Consumers' Search Cost and Emerging Structure of Web Sites Coalitions: a Multi-agent Based Simulation of an Electronic Market[1]

Jacques LAYE[2], Charis LINA[3], Herve TANGUY[4]

Abstract

We have developed an agent-based computational model, extension of an analytical model[5] that studies the structure of coalitions of B-to-C web sites, when Internet buyers incur search costs for finding the good that matches their preferences, and coalitions of sites reduce this cost through specialized search engines. This multi-agent model consists of heterogeneous, bounded rational agents: sites and web users, which have states and rules of behavior. Our goal is to run simulations by instantiating two agent populations (a population of sites/sellers and a population of web users/consumers), let the agents interact through the consumer search process and the coalition formation process, and monitor the evolution of the simulations in order to study the emerging dynamics, in particular the emerging coalition structure. The agent-based model extends the analytical results for less restricting assumptions. Moreover, beyond the coalition formation process, the analytical model is enriched by additional behaviors for the sites (entry and death), which give insights on the dynamics of competition when building coalition matters for increasing demand at the expense of non or less coalesced sites.

Key Words: B-to-C, coalition formation, multi-agent.
JEL: L11, C63, D83, D40.

Résumé

On développe un modèle multi-agent, extension d'un modèle analytique qui étudie la structure des coalitions de sites B-to-C, lorsque les consommateurs-internautes ont un coût de recherche pour trouver le bien qui correspond à leur préférence, et lorsque les coalitions de sites permettent de réduire ces coûts de recherche *via* des moteurs de recherche spécialisés. Ce modèle multi-agent comporte des agents hétérogènes à rationnalité limitée: sites et internautes, dont on défini l'état et les règles de comportement. Notre objectif est de faire des simultations qui mettent en jeu ces deux types d'agents (une populations de sites marchands et une population de consommateurs), de les faire intéragir à travers une procédure de recherche des consommateurs et un processus de formation des coalitions des sites, et d'observer la dynamique du système, en particulier en ce qui concerne la structure de coalition émergente. Le modèle multi-agent étends ainsi les résultats obtenus dans le modèle analytique, avec des hypothèses moins restrictives. En plus du processus de formation des coalitions, le modèle théorique se voit aussi enrichi par d'autres règles de comportement pour les sites (entrée sur le marché, faillite), ce qui apporte des enseignements sur la dynamique de la concurrence lorsque la formation des coalition permet d'accroître la demande au détriment des sites peu ou non coalisées.

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[2] Laboratoire d'Economie Forstière (L.E.F.) UMR INRA-ESR/ENGREF, 14 rue Girardet, CS 4216, 54042 Nancy Cedex. e-mail: laye@nancy-engref.inra.fr.
[3] ARMINES and University of Crete. e-mail : lina@csd.uoc.gr.
[4] Laboratoire d'Econometrie de l'Ecole Polytechnique ,1, rue Descartes, 75005, Paris, France. e-mail: tanguy@poly.polytechnique.fr.
[5] Laye (2003), Laye and Tanguy (2004).

1. Introduction

As R. Axtell points out (see [1, 2, 3]), "an agent-based computational model is valuable to study the non-equilibrium dynamics, in which structure is perpetually born, growing and perishing" , which is the case of the constantly evolving Internet Landscape. We develop a agent-based simulation model in order to study the forces that drive the Web sites aggregation phenomena, following the principles described in the related litterature (see for instance the work of Wooldridge, Jennings, and Kinny [11], [13-15], [24]). We study a virtual environment consisting of Internet consumers, online sellers (i.e. B-to-C Web sites competing in a differentiated electronic market), and rules of interaction among these economic agents. Placing agents who make individual decisions while interacting with other agents in such an environment allows us to study the emergence of coalition structures.

Most of the agent-based approaches that focus on the Internet economy consider that the determinating factor of the evolution of the Internet landscape are network effects and increasing returns. For example Lelis *et alii* (2003) explain regularities in the distribution of visitors per Web site using word-of-mouth-based informational feedbacks, while taking in account the existence of guiding navigation paths (Web linkages). Lina (2003) investigates agglomeration economies, modelizing increasing returns as frequency-dependent choices, while allowing Web sites to ameliorate their performance through investment strategies.

Our approach differs from these contributions in two aspects. Firstly, our analysis does not take into consideration such characteristics of the Internet landscape (information feedback effects, sites' underlying network, presence of Web communities and so on). In fact we focus on the aggregation strategy of pure merchant sites, trying to capture consumers willing only to buy online. In other words, the Web here is just an alternative channel of distribution and not an entertainment good. Secondly, we base our multi-agent modelization over a theoretical model that is related to the litterature of Industrial Organization, which allows us to take into account economic behaviors (agents are maximizing their utility, taking somehow into account the strategic impact of their decisions) rather than pure behavioral rules.

As a basis for the multi-agent model, we first present here a short version of the analytical model that studies the phenomenon of coalitions of B-to-C sites available in Laye (2003) and Laye and Tanguy (2004). In this model, coalitions are studied within the framework of a circular model of spatial differentiation that represents both the characteristics of the differentiated goods offered by Web sites and the tastes of consumers in a given sector. These consumers incur a disutility (adaptation cost) when buying a good offered at a distant location, *i.e.* that does not match perfectly with their preferences. As in Gabszewicz and Garela (1986) or Bakos (1997), we suppose that sites compete in price facing a population of imperfectly informed consumers. We introduce search costs for the consumers in order to find the characteristic of the good offered by a particular site and its price, which are independent of their adaptation/transportation. Next, we consider that sites have the possibility of forming coalitions and that this leads to a reduction of the search cost for a consumer that visits a site within a coalition. The reduction of search costs is considered to be the result of the coordinated efforts of the coalesced sites to develop more efficient search tools in order to facilitate the finding of the goods closest to the tastes of Internet consumers (mutual electronic linkage among sites or through the development of specialized search engines). This leads to an increase in the probability for a coalesced site to be discovered and selected by a consumer. This defines the notion of coalition we are interested in: reduction of search cost rather than price coordination or merging. Two main questions motivate this analytical model: absent price coordination, what is the economic rationale behind the coalition formation of specialized commercial Web sites and, what is the more preferable type of coalition: firms with close products (defined as *connex*) or highly differentiated ones (*non-connex*)?

However, the strength of the assumptions of the analytical model (a market consisting of 4 sites symmetrically located with only consecutive natural territories intersecting) requires a more flexible modelling approach, in particular because by construction competition between coalitions was not modelized nor the process of coalition formation. An agent-based extension of the analytical model of coalition formation mentioned above is designed and implemented. In this computational model, agents mimicking the behavior of Web sites and consumers interact through two basic processes: the *consumers' search process* and the *coalition formation process*. An agent in this model is a logical entity that performs, within the model, a series of actions and is implemented by a software entity that has a state and specific behavior rules[6].

Extending the analytical model by introducing dynamics and heterogeneity on the agents' characteristics and behavior enables us to study more systematically the emergent coalition structures in a more complex, and thus more realistic, environment. Conclusions based on agent-based simulations can serve as a complement to the existing formal theory. Our goal is, first, to verify the robustness of the results of the analytical model when its simplifying assumptions are relaxed and, second, have a tool that allows us to execute other scenarios on the simulated Web market.

The developed agent-based model incorporates a dynamic process of coalition formation that is closer to the constantly evolving environment of a differentiated Web market where coalitions can be formed in parallel and without a bound on their size *a priori*. Contrary to the analytical model, the number of sites and consumers in the simulated market grows over time. Moreover, sites have the ability to dynamically adjust their price using a trial and error algorithm based on random experimentation and experience. Furthermore, a process of market entry and market exit was added to the agent-based model: sites incur an initial cost to enter the market and unless they are able to repay it in a specified amount of time, they are forced to exit the market. Finally, the search cost of consumers in the agent-based model lowers with the time they have been on the market under the assumption that they get more efficient in using the Web for finding products.

We are interested in looking at the economical result that is produced from various configurations of the agent-based model with respect to:

- The degree of differentiation of the coalesced sites
- The number and size of the formed coalitions
- Market shares and profits of sites belonging or not to coalitions
- The type of sites that manage to survive given the market exit process (coalesced or non-coalesced, and if coalesced then connex or non-connex etc.)

We also intend to study the dependence of the above economic results to the parameters of the model, as well as the impact of the model's assumptions in terms of agents' heterogeneity and growth rate and the dynamic

[6] Our agents belong to the category of "weak agents" according to Wooldridge and Jennings (1995).

procedures of coalition formation and price adjustment of the sites.

The rest of the paper is organized as follows: after a presentation of the modelization of the differentiated market, the search procedure of the consumer, and the different coalition structures, we summarize the results of the analytical model[7] in section 2. The agent-based model is presented in section 3 and the results of the simulations are presented in section 4.

2. Analytical Model of Coalition Formation

As in Bakos (1997) model, we consider a market with a continuum of Internet consumers and *m* B-to-C sites. *m* is supposed to be common knowledge. Each site *j* sells a unique good at price p_j and the characteristics x_j of the goods are differentiated along the unit circle[8] as in Eaton's (1976) pioneering work and subsequent models (D'Aspremont, Gabszewicz, Thisse (1979), Salop (1979), Novshek (1980), Eaton (1982), Stahl (1982), Economides (1989)). The tastes x_i of the consumers are heterogeneous and uniformly distributed along the same circle. By buying a unit of good that does not match exactly with its preference, the consumer incurs an adaptation cost *t* per unit of distance ($t>0$) between its location (*i.e.* its preferred product) and the location on the circle of the chosen site *i.e.* the good offered) for the transaction. Therefore, the utility function if consumer *i* buys a unit offered by site *j* is: $U(i,j)=r-p_j-t|x_i-x_j|$, where *r* is the reservation utility of each consumer.

Consumers' search procedure. Consumer *i* acquires information on the location and the price of one of the *m* sites of the electronic market by incurring a constant search cost $c>0$. We consider this search cost to be both the cost associated with the discovery of the site on the Web, for example through a search engine, and the cost of visiting the site to find out about its characteristics: sell price *S*, and distance *D*. The utility of the consumer in case of a transaction is $U(S,D)=r-S-tD$. If the consumer decides to search further and finds another site located at distance *x* and with price *p*, the utility in this case is $U(p,x)=r-p-tx$. Thus, $(U(x,p)-U(S,D))^+=(S+tD-xt-p)^+$ represents the increase of utility for the consumer if $U(x,p)-U(S,D)>0$ (otherwise it is 0). We suppose that the consumers are risk neutral. The calculation of the expected gain in utility based on the priors on the distributions of sites' locations and prices allows the consumer to decide on the opportunity to continue the search procedure. This defines the space of strategy of the consumer.

Consumers' priors. Concerning the priors on prices the consumers believe that at equilibrium all sites choose the same price p^*. More precisely the distribution of prices is such that $f(p)=1$ if $p=p^*$, and $f(p)=0$ otherwise. Concerning the priors on locations, the consumers believe that sites locate according to a uniform distribution over the unit circle. We also suppose that consumers find sites according to a random trial with replacement. These assumptions are related to the fact that consumers are considered to not change their priors on the

[7] This approach shows in particular that it is optimal for a site willing to form a new coalition to choose a site whose characteristics are highly differentiated.

[8] We choose a circular differentiation model like in the model of Bakos (1997) rather than the classical linear one, first proposed by Hotelling (1929), since, in our model, it is more convenient that all the locations of the differentiated market are *a priori* equivalent.

distributions of locations or prices after finding each site.

Stopping rule. The expected gain in utility obtained in this case is:

$$g(S,D)=\int_{x=0}^{1}\int_{\Re}(S+tD-xt-p)^{+}f(p)dp\,)dx$$

According to the priors of the consumers on the locations, we find like in Bakos (1997) that $g(S,D) = (S+tD-p^*)^2/t$. Next, the consumer has only to compare its expected gain in utility with the search cost c. If $g(S,D)>c$, the consumer will prefer to continue its search. If $g(S,D)<c$, the consumer will choose to buy a unit of the good located at a distance D and at price S. At equilibrium with rationale expectations for the consumers, $S=p^*$. For each consumer i located in x_i, we have that $g(p^*,D)<c$ on the interval $[x_i-L, x_i+L]$, where $L=\sqrt{c/t}$. Consequently, if the consumer discovers a site at a distance smaller than L, the transaction will take place. Symmetrically, from the point of view of a site, the more distant potential client is located at distance L. We obtain an interval of length $2L$ around any site, which will be referred to as "natural territory".

Definition 1. *The natural territory of a site corresponds to the interval around its location in which consumers stop their search and buy from this site if they find it.*

Let us now describe the simplest framework needed to capture the effects we want to describe once it is possible for sites to coalesce. We consider that $m=4$, that these sites are located according to the principle of maximum differentiation[9], and that they sell at price p^*, which is also the price anticipated by the consumers. We restrict the study in terms of length of natural territories by supposing that $\underline{L} < L \leq \overline{L}$ such that no consumer is priced out of the market and the natural territory of a site only intersects with those of its neighbors. In the case of 4 sites, we have $\underline{L} = 1/8$ and $\overline{L} = 1/4$.

Coalition structures. The setting we described is also the minimal setting required to differentiate coalition structures: a site willing to coalesce can choose two kind of partners defining the two different categories of coalitions.

Definition 3. *A coalition will be called "connex" if the natural territories of its members intersect, otherwise the coalition will be called "non-connex".*

[9] Given the assumptions of our model, we can prove that in a two-stage game in which sites choose locations in the first stage of the game and set prices in the second stage, sites choose the same price (symmetric equilibrium) and locate so as to maximize the distance between their locations.

For $\underline{L} < L \leq \overline{L}$, a coalition is connex if its members are located consecutively on the circle (little differentiation), and non-connex otherwise (high differentiation).

From the point of view of the consumers, visiting a coalesced site allows the consumer, which has only incurred the search cost *c* for a search on the entire Web, to visit other sites by incurring a lower cost *c'*<*c* within the coalition. We normalize *c'* to zero. We suppose that the consumer benefits only *ex post* from the reduction of the search cost: the consumer do not anticipate the presence of a coalition on the market and if one coalition is discovered the search procedure continues without modifying its priors accordingly. Modifying the priors would be equivalent to consider that after discovering a coalition (without finding a site to make the transaction), the search cost is lower for the rest of the search procedure. In other words, the expected gain is increased since there is more chance to find a site that matches the preferences of the consumer from now on, in a market with coalitions. Therefore after the first discovering of a coalition and for the rest of the search procedure we would be in an equivalent situation than without this simplifying assumption. Therefore the results are not affected qualitatively by this assumption. From the point of view of a coalesced site, a coalition is a possibility to increase its expected demand.

2.1 Results of the analytical model[10]

The static comparative of the two coalition structures, putting aside price competition shows that with fixed prices, a site willing to coalesce has more incentives to choose a non-connex partner.

Concerning the way in which strategic pricing for the sites influences the choice of the coalition structure, it is shown that for both coalition structures (connex and non-connex), coalesced sites have an incentive to lower their prices from the one obtained without coalitions in order to increase their natural. The opposite tendency is observed for the non-coalesced sites: they increase their price in order to decrease the length of this territory. Furthermore, the non-connex coalition is more aggressive than the connex one. The fact that non-connex partners decrease more their prices than is they were in a connex coalition shows that it is not the increase in the competition between the coalesced sites that drives the price decrease. Decreasing the price reflects only the opportunity to gain market share from the non-coalesced sites. When the coalition is connex, a coalesced site gains market shares from the territory shared with a non-coalesced site (on one side of its location) and shares equally the rest of the consumers with its partner (on the other side of its location), which brings no additional demand. The gain of market shares is far better exploited when the coalition is non-connex since it occurs on both sides of coalesced sites' locations, without interacting with their partner.

Finally, by comparing the profit of the coalesced sites depending on the coalition structure, with price competition, the result we obtain with fixed prices still holds: the non-connex structure is always preferred by the initiator of a new coalition.

Given that the existence of search costs that are independent from the adaptation costs is a specific characteristic of Internet distribution, the analysis carried out above can enlighten the discussions on the emerging structures of B-to-C coalitions, as part of the evolving Internet landscape. However, we are interested in the way the heterogeneity of sites and consumers and the dynamics of the coalition formation can influence the mechanisms captured by the analytical model, which is a motivation for the agent-based model presented in the next sections.

[10] These results are available in Laye(2003) and Laye, Tanguy (2004)

3. Agent-based Model

We have developed an agent-based computational model, extension of the analytical model described earlier. This model consists of heterogeneous, bounded rational agents: sites and Web users, which have states and rules of behavior. Our goal is to run simulations by instantiating two agent populations: a population of sites/sellers and a population of Web users/consumers, let the agents interact through the consumer search process and the coalition formation process, and monitor the evolution of the simulations in order to study the emerging dynamics.

3.1 The rational behind

Our major motivation in developing an agent-based computational model comes from the fact that agents with diverse and heterogeneous characteristics and behaviors will facilitate the systematic study and advance our understanding of the emergent coalition structures in the Web as a complement to the formal theory. We are interested in seeing the economical result that can come out from various configurations of the model with respect to:

- the degree of differentiation of the coalesced sites (connexity)
- the number and size of the formed coalitions
- market share and profits of sites inside and outside of coalitions

We intend to study the dependence of the above economic results to the parameters of the model, as well as the impact of the model's assumptions in terms of heterogeneity of the agents, rationality and search process of the consumers and the coalition formation process of the sites.

3.2 Set-up of the model

Assumption 1: Initial Conditions

Sites select their locations on the market. In the model two alternative selection schemes are considered. In the first, each site j selects its location y_j randomly from a uniform distribution ($y_j \sim U[0,1]$, $\forall j = 1..M_0$, where M_0 is the initial number of sites in the market), while in the second sites are positioned in the market according to the principle of maximal differentiation ($y_j = j/M_0$, $\forall j = 1.. M_0$). We also assume that there are two alternative schemes for the setting of prices. Sites can either select the price of their product randomly from a uniform distribution between p_{min} and p_{max} ($p_j \sim U[p_{min}, p_{max}]$, $\forall j = 1.. M_0$) or all sites set their price to a constant price p^*. Furthermore, consumers anticipate that the prices set by sites are uniformly distributed between pA_{min} and pA_{max}.

Consumers are uniformly distributed along the circle. Consumer i's location is denoted by x_i, $\forall i = 1..N_0$, where N_0 is the initial number of consumers in the market. Consumer i's reservation utility R_i is selected randomly from a uniform distribution between R_{min} and R_{max}, $\forall i = 1.. N_0$. The adaptation cost t_i (utility loss per distance unit between consumer i's location and the site's location) is also selected randomly between t_{min} and t_{max}, for each consumer. Each consumer i has associated with him a search cost c_i for discovering a new site, selected randomly between c_{min} and c_{max}. Moreover, consumer i will have a reduced search cost c_i', when discovering sites within a coalition, selected randomly from a uniform distribution between *0* and c_i $(c_i' \sim U[0, c_i]\ ,\ \forall i=1..\ N_0)$.

Assumption 2: Market Entry and Exit

We assume that each site incurs an initial cost in order to enter the market. This cost, denoted by C_{entry}, represents an initial capital borrowed from a bank for the set up of the site (network equipment, hardware and software, advertisement expenses, etc.). The goal of each site is to repay this initial dept and be able to be profitable within a specified amount of time denoted by T_B. More precisely, in each step a site has to be able to repay the interest over the current dept from the profit made in this step. The current dept is reduced by the difference of its profit and the interest it had to pay. If after a period of T_B steps since it entered the market, the site is not able to fully repay its current dept to the bank, the site is forced to exit the market. This defines the dying process of the sites. If a site that belongs to a coalition[11] dies the coalition's size is reduced by one.

Assumption 3: Agents' growth rates and localisation

We assume that the number of consumers and the number of sites grow exponentially with time at rates g_c and g_s respectively. Therefore at time step *t* the number of consumers *M(t)* and sites *N(t)* are given by the following formulas:

$$M(t) = (1+g_c)^t M_0$$
$$N(t) = (1+g_s)^t N_0$$

New consumers and sites are initialised in the same way as the initial populations (see *3.2.1*). The locations of the new sites that are added approximate the principle of maximal differentiation in the following way: the first site that is added is located equidistantly between a pair of existing sites randomly selected, the second site to be added in the market is located diametrically opposite to the previous site, the third site is located also in the middle of another pair of sites randomly selected and so on, such as shown in the following figure.

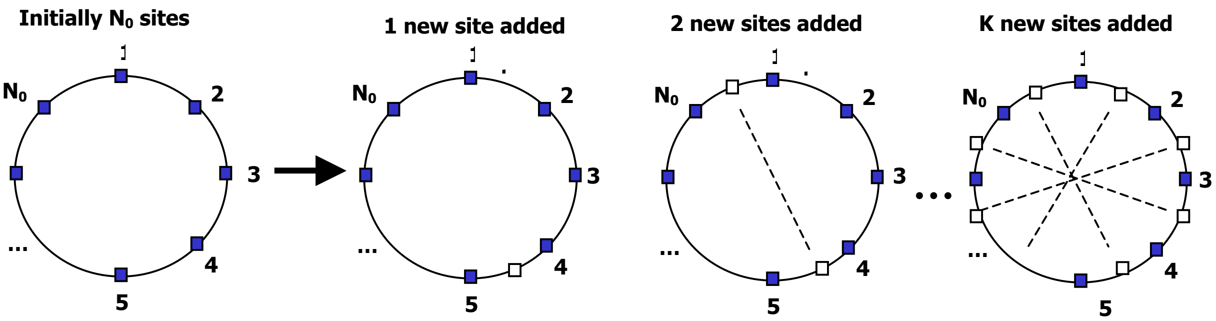


Figure 8

Concerning the locations of the users as their population grow over time, we assume that each time new users are added to the market, the locations of all users (the new ones included) are rearranged so that they follow the principal of maximal differentiation. This assumption is related to the fact that consumers have no "identity" in this model, since their influence lies in the level of aggregate demand. On the contrary, sites cannot change

[11] See Section 3.2.6 for an explanation of the coalition formation process.

location (characteristic of the good) with the entry of each new site.

Assumption 4: Coalition Formation Process

Every *Tc* time steps a fraction α of the single sites and a fraction β of coalesced sites is randomly activated to engage in a new coalition formation process. Depending on whether the activated site is single or coalesced, one of the following will occur:

- If the site is single, then it becomes a coalition initiator by selecting a partner randomly from all the sites in the market. If the selected partner is single as well, they form a new coalition of size 2. Otherwise, the selected coalesced site "proposes" to the single site to join this coalition, thereby increasing the size of the existing coalition by one.
- If the site already belongs to some coalition, then it randomly selects a *single* partner in order to expand its coalition by one member. In our model, we do not allow the merging of two existing coalitions into a new one.

Once a new coalition is formed, in one of the ways mentioned above, it is tested on the market for the next *Tc* time steps. At the end of the test period, each member of the new coalition compares the sum of its individual profits over the last *Tc* time steps with the equivalent sum in the previous *Tc* time steps (whether they belonged to a coalition or not). If profits have increased for each site of the new coalition then we consider that the criterion of individual satisfaction is satisfied (the coalition is considered profitable for all participants) and the coalition is permanently adopted (the last site to enter is definitively accepted as a member). Otherwise, the last member added in the coalition is not accepted. Sites that have been rejected by a given coalition, will not be candidates for acceptance by the same coalition ever again in the course of the simulation.

Remark: In our simulations we have included the possibility that there is only one site that acts as initiator and selects other sites to participate in its coalition. In this way, instead of having many coalitions growing in parallel, we obtain a single coalition growing which has the advantage of being more simple to study and, afterwards, we can compare the results obtained when there is only one coalition with those obtained when we allow multiple coalition formation.

Assumption 5: Consumers' Search Process

In each time step a fraction γ of consumers is randomly activated to undertake a new search process that may result in the purchase of one product unit. Each consumer starts its search process by randomly selecting a site to visit from the whole population of sites. During the visit, the consumer observes the site's product offering and price (price *S*, distance *D* between the consumer's location and the product's characteristic) by incurring a constant search cost c_i. The consumer computes the utility *U(S,D)* it will derive if it buys from this particular site and the expected gain in utility *g(S,D)* from continuing its search process by selecting another site to visit. The comparison of *g(S,D)* with the search cost defines the opportunity of continuing the search process as follows:

- If the current site does not belong to any coalition, the consumer's search cost is c_i. If *g(S,D)* is less than c_i the consumer stops its search process and buys from the current site, else the consumer continues the search process by selecting randomly another site.
- If, however, the current site belongs to some coalition then the consumer can discover at a search cost

c_i', lower than c_i, the offerings of the rest of the members of the coalition. In this case, the consumer compares $g(S,D)$ to c_i' and if the computed expected gain is larger, then the consumer continues the search process by randomly selecting another member of the current coalition. If the coalition has no members left to visit the consumers selects randomly from the rest of the population of sites.

Consequently, the search process stops in two ways:

(i) the expected gain in utility from continuing the search is less than the search cost in which case the consumer buys from the current site

(ii) the consumer has visited every site on the market but the expected gain in utility is still higher than the search cost. In this case, the consumer cannot continue its search process but does not perform a transaction at any site: consumer is "priced out".

Assumption 6: Sites' Price Adjustment Procedure

Sites periodically adjust their prices according to the following procedure: Each T_P time steps, where T_P is a model parameter, each site readjusts its price after comparing the sum of profits that it had during the last T_P steps with the one of the previous T_P steps. If there has been an increase in profits, the site makes a price adjustment selected randomly in $\pm[\delta p_{min}, \delta p_{max}]$, otherwise the site readopts the price it had before the last price adjustment. In the rest of this document we will refer to this algorithm as *RND*.

We also study the results of the *Derivative Following Algorithm* (see Kephart *et alii* (1998)) for the price adjustment process. According to this algorithm, each site makes its first price adjustment randomly ($\pm U[\delta p_{min}, \delta p_{max}]$). If (after T_P steps), the sites finds that its profits have increased, it keeps moving the price in the same direction; otherwise it reverses direction. In the rest of this document we will refer to this algorithm as *DFA*.

For example, the following figure shows the effect of the RND adjustment algorithm in an experiment with 4 sites that started from random prices, when no coalitions are formed. We can observe that gradually the price of the sites gets closer to the theoretical equilibrium price, which for that experiment was 1000. This price adjustment algorithm will be also used in experiments where coalition formation is allowed.

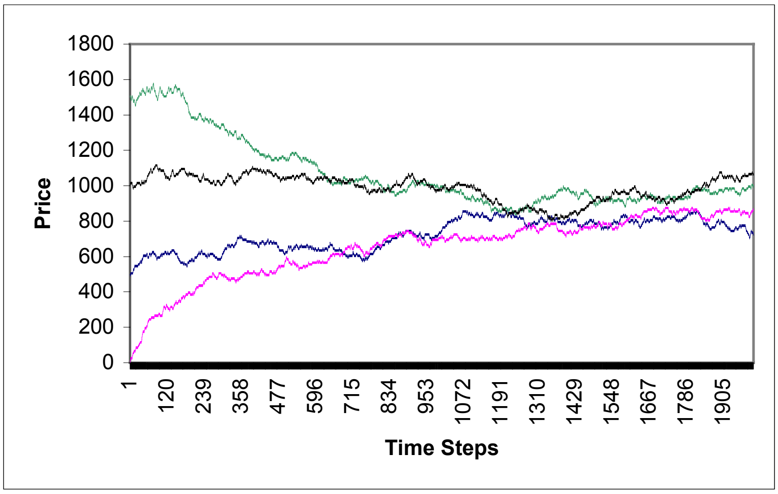


Figure 9

Assumption 7: Consumers' expertise and reduction of search cost

In order to take into consideration the fact that consumers improve with time their ability to search and find products on the Web, we consider that the search cost of the consumers decreases linearly with time according to two alternative ways:

(i) each consumer *i* has an initial search cost of c_i^0 when it enters the market and this search cost decreases linearly with its age reaching the value of c_i^{end} at the end of the simulation. This assumption reflects the fact that each consumer gets more familiar with the Web with time thereby improving its ability to search and locate products.

(ii) all consumers have the same search cost at each time step of the simulation. The consumers' search costs starts from c_i^0 at time step 0 and decreases linearly with time reaching the value of c_i^{end} at the end of the simulation. This assumption reflects the fact that the population of Internet users as a whole improves its searching abilities with time, since a new user that is interested in consuming products can profit from the accumulated experience of previous users.

Table of model parameters

N	Final number of sites
M	Final number of consumers
Activation	
α	Fraction of the single sites' population activated for coalition formation every K time steps
β	Fraction of the coalesced sites' population activated for coalition formation every K time steps
γ	Fraction of the consumers' population activated in each time step
Agents' Growth	
M_0	Initial number of consumers
N_0	Initial number of sites
g_s	Sites' growth rate
g_c	Consumers' growth rate
Search process	
c_{min}	Lower bound for consumers' search cost
c_{max}	Upper bound for consumers' search cost
c'	Reduced search cost
R_{min}	Lower bound for consumers' reservation utility
R_{max}	Upper bound for consumers' reservation utility
T_{min}	Lower bound for consumers' adaptation cost
T_{max}	Upper bound for consumers' adaptation cost
Coalition formation process	
T_C	Number of time steps between site's activation for coalition testing and formation
Prices	
p_{min}	Lower bound for sites' price
p_{max}	Upper bound for sites' price
pA_{min}	Lower bound for consumers' anticipated price
pA_{max}	Upper bound for consumers' anticipated price
T_P	Number of time steps between site's activation for price adjustment
$\delta_{\min}$	Lower bound for sites' price adjustment
$\delta_{\max}$	Upper bound for sites' price adjustment
Market entry/exit	
C_{entry}	Sites' market entry cost
i	Interest rate for sites
T_B	Number of time steps in which each site has to repay his initial dept

4. Simulation Results

4.1 Simulation plan

The analytical result presented earlier, serves as the starting point for a series of experiments with the agent-based model whose results will be the subject of the rest of this document. One of our primary goals is to verify the robustness of the analytical result in the agent-based environment that has been built. The first step of the simulation plan consists of keeping the symmetry assumptions that have been used in the analytical model in order to study the impact of the coalition formation procedure on the market structure (first when there is only one coalition growing and then when there are multiple coalitions growing in parallel). In this context, we analyse the role of the structural parameters of the model (*i.e.* adaptation cost and search cost) on the results. The next steps of the simulation plan consist of exploring the effect of the market entry and exit processes on the market structure, as well as the effect of the decrease of the consumers' search cost and the sites' price adjustment procedure. More precisely, we are interested in identifying those forces that are capable of creating *aggregation* in our simulated market. Aggregation, here, is defined as the existence of a small number of big coalitions and a large number of single sites or sites participating in very small coalitions. Finally, we introduce heterogeneity in the simulations and the ability for the sites to adjust their prices.

The simulation plan described above provides an overview of the elementary economic mechanisms that drive the emerging structure of the coalitions by analyzing the combined effects of the structural parameters, the different coalition formation procedures, the processes of market entry and exit and the price adjustment mechanism.

4.2 Reference Simulation

To study the impact of the structural parameters on the results of the model in terms of emerging coalition structure we start by a simple case:

- Consumers are identical (same search cost, adaptation cost and reservation utility, symmetric locations)
- Sites are symmetric (same price, symmetric locations)
- Inside a coalition the search cost of the consumers is c'=0

Given these assumptions, we test two different coalition formation procedures:

(i) A unique site is activated as initiator of a single coalition that grows throughout the simulation

(ii) All sites can be activated and are able to form coalitions in parallel throughout the simulation

The results in terms of size obtained with the first coalition formation procedure can be used as a reference for those obtained with the second.

In the scenarios presented here, all sites are located according to the principal of maximum differentiation whereas the consumers expect the distribution of sites' locations to be uniform. All sites set the same price p^*, which is the price anticipated by consumers ($pA_{min}=pA_{max}=p^*$) .

4.2.1 Size and number of coalitions

In this section we study the size and the number of coalitions formed and how these are affected by the structural parameters of the model that define the configuration of the configuration of natural territories in the market. We also study the effect of two different criteria that sites can use for accepting a coalition. Finally, we analyze the emerging coalition structures which, like in the theoretical model, are most of the times non-connex and, in addition to this, we explain the presence of connex components in the coalitions formed.

4.2.1.1 Only consecutive natural territories intersect

To begin with, we present the results of an experiment that is the generalisation of the 4 sites case of the analytical model to 20 sites and where only the natural territories of consecutive sites intersect. The characteristic of this setting is that the notion of connexity corresponds to consecutive locations on the circle.

There are 1000 consumers in the market. Consumers have a reservation utility of 10000 and an adaptation cost per unit of distance equal to 1000, meaning that they will pay at most 500 units to adapt for the version that is more far away from their ideal one (when distance=1/2). Furthermore, consumers incur a fixed search cost of 1 unit. All sites offer their products at the same price equal to 10 and consumers anticipate this price.

The following table contains the values of the most important simulation parameters for this run.

Parameter	Value
Number of sites	20
Number of users	1000
Reservation Utility	10000
Search Cost	1
Reduced Search Cost	0
Adaptation Cost	1000
Consumers' anticipated price	p*= 10
Consumers' priors on locations	Uniform
Distribution of prices	p_{min}=p_{max}= 10

<u>*Remark:*</u> The length of the sites' natural territory in this case is $2\sqrt{c/t}=0.063$.

We examine the case of sites using the individual satisfaction criterion in order to decide whether to coalesce or not. We compare the results obtained with two different coalition formation processes: (*i*) when a single coalition is growing and *(ii)* when multiple coalitions are growing in parallel.

(*i*) Figure 10a shows a typical run of simulation where only one coalition is allowed to grow (there is only one initiator site) and sites are tested for participation in this coalition. In this simulation, the single coalition formation lead to a coalition of size 5.

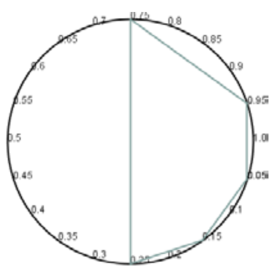


Figure 10a

(ii) Figure 10b shows a typical result of a simulation using the same parameters but allowing, this time, the formation of multiple coalitions with different initiators. The simulation ended when there was no way for the existing coalitions to grow more. For presentation reasons, instead of drawing all the coalitions on the same circle, coalitions are grouped by size. In the table below the circles, the number and the size of the formed coalitions are mentioned.

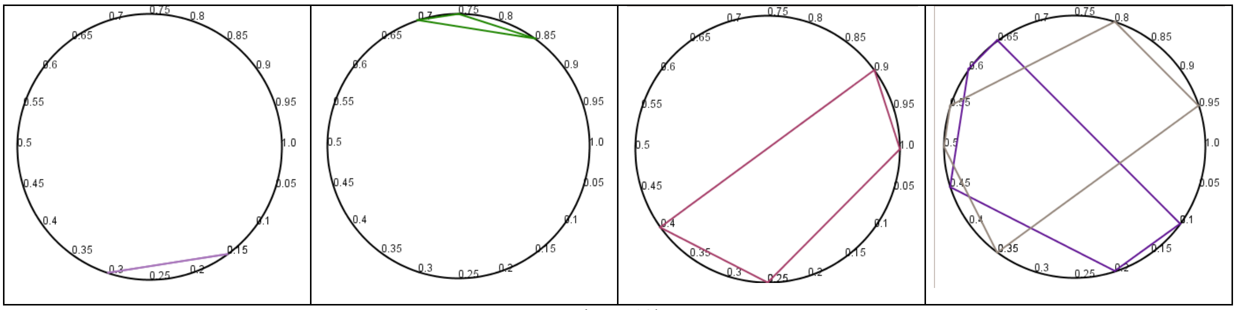


Figure 10b

In this simulation, the multiple coalition formation lead to the emergence of 5 coalition (1 of size 2, 1 of size 3, 1 of size 4 and 2 of size 5).

As we can see from Figure 10a, the single coalition grew by adding a non-connex member each time. The only way in this case to obtain a "connex component" (two members of the coalition are connex) is that the coalition starts (from single to size 2) by this connex coalition. Furthermore, Figure 10b shows a big majority of non-connex components in the case of multiple coalition formation.

1st outcome of the agent-based approach: *The dynamic process coalition formation confirms and generalises the theoretical result in terms of coalition structure: non-connex partners are preferred when forming a coalition*[12].

In addition to this, in Figure 10a the size of the coalition obtained is 5, which is much smaller than what could be expected (a single coalition of size 10, see Figure 11). In fact, by extending the result of the theoretical model (non-connex partners are preferable to connex ones), we would expect in the case of a single coalition growing in a market with 20 sites to obtain a non-connex coalition of size 10.

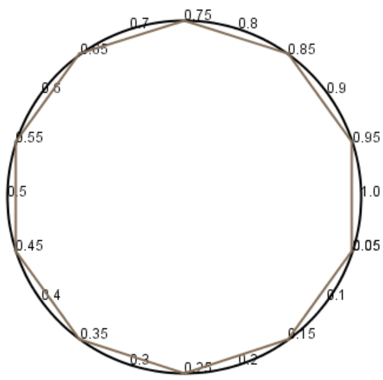


Figure 11

This can be attributed to the fact that the number of consumers inside the natural territory of any site is very

[12] This phenomenon is systematically observed in all the different simulation scenarios described in the rest of the document.

small, only 1000*0.063 = 63 consumers. As a result, the intersection of natural territories contains 1000*0.013 = 13 consumers. Without coalitions, consecutive sites would share equally these 13 consumers. As explained in the analysis of theoretical model, a coalition of size 2 with a non-connex site increases the probability to obtain these consumers from 1/2 to 2/3. This amounts to 8 consumers on expectation instead of 6. However, for larger coalitions, for example when the coalition passes from size 5 to 6, the expected amount of consumers is now (6/7-5/6)*13 = 1.53. In other words, due to the combined effect of the number of consumers on the market (discrete distribution, instead of continuous distribution as in the theoretical model) and the length of natural territories, the advantage of being coalesced may not appear.

Added to this random effect, the marginal increase in profits to the last entrant in a coalition decreases with the size of the coalition. Figure 12 shows the average profit increase for the members of the single coalition of Figure 10a in function of the size of the coalition during the simulation. The average profit increased by 23,7% when the initiator found another single site to coalesce with, but as new sites were being added to the coalition the average profit increase became less and less, e.g. the addition of the last member of the coalition brought an average profit increase of 4,3%.

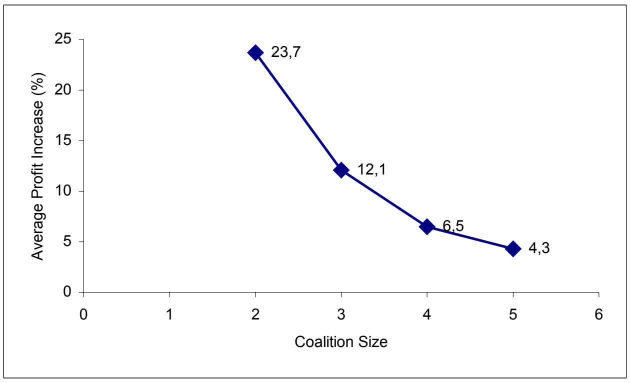


Figure 12

In the case of multiple coalition formation (Figure 10b), for the reasons described above, the maximum coalition size obtained was also 5. For the case of multiple coalition formation in particular, another phenomenon can have impact on the stopping of the growth of a coalition: the "competition" among existing coalitions for new, highly differentiated members. The more the different coalitions grow in size, the fewer the remaining highly differentiated sites that could be accepted by these coalitions.

2nd outcome of the agent-based approach: *The growth of the coalition is halted before it reaches the size of the maximum possible non-connex coalition.*

The importance of this result lies in the fact that a discrete distribution of consumer preferences is a more realistic setting than the continuum considered in the theoretical model. The outcome of the theoretical model according to which it is always profitable to coalesce has to be reconsidered since the way in which the number of consumers interacts with the values of the search cost and adaptation cost (that define the length of the natural territories) influences dramatically the size of the coalitions.

4.2.1.2 Impact of the model's structural parameters

In the previous section the extreme case of natural territories intersecting only for consecutive neighbors. In this section, we present the typical results from an experiment in which the values of the parameters concerning the adaptation cost of the consumers are such that the natural territory of each site is so large that it almost covers the whole circle. More precisely, in this experiment the intersection of the natural territories concerns not only consecutive sites: each site is little differentiated with the 8 sites that are consecutively located before and after this site on the circle.

In fact, the analysis of the impact of the parameters of the model reveals that only the values of search cost and adaptation cost have a dramatic impact on the emerging coalition structures in the market since they define the configuration of sites in terms of natural territories (every pair (c,t) interacting with the priors of consumers and the number of sites defines the way the natural territories of sites intersect).

The following table contains the values of the most important simulation parameters for this run.

Parameter	**Value**
Number of sites	20
Number of users	1000
Reservation Utility	100
Search Cost	1
Reduced Search Cost	0
Adaptation Cost	5
Consumers' anticipated price	p*= 10
Consumers' priors on locations	uniform
Distribution of prices	$p_{min}=p_{max}= 10$

<u>*Remark:*</u> The length of the sites' natural territory in this case is $2\sqrt{c/t}=0.89$. This means that in this particular experiment almost only sites whose locations are diametrically opposed on the circle are non-connex, while in the theoretical model and in the previous experiment there was a equivalence between being connex and being neighbor for a site.

Figure 13a shows the coalitions obtained. when a single is growing. In this simulation, the process stopped at size 2.

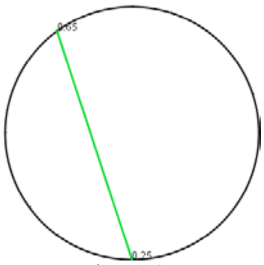


Figure 13a

In Figure 13b we also see that only coalitions of small size emerged, consisting of 2-3 members (5 coalitions of size 2 and 3 coalitions of size 3).

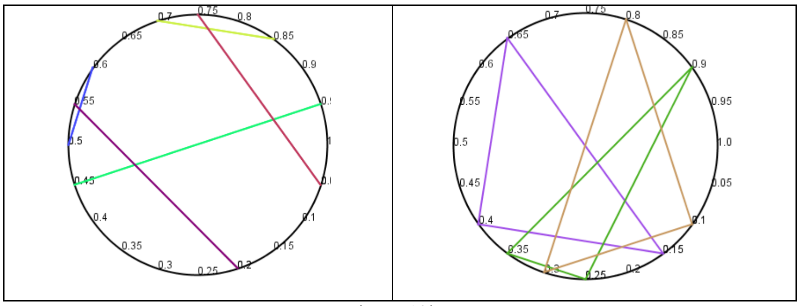


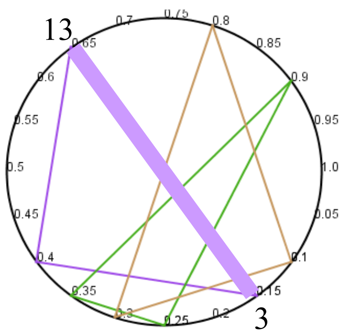
Figure 13b

Given that there is extended overlapping among the natural territories in this experiment, one would expect that there would be no coalitions of size greater than 2 in the case of multiple coalitions, just like in the case of a single coalition growing, since only diametrically opposed sites have territories that do not intersect.

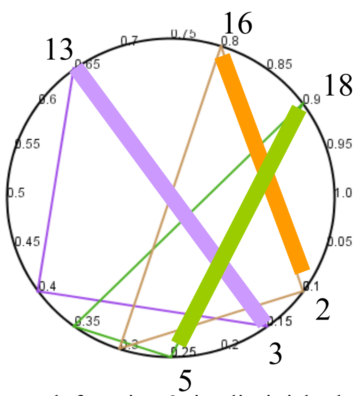
Existence of connex components

In the case of multiple coalition formation, we observe that connex partners are accepted in coalitions. To explain this phenomenon we analyze the history of the formation of the coalition (3,13,8) of Figure 14b.

Chronologically, sites 3 and 13, which are non-connex sites, formed a coalition of size 2. This allows site 3, for example, to attract more consumers on expectation over those that belong to the intersection of territories with connex sites that are not yet coalesced.

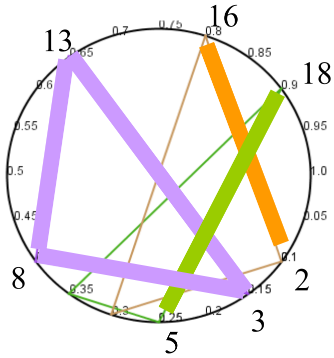


Next, coalitions (2,16) and (5,18) are formed during the dynamic process of multiple coalition formation. So, site 3 is now surrounded by two connex neighbors that participate in coalitions of size 2.



Therefore, the advantage of being coalesced for site 3 is diminished: the probability to obtain consumers belonging to the intersections of natural territories of sites 2, 3 and 5 is now identical for these three sites.

Afterwards, site 8 enters coalition (3,13). Thus, we obtain a coalition of size 3 (3, 13, 8) with 2 connex components (13,8) and (3,8).



This phenomenon, which is impossible in the case of one coalition that grows (see Figure 14a), is made possible due to the dynamic process of multiple coalition formation. Even though this site is connex with site 3, the participation of site 8 in the coalition is beneficial for site 3 which can now attract more consumers than sites 2 and 5. For similar reasons site 13 accepts the entry of site 8 in the coalition since it is surrounded by coalitions (10,12) and (1,14) that are not drawn for presentation reasons (see Figure 14b).

3rd outcome of the agent-based approach: *Although non-connex partners are always preferable for coalescing with, the dynamic process of multiple coalition formation allows the emergence of coalitions containing connex components as a reply to the competition from existing coalitions in the market.*

4.3 Adding agents' growth and market exit

In this section we study scenarios in which we keep the assumptions of symmetry but the population of users and sites grow over time with exponential rate (Assumption 3). Furthermore, sites have a procedure for market exit in order to study in a more realistic context the resulting market structure (Assumption 2).

The sequence of events in the simulations presented in the rest of this document is shown in Figure 16. One can observe that the events of coalition formation and of market entry/exit are separated in time in order to avoid the "noise" that will be produced by the combined effect of these events. In this way, when a coalition is tested in the market, the structure of the market does not change (no competitors enter or die and the number of consumers does not change) and, as a result, the members of the coalition decide based on the pure effect of the coalition.

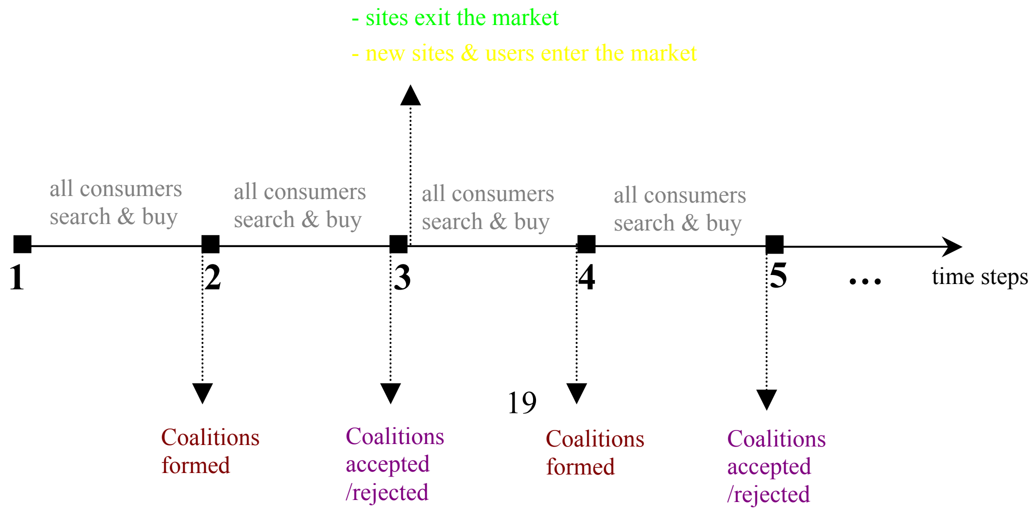


Figure 14

The following table contains the values of the most important simulation parameters for the experiments presented in this section. The values of the consumers' adaptation and search cost are such that the natural territories of each pair of consecutive sites that belong to the initial population of sites intersect. Concerning the locations of the new sites and users they follow the principal of maximal differentiation as described in Assumption 3.

Parameter	Value
Number of sites	100
Number of users	5000
Initial number of sites	20
Initial number of users	1000
Reservation Utility	10000
Search Cost	1
Reduced Search Cost	0
Adaptation Cost	1000
Consumers' anticipated price	p*= 10
Consumers' priors on locations	uniform
Distribution of prices	$p_{min}=p_{max}= 10$

In order to have a reference for our experiments when new sites and users enter the market and sites exit the market, we present firstly the results of an experiment where there is no dying process and the numbers of sites and users grow at exponential rate. Next, we show the results when sites incur an initial cost to enter the market and die if they cannot repay it in a predefined time. Finally, we explore the effect of different combinations of the growth rates of sites and users on these results.

4.3.1. Agent growth without market exit

In this section, we study the results obtained when the number of users and sites grows exponentially with time in such a way that the ratio of users to sites is constant in each time step of the simulation.

The following table shows the number of coalesced and non-coalesced sites in the end of the simulation. We see that the majority of sites (79%) participate in some coalition. Most of the sites that did not coalesce are sites that entered the market towards the end of the simulation (they did not have time to test their participation in enough coalitions to find a profitable one). In addition to this, sites that enter late during the simulation are systematically connex with existing sites (since the location of a new site is always between the locations of two consecutive sites on the circle), which makes them less attractive for adherence in the existing coalitions.

Figure 15 shows the average profit of the coalesced sites, the average profit of the non-coalesced sites and the average profit over all sites in each simulation time step.

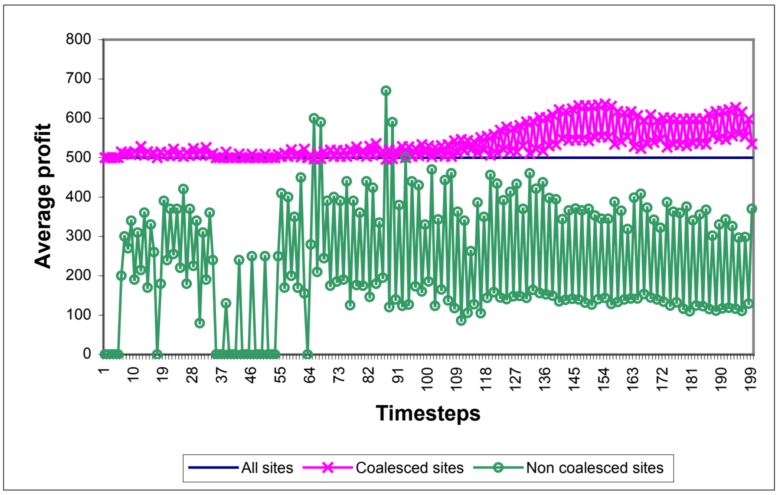


Figure 15

When a category of sites (coalesced or non-coalesced) is absent from the market in some step of the simulation, the average profit of this category is zero. It is evident in Figure 17 that the average profit of non-coalesced sites is almost always lower than that of the coalesced sites throughout the simulation. In the rare occasion in which the average profit of non-coalesced sites surpasses that of coalesced sites (time steps 65, 68, 87 and 88), it happened that there is only 1 non-coalesced site in the market and its profit was higher that the average profit of all the coalesced sites. The dynamic process does not alter the result obtained in Proposition 8 of the theoretical model: the average profit of coalesced sites is better than the profit of non-coalesced sites.
The average profit over all sites (coalesced and non coalesced) is constant since the ratio of consumers to sites is constant during the whole simulation and that no consumer is priced-out (since there is no dying process and the territories of the initial population of sites intersect).

The following figure shows the number of coalitions that exist in the market in each time step of the simulation.

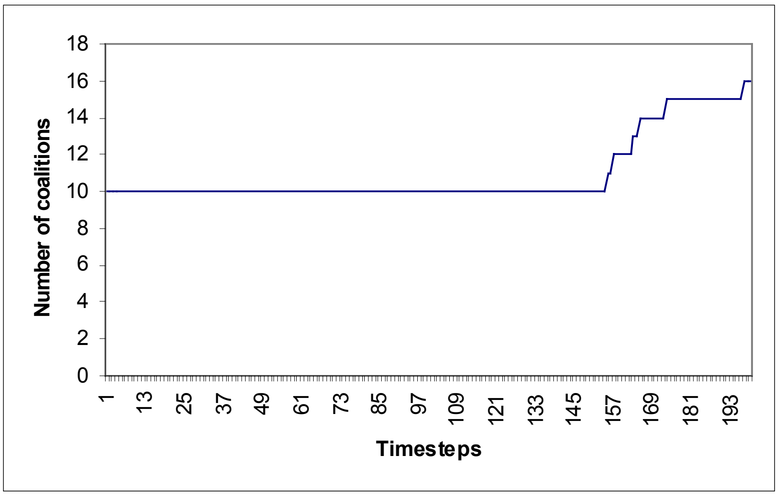


Figure 16

In time step 1 when there were 20 sites in the market, 10 coalitions were formed. The number of coalitions remained 10 until time step 155. This shows that most of the new sites that entered in the market participate in one of the initial coalitions, while the rest of the new entrants remained single. The explanation of this phenomenon is related to the activation for coalition formation and the exponential growth rate of sites:

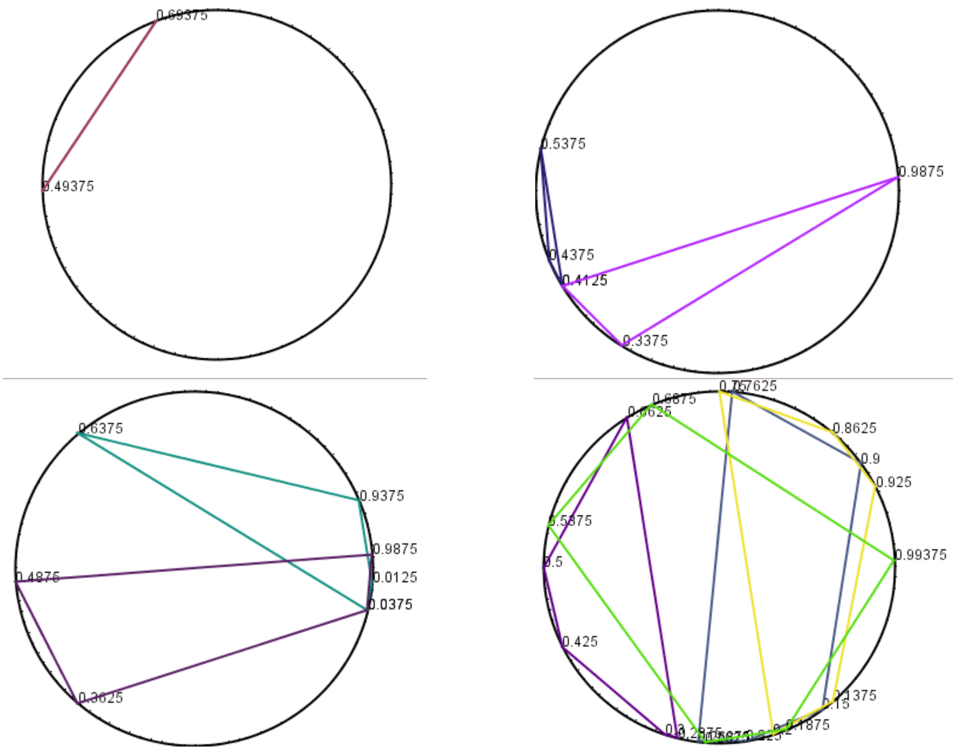
- When a site belonging to 1 of these 10 coalitions is activated it will chose a single site to coalesce with, and since sites enter 1 or 2 at a time (at big intervals in the beginning due to the exponential rate) the probability for new sites to be integrated in the existing coalitions is very high.
- When a new site is activated it has more chances to choose to coalesce with a site that already belongs to some coalition than with another single site.

After time step 155, new entrants to be rejected by the existing coalitions that have, by that time, reached their maximum size. This creates the possibility for these rejected sites to choose or be chosen by new entrants and form new coalitions.

In this experiment 16 coalitions of different sizes ranging from 2 to 7 were formed. Coalitions of final size from 5 to 7 were formed in the beginning of the simulation, whereas the majority of smaller size coalitions were formed in relatively late stages of the run (after time step 155 as mentioned above).

	Coalition size					
	2	3	4	5	6	7
Number of coalitions	1	2	2	4	6	1

Figure 17 shows the coalitions that emerged in this experiment grouped by size on the same circle.



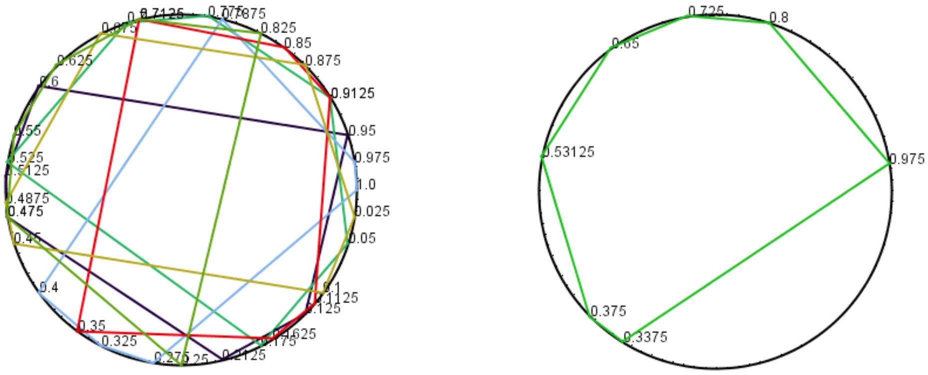


Figure 17

The first interesting phenomenon is that coalitions often surpass in size the maximum size found in the experiment where there was not growth in the agent populations. This can be attributed to the fact both to the growth of the number of sites and the growth of consumers:

- The growth of consumers reduces the "random effect", described in the *1rst outcome* of our approach, thus allowing a coalition to grow more towards the size of the maximum non-connex coalition (size 10).
- The growth of sites changes the structure of the market in terms natural territories, making that more sites are connex. Therefore, as explained in the *3rd outcome* of our approach connex components are now likely to appear increasing the size of a coalition.

4th outcome of the agent-based approach: *The agents' growth allows the coalitions to grow more through the reduction of the random effect (consumers' growth) and the possibility to accept connex components (sites' growth).*

4.3.2 Agent growth with market exit

In the following sections we study the results obtained when the number of users and sites grows exponentially with time and when there is the process so that sites exit the market described in Assumption 2. The related parameters are the following:

Entry Cost	3600
Time to repay	8 time steps
Interest rate	5%

In this setting we will study the effect of different combinations of consumers-users ratio:

- Constant ratio of consumers to sites over time: the ratio of consumers to sites is constant during the simulation
- Increasing ratio of consumers to sites over time: the ratio of consumers to sites increases during the simulation
- Decreasing ratio of consumers to sites over time the ratio of consumers to sites decreases during the simulation

4.3.2.1 Constant ratio of consumers to sites over time

First we examine the case in which sites and users enter the market with exponential growth rates but in such a way that the ratio of users to sites is constant and equal to 50 in each time step of the simulation.

The following table contains aggregate statistics concerning the number of alive and dead sites and whether they are coalesced or not in the end of the simulation. We see that almost half of the sites (52%) do not manage to repay their initial dept and exit the market. Furthermore, most of the sites that survived belong to some coalition, 38 of them are coalesced in the end of the simulation and only 10 of them are not coalesced. This is due to the fact that, on one hand, it is better for a site to belong to a coalition than being single and, on the other hand, the entry and exit of sites in the market creates constantly new opportunities for sites to coalesce.

Number of alive coalesced sites	Number of alive non coalesced sites	Number of dead coalesced sites	Number of dead non coalesced sites
38 (38%)	10 (10%)	40 (40%)	12 (12%)

The following figure shows the average profit of the coalesced sites and non-coalesced sites, as well as the average profit over all sites in each simulation time step.

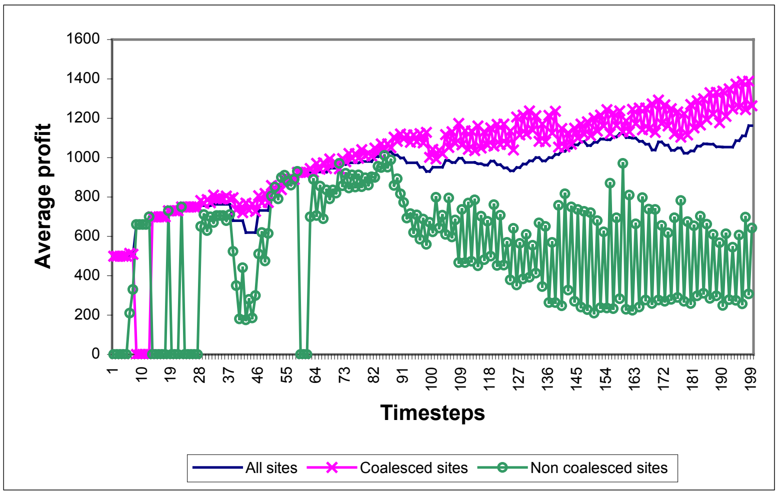


Figure 18

As when there is no market exit, we see that the average profit of coalesced sites is higher than the average profit of the non-coalesced sites. The average profit over all sites and over the coalesced sites increases with time, since the death of some sites increases the demand of the remaining sites (for what concerns non priced-out consumers). More precisely, the average profit of coalesced sites is 2,3 times higher than that of the previous experiment, while the average profit of the non-coalesced sites is 1,7 times higher than before.

The two lines in the next figure represent the number of alive and dead sites in each step of the simulation. The thicker line represents the number of alive sites.

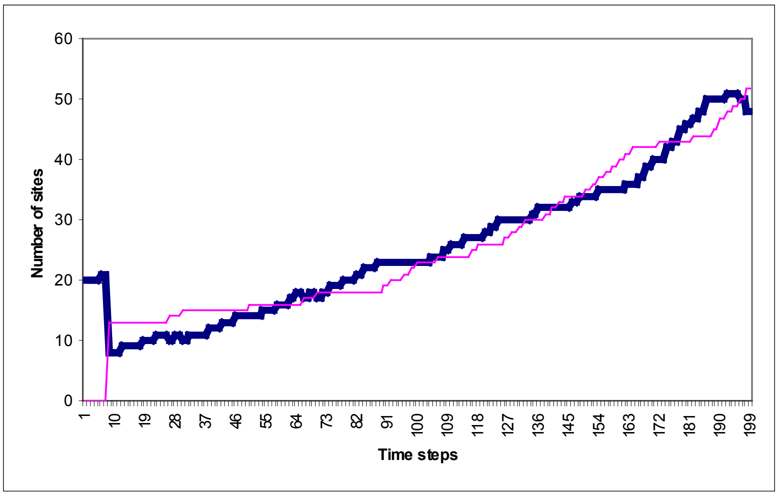


Figure 19

The population of sites starts by 20 and throughout the simulation new sites enter the market while others exit the market. In time step 9, which is the time step in which each site of the initial population of sites has to fully repay its entry dept, 13 out of the 20 initial sites exit the market. We present here this experiment because neither all the sites of the initial population exit from the market, nor all of them survive. When all the initial sites exit the market, the simulation describes a situation in which sites enter progressively increasing the size of a single coalition without any competition. In this case a single, big coalition is obtained in a trivial way. When no sites exit the market, the influence of the dying process is invisible. The choice of parameters of the less violent intermediary case is therefore more interesting.

The number of existing coalitions in each time step is shown in the Figure 20.

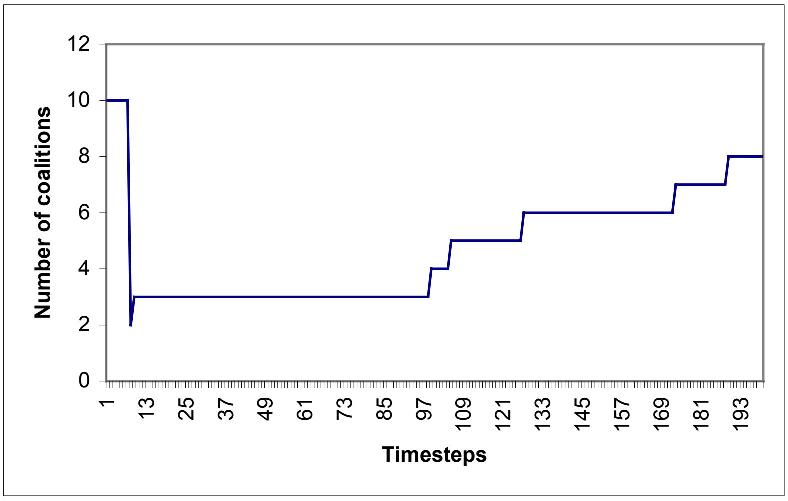


Figure 20

In the beginning of the simulation (time step 1) 10 coalitions are formed among the 20 sites of the initial population. However, this number is reduced to 2 at time step 9 as some coalition members failed to repay their dept and exited the market. We see that 8 coalitions are finally in the market, while in the previous experiment, with no market exit, the final number of coalitions was considerably higher (18). For the same reasons, the number of coalitions remains 3 for a large period of time steps, meaning that the new entrants are integrated to the existing coalitions rather than forming new ones.

Figure 21 shows the average profit (*i.e.* the sum of profits of the members of coalition divided by the coalition size) in function of the coalition size.

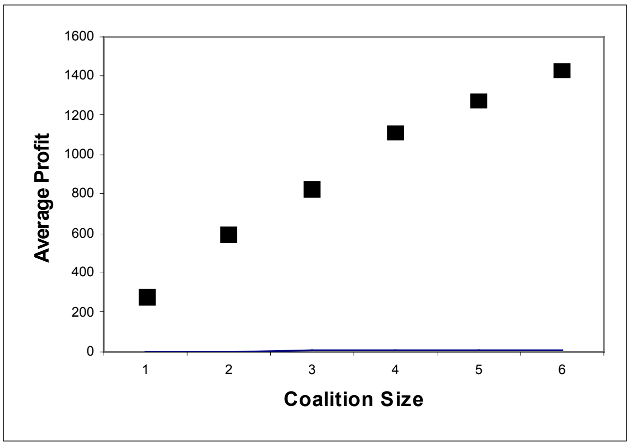


Figure 21

The 8 coalitions that formed in this experiment had sizes from 2-8 as we can see in the following table.

	Coalition Size					
	2	3	4	5	6	8
Number of coalitions	1	1	1	2	2	1

From the output of this run we can observe the coexistence of a relatively high number of relatively large size coalitions (5 coalitions of size more than 5).

Definition 3. We define a notion of *aggregation* related to the proportion of coalitions of each size:

- A market structure in which we observe the emergence of a high number of big coalitions coexisting is considered to be *dispersed* resulting in an *oligopolistic* market.
- A market structure in which there is only a small number of big coalitions while the rest of the sites are either single or participate in small coalitions is considered to be *concentrated*. In this case, the market structure goes towards *duopoly* or *monopoly* structures.

As a result of this experiment we observe the emergence of an oligopolistic market structure.
In a larger scale experiment where the number of sites that enter the market goes to 1000 and the number of consumers to 50000, we obtained the same phenomenon of dispersion in the market. Out of 86 coalitions 33 have size larger than 5:

	Coalition Size						
	2	3	4	5	6	7	8
Number of coalitions	13	24	16	17	11	4	1

4.3.2.2 Increasing ratio of consumers to sites over time

Here we examine the case in which sites and users enter the market with exponential growth rates but in such a way that the ratio of users to sites is increasing with time as shown in the following figure.

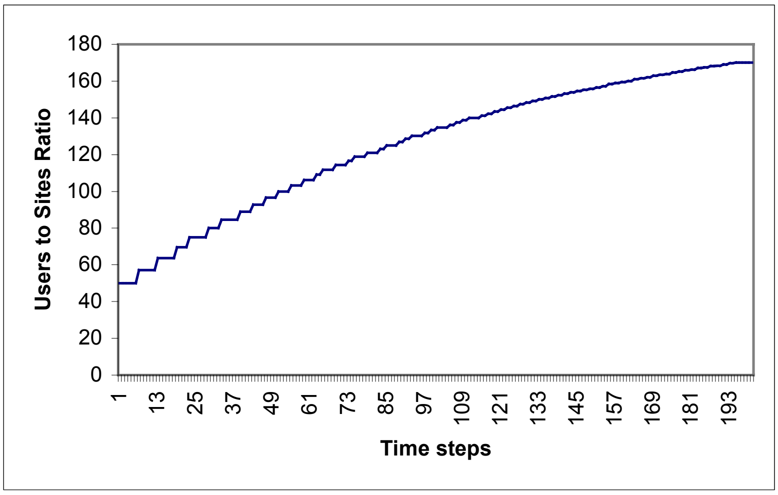


Figure 22

The number of sites that entered the market in this experiment is 100 and the number of consumers 17000.
The following table contains aggregate statistics concerning the number of alive and dead sites and whether they are coalesced or not in the end of the simulation.

Number of alive coalesced sites	Number of alive non coalesced sites	Number of dead coalesced sites	Number of dead non coalesced sites
75 (75%)	15 (15%)	5 (5%)	5 (5%)

In this case, only 10 sites (*i.e.* 10%) that entered the market do not manage to repay their initial dept and exit the market because the growing ratio of consumers to users creates favorable circumstances for the prosperity of the sites. Like before, most of alive sites belong to some coalition, 75 alive sites are coalesced in the end of the simulation while only 15 alive sites are not coalesced.

The following figure shows the average profit of the coalesced sites, the average profit of the non-coalesced sites and the average profit over all sites in each simulation time step.

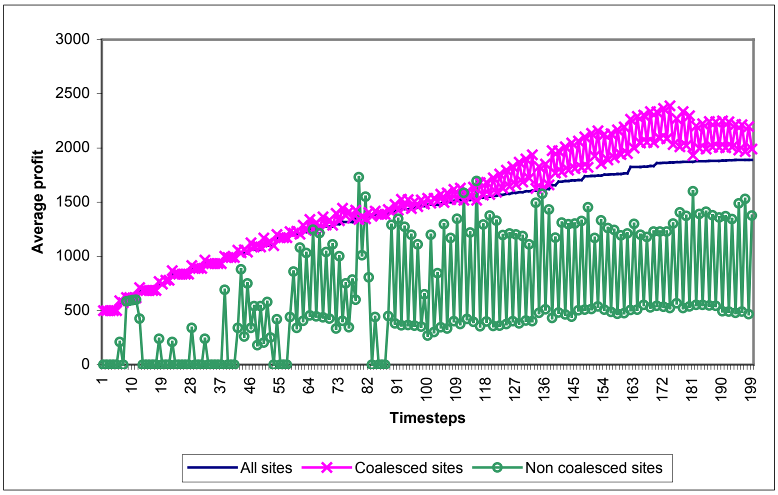


Figure 23

Like in the previous experiments, the average profit of coalesced sites is always higher than that of the non-coalesced sites. The average profit over all sites and over the coalesced sites increases not only because of the death of existing sites but also because of the fact that the consumers to sites ratio also increases with time in this case. In the end of the simulation the average profit of the coalesced sites are almost 1,6 times the ones when the consumers to sites ratio remained constant over time, while the average profit of the non-coalesced sites is 2,1 times higher.

The two lines in the next figure represent the number of alive and dead sites in each step of the simulation. The thicker line represents the number of alive sites.

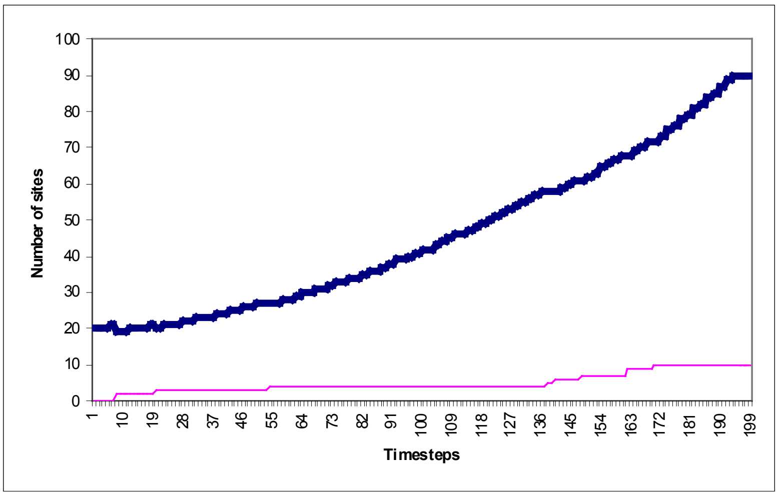


Figure 24

In this experiment, the number of sites that survive is constantly higher than the number of sites that exit the market and there is a big difference between them as already mentioned.

In this experiment 16 coalitions are formed (which is the double than what was obtained with constant consumers-sites ratio) in the end of the simulation and their size varied from 2 to 7.

	Coalition Size					
	2	3	4	5	6	7
Number of coalitions	3	2	2	2	4	3

In the following figure we can see the coalitions that are formed in this experiment.

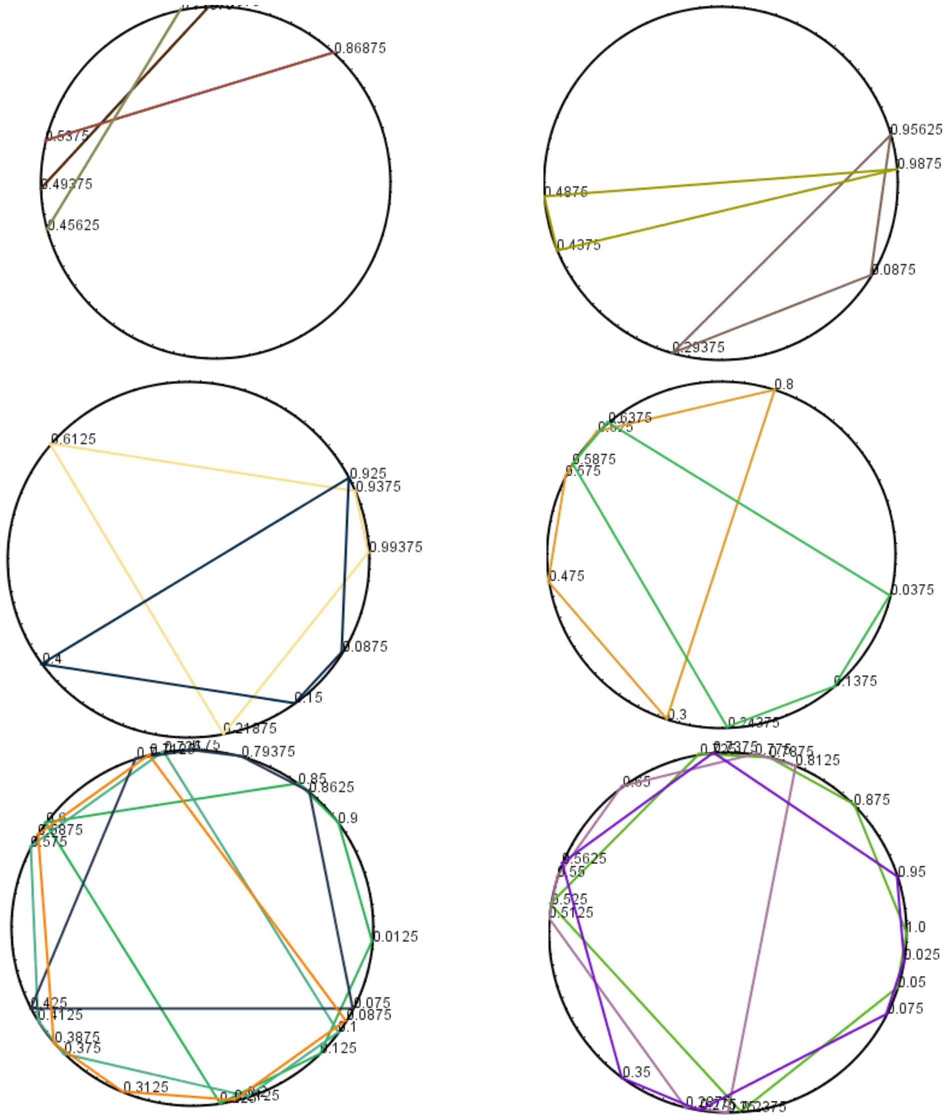


Figure 25

It is evident that there is a significant number of big size coalitions (size 5-7, which are, as we have seen before, the more profitable). 7 of these coalitions have a size greater than 6.

5th outcome of the agent-based approach: *An increasing ratio of consumers to sites encourages the emergence of dispersion in the market resulting to an oligopoly of big coalitions coexisting in the market.*

4.3.2.3 Decreasing ratio of consumers to sites over time

Finally we examine the case in which sites and users enter the market with exponential growth rates but in such a way that the ratio of users to sites is decreasing with time as shown in the following figure.

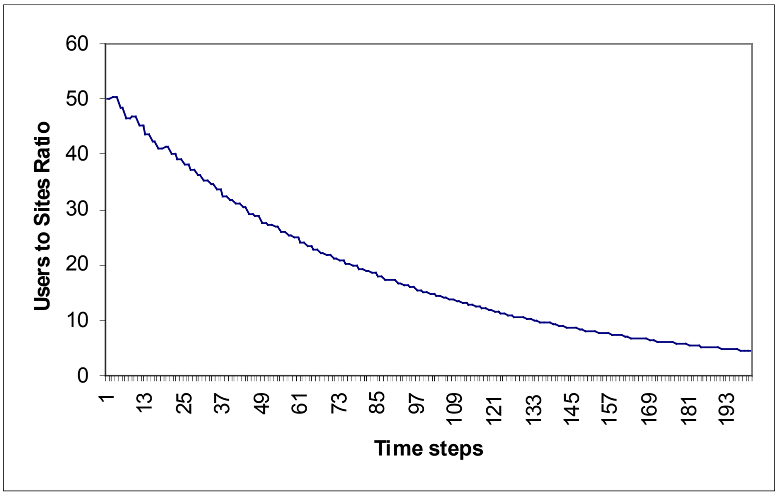


Figure 26

The number of sites that entered the market in this experiment is 517 and the number of consumers 2277. In this experiment only 16% of sites that entered the market survive and 71% of these sites are coalesced.

Number of alive coalesced sites	Number of alive non coalesced sites	Number of dead coalesced sites	Number of dead non coalesced sites
59 (11.4%)	24 (4.6%)	243 (47%)	191 (37%)

Figure 27 shows the average profit of the coalesced sites, the average profit of the non-coalesced sites and the average profit over all sites in each simulation time step.

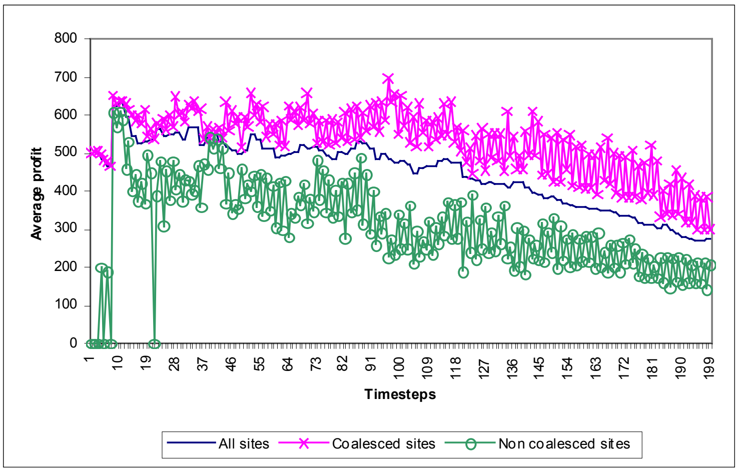


Figure 27

The average profit of coalesced sites is still higher than the average profit of non-coalesced sites, although this time, their difference is less than before. The average profit of all alive sites decreases, contrary to the previous experiments, due to the fact that in this experiment the ratio of consumers to sites is decreasing. In the end of the simulation the average profit of coalesced sites is almost 4,2 times lower than the ones when the consumers to sites ratio remains constant over time, while the average profit of the non-coalesced sites is 3 times lower.

The two lines in the next figure represent the number of alive and dead sites in each step of the simulation. The thicker line represents the number of dead sites.

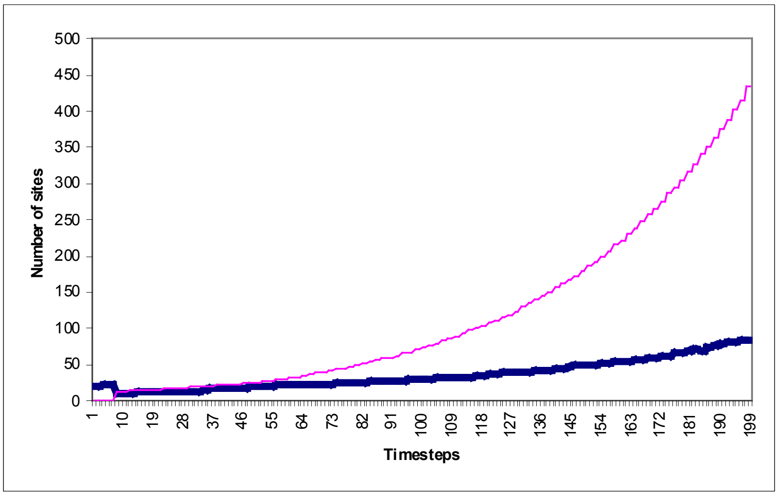


Figure 28

One can easily see that the number of sites that exit the market is much higher than in the previous experiments due to the decrease of consumers-sites ratio, which makes more difficult with time the survival of sites.

The number of existing coalitions in each time step is shown in the following figure.

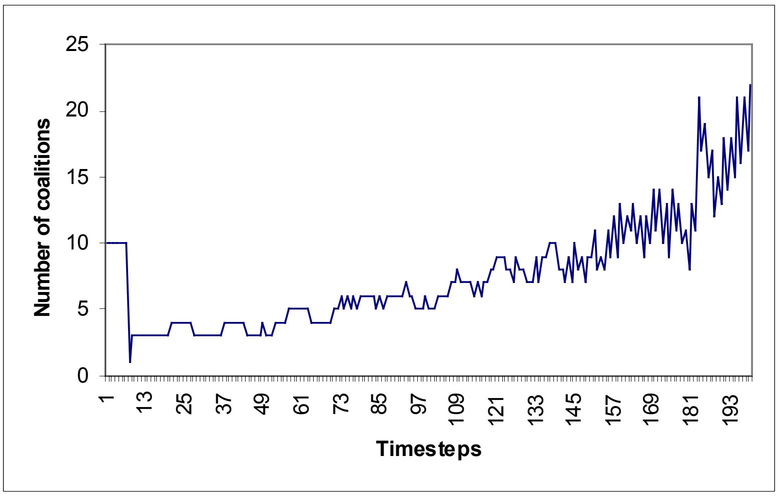


Figure 29

Contrary to all the previous experiments the number of coalitions constantly increases over time because many sites exit the market, consequently a lot of coalitions disappear before they have the chance to grow in size.

For the first time in this experiment we observe the existence of aggregation in the emerging market structure: only 2 coalitions of size greater than 3 are formed, while there is a large number of small coalitions of size 2 and 3.

	Coalition Size			
	2	3	5	7
Number of coalitions	13	7	1	1

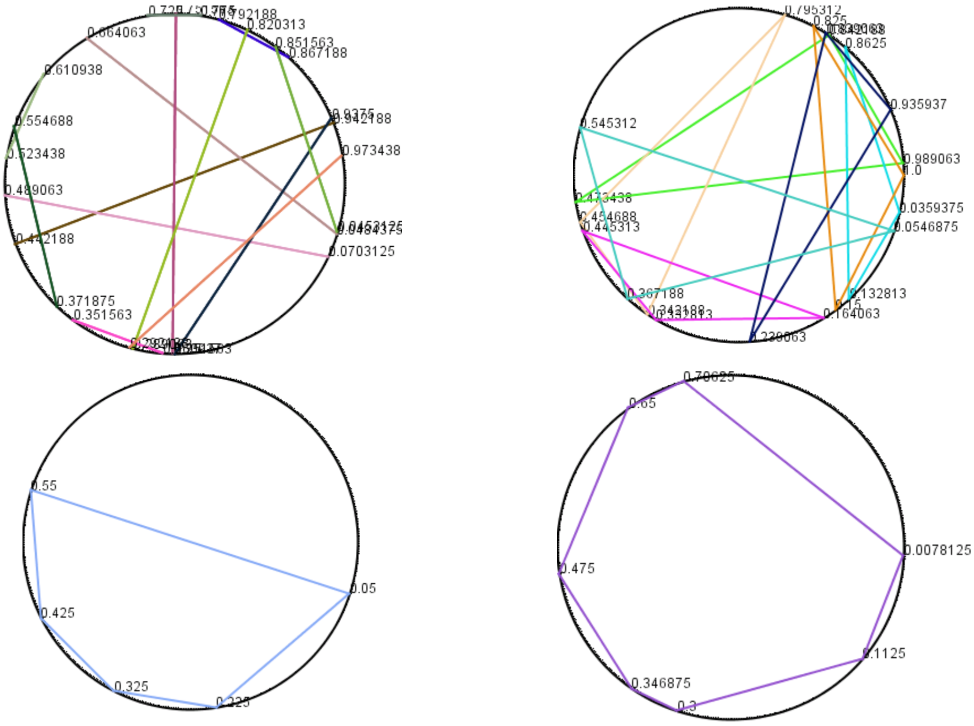


Figure 30

In this experiment, we see that only 2 big coalitions dominate the market. The structure is more concentrated since the duopoly faces the competition of many small-size coalitions and single sites, which are therefore less efficient.

6th outcome of the agent-based approach: *A decreasing ratio of consumers to sites encourages the emergence of concentration in the market resulting to a market structure that goes towards a monopoly.*

In a larger scale experiment where the number of sites that enter the market goes to 1000 and the number of consumers goes to 4240, we obtained the same phenomenon of concentration in the market. Only 2 coalitions of size larger than 4 are formed:

	Coalition Size				
	2	3	4	6	7
Number of coalitions	24	9	3	1	1

4.4 Decreasing search cost

In the experiments presented in this section the search cost decreases linearly for all consumers with time as explained in Assumption 10. The search cost at time step 1 is 1.0 for all consumers in the market and at the end of the simulation it becomes 0.25.

The reduction of search cost has two implications in the behavior of model. On one hand, the natural territories of sites, which are given by the formula $\sqrt{c/t}$, decrease with the search cost, meaning that with time sites become less connex among them (the intersection of their natural territories diminishes) and, as a result, it is more easy for a site to accept to coalesced with another site.

The following table contains aggregate statistics concerning this experiment. The decrease in the search cost did not affect the final numbers of alive and dead sites (coalesced or not) in the market.

Number of alive coalesced sites	Number of alive non coalesced sites	Number of dead coalesced sites	Number of dead non coalesced sites
35 (35%)	10 (10%)	36 (36%)	19 (19%)

The same number of coalitions is formed as when the search cost is constant, but we do not observe the presence of a larger size coalition, like 7 or 8. On the contrary, we observe more medium-sized coalitions.

	Coalition Size			
	2	3	4	6
Number of coalitions	2	1	1	4

Finally, the table below contains the results of an experiment where the search cost decreases just like described above, but when the ratio of users to consumers also decreases with time (this setting lead to a concentrated market structure as mentioned in the end of the previous section). We see that the market is less concentrated when the search cost decreases.

		Coalition Size					
		2	3	4	5	6	7
Number of coalitions	*Decreasing search cost*	16	20	13	18	11	2
	Constant search cost	24	9	3	-	1	1

This result is due to the decrease of the natural territories that decreases the competition among sites since it makes them less connex. This creates more opportunities for coalitions to be formed resulting in a market in which more coalitions manage to grow.

4.5 Price adjustment

In this section we show the results of experiments in which the sites have the possibility to adjust their prices according to the two alternative algorithms described in Assumption 6. We are interested in seeing whether or not the concentration that emerges in the market under the conditions described in the 7th outcome of our model is affected.

The new sequence of events when sites adjust their prices in the beginning of each time step is shown in the following figure.

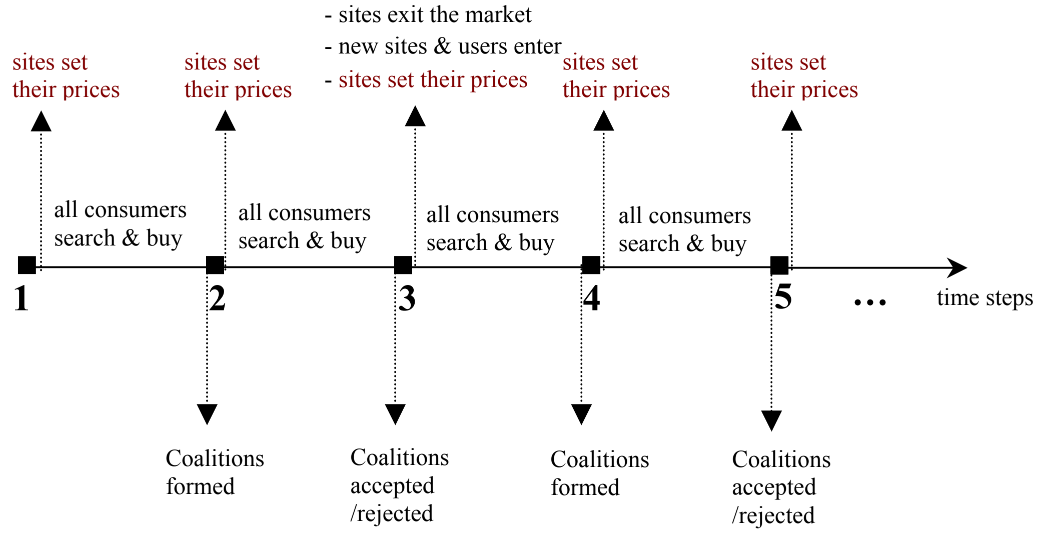


Figure 31

The values of the more important simulations parameters in the following experiments are shown in the table below. The initial price of each site entering the market is randomly selected in [0.1, 20.0].

Parameter	Value
Number of sites	517
Number of users	2277
Initial number of sites	20
Initial number of users	1000
Reservation Utility	10000
Search Cost	1
Reduced Search Cost	0
Adaptation Cost	1000
Consumers' anticipated price	p*= 10
Consumers' priors on locations	uniform
Price adjustment	$\delta p_{min} = \delta p_{max} = 1.0$

The experiments with the different price adjustment algorithms show that both kinds of price adjustment algorithms give better average profits for all sites, DFA being more efficient as we can see in the following figure:

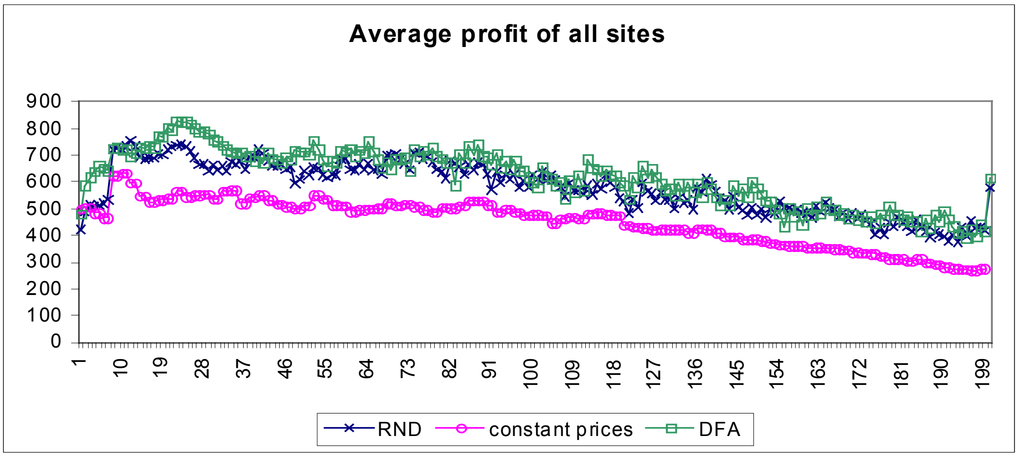


Figure 32

However, we can see that the price adjustment is more beneficial for coalesced sites than for non-coalesced. Figure 44 shows the average profit with price adjustment is clearly separated from the average profit without price adjustment:

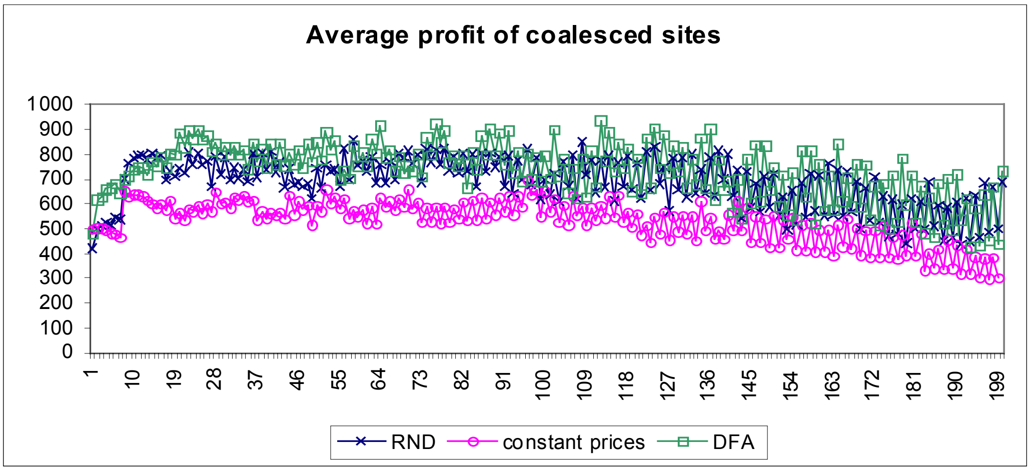


Figure 33

While for non-coalesced sites the price adjustment has little not a significant effect:

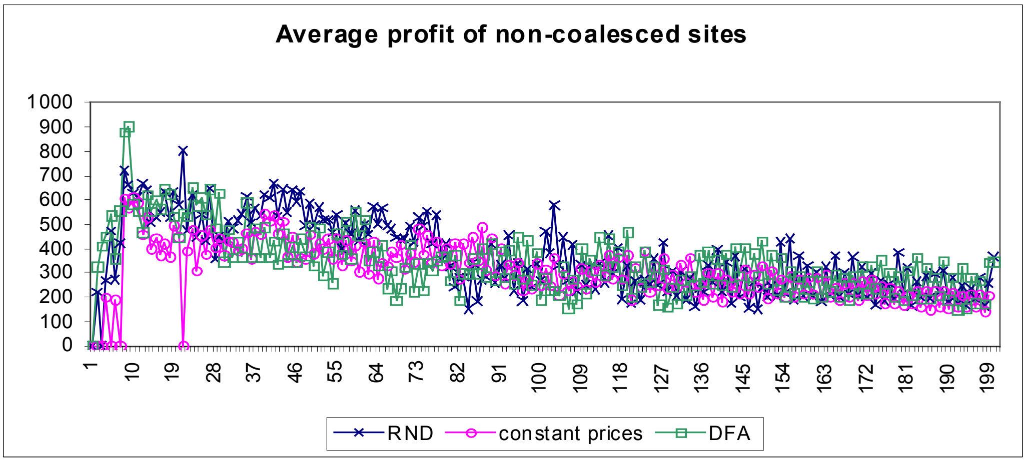


Figure 34

7th outcome of the agent-based approach: *During the dynamic process of coalition formation with price adjustment, the average profit of coalesced sites is better than the profit of non-coalesced sites, confirming the result of the static comparative model.*

Finally, in terms of the aggregation, as we can see in the following table, the DFA price adjustment algorithm diminishes less the concentrated character of the market compared to the RND price adjustment algorithm.

		Coalition Size					
		2	3	4	5	6	7
Number of coalitions	*Constant prices*	24	9	3	-	1	1
	RND price adj	8	5	3	2	-	-
	DFA	11	4	-	3	1	-

4.6 Agent Heterogeneity

All the runs described in the previous sections have been repeated with heterogeinity in the agents' characteristics (the corresponding values were selected following a uniform distribution each time): search cost, reduced search cost, reservation utility, adaptation cost, prices (constant) and entry cost.
However, the determining value for these runs appeared to be the average value of these parameters resulting in similar results than those obtain with symmetric agents. The relative "noise" brought by variations in these parameters do not change the qualitative results presented in the previous sections.

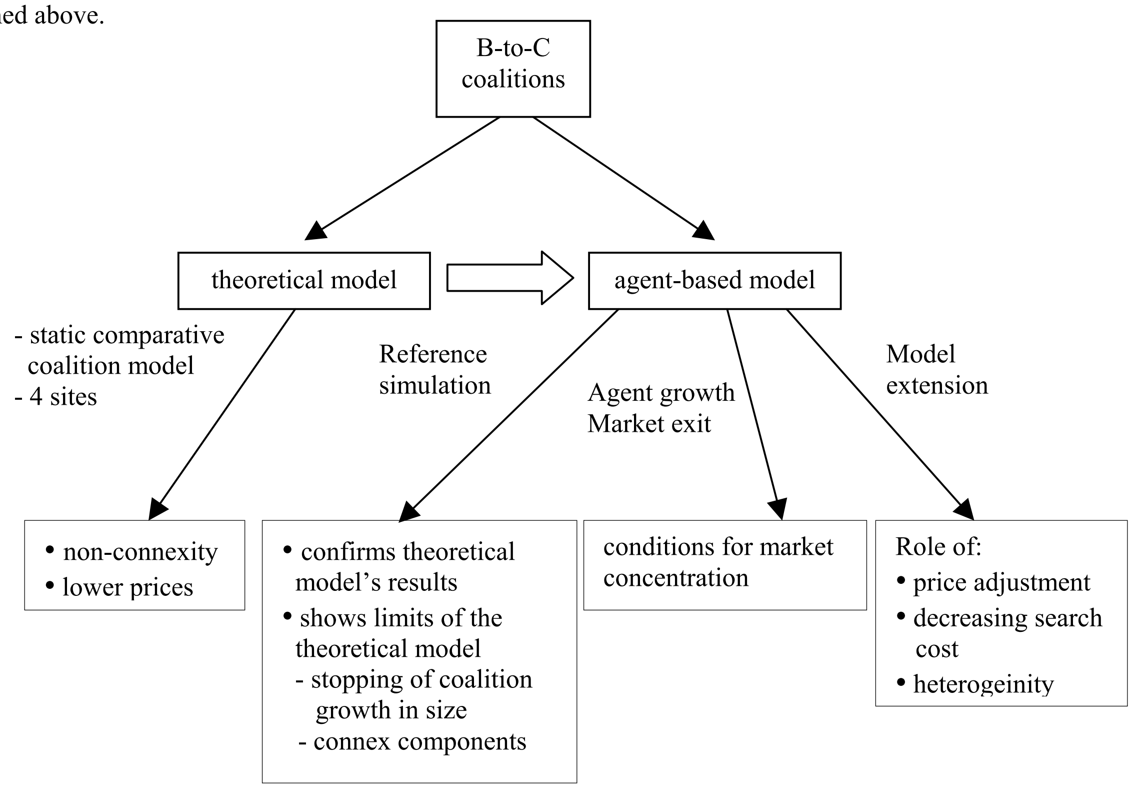
5. General conclusion

The analytical model showed that the presence of search costs for the consumers (to find a good in a differentiated market) that are independent from the adaptation costs they incur (when the good they find does not match with their preference) provides original and interesting results: *(i)* coalesced sites have an incentive to lower their prices and *(ii)* sites choose a highly differentiated partner to form a coalition.
The agent-based model extends the analytical model with less restricting assumptions that help in the understanding of the dynamics of the formation of coalitions on the Web. The agent-based model was enriched by additional behaviors for the consumers and the sites, which make the representation of an electronic B-to-C market more realistic compared to the theoretical model.
The basic findings of the agent-based model are the following:

- The dynamic process coalition formation confirms and generalises the theoretical result in terms of coalition structure: non-connex partners are preferred when forming a coalition.
- The growth of the coalition is halted before it reaches the size of the maximum possible non-connex coalition.
- Although non-connex partners are always preferable for coalescing with, the dynamic process of multiple coalition formation allows the emergence of coalitions containing connex components as a reply to the competition from existing coalitions in the market.
- The agents' growth allows the coalitions to grow more through the reduction of the random effect (consumers'

growth) and the possibility to accept connex components (sites' growth).

- An increasing ratio of consumers to sites encourages the emergence of dispersion in the market resulting to an oligopoly of big coalitions coexisting in the market.
- A decreasing ratio of consumers to sites encourages the emergence of concentration in the market resulting to a market structure that goes towards a monopoly
- During the dynamic process of coalition formation with price adjustment the average profits of coalesced sites is better than the profit of non-coalesced sites, confirming the result of the static comparative model.
- The decrease in the consumers' search cost with time causes a lowering in the market concentration.
- Experiments performed on the role of heterogeinity proved to have little or no impact on qualitative results mentioned above.



The figure above summarises the different approaches used to study B-to-C coalitions and their outcomes.

If we look at the geographical economy literature applied to the aggregation of shops in some locations, two main forces are present : the lowering of search costs for consumers that may ignore the prices and characteristics of goods but *a priori* know the number of products that are offered in a given location, on one side, and the increased competition due to the proximity of shops, on the other side. The first force drives aggregation by directly increasing the demand through the number of consumers that want to economize on search costs whose main component is transportation cost. The second one may limit this aggregation when price decrease due to competition is no more counterbalanced by the increased market accruing to each shop, except when price coordination is made possible, for instance through mergers. In the world of B-to-C merchant internet sites, these mechanisms are severely modified because search cost are not transportation costs and because aggregation may arise through coalition of independent sites at (almost) no cost, what ever the type of goods offered by each firm (or the location of shops in a differentiated space along consumers preferences).

Mono-product firms have to coalesce in order to gain market share on non coalesced firms through a demand effect (number of visitors) even when consumers have no priors about the existence of such coalitions (they ignore that they are entering a city), firms prefer to coalesce with different firms, not for avoiding stronger competition but for maximizing the increased market share effect in the competition with non coalesced firms. The multi-agent simulation confirms the qualitative results of the analytical model (preference for coalitions and coalitions of non connex sites) and give some insights on the dynamic process that may lead to more or less concentrated market structure on the (merchant side) of the Web, without taking into account merges that may arise for anticompetitive reasons nor network effects on the consumers side : the larger number of entry of new Web merchant sites compared to the growth of consumers buying on the Web seems to be the main factor explaining the concentration (few number of very big coalitions). Within this framework, the venture capitalism bubble for Web sites that excessively favoured the entry of new sites before the market was there would have dramatically favoured this form of concentration.

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