Fiscal Policy and Microstructure of Treasury Bonds

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June 1, 2006

Abstract

This paper presents an alternative approach to understand the role of insurer in an economy with incomplete market. Based in a simple Stokey-Lucas framework. I construct a model with microstructure in the treasury bond markets with heterogenous bidders.

The quantities and prices of the treasury bonds are a result of an auction mechanism, where the agents infer the private valuation distribution of others agents in order to obtain individual valuations. In this environment, the government's borrowing constraint is endogenously determined by strategic behavior, and therefore the government insurer role depends on the size of incompleteness of public debt markets.

Jel Classification : E62,H31, O15

Key Words: Optimal public debt and Treasury Auctions

1 Introduction

When public debt markets are exogenously incomplete a non contingentdebt profile are implementable according with state- competitive allocations [Angeletos 2001, Buera and Nicolli 2004]. Optimal structure of public debt

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plays a roll of insurer to face exogenous shocks [Mendoza 2004]. Nevertheless this result is sensitive to agent's perceptions and frictions in treasury-bonds markets. Sleet and Yeltekin [2003], Sleet and Albanessi [2004] suggest a imperfect government insurance by moral hazard problems and highlighted labor taxes persistence, positive covariance between government spending and value of public debt are signals of endogenous incomplete marktet. This paper is closely related with Sleet and Yeltekin [2004] with others characteristics of the treasury bonds market. Aspects as participation, collusion and expectations of the agents into public debt markets are determined in the incompleteness size of the market. This article shows a particular case where the government fails to achieve a full insurance. It also introduces a microstructure of treasury bonds markets in a stochastic Stokey-Lucas economy [1983] with heterogeneous agents. The heterogeneity comes from the amount of information that agents know. Level and prices of treasury bonds are a result of the auction mechanism, where agents infer the private valuation distribution of others agents in order to obtain individual valuations. The number of bidders is known by agents and the dynamic behavior of the agents is modeled as a Markovian game with incomplete information.

Related literature about the auctions in treasury bonds markets show the importance of pricing rules on revenues, level participation and incentives to collusion as determinants in allocations mechanism of the resources into the auctions. Bikhchandani and Huang [1993] analyzes the revenues behind uniform auction and discriminatory auctions and found that in the case of uniform auction format, these have a higher revenue because the winner's curse leads to lower demand a behind discriminatory scheme. Similar results are supported by Ausbel and and Cramton [1998], Chari and Weber [1992], and Nyborg and Sundaresan [1995] . Contrary, Wilson [1979] and Daripa [2000] argue that two auction regimes lead to the same revenue. As presented in Sareen [2004] the results are not conclusive and depend on the *price reveladed mechanism* used by the government in order to maximize the revenues.

Another crucial topic in treasury auction markets is participation. Sareen [2004] presents two different definitions of participation according with the market structure. The first definition is related to a number of bidders who participate in government securities auctions which in most cases are regulated by institutional features. The second definition is refer to "coverratio" which is defined as total bid amount related to issues amount. In this sense, when the cover ratio is high the difference between average yield of all accepted binds and cutoff yield are increases. One reason, is the aggressive behavior of bidders which lead to the probability of an eligible bidder winning positive amount in the auctions, this probability increases as the number of aggressive agents increases in participation by this bidder. An important feature are different auction mechanisms that lead to changes in the participation. In the discriminatory first price securities auctions, the agents try to infer the cutoff yield in the primary market and resale in the second market. This arbitrage opportunities lead to "winner's curse" between informed and uninformed players. If both types agents taken into account this problem, the informed bidders will shade theirs bids with respect to uninformed agents.

On the other side, the literature about the implications of public debt over the allocations of resources show how sensible is the optimal level of debt to the market structure. A first article in this line was Barro [1974]; he analyzed if public bonds induce changes in net wealth, and enunciated the Ricardian equivalence proposition, where the optimal debt depends on the initial level and have neutrality over all intertemporal path. This proposition was a good starting point to analyze the performance of public debt with other specifications, as distorsionary taxation, inheritance, finite horizons other features. The question arising behind those characteristics is whether the same results can be maintained. Other papers as Bernheim [1987] show how Ricardian equivalence fall behind other assumptions as finite horizons, liquidity constraints and distorsionary taxation. Blanchard and Fisher [1985] present a model of perpetual youth where the agents face a probability of dying at different moments of life cycle. In this model the taxes are partly shifted to the future generations and Ricardian equivalence fall. Marc Hayford [1989] explore the implications of Ricardian equivalence where consumption is very sensitive to current income. Behind imperfect market, the Ricardian equivalence holds if the quantity of borrowers pay taxes in the future.

Chari, Chrisitano and Kehoe [1993] analyze the optimal fiscal policy in a business cycle model and found that in cycle the optimal labor taxes are constant across the optimal path. The optimal expected capital taxes are zero and the role of debt is to absorb the macroeconomic shocks. The contributions of this paper are that the optimal labor tax do not respond to non anticipate shocks, therefore this type of taxes are effective to smooth taxation. In this model the distortions could be smoothing with state-contingent capital taxes equal to return state contingent debt. Aiyagari and McGrattan [1998] analyze the optimal debt where the markets are incomplete. In this model the public debt generate liquidity effects on the intertemporal allocations. The optimal debt merge a trade-off between liquidity effects and distortionary taxation. They found that the optimal level of debt of US is small. This model is based on Aiyagari's [1994] tradition where the agents face borrowing constraints and precautionary saving motives. The model show that the public debt lead to crowding out capital with higher interest rates a lowers consumption level.

Buera and Nicolini [2000] show how different maturity structures improve the incompleteness of the asset markets. The government use different maturity structure as an strategy to complete the assets market. A similar study is exposed by Angeletos [2001] who explores the implications of the contingent debt over the optimal maturity structure, in this case the maturity structure is an insurance to uncertainty. The Ramsey outcome is supported by the optimal maturity structure and by smoothing taxation behavior of the government.

Aiyagari et al [2002] create a model similar to Lucas and Stokey, where the markets are incomplete. In this case the debt and taxes process with limits of debt replicate the same results of smoothing intertemporal consumption with debt. In this case the contingent debt is a characteristic of optimal fiscal policy behind complete markets. Villaverde and Tsyvinski [2002] analyze the optimal fiscal policy without commitment. They use recursive approach of repeated game theory for characterizing the set of outcome equilibrium and the strategies that support the equilibria payoff. The model compares the solutions behind Ramsey policy and the Markov solution behind two environments. Another work is presented by Guerson [2003] who shows that the procyclical policy increases the volatility and reduce the welfare in the case where the government do not have a technology commitment. The author shows how the volatility is minimized if the government reduces the expenditure in the bad nature states evading the excessive accumulation of debt. As mentioned earlier Sleet and Yeltekin [2004] create a model where the government has private information and limited commitment, the implication is that the treasury bond markets are endogenously incomplete, which generate high volatility in the asset prices and reduce the possibility that the government uses the public debt an insurance to exogenous shocks.

In this article, optimal intertemporal consumption rules are analyzed taking into account different agents valuations. The auctions determine bond prices and therefore saving allocations. When an amount of bidders and private valuations change volatility in bond prices and returns, this generates a wealth effect not considered before in macroeconomic intertemporal models. In this model, the government losses a full control of the interest rate and therefore optimal structure of public debt is missed. This result show that the government's borrowing constraint is endogenously determined by the strategic behavior, and therefore the "government's insurer role" depends on the size of incompleteness of public debt markets. The rest of this paper is organized as follows. In the second section the model framework is presented. The third section is dedicated to explore different implications of the incomplete markets where risks are endogenously determined by agents. The model is parametrized in the section fourth and concluding remarks are given in the last section.

2 The Model

In this model we assume a simple microstructure of the public debt market. Following the Glosten-Milgrom model [1985] there exists a number of heterogenous informed participants. The market selection mechanism is based on the bid-ask process which explained the spread between different periods. We identify two types of participants: specialist and investor. The specialists set a bid and asking price with the interpretation that they are willing to sell one unit of stock at the ask and buy on unit of the public bond at the bid. An investor arrives at the market and is informed of the biding prices and asking prices at which time he is free to buy one unit at the ask or sell one unit at the bid or leave. This microstructure of the government securities is connected with intertemporal model in order to study welfare effects of public debt dynamic.

2.1 Environment

Consider an exchange intertemporal economy populated by N agents infinitely lived. In each period the economy faces a stochastic shock in the government expenditure which is modeled as a Markov process as Aiyagari et al [2002] $\pi(s'|s) = prob\{s_{t+1} = s'|s_t = s\}$. The history shocks are the realization of the past values $s^t = (s_0, s_1, ..., s_t) \forall s \in S$. In each period each agent has access to the treasury bonds market and her is a potential buyer in this market. Each i-buyer receives a private signal $m_i \in [\underline{m}, \overline{m}]$, with this information and given a specific state s, each i-agent demands a set of price-quantity (b_i^g, p) . This set is used to transfer consumption over time. Preferences of each agent are sequences of consumption and labor and they are represented as standard form:

$$\sum_{t=0}^{\infty} \sum_{s^{t}} \beta^{t} \pi \left(s^{t} | s \right) u \left(c \left(s^{t} \right), l \left(s^{t} \right) \right)$$

where β is the discount factor and the utility function is strictly concave. Technology is lineal with labor distorsionary taxation:

$$c_t\left(s^t\right) + g_t\left(s^t\right) = \left(1 - l_t\left(s^t\right)\right)\left(1 - \tau_t^l\left(s^t\right)\right)$$

The individual budget constraint is

$$c_t(s^t) + p_t(s^t)b_t^g(s^t) = (1 - l_t(s^t))(1 - \tau_t^l(s^t)) + p_{t-1}(s^t)b_{t-1}^g(s^t)$$

In each period, the government constraint is given by:

$$g_t\left(s^t\right) + p_t\left(s^t\right)b_{t+1}^g\left(s^t\right) = \left(1 - l_t\left(s^t\right)\right)\tau_t^l\left(s^t\right) + p_t\left(s^t\right)b_{t-1}^g\left(s^t\right)$$

2.2 Treasury Bond Market Aggrements

In each period of time, the government sells securities to i-agents. The microstructure of treasury bonds is analyzed through first price auction mechanism where many units of treasury bonds are offered to sale (multi-unit auction) and the bidders bids submit a demand function which represent the willingness to pay. Two different mechanism is used in the allocation bonds between agents, the first is discriminatory action or multiple price mechanism. In this auction format each bidder may submit multiple price quantity pair as theirs bids, then the market is cleared from the highest price for each point in the demand curve, in this case all winning biders pay their own bid price "pay- as- bid". Another hand, other mechanism used in the treasury market is the *uniform price auction* where the quantity of public bonds that government wishes to sell is a result of clearing conditions in the market.

We study the welfare implications of the fiscal policy when the heterogenous informed agents about the prices and return of the public bonds, in this sense the microstructure of public debt is modeled. The interaction of different agents is given by first price auction mechanism. The prices and quantities of bonds as a result of Nash equilibrium between market makers and government. An auction has N symmetric bidders which are denoted by (i = 1, ..., N,) each bidder have a valuation ϕ_i for the single bond and he receive a private signal X_i about ϕ_i . Bidder i only observe X_i prior to begins to auction but she face asymmetrical information because don't observe other valuations X_i .

The private signals and bidder's valuations are jointly distributed according to probabilistic distribution function $F(\phi_1, ...\phi_N, X_1, ..., X_N)$, the paradigm used in this case is the private value which implies that $(\phi_i = X_i)$. In this sense, the key point is estimate a structural model in order to identify the F distribution according with the bids winning data $h(p_1^b, ..., p_N^b)$. A bid function is a $b_i : F(\phi_1, ...\phi_N, X_1, ..., X_N) \to h(p_1^b, ..., p_N^b)$, we assume monotonicity assumption in such way the bid function ensure the mapping is one to one. The bid equilibrium function is strictly increasing and differentiable and the inverse function exist $\psi(b_i)$. Therefore, the first price auction mechanism implies given a observed object, the bidders solve the following problem according with the behavior of the other bidders. Demand function of each bidder is denoted $b_i^g(p(s^t)) = \varphi_i(p, x_i)$, in this sense the aggregate demand function is:

$$\widehat{b}^{g}\left(p\left(s^{t}
ight)
ight) = \sum_{i} b_{i}^{g}\left(p\left(s^{t}
ight)
ight)$$

the market clearing conditions satisfied:

$$Z^{g}\left(s^{t}\right) = \arg\max_{p^{*}}\left(\widehat{b}^{g}\left(p\left(s^{t}\right)\right) - \sum_{i} b_{i}^{g}\left(p\left(s^{t}\right)\right)\right)$$

Nevertheless, the demand function express the stochastic residual supply according with the following probability distribution over set market clearing prices:

$$\psi\left(p\left(s^{t}\right), b_{i}^{g}\left(p\left(s^{t}\right)\right)\right) = \Pr\left\{b_{i}^{g}\left(p\left(s^{t}\right)\right) \leq \widehat{b}^{g}\left(p\left(s^{t}\right)\right) - \sum_{i \neq j} b_{j}^{g}\left(p\left(s^{t}\right)\right)\right\}\right\}$$

For each (s), the bidder solve the following static stochastic problem:

$$\delta^{*}(s^{t}) = \max_{b_{i}(s^{t})} \int_{0}^{\infty} \left\{ \int_{0}^{b_{i}^{g}(p^{*})} \left[\mu(p, m_{i}) - b_{i}^{-1}(p(s^{t})) \right] dp \right\} d\psi(p, b_{i}^{g}(p))$$

Taken as given the state (s) the Euler condition for this optimization problem is:

$$p(s^{t}) = \mu\left(p\left(s^{t}\right), m_{i}\right) - \frac{\psi\left(p\left(s^{t}\right), b_{i}^{g}\left(p\left(s^{t}\right)\right)\right)}{\frac{\partial}{\partial p}\psi\left(p\left(s^{t}\right), b_{i}^{g}\left(p\left(s^{t}\right)\right)\right)}$$

This equation mean that in the optimum bid price is the spread between own bidders valuation and average yield or "bid-shading factor" Hortacsu [2002]. Then the Euler condition is a symmetric Bayesian Nash equilibrium, the bid shading factor reflect the "markup" associated to the best response of the other agents. Observe, that in the price bonds face a high variability given that the perceptions of the agents in to the auction. As address in the next subsection, the fluctuation prices introduce another channel in the understand the endogenous incomplete markets in a intertemporal economy.

2.3 Individual Intertemporal Optimization Problem

As mentioned above, each agent use public debt to smoothing consumption over time. The timing sequences of the events is as follow: Firstly agents participate in the auction in order to get public bonds and after her make saving decisions note that the natural limit is generated endogenously by the auction interaction. The problem that face each agent is:

The problem that solve each agent is:

$$\max_{\{c_{t}^{i}, b_{t+1}^{i}\}_{t=0}^{\infty}} E \sum_{t=0}^{\infty} \beta^{t} \pi \left(s'|s\right) u_{t}\left(c_{t}, l_{t}\right)$$

subject to:

$$c_{i}(s^{t}) + p(s^{t})b_{t}^{g}(s^{t}) = (1 - l_{t}(s^{t}))(1 - \tau^{l}(s^{t})) + p_{t-}(s^{t})b_{t-1}^{g}(s^{t})$$

where:

$$n(s^t) + l\left(s^t\right) = 1$$

with

$$\overline{b \sup} = \max\left[b_i^{g*}\right]$$

For simplicity 's denote the next value of variables. A Markov game framework applies to first price auction and it's represented using the Bellman equations as represent recursive decisions process. The Markov equilibrium concept involve best response of the bid function according with the intertemporal plans.

Definition 1 A Markov game $\langle S, I, A, P, M \rangle$ consists a nonempty sets of states S a finite sets of agents I and actions A, a transition function P : $S \times A \rightarrow S$ a reward function $M : S \times A \rightarrow R$ where:

$$M_i = u_t^i\left(c_t^i | \left(\Theta, F\right)\right)$$

That solve the following problem

$$V^{i}\left(s,b^{i}\right) = \sup_{\mu^{i}\left(b^{i}_{t}\right),b^{i'}}\left(u^{i}\left(c^{i}\right|\left(\Theta,F\right)\right)\right) + \beta \sum_{s'\in S}\int\pi\left(s'|s\right)V'\left(s',b^{i'}\right)dF\left(\psi\left(b^{i},b^{i'}\right)\right)$$

The Euler equation for this optimization problem are:

$$E_{t}\beta\frac{u_{c}\left(t\right)}{u_{c}\left(t+1\right)} \geq \mu\left(p\left(s^{t}\right), m_{i}\right) - \frac{\psi\left(p\left(s^{t}\right), b_{i}^{g}\left(p\left(s^{t}\right)\right)\right)}{\frac{\partial}{\partial p}\psi\left(p\left(s^{t}\right), b_{i}^{g}\left(p\left(s^{t}\right)\right)\right)} \quad \forall s, t \quad b > (1)$$

$$u_{l,t}^{i} = u_{c,t}^{i}\left(\left(1-\tau^{l}\left(s^{t}\right)\right)\right) \forall s, t \qquad (2)$$

Observe that agents valuation lead to in the intertemporal rate of substitution lead to "income fluctuation problem". In addition, labor taxation induce distortion in the allocation between leisure and consumption. Intuitively, strategic interaction between agents in the auction markets does influence a "endogenous risk premium" which restrict the set of asset trades the government can used in to order to insurance face to stochastic shocks, that is intrinsic principle of the model.

When replace the price-Kernel stochastic process in the government budget constraint the pseudo-implementability budget constraint:

$$b_{t}^{g}(s^{t}) = \left[\frac{u_{lt}(s^{t})}{u_{ct}(s^{t})}l_{t}(s^{t}) - g_{t}(s^{t})\right] + \left[E_{t}\beta\frac{u_{c}^{i}(t-1)}{u_{c}^{i}(t)} - \mu\left(p\left(s^{t}\right), m_{i}\right) - \frac{\psi\left(p\left(s^{t}\right), b_{i}^{g}\left(p\left(s^{t}\right)\right)\right)}{\frac{\partial}{\partial p}\psi\left(p\left(s^{t}\right), b_{i}^{g}\left(p\left(s^{t}\right)\right)\right)}\right]b_{t-1}^{g}(s^{t}) + \left[E_{t}\beta\frac{u_{c}^{i}(t-1)}{u_{c}^{i}(t)} - \mu\left(p\left(s^{t}\right), m_{i}\right) - \frac{\psi\left(p\left(s^{t}\right), b_{i}^{g}\left(p\left(s^{t}\right)\right)\right)}{\frac{\partial}{\partial p}\psi\left(p\left(s^{t}\right), b_{i}^{g}\left(p\left(s^{t}\right)\right)\right)}\right]b_{t-1}^{g}(s^{t}) + \left[E_{t}\beta\frac{u_{c}^{i}(t-1)}{u_{c}^{i}(t)} - \mu\left(p\left(s^{t}\right), m_{i}\right) - \frac{\psi\left(p\left(s^{t}\right), b_{i}^{g}\left(p\left(s^{t}\right)\right)\right)}{\frac{\partial}{\partial p}\psi\left(p\left(s^{t}\right), b_{i}^{g}\left(p\left(s^{t}\right)\right)\right)}\right]b_{t-1}^{g}(s^{t}) + \left[E_{t}\beta\frac{u_{c}^{i}(t-1)}{u_{c}^{i}(t)} - \mu\left(p\left(s^{t}\right), m_{i}\right) - \frac{\psi\left(p\left(s^{t}\right), b_{i}^{g}\left(p\left(s^{t}\right)\right)\right)}{\frac{\partial}{\partial p}\psi\left(p\left(s^{t}\right), b_{i}^{g}\left(p\left(s^{t}\right)\right)\right)}\right]b_{t-1}^{g}(s^{t}) + \left[E_{t}\beta\frac{u_{c}^{i}(t-1)}{u_{c}^{i}(t)} - \mu\left(p\left(s^{t}\right), m_{i}\right) - \frac{\psi\left(p\left(s^{t}\right), b_{i}^{g}\left(p\left(s^{t}\right)\right)\right)}{\frac{\partial}{\partial p}\psi\left(p\left(s^{t}\right), b_{i}^{g}\left(p\left(s^{t}\right)\right)\right)}\right]b_{t-1}^{g}(s^{t}) + \left[E_{t}\beta\frac{u_{c}^{i}(t-1)}{u_{c}^{i}(t)} - \mu\left(p\left(s^{t}\right), m_{i}\right) - \frac{\psi\left(p\left(s^{t}\right), b_{i}^{g}\left(p\left(s^{t}\right)\right)\right)}{\frac{\partial}{\partial p}\psi\left(p\left(s^{t}\right), b_{i}^{g}\left(p\left(s^{t}\right)\right)\right)}\right]b_{t-1}^{g}(s^{t}) + \left[E_{t}\beta\frac{u_{c}^{i}(t-1)}{u_{c}^{i}(t)} - \mu\left(p\left(s^{t}\right), m_{i}\right) - \frac{\psi\left(p\left(s^{t}\right), b_{i}^{g}\left(p\left(s^{t}\right)\right)}{\frac{\partial}{\partial p}\psi\left(p\left(s^{t}\right), b_{i}^{g}\left(p\left(s^{t}\right)\right)}\right)}\right]b_{t-1}^{g}(s^{t}) + \left[E_{t}\beta\frac{u_{c}^{i}(t-1)}{u_{c}^{i}(t)} - \mu\left(p\left(s^{t}\right), m_{i}\right) - \frac{\psi\left(p\left(s^{t}\right), b_{i}^{g}\left(p\left(s^{t}\right)\right)}{\frac{\partial}{\partial p}\psi\left(p\left(s^{t}\right), b_{i}^{g}\left(p\left(s^{t}\right)\right)}\right)}\right]b_{t-1}^{g}(s^{t}) + \left[E_{t}\beta\frac{u_{c}^{i}(t-1)}{u_{c}^{i}(t)} - \mu\left(p\left(s^{t}\right), m_{i}\right) - \frac{\psi\left(p\left(s^{t}\right), b_{i}^{g}\left(p\left(s^{t}\right)\right)}{\frac{\partial}{\partial p}\psi\left(p\left(s^{t}\right), b_{i}^{g}\left(p\left(s^{t}\right)\right)}\right)}\right)$$

Let $\Delta = \mu\left(p\left(s^{t}\right), m_{i}\right) - \frac{\psi\left(p\left(s^{t}\right), b_{i}^{g}\left(p\left(s^{t}\right)\right)\right)}{\frac{\partial}{\partial p}\psi\left(p\left(s^{t}\right), b_{i}^{g}\left(p\left(s^{t}\right)\right)\right)}$, $\theta = \frac{u_{lt}\left(s^{t}\right)}{u_{ct}\left(s^{t}\right)}l_{t}\left(s^{t}\right) - g_{t}\left(s^{t}\right)$, and = $\frac{u_{c}^{i}(t-1)}{u_{ct}\left(s^{t}\right)}$

 $\Xi = \frac{u_c^i(t-1)}{u_c^i(t)}$

Then path of debt following the stochastic sequence:

$$b_t^g\left(s^t\right) = \theta_k + \beta E_t \left(\Xi_t - \Delta_t\right)$$

Iterating debt equation:

$$b^{g}\left(s^{t}\right) = E\sum_{k=0}^{\infty}\beta^{k}\left(\Xi_{k}-\Delta_{k}\right)\theta_{k}$$

Whether a positive correlation between intertemporal consumption and budget supply then the government issues more debt in order to hedge against fiscal shocks. In the similar way when the agents have high uncertainly, the government debt play a role provide liquidity to agents in order to smoothing distortions over time. Nonetheless, the model show when the perceptions about fiscal situation contain high variability the insurer role of government debt decreases, that is partial insurer role of government. The timing of treasury contracts influence the final price. The optimal timing taking the endogenous set of bonds choices into account may save costs. In addition, The design of auction due intertemporal constraints and bidder heterogeneity which affect the optimal saving in the economy.

Consider $\overline{\sigma_t}^h$ as the strategy profile which consist in a set of optimal bids space $s \in S$ described by #. The value function associated to this problem is :

$$V^{h}\left(\overline{\sigma}^{h}\right) = \arg\max\sum_{t=0}^{\infty}\beta^{t}W\left(\overline{\sigma}^{h}\right)\left(\pi_{t}\right)$$

In similar way, the government has a sequence of treasury bonds which are issued taking account private information about the endowment shock in the economy $(s) \ \overline{\sigma_t}^g(s)$. The maximum value of government strategies are:

$$V^{g}(\overline{\sigma}^{g}) = \arg \max \sum_{t=0}^{\infty} \beta^{t} W(\overline{\sigma}^{g})(\pi_{t})$$

A subgame perfect equilibrium is a strategy profile $\sigma = (\sigma^h, \sigma^g)$ such that is a competitive equilibrium and the government plans are consistent over time, that is:

$$V^{g}\left(\overline{\sigma}^{g}\right) \geq V^{g*}\left(\overline{\sigma}^{g}\right)$$

2.4 Computational Method

The optimization problem described above is solved using a simple extension of the parametrized expectation approach (therefer PEA). This method is coherent with used by Macfadden [1989], Pakes and Pollard [1989] and Laffont, Ossard and Voung [1995]. The basic idea is develop a estimation method for the distribution of the private values. The algorithm is proposed in the way of simulated non linear least square objective function which is used in order to obtain the fundamental parameters of the private value distributions.

When the microstructure is modeled both with the intertemporal decisions of the agents, the optimal conditions is a set of the individual policy rules that involve intertemporal consumption and bid functions, then, the expectation of the agents is parametrized according with the state variables. This method could be used with other specification of the simulated econometric methods which are coherent with the PEA approach, for example adaptative models in auction is proposed by Bajari and Hortacsu [2005] and its is extended by learning dynamic behind PEA algorithm [see Heer and Maussner 2005].

The extension which is proposed in this papers follow two parts: 1) A simulated estimation of the private values in the auction. 2) Parametrized expectation of the state variables. The first part implies a coherent distribution

with observed winning auctions and the second integrate this distribution with invariant policy rules

The first order condition is transformed in the following problem in a least squares objective function which minimizes the sum of squared deviations between moments in the data and the theoretical sample moments of Z distributions.

$$Q^{*}(\theta^{*}) = \arg\min_{\theta^{*}} \frac{1}{J} \sum_{j=1}^{J} \left[\left(b_{j,t}^{w*} - E_{g,t}(b(\theta)) \right)^{2} \right]$$

Where J is the number of auctions, N number of the participants in the auctions. $b_{j,t}^{w*}$ is the winning observed in the J auction, and the $E_{g,t}(b(\theta))$ is the expected bid which is calculated by Z distribution. The procedure to obtain $E_{g,t}(b(\theta))$ for j = 1, ..., J is:

- 1. Draw $(X_1^s, ..., X_N^s)$ using the marginal distribution F_x for each parameter θ , holding a seed constant for random numbers across different values of θ .
- 2. Given the parameter value, evaluate the bids which correspond to the drawn signals: $(b_1^{w*} = \nu_1 X^s(\theta), ..., b_n^{w*} = \nu_n (X^s(\theta)))$
- 3. if $b_u^{w*} \neq \nu_u X^s(\theta)$ for any u auction, we need to repeat the step 1 to converge.

The first order conditions described above are expressed in the following system equation:

$$\Omega \left\{ \begin{array}{l} E_{t}\beta \frac{u_{c}(t)}{u_{c}(t+1)} - \mu \left(p\left(s^{t}\right), m_{i}\right) + \frac{\psi(p(s^{t}), b_{i}^{g}(p(s^{t})))}{\frac{\partial}{\partial p}\psi(p(s^{t}), b_{i}^{g}(p(s^{t})))} \\ u_{l,t}^{i} = u_{c,t}^{i} \left(\left(1 - \tau^{l}\left(s^{t}\right) \right) \right) \\ p(s^{t}) - \mu \left(p\left(s^{t}\right), m_{i} \right) + \frac{\psi(p(s^{t}), b_{i}^{g}(p(s^{t})))}{\frac{\partial}{\partial p}\psi(p(s^{t}), b_{i}^{g}(p(s^{t})))} \end{array} \right\} = 0 \\ = 0$$

According to the simulated estimation solve the system equation described above and found the optimal path for consumption, public debt, bonds prices and valuations.

3 Simulated estimation and Numerical Example

Based in the Colombian data, I consider the following treasury 54 bids which have the following statistics: the mean of bids are 108.375, Standard Deviation 5.803 and number of bidders are 70. The frequency is described in the next figure:



Graph 1

The stationary distribution of bonds and consumption are computed using a fixed point of the Euler conditions, and it is behavior is reported in appendix graph. The experiments is based in the stochastic shocks of the magnitude of one standard deviation, and the results show that the level of participation alters the free-risk return and prices of new debt. The consumption is less volatile and the mean value of household's asset (i.e public bonds) is increased. The intuition behind that, is when the participation is increased the agents have incentive to smoothing consumption, in this sense the public bonds react to insurer in the economy¹. Contrary, if the a few participation in the bond market lead to high asymmetrical market which is traduced in liquidity constraints and consumption volatility and less welfare.

¹Similar arguments are presented by Angeletos[1999], Nicolli[1999] and Mendoza [2001]



Figure 1: Figure 1

As illustrated in the different simulations, the bidders and policy functions shows precautionary behavior of agents when face high volatility. Different perceptions of agents induce a collusive behavior as showed in figures 3,5,6 which is coherent of trend in the policy functions. This particular flows of information induce to agents suffer of "winner-course" and therefore high variance in the consumption. The optimal bids function in the discriminatory case are presented in the graph 7 and benefits of auctions are reported in figure 8.

4 Appendix Graph



Figure 2: Figure 2



Figure 3: Figure 3



Figure 4: Figure 4



Figure 5: Figure 5





Figure 6: Figure 6



Figure 7: Figure 7



Figure 8: Figure 8



Figure 9

5 Concluding Remarks

This paper presents a model of intertemporal fiscal policy with a microstructure of the public debt markets. Different valuations in the treasury bond markets affect intertemporal consumption. When an economy face incomplete markets, the agent perceptions in the treasury market lead to endogenous borrowing limit. In this sense, if the government would like implemented an optimal public debt, the strategic interaction affect this level and induce changes in the optimal taxes and in this way the credible equilibria in the long run.

Different auction mechanism affect the strategic behavior in the treasury bond market. In this paper, I explore the case of the discriminatory auction and found irregular trends for different levels of participation. In this sense while the participation is increased, the volatility of consumption is reduced and the welfare increases. This a effect of government as insurer, but a difference with other papers the incompleteness is endogenous and its determined by a strategic behavior of the agents.

6 Bibliography

- 1. Atkeson, A. (1991). International lending with moral hazard and risk of repudiation. Econometrica 59, 1069-1090.
- Abreu, D., D. Pearce, and E. Stacchetti (1990). Toward a theory of discounted repeated games with imperfect monitoring. Econometrica 58 (5), 1041-1063.
- 3. Benhabib, J. and A. Rustichini (1997). Optimal taxes without commitment. Journal of Economic Theory 77 (2), 231-259.
- 4. Chamley, C. (1986). Optimal taxation of capital income in general equilibrium with infinite lives. Econometrica 54 (3), 607-622.
- Chang, R. (1998). Credible monetary policy in an infinite horizon model: Recursive approaches. Journal of Economic Theory 81 (2), 43167.
- Chari, V. V., L. J. Christiano, and P. J. Kehoe (1994). Optimal fiscal policy in a business cycle model. Journal of Political Economy 102 (4), 617-652.
- Chari, V. V., L. J. Christiano, and P. J. Kehoe (1995). Policy analysis in business cycle models. In T. F. Cooley (Ed.), Frontiers of Business Cycle Research, Chapter 12. Princeton, N. J.: Princeton University Press.

- Chari, V. V. and P. J. Kehoe (1990). Sustainable plans. Journal of Political Economy 98 (4), 783-802
- 9. Chari, V. V. and P. J. Kehoe (1993). Sustainable plans and debt. Journal of Economic Theory 61 (2), 230-261.
- 10. Fernandez-Villaverde, J., and Aleh Tsyvinski (2002). Optimal fiscal policy in a business cycle without commitment. Mimeo.
- Judd, K. L. (1985). Redistributive taxation in a simple perfect foresight model. Journal of Public Economics 28 (1), 5983.
- 12. Klein, P., P. Krusell, and J.-V. Rios-Rull (2004). Time-consistent public expenditures Mimeo.
- 13. Klein, P. and J.-V. Rios-Rull (2003). Time-consistent optimal fiscal policy. International Economic Review 44 (4), 1217-1246.
- Krusell, P., F. M. Martin, and J.-V. Rios-Rull (2004). Time-consistent debt. Mimeo.
- Kydland, F. E. and E. C. Prescott (1977). Rules rather than discretion: The inconsistency of optimal plans. Journal of Political Economy 85 (3).
- Kydland, F. E. and E. C. Prescott (1980). Dynamic optimal taxation, rational expectations, and optimal control. Journal of Economic Dynamics and Control 2, 78-91.
- 17. Ljungqvist, L. and T. J. Sargent (2004). Recursive Macroeconomic Theory, 2nd edition. The MIT Press.
- Lucas, R. E. J. and N. L. Stokey (1983). Optimal fiscal and monetary policy in an economy without capital. Journal of Monetary Economics 12 (1), 5593.
- 19. Phelan, C. and E. Stacchetti (2001). Sequential equilibria in a Ramsey tax model. Econometrica 69 (6), 1191-1518.
- Stokey, N. L. (1989). Reputation and time consistency. American Economic Review, Papers and Proceedings, 79 (2), 134-139.

- 21. Stokey, N. L. (1991). Credible public policy. Journal of Economic Dynamics and Control 15 (4), 627-656.
- 22. Stockman, D. (2001). Balance-budget rules: Welfare loss and optimal policies. Review of Economic Dynamics 4 (2).
- 23. Zhu (1992). Optimal fiscal policy in a stochastic growth model. Journal of Economic Theory 58 (2), 250-289.