \in M P C$. The judgments are assumed to be independent, i.e. $\operatorname{cov}\left(p_{h, j}, p_{z, q}\right)=0 \forall h \neq z, j \neq q$ and $h, z \in H, j, q \in M P C$, and the distributions generating the judgments are unbiased. ${ }^{13}$ We then have the following:

Lemma 1 If the decision rule is linear in $p$, then
-The probability of a dilemma $\left(\operatorname{Pr}\left(i^{c b} \neq i^{p b}\right)\right)$ is
independent of $\alpha_{0}$ and $\mathbf{p}^{*}$,
homogenous to degree 0 in the vector $\left(\alpha_{1} \sigma_{1}, \alpha_{2} \sigma_{2}, \ldots, \alpha_{k} \sigma_{k}\right)$.
-The size of the dilemma measured as $\sqrt{E\left(i^{c b}-i^{p b}\right)^{2}}$ is
independent of $\alpha_{0}$ and $\mathbf{p}^{*}$,
homogenous to degree 1 in the vector $\left(\alpha_{1} \sigma_{1}, \alpha_{2} \sigma_{2}, \ldots, \alpha_{k} \sigma_{k}\right)$.
Proof. See Appendix A
Lemma 1 implies that for linear decision functions the probability of a dilemma only depends on $n, k$, the relative relations between the coefficients, and the relative variances of judgment errors, i.e., $\sigma_{h}^{2} / \sigma_{g}^{2}, h \neq g \in H$. The size of the dilemma will in addition be increasing in $A$ and $\Sigma$. To learn more about the relevance of the discursive dilemma for linear decision functions we pursue Monte-Carlo simulations on the following model

$$
\begin{equation*}
i=p_{1}+p_{2}+\ldots+p_{k}, p_{h}^{*}=0 \forall h \in H \tag{3}
\end{equation*}
$$

where $n, k$, and the sigmas are allowed to vary. ${ }^{14}$ The simulations are documented in Appendix A.

Probability of a dilemma We find that the probability of a discursive dilemma, i.e., $\operatorname{Pr}\left(i^{c b} \neq i^{p b}\right)$, is monotonically increasing in the number of MPC members $n$ and and in the number of premise-variables $k$ for any value of the relative variances of judgment errors, i.e., $\sigma_{h}^{2} / \sigma_{g}^{2}, h \neq g \in H$. For a given $n$ and $k, \operatorname{Pr}\left(i^{c b} \neq i^{p b}\right)$ is concave around its maximum at $\sigma_{h}^{2}=\sigma^{2}$ $\forall h \in H$. If $n=3, k=2$ and the judgments are normally distributed, then $\operatorname{Pr}\left(i^{c b} \neq i^{p b}\right)$ approaches 66,7 per cent as the relative variances of

[^7]Table 2: Probability of a discursive dilemma in the linear model

|  | $n=3$ | $n=5$ | $n=7$ | $n=9$ | $n=11$ | $n=13$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $k=2$ | 76 | 89 | 94 | 96 | 97 | 100 |
| $k=3$ | 92 | 98 | 99 | 100 | 100 | $\ldots$ |
| $k=4$ | 97 | 100 | 100 | $\ldots$ | $\ldots$ | $\ldots$ |
| $k=5$ | 99 | 100 | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| $k=6$ | 100 | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | 100 |

Note: $i=p_{1}+p_{2}+\ldots+p_{k} . n=3,5, \ldots, 19, k=1,3, \ldots, 10, p_{h, j} \sim N\left(0, \sigma^{2}\right), \forall h, j$. No. of simulations is 100000 . Combinations of $k$ and $n$ not reported give a probability of 100 per cent.

Table 3: Size of a discursive dilemma in a linear model

|  | $n=3$ | $n=5$ | $n=7$ | $n=9$ | $n=11$ | $n=13$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $k=2$ | 0.05 | 0.05 | 0.05 | 0.04 | 0.04 | 0.04 |
| $k=3$ | 0.07 | 0.07 | 0.06 | 0.05 | 0.05 | 0.05 |
| $k=4$ | 0.09 | 0.08 | 0.07 | 0.06 | 0.05 | 0.05 |
| $k=5$ | 0.1 | 0.09 | 0.08 | 0.07 | 0.06 | 0.06 |
| $k=6$ | 0.01 | 0.1 | 0.09 | 0.08 | 0.07 | 0.07 |
| $=$ |  |  |  |  |  |  |$=p_{1}+p_{2}+\ldots+p_{k} . p_{h, j} \sim N\left(0,0.1^{2}\right), \forall h, j$. No. of simulations is 100000.

the judgment errors is increased. Consequently, we can conclude that the probability of a dilemma is large for linear decision functions. Table 2 gives the probability of a discursive dilemma for (3) when $\sigma_{h}^{2}=\sigma^{2} \forall h \in H$. We see that for most values of $n$ and $k$, the probability of a discursive dilemma is close or equal to 100 per cent.

The size of the dilemma For large committees the variance of the median judgment on $p_{k}$ is given by $\sigma_{k}^{2} \pi / 2 n$ (see Kenney and Keeping (1962)). There exists no expression for the variance of the median in small samples, but the estimates in? show that the behavior of the median of a smaller sample is similar to the variance of the median of larger samples. A priory we therefore know that the size of the dilemma must be decreasing in $n$. This is confirmed by our simulations. Furthermore, our simulations show that the size of the dilemma is increasing in $k$. The size of a dilemma reaches its maximum for $\sigma_{h}^{2} / \sigma_{g}^{2}=1 h, g \in H$. Table 3 give the size of the dilemma in the benchmark case when $\alpha_{h}=1$ and $\sigma_{h}^{2}=0.1^{2} \forall h \in H$. Remembering that the absolute size is homogenous to degree 1 in $A$ and $\Sigma$, the table indicates that the size of the dilemma tends to be large for linear decision rules for small to mediumsized MPCs. Since the absolute size of the dilemma depends so much on the parameterization of the general model it is useful also to crosscheck against some concrete examples. We consider two examples, a Taylor rule and an estimated reaction function for the FOMC.

Suppose first that the MPC sets the interest rate according to the Taylor
rule (1) and let the neutral real interest rate $r^{*}$ and the output gap $y$ be the premise-variables. Let the distribution of the judgments be $r_{j}^{*} \sim N\left(2,0.5^{2}\right)$ and $y_{j} \sim N\left(0,0.5^{2}\right)$ where the expectations are the full information values of the premise variables. Monte-Carlo simulations on this model show that the absolute size of the dilemma is 0.17 percentage points if $n=3$ and falls to 0.10 percentage points when $n=19$. Table 11 in Appendix A gives the size of the dilemma for different committee sizes and different degrees of dispersion of the judgments. The table shows that the size of the dilemma is 0.1 percentage points or more for a large set of plausible parameter values. Thus, we may conclude that if the MPC's decision function can be approximated by a Taylor rule, the size of the dilemma will tend to be large.

We now turn to the example using actual interest decisions. Chappell et al. (2005), have estimated the individual reaction functions of the FOMC members under the chairman periods of Arthur Burns and Alan Greenspan based on the Memoranda of Discussion and the FOMC Transcripts. The estimated reaction functions in Chappell et al. (2005) are specified as follows:

$$
i_{j, t}=\beta_{j, 0}+\beta_{j, 1} i_{t}^{f}+\beta_{j, 2} \Delta m_{1 t}+\beta_{j, 3} U_{t}+\beta_{j, 4} \pi_{t}+\beta_{j, 5} \Delta y_{t}
$$

where $i^{f}$ is the pre-meeting funds rate, $\Delta m_{1}$ is the three-months average money growth rate, $U$ is the civilian unemployment rate, $\pi$ is the rate of inflation, and $\Delta y$ is growth rate of real GNP. The Memoranda and the Transcripts do not give detailed information about the FOMC members' judgments on unobservable variables like the output gap, neutral real interest rate etc, and on economic mechanisms. These potential judgment differences are captured in a crude way by the individually diverging coefficients on the (known) macroeconomic variables in the reaction function. Given the potential misspecification due to data limitations, as well as the uncertainty associated with the estimated coefficients, the individual reaction functions must obviously be interpreted with considerable caution. The estimations by Chappell et al. (2005) may still give some indication of the relevance of the discursive dilemma. ${ }^{15}$ Since the the FOMC members differ in the coefficients in the individual reaction functions, the set of premise-variables in the implied policy rule is $P=\left(\beta_{0}, \beta_{1}, \beta_{2}, \beta_{3}, \beta_{4}, \beta_{5}\right)$, while the set of macroeconomic variables $\left(i_{t}^{f}, \Delta m_{1 t}, U_{t}, \pi_{t}, \Delta y_{t}\right)$ serves as coefficients in the implied decision function. In our calculations, we have assumed that the aggregation method used by the FOMC is majority voting, ${ }^{16}$ and calculated CBP by

$$
i_{t}^{c b}=i_{t}^{m}
$$

[^8]Table 4: FOMC and the discursive dilemma Burns period Greenspan period

| $\operatorname{Pr}\left(i^{c b} \neq i^{p b}\right)$ | 100 | 100 |
| :---: | :---: | :---: |
| $\sqrt{E\left(i^{c b}-i^{p b}\right)^{2}}$ | to be added | to be added |

Note: To be added
and PBP by

$$
i_{t}^{p b}=\beta_{0}^{m}+i_{t}^{f} \beta_{1}^{m}+\Delta m_{1 t} \beta_{2}^{m}+U_{t} \beta_{3}^{m}+\pi_{t} \beta_{4}^{m}+\Delta y_{t} \beta_{5}^{m}
$$

Summarizing the Burns and Greenspan periods used in the estimations by Chappell et al. (2005) gives altogether 173 FOMC meetings. Table 4 shows that there exists a discursive dilemma in all of these meetings. Furthermore, the dilemma is quite large. Although these calculations should be interpreted with considerable caution, they suggest that size of the discursive dilemma is non-negligible for FOMC decisions under the Burns and Greenspan periods.

We may conclude that the size of the dilemma seems to be large enough to render the discursive dilemma relevant for situations where the MPC's decisions can be approximated by a linear decision function.

Truth-tracking Since the distributions generating the judgments are assumed unbiased, and since the decision function is linear, it follows that both $i^{c b}$ and $i^{p b}$ are unbiased estimators $i^{*}$ and none of the procedures have a policy bias, i.e. $E\left(i^{c b}-i^{*}\right)=E\left(i^{p b}-i^{*}\right)=0$. Thus, the only source of differences in truth-tracking must be if one procedure generates more variance in the interest rate than the other. For large committes we know that $\operatorname{var}\left(i^{c b}\right)=\operatorname{var}\left(i^{p b}\right) .{ }^{17}$ Our simulations confirm that $\operatorname{var}\left(i^{c b}\right)=\operatorname{var}\left(i^{p b}\right)$ also for small samples. Thus, there is no difference in the truth-tracking capacity of the two procedures, and none of the two decision procedures gives a better policy. As we will show in the next section, this is not the case with non-linear decision functions.

## 4 Non-linear decision functions

The set of non-linear decision functions is in principle infinite. We have chosen to focus on decision functions that can be derived from a standard monetary policy model. Specifically, we consider the canonical New Keynesian model which is well established as a tool for monetary policy analysis

[^9]and summarizes important features of macroeconomic models in many central banks. See e.g. Clarida et al. (1999) and Walsh (2005) for a discussion of the model.

### 4.1 The model

The canonical New Keynesian model can be derived from optimizing agents and is represented by the following equations:

$$
\begin{gather*}
y_{t}=E y_{t+1}-\alpha\left(i_{t}-E_{t} \pi_{t+1}\right)+g_{t}  \tag{4}\\
\pi_{t}=\beta E_{t} \pi_{t+1}+\kappa y_{t}+u_{t} \tag{5}
\end{gather*}
$$

where $y_{t}$ is the output gap, $i_{t}$ is the one-period nominal interest rate, and $\pi_{t}$ is the rate of inflation. The first equation is a linearization of the Euler equation representing optimal intertemporal consumption decisions, where the neutral real interest rate for simplicity is set equal to zero. $\alpha$ represents the intertemporal elasticity of substitution. The second equation is the New Keynesian Phillips curve, which can be derived from optimal price setting under monopolistic competition and price rigidity. $g_{t}$ and $u_{t}$ are shocks to output and inflation respectively, and they are assumed to be white noise. The objectives of the central bank are represented by the following (period) loss function:

$$
\begin{equation*}
L_{t}=\frac{1}{2} E\left[\left(\pi_{t}-\pi^{*}\right)^{2}+\lambda y_{t}^{2}\right] \tag{6}
\end{equation*}
$$

where $\pi^{*}$ is the desired rate of inflation (inflation target). $\lambda$ is the relative weight on output stability relative to inflation stability. There are two approaches to the loss function in the literature. The first approach is to let the loss function represent the preferences of the political authorities. In the recent years, another approach has become increasingly popular, namely to derive the true welfare loss function based on an approximation of the representative consumer's utility loss. ${ }^{18}$ Then, $\lambda$ is a function of the deep parameters of the model. In order to keep the analytical solution tractable, we will take the first approach and treat $\lambda$ as exogenous and representing the policy preferences. We open up for the possibility that the MPC members may have different judgments on $\lambda$ given by $\lambda_{j}$, where $\lambda_{j} \in \mathbb{R}^{+}$.

Although the members of the committee may agree to describe the 'structure' by (4) and (5)), they may have different judgments on the parameters $\alpha, \kappa$, and the shocks $g_{t}$ and $u_{t}$. Let $\alpha_{j}$ and $\kappa_{j}$ be committee member $j$ 's judgments on the parameters $\alpha, \kappa$. We assume that each MPC member receives a noisy signal on the true parameters, and that the judgments MPC member $j$ are given by

[^10]\[

$$
\begin{aligned}
\alpha_{j} & =\alpha^{*}+\varepsilon_{\alpha, j} \\
\kappa_{j} & =\kappa^{*}+\varepsilon_{\kappa, j}
\end{aligned}
$$
\]

where $\alpha^{*}$ and $\kappa^{*}$ are the true values and $\varepsilon_{j}^{\sigma}$ and $\varepsilon_{j}^{\kappa}$ are white noise processes with standard errors $\sigma_{\alpha}$ and $\sigma_{\kappa}$ respectively.

Similarly, let $g_{t, j}$ and $u_{t, j}$ be committee member $j$ 's judgments on the shocks $g_{t}$, and $u_{t}$ in period $t$. The MPC members' judgments on the shocks are the sum of the true shocks and a noisy signal, i.e.

$$
\begin{aligned}
g_{t, j} & =g_{t}^{*}+\varepsilon_{y, t, j} \\
u_{t, j} & =u_{t}^{*}+\varepsilon_{\pi, t, j}
\end{aligned}
$$

where $g_{t}^{*}$ and $u_{t}^{*}$ are the true shocks and $\varepsilon_{t}^{y}$ and $\varepsilon_{t}^{\pi}$ are white noise processes with standard errors $\sigma_{y}$ and $\sigma_{\pi}$ respectively. Also $g_{t}^{*}$ and $u_{t}^{*}$ are white noise processes with standard errors $\sigma_{g}$ and $\sigma_{u}$ respectively.

The decision problem of the MPC is to decide the interest rate that gives the minimum loss given the information available. We assume that the MPC is not able to enforce the commitment solution and thus conducts time-consistent (discretionary) monetary policy. This assumption is only for analytical convenience, since assuming instead that the MPC could commit to, e.g., optimal policy in a 'time-less perspective', would not add anything to the analysis of the discursive dilemma, but would make the analytical expressions slightly more complicated since optimal policy no longer could be treated as a static optimization problem. It may also be argued that in practice central banks tend to re-optimize each period, see Svensson (2005).

The first-order condition for optimal time-consistent policy, based on MPC member $j$ 's judgment is ${ }^{19}$

$$
\begin{equation*}
E_{t, j}\left[\kappa_{t} \pi_{t}+\lambda y_{t}\right]=0 \tag{7}
\end{equation*}
$$

The full information first-order condition is given by the brackets, but since we have assumed that the MPC members do not observe the parameters and the shocks perfectly, the first-order condition must be conditioned on member $j$ 's information and judgments, where $E_{t, j}$ is the expectation operator conditional on MPC member $j$ 's information at period $t$. Inserting equation (5) and (4) into (7), taking the expectation through (7) and solving for $y_{t}$ gives ${ }^{20}$

$$
\begin{gather*}
E_{t, j}\left[\alpha_{t} \kappa_{t}\left(\beta E_{t} \pi_{t+1}+\kappa_{t}\left(E_{t} y_{t+1}-\alpha_{t}\left(i_{t}-E_{t} \pi_{t+1}\right)+\varepsilon_{y, t}\right)+\varepsilon_{\pi, t}\right)\right.  \tag{8}\\
\left.+\lambda \alpha_{t}\left(E_{t} y_{t+1}-\alpha_{t}\left(i_{t}-E_{t} \pi_{t+1}-r_{t}^{*}\right)+\varepsilon_{t}^{y}\right)\right]=0
\end{gather*}
$$

[^11]Taking the expectation through this expression and noting that $E_{t} \pi_{t+1}=$ $E_{t} y_{t+1}=0$, since there is no autocorrelation in the shocks and the MPC re-optimizes on each MPC meeting, we get the following solution for the interest rate preferred by member $j$ :

$$
\begin{equation*}
i_{t, j}=\frac{\alpha_{j}}{\alpha_{j}^{2}+\sigma_{\alpha}^{2}}\left(g_{t, j}+\frac{\kappa_{j}}{\kappa_{j}^{2}+\sigma_{\kappa}^{2}+\lambda_{j}} u_{t, j}\right) \tag{9}
\end{equation*}
$$

Note that the solution for interest rate in equation (9) cannot not be implemented directly as a policy rule, because it would lead to indeterminacy due to the fact that only exogenous (state-)variables enter. To ensure determinacy, it suffices to add a term with a nominal endogenous variable with an appropriate coefficient. ${ }^{21}$ For example, adding a term with $E_{t} \pi_{t+1}$ with a coefficient larger than unity would give determinacy, but since $E_{t} \pi_{t+1}=0$ in equilibrium anyway, we will for simplicity disregard this term when considering the decision function, such that we treat equation (9) as the decision function following from minimizing the expected loss.

### 4.2 Multiplicative uncertainty

The literature on monetary policy distinguish between additive and multiplicative uncertainty. In our model uncertainty regarding the size of the shocks $u_{t}$ and $g_{t}$ represents the additative uncertainty, while uncertainty regarding the effects of policy $(\alpha, \kappa)$ represents multiplicative uncertainty. Brainard (1967) showed that if there is uncertainty about the effects of policy, which is an example of multiplicative uncertainty, certainty equivalence no longer holds, and optimal policy should respond less to shocks. In our model this is captured by the two fractions in (9) which are smaller the larger $\sigma_{\alpha}^{2}$ and $\sigma_{\kappa}^{2}$ are. The distinction between additive and multiplicative uncertainty is also important for the relevance of the discursive dilemma. If there is certainty regarding the effects of policy, the decision function becomes linear in the two shocks $g_{t, j}$ and $u_{t, j}$, and the results from section 3 applies. ${ }^{22}$ If there is uncertainty regarding the effects of policy, the decision function becomes non-linear in some premise-variables because the fractions in (9) are non-linear in $\alpha_{j}, \kappa_{j}$ and $\lambda_{j}$. Furthermore, the decision function is non-monotonic in $\alpha$ and $\kappa$. Consequently (ii) in proposition 1 applies, and a discursive dilemma cannot be ruled out even if the MPC-members have the same judgments for all premise variables except $\alpha$ or $\kappa$. In this subsection we study the effect of this type of uncertainty for the discursive dilemma.

[^12]Figure 1: Response coefficients for cost-push shocks for $\sigma_{\kappa}^{2}+\lambda=1$ (solid) and $\sigma_{\kappa}^{2}+\lambda=4$ (dashed)


We focus on the case where the only premise-variable is the slope of the Phillips curve, $\kappa$. The analysis of the case when there is disagreement on only $\alpha$, is parallell. Note also that an analysis of the relevance of the discursive dilemma in Brainard (1967) model would be similar to the analysis we perform in this subsection.

For analytical convenience, and without loss of generality, we normalize the rule of inference by setting $g_{t}^{j}=0$ and $u_{t}^{j}=1$ for $j=1,2, \ldots, n$. The rule of inference can then we written as

$$
\begin{equation*}
i_{t}=f(\kappa)=\frac{\kappa}{\kappa^{2}+\sigma_{\kappa}^{2}+\lambda}, \tag{11}
\end{equation*}
$$

which we in the following call the response coefficient for cost push shocks, or simply the response coefficient. The response coefficient is plotted in figure 1.

The response coefficient is clearly non-monotonic. This is because $\kappa$ affects the optimal monetary policy response through two separate effects. First, monetary policy is more effective in stabilizing inflation the higher $\kappa$ is. In isolation, this speaks for responding aggressively to cost-push shocks and allowing higher output variability. However, when $\kappa$ is high, the need for a strong interest rate response in order to stabilize inflation is less, which in isolation reduces the response coefficient. The figure shows that the first effect dominates up the point $\kappa<\left(\lambda+\sigma_{\kappa}^{2}\right)^{1 / 2}$, while the second effect dominates when $\kappa>\left(\lambda+\sigma_{\kappa}^{2}\right)^{1 / 2}$.

Table 5: Probability a dilemma.
NK model with one-dimensional multiplicative uncertainty.

|  | $n=3$ | $n=5$ | $n=7$ | $n=9$ | $n=15$ | $n=19$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $\operatorname{Pr}\left(i^{c b} \neq i^{p b}\right)$ | 19 | 26 | 31 | 36 | 50 | 58 |
| $\sqrt{E\left(i^{c b}-i^{p b}\right)^{2}}$ | 0.04 | 0.04 | 0.03 | 0.03 | 0.03 | 0.02 |
| CBloss-PBloss $(* 100)$ | 0.2 | 0.2 | 0.2 | 0.3 | 0.3 | 0.3 |
| Note: Simulation based on: $t=10000, \lambda^{*}=0.1, \kappa \sim$ | $\sim\left(0.5,0.15^{2}\right), g=0, \varepsilon_{y}=$ |  |  |  |  |  |
| $0, u=1, \varepsilon_{u}=0$ |  |  |  |  |  |  |
| (matlabscript: H/MPC1/matlab/NKkappageneral.m) |  |  |  |  |  |  |

All simulations discussed and referred to in the following are documented in Appendix B.

Probability of a dilemma A situation with a dilemma for $n=3$ is illustrated in the figure. With these judgments on $\kappa$, PBP would give $i_{2}$, while CBP would give $i_{3}$. The figure also illustrates that the existence of a dilemma depends on the domain for $\kappa, \sigma_{\kappa}$ and $\lambda$. If, for example, $\kappa^{\min } \geq \kappa_{2}$, there cannot be a dilemma if $\lambda+\sigma_{\kappa}^{2} \leq 1$, whereas there can if $\lambda+\sigma_{\kappa}^{2}<1$. Thus, the probability of a dilemma will depend on all parameters of the model and it is not possible to say anything without a further calibration. Table 5 show the results from Monte-Carlo simulations on one calibration of the model. The first row of Table 5 shows that the probability of a dilemma can be quite large even if there is only one premise-variable. We have run simulations for a large set of parameter values. These simulations show that the probability of a dilemma tends to be large for a large set of parameter values. However, if $\lambda>\kappa^{*}$ and $\kappa^{*}$ is not too large, a dilemma can be ruled out for all committee sizes. Note that also here the probability of a dilemma is increasing in $n$. <Explanation: to be added $>$

Size of the dilemma The differences in policy following from the PBP and the CBP is small for most parameter values. The second row of table 5 give the standard deviation of the difference between CBP and PBP for a reasonable parameterization of the model. Simulations on a large set of parameter values and committee sizes show that if there is disagreement about $\kappa$ only, then the absolute size is larger than 0.1 percentage points only if $\lambda$ is close to zero and there is much uncertainty measured as $\sigma_{\kappa}$. Note that such a situation is parallell to a situation where the differences in judgments on the interest rate elasticity in the IS-curve $(\alpha)$ tends to be large. ${ }^{23}$

[^13]Table 6: Probability a dilemma.
NK model with disagreement about $\kappa$ and cost push shock.

|  | $n=3$ | $n=5$ | $n=7$ | $n=9$ | $n=15$ | $n=19$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $\operatorname{Pr}\left(i^{c b} \neq i^{p b}\right)$ | 71 | 85 | 91 | 94 | 97 | 98 |
| $\sqrt{E\left(i^{c b}-i^{p b}\right)^{2}}$ | 0.22 | 0.22 | 0.22 | 0.21 | 0.19 | 0.18 |
| CBloss-PBloss $(* 100)$ | 2.7 | 3.9 | 4.4 | 4.6 | 5.2 | 5.3 |
| Note: Simulation based on: $t=100000, \lambda^{*}=0.3, \kappa \sim N\left(0.5,0.2^{2}\right), g=0, \varepsilon_{y}=$ |  |  |  |  |  |  |
| $0, u \sim N(0,1), \varepsilon_{u} \sim N(0,0.5)$ |  |  |  |  |  |  |
| (matlabscript: H/MPC1/matlab/NKkappa.m) |  |  |  |  |  |  |

Truth tracking Note first that since $f(\kappa)$ has one local maximum, CBP would give a more cautious policy response than PBP when there is a dilemma (see Claussen and Røisland (2005) for a generalization and proof). Thus we have that $E\left(i^{c b}\right)<E\left(i^{p b}\right)$, which means that the interest rate response to the shock is more cautious under CBP than under PBP. This is a cautiousness that comes from an aggregation bias, and it represents an additional source of cautiousness that comes in addition to the (intended) cautiousness stemming from multiplicative Brainard-uncertainty. While a cautious response in the standard Brainard model is needed to minimize the expected loss, the additional cautiousness caused by judgment aggregation gives an output stabilization bias in monetary policy. There exists no analytical expression for $\operatorname{var}\left(i^{c b}\right)$ and $\operatorname{var}\left(i^{p b}\right)$, and a priory it is hard to say whether one is larger than the other. To investigate the truth-tracking capacity of the two procedures we must therefore rely on numerical simulations. Our simulations show that PBP has better truth-tracking capacity than CBP in this case.

### 4.3 Additive and multiplicative uncertainty

We now return to the situation where there is both multiplicative and additive uncertainty. Table 6 show the result of a simulation for a situation where $p=\left(\kappa, u_{t}\right)$. Table 7 shows the results for a simulation where $p=\left(\kappa, u_{t}, \alpha, g_{t}, \lambda\right)$ and there is is little uncertainty and differences in judgments among the members of the MPC. Table 8 shows the results from a simulation where $p=\left(\kappa, u_{t}, \alpha, g_{t}, \lambda\right)$ and there is more uncertainty and differences in judgment, but where the degree of disagreement is still reasonable (se note to table). All simulation results referred to in this subsection are documented in Appendix B.

Probability of a dilemma From the analysis on linear rules we would expect that the probability of a dilemma would be large when $p=\left(\kappa, u_{t}, \alpha, g_{t}, \lambda\right)$. And as expected the probabilities are large in this case. But note also that that while the probability of a dilemma was quite small for the case when

Table 7: NK-model with little disagreement

| Committe size | 3 | 5 | 9 | 15 | 17 | 19 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Probability of dilemma | 99,07 | 99,90 | 100,00 | 100,00 | 100,00 | 100,00 |
| Dilemma size | 0,21 | 0,20 | 0,16 | 0,13 | 0,13 | 0,12 |
| (cbloss-pbloss)/pbloss | 1,70 | 3,69 | 6,13 | 11,13 | 11,77 | 12,86 |

Note: Simulation based on: $t=100000, \lambda^{*}=0.25, \sigma_{\lambda}=0.1, \alpha \sim N\left(0.5,0.1^{2}\right), \kappa \sim$ $N\left(0.5,0.1^{2}\right), g \sim N(0,1), \varepsilon_{y} \sim N(0,0.1), u \sim N(0,1), \varepsilon_{u} \sim N(0,0.1)$
(matlabscript: H/MPC1/matlab/NKmodellittl.m)
Table 8: NK-model with more disagreement

| Committe size | 3 | 5 | 9 | 15 | 17 | 19 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Probability of dilemma | 99,17 | 99,90 | 100,00 | 100,00 | 100,00 | 100,00 |
| Dilemma size | 0,65 | 0,58 | 0,47 | 0,39 | 0,37 | 0,36 |
| (cbloss-pbloss)/pbloss | 6,05 | 13,35 | 24,89 | 38,25 | 40,90 | 43,40 |

Simulation based on: $t=100000, \lambda^{*}=0.25, \sigma_{\lambda}=0.1, \alpha \sim N\left(0.5,0.2^{2}\right), \kappa \sim$ $N\left(0.5,0.2^{2}\right), g \sim N(0,1), \varepsilon_{y} \sim N(0,0.5), u \sim N(0,1), \varepsilon_{u} \sim N(0,0.5)$
(matlabscript: H/MPC1/matlab/NKmodelmore.m)
$p=(\kappa)$ it increases drastically when we add differences in judgments about one the cost push shock, i.e. when $p=\left(\kappa, u_{t}\right)$.

Size of the dilemma The second row of the tables shows the size of the dilemma. The size is large in all of the three tables and as expected the dilemma is larger for smaller committees. We have pursued numerical simulations for a large set of parameter values and find that the size of a dilemma tends to be rather large when $p=\left(\kappa, u_{t}, \alpha, g_{t}, \lambda\right)$. Note also that although the size of the dilemma was small when there was disagreement about $\kappa$ only, the effect of this kind of disagreement become very important when there is uncertainty also about the size of the relevant shock as table 6 illustrates. Although there is very little uncertainty and disagreement in this case, the size of the dilemma is large.

Truth tracking The PBP outperforms the CBP for all the examples in table 6-8. We also see that the gain from using the PBP relative to the CBP is increasing in the size of the committee. This is because the aggregation bias is increasing in the committe size. We see this directly from figure 1. Our simulations show that the gain from using the PBP can be large. In table 7 , for example, we see that even if there is not much disagreement, the gain is more than 6 per cent if $n=9$. If the degree of disagreement is increased, the gains from using the PBP increases, c.f. Table 8. Simulations based on a broarder set of parameter values shows that there PBP clearly
outperforms the CBP.

## 5 Institutional design

The results in Section 4 show that a premise-based decision-making procedure gives better outcomes than a conclusion-based procedure. This result has institutional implications, since it gives a case for designing institutions that support premise-based decisionmaking. It can be argued that monetary policy decisions are to some extent premise-based, as MPCs spend considerable time on discussing premises like the state of the economy and the inflation outlook. However, they vote directly on the interest rate, and it is not reasonable to assume that the individual members feel committed to the aggregate judgment on the premises when voting on the interest rate. This is in particular the case if the MPC members are individually responsible for their interest votes, which is the case for, e.g., members of the MPC at the Bank of England.

Another practical difficulty with implementing a premise-based procedure is discussed by Faust and Henderson (2004): MPC members with different views may have less difficulties of agreeing on and implementing policy directly than agreeing on all the premises for the policy. We find, however, this argumenet less convincing, since the premise-based procedure does not require that the MPC members agree on all the premises. They may agree to disagree, and still reach a collective judgment, for example by majority voting.

A more fundamental problem of a premise-based decision-making procedure is strategic voting. The MPC members can manipulate the result of premise-based decision-making by reporting false judgments on the premise variables. In principle, there is no way of preventing policymakers to act strategically. In practice, however, there is some collective discipline among MPC members that may reduce the scope for strateging voting. Moreover, there exist institutional devices, such as core models, inflation reports [and MPC minutes] that support a premise-based decision-making procedure. We will discuss these devices in turn.

We will point out that the distinction between a premise-based procedure and a conclusion-based procedure is in practice not completely dichotomous. There could be various degrees of premise-based decisions, for example by making collective judgments on some, but not all, premise-variables.

### 5.1 The internal organization of interest rate decisions

### 5.1.1 Core model

Central banks use models to guide their forecasts and interest rate decisions. Most central banks have not only one model, but rather a suite of
models. The advantage of having a suite of models is obvious. To cite George Box (1979): "all models are wrong, but some are useful". Different models have different strenghtes and are useful for different purposes. Although a suite of models approach is advantageous, there is also a danger that each policymaker can "pick a model" to justify his/her judgment on the policy conclusion. Then, the MPC members' preferred interest rates may be based on different models and views on economic mechanisms. In one sense, it is advantageous that policymakers have different views on economic mechanisms, since this may make the monetary policy decisions more robust. However, we have shown that the MPC members should not take their different views on the model the whole way to their interest rate votes, but instead aggregate the views into one "model" that reflects the median views.

Many central banks have chosen to let one particular model - often called the 'core model' - play a dominant role within the suite of models. The main reason for having a specific core model is probably that it helps coordination the analysis and forecasting process within the bank. Our results suggest that there is a rationale for having a core model that goes beyond its practical use as a coordination tool: It can be viewed as an institutional device to support a premise-based decision-making procedure. In order to support a premise-based procedure, it is advantageous if the core model is "owned" by the MPC, and not only by the central bank staff.

### 5.1.2 Inflation reports

Most inflation targeting central banks publish inflation reports. These reports have both an external and an internal role. The external role has to do with providing transparency and accountability. The internal role is to provide a common analytical framework for analyzing the state of the economy and forecasting economic developments. An important part of inflation reports is a description and analysis of the current state. In the language of the theoretical model considered in section 4.1 , this part of the analysis is to identify and estimate the shocks that have hit the economy. If the inflation report is "owned" by the MPC, and the MPC members have different judgment of the current state of the economy, they have to reach an aggregate judgment of the state in order to present a consistent analysis in the inflation report. The description of the state of the economy in MPC-owned inflation reports can thus be interpreted as the MPCs aggregate judgments on a set of important premise-variables for the interest rate decisions.

In addition to identifying and estimating shocks, the inflation report presents forecasts of inflation and other macroeconomic variables. The core macroeconomic model plays a key role in the forecasting process. One could argue that when the MPC has agreed on a certain forecast, it has then also agreed on an implicit model, since the forecasts rest on certain assump-

Figure 2: FPAS

tions and specifications of the economic process. From the point of view of premise-based decision-making, it may thus not be necessary to agree on a specific core model in addition to the forecasts in the inflation report. It is, however, possible that MPC members can agree on the forecasts, but disagree on the economic mechanisms, since different models, or different calibrations of the same model, can give the same forecast. The implications for monetary policy might, however, be different even if they give identical forecasts. ${ }^{24}$ Therefore, inflation reports do not make a core model fully superfluous as an institutional device for premise-based decisionmaking.

### 5.1.3 Premise-based decisionmaking model in practice

It is probably both difficult and impractial to make the decision procedure perfectly premise-based. For instance, it is very time-consuming to reach an aggregate view on each premise-variable in a large set of premise-variables Moreover, all premise-variables are not always possible to identify and specify in a presice way, which may be required in order to conduct, e.g., majority voting. Realistically, one may talk about a predominantly premise-based procedure, where the MPC has aggregated the individual judgments on the most important premise-variables.

[^14]A typical forecasting and policy analysis system (FPAS) in a central bank is illustrated in figure 2. The analysis and forecasts presented in the inflation report are key inputs to interest rate decisions. However, except from the Norges Bank and the Reserve Bank of New Zealand, the inflation report does not give an unconditional forecast of the interest rate. Therefore, the MPC has to use judgments in in addition to what is present in the inflation report.

One premise-variable that may be difficult to vote on before the actual interest rate decision is the relative weight on stability in the real economy, that is, $\lambda$ in our model. Most policymakers have judgments or preferences as regard a fair trade-off between conflicting objectives, but in practice the judgments on the appropriate weighing cannot be reduced to voting on a $\lambda$. One way to get around this is to have a premise-based procedure on the conditional forecasts, i.e., forecasts based on alternative interest rate assumptions, and let the MPC members vote on the interest rate path that is the one in accordance with his/her judgment on the weighing of objectives. We see from equation (9) that the decision function is non-linear, but monotonic in $\lambda$. This implies that voting on $\lambda$ and voting on $i_{t}$ gives the same conclusion.

To conclude, a good decision model for interest rate decisions is have a semi-PBP. First, the MPC should reach an aggregate judgment on the main "economic" premises, that is, the shocks and the "model", and let the staff produce alternative scenaries based on different interest rate path. Then, in the last stage the MPC should use a conclusion-based procedure and vote on the interest rate (path). It is important the in this stage the MPC members votes should be based on the aggregate views on the "economic" premises represented in the different scenarios and not by his/her own judment on these premises.

### 5.2 Delegation

A large strand of the monetary policy literature considers optimal delegation. This literature was sparked off by Rogoff (1985), who showed that monetary policy would be improved upon if the government delegated monetary policy to a conservative central banker. ${ }^{25}$ The motivation for appointing central bankers with certain characteristics is to remove a policy bias that results if the central bankers have the same characteristics as the government.

A new sort of policy bias with implications for delegation may arise when there is multiplicative uncertainty and the monetary policy decisions

[^15]Table 9: Optimal correction of lambda.
NK model with disagreement about $\kappa$.

| CBP (PBP) | $\lambda=0.1$ | $\lambda=0.2$ | $\lambda=0.3$ | $\lambda=0.4$ |
| :--- | :--- | :--- | :--- | :--- |
| $\sigma_{\kappa}=0.10$ | $0.91(0.91)$ | $0.93(0.94)$ | $0.95(0.96)$ | $0.97(0.97)$ |
| $\sigma_{\kappa}=0.20$ | $0.55(0.64)$ | $0.71(0.78)$ | $0.80(0.84)$ | $0.86(0.87)$ |
| $\sigma_{\kappa}=0.30$ | $0.00(0.15)$ | $0.36(0.49)$ | $0.55(0.64)$ | $0.66(0.72)$ |

Note: Simulation based on: $t=10000, p=(\kappa), \kappa \sim N\left(0.5, \sigma_{\kappa}^{2}\right), g=0, \varepsilon_{y}=0, u=$ $0, \varepsilon_{u}=0, n=9$
(matlabscript: H/MPC1/matlab/beregning av korreksjoner/optlmd.m)
are made by a committe and not by a single individual. ${ }^{26}$ Figure 1 may serve as an illustration. Suppose first that $\kappa^{*}=\kappa_{3}$ and $\kappa^{\max } \geq \kappa_{2}$ so that there is no discursive dilemma. Let $\sigma_{\kappa}>0$ and $\sigma_{\kappa}^{2}+\lambda=1$ (response coefficient is given by the solid line). Since $f^{\prime}(\kappa)<0$ over its domain and $f(\kappa)$ is strictly convex over an intreval around $\kappa^{*}$ it follows that $\left|f\left(\kappa^{*}+\varepsilon\right)\right|<\left|f\left(\kappa^{*}-\varepsilon\right)\right|$. Consequently there will be a policy bias rendering the policyresponse to cost-push shocks $\left(u_{t}\right)$ too aggressive. Since there is no discursive dilemma the size of the bias is independent of decision procedure. Suppose now that $\kappa^{*}=\kappa_{2}\left(\right.$ and still $\sigma_{\kappa}>0$ and $\left.\sigma_{\kappa}^{2}+\lambda=1\right)$. We see from the figure that there will be a policy bias also in this situation, but now it renders the response to cost-push shocks $\left(u_{t}\right)$ too cautious. Furtermore, in this situation the CBP will tend to give a more cautious policy than the PBP (See Claussen and Røisland (2005)).

The policy bias arising can be removed by appointing MPC members with appropriate characteristics. In the second example babove the government may appoint 'conservative' or 'overconfident' MPC members. A conservative MPC member is a person that has lower $\lambda$ than the government. An 'overconfident' MPC member is a person who believes he has a smaller observation error than his or her true observation error $\sigma_{k}$. Since the policy bias induced by the CBP and the PBP may differ, the optimal degree of conservativism or overconfidence will depend on the decision procedure used. Table 9 give the optimal degree of conservativism for different values of $\sigma_{\kappa}$ and $\lambda$. The table indicates that optimal delegation implies $\lambda^{c b} \leq \lambda^{p b}<\lambda$. This finding is confirmed in simulations on a broarder set of realistic parameter values. Table 10 give the optimal degree of overconfidence for the MPC members. The table show that $\sigma_{k}^{c b} \leq \sigma_{k}^{p b}<\sigma_{k}$, which also is the case for a broarder set of realistic parameter values.

In more realistic monetary policy decisions there may also be uncertianty and disagreement about the intertemporal elasticity of substitution $(\alpha)$. This will be another source of policy bias. Since $\lambda_{j}$ does not enter the

[^16]Table 10: Optimal correction of sigma.
NK model with disagreement about $\kappa$.

| CBP (PBP) | $\lambda=0.0$ | $\lambda=0.1$ | $\lambda=0.2$ | $\lambda=0.3$ | $\lambda=0.4$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\sigma_{\kappa}=0.10$ | $0.74(0.74)$ | $0.63(0.63)$ | $0.52(0.52)$ | $0.42(0.42)$ | $0.31(0.31)$ |
| $\sigma_{\kappa}=0.20$ | $0.78(0.78)$ | $0.67(0.67)$ | $0.57(0.57)$ | $0.46(0.47)$ | $0.34(0.36)$ |
| $\sigma_{\kappa}=0.30$ | $0.78(0.78)$ | $0.66(0.67)$ | $0.54(0.57)$ | $0.39(0.47)$ | $0.07(0.37)$ |

Note: Simulation based on: $t=10000, p=(\kappa), \kappa \sim N\left(0.5, \sigma_{\kappa}^{2}\right), g=0, \varepsilon_{y}=0, u=$ $0, \varepsilon_{u}=0, n=9$
(matlabscript: H/MPC1/matlab/beregning av korreksjoner/optsigma.m
response coefficient on demand shocks $\left(g_{t}\right)$, appointing 'conservative' MPC members does not reduce this policy bias, and the only option is to consider the MPC-members confidence. Since the variances of the judgment errors $\alpha$ and $\kappa$ are likely to be different, and since the response coefficients on demand shocks and cost-push shocks are somewhat different, optimal response to both demand shocks and cost-push shocks implies that the degree of overconfidence on $\alpha$ might be different than the optimal degree of overconfidence on $\kappa$. However, it is reasonable to assume that the degree of overconfidence relates to both uncertainty about $\alpha$ and $\kappa$, such that $\tilde{\sigma}_{\alpha}^{2}=\rho \sigma_{\alpha}^{2}$ and $\tilde{\sigma}_{\kappa}^{2}=$ $\rho \sigma_{\kappa}^{2}$, where $\tilde{\sigma}_{i}^{2}$ is the members' judgment on the variance and $\rho$ is the inverse of the degree of overconfidence. Table xx give $\rho$ for $n=9$ and $\sigma_{\kappa}=0.2$.

- < More results to be added $>$


## 6 Conclusion and issues for further research

In this paper we have analyzed the relevance of the discursive dilemma for monetary policy decisions in MPCs. We find that the dilemma is indeed relevant. It is likely to occure, and likely to be large. Furthermore, we find that under multiplicative (Brainard) uncertainty, a premise-based decision procedure tends to outperform a conclusion-based procedure because the latter gives too cautious policy decisions. The results have implications for how central banks (and other organizations) should organize the decision process. If there is multiplicative uncertainty it will be particularly important to have a core forecasting model, which reflects the MPC members aggregate views on the main economic mechanisms, and an inflation report, which reflects the MPC members aggregate judgments on the shocks. A core model and inflation reports serve as institutional devices that support a premise-based decisionmaking procedure. Our results also have implications for central bank governance, such as delegation and optimal appointments. For example, the bias towards excessive cautiousness, in particular under CBP, may be removed if the government appoints "over-confident" MPC members, i.e., members who tend to underestimate the actual degree of uncertainty.

In the paper we focus on situations where the monetary policy committee aggregate judgments by majority voting. The motivation is that many monetary policy committees seems to use this aggregation method. However, there are also MPC that claim to aggregate judgments by consensus formation. An interesting extention to this paper would be to study the relevance of the discursive dilemma in situations where the MPC aggregate judgments by consensus formation. To further check the robustness of our results it would be useful to use decision functions from other macroeconomic models than the New Keynesian model considered here. In addition, it will be interesting to investigate whether our result is robust to other types of multiplicative uncertainty than the one that implies optimal cautiousness. For example, is the a premise-based procedure still better if there is uncertainty about the degree of inflation persistence which, as shown by Söderström (2002), implies that optimal policy should be more aggressive?

In addition, there is a role for future research in analyzing the relevance of the discursive dilemma in other types of collective economic decisions than monetary policy decisions, for example for fiscal policy decisions and for decisions in corporate boards.

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## A Proofs and simulations to section 3

## A.0.1 Proof of Lemma 1

Since the $\alpha_{h} p_{h}$ are monotonic transformations of $p_{h}$, it follows that when we consider voting decisions we can write (2) as

$$
\begin{equation*}
i=\omega_{1}+\omega_{2}+\ldots+\omega_{k}+\Theta \tag{12}
\end{equation*}
$$

where $\omega_{h}=\alpha_{h}\left(p_{h}-p_{h}^{*}\right), \Theta=\alpha_{0}+A^{\prime} \mathbf{p}^{*}, E\left(\omega_{h}\right)=0$ and $\operatorname{var}\left(\omega_{h}\right)=$ $\alpha_{h}^{2} \sigma_{h}^{2}$. Since $i^{c b}-i^{p b}$ is independent of the $\Theta$-term we have that $\operatorname{Pr}\left(i^{c b} \neq i^{p b}\right)$ and $\sqrt{E\left(i^{c b}-i^{p b}\right)^{2}}$ is independent of $\alpha_{0}$ and $\mathbf{p}^{*}$. Furthermore we have that both $i^{c b}$ and $i^{p b}$ is homogenous to degree 1 in $A$. Consequently, $\operatorname{Pr}\left(i^{c b} \neq i^{p b}\right)$ is homogenous to degree 0 in $A$ and $\sqrt{E\left(i^{c b}-i^{p b}\right)^{2}}$ is homogenous to degree 1 in $A$. Since $\operatorname{var}\left(\omega_{h}\right)=\alpha_{h}^{2} \sigma_{h}^{2}$ it also follows that $\operatorname{Pr}\left(i^{c b} \neq i^{p b}\right)$ is homogenous to degree 0 in the vector $\left(\alpha_{1} \sigma_{1}, \alpha_{2} \sigma_{2}, \ldots, \alpha_{k} \sigma_{k}\right)$, and $\sqrt{E\left(i^{c b}-i^{p b}\right)^{2}}$ is homogenous to degree 1 in $\left(\alpha_{1} \sigma_{1}, \alpha_{2} \sigma_{2}, \ldots, \alpha_{k} \sigma_{k}\right)$.

## A.0.2 Simulations on the general linear model

In the numerical simulations we allow $n$ to take the values $3,5, \ldots, 19$ and $k$ to take the values $1,2, \ldots, 10$. In the case when $n=3$ and $k=2$, $\sigma_{h}^{2} / \sigma_{m}^{2}, m \neq h, m, h \in H$ is alowed to take the values $1 / 100000,1 / 10000$, $1 / 1000,1 / 100,1 / 10,2 / 10, \ldots, 1$. For $n>3$ and $k>2$ we have only simulated for $\sigma_{h}^{2}=\sigma^{2} \forall h \in H$. For each combination of parameters we have simulated the model 10000 times. The judgements are generated from a normal distribution. The malab codes used is available from the authors upon request (and are forthcoming on the corresponding author's homepage, http://www.norges-bank.no/english/research/claussen.html).
(memo: h: $\backslash \mathrm{mpc} 1 \backslash$ matlab $\backslash$ taylor_norm_model.m)

## A.0.3 Simulations on the Taylor rule

Model simulated: $i=r^{*}+2+1.5(0-0)+0.5 y=r^{*}+2+0.5 y, r^{*} \sim$ $N\left(2, \sigma_{r}^{2}\right), \sigma_{r}=(0.1,0.2, \ldots, 0.5), y \sim N\left(0, \sigma_{y}^{2}\right), \sigma_{y}=(0.1,0.2, \ldots, 0.5), n=$ $3,5, \ldots, 19$.The results for $n=7$ are in table 11

The malab code used is available from the authors upon request (and are forthcoming on the corresponding author's homepage, http://www.norgesbank.no/english/research/claussen.html).
(memo: h: $\backslash \mathrm{mpc} 1 \backslash$ matlab $\backslash$ taylor_norm_model.m)

## B Documentation of simulations in section 4

The simulations in the tables in the text are documented in the note to each table. In addition we have pursued Monte Carlo simulations on the

Table 11: Taylor rule. Size of the discursive dilemma for different committee sizes and diffrent degree of dispersion of judgments

| $\mathbf{n}=\mathbf{3}$ |  |  |  | sigma $y$ |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | :---: | :---: | :---: |
|  |  | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |  |  |  |
| sigma $r$ | 0.1 | 0,03 | 0,05 | 0,06 | 0,07 | 0,07 |  |  |  |
|  | 0.2 | 0,04 | 0,07 | 0,09 | 0,11 | 0,12 |  |  |  |
|  | 0.3 | 0,04 | 0,08 | 0,10 | 0,13 | 0,14 |  |  |  |
|  | 0.4 | 0,04 | 0,08 | 0,11 | 0,14 | 0,16 |  |  |  |
|  | 0.5 | 0,04 | 0,08 | 0,12 | 0,15 | 0,17 |  |  |  |


| $\mathbf{n}=\mathbf{7}$ |  |  | sigma $y$ |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | :---: | :---: |
|  |  | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |  |  |
| sigma $r$ | 0.1 | 0,03 | 0,05 | 0,05 | 0,06 | 0,07 |  |  |
|  | 0.2 | 0,04 | 0,06 | 0,08 | 0,09 | 0,10 |  |  |
|  | 0.3 | 0,04 | 0,07 | 0,09 | 0,11 | 0,12 |  |  |
|  | 0.4 | 0,04 | 0,08 | 0,10 | 0,12 | 0,14 |  |  |
|  | 0.5 | 0,04 | 0,08 | 0,11 | 0,13 | 0,15 |  |  |


| $\mathbf{n}=\mathbf{1 5}$ |  | sigma $y$ |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | :---: |
|  |  | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |  |
|  | 0.1 | 0,02 | 0,03 | 0,04 | 0,05 | 0,05 |  |
|  | 0.2 | 0,03 | 0,05 | 0,06 | 0,07 | 0,07 |  |
| sigma $r$ | 0.3 | 0,04 | 0,06 | 0,07 | 0,08 | 0,09 |  |
|  | 0.4 | 0,04 | 0,06 | 0,08 | 0,09 | 0,11 |  |
|  | 0.5 | 0,04 | 0,07 | 0,09 | 0,10 | 0,12 |  |

following model:

$$
i_{t, j}=\frac{\alpha_{j}}{\alpha_{j}^{2}+\sigma_{\alpha}^{2}}\left(g_{t, j}+\frac{\kappa_{j}}{\kappa_{j}^{2}+\sigma_{\kappa}^{2}+\lambda_{j}} u_{t, j}\right)
$$

where $\alpha_{j} \sim N\left(\alpha^{*}, \sigma_{\alpha}^{2}\right), \lambda_{j} \sim N\left(\lambda^{*}, 0\right), \kappa_{j} \sim N\left(\kappa^{*}, \sigma_{\kappa}^{2}\right), g_{t} \sim N(0,1), \varepsilon_{t, y j} \sim$ $N\left(0, \sigma_{\varepsilon_{y}}^{2}\right), u_{t} \sim N(0,1), \varepsilon_{t, y j} \sim N\left(0, \sigma_{\pi}^{2}\right)$ and $t=10000$.

This model is simulated for all combinations of the following parameter values:

$$
\begin{aligned}
& \alpha^{*}=0.5,0.6, \ldots, 1, \sigma_{\alpha} / \alpha^{*}=0,0.1,0.2, \ldots, 0.5 \\
& \kappa^{*}=0.5, \sigma_{\kappa} / \kappa^{*}=0,0.1,0.2, \ldots, 0.5 \\
& \varepsilon_{y}=0.1,0,2 \\
& \varepsilon_{\pi}=0.1,0,2 \\
& n=3,5, \ldots, 17
\end{aligned}
$$

The malab code used is available from the authors upon request (and are forthcoming on the corresponding author's homepage, http://www.norgesbank.no/english/research/claussen.html). (memo: h: $\backslash \mathrm{mpc} 1 \backslash$ matlab $\backslash$ NKfull.m)
$<$ More simulations to be added>


[^0]:    *We are grateful for comments and suggestions from participants at seminars at Norges Bank. The views presented are our own and do not necessarily represent those of Norges Bank.
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[^1]:    ${ }^{1}$ References to be added
    ${ }^{2}$ See, e.g, Waller (1989), Von Hagen and Süppel (1994), Brükner (2000), Aksoy et.al. (2002), Gros and Hefeker (2002), Hefeker (2003), Sibert (2003), and Matsen and Røisland (2005).
    ${ }^{3}$ Gerlach-Kristen (2006)considers how the monetary policy committee (MPC) should aggregate judgments, with particular focus on whether to use voting or averaging. If the MPC members have equal skills, averaging is found to give the better decisions, while voting can be superior if there are considerable asymmetries in skills among the MPC members. The gains from delegations within the committee are analyzed in GerlachKristen (2003).

[^2]:    ${ }^{4}$ It is also sometimes referred to as the "doctrinal paradox".
    ${ }^{5}$ See http://personal.lse.ac.uk/LIST/doctrinalparadox.htm ffor a comprehensive overview of the literature.
    ${ }^{6}$ It is possible to formulate judgments on continuous variables in a binary choice model. This is, however, cumbersome and hides the conditions for the dilemma to apply. Specifically, the functional form of the policy rule is crucial, and this aspect is difficult to capture in a binary framework.

[^3]:    ${ }^{7}$ ? is related to our work in study the Probability of a dilemma. However, he focuses on dichotomous decisions.

[^4]:    ${ }^{8}$ What we call aggregation method corresponds to 'constitution', or 'judgment aggregation function' the social choice literature.
    ${ }^{9}$ The ordering of arguments in the $g(\cdot)$-function does not matter.
    ${ }^{10}$ Note that this definition differs somewhat from the definition in the literature on judgment aggregation. See Claussen and Røisland (2005) section 4.

[^5]:    ${ }^{11}$ An alternative could be to consider consensus decisions modelled as the average judgment (see e.g. DeGroot (1974))

[^6]:    ${ }^{12}$ Follows from a Taylor approximization around the minimum loss: $L(i)=L\left(i^{*}\right)+$ $L^{\prime}\left(i^{*}\right)\left(i-i^{*}\right)+1 / 2 L^{\prime \prime}\left(i-i^{*}\right)=L\left(i^{*}\right)+1 / 2 L^{\prime \prime}\left(i-i^{*}\right)$ since $L^{\prime}\left(i^{*}\right)=0$ per definition.

[^7]:    ${ }^{13}$ The assumption of uncorrelated judgment errors is not essential for the results, but makes the results easier to interpret. In practice, judgments errors are likely to be positively correlated, in particular if the MPC members have similar background and education.
    ${ }^{14}$ It follows from lemma 1 that differences in $\sigma_{h}^{2}$ can be interpreted eighet as differences in coefficients or in the dispersion of judgments.

[^8]:    ${ }^{15}$ The discursive dilemma is not discussed by Chappell et al. (2005)
    ${ }^{16}$ Chappell et al. (2005) do not, however, find conclusive evidence on that majority voting is the relevant aggregation method, and they consider also 'averaging'.

[^9]:    ${ }^{17}$ The variance of the median judgment on $p_{k}$ is given by $\sigma_{k}^{2} \pi / 2 n$ (see discussion of the size of the dilemma above). Then $\operatorname{var}\left(c^{p b}\right)=\pi / 2 n\left(\alpha_{1}^{2} \sigma_{1}^{2}+\alpha_{2}^{2} \sigma_{2}^{2}+\ldots+\alpha_{K}^{2} \sigma_{K}^{2}\right)$ and $\operatorname{var}\left(c^{c b}\right)=\pi / 2 n \operatorname{var}(c)=\pi / 2 n\left(\alpha_{1}^{2} \sigma_{1}^{2}+\alpha_{2}^{2} \sigma_{2}^{2}+\ldots+\alpha_{K}^{2} \sigma_{K}^{2}\right)$.

[^10]:    ${ }^{18}$ Benigno and Woodford (2004) showed that approximating the utility loss in the standard New Keynesian model results in a loss function like (6). See also Walsh (2005)for a discussion of this approach to the loss function.

[^11]:    ${ }^{19}$ Under commitment to the timeless perspective, the level of the output gap is replaced by the change in the output gap, see Clarida et al. (1999).
    ${ }^{20}$ Note that $E_{t} \pi_{t+1}=0$ since there is no autocorrelation in the shocks and the MPC operates under discretion.

[^12]:    ${ }^{21}$ See, e.g., Gali (2003), section 5.2 .1 for a discussion of this point.
    ${ }^{22}$ The rule is now

    $$
    \begin{equation*}
    i_{t, j}=a g_{t, j}+b u_{t, j} \tag{10}
    \end{equation*}
    $$

    where $a \equiv \alpha / \alpha^{2}+\sigma_{\alpha}^{2}$ and $b \equiv \alpha \kappa /\left(\alpha^{2}+\sigma_{\alpha}^{2}\right)\left(\kappa^{2}+\sigma_{\kappa}^{2}+\lambda\right)$. This is a linear decision function. Linear decision functions were discussed in section 3.

[^13]:    ${ }^{23}$ In section 4.3 below we will show that these differences albeit small have large effects when there is more than one premise variable.

[^14]:    ${ }^{24}$ The might, for instance, have different transmission mechansims.

[^15]:    ${ }^{25}$ Other contributions to this literature is Røisland (2001), who showed that the optimal central banker is "liberal" if output is persistent and the government assigns an output target that is equal to (short-run) potential output. Leitemo (2005) shows that it is optimal to appoint a governor who perceives inflation persistence to be zero.

[^16]:    ${ }^{26}$ The issue of optimal delegation does not apply if there is only additive uncertainty, since then the decision function is linear.

