

Monetary Policy and the Distribution of Money and Capital *

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Abstract

Existing search-theoretical models of money have in general abstracted from the existence and accumulation of other assets, in particular, capital. In this paper we present a model where the optimal portfolio allocation decision of agents is explicitly modeled. Trade frictions in a decentralized consumption goods market give rise to an endogenous role for money. Capital goods are assumed to be type-specific and traded in a centralized market. Uninsurable idiosyncratic uncertainty in production and trading opportunities leads to a non-degenerate distribution of wealth. By focusing on stationary equilibria we characterize numerically the wealth distribution and its composition. We further analyze the effects of monetary policy on the equilibrium patterns of exchange, the distribution of wealth, capital accumulation, and welfare. In particular, we show that a moderate expansionary policy, accomplished via lump-sum transfers, can lead to a steady-state increase in aggregate output, aggregate consumption, capital accumulation, and welfare.

JEL Classification: E40, E50

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1 Introduction

The question of what is the optimal rate of inflation has interested and puzzled economist for a long time. While most traditional representative agent models used for monetary policy analysis (for example, the money in the utility function or cash-in-advance models) prescribe the Friedman's Rule as the optimal monetary policy rule, the discussion in most Central Banks seems to be centered on what non-negative, but moderate, rate of inflation to target. Several explanations have been proposed for this apparent paradox. Some authors (e.g. Phelps (1973), Cooley and Hansen (1991)) have suggested that, given the need to finance some level of government expenditures, the inflation tax collected by following an expansionary monetary policy might be less distortionary than alternative forms of taxation.¹ Other authors have stressed the difficulties in implementing a contractionary monetary policy.

A recent branch in the literature has suggested an alternative explanation which representative agent models are unable to capture. A common feature to most of the literature that prescribes the optimality of the Friedman's Rule is that, in those models, there is completeness of contingent claims markets allowing agents to perfectly insure against any form of idiosyncratic risk (and thus the justification for a representative agent assumption). However, recent fundamental models of monetary economies based on microfoundations (Levine(1991), Kehoe, Levine, and Woodford (1992), Molico(1997), Deviatov and Wallace(2001)) have stressed that in an economy subject to uninsurable (due to the frictions that make money valuable) idiosyncratic risk (whether modeled as preference shocks, or uncertainty regarding consumption, production, or trading opportunities) a distributional expansionary

¹There is a large literature on optimal taxation and seignorage which we will not discuss here.

monetary policy can be welfare improving.² The presence of uninsurable idiosyncratic risk gives rise to endogenous wealth heterogeneity and a potential insurance role for expansionary monetary policy through redistribution. In these models, although the same forces that arise in the models for which the Friedman Rule is optimal are at play, the additional redistributive effect of monetary policy can potentially lead to an inflationary policy being beneficial.³

A limitation of this work is that it assumes an extreme degree of market incompleteness - money is the only asset.⁴ In this paper, we extend this literature by considering a search-theoretical model of money where agents can hold and accumulate both money and capital. By allowing agents to accumulate capital (and thus partially self-insure) we might reduce the insurance role provided by an expansionary monetary policy. On the other hand, by affecting the asset's portfolio allocation decision of agents, monetary policy can potentially affect the productive capacity of the economy. The goal of this paper is to characterize the portfolio allocation decision of the agents, and analyze and quantify the effects of expansionary monetary policy on the stationary equilibrium patterns of exchange, distribution of prices, wealth portfolio allocation, distributions of money and capital, and welfare.

We consider a search-theoretical model of money where both goods and money

²For other examples in which a redistributive expansionary monetary is beneficial in the presence of uninsurable idiosyncratic risk, see, for example, Edmond (2002), Green and Zhou (2004), Bhattacharya, Haslag and Martin (2004), Berentsen, Camera and Waller (2005).

³Another branch that has focused on the distributional effects of monetary policy is the limited participation literature. This literature is concerned with the role of monetary policy in the short run. Recent papers representative of the state of the art in such literature include Alvarez, Atkeson and Kehoe (2002), Alvarez, Atkeson, and Edmond (2003), Chiu (2004), and Williamson (2005).

⁴Other papers in the search monetary literature that have incorporated capital into the model include Shi (1999a), Shi (1999b), and Arouba, Waller and Wright (2004). In these models, however, given their tractability assumptions, agents are able to fully insure against any idiosyncratic risk, leading to a degenerate distribution of wealth (money and capital). In this paper agents have no access to such risk-sharing mechanism.

are perfectly divisible. Unlike most of the models in the literature we allow consumers to store their own (and only their own) consumption good, which can be used for consumption or as an input (capital) in the production of an agents type-specific production good. By this assumption, we preclude capital (the stored consumption good) to compete with money as a medium of exchange, allowing it however to still serve as a store of value and to serve a productive role.⁵ We assume that there are two types of markets that open sequentially. A decentralized goods market where production and trade take place, and a centralized capital market where agents can re-optimize their asset's portfolio but no production can take place.⁶ Trade frictions in the decentralized consumption goods market give rise to an endogenous role for money. In this market agents meet randomly and bilaterally and bargain over the amounts of money and goods to be traded. In the centralized market agents are anonymous and trade money for capital (consumption good).⁷

The presence of uninsurable idiosyncratic risk in trade opportunities leads to heterogeneity in wealth and in asset's portfolio choices. Thus the state of the economy will be described by the joint distribution of money and capital among the agents in the economy, making it impossible to provide an analytical solution to the model. As such, we develop a numerical algorithm that allows us to solve for and characterize stationary equilibria. Furthermore, we use these numerical

⁵In this paper we abstract from the issue of co-existence of different media-of-exchange and potential rate-of-return dominance. A recent paper that attempts to address those issues while abstracting from the redistributive effects of monetary policy is Lagos and Rocheteau (2004).

⁶The role of the centralized market is very different that the one in Lagos and Wright(2005). There production of general goods takes place in the centralized market. Given the assumption that current-period utility is linear in labor supply in that market, agents optimally choose to take the same amount of money holdings into the decentralized market leading to a degenerate distribution of money. The linearity of preferences over labor supply serves as an insurance against the idiosyncratic risk the agents face in the decentralized market. Still, in both models the centralized market serves as a market for liquidity.

⁷We assume that investment is reversible, that is, agents can eat their stored consumption good (capital) at the end of any period. So far, we have not analyzed the implications of the alternative assumption - irreversibility of investment.

methods to provide quantitative answers to monetary policy questions.

Regarding the effects of monetary policy, we show that money is always neutral in the long-run and also in the short-run if monetary injections are accomplished via proportional transfers. However, if the transfer mechanism generates a redistribution of wealth, increases in the level of the money supply will have short-run real effects. Furthermore, regarding the effects of changes in the rate of monetary growth, we show that proportional transfers are superneutral but that redistributive lump-sum transfer will have permanent real effects. Our preliminary numerical results suggest that a moderate rate of monetary expansion can lead to an increase in steady-state aggregate output, aggregate consumption, capital accumulation, and welfare. Also, the average fraction of time spent working might decrease. As such, our example suggests that Friedman's Rule might not be optimal in such environment.⁸

The remainder of the paper is organized as follows. In section 2, we describe the model economy. In section 3, we define a recursive competitive equilibrium for this economy. We then proceed to develop and present a numerical algorithm that allows us to compute stationary equilibria of the model. In section 5, we use this algorithm to characterize the stationary equilibria of the model and to illustrate the effects of a redistributive expansionary monetary policy. Section 6 presents some discussion and concludes.

⁸Note that, due to computational restrictions, we only compare across steady states and thus ignore potential important effects along the transition path. In particular, given the endogenous capital accumulation and labor supply decisions, particular care in the interpretation of the results is required.

2 The Model

Population, Goods and Specialization

Time is discrete. There is a $[0, 1]$ continuum of infinitely lived agents who specialize in the consumption, storage, and production of perfectly divisible goods. There are N varieties of goods and N types of agents, $i = 1, \dots, N$ ($N \geq 3$), with an equal measure $\frac{1}{N}$ of agents of each type. Agent type i consumes good i , has the ability to store solely good i , and produces good $i + 1 \pmod{N}$ ⁹.

Technology

Production requires the use of capital (an agent's stored consumption good) and labor inputs. Capital is production good specific. The production of good $i + 1$ requires the input of good i . Agent type i combines good i and his own labor effort to produce good $i + 1$ according to a Cobb-Douglas production function:

$$y_{i+1} = f(k_i, l) = Ak_i^\alpha l^{1-\alpha}, \quad 0 < \alpha < 1, A > 0.$$

Capital depreciates at rate $0 < \delta < 1$ ¹⁰.

Preferences

Agents derive utility $u(c)$ from consuming c units of their type-specific consumption good and disutility $g(l)$ from providing l units of labor effort. Assume u is twice continuously differential with $u(0) = 0$, $u'(c) > 0$, and $u''(c) < 0$. Also, assume g is twice continuously differentiable with $g(0) = 0$, $g'(l) > 0$, $g''(l) > 0$,

⁹Note that, unlike what it is commonly assumed in the search-theoretical literature, e.g. Trejos and Wright (1995), we allow agents to store goods which can be used as an input in production. However, each type of agent can **only** store his/her own consumption good which prevents goods from being used as a medium-of-exchange, although they can serve as a store-of-value. The issues of coexistence of different media-of-exchange and potential rate of return dominance are interesting and important but we will not pursue them in this paper.

¹⁰For simplicity, we assume capital depreciates at the same rate independently of whether it has been used in production or simply stored. More generally, one could consider different rates of depreciation.

$\lim_{l \rightarrow 0} g'(l) = 0$, and $\lim_{l \rightarrow 1} g'(l) = +\infty$. The per-period utility is given by

$$U(c, l) = u(c) - g(l).$$

Agents discount the future at discount factor $0 < \beta < 1$.

For notational convenience, let $v(y, k)$ denote the disutility of producing an amount y of output when using an amount k of capital.

$$v(y, k) \equiv g \left[\left(\frac{y}{Ak^\alpha} \right)^{\frac{1}{1-\alpha}} \right]$$

Assets

In addition to capital, in this economy there is another, perfectly divisible, and costlessly storable, object which cannot be produced or consumed by any private individual, called *fiat money*. Agents can hold any nonnegative amount of money $\hat{m} \in \mathfrak{R}_+$. Let M_t denote the money supply at the beginning of period t .

Markets

There are two markets which open sequentially during each period. A decentralized goods market and a centralized capital market. In the decentralized goods market agents meet randomly and bilaterally, bargain, produce, and trade. In the centralized capital market agents are anonymous, and can optimize their assets portfolio by purchasing or selling capital, but cannot produce. Note that, the centralized market is a market for liquidity where stored goods can be traded for money but where no production is allowed. Allowing for production in this market would eliminate the need for a medium of exchange.¹¹

Monetary Policy

¹¹The role of the centralized market is very different than the one in Lagos and Wright(2005). There agents can produce, trade and consume general goods. Given the assumption that utility is linear in labor supply, agents are able to fully insure against the idiosyncratic uncertainty they face in the decentralized market, leading in equilibrium to a degenerate distribution of wealth.

Agents receive monetary transfers, $\hat{\tau}_t(\hat{m}, k)$ at the entrance of the centralized market, before trade occurs. In what follows we express all nominal variables as fraction of the beginning of the period money supply (before the current period's monetary transfer).

$$m \equiv \frac{\hat{m}}{M}$$

$$\tau(m, k) \equiv \frac{\hat{\tau}(\hat{m}, k)}{M}$$

The aggregate money supply grows at a constant rate μ ,

$$M_{t+1} = (1 + \mu)M_t.$$

Decision Timing

The timing of the model is as follows. At the beginning of each period agents are randomly matched with a potential trading partner in the decentralized goods market. If in the meeting there is single-coincidence-of-wants,¹² agents bargain over the quantity of money and goods to be exchanged, and production and exchange take place. For convenience, we assume that capital depreciates after the decentralized market closes. After that, the centralized capital market opens. Monetary transfers take place, agents choose consumption and the asset's portfolio allocation for next period, trade, and consume.¹³

This concludes the description of the environment. In what follows, we will build gradually towards the definition of equilibrium.

¹²Note that, by assumption, we excluded the possibility of double-coincidence-of-wants meetings and thus barter. Allowing for barter does not change qualitatively the analysis of the paper.

¹³We assume that investment is reversible, i.e., agents can eat their stored consumption/capital good. Furthermore, they can potentially purchase consumption good from other agents of their type. The price of the good however, will never be cheaper than the price at which an agent can acquire the good in the decentralized market, as will be shown below.

3 Equilibrium

In this section we define a recursive equilibrium for this economy. We begin by describing the individual and aggregate state variables. An individual's state variable consists of his current portfolio of money balances, as fraction of the current money supply, and capital. For notational convenience, we denote an individual's state variable by $s \equiv (m, k)$, where $s \in S \equiv \mathbb{R}_+ \times \mathbb{R}_+$. The aggregate state variable is, in turn, defined as the current probability measure over money holdings (measured as fraction of the current money supply) and capital, denoted λ . Let $\lambda : \mathcal{S} \rightarrow [0, 1]$, where $\mathcal{S} \equiv \mathcal{B}_{\mathbb{R}_+} \times \mathcal{B}_{\mathbb{R}_+}$ denotes the Borel subsets of S .

The agent takes as given the law of motion of the aggregate state variable, $\lambda' = H(\lambda)$, where prime denotes the future period. Later, we will describe in detail the law of motion. Also, the agent takes as given the price of capital in the centralized market, p_k , as a function of the current aggregate state, that is, $p_k : \Lambda \rightarrow \mathbb{R}_+ \setminus \{0\}$, where Λ denotes the space of probability measures over \mathcal{S} .

In what follows we describe the value functions, taking as given the terms of trade in the decentralized market. Let $y(s_b, s_s; \lambda)$ and $d(s_b, s_s; \lambda)$ denote, respectively, the amount of goods and money (measured as a fraction of the current money supply) traded in the decentralized goods market between a buyer with individual state s_b , and a seller with individual state s_s , when the aggregate state is λ , taking as given the pricing function p_k . Later, we will specify the determination of the terms of trade.

Let $V(s; \lambda)$ be the value function for an agent holding portfolio s , when the aggregate state is λ , at the entrance of the decentralized market. Assume $V(\cdot; \lambda)$ is a continuous function. Also, let $W(\bar{m}, \bar{k}; \lambda)$ be the value of entering the centralized capital market holding the portfolio (\bar{m}, \bar{k}) when the aggregate state is λ .

We can then write the following functional equation expressing the relationship between the two value functions.¹⁴

$$\begin{aligned}
V(s; \lambda) = & \frac{1}{N} \int_S W[m - d(s, s_s; \lambda), (1 - \delta)k + y(s, s_s; \lambda); \lambda] \lambda(ds_s) \\
& + \frac{1}{N} \int_S \{-v[y(s_b, s; \lambda), k] + W[m + d(s_b, s; \lambda), (1 - \delta)k; \lambda]\} \lambda(ds_b) \\
& + \frac{N - 2}{N} W[m, (1 - \delta)k; \lambda]. \quad (1)
\end{aligned}$$

The first integral term on the r.h.s. is the expected value of being a buyer, the second integral term is the expected value of being a seller, and the last term is the expected value of a no single-coincidence-of-wants meeting, in which case no trade occurs.

The value of entering the centralized market holding portfolio \bar{s} , given the aggregate state λ , is defined as

$$W(\bar{s}; \lambda) = \max_{c \geq 0, s' \geq 0} u(c) + \beta V(s'; \lambda') \quad (2)$$

subject to:

$$\frac{\bar{m}}{p_k(\lambda)} + \bar{k} + \frac{\tau(\bar{m}, \bar{k})}{p_k(\lambda)} = c + \frac{m'}{p_k(\lambda)}(1 + \mu) + k' \quad (3)$$

$$\lambda' = H(\lambda). \quad (4)$$

By the *Theorem of the Maximum*, $W(\cdot; \lambda)$ is a continuous function and set of optimizers is a nonempty, compact-valued, and u.h.c correspondence. By the *Measurable Selection Theorem*, this correspondence permits a measurable selection.

¹⁴For now, assume $d(\cdot, \cdot; \lambda)$, $y(\cdot, \cdot; \lambda)$, and $W(\cdot; \lambda)$ are measurable functions. We will later show that, given $V(\cdot; \lambda)$ is continuous, $W(\cdot; \lambda)$ is continuous, and thus measurable, and that $d(\cdot, \cdot; \lambda)$ and $y(\cdot, \cdot; \lambda)$ can be defined as being measurable selections from a u.h.c. correspondence.

Define χ , Ψ , and κ as (measurable) sections of such correspondence, with

$$c = \chi(\bar{s}; \lambda) \quad (5)$$

$$m' = \Psi(\bar{s}; \lambda) \quad (6)$$

$$k' = \kappa(\bar{s}; \lambda). \quad (7)$$

We will now describe the determination, by bargaining, of the terms of trade in the decentralized market. We will adopt the generalized Nash bargaining solution where the buyer has bargaining power θ , $0 < \theta < 1$, and with the threat point for an agent given by his continuation value $W(\bar{s}; \lambda)$.

For notational convenience, define $\Sigma_b(\bar{y}, \bar{d}, s_b, s_s; \lambda)$ and $\Sigma_s(\bar{y}, \bar{d}, s_b, s_s; \lambda)$ to be, respectively, the net surplus from trading an amount of good \bar{y} for a fraction of the current money supply \bar{d} , for a buyer and a seller holding respectively, the portfolios s_b and s_s , given the aggregate state λ .

$$\Sigma_b(\bar{y}, \bar{d}, s_b, s_s; \lambda) \equiv W[m_b - \bar{d}, (1 - \delta)k_b + \bar{y}; \lambda] - W[m_b, (1 - \delta)k_b; \lambda]$$

$$\Sigma_s(\bar{y}, \bar{d}, s_b, s_s; \lambda) \equiv -v(\bar{y}, k_s) + W[m_s + \bar{d}, (1 - \delta)k_s; \lambda] - W[m_s, (1 - \delta)k_s; \lambda].$$

Consider the following generalized Nash bargaining problem.

$$\Sigma(s_b, s_s; \lambda) = \max_{(\bar{y}, \bar{d}) \in G(s_b, s_s; \lambda)} \Sigma_b(\bar{y}, \bar{d}, s_b, s_s; \lambda)^\theta \Sigma_s(\bar{y}, \bar{d}, s_b, s_s; \lambda)^{1-\theta}, \quad (8)$$

where $G : S \times S \rightarrow S$ is a correspondence defined by $G(s_b, s_s; \lambda) \equiv \{(\bar{y}, \bar{d}) \in S : 0 \leq \bar{d} \leq m_b, 0 \leq \bar{y} \leq Ak_s^\alpha\}$. Once again, by the *Theorem of the Maximum*¹⁵ and the *Measurable Selection Theorem* the correspondence $D(s_b, s_s; \lambda) = \{(\bar{y}, \bar{d}) \in G(s_b, s_s; \lambda) : \Sigma_b(\bar{y}, \bar{d}, s_b, s_s; \lambda)^\theta \Sigma_s(\bar{y}, \bar{d}, s_b, s_s; \lambda)^{1-\theta} = \Sigma(s_b, s_s; \lambda)\}$ permits a measurable selection. Define $y(s_b, s_s; \lambda) : S \times S \rightarrow \mathbb{R}_+$ and $d(s_b, s_s; \lambda) : S \times S \rightarrow \mathbb{R}_+$

¹⁵Note that, given $W(\cdot, \cdot; \lambda)$ is continuous, the objective function is continuous. Furthermore, it is easy to show that the correspondence G is compact-valued and continuous.

to be (measurable) sections of such correspondence.¹⁶

Before defining a recursive equilibrium for this economy, we describe the law of motion $\lambda' = H(\lambda)$. We begin by describing the evolution of the aggregate state from the beginning of the centralized market to the beginning of the next decentralized market. Define $\Pi : S \times \mathcal{S} \rightarrow [0, 1]$ to be

$$\Pi(\bar{s}, B; \lambda) = \begin{cases} 1, & \text{if } [\Psi(\bar{s}; \lambda), \kappa(\bar{s}; \lambda)] \in B; \\ 0, & \text{otherwise.} \end{cases} \quad (9)$$

Given that, for each \bar{s} , $\Pi(\bar{s}, \cdot; \lambda)$ is a probability measure on (S, \mathcal{S}) , and, for each $B \in \mathcal{S}$, $\Pi(\cdot, B; \lambda)$ is a \mathcal{S} -measurable function¹⁷, Π is a well defined transition function. Let $\Omega(\bar{s})$ denote the distribution at the entrance of the centralized market, then

$$\lambda'(B) = \int_S \Pi(\bar{s}, B; \lambda) \Omega(d\bar{s}) \quad \forall B \in \mathcal{S}. \quad (10)$$

We now describe the evolution of the aggregate state from the beginning of the decentralized market to the beginning of the centralized market. Let $T = \{buyer, seller, neither\}$ and define the space (T, \mathcal{T}) , where \mathcal{T} is the σ -algebra. Define the probability measure $\tau : \mathcal{T} \rightarrow [0, 1]$, with $\tau(buyer) = \tau(seller) = \frac{1}{N}$, and $\tau(neither) = \frac{N-2}{N}$. Then, (T, \mathcal{T}, τ) is a measure space. Define an *event* to be a pair $e = (t, s)$, where $t \in T$ and $s \in S$. Intuitively, t denotes and agents trading status and s the portfolio of his current trading partner. Let (E, \mathcal{E}) be the space of such *events*, where $E = T \times S$ and $\mathcal{E} = \mathcal{T} \times \mathcal{S}$. Furthermore, let $\xi : \mathcal{E} \rightarrow [0, 1]$ be

¹⁶Note that, since $(0, 0) \in G(s_b, s_s; \lambda)$ any solution to the Nash bargaining problem satisfies the participation constraints, that is, $\Sigma_b[y(s_b, s_s; \lambda), d(s_b, s_s; \lambda), s_b, s_s; \lambda] \geq 0$ and $\Sigma_s[y(s_b, s_s; \lambda), d(s_b, s_s; \lambda), s_b, s_s; \lambda] \geq 0$.

¹⁷This follows immediately from the measurability of $\Psi(\cdot; \lambda)$ and $\kappa(\cdot; \lambda)$.

the product probability measure. Define the mapping $\phi(s, e) : S \times E \rightarrow S$, where

$$\phi(s, e) = \begin{cases} [m - d(s, \cdot; \lambda), (1 - \delta)k + y(s, \cdot; \lambda)], & \text{if } e = (\text{buyer}, \cdot); \\ [m + d(\cdot, s; \lambda), (1 - \delta)k], & \text{if } e = (\text{seller}, \cdot); \\ [m, (1 - \delta)k], & \text{otherwise.} \end{cases} \quad (11)$$

We can now define $\Gamma : S \times \mathcal{S} \rightarrow [0, 1]$ to be

$$\Gamma(s, B; \lambda) \equiv \xi(\{e \in E | \phi(s, e) \in B\}). \quad (12)$$

Again, Γ is a well defined transition function.¹⁸ Then,

$$\Omega(B) = \int_S \Gamma(s, B; \lambda) \lambda(ds) \quad \forall B \in \mathcal{S}. \quad (13)$$

Finally, we can describe the law of motion of the aggregate state as

$$\lambda'(B) = H(\lambda)(B) \equiv \int_S \int_S \Pi(\bar{s}, B; \lambda) \Gamma(s, d\bar{s}; \lambda) \lambda(ds) \quad \forall B \in \mathcal{S}. \quad (14)$$

We are finally ready to define a recursive equilibrium for this economy.

Definition 1 (*Recursive Equilibrium*) *A recursive equilibrium is a list of:*

Pricing function: $p_k : \Lambda \rightarrow \mathbb{R}_+ \setminus \{0\}$;

Law of motion: $H : \Lambda \rightarrow \Lambda$;

Value functions: $V : S \times \Lambda \rightarrow \mathbb{R}_+$ and $W : S \times \Lambda \rightarrow \mathbb{R}_+$;

Policy functions: $\chi : S \times \Lambda \rightarrow \mathbb{R}_+$, $\Psi : S \times \Lambda \rightarrow \mathbb{R}_+$, $\kappa : S \times \Lambda \rightarrow \mathbb{R}_+$; and

Terms of trade $y : S \times S \times \Lambda \rightarrow \mathbb{R}_+$, and $d : S \times S \times \Lambda \rightarrow \mathbb{R}_+$;

such that:

¹⁸By construction, for each s , $\Gamma(s, \cdot; \lambda)$ is a probability measure on (S, \mathcal{S}) . Furthermore, given the measurability of $d(\cdot, \cdot; \lambda)$ and $y(\cdot, \cdot; \lambda)$, $\Gamma(\cdot, B; \lambda)$ is a \mathcal{S} -measurable function.

1. given the pricing function, the terms of trade, and the policy functions, the value functions satisfy the functional equations (1) and (2);
2. given the value functions and the law of motion of the aggregate state, the policy functions solve (2);
3. given the value functions, the terms of trade solve (8);
4. given the terms of trade the law of motion of the aggregate state is defined by (14);
5. the centralized market clears, $\int_S \int_S \Psi(\bar{s}, \lambda) \Gamma(s, d\bar{s}; \lambda) \lambda(ds) = 1$.

In the remainder of the paper we will only focus on *stationary equilibria*, where $\lambda = H(\lambda)$.

4 The Numerical Algorithm

In this section we briefly present the numerical algorithm developed for finding stationary monetary equilibria of the model and discuss some computational considerations.

The basic strategy of the algorithm is to iterate on a mapping defined by the value function equations (1) and (2) and the law of motion of the aggregate state given by equation (14). Special care is taken in keeping track of the distribution of wealth and its composition across iterations. In particular, we keep track of a large sample of agent's wealth portfolios and use non-parametric density estimation methods. We begin the algorithm at the entrance of the centralized market.

A brief description of the algorithm follows:

- Step 1.** Given an initial guess for the distribution of money and capital at the entrance of the centralized market, draw a large sample of agent's wealth portfolios.¹⁹
- Step 2.** Define a grid on the state space of money and capital holdings, (m, k) and an initial guess for the value function at the entrance of the decentralized market, $V^0(m, k)$, by defining the value of the function at the gridpoints and using interpolation methods to evaluate the function at any other point.²⁰
- Step 3.** Given the sample of wealth portfolios at the entrance of the centralized market and the value function at the entrance of the decentralized market, find the market clearing price by solving the centralized market problem (2) for all agents in the sample and iterating on the price, p_k , given an initial guess, until the market clears.
- Step 4.** Given the market clearing price and the sample of wealth portfolios, define the maximum value of real wealth, at the entrance of the centralized market (post-monetary injection), $wmax$. Compute $W(\cdot)$ by solving the centralized market problem on a grid on $[0, 4 * wmax]$, and using interpolation methods to evaluate the function at any other point.²¹
- Step 5.** Given the market clearing price, update the wealth portfolios of the agents by solving their optimization problem. For the steps that follow, it turns

¹⁹A sample of 10,000 observations is used.

²⁰The grid must be made large enough so that no attempts of evaluation of V outside the grid are made while solving the centralized market problem, given the guess for the distribution of wealth at its entrance. A grid of 100 by 100 points and bi-cubic spline interpolation are used.

²¹Note that, given p_k and the types of monetary transfers studied (proportional and lump-sum), only the total amount of real wealth, and not its composition, matter at the entrance of the centralized market. Since strict concavity of both V and W cannot be assured, special care must be taken in solving the optimization problem and keeping track of W . Furthermore, when estimating W agents are constrained to consume any amount of real wealth (post-monetary transfer) in excess of $2 * wmax$. By construction, in equilibrium this constraint will never be binding.

out to be computationally more convenient to keep track of the distribution of real wealth at the entrance of the decentralized market, and a portfolio allocation rule, instead of the joint distribution of money and capital. Again, we keep track of the portfolio allocation rule by defining a grid on $[0, wmax]$ (the space of real wealth) and using interpolation methods, being careful to keep track of any possible discontinuity points. The distribution of real wealth is estimated using Gaussian kernel non-parametric density estimation methods.²²

Step 6. Given $wmax$, redefine the grid on (m, k) by setting the maximum capital holdings to $2 * wmax$ and the maximum money holdings to $2 * wmax * p_k$. These bounds are large enough to guarantee that the soon to be updated function V will never be evaluated outside the grid at any iteration.

Step 7. Given the market clearing price at the centralized market, the value function $W(\cdot)$, the distribution of real wealth at the entrance of the decentralized market, and the portfolio allocation rule, update the value function $V(m, k)$ by using the mapping defined by equation (1) to compute its value at the new gridpoints and re-estimating the interpolant coefficients. Note that, the numerical computation of the integrals requires solving the bargaining problem (8) numerous times.

Step 8. For each individual on the sample, update their wealth portfolio by simulating their meetings. Randomly determine their trading status in a meeting (buyer, seller, or neither) and, using the estimated distribution of real wealth and the portfolio allocation rule, the wealth portfolio of his trading partner.

²²The reason for keeping track of the distribution of real wealth and the portfolio allocation rule instead of estimating directly the joint distribution of money and capital is that it makes the computation of the expectations (integrals) in equation (1) much more expedite. Still, the joint and marginal distributions can easily be computed if desired.

Repeat steps 3 to 8 until convergence is achieved.

A parallel Fortran 90 implementation of this algorithm is available from the authors by request. Given the computational complexity of the problem, for each parametrization of the model, convergence requires several days (depending critically on the value of the discount factor) using approximately 20 Opteron processors.

5 Preliminary Numerical Results: An Illustrative Example

In what follows we use the numerical algorithm presented in the last section to find and characterize stationary equilibria of the model. Although we are currently in the process of calibrating the model to certain data observations, like the average level of velocity, capital-output ratio, fraction of time spent working, etc, for which the model delivers predictions, for now we will take the parametrization below as an illustrative example. We use this example to characterize a typical features of a stationary equilibrium of the model and illustrate the potential effects of monetary policy.

To solve numerically the model we adopt the following functional forms for the utility of consumption and the disutility of supplying labor effort.

$$u(c) = \frac{(c + b)^{1-\eta} - b^{1-\eta}}{1 - \eta}$$

$$g(l) = \frac{l}{1 - l}$$

In addition, for now, we define a model period to be one year and pick parameter values reported in Table 1.

Parameter	Value
N	3
β	0.96
η	5
b	0.2
α	0.36
A	1
δ	0.07
θ	0.5

Table 1: Parameter Values

Some of these parameters, given we take a model period to be one year, are standard. For example, β and δ imply respectively, an annual interest rate of 4% and an annual depreciation rate of 7%. The others are taken from the original calibration in Lagos and Wright(2005). Ideally, we would like the model to match the average annual consumption velocity of money in the data. For the period 1959-2002, depending on whether one uses the monetary base or M1 as a measure of the money supply, these values are respectively, 11.2 and 4.0. As it will be clear below, to match the consumption velocity of money will require to calibrate the model to a shorter period than a year. In fact, by construction, the annual version cannot generate a velocity above $\frac{1}{N}$ given that only $\frac{1}{N}$ of the agents are buyers in each period of the model. Thus, even setting $N = 3$, the smallest possible number, will imply that a model period of one year cannot generate high enough velocity. We are in the process of addressing this issue.

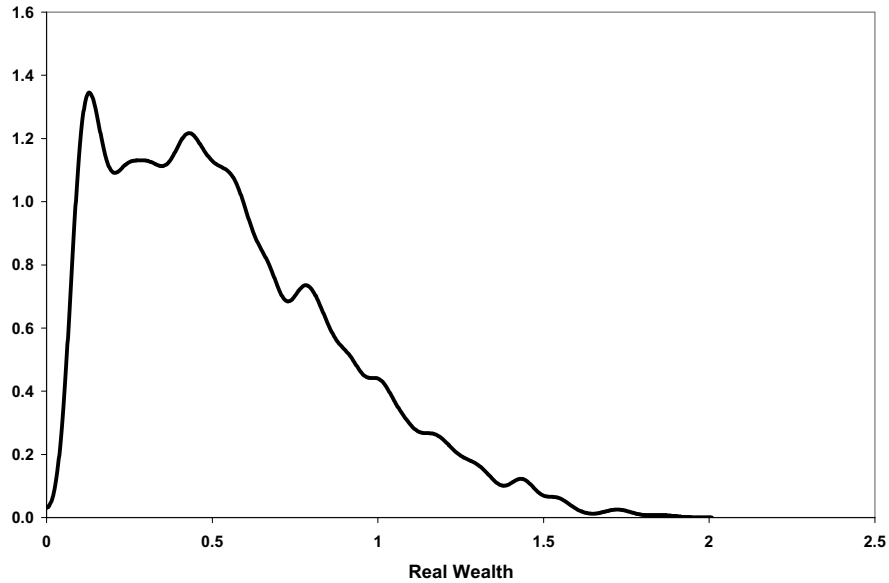


Figure 1: The Distribution of Wealth at the Entrance of the Decentralized Market

5.1 Equilibrium Characterization

Given the functional forms and parameter values described above, we begin by analyzing the distribution of wealth and the portfolio allocation decision of the agents. Unless otherwise noted, we consider the case where there is no monetary growth, $\mu = 0$. Figure 1 illustrates the typical distribution of wealth at the entrance of the decentralized market. Note that, not surprisingly, the distribution of wealth is non-degenerate and, for the parameter values we use, tends to be relatively jagged²³. The jaggedness of the wealth distribution can be better understood once one looks at the distribution of prices in the decentralized market and the portfolio allocation decision of the agents.

²³The jaggedness of the wealth distribution is robust and is not a result of undersmoothing in the non-parametric density estimation process.

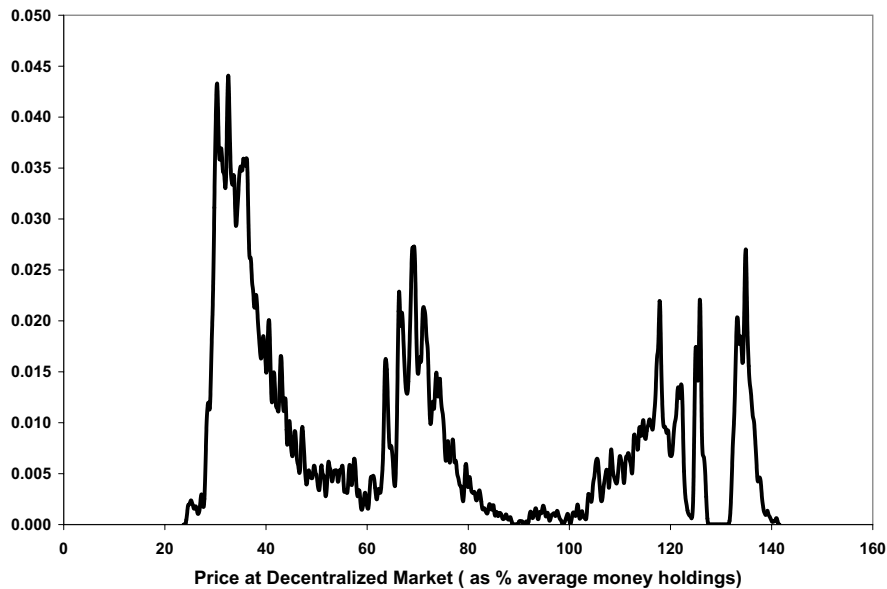


Figure 2: The Distribution of Prices in the Decentralized Market

Figure 2, shows the distribution of prices in the decentralized market, measured as percentage of the current money supply. The fact that the model generates price dispersion is not surprising if one notes that, in general, the terms of trade in the decentralized market will depend on both the wealth of the buyer and the seller as well as on their portfolio composition. For a given portfolio of a seller, the wealthier the buyer, holding constant his money holdings, the lower the price at which trade occurs. Also, for a given portfolio of the buyer and as long as the money constraint of the buyer is not binding, the wealthier the seller, keeping constant the amount of capital she holds, the higher the price at which trade occurs. (However, after a certain level of wealth of the seller, the money constraint of the buyer will be binding which will allow him to extract a higher fraction of the surplus.) This is a result of the fact that there is diminishing marginal value

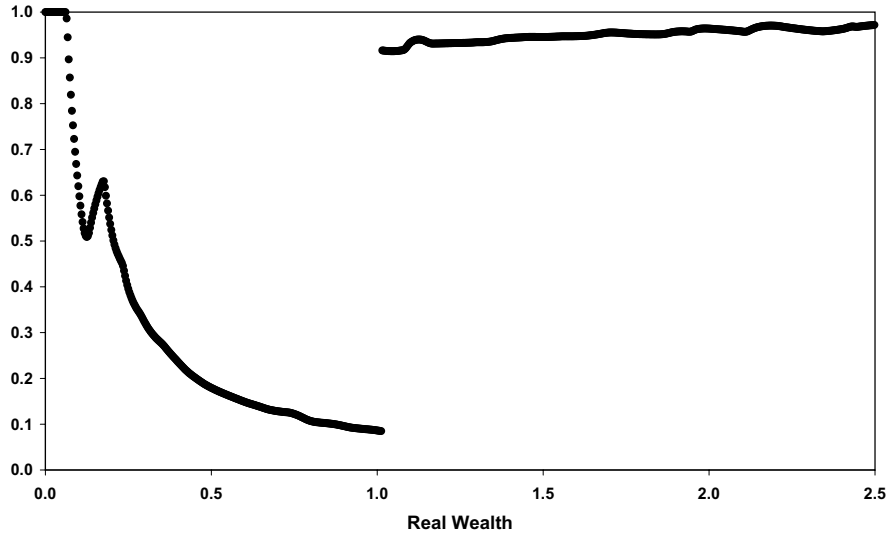


Figure 3: Fraction of Wealth Held as Real Money Balances

of wealth. On the other hand, the terms of trade will depend on the amount of money the buyer is holding and the amount of capital the seller has, and thus on their portfolio allocation decision. For a given seller's portfolio, the more money the buyer brings to the bargaining table, holding fix his wealth, the worst terms of trade he will face (higher price). In particular, it is important to note that, if the money constraint of the buyer is binding, he is able to extract a higher fraction of the total surplus. Also, for a given buyer's portfolio, the higher the amount of capital the seller brings into the bargaining, holding fixed his wealth, the more he will be willing to produce and thus the lower the price. Agents, are fully aware of all these considerations when making their optimal portfolio allocation decision. Thus the equilibrium price distribution depends on the equilibrium portfolio allocation decision and vice versa.

Figure 3 depicts the fraction of wealth held as real money balances at the entrance of the decentralized market. In what follows, I will try to provide some intuition to this optimal portfolio allocation decision. First consider the problem faced by a relatively poor agent. As a buyer, it is most likely that even if he holds a large fraction of his wealth as money, his money holdings are going to be binding most of the time²⁴. As such he will be able to extract a larger fraction of the total surplus from trade. On the other hand, by holding a large fraction of his wealth as money he maximizes the expected value of the total surplus from trade. As a seller, even if she were to hold a large fraction of her wealth as capital, the cost of production would be high, and thus she would not be willing to produce much output. As a result the expected value of the total surplus from trade is small. Furthermore, since the agent is relatively poor, she would be able to extract a small fraction of the total surplus. Thus, a relatively poor agent will choose to hold a large fraction of their wealth as real money balances. As an agents wealth increases however, there is less incentive to leave a large fraction of that wealth as money. Note that, if the agent were to keep a constant fraction of his wealth as money, eventually, as his wealth increased, as a buyer, his money holdings constraint would be seldom binding. This would mean that the agent would be able to extract a smaller fraction of the total surplus. On the other hand as a seller, by holding more capital, the agent reduces the cost of production and increases the expected value of the total surplus from trade. As a result, in general, the wealthier the agent, the higher fraction of wealth he will hold as capital. However, given that there is diminishing marginal productivity of capital, eventually the expected gain of holding any additional capital is dominated by the depreciation cost of capital. After that point, as long as the rate of inflation is low enough, agents rather store

²⁴Note that, on average other agents are going to be richer than him and thus have more (endogenous) bargaining power.

any additional wealth as money.²⁵

Note that, in general, there are two types of situations that can arise in a bilateral meetings: either the money constraint of the buyer is binding or it is not. If the constraint is binding, agents will trade on average at a lower price, if it is not the price on average will be higher. This fact, together with the optimal asset's portfolio allocation and the distribution of wealth imply that the price distribution might not be unimodal. In turn, the fact that the distribution of prices is not unimodal leads to the jaggedness of the equilibrium distribution of wealth.

Finally, figure 4 shows consumption as a function of the wealth of the agent at the entrance of the centralized market.

5.2 The Effects of Monetary Policy

We will now address the effects of monetary policy. First, it is easy to show that changes in the level of the money supply are neutral in the long-run. To see this, note that although changes in the level of the money supply will have, in general, real effects in the short-term due to a one time redistribution of real wealth among agents in the centralized market, that change will not have permanent effects (the distribution of real wealth will eventually converge back to the same invariant distribution) and thus there will be no long-run real effects. Moreover, it is easy to see from equation (3) that, if the increase in the money supply is accomplished via proportional transfers, there will be no redistribution of wealth and thus money will be both short-run and long-run neutral. It follows that, if we consider the case

²⁵Note that, since capital depreciates, as long as the rate of inflation is low enough, money is a better store of value. For high rates of inflation though, capital is a better store of value. However, money also plays a role as medium of exchange and capital has a productive role. At each agent's equilibrium portfolio allocation, the marginal value of each asset must be the same.

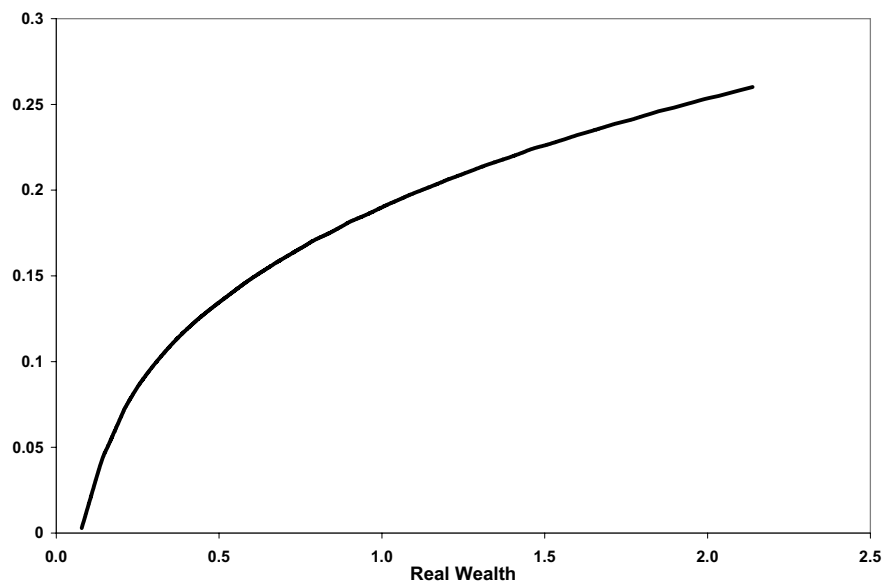


Figure 4: Consumption as Function of the Wealth at the Entrance of the Centralized Market.

of proportional transfers, money is also short-run and long-run superneutral, i.e. changes in the rate of monetary growth will have no short-run or long-run effects on any real variable.

A more interesting case for our analysis is the one where money is injected via lump-sum transfers. These will have redistributive effects and, in general, lead to a permanent change in the distribution of wealth. In this case, changes in the rate of monetary growth will have both short-run and long-run real effects. At this moment, we are only able to solve the model for stationary equilibria of the model and thus we will not address any short-run effects of inflation. Furthermore, we will simply compare across steady-states with different rates of monetary growth/inflation ignoring the transition path. The following table illustrates the effects of different inflationary policies.

Rate of Inflation (μ)	0%	2%	5%	10%	20%
Output (Y)	0.162	0.169	0.179	0.182	0.176
Consumption (C)	0.138	0.142	0.146	0.147	0.142
Capital (K)	0.341	0.390	0.461	0.503	0.493
Average Hours	0.248	0.239	0.236	0.231	0.239
Wealth ($W = K + \frac{M}{P_k}$)	1.153	0.527	0.542	0.567	0.547
C/Y	0.85	0.84	0.82	0.81	0.81
K/Y	2.10	2.31	2.58	2.78	2.80
M/P_c	0.978	0.655	0.542	0.508	0.462
M/P_k	0.812	0.137	0.080	0.064	0.054
P_k/P_c	1.20	4.78	6.77	7.94	8.55
Velocity	0.17	0.26	0.33	0.33	0.33
$\frac{M/P_k}{W}$	0.42	0.26	0.15	0.13	0.10
Ex-ante Welfare	138.78	139.32	139.87	140.31	140.27

Table 2: Effects of Expansionary Monetary Policy - Lump-sum Transfers.

Note that moderate rates of inflation lead to a higher level of aggregate output, aggregate consumption, aggregate capital stock, and ex-ante welfare. It also leads to a reduction in the average fraction of time spent working.

In what follows we will try to provide some intuition for these results by analyzing the different effects at work here. First note that the lump-sum money creation has a distributional effect. By receiving the same amount of real money balances, poor individuals become relative wealthier and wealthier individuals relatively poorer. This tends to decrease the dispersion of wealth. *Ceteris paribus*, since poor individuals have a higher marginal propensity to consume out of their wealth, this leads to an increase in aggregate consumption and welfare. However, the portfolio allocation decision is also affected. For higher rates of inflation, money becomes a worse store of value, leading all agents to increase their holdings of capital and decrease their real money balances (Tobin effect). The increase in capital holdings leads sellers to be willing to produce more output while decreasing the cost of production (and hours worked - effort) and thus to increase the amount of aggregate wealth, which leads to an additional increase in consumption. However, as inflation increases, money becomes less valuable and thus agents are less willing to hold real money money balances for transaction purposes (real balance effect) and to produce for money. This discourages the accumulation of capital since its productive role becomes less important. Eventually, the decrease in the value of money and capital and decrease in capital accumulation lead to less trade occurring in the decentralized market, less output being produce, increase in the hours worked, and a decrease in consumption and welfare. It is interesting to note that, as inflation increases, the average price in the decentralized market grows slower than the price in the centralized market. This is due to the adjustments in the wealth portfolios and its effects on the terms of trade in the decentralized

market.

Looking at the results for velocity, real money balances and the fraction of aggregate wealth held as real money balances it becomes clear that by considering a period to be an year we made search frictions too severe and magnified the role of money as a medium of exchange. This explains the large magnitudes of the effects and the high level of welfare maximizing inflation. We expect that with a proper calibration these numbers will become more reasonable. Still, the model suggested that velocity is increasing with inflation as the extensive margin of trade increases.

6 Conclusion

In this paper we extended the literature on fundamental models of money by considering a search-theoretical model where agents were allowed to hold and accumulate both money and capital. In the model, the frictions that give rise to a role for money lead to uninsurable idiosyncratic uncertainty regarding trading opportunities and a non-degenerate distribution of wealth. We developed a numerical algorithm that allowed us to numerically characterize stationary equilibria and to analyze the effects of monetary policy on the equilibrium patterns of exchange, the distribution of wealth, its composition, capital accumulation, and welfare, among others.

Regarding the effects of monetary policy, we have shown that money is always neutral in the long-run and also in the short-run if monetary injections are accomplished via proportional transfers. However, if the transfer mechanism generates a redistribution of wealth, increases in the level of the money supply will have short-run real effects. Furthermore, regarding the effects of changes in the rate of

monetary growth, we show that proportional transfers are superneutral but that redistributive lump-sum transfer will have permanent real effects. Our preliminary numerical results suggest that a moderate rate of monetary expansion can lead to an increase in steady-state aggregate output, aggregate consumption, capital accumulation, and welfare. Also, the average fraction of time spent working might decrease. These results are suggestive that moderate inflationary policies might be beneficial for welfare and help conciliate the apparent paradox between the optimal deflationary monetary policy prescribed by most traditional monetary models, the Friedman Rule, and the mildly inflationary practice of most Central Banks. Another contribution of this paper is to bring fundamental models of money closer to being able to address monetary policy questions, including providing quantitative answers, that previously could only be addressed using reduced form models.

One thing to note is that, in this model, we precluded agents from entering into credit arrangements in the centralized market or even to trade claims on capital, by assuming anonymity. If allowed, agents would certainly want to do so. However, the point of the paper is exactly to analyze the effects of monetary policy in a world where such risk-sharing mechanisms might be absent for some forms of idiosyncratic uncertainty.

References

Alvarez, F., A. Atkeson, and C. Edmond. (2003). "On the Sluggish Response of Prices to Money in an Inventory-Theoretic Model of Money Demand," NBER working paper 10016.

Alvarez, F., A. Atkeson, and P. Kehoe. (2002). "Money, Interest Rates, and Exchange Rates with Endogenously Segmented Markets," *Journal of Political Economy*, 110: 73-112.

Arouba, B., C. Waller, and R. Wright. (2004). "Money and Capital," mimeo, University of Pennsylvania.

Berentsen, A., G. Camera and C. Waller. (2005). "The Distribution of Money Balances and the Non-Neutrality of Money," Institute for Empirical Research in Economics, University of Zurich, Working Paper Series 220.

Bhattacharia, J., J. Haslag, and A. Martin. (2004). "Heterogeneity and Redistribution, and the Friedman Rule," *International Economic Review*, forthcoming.

Chiu, J. (2004). "Endogenously Segmented Asset Market in an Inventory Theoretic Model of Money Demand," mimeo, The University of Western Ontario.

Cooley, T. F. and G.D. Hansen. (1991). "The Welfare Costs of Moderate Inflation," *Journal of Money, Credit and Banking*, Vol. 23. No.3, Part 2: Price Stability (August): 483-503.

Deviatov, A. and N. Wallace. (2001). "Another Example in which Lump-sum Money Creation is Beneficial," *Advances in Macroeconomics*, 1, Article 1.

Edmond, C. (2002). "Self-Insurance, Social Insurance, and the Optimal Quantity of Money," *American Economic Review: Papers and Proceedings* (May): 141-147.

Green, E. and R. Zhou. (2004). "Money as a Mechanism in a Bewley Economy," *International Economic Review*, forthcoming.

Kehoe, T., D. Levine, and M. Woodford. (1992). "The Optimum Quantity of Money Revisited," *Economic Analysis of Market Games: Essays in Honor of Frank Hahn*, ed. by P. Dasgupta et al. Cambridge and London: MIT Press, 1992 :501-26.

Lagos, R. and G. Rocheteau. (2004). "Money and Capital as Competing Media of Exchange," Federal Reserve Bank of Minneapolis, Research Department Staff Report 341, August 2004.

Lagos, R. and R. Wright. (2005). "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy*, 113: 463-484.

Levine, D. (1991). "Asset Trading Mechanisms and Expansionary Policy," *Journal of Economic Theory*, 54: 148-64.

Molico, M. (1997). "The Distribution of Money and Prices in Search Equilibrium," Ph.D. dissertation. University of Pennsylvania.

Phelps, E. S. (1973). "Inflation in Theory of Public Finance," *Swedish Journal of Economics*, Vol. 75, No. 1 (March): 67-82.

Shi, S. (1999a). "Search, Inflation, and Capital Accumulation," *Journal of Monetary Economics*, 44: 81-103.

Shi, S. (1999b). "Money, Capital, and the Redistributive Effect of Monetary Policies," *Journal of Economic Dynamics and Control*, 23: 5656-590.

Trejos, A. and R. Wright. (1995). "Search, Bargaining, Money and Prices," *Journal of Political Economy*, 103: 118-41.

Williamson, S. (2005). "Search, Limited Participation, and Monetary Policy,"

mimeo, University of Iowa.