The trade-off technological Vs environmental efficiency at glance: Lessons from optimal switching models

Raouf Boucekkine* Thomas Vallée†

1 The benchmark optimal control problem: The one-country case

We first consider the case of an isolated economy, which therefore takes its decisions in exclusive accordance with its own preferences and constraints.
Consider an economy whose benevolent central has to maximize the following intertemporal utility function:

$$\int_0^T u\left(C(t), P(t)\right)\ e^{-\rho t} dt,$$

where C is aggregate consumption, P is the aggregate stock of pollution, and $u(.)$ is a concave utility function with $u_C^{'} > 0$ et $u_P^{'} < 0$. ρ is the time discounting rate, and T is the time horizon, assumed finite in our framework.

On the production side, we have an elementary one-sector structure: the production function is assumed to be of the AK type, and output is either used for consumption or as an input, X:

$$Y = C + X = F(X) = A_i\ X \Rightarrow C \equiv X(A_i - 1)$$

where $A_i > 0$ is the marginal productivity of capital.
The stock of pollution is assumed to evolve proportionally to the production level:

$$\dot{P} = \alpha_i A_i X,$$

with $P(0) \geq 0$ given. Notice that α_i measures the marginal pollution of an additional production unit. Clean technologies would therefore be associated with low values for α_i. We finally, assume in this benchmark case that $P(T)$ is free.

Hereafter, we shall represent any technical menu by a pair of positive numbers (A_i, α_i). We assume that the economy starts with a menu (A_1, α_1). However,

*Corresponding author. CORE and Department of Economics, Université catholique de Louvain, Voie du Roman-Pays, 34, B-1348 Louvain-la-Neuve (Belgique). boucekkine@core.ucl.ac.be

†LEN, Faculté d'Economie et de Gestion, Chemin de la Censive du Tertre 44322 Nantes (France). Thomas.Vallee@univ-nantes.fr

another menu (A_2, α_2) is available from $t = 0$ involving: $\alpha_2 > \alpha_1$ but $A_2 < A_1$. The planner has to decide whether the economy has to switch to the new technological regime, and if he believes so, he has to fix the optimal switching time, say t_1, with $0 \leq t_1 \leq T$.
Summarizing the discussion just before, our optimal control problem is

$$\max_{X,t_1} \int_0^T u(X(t), P(t))e^{-\rho t}dt,$$

subject to (1) and (2), P_0 given and $P(T) \geq 0$ free. Note that we can rewrite our objective function as follows:

$$U(X, P, t_1) = \int_0^{t_1} u(X, P)e^{-\rho t}dt + \int_{t_1}^T u(X, P)e^{-\rho t}dt. \tag{1}$$

In order to get **analytical** solutions, we shall restrict our study to the following class of utility functions:

$$u(C, P) = \ln(C) - \beta\ P,$$

where β measures the marginal disutility due to pollution, which is assumed in our analytical case independent of the level of the pollution stock.

2 Two-country extensions

After characterizing the solution of the one-country model, we move to the more interesting two-country case. We consider the elementary situation where the two countries (a and b) don't trade in goods but share the same pollution stock.

$$\dot{P} = \alpha_i^a A_i^a X^a + \alpha_i^b A_i^b X^b$$

Each country has the same type of objective function and technology as before but faces now the new state equation just above. We study their optimal switching policies in three game-theoretic configurations:

i) Nash games

ii) Cooperative games

iii) Stackelberg games