# Can Structural Small Open Economy Models Account for the Influence of Foreign Disturbances?*

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**Abstract**

This paper evaluates whether an estimated, structural, small open economy model of the Canadian economy can account for the substantial influence of foreign-sourced disturbances identified in numerous reduced-form studies. The analysis shows that the benchmark model — and a number of variants which include a range of market imperfections — imply cross-equation restrictions that are too stringent when confronted with the data, yielding implausible parameter estimates. While appropriate choice of ad hoc disturbances can relax these cross-equation restrictions and therefore capture certain properties of the data — for instance, the volatility and persistence of the real exchange rate — and yield plausible parameter estimates, this success is qualified by the model's inability to account for the transmission of foreign disturbances to the domestic economy: less than one percent of the variance of output is explained by foreign shocks.

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# 1 Introduction

Reduced form analyses identify a significant influence of foreign disturbances on small open economy business cycles. In the case of Canada, for instance, using a vector autoregression Cushman and Zha (1997) estimate that U.S. disturbances account for 74 percent of domestic output variation in the long run. Kose, Otrok, and Whiteman (2003) and Justiniano (2004) find similarly large contributions of foreign-sourced disturbances on the evolution of Canadian macroeconomic variables using factor analysis. Indeed, the existence of substantial comovements in macroeconomic fluctuations across the U.S. and Canada, as well as other countries, has been well known since the celebrated work of Backus and Kehoe (1992) based on bivariate correlations.

These stylized facts have motivated ample theoretical work seeking to replicate the observed comovements in economic activity across countries. However, the empirical validation of these models has largely relied on calibrations aimed at matching selected moments in the data, as in the contributions of Backus, Kehoe and Kydland (1992, 1995) and Stockman and Tesar (1995). Particular interest has been given to the real exchange rate and the terms of trade due to the prominent role these variables assume in the transmission of business cycle fluctuations. In contrast, the objective of this paper is to evaluate the ability of an *estimated* small open economy DSGE model to explain the dynamics in a larger number of data series by looking at persistence, volatility and in particular the transmission of foreign shocks. The likelihood-based estimation procedure seeks to match *all* second order moments of the model to the data.

The analysis is pursued using generalizations of the small open economy framework proposed by Gali and Monacelli (2005) and Monacelli (2003), in which a small and large country each specialize in the production of a continuum of goods subject to imperfect competition and price rigidities.[1] Imports are subject to local currency pricing (through what could be considered a retail sector providing distribution services) giving rise to deviations from the law of one price. We depart from their framework by considering incomplete asset markets and also incorporate other nominal rigidities — such wage stickiness, indexation and habits — as well as a large set of disturbances which have been found crucial in taking closed economy models to the data as documented by,

[1] The model is technically a semi-small open economy model, as domestic goods producers have some market power. The model shall nonetheless be referred to as a small open economy. Note also that our analysis appeals to an earlier interpretation of the Gali and Monacelli (2005) of a small-large country pair, rather than as an analysis of a continuum of small open economies.

inter alia, Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2002). The model is estimated using Bayesian methods taking Canada and the United States as the small-large country pair.

Our enquiry is framed by identifying which cross-equation restrictions are most stringent when confronted with the data. As such, we seek to highlight the dimensions along which the model fails to explain the observed properties of the data, moving beyond sole reliance on matching specific moments through calibration. Furthermore, while we consider alternative model specifications, we do not base our analysis solely on formal model comparison methods — such as posterior odds — but rather complement this approach by contrasting the empirical second order moments with those implied by the model.

Our analysis yields several insights. First, it is possible to identify a model specification that yields plausible estimates for all parameters and largely accounts for the persistence and volatility properties of all individual observable series used in estimation, including the terms of trade and the real exchange rate. Second, success in this dimension hinges on the inclusion of two disturbances to ease stringent cross-equation restrictions imposed by the model: i) a cost-push shock in the retail sector for imported foreign goods (to induce through retail sector pricing sufficient persistence in the terms of trade) and ii) a risk premium shock (to break the restriction on relative movements of real interest rates and the real exchange rate implied by uncovered interest rate parity and to impart sufficient persistence in the latter variable). While the empirical failure of uncovered interest parity is well known — see Lewis (1995) — it is notable that these two disturbances prove crucial in fitting the model despite the inclusion of several additional shocks, market imperfections, mechanisms of persistence, as well as allowing for deviations from the law of one price.

Third, the role of these ad hoc shocks is to relax the model's cross-equation restrictions to provide exchange rate disconnect — the property that real exchange rate movements are unrelated to fundamentals — as discussed, for instance, by Obstfeld and Rogoff (2000). Absent these disturbances, or even a very high degree of persistence in these shocks, the model cannot explain the dynamics of the real exchange rate and terms of trade. Furthermore, when these disturbances are excluded, estimates converge to a configuration that either: resembles a closed economy model; minimizes expenditure switching effects by having an elasticity of substitution between foreign and domestically produced goods equal to zero; or has implausible estimates for a range of parameters and implied volatilities.

Fourth, analyzing the implied structural variance decompositions, our main contribution is to document that the estimated model shuts down the channels of transmission of international business cycles to the domestic economy. Consequently, the model is unable to account for any comovements across countries. Indeed, only one percent of the variance of Canadian output, interest rates or inflation can be sourced to the combined effect of all U.S. disturbances. Evidence is adduced demonstrating that this finding is robust to the role of priors, detrending of the data or the characterization of the foreign block.

Importantly, and consistently with earlier cited literature, significant comovements and transmission of foreign disturbances are a property of our data, here documented with the estimation of seemingly unrelated regressions and factor models. However, despite the DSGE model's success in matching volatility and persistence of each individual variable, simple measures of comovement, such as the model predicted cross-correlation functions, are strikingly different to corresponding empirical counterparts. This underscores the importance of using measures of fit other than posterior odds ratios for validating DSGE models.

The model's failure in explaining the international transmission of business cycles suggests that empirical work should consider alternative modeling structures to account for comovement. Our findings are consistent with Devereux and Engel (2002) which argues that matching the persistence and volatility of the real exchange requires three ingredients: i) incomplete markets, ii) production/pricing structure that limits expenditure switching effects and iii) non-rational exchange rate expectations. Indeed, we have found (although not reported) that departing from complete markets helps in matching dynamics, with the exception of comovement, and that the model's simple production/pricing structure tends to favor parameter estimates with a near zero elasticity of substitution between domestic and foreign goods to mitigate expenditure switching effects.[2]

However, it is not immediate that the exchange rate disconnect inherent in estimation need be fully responsible for the identified absence of comovement. In fact, reduced form evidence reported here suggests that variations in the real exchange rate and the terms of trade are to a large extent

[2] While not reported, the estimation of several complete markets models demonstrates that even in the presence of significant nominal rigidities, the model cannot explain key properties of the data (i.e. the volatility and persistence of the real exchange rate) without implying implausible parameter estimates. Incomplete financial markets therefore assist to some degree in obtaining plausible parameter estimates and capturing the standard deviation and autocorrelation properties of the data. Such findings concur with Rabanal and Tuesta (2005) using a 2-country model.

not driven by foreign disturbances — yet foreign disturbances still significantly influence domestic variables. Therefore, further work needs to be done to isolate the precise source of model misspecification that engenders the failure to account for transmission mechanisms. Whatever the source, our results cast doubt on the ability of these models to be used in their present form for policy analysis in an open economy.

This paper belongs to an earlier literature (Backus, Kehoe and Kydland (1992, 1995) and Stockman and Tesar (1995)) documenting puzzles in international business cycle models, although these calibration-based studies sought only to match a subset of the second order moments that our estimation procedure seeks to fit. The failure of this class of models with nominal rigidities to explain the real exchange rate is consistent with the observations of Chari, Kehoe, and McGrattan (2002). Nonetheless, our contribution is to present evidence that this remains true even when the model is given every opportunity to fit the data through the inclusion of a range of market imperfections, ad hoc persistence mechanisms and several disturbances, as well as by restricting asset trade to one period domestic and foreign debt.

Our paper also relates to numerous recent efforts to evaluate and estimate New Open Economy Models (NOEM). These include: Ambler, Dib, and Rebei (2004), Bergin (2003, 2004), Del Negro (2003), Dib (2003), Ghironi (2000), Justiniano and Preston (2004), Lubik and Schorfheide (2003, 2005), Lubik and Teo (2005) and Rabanal and Tuesta (2005). However, to the extent of our knowledge, the absence of comovement and transmission of foreign shocks has neither been previously documented nor systematically analyzed in this literature. However, parallel and independent work by Adolfson, Laseen, Linde, and Villani (2005) presents a more richly specified model than considered here, in which reported variance decompositions also reveal little transmission of foreign sourced disturbances from the European Union to Sweden — a property that is not remarked upon. Similar observations apply to recent work by de Walque, Smets, and Wouters (2005) in a two country model. More in line with the focus of our analysis, we build on Schmitt-Grohe (1998) which evaluates whether a calibrated small open economy real business cycle model is able to replicate impulse responses to a single foreign output shock extracted from a U.S.-Canada vector autoregression model.

The remainder of the paper proceeds as follows. Section 2 lays out the theoretical model. Section 3 develops the estimation methodology giving particular regard to identification issues that plague estimation of large scale DSGE models. Section 4 presents the estimation results. Section 5 demonstrates that the adopted model fails to account for the influence of foreign disturbances. Section 6 presents robustness checks and elucidates the dimensions of the model's failure by comparison with various kinds of reduced form evidence. Section 7 concludes.

# 2 The Model

Building on Gali and Monacelli (2005), Monacelli (2003) and Justiniano and Preston (2004), the following section sketches the derivation of key structural equations allowing for habit formation, indexation of prices, labor market imperfections and incomplete markets. The former papers extend the microfoundations of the kind described by Clarida, Gali, and Gertler (1999) and Woodford (2003) for analyzing monetary policy in a closed-economy setting to an open economy context. For additional details the reader is encouraged to consult Monacelli (2003).

## 2.1 Households

Each household $k$ maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \tilde{\varepsilon}_{g,t} \left[ \frac{\left(C_t^k - H_t\right)^{1-1/\sigma}}{1-1/\sigma} - \frac{\tilde{\varepsilon}_{l,t} \left(N_t\left(k\right)\right)^{1+\varphi}}{1+\varphi} \right]$$

by choice of $C_t^k$, $W_t^k$ and the bond portfolio described below. $N_t\left(k\right)$ is the labor input; $H_t \equiv hC_{t-1}$ corresponds to external habit taken as exogenous by the household and $0 < h < 1$; $\sigma^{-1}, \varphi > 0$ are the inverse elasticities of intertemporal substitution and labor supply respectively; while $\tilde{\varepsilon}_{g,t}$ and $\tilde{\varepsilon}_{l,t}$ denote preference and labor supply shocks respectively. $C_t$ is a composite consumption index

$$C_t^k = \left[ (1-\alpha)^{\frac{1}{\eta}} \left(C_{H,t}^k\right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left(C_{F,t}^k\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

where $C_{H,t}$ and $C_{F,t}$ are Dixit-Stiglitz aggregates of the available domestic and foreign produced goods given by

$$C_{H,t}^k = \left[ \int_0^1 C_{H,t}^k\left(i\right)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \quad \text{and} \quad C_{F,t}^k = \left[ \int_0^1 C_{F,t}^k\left(i\right)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}.$$

$\eta > 0$ therefore gives the elasticity of substitution between domestic and foreign goods and $\theta > 1$ is the elasticity of substitution between types of differentiated domestic or foreign goods.

Assuming the only available assets are one period domestic and foreign bonds, optimization occurs subject to the flow budget constraint

$$P_tC_t^k + D_t^k + e_tB_t^k = D_{t-1}\left(1+\tilde{\imath}_{t-1}\right) + e_tB_{t-1}^k\left(1+i_{t-1}^*\right)\phi_t\left(A_t\right) + P_t^HY_t^H + \left(P_t^F - \tilde{e}_tP_t^*\right)C_t^F + T_t$$

for all $t > 0$, where $D_t^k$ denotes the household's holding of one period domestic bonds, and $B_t^k$ one period foreign bonds with corresponding interest rates $i_t$ and $\tilde{\imath}_t$. The price indices $P_t$, $P_{H,t}$ and $P^*$ correspond to the domestic CPI, domestic goods prices and foreign prices respectively and are formally defined below. $T_t$ denotes taxes and transfers. Following Benigno (2001), Kollmann (2002) and Schmitt-Grohe and Uribe (2003), the function $\phi_t(\cdot)$ is interpretable as a debt elastic interest rate premium given by

$$\phi_t = \exp\left[-\chi\left(A_t + \tilde{\phi}_t\right)\right]$$

where

$$A_t \equiv \frac{\tilde{e}_{t-1}B_{t-1}^f}{\bar{Y}P_{t-1}}$$

is the real quantity of outstanding foreign debt expressed in terms of domestic currency as a fraction of steady state output and $\tilde{\phi}_t$ a risk premium shock. The adopted functional form ensures stationarity of the foreign debt level.

Implicitly underwriting this expression for the budget constraint is the assumption that all households in the domestic economy receive an equal fraction of both domestic and retail firm profits and that labor income risk is pooled across agents. Hence the final two terms in the flow budget constraint represent the income received from operation of the domestic and imported goods sector firms discussed below. Absent this assumption, which imposes complete markets within the domestic economy, the analysis would require modeling the distribution of wealth across agents. That same assumption also ensures that households face identical decision problems and therefore choose identical state-contingent plans for consumption so that $C_t = \int C_t^k dk = C_t^k$. The superscript $k$ is dropped henceforth.

The household's optimization problem requires allocation of expenditures across all types of domestic and foreign goods both intratemporally and intertemporally. This yields the following set of optimality conditions. The demand for each category of consumption good is

$$C_{H,t}(i) = \left(P_{H,t}(i)/P_{H,t}\right)^{-\theta}C_{H,t} \quad \text{and} \quad C_{F,t}(i) = \left(P_{F,t}(i)/P_{F,t}\right)^{-\theta}C_{F,t}$$

for all $i$ with associated aggregate price indexes for the domestic and foreign consumption bundles given by $P_{H,t}$ and $P_{F,t}$. The optimal allocation of expenditure across domestic and foreign goods implies the demand functions

$$C_{H,t} = (1-\tau)\left(P_{H,t}/P_t\right)^{-\eta} C_t \quad \text{and} \quad C_{F,t} = \tau \left(P_{F,t}/P_t\right)^{-\eta} C_t \tag{1}$$

where $\tau$ is the share of foreign goods in the domestic consumption bundle and $P_t = \left[(1-\tau) P_{H,t}^{1-\eta} + \tau P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}$ is the consumer price index. Allocation of expenditures on the aggregate consumption bundle satisfies

$$\lambda_t = \tilde{\varepsilon}_{g,t} \left(C_t - H_t\right)^{-1/\sigma} \tag{2}$$

and portfolio allocation is determined by the optimality conditions

$$\lambda_t \tilde{e}_t P_t = E_t \left[\left(1+\tilde{\imath}_t^*\right) \beta \phi_{t+1} \lambda_{t+1} \tilde{e}_{t+1} P_{t+1}\right] \tag{3}$$

$$\lambda_t P_t = E_t \left[\left(1+\tilde{\imath}_t\right) \beta \lambda_{t+1} P_{t+1}\right] \tag{4}$$

for Lagrange multiplier $\lambda_t$. The latter condition when combined with (2) gives the usual Euler equation.

The household problem in the foreign economy is similarly described with the exceptions now noted. Because the foreign economy is approximately closed (the influence of the domestic economy is negligible), the available consumption bundle comprises the continuum of foreign produced goods $C_{F,t}^*(j)$ for $j \in [0,1]$. Foreign households need only decide how to allocate expenditures across these goods in any time period $t$ and also over time. Furthermore, foreign debt in the foreign economy is in zero net supply — using the property that the domestic economy engages in negligible financial asset trade — and there is no access to domestic debt markets for foreign agents. Conditions (2) and (4) continue to hold with all variables taking superscript "*".

## 2.2 Optimal Labor Supply

Following Erceg, Henderson, and Levin (2000) and Woodford (2003, chap. 3), assume a single economy wide labor market and that producers of the domestic good hire the same bundle of labor inputs at common wage rates. Firm $j$ produces good $j$ according to the production technology

$$Y_t(j) = \tilde{\varepsilon}_{a,t} f\left(N_t(j)\right)$$

where $\tilde{\varepsilon}_{a,t}$ is a technology shock and $f(\cdot)$ satisfies the usual Inada conditions. The labor input used in the production of each good $j$ and associated aggregate wage index are given by the CES aggregators

$$N_t(j) \equiv \left[\int_0^1 N_t(k)^{\frac{\theta_w - 1}{\theta_w}} dk\right]^{\frac{\theta_w}{\theta_w - 1}} \quad \text{and} \quad W_t = \left[\int_0^1 W_t(k)^{1-\theta_w} dk\right]^{\frac{1}{1-\theta_w}}$$

for $\theta_w > 1$. Firm $j$'s demand for each type of labor $k$ is determined by maximizing the former index for a given level of wage payment. This gives the demand function

$$N_t(k) = N_t(j)\left(\frac{W_t(k)}{W_t}\right)^{-\theta_w}. \tag{5}$$

Households supply their labor under monopolistic competition. They face a Calvo-style price setting problem, having the opportunity to re-optimize their wage with probability $1 - \alpha_w$ each period, where $0 < \alpha_w < 1$. As in Christiano, Eichenbaum, and Evans (2005) and Woodford (2003, chap. 3), households not re-optimizing adjust their wage according to the indexation rule

$$\log W_t(k) = \log W_{t-1}(k) + \gamma_w \pi_{t-1}$$

where $0 \leq \gamma_w \leq 1$ measures the degree of indexation to the previous period's inflation rate and $\pi_t = \log(P_t/P_{t-1})$. Since all households having the opportunity to reset their wage face the same decision problem they set a common wage $W_t'$.

The household's wage setting problem in period $t$ is to maximize

$$E_t \sum_{T=t}^{\infty} (\alpha_w \beta)^{T-t} \left[\lambda_T W_t(k)\left(\frac{P_{T-1}}{P_{t-1}}\right)^{\gamma_w} N_T(k) - \frac{\tilde{\varepsilon}_{l,t} N_T(k)^{1+\varphi}}{1+\varphi}\right]$$

by choice of $W_t(k)$ subject to the labor demand function (5). The first order condition for this problem is

$$E_t \sum_{T=t}^{\infty} (\alpha_w \beta)^{T-t} \left[\lambda_T \left(\frac{P_{T-1}}{P_{t-1}}\right)^{\gamma_w} \left(N_T(k) + W_t(k)\frac{\partial N_T(k)}{\partial W_t(k)}\right) - \tilde{\varepsilon}_{l,t} N_T^{\varphi} \frac{\partial N_T(k)}{\partial W_t(k)}\right] = 0. \tag{6}$$

Households in the foreign block face an identical problem, with appropriate substitution of foreign variables and technology and preference parameters.

## 2.3 Domestic Producers

There is a continuum of monopolistically competitive domestic firms producing differentiated goods. Calvo-style price-setting is assumed allowing for indexation to past domestic goods price inflation.

Hence, in any period $t$, a fraction $1-\alpha_H$ of firms set prices optimally, while a fraction $0<\alpha_H<1$ of goods prices are adjusted according to the indexation rule

$$\log P_{H,t}(i) = \log P_{H,t-1}(i) + \gamma_H \pi_{H,t-1} \tag{7}$$

where $0 \leq \gamma_H \leq 1$ measures the degree of indexation to the previous period's inflation rate and $\pi_{H,t} = \log(P_{H,t}/P_{H,t-1})$. Since all firms having the opportunity to reset their price in period $t$ face the same decision problem, they set a common price $P'_{H,t}$. Firms setting prices in period $t$ face a demand curve

$$y_{H,T}(i) = \left( \frac{P_{H,t}(i)}{P_{H,T}} \cdot \left( \frac{P_{H,T-1}}{P_{H,t-1}} \right)^{\gamma_H} \right)^{-\theta} \left( C_{H,T} + C^*_{H,T} \right) \tag{8}$$

for all $t$ and take aggregate prices and consumption bundles as parametric.

The firm's price-setting problem in period $t$ is to maximize the expected present discounted value of profits

$$E_t \sum_{T=t}^{\infty} \alpha_H^{T-t} Q_{t,T} \left[ P_{H,t}(i) \left( \frac{P_{H,T-1}}{P_{H,t-1}} \right)^{\gamma_H} y_{H,T}(i) - W_T f^{-1} \left( \frac{y_{H,T}(i)}{\tilde{\varepsilon}_{a,t}} \right) \right]$$

subject to the demand curve (8), implying the first order condition

$$E_t \sum_{T=t}^{\infty} \alpha_H^{T-t} Q_{t,T} y_{H,T}(i) \left[ P_{H,t}(i) \left( \frac{P_{H,T-1}}{P_{H,t-1}} \right)^{\gamma_H} - \frac{\theta}{\theta-1} P_{H,T} MC_T \right] = 0. \tag{9}$$

where $MC_t$ is the marginal cost function of firm $i$.

Foreign firms face an analogous problem. Thus the optimality condition takes an identical form, with all variables taking the superscript "*" and the subscript $H$ being changed to $F$. Preferences and shocks are allowed to differ and the small open economy assumption implies that $P^*_t$ is equivalent to $P^*_{F,t}$.

## 2.4 Retail Firms

Retail firms in the small open economy import foreign differentiated goods for which the law of one price holds at the docks. However, in determining the domestic currency price of imported goods they are assumed to be monopolistically competitive. This small degree of pricing power leads to a violation of the law of one price in the short run.

In determining prices, retail firms face a Calvo-style price-setting problem allowing for indexation to past inflation. Analogously to the optimization problem for domestic goods producers, a fraction

$1-\alpha_F$ of firms set prices optimally, while a fraction $0<\alpha_F<1$ of goods prices are are adjusted according to an indexation rule analogous to (7) with indexation parameter $0<\gamma_F<1$. Firms setting prices in period $t$ face a demand curve

$$C_{F,T}(i)=\left(\frac{P_{F,t}(i)}{P_{F,T}}\cdot\left(\frac{P_{F,T-1}}{P_{F,t-1}}\right)^{\gamma_F}\right)^{-\theta}C_{F,T} \tag{10}$$

for all $t$ and take aggregate prices and consumption bundles as parametric. The firm's price-setting problem in period $t$ is to maximize the expected present discounted value of profits

$$E_t\sum_{T=t}^{\infty}\alpha_F^{T-t}Q_{t,T}C_{F,T}(i)\left[P_{F,t}(i)\left(\frac{P_{F,T-1}}{P_{F,t-1}}\right)^{\gamma_F}-\tilde{e}_TP_{F,T}^*(i)\right]$$

subject to the demand curve, (10), and implies the first order condition

$$E_t\sum_{T=t}^{\infty}\alpha_F^{T-t}Q_{t,T}\left[P_{F,t}(i)\left(\frac{P_{F,T-1}}{P_{F,t-1}}\right)^{\gamma_F}-\frac{\theta}{\theta-1}\tilde{e}_TP_{H,T}^*(i)\right]=0.$$

Note that in the foreign economy there is no analogous optimal pricing problem. Because imports form a negligible part of the foreign consumption bundle, variations in the import price have a negligible effect on the evolution of the foreign price index, $P_t^*$, and therefore need not be analyzed.

### 2.5 International Risk Sharing and Prices

From the asset pricing conditions that determine domestic and foreign bond holdings, the uncovered interest rate parity condition

$$E_t\lambda_{t+1}P_{t+1}[(1+\tilde{i}_t)-(1+\tilde{i}_t^*)(\tilde{e}_{t+1}/\tilde{e}_t)\phi_{t+1}]=0 \tag{11}$$

follows, placing a restriction on the relative movements of the domestic and foreign interest rate, and changes in the nominal exchange rate.

The real exchange rate is defined as $\tilde{q}_t\equiv\tilde{e}_tP_t^*/P_t$. Since $P_t^*=P_{F,t}^*$, when the law of one price fails to hold, we have $\tilde{\Psi}_{F,t}\equiv\tilde{e}_tP_t^*/P_{F,t}\neq 1$, which defines what Monacelli (2003) calls the law of one price gap. The models of Gali and Monacelli (2005) and Monacelli (2003) are respectively characterized by whether or not $\tilde{\Psi}_{F,t}=1$.

### 2.6 General Equilibrium

Equilibrium requires that all markets clear. In particular goods market clearing requires

$$Y_{H,t} = C_{H,t} + C^*_{H,t} \text{ and } Y^*_t = C^*_t \tag{12}$$

in the domestic and foreign economies respectively. The model is closed assuming foreign demand for the domestically produced good is specified as

$$C^*_{H,t} = \left(\frac{P^*_{H,t}}{P^*}\right)^{-\lambda} Y^*_t$$

where $\lambda > 0$. This demand function is standard in small open economy models (see Kollmann (2002) and McCallum and Nelson (2000)) and nests the specification in Monacelli (2003) by allowing $\lambda$ to be different from $\eta$, the domestic elasticity of substitution across goods in the domestic economy, in order to give additional flexibility in the transmission mechanism of foreign disturbances to the domestic economy. However, our results are unaffected by the parametrization of this demand function.[3] The dynamics of $Y^*_t$ and other foreign variables remain specified by the structural relations developed above. Domestic debt is assumed to be in zero net supply so that $D_t = 0$ for all $t$.[4]

The analysis considers a symmetric equilibrium in which all domestic producers setting prices in period $t$ set a common price $P_{H,t}$. Similarly, all domestic retailers and foreign firms each choose a common price $P_{F,t}$ and $P^*_t$ respectively. Analogous conditions hold for wage setters in the domestic and foreign economies. Finally households are assumed to have identical initial wealth, so that each faces the same period budget constraint and therefore makes identical consumption and portfolio decisions.

Finally, monetary policy is assumed to be conducted according to a Taylor-type rule discussed in the subsequent section. Fiscal policy is specified as a zero debt policy. While not explicited noted, the log-linearization assumed that taxes are equal to the subsidy required to eliminate the distortion induced by imperfect competition in the domestic and imported goods markets.

[3] Constraining $\lambda$ to equal $\eta$ results in identical insights from the estimation, and therefore we report results based on this more general specification.

[4] A similar condition holds for the foreign economy once it is noted that domestic holdings of foreign debt, $B_t$, is negligible relative to the size of the foreign economy.

## 2.7 Log-linear approximation to the model

For the purpose of the empirical analysis, a log-linear approximation to the model's optimality conditions around a non-stochastic steady state with a zero net asset position is employed. For any variable $X$ define $x_t = \log(X_t/\bar{X})$ and for any variable $\tilde{x}$ define $x_t = \log(\tilde{x}_t/\bar{x})$ as the respective log deviations from steady state. For details of the steady state solution see Monacelli (2003).

A log linear approximation to the domestic household's Euler equation (4) provides

$$c_t - hc_{t-1} = E_t(c_{t+1} - hc_t) - \sigma(1-h)(i_t - E_t\pi_{t+1}) + \sigma(1-h)\left(\varepsilon_{g,t} - E_t\varepsilon_{g,t+1}\right). \qquad (13)$$

In the absence of habit formation, when $h = 0$, the usual Euler equation obtains. To derive a relationship in terms of domestic output, a log-linear approximation to the goods market clearing condition implies:

$$y_t = (1-\tau)\,c_t + (\tau\lambda + \tau\eta\,(1-\tau))s_t + \tau\lambda\psi_{F,t} + \tau y_t^* \qquad (14)$$

where

$$\psi_{F,t} \equiv (e_t + p_t^*) - p_{F,t}$$

denotes the law of one price gap, the difference between the world currency price and the domestic currency price of imports, and $s_t = p_{F,t} - p_{H,t}$ gives the terms of trade.[5] Time differencing the terms of trade definition implies

$$\Delta s_t = \pi_{F,t} - \pi_{H,t}. \qquad (15)$$

Thus, equilibrium domestic consumption depends on domestic output and three open economy sources of fluctuation: the terms of trade, deviations from the law of one price and foreign output.

The terms of trade and the real exchange rate are related according to

$$q_t = e_t + p_t^* - p_t = \psi_{F,t} + (1-\tau)\,s_t \qquad (16)$$

so that the real exchange rate varies with deviations from the law of one price and also differences in the consumption bundles across the domestic and foreign economies.

A log-linear approximation to domestic firms' optimality conditions for price setting implies the relation

$$\pi_{H,t} - \gamma_H\pi_{H,t-1} = \xi_H\left(w_t + s_t + \psi_t\right) + \beta E_t\left(\pi_{H,t+1} - \gamma_H\pi_{H,t}\right) + \varepsilon_{ch,t} \qquad (17)$$

[5] In deriving equation (14) we make use of the fact that in steady state imports equal exports. This imposes $\left(\bar{Y}^*\right)^\gamma/\bar{Y} = \tau$ as the numerator gives foreign export demand in steady state.

where

$$\begin{aligned}\psi_t &= (1+\omega_p)\,\varepsilon_{a,t} - \omega_p y_t \quad (18)\\ \xi_H &= \alpha_H^{-1}(1+\omega_p\theta)^{-1}(1-\alpha_H)(1-\alpha_H\beta)\\ \omega_p &= -f''\bar{Y}/\left(f'\right)^2 > 0\end{aligned}$$

and $\varepsilon_{ch,t}$ is a cost-push shock added for estimation purposes. Thus domestic price inflation, $\pi_{H,t} = p_{H,t} - p_{H,t-1}$, is determined by current marginal costs, expectations about inflation in the next period and the most recent observed inflation rate. The latter appears as a result of price indexation. In the case of zero indexation to past inflation, $\gamma_H = 0$, the usual forward looking Phillips curve arises.

The optimality conditions for the retailers' pricing problem yields

$$\pi_{F,t} - \gamma_F\pi_{F,t-1} = \xi_F\psi_{F,t} + \beta E_t(\pi_{F,t+1} - \gamma_F\pi_{F,t}) + \varepsilon_{cf,t} \quad (19)$$

where

$$\xi_F = \alpha_F^{-1}(1-\alpha_F)(1-\alpha_F\beta)$$

and augmenting the model with a cost-push shock, $\varepsilon_{cf,t}$. Here, inflation in the domestic currency price of imports, $\pi_{F,t} = p_{F,t} - p_{F,t-1}$, is determined by current marginal cost conditions given by $\psi_{F,t}$ and expectations about next period's inflation rate. Again, that prices are indexed to past inflation induces a history dependence on the most recent observed inflation rate.

Optimal wage setting implies

$$\pi_t^w - \gamma_w\pi_{t-1} = \beta E_t\left(\pi_{t+1}^w - \gamma_w\pi_t\right) + \xi_w(v_t - w_t) \quad (20)$$

where

$$\begin{aligned}v_t &= \phi(y_t - \varepsilon_{a,t}) + \frac{\sigma}{1-h}(y_t - hy_{t-1}) \quad (21)\\ \xi_w &= \alpha_w^{-1}(1+\phi\theta_w)^{-1}(1-\alpha_w)(1-\alpha_w\beta).\end{aligned}$$

Furthermore, nominal wage inflation and the real wage satisfy the identity

$$w_t = \pi_t^w - \pi_t - w_{t-1}. \quad (22)$$

The uncovered interest-rate parity condition gives

$$(i_t - E_t\pi_{t+1}) - \left(i_t^* - E_t\pi_{t+1}^*\right) = E_t\Delta q_{t+1} - \chi a_t - \phi_t \quad (23)$$

while the flow budget constraint implies

$$c_t + a_t = \beta^{-1} a_{t-1} - \tau \left( s_t + \psi_{F,t} \right) + y_t \tag{24}$$

where $a_t = \log(e_t B_t^f / (P_t \bar{Y}))$ is the log real net foreign asset position as a fraction of steady state domestic GDP.

Conditional on the evolution of the world economy and other exogenous disturbances, closing the model requires specification of monetary policy. It will be assumed that monetary policy is conducted according to the Taylor-type rule

$$i_t = \theta_i i_{t-1} + (1 - \theta_i) \left( \theta_\pi \pi_t + \theta_y y_t \right) + \varepsilon_{i,t} \tag{25}$$

so that the nominal interest rate is determined by past interest rates and also responds to the current CPI inflation rate and output.[6] The final term, $\varepsilon_{i,t}$, is a monetary policy shock or implementation error in the conduct of policy. Finally, the CPI, domestic goods prices and the terms of trade are related according to

$$\pi_t = \pi_{H,t} + \tau \Delta s_t. \tag{26}$$

The domestic block of the economy is therefore given by equations (13)-(26) in the 14 unknowns $\left\{ c_t,\ y_t,\ i_t,\ a_t,\ q_t,\ s_t,\ \pi_t,\ \pi_{H,t},\ \psi_{H,t},\ \pi_{F,t},\ w_t,\ \pi_t^w,\ v_t,\ \psi_t \right\}$. Given processes for the exogenous disturbances $\{\varepsilon_{a,t},\ \varepsilon_{i,t},\ \varepsilon_{g,t},\ \varepsilon_{l,t},\ \varepsilon_{ch,t},\ \varepsilon_{cf,t}\}$ and $\{\pi_t^*,\ x_t^*,\ i_t^*\}$ this linear rational expectations model can be solved using standard methods. The disturbances $\{\varepsilon_{a,t},\ \varepsilon_{g,t},\ \varepsilon_{l,t},\ \varepsilon_{ch,t},\ \varepsilon_{cf,t}\}$ are assumed to be independent AR(1) processes and $\{\varepsilon_{i,t}\}$ an i.i.d. process. The determination of the foreign block is discussed in the appendix, although we note here that the four shocks driving fluctuations in the U.S. block correspond to technology, preference, and cost push shocks and an innovation to the Taylor Rule.

# 3 Estimation Methodology

## 3.1 Estimation

Our objective is to estimate the parameters of the DSGE model specified in the previous section using Bayesian methods. Therefore, we aim to characterize the posterior distribution of the model

[6] We have also allowed for an exchange rate term in the policy rule following Lubik and Shorfheide (2003) without any impact on the results.

parameters $\theta \in \Theta$. Given a prior, $\pi(\theta)$, the posterior density is proportional to the product of the likelihood and the prior. As described by Schorfheide (2000), posterior draws for this density can be generated using a random walk metropolis algorithm and the state-space representation implied by the solution of the linear rational expectations model and the Kalman filter. Measures of location and scatter are obtained from the draws by computing, for instance, the median and posterior probability bands. Furthermore, given the draws it is possible to characterize the posterior distribution of any functional of interest, by computing the corresponding functional for each of the draws. This convenient feature of the estimation will later be exploited to analyze the model implied variance decompositions.

An optimization algorithm (Christopher Sims' csminwel) is used to obtain an initial estimate of the mode. We start the maximization algorithm from a number of random draws from the prior — before launching the MCMC chains — and check that the optimization routine always converges to the same value.[7] This is a useful diagnostic for the presence of identification problems. Indeed, this helped resolve an identification issue in the parameters relating to the domestic currency price of imported goods, resulting from the interaction of Calvo pricing, price indexation and correlated mark-up shocks, which together likely overparameterized the persistence of this variable. Therefore, for the remainder of the paper, the model is estimated assuming for imported goods price inflation either the presence of persistent (as opposed to white noise) markup shocks and no indexation or vice versa. This is inconsequential for our main results.

Having ensured a unique mode, the Hessian from the optimization routine is used as a proposal density, properly scaled to yield a target acceptance rate of 25%. For the MCMC results, seven chains of 100,000 draws each were initialized by randomly selecting starting values (using an over dispersed normal density centered at the mode with a scaled-up Hessian as variance covariance matrix). For each chain, following a burn-in phase of 40,000 draws, convergence is monitored using CUMSUM plots and, for the overall chains, the potential scale reduction factors and confidence interval variants of Brooks and Gelman (1998).

The first column in Table 1 presents the priors for the DSGE coefficients indicating the density, median and standard deviation. They are motivated by earlier work reported in Justiniano and Preston (2004), are fairly uncontroversial and accord with other studies adopting a Bayesian approach

[7] For the baseline model over 50 optimization chains were launched from the prior draws, all converging to the same mode. For other specifications, 10 or more initial searches for the mode were used.

to inference. Note, however, that we wish to remain fairly agnostic on the sources of persistence and business cycle fluctuations and for this reason adopt identical reasonably uninformative priors for the autoregressive coefficients and standard deviations of all shocks.[8] One prior that deserves comment is $\tau$, the degree of openness, which could have been calibrated at the mean of the trade shares in Canadian output. Given the importance of this parameter for the possible influence of foreign shocks, we preferred to specify a prior distribution, although rather tight, around this value.[9]

Several other parameters are not well identified and therefore calibrated. The discount factor is fixed at 0.99. The elasticities of demand across varieties of goods and labor input in both the domestic and foreign block are set equal to 8 following results reported in Woodford (2003, Ch. 3). Finally, the parameter governing the elasticity of interest rate to debt is fixed at 0.01 following Beningo (2001). The sensitivity of our results to these calibration assumptions is discussed later.

### 3.2 Data

We estimate the model using ten observable series corresponding to Canadian and U.S. output, inflation, interest rates, real wages as well as both the terms of trade and the real exchange rate. Some empirical open economy DSGE papers include one of either the real exchange rate or the terms of trade, or alternatively treat one of these series as exogenous. Confronting the model with less data exploits fewer model implied cross-equation restrictions and therefore delivers a less stringent test of the model.

For the United States all series are downloaded from Haver analytics. Output corresponds to per capita real GDP. Inflation is the annualized log-difference of the GDP deflator (JGDP) and the nominal interest rate the annualized effective funds rate. Real wages are measured by the nominal compensation per hour in the nonfarm business sector (LXNFR) divided by the GDP deflator. For Canada, the gross domestic product (millions of 97 chained dollars) published by the OECD is used for per capita real GDP (StatCan). The annualized quarterly log difference in the consumer price index excluding food and energy (StatCan) corresponds to the measure of overall inflation.[10]

[8] In addition the same priors are used for all coefficients of the domestic and foreign economy.

[9] Furthermore, we also note that calibrating $\tau$ at a reasonable value, such as 0.2, does not in any way alter our conclusions regarding the anomalies found in accounting for the influence of foreign shocks that is the central topic of section 5.

[10] Canadian inflation series, even excluding food and energy, exhibit a sharp spike in 1991 due to the effects of indirect taxes. Hence for that year only we use a measure of inflation which excludes energy, food, as well as indirect taxes.

The interest rate is measured by the official discount rate published by the Bank of Canada. The series on hourly earnings published by International Financial Statistics (IMF) divided by the GDP deflator gives our measure for real wages. The real exchange rate is the bilateral Canada-US real exchange rate series constructed by the IMF. Finally, we take the ratio of the deflator for imports to exports published by the OECD as the measure for the terms of trade.

Output and real wages are included in log-deviations from a linear trend, although we later assess the sensitivity of our results to an alternative data set in first differences. The real exchange rate and the terms of trade are in log differences. All variables are demeaned for the estimation. The sample runs from 1984q1 to 2004q3.[11]

# 4 Results

This section reports estimation results. The benchmark model is presented first and a comparison of data versus model implied volatilities and persistence follows. Our analysis then seeks a deeper understanding of the role of various model disturbances in the estimation and, in particular, the importance of risk premium and import inflation cost-push shocks for easing cross-equation restrictions. The central analysis on the success in accounting for international linkages (i.e. covariances) is discussed in the sequel.

## 4.1 Estimates and Model Fit

Table 1 reports parameter estimates for the benchmark model, determined after extensive investigation of alternative specifications. These included various combinations of endogenous persistence mechanisms — habit formation, price indexation in the domestic goods, imported goods and labor markets — as well as variation in the parameterization of the exogenous sources of persistence. We also experimented with various combinations of shocks which included, among others, cost-push shocks in price setting of all three markets, labour supply shocks, correlated preference shocks, and correlated technology shocks. It is worth emphasizing that our main results do not depend on these alternative specifications, which, as shown later are always associated with a poorer fit (both in terms of posterior odds and the comparison of implied versus empirical second order moments).

[11] We use the 1981q1 to 1983q4 for the initialization of the Kalman filter.

Furthermore, this specification search was performed while retaining the same reasonably agnostic prior ensuring resolution of any possible identification issues.

Estimates are for the most part reasonable. The degree of price stickiness in the production of home produced goods, both in the domestic and foreign blocks of the model, seem somewhat high. However, it is worth emphasizing both the cost-push innovations to the domestic and foreign Phillips' curve are assumed white noise, as opposed to persistent processes and also that price indexation implies prices are changed every quarter. The Calvo adjustment parameters in the domestic and foreign economies imply wages are re-optimized very 4-5 quarters. Imported goods prices are re-optimized most frequently every 2-3 quarters. The intertemporal elasticity of substitution and elasticity of labor supply accord with earlier macroeconomic studies of this kind. The estimated coefficients of the Taylor rule align with conventional wisdom while preference, risk premium and imported goods cost-push shocks are revealed to be highly persistent. Technology shocks are persistent, though less so than typically assumed in calibrated models. The median estimate for the elasticity of substitution across home and foreign goods is 1.8, and in line with the value of 1.5 used in calibrations by Chari, Kehoe, and McGrattan (2002).

Table 2 shows the standard deviations and autocorrelation of both the data and the corresponding estimates of these quantities implied by the benchmark model. The model implied statistics are constructed using the median parameter estimates by simulating $10,000$ series of length 100 and computing the moment of interest. The second column of each panel presents the median value of each statistic while the third column gives the implied 5th and 95th percentiles.[12] Overall, it is clear that the incomplete markets baseline model does well in replicating these features of the data. The 90 percent intervals for the model implied volatilities encompass the empirical moments for all but one series. The only exception is the terms of trade, for which the model actually over-predicts the standard deviation.

Turning to the autocorrelations, it is immediate the model provides a remarkably good characterization of the persistence properties of the data. The persistence of inflation is perfectly matched, while the persistence of the remaining series is slightly under predicted by the model. Given the agnostic priors assigned to the persistence of exogenous disturbances relative to other recent studies, this is certainly an appealing feature of the estimation results.

[12] We report empirical and theoretical moments on filtered data for the real exchange rate and the terms of trade to facilitate comparisons with previous work that has investigated the performance of these models along these dimensions.

Backus, Kehoe, and Kydland (1995) show in a two country real business cycle model that the implied standard deviation for the terms of trade is small relative to the data and identify this discrepancy as a puzzle. While they dismiss as potential remedies the introduction of additional shocks (oil and preference shocks); incomplete markets; money; and imperfect competition when taken individually, it is evident that when combined these assumptions may account for this price anomaly. Of course the model presented here over predicts the volatility of the terms of trade and hence does not fully resolve this puzzle. Moreover, the cost push import shock and risk premium shock are weakly motivated by economic theory. We now discuss their importance for model parameter estimates and dynamics.

## 4.2 Cost-push and risk premium shocks

This section demonstrates that the addition of risk premium and import cost-push shocks is required to loosen certain cross-equation restrictions which would otherwise result in implausible parameter estimates and dynamics. While risk premium shocks have become common in open economy models (see Kollman (2002) and McCallum and Nelson (2002)) an insight emerging from this exercise is that these two disturbances serve to break the theoretical link between both the terms of trade and the real exchange rate with the remaining variables. For the real exchange rate, such findings resonate with the exchange rate disconnect puzzle. As for the terms of trade, we provide evidence that its close comovement with the real exchange rate is well captured in the model thanks to the risk premium shock, which in part seems to proxy for fluctuations in commodity prices.

### 4.2.1 Parameter Estimates Absent These Disturbances

Table 3 presents the estimated parameters when one or both of these shocks are absent. The first column shows that when both disturbances are excluded from the estimation, the elasticity of substitution between domestic and foreign goods, $\eta$, takes a value close to zero. Moreover, the domestic intertemporal elasticity of substitution is negligible and the model converges to a closed economy representation ($\tau$ is 0.01). The marginal likelihood deteriorates dramatically, although posterior odds should be interpreted with care as the values attained by these parameters close to the boundaries induced convergence problems in the MCMC chains which we take as additional evidence of problems with this specification.[13] In addition, with this caveat in mind, the model implied standard deviations (not shown) for the domestic nominal interest rate and inflation are as much as ten times larger than in the data.

To gain further insight into the role of each shock, we re-estimate the model excluding one disturbance at a time. The second column reports the estimates for a model without risk premium shocks. In this case, both the persistence and volatility of the cost push shock in imported goods prices rise. Most notable is the estimate for the intertemporal elasticity of substitution which, once again, is virtually zero.

To give some intuition, recall the uncovered interest rate parity condition (23) when written in terms of the real exchange rate.[14] Substitution of the domestic Euler equation and rearranging implies the relation

$$
\begin{aligned}
& E_t\left(c_{t+1} - hc_t\right) - \left(c_{t+1} - hc_t\right) + \sigma\left(1-h\right)\left(\varepsilon_{g,t} - E_t\varepsilon_{g,t+1}\right) - \sigma\left(1-h\right)\left(i_t^* - E_t\pi_{t+1}^*\right) \\
= \quad & \sigma\left(1-h\right)\left(E_t\Delta q_{t+1} - \phi a_t - \phi_t\right).
\end{aligned}
$$

Hence a value of $\sigma$ close to zero serves to break the tight connection between the real exchange rate and domestic consumption movements (and therefore domestic output movements given relation (14)). An immediate implication is that the incomplete markets assumption is, in and of itself, insufficient to give plausible parameter estimates — even though trade in assets is restricted to one period domestic and foreign debt. While the failure of uncovered interest parity is well known, it is nonetheless remarkable that neither the addition of frictions nor a range of disturbances (particularly the preference shocks which explicitly enter this expression) help alleviate the constraint imposed by this restriction. Taking the model to the data still requires a purely exogenous stochastic component to the risk premium.

The last column shows results absent the cost push shock in the domestic price of imported goods, yet allowing for indexation in the retail sector. As in the case when both disturbances are excluded, $\eta$ becomes very small. To understand this result, note that the terms of trade necessarily

[13] Launching several minimization algorithms from random prior draws led to modes very similar to one another but differing only on the magnitude of these particular coefficients which in all cases ware negligible.

[14] When these disturbances are dropped, we introduce shocks to the market clearing condition and to the preference for leisure to avoid stochastic singularity in estimation.

satisfies the following approximate log-linear relationships:

$$\begin{aligned}(1-\tau)\,\Delta s_t &= \pi_{F,t} - \pi_t \\ c_{H,t} - c_{F,t} &= \eta s_t. \end{aligned} \tag{27}$$

By inducing fluctuations in import goods inflation, the cost-push disturbance in principle enables the model to loosen the first relationship and to disentangle movements in the terms of trade from other domestic series. Implied cross-correlations from our baseline model reveal that the estimated model seeks this disconnect and that fluctuations in this disturbance, import prices and the terms of trade are closely linked (the latter series have a cross correlation of 0.95).

When the cost-push shock is excluded, terms of trade fluctuations need to be explained by disturbances linked to other domestic series as seen from (27). The model attempts to circumvent this restriction by driving the elasticity of substitution between domestic and foreign goods to zero, which serves to diminish the tight link between relative world price movements and fundamentals. Nonetheless, the model ultimately overshoots the implied variability of the terms of trade, as well as other series, in particular domestic interest rates and inflation, whose volatilities exceed those in the data by an order of magnitude (results omitted due to space considerations).

#### 4.2.2 Variance decompositions for cost-push and risk premium shocks

To gain further insight into the crucial role played by the cost-push and risk premium shocks, Table 4 presents the stationary variance decomposition of the baseline incomplete markets model for domestic variables when both these disturbances are included. The share explained by foreign shocks is extensively analyzed in the next section. Observe that these two disturbances explain the bulk of the variation in the terms of trade and the real exchange rate, while only accounting for a very modest share of the fluctuations in domestic output, inflation and interest rates. This further reinforces the discussion above on how these shocks ease cross-equation restrictions of the model and disconnect these two series from the remaining observable series used in estimation. Moreover, it accords well with the exchange rate disconnect puzzle and the inability to link exchange rate volatility to disturbances affecting other real variables (such as domestic and foreign technology and preference shocks).[15]

[15] Furthermore, the model does a very poor job in forecasting the log difference in the real exchange rate. In fact, this series is mostly driven by the innovations (obtained with a disturbance smoother) to the stochastic process labeled

The law of one price gap helps the model, in principle, to break the close link between the terms of trade and the real exchange rate. However, the comovement between these two series is perfectly captured, as the model implied cross-correlation is 0.421, almost identical to the sample estimate 0.415.[16] This also becomes evident from the first five panels in Figure 1 which plot the terms of trade and the real exchange rate using both the model implied and raw data series in levels and differences, together with the filtered time series for these two shocks.[17] Consistent with the variance decomposition, these plots suggest the risk premium shock is crucial in explaining the linkages between these two series.[18]

#### 4.2.3 Interpretation

The inclusion of risk premium shocks has become standard in open economy models, and can be variously interpreted as a time varying risk premium (Obstfeld and Rogoff, 2002), heterogenous expectations regarding the exchange rate by noise traders (Jeanne and Rose, 2002) and portfolio shifts in asset markets (Bergin, 2004). In the present analysis, given the importance of commodities in Canadian exports, it seems plausible that one of these disturbances represents innovations to commodity prices, which may in fact be exogenous to the economy and originate in world markets.[19] Indeed, Chen and Rogoff (2003) find evidence of a long-run relationship between the Canadian-U.S. dollar real exchange rate and non-energy commodity prices, which has also been a staple in the exchange rate forecasting equations of the Bank of Canada (see Amano and van Norden (1993)).

To explore this possibility, Figure 2 plots the log-differences in non-energy real commodity prices used in the forecasting equations of the Bank of Canada, together with the first differences in the import cost-push and risk premium shocks.[20] Table 5 further presents the correlation between these

the risk premium shock. Similarly, the one-step-ahead forecast errors for these series from the Kalman filter are quite large and volatile. Consistent with the random walk hypothesis, the real exchange rate in the model is being driven by a very persistent exogenous process, which yields first differences that are essentially white noise (the model implied first order autocorrelation for the real exchange rate growth is -0.016, in contrast to 0.29 in the data).

[16] The minor role played by technology in the variance decomposition is perhaps striking, but is consistent with other recent empirical work in closed models that highlights the relative importance of preference shocks — see Primiceri, Schaumburg, and Tambalotti (2005).

[17] The shocks correspond to the filtered states from the forward Kalman filter using the solution to the DSGE model at the median of the parameter estimates.

[18] From the last panel in Figure 1 it can be seen that changes in the risk premium and import cost-push shocks are largely uncorrelated, in accordance with the orthogonality assumption imposed in the estimation. The cross-correlation between these two processes is -0.006.

[19] Based on nominal national accounts data produced by Statistics Canada, agricultural, energy and forestry exports account on average for 26 percent of Canada's total exports over our sample.

[20] This real series is constructed as the ratio of the nominal non-energy commodity price index published by Statistics

series, together with the terms of trade and real exchange rate. There is little evidence of comovement between non-energy commodity prices and the import cost-push shock. In contrast, the second panel of Figure 2 and the cross correlations suggest negative comovements between the real non-energy commodity price and the real exchange rate, terms of trade and risk premium disturbance.[21] This is consistent with Bank of Canada regression estimates that indicate positive innovations in real non-energy commodity prices are associated with real appreciations. It also rationalizes why this shock can match well the comovements between the real exchange rate and the terms of trade.

This exercise suggests two additional comments. First, it highlights the risks in interpreting disturbances estimated from DSGE models. Given that the risk premium shock helps break the restriction imposed by the uncovered interest parity condition, it is more natural to associate this disturbance with developments in financial markets that have proven hard to model under rational expectations. While it is certainly plausible that what has been labelled the risk premium shock captures innovations of this nature, we have also provided evidence linking it to fluctuations in non-energy commodity prices.

Secondly, and most important for the ensuing analysis, we do not refer to the import cost-push and risk premium shocks as foreign disturbances. This classification is of course arbitrary and it is unclear whether these shocks truly represent developments outside of the Canadian economy. The preceding discussion, though, suggests that at least some component of the so called risk premium shock may be driven by fluctuations in commodity markets which may well be exogenous to Canada. Nonetheless, from the point of view of the DSGE model that these ad hoc exogenous processes are, by construction, unrelated to the U.S. block of the model and hence unable to explain any comovements with U.S. series. Furthermore, as seen in Table 4, they play a very minor role in explaining Canadian output, inflation and interest rate fluctuations and therefore, at least in the context of this model, seem unlikely to proxy for the transmission of foreign shocks that we now turn to.[22]

Canada (V36383) to the U.S. GDP deflator. Helliwell, Issa, Lafrance, and Zhang (2004) describe the recently updated equations of the Bank of Canada which still include non-energy real commodity prices but abstract from energy. Incidentally, the inflation measure used in the estimation here removes energy components from the CPI which further suggests focusing solely on the non-energy series.

[21] Recall the terms of trade are defined as the ratio of import to export prices, so that a negative correlation suggests an associated terms of trade improvement.

[22] In other words, these results *cannot* be interpreted as suggesting that exogenous foreign disturbances, such as commodity price shocks, associated with terms of trade and real exchange rate movements do not lead to important fluctuations in output and inflation. The calibrated analysis of Mendoza (1991) and Kose (2001), for example, advocates trade shocks as crucial drivers of business cycles. Here, in contrast, estimation consistently attempts to separate the fluctuations in the terms of trade and the real exchange rate from the remaining domestic series.

# 5 Accounting for the Influence of Foreign Shocks

The previous section presented the first result of the paper: it is possible to design and estimate structural models that can account for the volatility and persistence of open economy data, provided ad hoc disturbances are used to relax certain cross-equation restrictions imposed by the theoretical framework. However, while some practitioners might not regard the addition of these shocks as problematic, there is an important qualification to the success of the estimated model. The second insight of this paper is that variance decompositions reveal the model to be unable to explain the influence of foreign disturbances on the domestic economy.

The following discussion adduces a range of evidence on the model's failure to account for the linkages between the U.S. and Canada, and corroborates — with the help of reduced form models — that comovement is indeed a salient property of our data set. An extensive robustness check of this result is postponed until the next section, where we consider whether our priors and calibrations, the characterization of the foreign block and the detrending of the data could be responsible for the model's drastic failure in this dimension. As a preview of our findings, all these variants leave our result unchanged.

## 5.1 Foreign shocks and comovements in the DSGE model

Based on the draws used for the baseline parameter estimates, we report the median fraction of variation in the domestic series, the real exchange rate and terms of trade that is attributable to all four foreign disturbances, at several forecast horizons. Here "foreign" denotes shocks originating in the U.S. block as only these shocks can potentially explain comovement across the two countries. The striking result in Table 6 is that at most one percent of variation in Canadian output, inflation and interest rates is explained by foreign disturbances. Furthermore, the 95 percentiles for the shares of these three series never exceeds 1 percent for any U.S. shock (percentiles have been omitted for space considerations). As for the real exchange rate and the terms of trade, the combined contribution of foreign disturbances is larger, with a median of 9 and 5 percent respectively. These observations are robust across forecast horizons.

Figure 3 provides an alternative perspective on the model's inability to account for the influence of foreign shocks, and therefore comovements, by plotting the cross correlations (contemporaneous through lag 15) between Canadian output, inflation and nominal interest rates with the U.S. foreign variables. Two sets of correlations are presented. The first one corresponds to the moments generated by the estimated baseline DSGE model over the parameter draws, for which we present medians and 90 percent confidence bands. These are contrasted with the empirical cross correlations in the data over the same sample used to estimate the model.

Most notable is the structural model predicts cross-correlations that are virtually zero at all lags. An immediate consequence is that the DSGE model cannot replicate the common fluctuations of domestic series with U.S. output, interest rates and inflation. However, the magnitude of the cross correlations is smaller for the real exchange rate.

Further evidence on the model's failure to capture the contribution of foreign disturbances comes from looking at the contemporaneous correlation matrix for the structural innovations. These are obtained under the baseline coefficient estimates using a disturbance smoother and are reported in Table 7. If the model properly accounted for dynamics, then these DSGE shocks should be orthogonal, an assumption imposed on the covariance matrix used to estimate the model. Yet a casual glance at this table reveals significant cross-correlations in the structural errors. This is particularly true for a number of correlations involving foreign preference and monetary policy shocks, as well as risk premium disturbances and monetary policy innovations. Given the importance of these disturbances in the uncovered interest rate parity condition, these observations suggest misspecification may in part be induced by this cross-equation restriction. Overall, some 15 of these correlations have absolute value greater than or equal to 0.15.

The model's departure from orthogonality in the estimated innovations underscores the existence of substantial misspecification and also serves to rationalize the implied structural variance decompositions and empirical cross-correlations analyzed earlier. Because the DSGE variance decompositions are constructed assuming orthogonality in the structural errors, the covariance embedded in the disturbances is not accounted for. Thus, a consequence of the model's inability to explain comovement and transmission is that the cross-correlation gets pushed into the structural shocks.

Worth stressing is that the lack of meaningful contribution from foreign shocks to the volatility of the domestic series is not an inherent feature of the structural model. On the contrary, random draws generated from our priors attribute a sizeable share of the domestic series' variance to foreign shocks, which can be as large as 90 percent. Moreover, the inability to explain the influence of foreign disturbances is not unique to the estimated model of this paper. In particular, parallel work by Adolfson, Laseen, Linde, and Villani (2005) also identifies negligible stationary variance shares for shocks originating in the rest of the world. While the authors do not comment on this issue, it is noteworthy that their estimated model includes features such as investment, variable capital utilization and even a working capital channel, whose absence here could have been suspected as the culprit for our results. Similarly, de Walque, Smets, and Wouters (2005) also fail to identify significant cross-country linkages in an estimated two-country model for the U.S. and the Euro area. This suggests that the small open economy assumption is not responsible for our findings.

### 5.2 Reduced-form Evidence

In the light of these results, it seems natural to ask: is comovement between domestic and foreign variables a property of our data? The sample cross-correlations previously discussed hinted the answer to this question. This section sheds further evidence on this issue, presenting reduced form estimates that affirms our insights and marks the sharp contrast between the structural model and the data regarding the influence of foreign shocks.

Two kinds of reduced form models are considered to capture the possible linkages between the United States and Canada. The first model corresponds to a VAR subject to exclusion restrictions, formally a seemingly unrelated regression (SUR), using the same data and sample as in the estimation of the DSGE model. Second, we turn to a dynamic moving average factor model which can analyze a larger number of series and may hence control for an erroneous variance decomposition induced by missing variables that are important for comovement.

We begin with the SUR specification that is subject to the same zero restrictions — imposing no feedback from Canada to the U.S. — as in the solution of the DSGE model. Our goal is to compute the variance shares for the domestic series corresponding to the U.S. disturbances. This is achieved by imposing a Cholesky decomposition on the estimated SUR innovations. We do not attempt to identify any particular shock, but simply seek to obtain the portion of the variance explained by the first four disturbances, which are the only ones that feed into the U.S. block. Therefore, the results obtained with this identification procedure are invariant to re-ordering of the series within the foreign block.

A one lag SUR model is estimated with the efficient block-recursive Gibbs algorithm proposed by

Zha (1999), using the same data set and sample as for inference on the DSGE model.[23] Initializing the Gibbs sampler at the classical SUR estimates, and after discarding 10,000 draws, the variance shares are computed for each of 50,000 draws, using the Cholesky decomposition for the SUR reduced form innovations. The second to last column in the bottom panel of Table 6 reports median shares corresponding to the sum of all four foreign shocks in the SUR. It is immediate that in contrast to the structural model, U.S. sourced disturbances account for around 2/3's of the variation in output and between 36 and 84 percent of the volatility in other domestic series. Slightly bigger shares are obtained using a SUR with two lags. At shorter forecast horizons the fractions explained by foreign disturbances are somewhat smaller, but never below 10 percent, and, hence, remain substantially larger than the corresponding decompositions for the DSGE model. These results are insensitive to the specification of the priors for the SUR coefficients and, furthermore, accord well with the findings of Cushman and Zha (1997) who use an overidentified structural SUR.

While the SUR confirms the large influence of U.S. shocks on Canadian business cycles, the analysis is limited by sample considerations to the estimation of a two lag model. Because the reduced form representation of the DSGE is a VARMA, it is desirable to extend the comparison of the estimated theoretical framework to a reduced form model allowing for richer dynamics. Furthermore, it also seems appropriate to work with a larger number of series, which can encompass a richer set of sources and channels of transmission of foreign shocks.

Based on these considerations, we also report variance decomposition estimates from a dynamic moving average factor model estimated for the U.S. and Canada on a similar sample. The analysis is based on Justiniano (2004) to which the reader is referred for further detail.[24] For our purposes it suffices to note that formal model comparison methods dictate the inclusion of four factors, two of which are common to both countries (foreign factors) with the remaining two exclusive to the

[23] A baseline prior specification considers a Normal (0,10), i.e. mean zero and standard deviation ten, for each off-diagonal element in the contemporaneous matrix, $A(0)$, following Zha's notation, of the structural representation underlying the SUR model. Own-lag coefficients are assigned a Normal (0.7,0.3) for each series, while a Normal (0,0.3) is chosen as a prior for off-diagonal parameters in the matrix of lags. Each structural innovation has an Inverse-Wishart prior with mean 0.4 and standard deviation 0.3, as for the DSGE innovations. Alternative priors, favoring less persistence in the (log-differenced) real exchange rate and terms of trade, with smaller or lager variance in the Inverse-Wishart ordinates do not affect any of the results, which indeed are very similar to those obtained with a classical SUR estimate.

[24] This is an update of the results presented in that paper allowing, here, for more general dynamics. In addition, the normalization of the factors is based in this case only on sign restrictions for the factor loadings. This in turn results in a normalization structure largely dictated by the data, as alternative series orderings always highlight the same series exhibiting greatest commonalities with the factors, which are then used for the normalization.

Canadian economy (domestic), in order to explain a panel of 32 series (16 for the United States and 16 for Canada).[25] The factors and idiosyncratic (series specific) components are allowed to follow independent autoregressive processes of order three. In addition, the comparison of marginal likelihoods across a wide array of specifications favors the inclusion of series specific moving average dynamics for the effect of the loadings, indicating that spillover effects may be important for some variables.

The last column in the bottom panel of Table 6 reports the median variance decompositions for Canadian output, inflation, interest rates, the terms of trade and the real exchange rate for the two foreign factors.[26] While differences in sample and data preclude accurate comparisons with the SUR results, it is noteworthy that the factors suggest an even more dominant role of foreign shocks in explaining Canadian business cycles. Kose, Otrok and Whiteman (2003, 2005) similarly find significant commonalities in output fluctuations across these two countries using factor analysis. Finally, the variance shares for the real exchange rate and the terms of trade in the SUR and factors are relatively more in agreement with those from the DSGE, although the latter are still smaller.

# 6 Robustness

## 6.1 Sensitivity to the choice of priors

A natural starting point to gauge the robustness of our results is to consider whether the priors are the culprit for the absence of linkages between domestic and foreign blocks. We begin by re-estimating the model for an alternative prior specification in which the Inverse-Wishart ($IW$) and Beta ($B$) densities for the disturbances are modified in order to tilt the volatility and persistence of Canada's fluctuations towards U.S. shocks.[27] As already mentioned, draws generated from the benchmark priors lead to large foreign variance shares in the domestic series, evidencing that the model has sufficient flexibility to capture the comovements in the data.

[25] The number of factors is determined by posterior model probabilities, computed using the marginal likelihood, and assigning equal prior probability to all models which differ in the number of latent common variables.

[26] Wages were not included in the panel. As in the dataset used for the DSGE, output is log-detrended (although from a quadratic trend) and the remaining series expressed in log differences, or differences in the case of interest rates. The sample runs from 1983 through 2003.

[27] More specifically the $IW$ densities for the standard deviation of foreign and domestic shocks now have means and standard deviations of $(0.7, 0.4)$ and $(0.3, 0.2)$ respectively, in contrast to the symmetric $IW(0.4, 0.3)$ baseline parametrization. Moreover, the Beta density for the autoregressive coefficients of foreign technology and preference shocks is now centered at 0.8 with a standard deviation 0.1. Compared with the original $B\,(0.5, 0.2)$ specification — preserved for the domestic disturbances — greater persistence is assigned a priori to U.S. disturbances.

The first column in Table 8 shows coefficient estimates are virtually unchanged relative to the baseline prior, repeating, therefore, a parameter configuration that substantially limits the model's ability to explain common U.S.-Canada fluctuations. Hence, it should not be surprising that the implied structural variance shares for the combined effect of all foreign shocks are identical to those reported earlier (omitted for space considerations).

Following standard practice in calibrated and estimated small open economy models, the share of foreign goods in the domestic economy, $\tau$, has until now been interpreted as a measure of openness. Consequently, our prior for this coefficient was centered close to the share of imports in Canada's output. However, it seems plausible that allowing for larger values of $\tau$ could dramatically alter the influence of U.S. shocks. Furthermore, in an alternative interpretation of small open economy models, this coefficient would be associated with the ratio of population shares in the large and small countries (Ghironi (2001)). These considerations motivate an alternative prior for $\tau$, a $B(0.9, 0.05)$, with a 95 percent prior probability band covering the 0.81 to 0.97 interval. The second column of table Table 8 evinces that most coefficient estimates remain largely unchanged with this prior, and the modal estimate of $\tau$ is 0.18, suggesting that the likelihood pushes this parameter towards substantially smaller values than plausible a priori. With those estimates, U.S. shocks combined account for roughly 1 percent of Canadian output and interest rate fluctuations, and less than 2 percent for inflation.

An alternative is to allow for correlation between foreign domestic and technology shocks, as sometimes assumed in international business cycle studies. However, this does not lead to different insights. For instance, setting the prior for the correlation coefficient as a $N(0.5, 0.25)$, yields an estimate of 0.17 but leaves unaltered the remaining parameter estimates (column 3). The resulting variance decompositions reveal negligible contributions of foreign disturbances to the variance of Canadian series.[28] This finding is not suprising in the light of the variance decompositions presented in Table 4 which showed that technology explained less than one percent of output variation in the long run. It is also consistant with recent closed economy empirical analyses such as Primiceri, Schaumburg, and Tambalotti (2005).

[28] In principle imposing a very high degree of correlation across the preference shocks may yield large comovements across foreign and domestic blocks. In addition to being another ad-hoc assumption imposed on the model, their interpretation does not seem straightforward.

Stockman and Tesar (1995) analyze the role of preference shocks in a two country calibrated business cycle model. While they conclude that they do serve to alleviate some counterfactual implications in international business cycles, they do not require these disturbances to be correlated across countries.

We have also experimented with alternative specifications assigning substantially greater prior probabilities to larger values of the elasticity of substitution between foreign and domestic goods, as well as the domestic and foreign intertemporal elasticity of substitution, given the importance of these parameters in the transmission of shocks. In all cases, the variance decompositions are largely unchanged and the share of domestic output, interest rates and inflation explained by all U.S. disturbances does not exceed 2 percent. Similar conclusions emerge under alternative calibrated coefficients governing markups in the goods and labor market (for example, reducing the elasticity of demand across varieties of goods or labor inputs from 8 to 4) or the sensitivity of foreign borrowing to the current account (decreased from 0.01 to 0.001).

## 6.2 Characterization of the foreign block and detrending

To ensure that our results are not being driven by our assumed structural model of the foreign block, the estimation is repeated for a specification in which U.S. series are modeled as an atheoretical first order vector autoregression. This alternative has the appealing feature that it is agnostic about the type of disturbances responsible for driving U.S. business cycles and should help correct for any misspecification in the foreign block. However, the first column in Table 9 shows that the variance decompositions of Canadian series are roughly unchanged, although a slightly higher fraction of the variation in the real exchange rate and terms of trade is now attributable to U.S. shocks.

We also experimented with a Taylor rule that includes the nominal exchange rate, following Lubik and Schorfheide (2003) and the Bank of Canada's focus on the Monetary Conditions Index (MCI) during part of the 1990s. This does not result in any changes regarding the model's inability to explain U.S.-Canada linkages.

A final robustness check is provided by estimating the model (appropriately transformed) using first differenced data. The variance decomposition in the second column of Table 9 is once again virtually identical to the benchmark model. In contrast, the last column shows the fraction of variation explained by U.S. shocks in a 1 lag SUR model with the same data and sample is substantial, albeit smaller than with detrended data.

# 7 Conclusion

This paper evaluates whether an estimated structural small open economy model can replicate the persistence and volatility of the data, and assesses its ability to capture the influence of foreign shocks on the domestic business cycle. A generalized version of the model presented by Gali and Monacelli (2005) and Monacelli (2003), that includes significant frictions in goods, labor and asset markets, is estimated using Bayesian methods for the United States and Canada.

Our first result is that an incomplete markets model combined with a wide range of nominal rigidities is not sufficient to provide plausible parameter estimates and capture the dynamics of each series. The estimation of various model specifications highlights that a cost-push shock in the retail sector and a risk premium disturbance are essential in order to avoid parameter configurations which display anomalies in the parameters and implied second order moments. This is because absent these disturbances the cross-equation restrictions embodied in the interest parity condition, as well as in the link between inflation and terms of trade prove too stringent when confronted with the data, despite the addition of other shocks and sources of rigidity. Not surprisingly, these two shocks account for most of the variation and autocorrelation in the terms of trade and the real exchange rate.

While this observation might be viewed as a qualified success in taking the model to the data, our second result raises a more fundamental caveat. We show that the model is unable to account for the influence of foreign shocks. Structural variance decompositions implied by a range of estimated models uniformly reveal that at most two percent of the variation in Canadian output, inflation and interest rates is attributable to all shocks driving U.S. fluctuations. Concomitantly, the analysis evinces that the implied cross correlations between domestic and foreign variables are predicted by the model to be negligible at all lag lengths. These observations are robust to the characterization of the foreign block in the model, the detrending of the data as well as to an extensive search over alternative specifications of the priors used in estimation.

Importantly, such lack of comovement is not a property of the data. A reduced form model (SUR) using the same data and sample indicates large common fluctuations between the United States and Canada, in accordance with previous empirical work. Similar conclusions emerge from a dynamic moving average factor model using a richer data set.

The introduction of the cost push and risk premium shocks in the structural model essentially unties the real exchange rate and the terms of trade from the remaining fundamental disturbances driving output, interest rates and inflation in the model, such as technology and preferences. It seems natural to interpret this disconnect as responsible for the estimated model's failure to explain comovement. In fact, we have provided evidence to indicate that at least some of the fluctuations in the risk premium shock are associated with variations in Canada's real non-energy commodity prices.

This suggests that a first step in attempting to solve the counterfactual implications for comovement of the estimated DSGE framework presented here would be to seek other modeling structures that serve to attenuate the link between the real exchange rate, inflation and output. The introduction of non-traded goods, particularly for distribution services, is a natural candidate, following Burstein, Eichenbaum, and Rebelo (2002) and Corsetti, Dedola, and Leduc (2005) in their respective work on low pass-through after large devaluations and the Backus-Smith puzzle.

However, it might be possible that real exchange rate disconnect need not imply an inability to explain comovement, an issue that we see as important for further research. Discrepancies in the variance decomposition across structural and reduced form models regarding the influence of foreign and domestic shocks are far less pronounced for the terms of trade and the real exchange rate. Nonetheless, these reduced form models still find a large degree of common fluctuations in output, inflation and interest rates across the domestic and foreign blocks, which the structural model cannot explain. To shed further light on this issue it would be of particular interest to evaluate and estimated small open economy model with an alternative trade structure in intermediate goods as in McCallum and Nelson (2000) combined with a different pricing structure as proposed by Devereux and Engel (2002).

The results in this paper indicate that the success in fitting closed economy estimated DSGE models may not easily translate to the open economy context when attempting to model international linkages. This remains a challenge for future work.

# A The Foreign Economy

The foreign economy is specified as the closed-economy variant of the model described above. Thus, the economy is given by the structural relations:

$$y_t^* - hy_{t-1}^* = E_t(y_{t+1}^* - hy_t^*) - \sigma(1-h)\left(i_t^* - E_t\pi_{t+1}^*\right) - \sigma(1-h)\left(1-\rho_g\right)\varepsilon_{g,t}^* \tag{28}$$

$$\pi_t^* - \gamma_p\pi_{t-1}^* = \beta E_t(\pi_{t+1}^* - \gamma_p\pi_t^*) + \xi_p(\psi_t^* + w_t^*) + \varepsilon_{cp,t}^* \tag{29}$$

$$\pi_t^{w*} - \gamma_w\pi_{t-1}^* = \beta E_t\left(\pi_{t+1}^{w*} - \gamma_w\pi_t^*\right) + \xi_w(v_t^* - w_t^*) \tag{30}$$

$$\psi_t^* = (1+\omega_p)\varepsilon_{a,t}^* - \omega_p y_t^* \tag{31}$$

$$v_t^* = \left(\phi + \frac{\sigma^{-1}}{1-h}\right)y_t^* - \phi\varepsilon_{a,t}^* - \frac{h\sigma^{-1}}{1-h}y_{t-1}^* \tag{32}$$

$$w_t^* = \pi_t^{w*} - \pi_t^* - w_{t-1}^* \tag{33}$$

$$i_t^* = \theta_{i^*}i_{t-1}^* + (1-\theta_{i^*})(\theta_{\pi^*}\pi_t^* + \theta_{y^*}y_t^*) + \varepsilon_{i,t}^* \tag{34}$$

with all parameters being similarly defined though taking possibly different values to the domestic economy, with "$*$" being dropped for simplicity of notation. These expressions are log-linear approximations to the household and firm optimality conditions (4), (6) and (9) with all variables appended by "$*$". Note that under the assumption that the world economy is approximately closed there is no distinction between the foreign CPI and foreign goods price inflation and also $c_t^* = y_t^*$. There is also no equation describing the evolution of foreign debt as it is assumed to be in zero net supply given that domestic holdings of this asset are negligible. The foreign block therefore comprises the seven equations (28)-(34) in the unknowns $\{x_t^*, \pi_t^*, i_t^*, w_t^*, v_t^*, \psi_t^*, \pi_t^{w*}\}$ and is exogenous to the domestic economy. Application of standard solution methods determines the paths of these variables as a linear function of lagged endogenous variables and the disturbances $\left\{\varepsilon_{a,t}^*,\ \varepsilon_{g,t}^*,\ \varepsilon_{i,t}^*,\ \varepsilon_{ch,t}^*,\ \varepsilon_{l,t}^*\right\}$.

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## Table 1: Baseline Incomplete Markets model, Prior Densities and Posterior Estimates

| Coefficients Domestic Economy | | Prior: Prior Density 1/ | Prior: Mean | Prior: Std | Posterior 2/: Median | Posterior 2/: Std | Posterior 2/: [5,95] Prob |
|---|---|---|---|---|---|---|---|
| Inverse Frisch | $\varphi$ | N | 1 | 0.3 | 1.186 | 0.26 | [ 0.795 , 1.649 ] |
| Intertemporal ES | $\sigma$ | N | 1 | 0.4 | 0.517 | 0.168 | [ 0.288 , 0.84 ] |
| Calvo domestic prices | $\alpha_H$ | B | 0.6 | 0.1 | 0.839 | 0.03 | [ 0.784 , 0.883 ] |
| Calvo import prices | $\alpha_F$ | B | 0.6 | 0.1 | 0.603 | 0.084 | [ 0.472 , 0.753 ] |
| Calvo wages | $\alpha_W$ | B | 0.6 | 0.1 | 0.759 | 0.045 | [ 0.677 , 0.822 ] |
| Indexation domestic prices | $\gamma_H$ | B | 0.5 | 0.15 | 0.658 | 0.084 | [ 0.516 , 0.792 ] |
| Indexation wages | $\gamma_W$ | B | 0.5 | 0.15 | 0.432 | 0.13 | [ 0.228 , 0.658 ] |
| Habit | h | B | 0.5 | 0.1 | 0.505 | 0.063 | [ 0.4 , 0.608 ] |
| Taylor rule, inflation | $\theta\pi$ | N | 1.8 | 0.3 | 1.723 | 0.211 | [ 1.4 , 2.097 ] |
| Taylor rule, output growth | $\theta x$ | N | 0.25 | 0.1 | 0.377 | 0.092 | [ 0.227 , 0.529 ] |
| Taylor rule, smoothing | $\theta i$ | B | 0.5 | 0.2 | 0.812 | 0.026 | [ 0.765 , 0.851 ] |
| Technology | $\rho_a$ | B | 0.5 | 0.2 | 0.362 | 0.086 | [ 0.218 , 0.503 ] |
| Preferences | $\rho_g$ | B | 0.5 | 0.2 | 0.95 | 0.027 | [ 0.886 , 0.976 ] |
| Openess | $\tau$ | B | 0.2 | 0.05 | 0.09 | 0.018 | [ 0.063 , 0.123 ] |
| Elasticity H-F goods | $\eta$ | N | 1.5 | 0.5 | 1.795 | 0.413 | [ 1.126 , 2.484 ] |
| Cost-push imports | $\rho_{cp,F}$ | B | 0.5 | 0.2 | 0.884 | 0.078 | [ 0.724 , 0.956 ] |
| Risk premium | $\rho_{rp}$ | B | 0.5 | 0.2 | 0.931 | 0.02 | [ 0.893 , 0.959 ] |
| sd technology | $\sigma_a$ | I | 0.4 | 0.3 | 0.269 | 0.059 | [ 0.185 , 0.381 ] |
| sd monetary policy | $\sigma_i$ | I | 0.4 | 0.3 | 0.237 | 0.02 | [ 0.209 , 0.274 ] |
| sd preferences | $\sigma_g$ | I | 0.4 | 0.3 | 0.599 | 0.121 | [ 0.457 , 0.848 ] |
| sd cost-push domestic | $\sigma_{cp,H}$ | I | 0.4 | 0.3 | 0.348 | 0.031 | [ 0.303 , 0.405 ] |
| sd cost-push imports | $\sigma_{cp,F}$ | I | 0.4 | 0.3 | 0.943 | 0.275 | [ 0.677 , 1.523 ] |
| sd risk premium | $\sigma_{rp}$ | I | 0.4 | 0.3 | 0.263 | 0.055 | [ 0.19 , 0.37 ] |

| Coefficients Foreign Economy | | Prior: Prior Density 1/ | Prior: Mean | Prior: Std | Posterior 2/: Median | Posterior 2/: Std | Posterior 2/: [5,95] Prob |
|---|---|---|---|---|---|---|---|
| Inverse Frisch | $\varphi^*$ | N | 1 | 0.3 | 1.116 | 0.27 | [ 0.682 , 1.556 ] |
| Intertemporal ES | $\sigma^*$ | N | 1 | 0.4 | 0.574 | 0.33 | [ 0.122 , 1.189 ] |
| Calvo prices | $\alpha_{H^*}$ | B | 0.6 | 0.1 | 0.888 | 0.02 | [ 0.854 , 0.918 ] |
| Calvo wages | $\alpha_{W^*}$ | B | 0.6 | 0.1 | 0.781 | 0.04 | [ 0.707 , 0.843 ] |
| Indexation prices | $\gamma_{H^*}$ | B | 0.5 | 0.15 | 0.597 | 0.1 | [ 0.435 , 0.762 ] |
| Indexation wages | $\gamma_{W^*}$ | B | 0.5 | 0.15 | 0.38 | 0.13 | [ 0.182 , 0.625 ] |
| Habit | hS | B | 0.5 | 0.1 | 0.536 | 0.07 | [ 0.406 , 0.65 ] |
| Taylor rule, inflation | $\theta\pi^*$ | N | 1.8 | 0.3 | 1.99 | 0.23 | [ 1.627 , 2.365 ] |
| Taylor rule, output growth | $\theta x^*$ | N | 0.25 | 0.1 | 0.47 | 0.11 | [ 0.293 , 0.639 ] |
| Taylor rule, smoothing | $\theta i^*$ | B | 0.5 | 0.2 | 0.841 | 0.03 | [ 0.789 , 0.885 ] |
| Elasticity foreign demand | $\lambda$ | N | 1.5 | 0.5 | 1.231 | 0.34 | [ 0.735 , 1.846 ] |
| Technology | $\rho_{a^*}$ | B | 0.5 | 0.2 | 0.292 | 0.09 | [ 0.155 , 0.435 ] |
| Preferences | $\rho_{g^*}$ | B | 0.5 | 0.2 | 0.906 | 0.04 | [ 0.817 , 0.953 ] |
| sd technology | $\sigma_{a^*}$ | I | 0.4 | 0.3 | 0.186 | 0.04 | [ 0.134 , 0.261 ] |
| sd monetary policy | $\sigma_{i^*}$ | I | 0.4 | 0.3 | 0.149 | 0.01 | [ 0.131 , 0.174 ] |
| sd preferences | $\sigma_{g^*}$ | I | 0.4 | 0.3 | 0.438 | 0.09 | [ 0.343 , 0.631 ] |
| sd cost-push | $\sigma_{cp^*}$ | I | 0.4 | 0.3 | 0.223 | 0.02 | [ 0.194 , 0.259 ] |

**Marginal Likelihood** -1112.77

Notes:

Calibrated coefficients: $\beta$ 0.99, $\theta$ and $\theta^*$ 8, $\omega_p$ 0.33, and $\chi$ is 0.01

1/ N stands for Normal, B Beta, G Gamma and I Inverted-Gamma1 distribution

2/ Median, std and posterior percentiles of 700,000 draws from the Random Walk metropolis algorithm

## Table 2: Actual and DSGE model implied standard deviations and auto-correlations

( Simulation at the media of the draws )

| | | INCOMPLETE MARKETS BASELINE | |
|---|---|---|---|
| | **Data Standard Deviation** | **Model Standard Deviation** | **[5%,95%]** |
| Inflation | 1.804 | 2.195 | [ 1.827 , 2.7492 ] |
| Interest Rate | 3.163 | 2.995 | [ 2.341 , 3.7134 ] |
| Output | 2.468 | 2.863 | [ 1.907 , 4.354 ] |
| Real Wage | 1.751 | 1.863 | [ 1.31 , 2.61 ] |
| Real Exchange Rate | 2.460 | 2.904 | [ 2.272 , 3.634 ] |
| Terms of Trade | 1.830 | 2.682 | [ 2.049 , 3.4 ] |

| | **Data Autocorrelations** | **Model Autocorrelations** | **[5%,95%]** |
|---|---|---|---|
| Inflation | 0.733 | 0.731 | [ 0.588 , 0.831 ] |
| Interest Rate | 0.943 | 0.908 | [ 0.822 , 0.953 ] |
| Output | 0.978 | 0.944 | [ 0.891 , 0.974 ] |
| Real Wage | 0.916 | 0.925 | [ 0.862 , 0.96 ] |
| Real Exchange Rate | 0.897 | 0.831 | [ 0.686 , 0.881 ] |
| Terms of Trade | 0.893 | 0.858 | [ 0.692 , 0.907 ] |

For the Real Exchange Rate and the Terms of Trade, the moments reported correspond to the band-pass filtered series with frequency cutoffs 6 and 32. Obtained by generating 10,000 series of length 100 at the median of the draws and computing the implied moments.

## Table 3: Model Estimates when Risk Premium and or Import Cost Push Shocks Removed

( Priors are identical to those in Table 1 )

| Coefficient | | No Risk Premium or Cost-push Shock | No Risk Premium Shock | No Cost-push Shock |
|---|---|---|---|---|
| Inverse Frisch | $\varphi$ | 1.36 | 1.41 | 1.15 |
| Intertemporal ES | $\sigma$ | 0.01 | 0.00 | 0.01 |
| Calvo domestic prices | $\alpha_H$ | 0.79 | 0.64 | 0.86 |
| Calvo import prices | $\alpha_F$ | 0.95 | 0.44 | 0.95 |
| Calvo wages | $\alpha_W$ | 0.88 | 0.77 | 0.90 |
| Indexation domestic prices | $\gamma_H$ | 0.96 | 0.73 | 0.97 |
| Indexation wages | $\gamma_W$ | 0.92 | 0.60 | 0.98 |
| Indexation import prices /4 | $\gamma_F$ | | | 0.06 |
| Habit | h | 0.61 | 0.51 | 0.53 |
| Openess | $\tau$ | 0.01 | 0.09 | 0.04 |
| Elasticity H-F goods | $\eta$ | 0.04 | 1.36 | 0.08 |
| Taylor rule, inflation | $\theta\pi$ | 1.05 | 2.36 | 2.04 |
| Taylor rule, output growth | $\theta x$ | 0.19 | 0.31 | 0.21 |
| Taylor rule, smoothing | $\theta i$ | 0.77 | 0.81 | 0.78 |
| Technology | $\rho_a$ | 0.99 | 0.98 | 0.98 |
| Preferences | $\rho_g$ | 0.90 | 0.96 | 0.90 |
| Cost-push imports | $\rho_{cp,f}$ | | 0.98 | |
| Risk premium | $\rho_{rp}$ | | | 0.98 |
| sd technology | $\sigma_a$ | 0.17 | 0.17 | 0.16 |
| sd monetary policy | $\sigma_i$ | 0.25 | 0.25 | 0.24 |
| sd preferences | $\sigma_g$ | 0.36 | 0.46 | 0.42 |
| sd cost-push domestic | $\sigma_{cp,H}$ | 0.50 | 0.42 | 0.50 |
| sd cost-push imports | $\sigma_{cp,F}$ | | 1.67 | |
| sd risk premium | $\sigma_{rp}$ | | | 0.11 |
| sd labor /1 /2 | $\sigma_{labor}$ | 0.70 | 0.67 | 0.67 |
| sd aggregate demand /1 /3 | $\sigma_{ag\ demand}$ | 0.26 | | |
| Inverse Frisch | $\varphi^*$ | 1.10 | 1.06 | 1.07 |
| Intertemporal ES | $\sigma^*$ | 0.47 | 0.40 | 0.44 |
| Calvo prices | $\alpha_{H^*}$ | 0.88 | 0.90 | 0.90 |
| Calvo wages | $\alpha_{W^*}$ | 0.76 | 0.79 | 0.79 |
| Indexation prices | $\gamma_{H^*}$ | 0.61 | 0.59 | 0.62 |
| Indexation wages | $\gamma_{W^*}$ | 0.38 | 0.36 | 0.37 |
| Habit | h* | 0.51 | 0.49 | 0.50 |
| Elasticity foreign demand | $\lambda$ | 1.10 | 0.89 | 1.12 |
| Taylor rule, inflation | $\theta\pi^*$ | 2.00 | 1.94 | 1.99 |
| Taylor rule, output growth | $\theta x^*$ | 0.45 | 0.45 | 0.45 |
| Taylor rule, smoothing | $\theta i^*$ | 0.84 | 0.85 | 0.85 |
| Technology | $\rho_{a^*}$ | 0.28 | 0.28 | 0.30 |
| Preferences | $\rho_{g^*}$ | 0.88 | 0.91 | 0.91 |
| sd technology | $\sigma_{a^*}$ | 0.18 | 0.17 | 0.17 |
| sd monetary policy | $\sigma_{i^*}$ | 0.15 | 0.15 | 0.15 |
| sd preferences | $\sigma_{g^*}$ | 0.42 | 0.40 | 0.41 |
| sd cost-push | $\sigma_{cp^*}$ | 0.23 | 0.22 | 0.22 |
| Marginal Likelihood /5 | | -1266.56 | -1182.03 | -1297.61 |

*Notes:*

1/ Prior is IG1(0.4,0.3), same as for all other shocks

2/ Shock to labor disutility

3/ Shock to aggregate demand

4/ Prior is Beta (0.5,0.15)

5/ For the first three columns, since some of the estimates lie close to the boundary of the parameter space, the marginal likelihood should be interpreted with care

**Table 4: Variance Shares of Canadian Series Attributed to Domestic Shocks in Baseline DSGE Model**

Obtained using the draws generated for the coefficient estimates in Table 1

| | Shocks | | | | | |
|---|---|---|---|---|---|---|
| **Variable** | Technology | Monetary Policy | Consumption Preferences | Domestic Inflation Cost Push | Import Inflation Cost Push | Risk Premium |
| Output | 0.04 | 0.04 | 0.83 | 0.02 | 0.06 | 0.02 |
| Inflation | 0.14 | 0.01 | 0.26 | 0.41 | 0.09 | 0.08 |
| Interest Rate | 0.12 | 0.20 | 0.39 | 0.14 | 0.08 | 0.08 |
| Real Wages | 0.44 | 0.00 | 0.07 | 0.19 | 0.13 | 0.16 |
| Real Exchange Rate | 0.01 | 0.12 | 0.01 | 0.03 | 0.07 | 0.66 |
| Terms of Trade | 0.02 | 0.04 | 0.01 | 0.01 | 0.45 | 0.37 |

**Table 5: Cross-correlations real non energy commodity prices, filtered risk premium and cost-push shocks**

( All series in growth rates )

| | Real Exchange Rate | Terms of Trade | Real Non-energy Commodities PI | Import Cost push shock | Risk premium shock |
|---|---|---|---|---|---|
| Real Exchange Rate | 1.00 | | | | |
| Terms of Trade | 0.43 | 1.00 | | | |
| Real Non-energy Commodities I | -0.34 | -0.43 | 1.00 | | |
| Import Cost push shock | -0.37 | 0.49 | -0.05 | 1.00 | |
| Risk premium shock | 0.86 | 0.61 | -0.35 | 0.00 | 1.00 |

The real non-energy commodity price corresponds to the log difference of the ratio between the nominal non-energy commodity price index and the U.S. GDP deflator.

Import cost push and risk premium shocks are obtained using the (forward) Kalman filter for the updated states, using the solution of the baseline incomplete markets model at the median of the estimated parameters. The inferred shocks are then then differenced.

## Table 6: Variance Shares of Canadian Series Attributed to All U.S. Shocks in Baseline DSGE and Reduced Form Models

Median Variance Shares, All U.S. shocks /1

**Baseline DSGE** /2

| Series | Forecast errors at 4 quarter horizon | Forecast errors at 16 quarter horizon | Stationary state-space variance |
|---|---|---|---|
| Output | 0.004 | 0.004 | 0.004 |
| Inflation | 0.003 | 0.007 | 0.009 |
| Interest Rate | 0.001 | 0.004 | 0.007 |
| Real Wages | 0.006 | 0.012 | 0.015 |
| Real Exchange Rate | 0.083 | 0.086 | 0.085 |
| Terms of Trade | 0.045 | 0.048 | 0.049 |

| | **SUR** /3 | | | **Factors** /4 |
|---|---|---|---|---|
| Series | Forecast errors at 4 quarter horizon | Forecast errors at 16 quarter horizon | Stationary state-space variance | Stationary state-space variance |
| Output | 0.363 | 0.501 | 0.664 | 0.714 |
| Inflation | 0.188 | 0.348 | 0.493 | 0.154 |
| Interest Rate | 0.508 | 0.73 | 0.844 | 0.316 |
| Real Wages | 0.248 | 0.425 | 0.537 | |
| Real Exchange Rate | 0.13 | 0.186 | 0.398 | 0.218 |
| Terms of Trade | 0.146 | 0.21 | 0.361 | 0.111 |

Notes:

/1 Variance shares cover [0,1] interval. Hence 0.045 corresponds to 4.5 percent

/2 Median of the sum of all four U.S. shocks computed over the draws for the coefficient estimates. These complement the variance decomposition in Table 3. Note however columns not need up to one since reporting medians.

3/ Media of the sum of all four U.S. shocks computed over the draws generated with the Gibbs sampler and a cholelsky decomposition of the reduced form variance covariance matrix

4/ Median variance decompositions using the draws of a two foreign and two domestic factor estimated by Gibbs sampling on a panel of 32 U.S. and Canadian series. All four factors and idiosyncratic errors follow an AR(3). Model also allows for an MA(1) factor loading structure, as suggested by posterior model odds. The foreign factors are normalized using U.S. output and U.S. intermediate producer price index. This ordering was largely dictated by the data as an agnostic normalization was imposed, based solely on the positive sign restriction of the contemporanoues factor loading for the first two U.S. series. Alternative normalizations picked these two series as exhibiting the highest comovements with the factors, regardless of oredering. The number of factors and the lag length of the loadings was selected by comparing posterior odds for models with up to 3 domestic and 3 foreign factors and MA(2) in the loadings.

**Table 7: Cross Correlations DSGE Innovations**

At the median of the parameters for the baseline incomplete market model /1

| Shocks | Foreign Technology | Foreign Monetary Policy | Foreign Preferences | Foreign Cost-Push | Technology | Monetary Policy | Preferences | Home Inflation Cost Push Shock | Import Inflation Cost Push Shock | Risk Premium |
|---|---|---|---|---|---|---|---|---|---|---|
| Foreign Technology | 1.00 | | | | | | | | | |
| Foreign Monetary Policy | 0.10 | 1.00 | | | | | | | | |
| Foreign Preferences | 0.22 | 0.26 | 1.00 | | | | | | | |
| Foreign Cost-Push | 0.10 | -0.12 | -0.13 | 1.00 | | | | | | |
| Technology | 0.13 | 0.14 | 0.16 | -0.19 | 1.00 | | | | | |
| Monetary Policy | -0.04 | 0.44 | 0.25 | 0.10 | -0.15 | 1.00 | | | | |
| Preferences | -0.13 | 0.08 | 0.10 | 0.08 | 0.07 | -0.08 | 1.00 | | | |
| Home Inflation Cost Push | 0.16 | 0.10 | -0.06 | 0.23 | 0.22 | -0.21 | 0.01 | 1.00 | | |
| Import Inflation Cost Push | 0.18 | -0.04 | 0.02 | -0.07 | -0.10 | 0.00 | -0.13 | -0.03 | 1.000 | |
| Risk Premium | 0.13 | -0.01 | 0.12 | -0.05 | -0.16 | 0.39 | -0.08 | -0.21 | -0.066 | 1.000 |

Note: 1/ cross-correlation of the smoothed innovations obtained with an invoation smoother at the mode of the baseline incomplete markets mode, using the estimated diagonal variance covariance matrix.

## Table 8: Model Estimates with Alternative Priors and Specifications

| Coefficient | | Domestic Coefficients: Prior Favoring Foreign Shocks /1 | High Tau /2 | Correlated Technology Shocks /3 |
|---|---|---|---|---|
| Inverse Frisch | $\varphi$ | 1.127 | 1.137 | 1.068 |
| Intertemporal ES | $\sigma$ | 0.452 | 0.846 | 0.501 |
| Calvo domestic prices | $\alpha_H$ | 0.855 | 0.814 | 0.840 |
| Calvo import prices | $\alpha_F$ | 0.623 | 0.520 | 0.611 |
| Calvo wages | $\alpha_W$ | 0.791 | 0.716 | 0.750 |
| Indexation domestic prices | $\gamma_H$ | 0.645 | 0.596 | 0.641 |
| Indexation wages | $\gamma_W$ | 0.420 | 0.386 | 0.382 |
| Habit | h | 0.503 | 0.485 | 0.499 |
| Taylor rule, inflation | $\theta\pi$ | 1.658 | 1.883 | 1.733 |
| Taylor rule, output growth | $\theta x$ | 0.365 | 0.414 | 0.374 |
| Taylor rule, smoothing | $\theta i$ | 0.817 | 0.794 | 0.818 |
| Technology | $\rho_a$ | 0.333 | 0.360 | 0.454 |
| Preferences | $\rho_g$ | 0.957 | 0.949 | 0.963 |
| Openess | $\tau$ | 0.069 | 0.178 | 0.087 |
| Elasticity H-F goods | $\eta$ | 1.845 | 1.501 | 1.762 |
| Correlation technology shocks | | | | 0.170 |
| Cost-push imports | $\rho_{cp,f}$ | 0.882 | 0.931 | 0.887 |
| Risk premium | $\rho_{rp}$ | 0.933 | 0.943 | 0.941 |
| sd technology | $\sigma_a$ | 0.298 | 0.264 | 0.223 |
| sd monetary policy | $\sigma_i$ | 0.231 | 0.242 | 0.232 |
| sd preferences | $\sigma_g$ | 0.566 | 0.846 | 0.637 |
| sd cost-push domestic | $\sigma_{cp,H}$ | 0.327 | 0.399 | 0.337 |
| sd cost-push imports | $\sigma_{cp,F}$ | 0.873 | 1.174 | 0.899 |
| sd risk premium | $\sigma_{rp}$ | 0.255 | 0.238 | 0.239 |

Notes: Priors are identical to those in Table 1 except as described below.
1/ Prior is IG1(0.7,0.4), for foreign shocks, IG1(0.3,0.2) for domestic disturbances, wehere IG1(A,B) refers to an Inverse Gamma 1 with mean A and standard deviation B. Also, prior persistence of foreign preference and technology shocks given by a Beta with mean 0.8 and standard deviation 0.1
2/ Prior for $\tau$ is a Beta with mean 0.9 and standard deviation 0.05.
3/ Prior for correlation coefficient is a Beta with mean 0.5 and standard deviation 0.25.

## Table 9: Variance Shares of Canadian Series Attributed to All U.S. Shocks Robustness Checks

Median Variance Shares, All U.S. shocks /1

| Series | DSGE VAR in Foreign Block<br>Stationary state-space variance | DSGE First Differenced Data<br>Stationary state-space variance | SUR First Differenced Data<br>Stationary state-space variance |
|---|---|---|---|
| Output | 0.006 | 0.008 | 0.362 |
| Inflation | 0.013 | 0.012 | 0.41 |
| Interest Rate | 0.017 | 0.01 | 0.763 |
| Real Wages | 0.018 | 0.019 | 0.296 |
| Real Exchange Rate | 0.12 | 0.101 | 0.171 |
| Terms of Trade | 0.092 | 0.067 | 0.22 |

Notes:

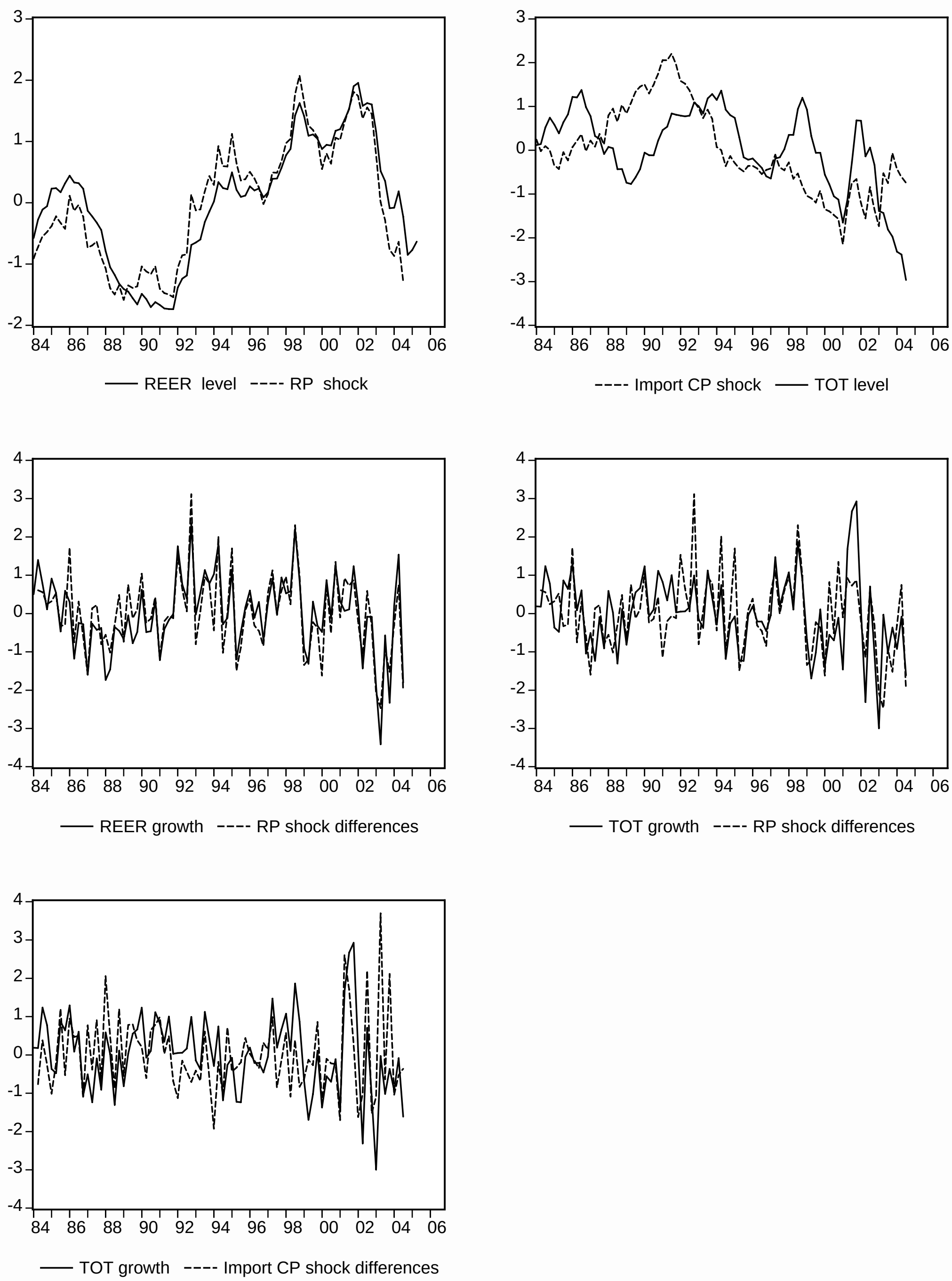
/1 Variance shares cover [0,1] interval. Hence 0.092 corresponds to 9.2 percent

/2 Median of the sum of all four U.S. shocks computed over the draws for the coefficient estimates when the foreign block is characterized by an atheoretical VAR.

/3 Median of the sum of all four U.S. shocks computed over the draws for the coefficient estimates when all domestic and foreign series, except interest rates and inflation rates, are in log first difference (multiplied by 100).

4/ Median sum of all four U.S shocks using the draws generated with the Gibbs sampler for a 1 lag SUR model with the same first difference dataset and sample used for the DSGE in column 2. Variance decompositions obtained with a Cholesky decomposition on the reduced form innovations.

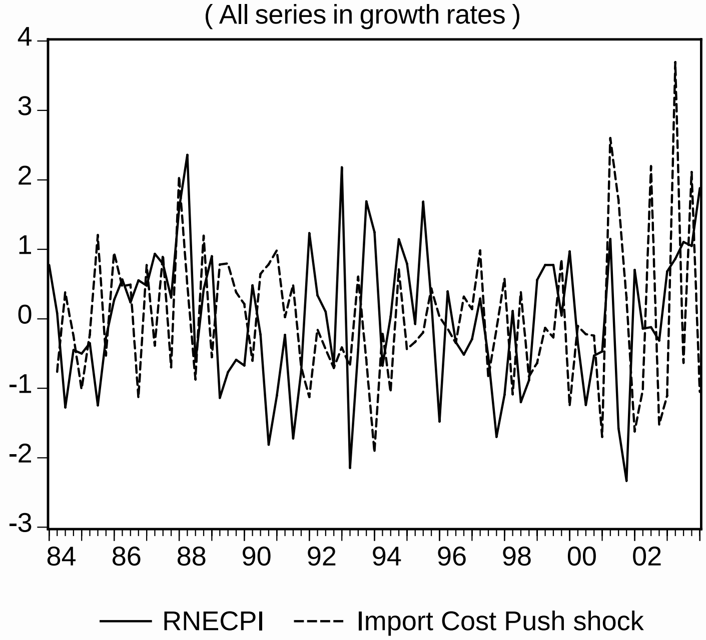
Figure 1: Levels and growth rates for the real exchange rate (REER), terms of trade (TOT) and filtered risk premium (RP) and import cost push (CP) shocks

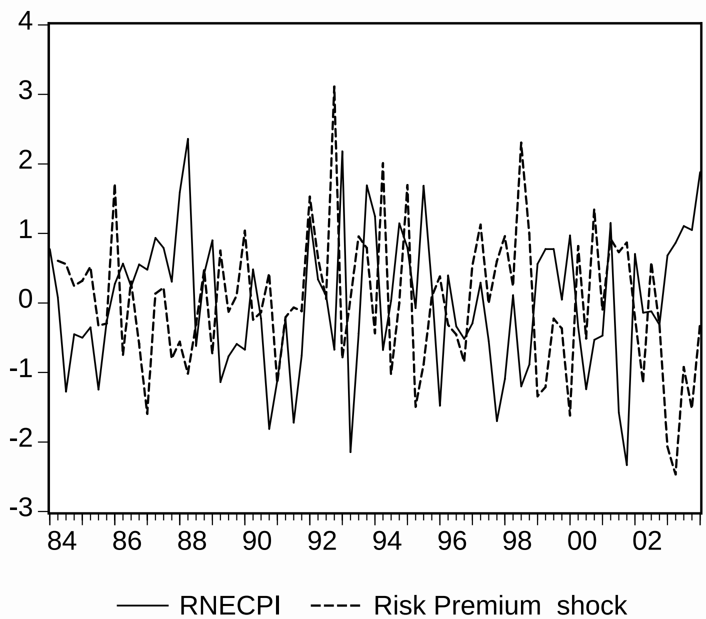



All series are demeaned and standarized to facilitate comparisons

Figure 2 : Real non-energy commodity price index, risk premium and import CP shocks

( All series in growth rates )

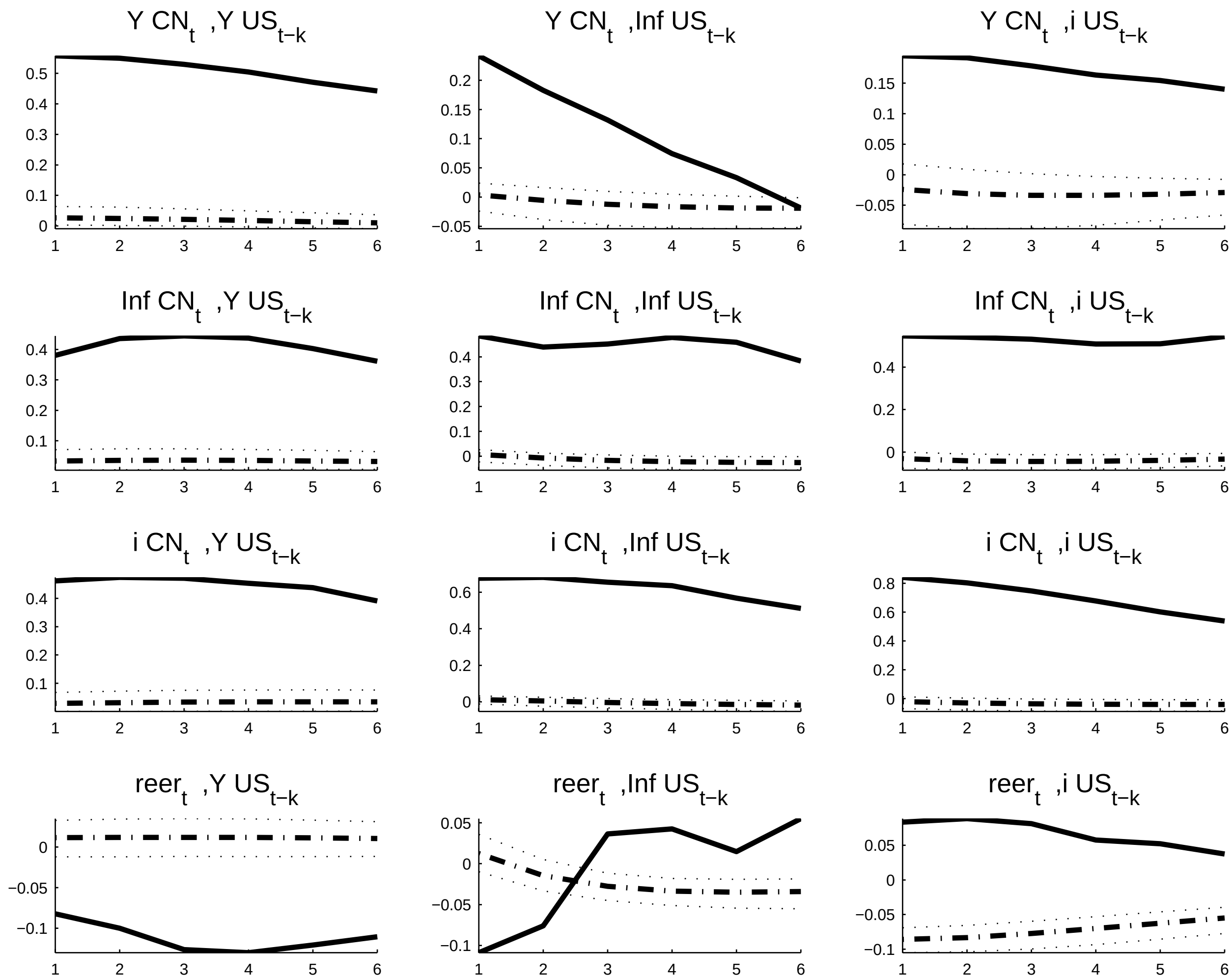



RNECPI: log-difference of real non-energy commodity price index

All series are demeaned and standardized

Figure 3: Cross−Correlations Data and Benchmark DSGE




Data (solid) , DSGE median (dashed) and 5−95 bands (dotted)