Government expenditure, capital adjustment, and economic growth

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Abstract

We analyze within a dynamic model the growth impact of private capital investment if the accompanying adjustment costs are a function of governmental activity. The impact of the productive public input is twofold: it (i) enhances private capital productivity and (ii) reduces adjustment costs. We derive the equilibrium in which the investment ratio is constant and determine the equilibrium growth rate. Carrying out comparative dynamic analysis allows us to show that better infrastructure endowment unequivocally spurs the equilibrium growth rate whereas the result becomes ambiguous with respect to the impact of rivalry. Since a reduction in congestion lowers the individually perceived capital productivity such a policy may reduce the equilibrium growth rate. While it is not possible to find closed solutions of the model we simulate the growth rate for different parameter constellations.

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1 Introduction

The impact of governmental activity on economic growth has been extensively studied in the last several years. Beginning with the seminal work of Aschauer (1989a) or Barro (1990) more recent models include aspects of uncertainty, congestion, excludability or adjustment costs (see e. g. Fisher and Turnovsky (1998), Turnovsky (1999a, 1999b), Eicher and Turnovsky (2000) or Turnovsky (2000a)). Within these models governmental activity consists of two parts: first, the provision of a productive input and second, the choice of the financing scheme that is required to finance a certain amount of the input (this amount has to be determined by the government) and to internalize external effects of capital accumulation that arise if congestion prevails. The public input acts as complement to private capital within the production process, increases private capital productivity and may have an impact of the firms' adjustment costs. Public policy thus plays an important role in the firm's capital investment decision. Turnovsky (1996, p. 363) argues that '. . . the degree of congestion is to some extent the outcome of a policy decision, and once determined, the degree of congestion turns out to be a critical determinant of optimal tax policy.'

This is to some extent the starting point of our paper. While most models focus on the role of the fiscal policy for economic growth we analyze the importance of the characteristics of the public input within the growth process and assume that private investment incurs adjustment costs that depend, among others, on the governmental input. What we have in mind is the following: Governmental activities –here interpreted as being the provision of freely available infrastructure– have an impact on investment in physical capital via different channels. Capital productivity is higher in regions with good infrastructure endowment. Additionally, the firms' overall investment costs are lower if the factory area is already developed. Thus the public input not only enhances private capital productivity but also decreases adjustment costs that arise within the investment process. Thus aside from fiscal policy implications the characteristics of the public input –namely amount of infrastructure and degree of rivalry– become especially important to assess public policy.

We show that the focus on the absolute amount of publicly provided infrastructure oversimplifies the context. Since the degree of congestion determines the individually available amount of the public input it can be shown that rivalry has an important and ambiguous impact on the capital investment decision. An increase in rivalry leads to a decrease in the individual's perceived available amount of infrastructure (e. g. traffic holdups) and thus spurs private capital investment. Nevertheless, there are counter working substitution effects between the private and the public input within the production process. Hence, in sparsely populated regions the low degree of rivalry may reduce capital investment whereas a high degree of rivalry, as e. g. in metropolitan areas like London, New York or Hamburg, may even encourage capital investment.

The paper adds to the existing literature as follows. While simple models assume that output might be transferred without additional costs into private capital the literature on investment theory which derives from the 'Tobin q' theory focuses on the impact of adjustment costs that arise e. g. due to an increase in demand. A survey of relevant approaches is given by Hamermesh and Pfann (1996) or Cooper and Haltiwanger (2003) whereas recent empirical studies can be found in Hall (2004). An industry specific discussion is done by Caballero and Engel (1999). Usually, those authors who focus on capital adjustment costs model them as relation between the investment in each period or time increment respectively and the firm's capital stock. An exception is the paper of Turnovsky (1996) who develops a one–sector endogenous growth model in which capital investment incurs adjustment costs that are related to governmental activity. This picks up the argument that firm specific aspects are not the unique determinants of capital adjustment. Aside from them also the economic environment like governmental activity gains importance.

Beginning with this thought we analyze within an dynamic model how a firm decides on capital investment if the accompanying adjustment costs are a function of governmental activity. Following Aschauer (1989b), Barro (1990) or Turnovsky (2000b) we interpret this governmental activity as being infrastructure. As a consequence thereof the adjustment costs are modeled as a function of the firm's investment and the governmental activity. Furthermore we include congestion effects of the infrastructure into the model as introduced by Edwards (1990) or Glomm and Ravikumar (1994) and also incorporated by Turnovsky (1996). Hence the individually available amount diminishes with an increase in aggregate economic activity. The impact of the public input within the dynamics of the model is twofold: on the one hand it enhances productivity of private capital. On the other hand the adjustment costs are also reduced by the extent of the available public input. With this respect our setup allows to disentangle the economic implications of infrastructure on the private investment decision in a production effect and an adjustment cost effect.

Until now the existing literature only refers to economic implications of the extent of infrastructure endowment whereas rivalry has been treated as exogenous and thus the impact of changes in the degree of congestion have not been analyzed in detail. But this is important since the degree of congestion is to some extent the outcome of a policy decision (see citation above). The dynamic equilibrium that consists of a constant investment ratio and a constant capital growth rate is derived. Carrying out comparative dynamics we show that a better regional endowment with infrastructure unequivocally spurs capital investment via the production and the adjustment cost effect. An ambiguous impact results from rivalry: A reduction of congestion also reduces the adjustment costs and with this stimulates private capital accumulation. But at the same time the marginal productivity of governmental expenditures increases due to enhanced individual availability and this leads to a crowding out of capital investment. The incentive for capital accumulation diminishes and the impact of the production effect on equilibrium capital becomes negative. While it is not possible to find closed solutions of the model we simulate the growth rate for different parameter constellations.

The paper is organized as follows: After presenting the analytical framework in part 2, the equilibrium in the decentralized economy is derived in Section 3. The first–best optimum is discussed in Section 4 and followed by policy implications in Section 5. Numerical simulations are carried out in Section 6. The paper closes with conclusions while formal details are relegated to the Appendix.

2 The analytical framework

Production technology and public inputs:
The economy is populated by N identical individuals who consume and produce a single good. Individual output is determined by privately owned capital, k, and the individually available amount of public services, G_s. The individual agent's production function

$$y = f(k, G_s) = \alpha\left(\frac{G_s}{k}\right)k \tag{1}$$

is homogenous of degree one in the two inputs.[1] It is assumed that the productive services derived by the representative individual from a given amount of public expenditure depend upon the usage of his individual capital stock relative to aggregate usage. This describes the situation of relative congestion that is introduced via a typical congestion function

$$G_s = Gk^{\sigma}K^{-\sigma} \quad 0 \leq \sigma \tag{2}$$

where K denotes the aggregate stock of private capital (see e. g. Barro and Sala-I-Martin (1992)) or Eicher and Turnovsky (2000). The case $\sigma = 0$ corresponds to a nonrival pure public input whereas $\sigma = 1$ reflects a situation of proportional (relative) congestion. Accordingly, the cases $0 < \sigma < 1$ correspond to situations of partial (relative) congestion, in the sense that given the individual stock of capital, government spending can increase at slower rate than does K and still provide a fixed level of services to the firm. The case $\sigma > 1$ describes a situation where congestion is so great that the public input must grow faster than the economy in order for the level of services provided to remain constant. This case is unlikely at the aggregate level, but may well be plausible for local public goods (see Edwards (1990)).

Introducing (2) into (1) the production function may be rewritten to be linear in capital and[2]

$$y = \alpha(gk^{\sigma-1}K^{1-\sigma})k \quad . \tag{3}$$

Capital accumulation, adjustment costs and the role of the public input:
Private investment, $\imath$, determines capital accumulation according to

$$\imath = \dot{k} - \delta k \tag{4}$$

with the rate of capital depreciation δ. The process of capital accumulation involves installation costs that are given by the function

$$i(1+\phi) = i\left(1+\phi\left(\frac{\imath}{G_s}\right)\right), \quad \phi' > 0, \quad \phi(0) = 0. \tag{5}$$

[1]We assume that labor is supplied inelastically.

[2]The relationship between the general production function and the intensive form can be found in Appendix 7 A.

Aside from private investment, (5) includes also the individually available amount of the public input, G_s. The linear homogeneity of this function is necessary if a steady–state equilibrium having ongoing growth is to be sustained. With specifying the adjustment cost function according to

$$\phi\left(\frac{\imath}{G_s}\right) = b\frac{\imath}{G_s}, \quad b \geq 0 \quad . \tag{6}$$

Installation costs are quadratic and amount to

$$\imath(1+\phi) = \imath\left(1 + b\frac{\imath}{G_s}\right) \quad . \tag{7}$$

The adjustment costs of private capital increase with the extent of investment and decrease with the amount of individually available public input, G_s. If $\phi' > 0$, the public input is not only productive but also facilitates private investment in such a way that installation costs are reduced. Another feature of (7) is that it introduces congestion effects in the adjustment costs. We shall analyze this in detail within the following sections.

The economic impact of public expenditure is crucially determined by the nature of the input and includes two dimensions:

- *Output and capital productivity*: As can be seen within production function (1) governmental expenditure is modelled as input that is complementary to private capital, $f_{k,G_s} > 0$. The productivity impact is mainly determined by the level of G_s and hence by the prevailing degree of congestion. Note that the level of G_s not only affects output directly but also acts indirectly via its impact on the marginal product of private capital.

- *Adjustment costs*: They are also affected by the public input via the ratio $\imath/G_s$. Again the nature of the public input gains importance: The adjustment costs increase with the level of congestion in the sense that less regions with less congestion (low ε) provide a more productive input to the firms and with this the adjustment costs are reduced. Aside from this the adjustment costs in the open countryside (low G) are higher than they are in areas that are already richly endowed with infrastructure (high G).

In the following part of the paper we refer to the term *adjustment cost effect* whenever we analyze effects that arise in the context of the adjustment costs. We use the term *production effect* to illustrate effects that influence output and/or capital productivity.

Lifetime utility and resource constraint:
The infinitely lived representative individual maximizes the intertemporal utility function

$$U = \int_0^\infty e^{-\rho t} \frac{c^{1-1/\varepsilon}}{1-1/\varepsilon} dt \tag{8}$$

with constant utility discounting, $\rho > 0$, and constant intertemporal elasticity of substitution, ε. [3] The individual decides about the utility maximizing consumption path, according to his budget constraint

$$(1-\tau_y)y = (1+\tau_c)c + (1-\tau_\imath)\imath(1+\phi) \tag{9}$$

where τ_y is the constant income tax rate, τ_c denotes the consumption tax rate, and $-\tau_\imath$ is the investment subsidy rate. The fiscal parameters are set by the government and are considered to be exogenous and constant within individual utility maximization.

3 Decentralized economy

The individual's problem is to choose the rate of consumption, c, of investment, $\imath$, and of capital accumulation, k, to maximize (8) subject to the budget constraint (9) and capital accumulation (4). The intertemporal maximization problem results in the Hamiltonian[4]

$$\mathcal{H} = e^{-\rho t}\left[\frac{c^{1-1/\varepsilon}}{1-1/\varepsilon} + \lambda\left[(1-\tau_y)f - (1+\tau_c)c - (1-\tau_\imath)\imath(1+\phi)\right] + q\lambda\left\{\imath - \delta k - \dot{k}\right\}\right]$$

[3] This specification of individual utility, which is quite usual in growth theory, is necessary in order to allow for steady state growth.

[4] We analyze households who are consumers and producers of the homogenous good at the same time. Equivalently, we could formulate households who supply capital and one unit of labor, and buy the consumption good, and firms which pay for the used production factors and supply the consumption good.

(10)

where λ is the shadow value of wealth in the form of new output and $q\lambda$ is the shadow value of the agent's capital stock. Analysis of the model is simplified by using the shadow value of wealth as numeraire. Consequently q is defined to be the market value of capital in terms of the (unitary) price of new output.

The necessary conditions which determine optimal consumption, investment and capital accumulation result in[5]

$$c^{-\frac{1}{\varepsilon}} = \lambda(1+\tau_c) \tag{11a}$$

$$(1-\tau_\imath)(1+\phi+\imath\phi_\imath) = q \tag{11b}$$

$$\frac{(1-\tau_y)f_k}{q} - \frac{(1-\tau_\imath)\imath\phi_k}{q} + \frac{\dot{q}}{q} - \delta = \rho - \frac{\dot{\lambda}}{\lambda} \tag{11c}$$

To fully specify the first order conditions, the transversality condition

$$\lim_{t\to\infty} q\lambda k e^{-\rho t} = 0 \tag{12}$$

has to be met, too.

Condition (11a) equates marginal utility of consumption to the shadow value of wealth, λ, which is given in units of new output. Condition (11b) equates the marginal installation costs to the market value of capital, q. Since marginal investment costs increase with $\imath/G_s$, either an increase in private investment or a decrease in the available public input raises the equilibrium market value of capital, $\partial q/\partial(\imath/G_s) > 0$. Condition (11c) determines optimal capital accumulation and reflects the results of the standard Keynes–Ramsey–Rule. Marginal return on consumption (RHS) is equilibrated with the rate of return on acquiring an additional unit of physical capital (LHS). The return on an additional unit of physical capital is composed of the following elements: (i) $\frac{(1-\tau_y)f_k}{q}$: after tax output per unit of installed capital (valued at the price q), (ii) $\frac{\dot{q}}{q} - \delta$: (net) rate of capital gain, (iii) $\frac{(1-\tau_\imath)\imath\phi_k}{q}$: reflects the fact that an additional resource of benefits of a higher capital stock is to reduce the installation costs associated with new investment.

We turn now to the equilibrium in a decentralized economy and derive the corresponding growth rate. While doing this we assume that the government sets its aggregate expenditure in proportion to the aggregate capital stock so that $g = G/K$.

[5] Note that – except in tax rates – indices refer to partial derivatives of a function with respect to the argument that is indexed.

The representative agent in making his individual investment decision assumes that he has a negligible impact on the aggregate capital stock and therefore ignores the linkages between its own investment decision and the resulting capital stock. This misperception is the source of a potential externality generated by the government expenditure.

In particular the firms perceive the individually available amount of the public input during the process of capital accumulation as

$$\frac{\partial G_s}{\partial k} = \sigma \frac{G_s}{k} \geq 0 \quad . \tag{13}$$

The firms' usage hence increases with a rise in the individual physical capital. This positive effect is reinforced via three channels: (i) the absolute size of the government, G, (ii) the prevailing degree of congestion, σ and (iii) the scale of the economy, N.[6] While the first effect is widely discussed within the congestion literature, the analysis usually is reduced to situations where $\sigma \leq 1$ and $N = 1$. Most of the existing models refrain from making statements about the economic impact of local public goods and of the size of the economy.

The economy's growth rate may be derived from the first order conditions (11c) and $\hat{\lambda}$ from (11a) which provides $\hat{c}$. Additionally we use (11b) to receive the relationship between $\imath/G_S$ and q

$$\frac{\imath}{G_s} = \frac{q/(1-\tau_\imath)-1}{2b} \tag{14}$$

The decentralized growth rate results as

$$\hat{c} = \varepsilon \left(\frac{(1-\tau_y) f_k}{q} + \frac{(q-(1-\tau_\imath))^2 g \sigma N^{1-\sigma}}{(1-\tau_\imath) q 4b} - \delta - \rho + \frac{\dot{q}}{q} \right) \quad . \tag{15}$$

It is composed of the net marginal product of capital including adjustment costs, the rate of time preference and the growth rate of the market value of capital.

[6]Note that in equilibrium with $K = Nk$ the available public input G_s does not increase with k. This misperception of the productivity impact of the public input leads to suboptimal overaccumulation by the individuals whenever $\sigma > 0$ since actually the individually available amount of the public input decreases with σ (see e. g.Turnovsky (1996)) who discusses this argument in the context of adjustment costs. In case of G_s being a pure public good ($\sigma = 0$) the value of (13) becomes zero.

The latter is assumed to be constant in equilibrium. This results directly from the definition of the equilibrium as a situation in which all macroeconomic variables grow at a constant and equal rate.

To specify q (11c), $\hat{\lambda}$ from (11a) and (14) are used and provide the equation of motion of the market value of capital as

$$\dot{q} = \frac{(2-\varepsilon\sigma)gN^{1-\sigma}}{4\varepsilon b(1-\tau_i)}q^2 + \left(\rho - \frac{1-\varepsilon}{\varepsilon}\delta - (1-\varepsilon\sigma)\frac{gN^{1-\sigma}}{2\varepsilon b}\right)q - \left((1-\tau_y)f_k + \frac{(1-\tau_\iota)\sigma g N^{1-\sigma}}{4b}\right) \quad . \tag{16}$$

Equalizing (16) to zero and solving the resulting quadratic equation for q implies[7]

$$\begin{aligned} q^d =& \frac{1-\tau_\iota}{2-\varepsilon\sigma}\left[1-\varepsilon\sigma - \frac{2b(\varepsilon\rho-(1-\varepsilon)\delta)}{gN^{1-\sigma}}\right. \\ & \left. + \sqrt{\left(1-\varepsilon\sigma-\frac{2b(\varepsilon\rho-(1-\varepsilon)\delta)}{gN^{1-\sigma}}\right)^2 + \frac{\varepsilon(2-\varepsilon\sigma)}{1-\tau_i}\left(\frac{4b(1-\tau_y)f_k}{gN^{1-\sigma}} + (1-\tau_\iota)\sigma\right)}\right] \end{aligned} \tag{17}$$

The decentralized equilibrium is defined by the growth rate (15) together with q^d from (17). Due to the discussed externality of capital accumulation it is not optimal.

4 First–best optimum

The first–best optimum reflects the decisions of the central planner who possesses complete information and chooses all quantities directly, taking into account the congestion caused by all agents and fixing the size of the governmental input. Using $K = Nk$ and $g = G/K$, the congestion function (2) modifies to

$$G_s = gN^{1-\sigma}k \tag{18}$$

[7]The quadratic equation (16) has two solutions. Within Appendix 7 it is shown that only q^d is a feasible solution.

and with this the planner's production function is given by

$$y = f(G_s, k) = \alpha(gN^{1-\sigma})k \quad . \tag{19}$$

The formal optimization is to maximize the agent's utility (8) subject to (4) and the economy–wide resource constraint

$$f = c + gk + \iota(1+\phi) \tag{20}$$

The resulting Hamiltonian is similar to (10) and the corresponding first–order conditions imply

$$c^{-1/\varepsilon} = \lambda \tag{21a}$$

$$1 + \phi + \iota\phi_\iota = q \tag{21b}$$

$$\frac{f_k - g}{q} - \frac{\iota\phi_k^p}{q} + \frac{\dot{q}}{q} - \delta = \rho - \frac{\dot{\lambda}}{\lambda} \tag{21c}$$

Equation (21b) determines the optimal ratio between private and public investment, $\frac{\iota}{G_s}$, as function of q. Note that the term ϕ_ι is also a function of this ratio. It is thus necessary to use the specified function (6) in order to solve (21b) for ι/G_s. As ratio between private and public investment results

$$\left(\frac{\iota}{G_s}\right)^* = \frac{q-1}{2b} \quad . \tag{22}$$

Together with $\phi_k^* = -\phi' \frac{\iota}{G_s}\frac{1}{k}$, $\frac{G_s}{k} = gN^{1-\sigma}$, and assuming $\dot{q} = 0$, the first–best growth rate is given by

$$(\hat{c})^* = \varepsilon\left[\frac{f_k - g}{q} + \frac{(q-1)^2 g^* N^{1-\sigma}}{q4b} - \delta - \rho\right] \quad . \tag{23}$$

It includes the market value of capital that may be derived from setting $\dot{q} = 0$ in (21c) and utilizing (21a). It is given by

$$q^* = \frac{1}{2-\varepsilon}\left[1-\varepsilon-\frac{2b(\varepsilon\rho-(1-\varepsilon)\delta)}{gN^{1-\sigma}} + \sqrt{\left(1-\varepsilon-\frac{2b(\varepsilon\rho-(1-\varepsilon)\delta)}{gN^{1-\sigma}}\right)^2+\varepsilon(2-\varepsilon)\left(\frac{4b(f_k-g)}{gN^{1-\sigma}}+1\right)}\right] \quad . \tag{24}$$

An optimum requires that both, the growth rate, $(\hat{c})^*$, as well as the size of the government, g^*, have to be set optimally. The latter is determined by the planner's optimization problem and leads to the additional optimality condition

$$f_g - k - \imath\phi_g = 0 \tag{25}$$

Utilizing (19), (22) and rearranging illustrates that the optimal value of g may be only determined implicitly as given by

$$\alpha'(g^* N^{1-\sigma}) + \frac{(q-1)^2}{4b} N^{1-\sigma} = 1 \tag{26}$$

The left–hand side measures the welfare benefits of a unit increase in government expenditure. These include: (i) the marginal benefits to the productivity of existing capital, f_g/k, and (ii) the marginal benefits from reducing the costs associated with installing new capital. An optimum requires that these marginal benefits equal the unit resource costs they absorb. Note that if $N = 1$ the optimal governmental size is independent from the degree of rivalry.[8] If instead $N > 1$ both, the population size and the nature of the public input as incorporated within the term $N^{1-\sigma}$, determine the resulting value of g^*. Two cases may be distinguished:

(i) $\phi = 0$: In case that the governmental input has no impact on investment costs, the corresponding benefits do not arise and optimal governmental expenditure is given if $\alpha'(gN^{1-\sigma}) = 1$. Increases (decreases) in $N^{1-\sigma}$ then induce a decreased

[8]This is a well–known result within congestion models that assume normalize population size to 1.

(increased) equilibrium level g^*. The nature of the public input becomes important since $N^{1-\sigma}$ increases (decreases) with N as long as $\sigma \leq 1 (\sigma > 1)$. Hence the optimal governmental size g^* decreases with a rise in N if the public expenditure is at most proportionally congested whereas g^* rises with N in case of regional public inputs.

(ii) $\phi > 0$: If adjustment costs arise, the marginal benefits from reduced installation costs allow for a decrease of the required marginal benefits from capital. The term α' is reduced thus increasing g^*. With respect to the impact of the nature of the public input the argumentation from above continues to hold.

Note that since the optimal value of g may not be determined explicitly it is not possible to find a closed–form solution of the first–best optimum. We solve this problem in Section **??** where we specify the production technology by a Cobb–Douglas production function.

5 Policy implications

The model has several policy implications: It allows to analyze optimal fiscal policy, i. e. to derive those tax rates that guarantee that the first–best optimum results as consequence of the decentralized decisions. Another implication refers to the nature of the public input and the consequences this has, in turn, on the resulting equilibrium. The public input may be characterized by the degree of congestion. Within the public good's literature it is argued that usually the amount and the type of governmental services are the outcome of voting mechanisms and thus represent the preferences of the individuals Although we do not endogenize the degree of congestion as consequence of a median voter model it is possible to carry out comparative analysis about how the decentralized decisions are influenced by the nature of the public input. Since these characteristics include different institutional arrangements (a highway ($\sigma = 1$) has to be provided in a different way than basic research ($\sigma = 0$) or a harbor ($\sigma > 1$)) we refer to the corresponding governmental activities as institutional policies. It is thus possible to compare this results with those of the first–best optimum and hence to assess alternative institutional policies with respect to their welfare implications. Note that due to the arising externalities growth maximization does not automatically imply welfare maximization. Hence the welfare effects of any policy depend considerably on the nature of the public input.

Fiscal policy:
Starting point are the components of the first–best optimum. We assume that the expenditure ratio is set optimally, $g = g^*$, neglect tax rates and analyze the components of both growth rates, $\hat{c}$ and $(\hat{c})^*$, as given by (15) and (23). These include identical rates of time preference, ρ, and depreciation, δ. The arising differences are due to alternative perceptions of the congestion function and affect the marginal product of capital, f_k, the market value of capital, q, and the marginal reduction of future adjustment costs, ϕ_k, in various ways. A survey of the details can be found in table 1.

Table 1: Optimal and decentralized determinants

	Planner	Individuals
congestion	$G_s = gN^{1-\sigma}k$	$G_s = gk^{\sigma}K^{1-\sigma}$
production	$y = \alpha(gN^{1-\sigma})k$	$y = \alpha(gk^{\sigma-1}K^{1-\sigma})k$
f_k	$f_k = \alpha(gN^{1-\sigma})$	$f_k = \alpha(gk^{\sigma-1}K^{1-\sigma}) \cdot (1-(1-\sigma)\eta_{y,g})$
capital value	see (24)	see (17)
ϕ_k	$\phi_k = -\phi' \frac{\iota}{G_s}\frac{1}{k}$	$\phi_k = -\phi' \frac{\iota}{G_s}\frac{1}{k}\sigma$
	$= -\frac{(q^*-1)^2}{2}\frac{1}{k} > 0$	$= -\frac{(q^d-1)^2}{2}\frac{1}{k} > 0$
g^*	$\alpha'(gN^{1-\sigma}) + \frac{(q-1)^2}{4b}N^{1-\sigma} = 1$	

(i) The marginal product of capital, as perceived by the individuals, exceeds the optimal one whenever the public input is characterized by rivalry. The gap between both unequivocally increases with σ since this spurs the incentive to accumulate capital.[9]

(ii) The decentralized market value of capital coincides with the optimal one if and only if $f_k^* - g = f_k^d$ and if the public production input is characterized by proportional congestion. The usage of the entire public input is equally distributed

[9] As argued above, this is the consequence of the external effect of capital accumulation as discussed within the congestion literature. If congestion is proportional and given that labor is supplied inelastically a distortionary income tax may be used to reduce the private capital accumulation activity. The corresponding governmental revenues suffice to finance the optimal amount of the public input (see e. g. **?** for an overview).

among the individuals and each may use $1/N$ parts. If $\sigma \neq 1$ and all things being equal the prevailing degree of congestion drives a wedge between q^d and q^*. While it is not possible to derive the relationships in case of a generalized production function explicitly, in case of a Cobb–Douglas production technology some results may be derived by numerical simulation.[10] The following relationship holds: $q^d \gtreqless q^*$ if $\sigma \gtreqless 1$ and the wedge increases with rising deviation from $\sigma = 1$. If the public input is less than proportionally congested ($\sigma < 1$) the decentralized market value of capital is suboptimally low and with this lowers the incentive to accumulate. The opposite applies in case of regional public inputs. Note that within the growth rates, the marginal product as well as the marginal reduction of future adjustment costs are valued at the price $q > 1$. This reduces the growth rates but while the reduction is suboptimally low as long as $\sigma < 1$ the opposite applies if governmental expenditure are used to provide a regional public good.

(iii) The marginal reduction of future adjustment costs, as measured by ϕ_k, may be suboptimally high or low, depending on the level of q and the prevailing degree of congestion. All things being equal the following results can be derived: If $\sigma < 1$ the corresponding ratio between private and public investment, $\imath/G_s$, is suboptimally low and with this spurs growth.[11] But note that this effect becomes weaker with increasing σ. If the public input is proportionally congested, $\sigma = 1$, the optimal ratio between private and public investment is realized and the individuals correctly perceive ϕ_k. However, if the governmental input is a regional public good, $\sigma > 1$, the individuals perceive the ratio $\imath/G_s$ as higher than the actual one and hence they do not realize future reductions of investment costs up to their full extent. This chokes capital accumulation and thus reduces the growth rate. Hence the growth impact of future adjustment cost reductions is considerably determined by the nature of the public input. Aside from positive growth effects of congestion, as those that also arise in the context of capital productivity, also negative effects of congestion may be identified.

It is neither for the general nor for the linear production technologies it is possible to derive the optimal value of g^*. With this we may not illustrate the first–best optimum and compare it with the decentralized decisions.[12]

[10]Details about the simulation are discussed in Section 6.

[11]This is hardly amazing as it follows the usual logic of the congestion models.

[12]However, the usual argumentation as carried out within the majority of growth models continues to hold: Congestion induces externalities thus driving a wedge between decentralized and

Institutional policies:
Aside from tax policy, the government may influence the decentralized decisions via changes of G_s. Governmental instruments thus include variations in the extent of the public input, G, as well as alternative institutional arrangements which end up in different σ. The impact of G_s on the equilibrium is twofold: as production input it determines the marginal product of capital, f_k, thus imposing a production effect on the growth rate. Additionally, due to the definition of ϕ, the public input also becomes important with respect to the resulting adjustment costs and hence governmental policy also induces adjustment cost effects.

It will be shown that an increase in G unequivocally spurs private capital productivity and reduces the adjustment costs. In contrast to this the impact of rivalry is ambiguous: while increases in σ always end up in higher adjustment costs, the growth effect of higher rivalry depends on the economy's size. The analysis is carried out via numerical simulations, assuming Cobb–Douglas production. Details can be found within the next section. It is shown that bigger economies imply lower adjustment cost effects whereas the productivity effect of the economy's size again is ambiguous and depends on the nature of the public input.

6 Numerical simulations

Since the first–best optimum may not be derived – even in the context of a specified production function – we are not able to carry out simulations with respect to that optimum. We therefore focus on the implications of alternative governmental sizes, g, and institutional arrangements, σ, on the decentralized equilibrium as given by (15) and (17).

The numerical simulations will be carried out for the Cobb–Douglas production technology

$$y = Ak^{\beta}G_s^{1-\beta}, \quad 0 < \beta < 1, \quad A > 0, \tag{27}$$

where A denotes the technological level of the economy. Since we analyze the decentralized equilibrium.

first–best optimum. Fiscal policy in form of taxes may then be used in order to correct for the market distortions and additionally to finance the provision of g^*.

$$f_k = [\beta + \sigma(1-\beta)]A(gN^{1-\sigma})^{1-\beta} \tag{28}$$

Taking the first derivative of (28) with respect to the degree of rivalry illustrates $\frac{\partial f_k}{\partial \sigma} \gtreqless 0 \Leftrightarrow \ln N \lesseqgtr 1/(\beta + \sigma(1-\beta))$. There exists a 'critical size' of the economy, determined by $\ln N$ and in the following denoted by $\tilde{N}$ that determines whether increased rivalry enhances or reduces private capital productivity.[13] To illustrate the different effects of governmental policy on the equilibrium growth rate we use the parameter specifications in table 2.

Table 2: Calibration parameters

production	$\beta = 0.7$
	$A = 0.4$
	$\delta = 0.05$
Taste and size of the economy	$\rho = 0.03$
	$N = 1, 2, 3$
Adjustment costs	$b = 0.5$
Governmental activity	$\tau_\iota = 0$
	$\tau_y = 0$
	$g = 0.0625, 0.125$
Congestion	$\sigma = 0, 1.5$

Economic impact of governmental size:

We begin the discussion with an analysis of the impact of alternative sizes of the government, g, on the resulting market value of capital, q^d and on the growth rate, $\hat{c}$.[14] A graphical illustration for alternative sizes of the economy is given within Figure 1. Bold lines represent a public input as local public good ($\sigma = 1.5$) while thin lines correspond to a pure public good ($\sigma = 0$).

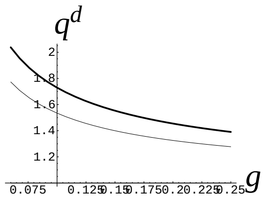
[13]For the assumed calibration parameters the critical values are given by $\tilde{N}(\sigma = 0) = 4.17$ and $\tilde{N}(\sigma = 1.5) = 2.39$ respectively.

[14]Due to the relation $g = G/K = G/Y * Y/K$, the level of g may be interpreted as representing the expenditure ratio.

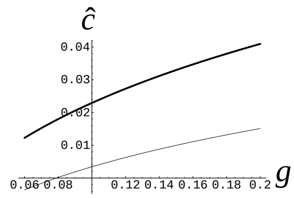
As can be seen within Figures 1(a), 1(c) and 1(e), the equilibrium market value of capital decreases with a rising expenditure ratio. This relationship holds for all sizes of the economy and for all types of public goods. Bigger governments imply a higher level of G_S thus reducing via the ratio $\imath/G_s$ thus lowering ϕ and hence q^d. Independent of the size of the economy, the equilibrium value of capital is always higher in case of the public input as regional public good (bold lines always above thin lines). This is caused by the fact that increased rivalry reduces $\imath/G_s$, thus increases ϕ and with this q^d. Besides, the wedge between the two functions in each Figure increases with the size of the economy. This result is due to 'critical' value of $\sigma = 1$ that separates the less than proportionally congested inputs from regional public goods: If $\sigma < 1$, an increase in N also increases G_s and thus reduces q. The thin lines shift downwards with an increased size of the economy. The opposite applies in case of the governmental input as regional public good, $\sigma > 1$. Then an increase in N reduces the individually available amount of the public input thus ending up in a higher market value of capita, q^d. The bold lines shift upwards and altogether the wedge between both lines rises.

The corresponding relationships between the growth rate and the government's size can be seen in Figures 1(b), 1(d) and 1(f). Again, $\hat{c}$ unequivocally increases with g. This is due to the complementarity of both production inputs; f_k unequivocally rises with g. But depending on the economy's size, either the growth impact of a regional public good ($N = 1$; bold lines above thin line) or of the pure public good (in case of $N = 3$; thin line above bold line) overweighs. This result is due to the ambiguous productivity effect the economy's size as mentioned above: If N lies below the 'critical value' the productivity enhancing effect of an increased size of the economy dominates. Then the growth rate increases with σ since the individuals accumulate more if rivalry arises. In case of a sufficiently big economy the negative scale effect within f_k becomes dominant. Since this effect is reinforced by σ the growth effect of a regional public good is smaller than it is the case if G_s is a pure public good.

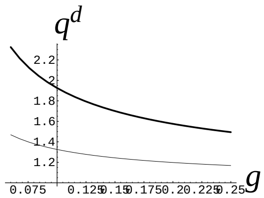
Economic impact of institutional arrangements:
Governmental policy also fixes the determinants of institutional arrangements and with this affect the equilibrium. These arrangements may be interpreted as the prevailing degree of congestion, i. e. the government may decide about the nature of the public input provided. Thus it is also possible to analyze the impact of institutional changes on the resulting equilibrium. Again we focus on the market



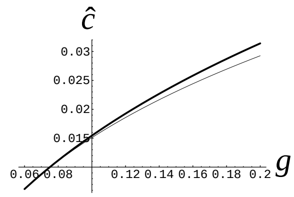
(a) $q(g)$ if $N = 1$



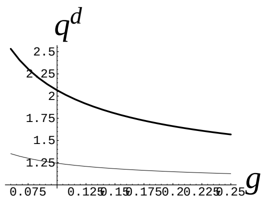
(b) $\hat{c}(g)$ if $N = 1$



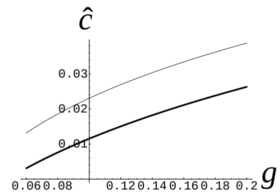
(c) $q(g)$ if $N = 2$



(d) $\hat{c}(g)$ if $N = 2$



(e) $q(g)$ if $N = 3$

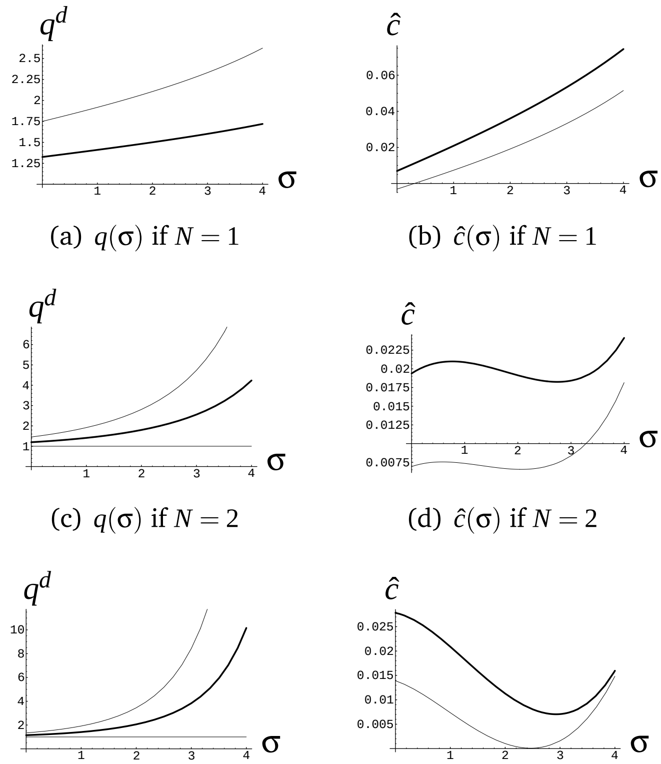


(f) $\hat{c}(g)$ if $N = 3$

Figure 1: The impact of governmental size on the level of q and the growth rate.
Bold lines correspond to local public goods ($\sigma = 1.5$) whereas thin lines reflect pure public goods ($\sigma = 0$)

value of capital as well as of the growth rate and discuss how they are determined via alternative σ. A graphical illustration can be found in Figure 2. Bold lines correspond to 'big' governments ($g = 0.125$) whereas thin lines reflect 'lean' governments ($g = 0.0625$).

As can be seen within Figures 2(a), 2(c) and 2(e), the market value of capital increases with σ for all sizes of the economy. Besides, for a given size of the economy, q^d is always higher in case of relatively small governments (thin lines above bold). This is due to the fact that not only an increase in σ but also a decrease in the size of the government (smaller g) reduce G_s and thus rises the corresponding level of q^d. If $\sigma = 0$, the initial value of q reduces with N since this rises G_s and with this reduces q^d. Independent of g the level of q^d rises more



(a) $q(\sigma)$ if $N=1$ (b) $\hat{c}(\sigma)$ if $N=1$

(c) $q(\sigma)$ if $N=2$ (d) $\hat{c}(\sigma)$ if $N=2$

(e) $q(\sigma)$ if $N=3$ (f) $\hat{c}(\sigma)$ if $N=3$

Figure 2: The impact of rivalry on the level of q and the growth rate. Bold lines correspond to 'big' government ($g = .125$) whereas thin lines represent a 'lean' government ($g = 0.0625$)

slowly with σ if N is small.

Figures 2(b), 2(d) and 2(f) again incorporate the conjunction of production and adjustment cost effect. Due to the complementarity of the inputs, the growth rate is always higher in case of a big government. If $N = 1$ the growth rate unequivocally increases with σ. This results because of the externality of private capital accumulation. The individuals perceive their capital productivity the higher the more rivalry arises since they do not realize the fact that in equilibrium the availability of the public input actually decreases if the input is used by more firms. Another result is obtained if $N > 1$. Then the growth rate increases until the above discussed 'critical value' $\tilde{N}$ is reached. A further increase of in N then induces a decreasing marginal product of capital. Altogether the production effect is positive until $\tilde{N}$ is reached and then becomes negative. While the production effect is ambiguous, the adjustment cost effect unequivocally increases with σ. Putting the

Table 3: Equilibrium values of q^d and $\hat{c}$; Cobb–Douglas technology (27)

		q		$\hat{c}\,(\%)$	
		$g = 0.0625$	$g = 0.125$	$g = 0.0625$	$g = 0.125$
$N = 1$	$\sigma = 0$	1.75024	1.45543	-0.310971	0.692866
	$\sigma = 1$	1.91767	1.56707	0.735463	2.08837
	$\sigma = 1.5$	2.00845	1.62607	1.30281	2.82587
$N = 2$	$\sigma = 0$	1.45543	1.27752	0.692866	1.93804
	$\sigma = 1$	1.91767	1.56707	0.734563	2.08837
	$\sigma = 1.5$	2.28690	1.79358	0.687331	2.0143
$N = 3$	$\sigma = 0$	1.34087	1.20755	1.39134	2.78304
	$\sigma = 1$	1.91767	1.56707	0.734563	2.08837
	$\sigma = 1.5$	2.48818	1.91261	0.37001	1.58621

effects together it is possible to derive a growth maximizing degree of rivalry. This latter is the smaller the higher N, since then the adjustment cost effect becomes more and more dominant.

7 Conclusions

This paper analyzes the growth impact of a congested public input. It is assumed that capital accumulation incurs adjustment costs that depend on the ratio between private and public investment. The analysis contributes to the existing literature via explicitly focussing on the impact of regional public inputs, on institutional versus fiscal policies and the size of the economy. After deriving the decentralized and the first–best optimum, fiscal and institutional policies are discussed. Since it is not possible to derive closed–form solutions of the equilibria, we specify the production function by a Cobb–Douglas technology and carry out numerical simulations. The main results may be summarized as follows. The governmental input not only enhances productivity of private capital but also affects the level of the adjustment costs. With this, productive governmental expenditure has a production effect and an adjustment cost effect. The extents of both are determined by the nature of the public input, i.e. by the degree of congestion. Another central feature is the size of the economy. The analysis is carried out with respect to two dimensions:

Fiscal policy:
First we compare the decentralized and the first–best optimum for a generalized production function. Due to congestion externalities, both do not coincide. The marginal product of capital as perceived by the individuals exceeds the optimal one whenever congestion arises. This ends up in overaccumulation of private capital thus reflecting the production effect of congestion. Basically taxing income may then be used as internalization instrument. This is a well–known result within the congestion literature.

However, with introducing adjustment costs, an optimal fiscal policy is more complex since the decentralized capital is only one determinant among others that have an impact on the resulting decentralized growth rate. The latter is also considerably determined by the market value of capital that itself is a positive function of the ratio between private and public investment and with this affects adjustment costs. These incorporate a static dimension (since the higher the ratio between private and public investment the higher are the costs in each period) and a dynamic dimension (since investment in one period contributes to an increase in the existing capital stock and with this reduces future investment costs). Hence, congestion also affects the arising adjustment costs. But in contrast to the production effect, the adjustment cost effect is ambiguous in the following way: As long as the governmental input is characterized by congestion that is at most proportional, the decentrally resulting market value of capital is suboptimally low. The opposite applies if the governmental input is a regional public good. Then a suboptimally high market value results and the incentive to capital accumulation is basically reduced. This reflects the static dimension of adjustment costs as argued above. But the model includes also a dynamic dimension: Since the market value of capital is negatively linked with future adjustment cost reductions, higher levels of that value spur growth. Hence the total growth impact of the market value of capital crucially depends upon which of the two dimensions overweighs.

With respect to the welfare economic implications it is shown that the individuals choose the first–best level of the capital value if the public input is proportionally congested. Then the individuals correctly perceive how the availability of the public input is affected by capital accumulation.

Merging the arguments, the welfare implications of a congested public input with adjustment costs may be summarized as follows: (i) The individually perceived marginal product of capital deviates from the optimal one whenever rivalry arises. The gap between both unequivocally increases with congestion. (ii) The optimal market value of capital only results in a market economy if the public input is proportionally congested. Hence increasing congestion reduces the gap between the decentralized and the optimal levels of capital value up to proportional congestion. Further increases in rivalry induce again deviations between the two parameters. (iii) Since a first–best optimum requires an optimal governmental size and since this may not be determined explicitly we abstract from deriving any optimal fiscal policy and reduce the argumentation to illustrate that decentralized and first–best optima deviate and that this is due to congestion.

Institutional policies:
We then focus on the growth effects of alternative institutional policies via carrying out comparative analysis. This is done in the context of a Cobb–Douglas production technology and for two alternative and exogenously given sizes of the government. Several results are derived from simulation: (i) Bigger governments reduce the market value of capital for all degrees of congestion and for all sizes of the economy. Hence increasing the size of the government unequivocally spurs growth. (ii) In smaller economies the provision of local public inputs go along with a bigger growth effect than the provision of pure public goods. However, the opposite applies to 'bigger' economies. Then the growth effect of pure public goods exceeds that of regional public goods. (iii) Independent of the size of the economy, the market value of capital increases with rivalry and decreases with the size of the government. (iv) The corresponding growth impact shows that bigger governments unequivocally go along with higher growth whereas the result becomes ambiguous with respect to the size of the economy: If the economy is relatively small, growth increases with congestion whereas for all sizes of the economy that exceed unity there exists a critical population size that separates the cases in which increasing congestion spurs growth from those that reduce growth.

To sum up: This paper analyzes the fiscal and institutional policies in case of a congested public input if capital accumulation incurs adjustment costs. Extending the existing literature with respect to regional public goods and the population size provides new and interesting results. We have not derived the optimal fiscal policy explicitly. Since this is an important feature much work to be done remains.

Appendix

A: Relationship between general production function and intensive form

As the production function (1) is assumed to be homogeneous of degree one in the two arguments k and G_s, Euler's theorem implies

$$y = f_k k + f_{G_s} G_s = k\left(f_k + f_{G_s}\frac{G_s}{k}\right) \quad . \tag{A.1}$$

Taking the total differential of (1) leads to

$$dy = f_k dk + f_{G_s} dG_s = k\left(f_k + f_{G_s}\frac{dG_s}{dk}\right) \quad . \tag{A.2}$$

Using the congestion function (2) together with $g = \frac{G}{K}$ and $K = nk$ implies

$$G_s = gN^{1-\sigma}k \tag{A.3}$$

and hence $\frac{G_s}{k} = \frac{dG_s}{dk}$. So any production function having the above homogeneity properties can be written in the 'Ak–form'

$$y = \alpha\left(gk^{-(1-\sigma)}K^{(1-\sigma)}\right)k = \alpha(gN^{1-\sigma})k \tag{A.4}$$

B: Derivation of (17)

Setting (16) equal to zero leads to the two solutions for the value of capital

$$\begin{aligned} q_{1,2} =& \frac{1-\tau_\iota}{2-\varepsilon\sigma}\Bigg[1-\varepsilon\sigma-\frac{2b(\varepsilon\rho-(1-\varepsilon)\delta)}{gN^{1-\sigma}} \\ & \pm\sqrt{\underbrace{\left(1-\varepsilon\sigma-\frac{2b(\varepsilon\rho-(1-\varepsilon)\delta)}{gN^{1-\sigma}}\right)^2+\frac{\varepsilon(2-\varepsilon\sigma)}{1-\tau_\iota}\left(\frac{4b(1-\tau_y)f_k}{gN^{1-\sigma}}+(1-\tau_\iota)\sigma\right)}_{\Delta}}\Bigg] \end{aligned} \tag{B.1}$$

We restrict on parameter values which lead to real values of q, that is to positive values of Δ. Since the second term of Δ ist positive, $\sqrt{\Delta} > 1-\varepsilon\sigma - 2b(\varepsilon\rho - (1-\varepsilon)\delta)/(gN^{1-\sigma})$. Hence, q_2 is negative and the unique solution for the steady state value of capital results in

C: Effects of a change in rivalry (institutional changes)

$$\frac{\partial \hat{k}}{\partial \sigma} = \frac{\partial \hat{k}}{\partial gN^{1-\sigma}} \frac{\partial gN^{1-\sigma}}{\partial \sigma} = -\frac{q/(1-\tau_l)-1}{2b} gN^{1-\sigma} \ln(N) < 0 \tag{C.1}$$

$$\frac{\partial \hat{k}}{\partial q} = \frac{gN^{1-\sigma}}{(1-\tau_l)2b} > 0 \tag{C.2}$$

$$\begin{aligned}\frac{\partial q}{\partial \sigma} =& \frac{\varepsilon q}{2-\varepsilon\sigma} - \frac{(1-\tau_l)\varepsilon}{(2-\varepsilon\sigma)^2} - \frac{(1-\tau_l)\varepsilon}{(2-\varepsilon\sigma)\sqrt{\Delta}}\left(1-\varepsilon\sigma - \frac{2b(\varepsilon\rho-(1-\varepsilon)\delta)}{gN^{1-\sigma}} - \frac{\sqrt{\Delta}}{2-\varepsilon\sigma}\right.\\ &\left. + \frac{(1-\tau_l)}{2-\varepsilon\sigma}\sqrt{\Delta}\left(\frac{4b(1-\tau_y f_k)}{gN^{1-\sigma}} + (1-\tau_l)\sigma\right)\right)\end{aligned} \tag{C.3}$$

$$\begin{aligned}\frac{\partial q}{\partial gN^{1-\sigma}} =& \frac{(1-\tau_l)}{(2-\varepsilon\sigma)(gN^{1-\sigma})^2}\left[2b(\varepsilon\rho-(1-\varepsilon)\delta)\right.\\ &\left. + \left(1-\varepsilon\sigma - \frac{2b(\varepsilon\rho-(1-\varepsilon)\delta)}{gN^{1-\sigma}}\right)\frac{2b(\varepsilon\rho-(1-\varepsilon)\delta)}{\sqrt{\Delta}} - \frac{\varepsilon(2-\varepsilon\sigma)4b(1-\tau_y)f_k}{2(1-\tau_l)}\right]\end{aligned} \tag{C.4}$$

$$\frac{\partial gN^{1-\sigma}}{\partial \sigma} = -gN^{1-\sigma}\ln(N) < 0 \tag{C.5}$$

$$\frac{\partial q}{\partial f_k} = \frac{\varepsilon 2b(1-\tau_y)}{\sqrt{\Delta} gN^{1-\sigma}} > 0 \tag{C.6}$$

$$\frac{\partial f_k}{\partial \sigma} = f_{kG_s} G\left(\frac{k}{K}\right)^{\sigma} \ln\left(\frac{k}{K}\right) < 0 \tag{C.7}$$

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