AGGREGATION OF HETEROGENEOUS BELIEFS AND ASSET PRICING: A MEAN-VARIANCE ANALYSIS

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The Sharpe-Lintner-Mossin Capital Asset Pricing Model (CAPM) plays a central role in modern finance theory. It is founded on the paradigm of homogeneous beliefs and rational representative agent. However, from the theoretical perspective this paradigm has been criticised on a number of grounds (e.g. Williams (1977), Varian (1985), Detemple and Murthy (1994), Cecchetti et al. (2000), Abel (1989, 2002), Calvet et al. (2004), Jouini and Napp (2006)), in particular concerning its extreme assumptions about information of the economic environment and computational ability on the part of the rational representative economic agent. It is found from the empirical literature that (see Harvey (1989)) the fundamental CAPM equation that relates expected returns on assets to their portfolio risk is not well supported. For instance the security market line does not have the predicted slope and the measure of portfolio risk, the so-called beta coefficient, is time varying in ways that are not consistent with the theory.

There has been some work on extending the mean-variance model to allow for differences in beliefs across agents (see Williams (1977), Varian (1985), Detemple and Murthy (1994) Cecchetti et al. (2000), Abel (1989, 2002), Calvet et al. (2004), Jouini and Napp (2006)). The impact of heterogeneous beliefs among agents on the market equilibrium price has been focused. In particular, we are more interested in how the market equilibrium price is determined when agents are bounded rational and heterogeneous in beliefs.

Recent research has witnessed a shift towards a paradigm of boundedly rational and heterogeneous agents (see Frankel and Froot (1987), Day and Huang (1990), Chiarella (1992), Kirman (1993), Lux (1995), Brock and Hommes (1998), Lux and Maechesi (1999), Chiarella and He (2001, 2002, 2003), Chiarella and Bohm (2004) and Chiarella et. al (2002, 2006)). This strand of research has essentially taken the framework of the standard paradigm of finance theory (the rational representative agent choosing a portfolio of risky assets so as to maximise expected utility) and added to it bounded rationality, agent heterogeneity, agents seeking to learn about the economic environment and agents using various heuristics to change financial decision strategies. The result is a model of financial markets as nonlinear adaptive evolutionary systems that is able to reproduce at least some of the stylised facts of financial markets that are not well-explained by the standard paradigm, we refer Hommes (2006) and LeBaron (2006)) for two excellent surveys of these developments.

There remain a number of key issues with which this literature has still to come to grips, these include: the implications for portfolio diversification strategies; the implications for the relationship between expected risk adjusted returns on different assets (in other words how does the traditional Capital Asset Pricing Model - CAPM - look in this new framework?); whether this framework can provide a theory of a time-varying beta coefficient.

In most of the literature, a portfolio of one risk-free asset and one-risky asset is considered and agents are heterogeneous in the mean, rather than the variance. In this paper, we consider a portfolio of one risk-free asset and many risky assets and extend the mean-variance model in the discrete time setting to allow for heterogeneity in not only the mean but also the variance across agents. By assuming a Walrasian auctioneer scenario, we show that the market aggregate expected payoffs of the risky assets are weighted averages of the heterogeneous expected payoffs of the risky assets across the agents, in which the weights are given by the heterogeneous covariance matrices adjusted by the risk aversion coefficients of the agents. We also show that the market equilibrium prices of risky assets are determined jointly by the aggregate mean and variance of the market portfolio. In particular, we derive an equilibrium relation between the market aggregate expected payoff of the risky assets and the market portfolio expected payoff, leading to a CAPM-like relation with heterogeneous beliefs. An exact formula for the so-called β coefficient is derived. When the agents are heterogeneous in means but homogeneous in covariances of the risky assets, we show that, in equilibrium, the market aggregate expected payoffs of the risky assets and the market portfolio are the risk aversion weighted averages of the heterogeneous expected payoffs of the risk assets and the market portfolio, respectively. A further assumption of the homogeneous beliefs in the means then leads to the standard CAPM.

Some numerical examples are used to illustrate the impact of the heterogeneous beliefs on the aggregate returns of the market portfolio. In particular we examine the influence of the heterogeneous beliefs on the β coefficient in the standard CAPM. Further analysis on the explanatory power of the result with respect to the risk premium puzzle will be conducted.

REFERENCES

Abel, A. (1989), Asset prices under heterogeneous beliefs: implications for the equity premium, working paper 09-89, Rodney L. White Center for Financial Research.

Abel, A. (2002), 'An exploration of the effects of pessimism and doubt on asset returns', *Journal of Economic Dynamics and Control* **26**, 1075–1092.

Bohm, V. and Chiarella, C. (2004), 'Mean variance preferences, expectations formation, and the dynamics of random asset prices', *Mathematical Finance* **15**, 61–97.

Brock, W. and Hommes, C. (1998), 'Heterogeneous beliefs and routes to chaos in a simple asset pricing model', *Journal of Economic Dynamics and Control* **22**, 1235–1274.

Calvet, L., Grandmont, J.-M. and Lemaire, I. (2004), Aggregation of heterogeneous beliefs and asset pricing in complete financial markets, working paper 2004-12, CREST.

Cecchetti, S., Lam, P. and Mark, N. (2000), 'Asset pricing with distorted beliefs: are equity returns too good to be true?', *American Economic Review* **90**, 787–805.

Chiarella, C. (1992), 'The dynamics of speculative behaviour', *Annals of Operations Research* **37**, 101–123.

Chiarella, C., Dieci, R. and Gardini, L. (2002), 'Speculative behaviour and complex asset price dynamics', *Journal of Economic Behavior and Organization* **49**, 173–197.

Chiarella, C., Dieci, R. and He, X. (2006), 'Heterogeneous expectations and speculative behaviour in a dynamic multi-asset framework', *Journal of Economic Behavior and Organization* . in press.

Chiarella, C. and He, X. (2001), 'Asset price and wealth dynamics under heterogeneous expectations', *Quantitative Finance* **1**, 509–526.

Chiarella, C. and He, X. (2002), 'Heterogeneous beliefs, risk and learning in a simple asset pricing model', *Computational Economics* **19**, 95–132.

Chiarella, C. and He, X. (2003), 'Heterogeneous beliefs, risk and learning in a simple asset pricing model with a market maker', *Macroeconomic Dynamics* **7**, 503–536.

Day, R. and Huang, W. (1990), 'Bulls, bears and market sheep', *Journal of Economic Behavior and Organization* **14**, 299–329.

Detemple, J. and Murthy, S. (1994), 'Intertemporal asset pricing with heterogeneous beliefs', *Journal of Economic Theory* **62**, 294–320.

Frankel, F. and Froot, K. (1987), 'Using survey data to test propositions regarding exchange rate expectations', *American Economic Review* **77**, 133–153.

Harvey, C. R. (1989), 'Time-varying conditional covariances in tests of asset pricing models', *Journal of Financial Economics* **24**, 289–317.

Hommes, C. (2006), *Handbook of Computational Economics, Vol. 2:Agent-based Computational Economics*, North-Holland, chapter Heterogeneous Agent Models in Economics and Finance. to appear.

Jouini, E. and Napp, C. (2006), 'Heterogeneous beliefs and asset pricing in discrete time: An analysis of pessimism and doubt', *Journal of Economic Dynamics and Control* .

Kirman, A. (1993), 'Ants, rationality, and recruitment', *Quarterly Journal of Economics* **108**, 137–156.

LeBaron, B. (2006), *Handbook of Computational Economics, Vol. 2: Agent-based Computational Economics*, North-Holland, chapter Agent-based Computational Finance. to appear.

Lux, T. (1995), 'Herd behaviour, bubbles and crashes', *Economic Journal* **105**, 881–896.

Lux, T. and Marchesi, M. (1999), 'Scaling and criticality in a stochastic multi-agent model of a financial markets', *Nature* **397**(11), 498–500.

Varian, H. (1985), 'Divergence of opinion in complete markets', *Journal of Finance* **40**, 309–317.

Williams, J. (1977), 'Capital asset prices with heterogeneous beliefs', *Journal of Financial Economics* **5**, 219–239.