Persistence of Monopoly, Innovation, and R&D Spillovers: Static versus Dynamic Analysis

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February 2006

Abstract

We build a dynamic duopoly model that accounts for the empirical observation of monopoly persistence in the long run. More specifically, we analyze the conditions under which it is optimal for the market leader in an initially duopoly setup to undertake pre-emptive R&D investment, ("strategic predation" strategy) that eventually leads to exit of the follower firm. The follower is assumed to benefit from the innovative activities of the leader through R&D spillovers. The novel feature of our approach is that we introduce explicit dynamic model and contrast it with its static counterpart. Contrary to the predictions of the static model, strategic predation that leads to persistence of monopoly is in general optimal strategy to pursue in a dynamic framework when spillovers are not large.

Keywords: dynamic duopoly, R&D spillovers, persistence of monopoly, strategic predation, accommodation
JEL Classification: L12, L13, L41

1

1   Introduction

Is monopoly an environment conducive to innovation? Is there persistence of monopoly or there is a change of the identity of innovating firm every while (leapfrogging in jargon)? This kind of questions is not quite new among economists, but it seems to be actual again. In a recent issue of the British *The Economist* (2004), the authors of already celebrated rubric "Economics Focus" – that time dedicated to monopoly and innovation – in the provocatively entitled text "Slackers or Pace-Setters: Monopolies may have more incentives to innovate than economists have thought" noticed that monopolies may have much more prominent role in generation innovations than have been previously thought. The authors further express doubts about the predominant economic theory according to which "monopolist should have far less incentives to invest in creating innovations than a firm in a competitive environment." Apparently, there is a controversial role of market power and monopolies in creating innovations and the key to the answer lies in the underlying incentives to undertake innovations. The recent empirical evidence seems to support this Schumpeterian allegations from *The Economist*: there is a positive relationship between market power and intensity of innovation (see, for instance, Blundell, et al., 1999, Carlin et al., 2004, Aghion and Griffith, 2004). Commenting on this empirical evidence, Etro (2004) noticed that it "is consistent with pre-emptive R&D investment by the leaders." As a consequence of such strategic behavior, there may be only one firm at the end of the day, but this firm would display far more competitive behavior than standard monopolist; it would respectively generate higher flow of R&D, charge lower price and produce more.

There are many real-world examples of monopolistic or dominant firms that invest more in innovation and R&D than their rivals (see Etro, 2004) and that persists over long period of time. Here we could, for instance, refer to AT&T[1] as an example of such pattern. Founded in 1885, the company is one of the largest telephone companies and cable television operators in the world. After becoming a first long distance telephone network in the US AT&T made huge investments in research and development. As a result, the company obtained near monopoly power on long distance phone services. Heavy investments in R&D together with aggressive behavior on the market allowed AT&T to

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[1]AT&T is a giant old American telecommunications company, publicly listed on the New York Stock Exchange under the ticker symbol T. AT&T provides voice, video, data, and Internet telecommunications services to businesses, consumers, and government agencies (source: `www.sciencedaily.com/encyclopedia/at_t`).

obtain crucial inventions and to spread its near monopoly power on other markets. The company was both buying patents for significant innovations[2] and making innovations itself[3]. Only after a suit against AT&T in 1982 followed by the breakup of the company into several local independent units called "Baby Bells" in 1984 the US telephone industry became competitive and other companies entered the market. While loosing part of its market power on the long distance telephone services, the company has continued its aggressive investments in R&D. For example, in 2004 AT&T introduced a facility allowing businesses to securely run their private networks without any interruptions on AT&T's leading global Internet Protocol network. This innovation assures the company to remain a leader in IP networking. Even AT&T characterizes itself as "backed by the research and development capabilities of AT&T labs, the company is a global leader in local, long distance, Internet and transaction-based voice and data services".

The above observations concerning the relation between innovativeness, leadership and market power motivate our paper in that we aim to describe and analyze a particular setup in which the persistence of monopoly can arise in the long run. More specifically, we study the situation in which the market leader undertakes pre-emptive R&D investment, (or, in our words, adopts "strategic predation" strategy) that eventually leads to exit of the follower firm or/and prevents or limits the entry of the new firms and we contrast this situation with the one in which the leader (within the same setup) "accommodates" the follower, that is, it co-exists with follower in a duopoly market structure. This comparison will enable us to study both positive aspects of the two main strategies- accommodation and strategic predation- (like, for instance, which strategy yields higher R&D intensity or R&D stock) and normative aspect (social welfare implications) of the two resulting market structures: duopoly versus (constrained or unconstrained) monopoly. The latter aspect, as we will see, carries important policy implications.

The novel feature of our approach is that we introduce explicit dynamic model and contrast it with its static (or quasi-dynamic) counterpart. This comparison can be considered as a topic *per se* of our paper. Since strategic innovations are inherently dynamic phenomenon, we argue that suitable model aimed at capturing both accommodating and the pre-emptive or predatory behavior of the dominant firm should be explicitly dynamic. Furthermore, to emphasize the role of the leader we assume that he is the only one that

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[2]During the early 1920s, AT&T bought Lee De Forest's patents on the "audion", the first triode vacuum tube, which let them enter the radio business (cf. `www.sciencedaily.com/encyclopedia/at_t`).

[3]AT&T commissioned the first commercial communications satellite, Telstar I in 1962 (source: `www.sciencedaily.com/encyclopedia/at_t`).

invests in innovation while the follower imitates through R&D spillovers. The rationale for introducing spillovers stems from the fact that innovations, in general, are subject to R&D spillovers for which the recipients need to have so-called "absorptive capacity," that is, the "ability to identify, assimilate, and exploit knowledge from the environment and to apply it to commercial ends" (Cohen and Leventhal, 1989 and 1990).[4] The importance of R&D spillovers, imitations and its economic implications seem to be well and broadly documented (in both theoretical and empirical literature; see, for instance, Griliches, 1992). However, most of the theoretical models are static in nature and they focus on the accommodation strategies. That is, strategic predation is simply ignored or precluded by assumptions (so that it is never optimal). In such situations unilateral R&D spillovers create disincentives to invest in R&D and consequently hamper innovations. However, as we will see soon, in the case when strategic predation is optimal, the economic implication of R&D spillovers is exactly opposite. They enhance the incentive to invest in R&D.

A static (or quasi-dynamic) simple two-stage duopoly model will serve as a benchmark for our subsequent dynamic analysis. This should come at no surprise since the concept of two stage (or n-stage competition) use to be a standard tool to tackle the above mentioned types of strategic interactions. That approach concentrates on identifying "strategic effects" that influences the first-period behavior and aims to characterize the resulting strategic rivalry. The concept has been proven successful in a way that the same strategic principles (e.g. "overinvestment" or "underinvestment") apply in so many economic environments and the comparative static results from the static oligopoly theory can be used to provide information about strategic behavior (see Fudenberg and Tirole, 1984, Tirole,1990, Shapiro, 1989, and Etro 2004). However, the described idea relies on the artificial time structure since the final (i.e., second or n-th) period is essentially one of static oligopoly (see Shapiro, 1989). From the perspective of the full-fledged dynamic model, it gives at best the "steady state values" of the true underlying dynamic game. Thus, it is lacking the explicit motion of the strategic variables over time and its accompanying comparative dynamics. More importantly, the set of strategies available to the firms may be richer than in the corresponding static model. Also, the dynamic adjustment process is neglected in the n-stage competition games.

In order to contrast the standard static two-stage competition approach with its dynamic counterpart, we first construct a specific two-stage game and then build its ex-

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[4]For an alternative approach that focuses on the incumbent's absorptive capacity see Wiethaus (2005). Under certain plausible conditions, Wiethaus (2005) demonstrates that monopolist is able to retain its persistence by strategically investing in excess absorptive capacity.

plicit dynamic version. More specifically, the benchmark model is two-stage asymmetric duopoly game in which one firm (say, Firm 1) has strategic advantage in a form of prior (first stage) investment in R&D that leads to unit costs decrease while the second firm benefits from the R&D spillovers. In the second stage the two firms compete in quantities. Thus both firms are assumed to be initially sustainable in the market. We then construct an explicit dynamic counterpart of that game. To concentrate on the strategic aspects within the dynamic model we push the tactical decision (i.e. selecting the optimal quantity) in the background and deal with so called reduced form profit function indicating the firms flow profits as functions of unit costs. Unit costs of the firms serve as so-called "state variables" that are governed through the "control" variable, namely R&D expenditures. Another important feature here is that a passage from the two-stage model to a dynamic model requires the introduction of a specific adjustment parameter that captures the speed with which the R&D investments translate into the unit costs reduction (see, for example, Fersthman and Kamien (1987) and Stenbacka and Tombak (1993) for usage of a similar approach). This is at the same time more realistic due to the unavoidable time delay between the R&D investment and corresponding R&D output. The dynamic approach enables us also to study a behavior of the strategic variable over time and some of its comparative dynamic effects as well as the importance of the adjustment process that are missing in the simple two-stage framework. Finally, and most importantly, in an explicit dynamic model we can analyze how the optimal strategy of the firm that possesses strategic advantage may lead to the change in market structure over the time and thus create persistence of monopoly. This phenomenon is not possible in the static 2-stage game. In other words, the strategic advantage of the Firm 1 would enable it to exhibit pre-emptive behavior (or strategic predation) on its rival turning initial duopoly market structure into monopoly.

Our analysis provides the following new insights:

a) Strategic predation becomes ever more attractive strategy to pursue when the adoption of the new technology accelerates. More specifically, the parameter space in which strategic predation is optimal increases with the speed of adjustment parameter and soon becomes dominant part of this region. The intuition is that after initial period (up to time T), the firm might be willing even to incur losses in order to enjoy monopoly profit till the end of time. Thus, unlike in a static game, in a fully dynamic model the costs of predation last only for a limited period and have to be contrasted to the infinite stream of the monopoly profit afterwards. As a conse-

4

quence, Firm 1 displays more aggressive behavior compared with its behavior in the two-stage game. Moreover, the innovative effort, and output are usually bigger (and price lower) compared to the situation in which the leading firm adopts accommodation strategy. This in turn results in a larger generated social welfare in monopoly than in related duopoly setup. However, for the comparison of these two strategies (accommodation versus strategic predation) to be possible, both strategies have to be initially feasible, and that in turn requires the level of R&D spillovers not to be "too large". Thus, our model generates results that are consistent with recent empirical findings reported in the "Economic Focus" (*The Economist*, May 2004). Etro (2004) developed an alternative model that is also consistent with the above stylized facts. In his model, the persistence of monopoly requires a large number of potential entrants.

b) The underling dynamic optimization problem in the case of strategic predation is rather different than in the case of duopoly where firms maximize their discounted profit over infinite time horizon. In the case of the strategic predation Firm 1 aims to minimize the time that leads to the expulsion of Firm 2 from the market. The idea here is that Firm 1 may even bear temporary losses in order to enjoy the monopoly position later on. Imposing an upper bound of the sustainable strategic losses that Firm 1 is willing to sacrifice over period from zero till T, suffices to determine the lower bound on the minimum time to force the exit. This approach formalizes the "long purse" story.

c) The time pattern of the R&D investment crucially depends of the equilibrium strategy: if accommodation is an optimal strategy, then Firm 1 commits to the R&D path which steadily increases over time towards the unique steady state value. When on the other hand the strategic predation is an optimal strategy, the time profile of R&D is reversed: that is, the shorter the target time, T, at which Firm 2 is forced to exit, the higher the "predatory" level of R&D investment has to be. In other words, the level of optimal R&D investment decreases with the increase in the target time. (Note that to force an immediate exit of Firm 2 is not viable since it would require an infinite amount of R&D, if the speed of adjustment is finite.)

d) As a finding of an independent interest, we show that the steady state values of the R&D investment in a dynamic model can be interpreted as a generalized values of the equilibrium values obtained in the two-stage approach. If the adjustment is

instantaneous (meaning that there is no time delay between the R&D input and its output) as implicitly assumed in a two stage game, or the rate of time preference (or interest rate) is neglected, then these two sets of values coincide.

2   The Two-Stage Competition

The basic static model is a two-stage game (see, for instance, Žigič 1998, 2000). In the first stage, Firm 1 chooses its R&D expenditure, x, that, at the same time, represents the level of R&D investment. In the second stage, the firms compete in quantities. Firm 1 has unit costs of production (c_1):

$$c_1 = c_0 - \sqrt{gx}, \qquad x \leq \frac{c_0^2}{g}, \tag{1}$$

where the parameters g and c_0 describe the efficiency of the R&D process and pre-innovative unit costs, respectively. The expression $\sqrt{gx}$ is an "R&D production function", where, as in Chin and Grossman (1990), $g \in (0, 4)$ (the upper bound of this interval is determined by the required positivity of the monopoly output).

Firm 2 benefits through spillovers from the R&D activity carried out by Firm 1. Its unit cost function is

$$c_2 = c_0 - \beta\sqrt{gx}, \qquad \beta \in [0, 1], \tag{2}$$

where β denotes the level of spillovers (which, say, reflects the strength of intellectual property rights (IPR) protection).

We assume the linear inverse demand function: $P = A - Q$. The parameter A captures the size of the market (where $A > c_0$), variables q_1 and q_2 denote the quantities of the two firms' production, and $Q \equiv q_1 + q_2$ represents the aggregate supply.

In the second stage, given Firm's 1 R&D investment, the two firms engage in Cournot-Nash competition. Firm 1 maximizes profit net of the R&D expenditures. The first-order condition for a maximum yields $A - 2q_1 - q_2 - c_1 = 0$. The optimization problem for the firm 2 is similar yielding the analogous first-order condition: $A - 2q_2 - q_1 - c_2 = 0$. Solving the "reaction functions" yields the Cournot outputs and price as functions of R&D investment:

$$q_1(x) = \frac{A - 2c_1 + c_2}{3}, \qquad q_2(x) = \frac{A + c_1 - 2c_2}{3}, \qquad P(x) = \frac{A + c_1 + c_2}{3}. \tag{3}$$

Substituting expressions (3) into the profit function yields Firm's 1 profit function expressed in terms of R&D investment:

$$\pi_1(x) = \frac{(A - 2c_1 + c_2)^2}{9} - x = q_1^2 - x. \tag{4}$$

6

In the first stage of the game, Firm 1 selects x to maximize its profit. By substituting expressions (1) and (2) for c_1 and c_2 into (4) and maximizing with respect to R&D investment,[5] we obtain:

$$x^* = \frac{(A - c_0)^2 (2 - \beta)^2 g}{(9 - (2 - \beta)^2 g)^2} \tag{5}$$

It is straightforward to check that Firm's 1 R&D effort decreases with an increase in spillovers, that is, $\frac{\partial x^*}{\partial \beta} < 0$.

Since spillovers are in general imperfect ($\beta < 1$), there is a critical value of R&D efficiency, g_d (leading to critical unit cost asymmetry between two firms, that in turn lead to zero profit of Firm 2), defined as a function of β by

$$g_d(\beta) = \frac{3}{(1 - \beta)(2 - \beta)}, \tag{6}$$

such that for $g > g_d$ duopoly ceases to exist (see Figure 1). Equivalently, for any given g, there exist a critical value β_d below which duopoly is not viable. This critical value is simply obtained by inverting (6).

When R&D efficiency exceeds g_d two possibilities may occur: unconstrained monopoly and monopoly constrained by the credible threat of entry by Firm 2 (or shortened "constrained monopoly"). To see this, let us look first at the optimal quantity, R&D expenditures and price if unconstrained monopoly emerges. Firm 1, which is now assumed to be a monopolist, maximizes

$$\max \pi_m = (A - q_m) q_m - c_1 q_m - x. \tag{7}$$

The first-order condition for a maximum yields $A - 2q_m - c_1 = 0$. Solving for q_m and substituting in (7) yields $\pi_m(x)$. Substituting expression (1) for c_1 into the $\pi_m(x)$ and maximizing with respect to the R&D investment (x), we obtain

$$x_m = \frac{(A - c_0)^2 g}{(4 - g)^2} \tag{8}$$

with the corresponding price as:

$$p_m = \frac{A(2 - g) + 2c_0}{4 - g} \tag{9}$$

(note that spillovers play no role in the case of monopoly).

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[5]We assume that c_0 is sufficiently large in all cases so that the non-negativity constraint on c_1 does not bind. The second-order condition is satisfied for all permissible values of parameters, so the optimal expenditure, x^* is always positive.

To find the parameters values which allow for pure monopoly to exist, we have to evaluate the reduced profit function of Firm 2, that is, $\pi_2^*(x)$, at Firm 1 optimal R&D investment level expressed in terms of parameters and determining the region of parameters that leads to $\pi_2^*(x_m) \leq 0$. Equivalently, for Firm 1 to acquire an unconstrained monopoly position, it is necessary that $p_m \leq c_0 - \beta\sqrt{gx_m}$. Such a post-innovative situation is that in which "drastic innovation" takes place (see Tirole, 1990). By substituting for p_m and x_m in the above expression we obtain the critical efficiency, g_p, as a function of β:

$$g_p(\beta) = \frac{2}{1 - \beta} \tag{10}$$

such that for $g > g_p$ the equilibrium market form is unconstrained monopoly. The critical spillovers level below which Firm 1 gains unconstrained monopoly position is labelled as β_p.

However, if we compare this critical condition with the one required to sustain an asymmetric duopoly, we see that there is a region of parameters β and g where there is neither pure monopoly nor sustainable duopoly (the area between g_p and g_d in Figure 1).

Insert $\quad$ *Figure 1* $\quad$ HERE

If the degree of spillovers and the efficiency of cost reductions happen to be in this region, Firm 1 exhibits so called "strategic predation," (i.e. it chooses R&D expenditures in such a way as to cause $q_2^* = 0$ in equilibrium and thus induces the exit of Firm 2). Note that the efficiency parameter g in this situation is in the range of

$$\frac{3}{(1 - \beta)(2 - \beta)} \leq g \leq \frac{2}{1 - \beta} \tag{11}$$

whereas β stays below $\frac{1}{2}$. There are two useful corollaries resulting from the above discussion: Firm 1 can enjoy the monopoly position only if spillovers are "small" ($\beta < \frac{1}{2}$) and the R&D efficiency is rather high, more specifically, $g \geq g_p(\beta)$ has to hold. Second, since strategic predation is an option always available to Firm 1, this strategy is optimal only if spillovers and the R&D efficiency are in the region described by (11). Note that the region is rather small. Furthermore, the optimal R&D level is given by

$$x_p^* = \frac{(A - c_0)^2}{(1 - 2\beta)^2 g}, \tag{12}$$

where subscript p stands for predation. Notably, in the region of optimality of predation x_p^* increases in β.

3 The Dynamic Counterpart of the Static Model: The Case of Duopoly

3.1 Setting of the Problem

We first consider the setup in which both firms operate over an infinite time horizon. Firm 1 aims to determine its optimal R&D path that maximizes its discounted stream of profit (or, equivalently, its market value) over time. In doing so it takes into account the effect of R&D spillovers on its competitor's unit costs. As already mentioned in the introduction, it is convenient to analyze this issue by relying on the reduced form profit function that depends only on the firms' respective unit costs. The unit costs are in turn the function of the central strategic variable R&D investment. In order to build the genuine dynamic model, we assume that it takes time for R&D investment to transform into unit costs' decrease (otherwise the problem would be inherently static, see discussion in footnote 12). Thus, there is a "speed of adjustment" coefficient that captures the above mentioned time delay (more precisely, the inverse of it). In this respect our model closely follows that of Stenbacka and Tombak (1993) (see also Fershtman and Kamien (1987) for a similar approach).

Technically, the problem for Firm 1 is represented as an infinite horizon optimal control problem with two state and one control variable. More specifically, the problem is given by

$$\max_{x(t)} I(x(t)) = \int_0^{+\infty} [\Pi_1(c_1(t), c_2(t)) - x(t)]e^{-rt}dt, \tag{13}$$

$$\text{subject to}$$

$$(a) \quad \frac{dc_1}{dt} = \mu(c_0 - c_1(t) - \sqrt{gx(t)}),$$

$$(b) \quad \frac{dc_2}{dt} = \mu(c_0 - c_2(t) - \beta\sqrt{gx(t)}),$$

with initial conditions $c_1(0) = c_2(0) = c_0$, where $x(t)$ is the control variable (R&D expenditures), $c_1(t)$ and $c_2(t)$ are the state variables (costs of production).[6,7]

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[6]Subscript 2 refers to Firm 2; subscript 1 refers to Firm 1 and will be omitted in what follows.

[7]Our model can be easily adopted to capture the effect of R&D subsidization by introducing a new term, sx, in the objective function where $s \in [0, 1]$ is the subsidization rate:

$$\max_{x(t)} I(x(t)) = \int_0^{+\infty} [\Pi_1(c_1(t), c_2(t)) - x(t) + sx(t)]e^{-rt}dt.$$

Qualitatively, little will change in our analytical approach (developed further in Sections 3 and 4) though.

Note that the laws of motions (a) and (b) require sufficient perpetual investments in order to prevent the costs from increasing. If the investment is not sufficient, the costs tend to reverse back to its initial value c_0. In particular, when there is no investment in R&D, the costs will converge to c_0. This can be interpreted as some kind of depreciation of knowledge or skills.[8]

At each point in time, gross profit function (that is, R&D costs are not subtracted) is given by

$$\Pi_1(c_1, c_2) = (p - c_1)q_1 = (A - (q_1 + q_2) - c_1)q_1, \tag{14}$$

with $q_1(t)$ and $q_2(t)$ denoting quantities of good produced, while constants c_0 (pre-innovative unit costs) and $\beta \in [0, 1]$ (the level of spillovers) are equivalent to those in the static model. New parameter is $\mu > 0$ – the speed of adjustment, while g – the efficiency of R&D process – now belongs to the interval $(0, 4\rho)$ due to the requirement for monopoly output to be positive in the dynamic context (see Section 4.4 or Vinogradov and Žigič, 1999). The parameter ρ can be viewed as a so-called "generalized discount factor" and is defined in the following subsections. Symmetrically to Π_1, instantaneous gross profit function for Firm 2, Π_2 is defined as

$$\Pi_2(c_1, c_2) = (p - c_2)q_2 = (A - (q_1 + q_2) - c_2)q_2.$$

If Π_i are maximized at each point in time, we can obtain q_1 and q_2 as functions of c_1 and c_2 from the first order conditions (which are the same as in the two-stage model):[9]

$$\frac{\partial \Pi_1}{\partial q_1} = A - 2q_1 - q_2 - c_1 = 0,$$
$$\frac{\partial \Pi_2}{\partial q_2} = A - q_1 - 2q_2 - c_2 = 0, \tag{15}$$

which yield the quantities

$$q_1 = \frac{1}{3}(A - 2c_1 + c_2),$$
$$q_2 = \frac{1}{3}(A + c_1 - 2c_2) \tag{16}$$

and price

$$p = A - (q_1 + q_2) = \frac{1}{3}(A + c_1 + c_2). \tag{17}$$

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[8]Then $\mu(c_0 - c_i(t))$ corresponds to the depreciation rate (with $i = 1, 2$).

[9]Here we implicitly assume that Firm 1 and Firm 2 form a duopoly irrespective of the value of x. However, later in Lemma 6 it will be shown that the duopoly is not necessarily sustainable.

On the other hand, given (15) we can also express c_i as functions of q_i:

$$c_1 = A - 2q_1 - q_2,$$
$$c_2 = A - q_1 - 2q_2. \tag{18}$$

Substituting (18) into (14), we get

$$
\begin{aligned}
\Pi_1 &= \left(\frac{1}{3}(A + c_1 + c_2) - c_1\right) \cdot \frac{1}{3}(A - 2c_1 + c_2) \\
&= \frac{1}{9}(A - 2c_1 + c_2)^2 \\
&= q_1^2, \tag{19}
\end{aligned}
$$

thus the linear transformation (16) brings the optimal Π_1 into symmetric diagonal quadratic form in the $(q_1 - q_2)$ plane.

Now we are in a position to re-formulate the initial problem. Applying transformation (16), state equations (13(a),(b)) read as follows:

$$
\begin{aligned}
\dot{q}_1 &= \frac{1}{3}(-2\dot{c}_1 + \dot{c}_2) \\
&= \mu(B - q_1 + \gamma_1\sqrt{x}), \tag{20} \\
\dot{q}_2 &= \frac{1}{3}(\dot{c}_1 - 2\dot{c}_2) \\
&= \mu(B - q_2 + \gamma_2\sqrt{x}), \tag{21}
\end{aligned}
$$

where $B = \frac{1}{3}(A - c_0)$, $\gamma_1 = \frac{1}{3}(2 - \beta)\sqrt{g}$, $\gamma_2 = \frac{1}{3}(2\beta - 1)\sqrt{g}$. Similarly, the price pattern can be described by the following differential equation

$$\dot{p} = \mu\left((B + c_0) + p(t) - (\gamma_1 + \gamma_2)\sqrt{gx}\right). \tag{22}$$

With state equations (20)–(21), the optimal control problem (13) loses one state dimension and takes the form

$$\max_{x(t)} I(x(t)) = \int_0^{+\infty} (q_1^2 - x(t))e^{-rt}dt, \tag{23}$$

$$\text{subject to}$$

$$\frac{dq_1}{dt} = \mu(B - q_1(t) + \gamma_1\sqrt{x(t)}). \tag{24}$$

3.2 Optimal Solution

3.2.1 The First Order Conditions

The Hamiltonian function associated with the above problem is set up as

$$\mathcal{H}(q_1, x, \lambda) = (q_1^2 - x(t))e^{-rt} + \lambda(t)\mu(B - q_1(t) + \gamma_1\sqrt{x(t)}). \tag{25}$$

11

The first-order conditions (from now on we drop the subscript 1 for convenience) are

$$\mathcal{H}_x = -e^{-rt} + \frac{\lambda\mu\gamma}{2\sqrt{x}} = 0, \tag{26}$$

$$\mathcal{H}_q = 2qe^{-rt} - \lambda\mu + \dot{\lambda} = 0, \tag{27}$$

and the transversality condition is

$$\lim_{t\to+\infty} \lambda(t)q(t) = 0. \tag{28}$$

Equation (26) relates λ to x, so we can eliminate λ and $\dot{\lambda}$ from the system (27), (24):

$$\lambda = \frac{2\sqrt{x}}{\gamma\mu}e^{-rt}, \qquad \dot{\lambda} = \frac{\dot{x}}{\gamma\mu\sqrt{x}}e^{-rt} - \frac{2r\sqrt{x}}{\gamma\mu}e^{-rt},$$

and thus equation (27) becomes

$$\dot{x} = 2(r + \mu)x - 2\gamma\mu q\sqrt{x}. \tag{29}$$

Equation (29) determines the dynamics of the optimal R&D path. Let us introduce the new control variable $z \equiv \sqrt{x}$. Since $\dot{x} = 2z\dot{z}$, the substitution $x = z^2$ linearizes the equation of motion of the control variable x (29); thus

$$\dot{z} = (r + \mu)z - \gamma\mu q. \tag{30}$$

Finally, the joint dynamics of the state and control variable is given by the following system of linear differential equations

$$\dot{z} = (r + \mu)z - \gamma\mu q, \tag{31}$$

$$\dot{q} = \mu(B + \gamma z - q), \tag{32}$$

which is investigated in detail in the remaining part of this section.

3.2.2 Existence of the Equilibrium (Steady State) in Duopoly

Let us define ρ as $\rho = \frac{r}{\mu} + 1$. The parameter ρ can be interpreted as a generalized discount factor, that is the interest rate corrected by the speed-of-adjustment coefficient: given r, the higher the level of μ is, the faster R&D investment materializes, and the more important the future becomes.

System (31), (32) has a unique equilibrium (steady state)

$$z^* = \frac{B\gamma\mu}{r + \mu(1 - \gamma^2)} = \frac{B\gamma}{\rho - \gamma^2} = \frac{(A - c_0)(2 - \beta)\sqrt{g}}{9\rho - g(2 - \beta)^2}, \qquad x^* = (z^*)^2, \tag{33}$$

$$q^* = \frac{B(r+\mu)}{r+\mu(1-\gamma^2)} = \frac{B\rho}{\rho-\gamma^2} = \frac{3(A-c_0)\rho}{9\rho - g(2-\beta)^2}. \tag{34}$$

Since $z(t)$ must be non-negative, the equilibrium may arise only in the positive quadrant (i.e., $z^* > 0$, $q^* > 0$). Therefore, the equilibrium exists if and only if $\gamma^2 < \rho$, or, equivalently, if and only if $r + \mu(1-\gamma^2) > 0$ or $(2-\beta)^2 < \frac{9\rho}{g}$, or $g < \frac{9\rho}{(2-\beta)^2}$.

It is straightforward to show that the values (33) and (34) coincide with the static equilibrium values in the corresponding two-stage game[10] if $\rho = 1$, what in turn requires that either discount rate is zero or that the impact of R&D investment is instantaneous (that is, the speed of adjustment is infinite, $\mu = \infty$). This is more general result than the one obtained by Kobayashy (2001) where in his somewhat different approach the steady state values collapses to the corresponding two-stage game equilibrium only if discount rate is zero.[11] Moreover, the steady-state value of investment monotonically increases with μ, since

$$\frac{\partial z^*}{\partial \mu} = \frac{B\gamma r}{(r+\mu(1-\gamma^2))^2} > 0.$$

Note that similar to the two-stage game the equilibrium values of R&D expenditures and the quantity of good produced by Firm 1 both are decreasing functions of spillovers β:

$$\begin{aligned}
\frac{\partial z^*}{\partial \beta} &= \frac{\partial}{\partial \beta}\left(\frac{(A-c_0)(2-\beta)\sqrt{g}}{9\rho - g(2-\beta)^2}\right) \\
&= \left(-\frac{(A-c_0)\sqrt{g}(9\rho + g(2-\beta)^2)}{(9\rho - g(2-\beta)^2)^2}\right) < 0, \qquad \text{thus } \frac{\partial x^*}{\partial \beta} = 2z^*\frac{\partial z^*}{\partial \beta} < 0,
\end{aligned}$$

$$\frac{\partial q^*}{\partial \beta} = \frac{\partial}{\partial \beta}\left(\frac{3(A-c_0)\rho}{9\rho - g(2-\beta)^2}\right) = \left(-\frac{3(A-c_0)\rho \cdot 2g(2-\beta)}{(9\rho - g(2-\beta)^2)^2}\right) < 0.$$

3.2.3   The Dynamics of R&D and Output

The existence of a unique equilibrium also implies the existence of an optimal path of R&D and output converging to this equilibrium (in particular, the existence of the "optimal control" path for x), as we show below:

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[10]E.g., see equation (5) for equilibrium R&D in a static version of our model.

[11]Kobayashy (2001) made a differential game version of the D'Aspremont and Jacquemin (1988) two-stage game where the dynamics of the model stems from a depreciation of R&D stock rather than from the speed of adjustment.

Lemma 1 *If $\gamma^2 < \rho$ (i.e. $g < \frac{9\rho}{(2-\beta)^2}$) then there exists a unique optimal path of x, converging to the steady state.*

For proof of Lemma 1 see Appendix.

Let us assume now that in what follows the inequality $\gamma^2 < \rho$ always holds true. It can be re-arranged as

$$\beta > 2 - \frac{3\sqrt{\rho}}{\sqrt{g}} \tag{35}$$

and has two important implications on the existence of the optimal control path independent on γ:

Lemma 2 *For any γ, the optimal control always exists if either (i) $g < \frac{9}{4}\rho$, or (ii) $\mu < \frac{9}{7}r$.*

For proof of Lemma 2 see Appendix.

Next we determine the analytical solution to system (31, 32). Straightforward computations imply the following close-form solution for output and R&D (see Appendix for technical details):

$$q_{opt}(t) = -\frac{B\gamma^2}{\rho - \gamma^2} e^{\frac{1}{2}\left(\rho - 1 - \sqrt{(\rho+1)^2 - 4\gamma^2}\right)\mu t} + q^*, \tag{36}$$

$$z_{opt}(t) = -\frac{B\gamma^2}{\rho - \gamma^2} \cdot \frac{2\gamma}{\rho + 1 + \sqrt{(\rho + 1)^2 - 4\gamma^2}} e^{\frac{1}{2}\left(\rho - 1 - \sqrt{(\rho+1)^2 - 4\gamma^2}\right)\mu t} + z^*. \tag{37}$$

Then, the price pattern induced by investments z_{opt} is

$$p_{opt}(t) = \frac{B\gamma(\gamma + \gamma_2)}{\rho - \gamma^2} e^{\frac{\rho - 1 - \sqrt{(\rho+1)^2 - 4\gamma^2}}{2}\mu t} + p^*, \tag{38}$$

where

$$p^* = (B + c_0) - (\gamma + \gamma_2)\sqrt{g}z^* = A - (A - c_0)\frac{6\rho - g(2 - \beta)(1 - \beta)}{9\rho - g(2 - \beta)^2}. \tag{39}$$

Note that if an adjustment takes place instantaneously ($\mu = \infty$), then

$$q(t) \equiv q^*, \qquad z(t) \equiv z^*,$$

as predicted by the static model.[12] In this case, Firm 1's maximal profit is

$$\int_0^\infty [(q^*)^2 - (z^*)^2]e^{-rt}dt = \frac{(q^*)^2 - (z^*)^2}{r} = \frac{(A - c_0)^2}{(9 - g(2 - \beta)^2)r}. \tag{40}$$

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[12]Alternatively, we can find the optimal solution under instantaneous adjustment scenario by applying so-called Euler's equation: See Appendix for details of derivation.

Also note that the optimal R&D investment monotonically increases and the price monotonically decreases over time towards their steady state values.[13] As noted earlier, the steady-state value of investments is higher with higher values of μ and coincides with its static counterpart when $\mu \to \infty$. In this case, the adjustment becomes instantaneous, and the speed of convergence (as measured by the absolute value of the exponent in (36), (37)) monotonically increases.[14] The rationale is that a higher rate of transformation of R&D inputs into lower unit costs (higher μ) decreases the time gap between the R&D investment and its benefits expressed in terms of future profits. As a consequence, for higher μ, the convergence of the R&D investments towards the steady-state is faster. Figure 2 displays time paths of R&D levels, while Figure 3 plots the growth rates of R&D expenditures (the speed of convergence of R&D to the steady state values), for different μ.

Insert   *Figure 2*   HERE

Insert   *Figure 3*   HERE

3.3   Feasibility of Duopoly

Let us now address the issue of feasibility of a duopoly. In a duopoly scenario, quantities of goods produced by both firms must be strictly positive: $q(t) > 0$, $q_2(t) > 0$ for all $t \geq 0$. (Recall the initial conditions: $q(0) = q_2(0) = \frac{1}{3}(A - c_0) = B$.)

It can be easily shown, that Firm 1 always produces a positive quantity of goods:

Lemma 3 $q(t) > 0$ *for all* $t \geq 0$ *and all feasible* β, g.

For proof of Lemma 3 see Appendix.

For Firm 2 to operate, however, it is sufficient (but not necessary) that $\beta > \frac{1}{2}$. In other words, neither strategic predation nor unconstrained monopoly are viable in this β-region: given the upper bound of technological efficiency ($g < 4\rho$) high technological spillovers prevent the critical gap in unit costs for monopoly to occur.

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[13]Obviously, the exponent is negative, since $\rho > \gamma^2$.

[14]Note that due to the condition $\gamma^2 < \rho = 1 + \frac{r}{\mu}$, the inequality $\gamma < 1$ is necessary in order to have the set of feasible values of μ unbounded.

Lemma 4 *If $\beta \in (\frac{1}{2}, 1)$ then $q_2(t) > 0$ for all $t \geq 0$ (no matter what the value of g is).*

Proof:

Since $\gamma_2 > 0$ for $\beta \in (\frac{1}{2}, 1)$, the proof repeats that of Lemma 3.

Q.E.D.

If the steady state value of q_2 were non-positive, $q_2^* \leq 0$, it would indicate that Firm 2 can not compete with Firm 1 in the long run. Thus the inequality $q_2^* \leq 0$ implies that the duopoly is not sustainable in the long run and strategic predation may become an optimal strategy. In other words, if exists T such that $q_2(T) = 0$ then Firm 2 may not be capable of survival after T (T can be either finite or infinite). If Firm 1 eliminates Firm 2 at T, we say that Firm 1 exhibits strategic predation.

Let us define

$$g_d = \frac{3\rho}{(1-\beta)(2-\beta)}. \tag{41}$$

Technically speaking, the condition of non-sustainability of duopoly can be formalized as follows:

Lemma 5 *For q_2^* to be non-positive, it is necessary that $\beta < \frac{1}{2}$ and sufficient that $g \geq g_d$.*

For proof of Lemma 5 see Appendix.

From Lemma 4 and Lemma 5 we conclude that Firm 1 coexists with Firm 2 if $\beta > \frac{1}{2}$ and Firm 1 eliminates Firm 2 if $g \geq g_d$. Note that for $g \leq 4\rho$ these conditions cannot hold simultaneously, since $\beta > \frac{1}{2}$ implies $g_d > 4\rho$ (see also Figure 5). The border case $\beta = \frac{1}{2}$ requires an additional comment. If $\beta = \frac{1}{2}$ then Firm 2 always produces the fixed quantity (equal to B) of the good, with no respect to R&D activity performed by Firm 1. If $g < g_d$ and $\beta \leq \frac{1}{2}$ then constrained monopoly may emerge.

Summing up we obtain the following lemma.

Lemma 6 *Let $\beta \geq 0$. Then in the steady state the following statements hold:*

1. *The condition $\beta > \frac{1}{2}$ implies that duopoly is sustainable.*

2. *The condition $g \geq g_d$ implies that duopoly is not viable in the long run.*

16

The equality $g = g_d$ represents an upper boundary on values of g where duopoly is feasible. It is easy to observe that g_d decreases in μ and becomes $\frac{3}{(1-\beta)(2-\beta)}$ as $\mu \to \infty$. Interestingly, this is the same critical value as in the static model (see expression (6)).

Note also that condition $g \geq g_d$ can be rewritten as a condition for β, in the form $\beta \leq \frac{1}{2}\left(3 - \sqrt{1 + 12\frac{\rho}{g}}\right)$. Furthermore, observe that viability of duopoly implies the existence of the optimal control (but not vice versa) and that the range of parameters in which dynamic duopoly is viable is broader than in its static counterpart due to the fact the difference in unit cost of the two firms does not occur immediately and due to the fact that discount rate is in general positive.

4 Achieving Monopoly Position

4.1 Phase I: Strategic Predation

Instead of dealing with an infinite-horizon problem, let us consider a time-optimal problem with a horizontal terminal line. The objective of Firm 1 now is to reach the target of elimination of Firm 2 in the minimum amount of time and to gain a position of constrained monopolist. The time-optimal nature of the problem is conveyed by the objective functional $\max\limits_{x(t)} \int_0^T -1 dt$ subject to the following constraints and terminal conditions:

$$\dot{q} = \mu(B - q + \gamma\sqrt{x}), \quad q(t) \geq 0, \quad q(0) = B, \tag{42}$$

$$\dot{q}_2 = \mu(B - q_2 + \gamma_2\sqrt{x}), \quad q_2(0) = B, \quad q_2(T) = 0, \quad T \text{ is free}, \tag{43}$$

given that the optimal control x lies within the interval $[0, x^u]$ (the value of x^u is to be computed later; this is the so-called big-bang control).

Since Lemma 3 implies that $q(t) > 0$ for all t, and $q(0) = q_2(0) = B$, the constraints in (42) do not bind. Therefore, on substituting z for $\sqrt{x}$, the time-optimal problem reads as follows:

$$\max_{z(t)} \int_0^T -1 dt \tag{44}$$

subject to $\dot{q}_2 = \mu(B - q_2 + \gamma_2 z)$,

$q_2(0) = B, \; q_2(T) = 0, \; T \text{ is free, and } z \in [0, z^u]$.

The Hamiltonian associated with this problem is

$$\mathcal{H} = -1 + \lambda(\mu(B - q_2 + \gamma_2 z)).$$

17

If $\lambda > 0$ then the optimal $z^o = z^u$, if $\lambda < 0$ then $z^o = 0$. The equation of motion is

$$\dot{\lambda} = -\frac{\partial \mathcal{H}}{\partial q_2} = \mu \lambda,$$

which integrates to $\lambda(t) = \kappa e^{\mu t}$, κ is an arbitrary constant.

The transversality condition reads as $[\mathcal{H}]_{t=T} = 0$, i.e.

$$-1 + \lambda(\mu(B + \gamma_2 z^o)) = 0. \tag{45}$$

If $\lambda < 0$ then $z^o = 0$ and (45) has no solution. Therefore, $\lambda > 0$, which yields $\kappa > 0$ and $z^o = z^u$.

With $z^o = z^u$ for all t we can express the equation of motion of the state variable as

$$\dot{q}_2 + \mu q_2 = \mu(B + \gamma_2 z^u),$$

which has the solution

$$q_2^o(t) = (q_2^o(0) - B - \gamma_2 z^u)e^{-\mu t} + B + \gamma_2 z^u = B + \gamma_2 z^u(1 - e^{-\mu t}). \tag{46}$$

Therefore, given $q_2^o(T) = 0$ we can find[15] T from $B + \gamma_2 z^u(1 - e^{-\mu T}) = 0$, i.e.

$$T = -\frac{1}{\mu} \ln\left(\frac{B + \gamma_2 z^u}{\gamma_2 z^u}\right) \tag{47}$$

Note that the positivity of T demands for the negativity of both the numerator and the denominator: $B + \gamma_2 z^u < 0$ and $\gamma_2 z^u < 0$. Again, this observation is consistent with the fact that predation can only take place with $\beta < \frac{1}{2}$ and $q_2^* \leq 0$.

We can also invert (47) and express z^u as a function of T:

$$z^u(T) = -\frac{B}{(1 - e^{-\mu T})\gamma_2} = \frac{A - c_0}{(1 - 2\beta)\sqrt{g}(1 - e^{-\mu T})}. \tag{48}$$

Once $z^u(T)$ is known, it also becomes possible to evaluate optimal output in strategic predation from the equation

$$\dot{q} = \mu(B - q + \gamma z^u(T)) \tag{49}$$

Equation (49) has a close-form solution (recall the initial condition $q(0) = B$):

$$q(t) = \gamma z^u(T)(1 - e^{-\mu t}) + B = \frac{A - c_0}{3}\left(1 + \frac{2 - \beta}{1 - 2\beta} \cdot \frac{1 - e^{-\mu t}}{1 - e^{-\mu T}}\right). \tag{50}$$

---

[15]For an alternative approach to the optimal predation problem, see Appendix.

The output of Firm 2 can be obtained after substitution of (48) into (46):

$$q_2(t) = \frac{A - c_0}{3} \cdot \frac{1 - e^{-\mu(T-t)}}{1 - e^{-\mu T}}.$$

Interestingly this pattern is independent on parameters β and g. However, the optimal predation time might depend on them (see part 5.2).

In particular, at time $t = T$, when Firm 2 is eliminated, we get

$$q(T) = (A - c_0)\frac{1 - \beta}{1 - 2\beta} \tag{51}$$

and $q_2(T) = 0$. Note that $q(T)$ is positive if and only if $\beta < \frac{1}{2}$. Hence only $\beta < \frac{1}{2}$ allows for predation. On the other hand, if $\beta \geq \frac{1}{2}$, the only feasible market structure is duopoly (see also Lemma 6).

With $q(T)$ given by (51), the firms' costs at time T are $c_1(T) = A - 2q(T)$ and $c_2(T) = A - q(T)$. Interestingly, all these values do not depend on time T. Hence, no matter at which time Firm 2 is eliminated, Firm 1 has the same starting point[16] as (constrained) monopolist.

Note that the optimal time profile of R&D investments is declining with T. The shorter the desired time for the elimination of the competitor is, the larger R&D investment should be. This functional relationship between z^u and T can be depicted as in Figure 4. This result deserves a special emphasis, for on the one hand (48) enables us to find the optimal level of R&D expenditures needed to eliminate Firm 2 after any given point in time T, and on the other hand it establishes the relationship between the static model and its dynamic counterpart, proving once again that the static model is in a sense just a limiting case of the dynamic one.

$$\boxed{\text{Insert} \quad \textit{Figure 4} \quad \text{HERE}}$$

In particular, if the speed of adjustment is unlimited ($\mu \to \infty$), then for any T, z^u converges to the optimal value of R&D predicted by the static model (cf. Žigič, 1998):

$$x_p^* = \lim_{\mu \to \infty} (z^u)^2 = \frac{(A - c_0)^2}{(1 - 2\beta)^2 g}.$$

Finally, note that, contrary to the case of duopoly, we have now $dz^u/d\mu < 0$. That is, the quicker the speed at which the R&D investment materializes in the unit cost reduction, the lower the predatory expenditures that lead to expulsion of Firm 2.

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[16]I.e., the unit costs and quantities of both firms are the same irrespectively of T.

19

The above analysis can be read as formalization of the "long-purse" story (see, for instance, Sherrer and Ross, 1990, and Tirole, 1990) in a distinctive way since the result is not based on the information asymmetry nor imperfections on the financial markets (cf. Tirole (1990), Telser (1966), Bolton and Scharfstein (1990), etc.).

In order for strategic predation to be feasible (in our model), it is necessary that $c_1(T) \geq 0$, which means that the costs do not become negative in the predation phase. This condition can be equivalently formulated as

$$\beta < \bar{\beta} \equiv 1 - \frac{A}{2c_0}. \tag{52}$$

The above restriction is also supported empirically by Griliches (1992) who, in his summary of the empirical work on R&D spillovers, finds that typical values of β range between 0.2 and 0.4. Note that $\bar{\beta} < \frac{1}{2}$ due to condition $A > c_0$.

4.2 Constrained and Unconstrained Monopoly

Suppose that Firm 2 is eliminated at time $T > 0$. At this point, Firm 2's unit costs become equal to the equilibrium price. Since then, it will not be active in the market, if Firm 1 sets price not exceeding Firm 2's unit costs. Obviously, Firm 1 will either choose price equal to Firm 2's current unit cost c_2 or the (unconstrained) monopoly price

$$p_m = \frac{A + c_1}{2}. \tag{53}$$

This price comes from maximization of gross instantaneous profit $\Pi_1 = (p - c_1)(A - p)$ and correspond to setting the monopoly quantity $q_m = \frac{1}{2}(A - c_1)$. Note that q_m is actually best response to $q_2 = 0$.

If $p_m < c_2$, then Firm 1 is free to set the monopoly price p_m which does not allow Firm 2 to re-enter the market. In this case we say that Firm 1 is an *unconstrained monopolist*. On the other hand, if $p_m \geq c_2$, Firm 1 can set c_2 as the highest price that will keep Firm 2 out of the market.[17] In this case, we say that Firm 1 is a *constrained monopolist*.

We will assume that, after time T, Firm 2's costs $c_2(t)$ follow the standard equation of motion $\dot{c}_2 = \mu(c_0 - c_2(t) - \beta\sqrt{gx(t)})$ with no spillovers available any further, that is, with $\beta = 0$, i.e.,

$$\dot{c}_2 = \mu(c_0 - c_2(t)). \tag{54}$$

---

[17]The profit of Firm 1 is increasing in p for $p \leq p_m$.

This reflects the fact, that an inactive firm does not acquire any new technology via spillovers and its costs behave as if there would be no spillovers, i.e., there is some depreciation of knowledge, skills, etc.[18] As a consequence, Firm 2's costs actually increase after time T.

Since c_2 is now independent on Firm 1's R&D investments, we can solve the equation (54) with the initial value $c_2(T) = c_{2T} = A - q(T) = A - (A - c_0)\frac{1-\beta}{1-2\beta}$. The solution can be written as

$$c_2(t) = Ke^{-\mu t} + c_0, \qquad \text{where } K = \frac{c_{2T} - c_0}{e^{-\mu T}} = -\frac{(A - c_0)\beta}{(1 - 2\beta)e^{-\mu T}}. \tag{55}$$

Recall that $c_2(t)$ is increasing over time and converges to c_0 as $T \to \infty$. Further note that at time T we have $c_2(T) = A - q(T) = \frac{1}{2}(A + c_1(T)) = p_m(T)$, i.e., the unconstrained and constrained monopoly prices are the same.[19] At this point, if Firm 1's R&D investments are high enough, so that the monopoly p_m price does not exceed c_2 (or equivalently its costs do not exceed $2c_2 - A$), may set price p_m and become unconstrained monopolist. On the other hand, if the R&D investments are low, the monopoly price p_m may be higher than c_2 and Firm 1 becomes a constrained monopolist.

4.3 Phase II: Constrained Monopoly Optimization Problem

If Firm 1 becomes a constrained monopolist, it sets price $p_{cm}(t) = c_2(t)$, corresponding to quantity $q_{cm}(t) = A - c_2(t)$. Then its instantaneous gross profit function is

$$\Pi_{cm} = (c_2(t) - c_{cm}(t))(A - c_2(t)).$$

This yields the following optimization problem

$$\max_{x(t)} I(x(t)) = \int_T^{+\infty} [(c_2(t) - c_{cm}(t))(A - c_2(t)) - x(t)]e^{-rt}dt, \tag{56}$$

subject to

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[18]There are several other specifications how the unit costs of Firm 2 change after time T. The first alternative would be that they do not change at all, i.e., $c_2(t) = c_2(T)$ for $t \geq T$. This can be interpreted as if the inactive firm is "frozen" and can enter the market in the same state as it exited in. Another plausible specification would be to consider $c_2(t)$ following after time T the same equation of motion as before (with unchanged spillovers), i.e., $\dot{c}_2(t) = \mu(c_0 - c_2(t) - \beta\sqrt{gx(t)})$. Such setting can be interpreted as if Firm 2 still acquires new technology via spillovers at the same rate as before and "waits" for a right time to enter the market again.

[19]Equivalently, $q_m(T) = q(T)$ since the monopoly quantity q_m is the same as the best response to rival's quantity 0.

$$\frac{dc_{cm}}{dt} = \mu(c_0 - c_{cm}(t) - \sqrt{gx(t)}),$$

$$\frac{dc_2}{dt} = \mu(c_0 - c_2(t)).$$

Note that the post-predatory price, which is $c_2(t)$, is now increasing over time. This is consistent with the empirical observation that the leader's price increases after the predation phase has been completed.

The solution to the above optimization problem is derived in the Appendix. As result we get

$$z_{cm}(t) = \frac{(A - c_0)\mu\sqrt{g}}{2}\left(\frac{1}{r + \mu} + \frac{\beta e^{-\mu(t-T)}}{(1 - 2\beta)(r + 2\mu)}\right). \tag{57}$$

Observe that $x_{cm}(t) = (z_{cm}(t))^2$ is monotonically decreasing over time and converging to $x^*_{cm} = \frac{(A - c_0)^2 g}{4(1 + r/\mu)^2}$ as $t \to \infty$.

4.4 Phase II: Unconstrained Monopoly Optimization Problem

If Firm 1 becomes an unconstrained monopolist, it sets the (unconstrained) monopoly price. This price is equal $p_m(t) = \frac{1}{2}(A + c_m(t))$ and corresponds to quantity $q_m(t) = \frac{1}{2}(A - c_m(t))$. These yield Firm 1's optimal instantaneous gross profit function

$$\Pi_m = (p_m(t) - c_m(t))q_m(t) = (q_m(t))^2.$$

This is to be maximized subject to the standard constraint $\frac{dc_m}{dt} = \mu(c_0 - c_m(t) - \sqrt{gx(t)})$. Using the above transformations, the optimization problem can be rewritten into a form identical to (23)–(24):

$$\max_{x(t)} I(x(t)) = \int_0^{+\infty} (q_m^2 - x(t))e^{-rt}dt, \tag{58}$$

subject to

$$\frac{dq_m}{dt} = \mu(B_m - q_m(t) + \gamma_m\sqrt{x(t)}), \tag{59}$$

with $B_m = \frac{1}{2}(A - c_0)$ and $\gamma_m = \frac{1}{2}\sqrt{g}$. However, the initial condition is $q_m(T) = q(T)$ in this case. Proceeding in the same way as in Section 3.2 we obtain a system of two differential equations analogous to (31)–(32). This yields the equilibrium values

$$z_m^* = \frac{B_m\gamma_m}{\rho - \gamma_m^2} = \frac{(A - c_0)\sqrt{g}}{4\rho - g},$$

$$q_m^* = \frac{B_m\rho}{\rho - \gamma_m^2} = \frac{2(A - c_0)\rho}{4\rho - g}.$$

Note that in order for the equilibrium values to be positive, it is necessary and sufficient that $g < 4\rho$. Under this condition we also have $g < (\rho + 1)^2$ and $\lambda_m < 0$. Note that the derivation of equilibrium values is independent on the initial conditions and even on the predation time T. Therefore, the same equilibrium values would be obtained if Firm 1 has a monopolistic position from the beginning (see Section 3.1 and Vinogradov and Žigič, 1999).

Finally, using the above initial condition, the optimal solutions are

$$q_m(t) = \frac{(A - c_0)(2\rho - g(1 - \beta))}{(1 - 2\beta)(4\rho - g)} e^{\lambda_m \mu(t-T)} + q_m^*, \tag{60}$$

$$z_m(t) = \frac{(A - c_0)(2\rho - g(1 - \beta))}{(1 - 2\beta)(4\rho - g)} \cdot \frac{\sqrt{g}}{\rho + 1 + \sqrt{(\rho+1)^2 - g}} e^{\lambda_m \mu(t-T)} + z_m^*, \tag{61}$$

where $\lambda_m = \frac{1}{2}\left(\rho - 1 - \sqrt{(\rho+1)^2 - g}\right)$. The resulting price can be then computed as $p_m(t) = A - q_m(t)$. Depending on the sign of $2\rho - g(1 - \beta)$, the R&D investment may be decreasing or increasing over time. Then, the monopoly quantity $q_m(t)$ is moving in the same direction, whereas the monopoly price $p_m(t)$ is moving in the opposite direction reflecting the fact that higher R&D investments lead to lower unit costs for Firm 1 and consequently to a lower the monopoly price.

4.5   Sustainability of Constrained and Unconstrained Monopoly

In order for unconstrained monopoly to be sustainable, it is necessary that Firm 2 does not re-enter the market after time T. This is the case when $p_m(t) < c_2(t)$. Note that the unconstrained monopoly optimization problem does not take at all into account the path of Firm 2's costs. In order to prevent Firm 2 from entering the market it is particularly necessary that the above inequality is satisfied for steady-state values, i.e., that $A - q_m \leq c_0$. This can be equivalently rewritten as

$$g \geq 2\rho. \tag{62}$$

Thus for $g \leq 2\rho$, Firm 1 by following the optimal path of investments gives Firm 2 a chance to re-enter the market sometimes after time T (or eventually at time T).

On the other hand, constrained monopoly is sustainable, when $p_{cm}(t) \leq p_m(t)$. Otherwise, it is profitable for Firm 1 to lower the price to the unconstrained monopoly level, which still prevents Firm 2 from entering the market. Again, it is necessary the above inequality is satisfied for steady-state values, i.e., $c_0 \leq \frac{1}{2}\left(A + c_0 - \sqrt{g x_{cm}^*}\right)$. After substituting the value of x_{cm}^* derived in Section 4.3, we obtain equivalent inequality $g \leq 2\rho$,

which is the reverse inequality as in (62). The following lemma summarizes the above results.

Lemma 7 *Assume that Firm 2 is eliminated at time T. Then:*

1. *For unconstrained monopoly to be sustainable, it is necessary that $g \geq 2\rho$.*

2. *For constrained monopoly to be sustainable, it is necessary that $g \leq 2\rho$.*

5 Accommodation *versus* Strategic Predation

5.1 Choosing Optimal Strategy

Finally, we are in a position to determine, what would be the best strategy for Firm 1. It can opt for one of the two basic strategies:

1. *Accommodation:* Optimize its duopoly profit over time.

2. *Strategic predation:* In the first stage minimize the time needed to eliminate Firm 2 for good (which incurs losses of profit) and then in the second round enjoy (constrained or unconstrained) monopoly position.

The comparison is straightforward: in both situations it is technically feasible to evaluate the overall profit and determine the optimal strategy. Note that for strategic predation the profit consists from two parts corresponding to the predation phase and constrained or unconstrained monopoly phase. However, depending on the underlying parameters some of the strategies might not be sustainable.

5.2 Assessing Optimal Predation Timing

Since the equilibrium and the optimal paths in duopoly are discussed in details in Section 3, we focus here on the second strategy of Firm 1. That is, the strategy that first aims to eliminate the competitor (we assume that the duopoly is initial market structure) and then Firm 1 enjoys the constrained or unconstrained monopoly position afterwards.

We start by evaluating the profit (loss) $I_p(T)$ associated with the first phase of the competition:

$$I_p(T) = \int_0^T [(q(t))^2 - (z^u(T))^2] e^{-rt} dt. \tag{63}$$

When Firm 1 attempts to gain the position of a monopolist by given time T by setting R&D expenditures at the (constant) level dictated by (48), and with $q(t)$ defined by (49). Substituting the closed form solution (50) together with (48) into the profit function (63), we arrive at the explicit expression for $I_p(T)$:

$$
\begin{aligned}
I_p(T) &= \int_0^T [(\gamma z^u(T)(1 - e^{-\mu t}) + B)^2 - (z^u(T))^2]e^{-rt}dt \\
&= (\gamma z^u(T))^2 \frac{1 - e^{-(2\mu+r)T}}{2\mu + r} - \\
&\quad -2\gamma z^u(T)(\gamma z^u(T) + B)\frac{1 - e^{-(\mu+r)T}}{\mu + r} + \\
&\quad +((\gamma z^u(T) + B)^2 - (z^u(T))^2)\frac{1 - e^{-rT}}{r}.
\end{aligned}
\tag{64}
$$

The resulting expression is rather complicated and will not be listed here. All details and results of computations (also for the following computations) can be obtained from authors upon request. Note that when the adjustment is instantaneous ($\mu \to \infty$), the optimal predation profit becomes

$$
\frac{(A - c_0)^2}{(1 - 2\beta)^2 rg}[(1 - \beta)^2 g - 1](1 - e^{-rT}).
\tag{65}
$$

In constrained monopoly optimization problem discussed in Section 4.3 the optimal R&D level was uniquely defined in equation (57). Thus the evolution of costs $c_{cm}(t)$ in program (56) can be explicitly derived from the equation $\dot{c}_{cm} = \mu\left(c_0 - c_{cm}(t) - \sqrt{gx_{cm}}\right)$, with $x_{cm} = z_{cm}^2$, as given by (57). Using the boundary condition $c_{cm}(T) = c_1(T)$, we can obtain a close-form solution and, together with c_2 as given by (55), substitute it into the profit function:

$$
I_{cm}(T) = \int_T^{+\infty} [(c_2(t) - c_{cm}(t))(A - c_2(t)) - x_{cm}(t)]e^{-rt}dt.
$$

This equation uniquely defines the optimal constrained monopoly profit as a function of time T when the position of a constrained monopolist is gained. Since $c_2(t)$, $z_{cm}(t)$, and also $c_{cm}(t)$ can be written as functions of $(t - T)$, the profit $I_{cm}(T)$ can be written in the form $X \cdot e^{-rT}$, with X being independent on T. This is a consequence of the fact that the initial values in the second phase (i.e., at time T) do not depend on T. Hence, the constrained monopoly profit depends on T only through the discount factor and X can be interpreted as the present value of all future profits at the time when Firm 2 is eliminated. Note that when the adjustment is instantaneous ($\mu \to \infty$), the optimal constrained monopoly profit becomes

$$
\frac{(A - c_0)^2 g}{4r} \cdot e^{-rT}.
\tag{66}
$$

Finally, in order to find optimal T (if any) that maximizes total profit of Firm 1 pursuing predation strategy we have to solve the following univariate unconstrained optimization problem:

$$\max_{T} \left[I_p(T) + I_{cm}(T) \right]. \tag{67}$$

The above maximization problem represents the trade-off between incurring high costs in order to eliminate Firm 2 early, or delaying high constrained monopoly profits when Firm 2 is eliminated later. Recall that in the predation phase the optimal R&D investments are decreasing in T. Hence, an early elimination (i.e., low T) requires significant R&D investments in the predation phase. This may even lead to instantaneous losses (i.e., the instantaneous profit is negative). These losses are compensated later when Firm 2 is eliminated. On the other hand, when elimination is delayed (i.e., T is high), Firm 1 invests less in R&D in the predation phase, but at the same time delays high profits earned in the constrained monopoly phase (as noted above the present value of those profits X is independent on T). The optimal value of T can be computed from the first order condition. However, the resulting equation (with T as unknown) is not solvable analytically, unless $\mu \to \infty$.

When $\mu \to \infty$, from (65) and (66) we obtain that the profit from predation strategy is decreasing in T whenever $g < 2$, i.e., constrained monopoly is sustainable[20] (see Appendix for details). Hence the optimal value of T is zero. This reflects the fact that when the adjustment is instantaneous, Firm 1 eliminates the rival immediately. In this case it has a profit given by (66) with $T = 0$.

Similarly we can proceed when Firm 1 becomes an unconstrained monopolist in the second phase. In this case the profit from the second phase is

$$I_m(T) = \int_T^\infty \left[(q_m(t))^2 - (z_m(t))^2 \right] e^{-rt} dt,$$

with $q_m(t)$ and $z_m(t)$ given by (60)–(61). Again, the above equation uniquely defines the optimal unconstrained monopoly profit as a function of time T when the position of an unconstrained monopolist is gained. Since $q_m(t)$ and $z_m(t)$ can be written as functions of $(t - T)$,[21] the profit $I_m(T)$ can again be written in the form $X \cdot e^{-rT}$, with X being independent on T. When the adjustment is instantaneous ($\mu \to \infty$), the optimal unconstrained monopoly profit becomes

$$\frac{(A - c_0)^2}{(4 - g)r} \cdot e^{-rT}. \tag{68}$$

---

[20]Note that $\rho \to 1$ as $\mu \to \infty$.

[21]In this case they are even linear functions of $e^{\lambda_m \mu (t - T)}$.

In order to find the optimal predation time T that maximizes total profit of Firm 1 pursuing predation strategy with unconstrained monopoly in the second phase, we have to solve the following univariate unconstrained optimization problem:

$$\max_T \left[I_p(T) + I_m(T) \right] \tag{69}$$

The first order condition for this problem is an equation with T as unknown which is not solvable analytically.

Again when $\mu \to \infty$, from (65) and (68) we obtain that the profit from predation strategy is decreasing in T (see Appendix for details). Hence the optimal value of T is zero, which reflects the fact that when the adjustment is instantaneous, Firm 1 eliminates the rival immediately. Then its profit is $\frac{(A-c_0)^2 g}{(4-g)r}$, as given by (68) with $T = 0$.

5.3 Long-run Optimality

In order to asses the optimal strategy for Firm 1, we now compare its two possible payoffs: accommodating strategy payoff (or duopoly payoff) defined by (13), and the payoff that arise from the dynamic strategic predation, with either constrained or unconstrained monopoly in the second phase. Lemma 7 implies that both constrained and unconstrained monopoly cannot be simultaneously sustainable (with exception of $g = 2\rho$, which defines a set of measure zero).

As $g > g_d$ yields clear-cut prediction of monopoly (Lemma 6), $\beta > \frac{1}{2}$ yields clear-cut prediction of duopoly (see Lemmas 4–6 and Section 4.1), we will further restrict our comparison of the accommodation and predation strategies to the region

$$\mathcal{R} = \{(\beta, g) \in \mathbb{R}^2 : 0 \leq \beta < \tfrac{1}{2}, \ 0 < g < g_d\}. \tag{70}$$

Further denote $\mathcal{R}_{cm} = \{(\beta, g) \in \mathcal{R} : g < 2\rho\}$ the region where only constrained monopoly can be sustained and $\mathcal{R}_m = \{(\beta, g) \in \mathcal{R} : g > 2\rho\}$ the region where only unconstrained monopoly can be sustained. We then need to compare profit from duopoly to the profit from strategic predation with constrained monopoly (given by problem (67)) in region $\mathcal{R}_{cm}$ and the profit from duopoly to the profit from strategic predation with unconstrained monopoly (which is given by problem (69)) in region $\mathcal{R}_m$. The regions are illustrated on Figure 5.[22] As indicated, the curve represents the equality $g = g_d$, which is the upper boundary for duopoly to be feasible. As mentioned in Section 3.3, this boundary shifts downwards with increasing μ. Thus as speed of adjustment increase,

---

[22]Regions $\mathcal{R}_{cm}$ and $\mathcal{R}_m$ are labelled as "CM/D" and "UM/D".

the region below, at which duopoly is feasible, shrinks. This is intuitive since a larger value of μ enables Firm 1 to attain cheaper and faster the critical unit cost difference that eventually may lead to monopoly. The horizontal line represents the equality $g = 2\rho$, which is the boundary between constrained and unconstrained monopoly. Again, this line shifts downwards as μ increases. Thus, unconstrained monopoly becomes easier to sustain compared to constrained monopoly (note that the upper boundary $g = 4\rho$ shifts downwards too). Note that the intersection of these boundaries, namely $g = 2\rho$ and $g = g_d$, is given by equation $\beta^2 - 3\beta + \frac{1}{2} = 0$, which implies $\beta = \beta_0 \equiv \frac{1}{2}(3 - \sqrt{7}) \approx 0.1771$.

$$\boxed{\text{Insert} \quad \textit{Figure 5} \quad \text{HERE}}$$

As mentioned in Section 4.1, our model of predation is feasible only if $\beta < \bar{\beta}$ as given by (52). However, note that in all cases the optimal values of q, z, and price margins (defined as the difference of price and unit costs) are homogeneous of degree 1 in (A, c_0).[23] Moreover, they can be written in form $(A - c_0) \cdot X$, where X is independent on both A and c_0. Therefore, in all cases the optimal profits, consumer surplus, social welfare, and present value of R&D investment are homogeneous of degree 2 in (A, c_0) and can be written in form $(A - c_0)^2 \cdot Y$, with Y being independent on both A and c_0. Hence, any comparison of those variables does not depend on A and c_0; it can depend only on β, g, μ and r.[24]

Using the simulation technique, we compare predation and accommodation in region $\mathcal{R}$ (recall that duopoly is feasible in this region). We find that when the speed of adjustment μ is small, accommodation is more profitable in almost everywhere in both regions $\mathcal{R}_{cm}$ and $\mathcal{R}_m$, as shown on Figure 6. However, with increasing speed of adjustment, predation (with either constrained or unconstrained monopoly in the second phase) becomes likely; see Figures 7, 8.[25]

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[23]In the paper we do not provide the exact formula only for Firm 1's price margin in constrained monopoly. All details about its computation and resulting expression can be obtained from authors upon request.

[24]Since $\bar{\beta}$ depends on A/c_0, in order to facilitate the comparison for all values of A and c_0, we disregard the feasibility condition $\beta < \bar{\beta}$ and consider $\frac{1}{2}$ as the upper bound for β (which is also the upper bound of $\bar{\beta}$).

[25]The simulations have been performed using the *Mathematica 5.0* software. The program code can be obtained from authors upon request. In all presented simulation results we used the values $r = 0.05$, $A = 1$, and $c_0 = 0.8$ (however, due to the discussion above, the results do not depend on values of A and c_0). Given the value of μ, Firm 1's profits from duopoly and strategic predation (for optimal T solving the problem (67) or (69)) were computed for values of (β, g) taken from a grid with density 0.0025×0.025

| Insert | *Figure 6* | HERE |

| Insert | *Figure 7* | HERE |

| Insert | *Figure 8* | HERE |

| Insert | *Figure 9* | HERE |

In Figures 7 and 8, the lower boundary represents the equality between Firm 1's profits from accommodation strategy (duopoly) and strategic predation (with constrained monopoly in region $\mathcal{R}_{cm}$ and unconstrained monopoly in region $\mathcal{R}_m$). In the region above the boundary (i.e., when R&D efficiency g is high), Firm 1 prefers strategic predation whereas in the region below, Firm 1 prefers the accommodation strategy. With increasing μ, this lower boundary shifts downwards as well, and apparently more than the upper boundary, resulting in ever increasing parameter space for which strategic predation is the optimal strategy. Figures 10 and 11 support this intuition.[26] The figures show the dependence of the area where strategic predation is optimal in regions $\mathcal{R}_{cm}$ and $\mathcal{R}_m$, respectively, on the speed of adjustment μ.

| Insert | *Figure 10* | HERE |

| Insert | *Figure 11* | HERE |

Figure 9 shows the set of parameters where predation is optimal, when adjustment is instantaneous (i.e., $\mu \to \infty$). This can be obtained by comparing the duopoly profit (40) with (66) and (68). As opposed to the previous cases, here it is possible to find an explicit formula for the lower boundary and compute the area where predation is optimal. We obtain that Firm 1 chooses strategic predation in the whole region $\mathcal{R}_m$. In region

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on the set $[0, \frac{1}{2}) \times (0, 4\rho)$. Figures 6, 7, and 8 show the results for values $\mu = 0.2$, $\mu = 2$, and $\mu = 20$ respectively.

[26]The simulations have again been performed using the *Mathematica 5.0* software, for the same values of parameters r, A, and c_0 as before. For each μ from the grid with density 0.2 on $[0.2, 25]$, we computed the area as integral of a piecewise linear function approximating the lower boundary (in 100 points with an absolute error lower than 10^{-4}).

$\mathcal{R}_{cm}$, Firm 1 chooses strategic predation in about 71% (see below). The lower bound is described by formula $g = g_\infty \equiv \frac{9+\sqrt{81-16(2-\beta)^2}}{2(2-\beta)^2}$, which can be obtained by equating profits (40) with (66).

It can be easily computed that with instantaneous speed of adjustment, region $\mathcal{R}_m$ has area 0.2640.[27] Figure 10 indicates that the area where predation is optimal converges towards this value as $\mu \to \infty$. Furthermore, the area of the whole region $\mathcal{R}_{cm}$ is 0.9524,[28] whereas predation is optimal in $\mathcal{R}_{cm}$ in a domain with area 0.6773.[29] The convergence is again indicated in Figure 11.

6 Welfare analysis

In the previous section we have found that Firm 1 prefers in most cases accommodation to predation when the speed of adjustment is small, but the relation becomes reversed when the speed of adjustment is large. In this section we analyze the welfare effects of predation. In particular, we are interested in comparison of consumers surplus and social welfare for strategic predation and accommodation. From the viewpoint of competition policy, strategic behavior is mostly prohibited since it eliminates the competing firm. However, from economic perspective, one needs to take into account the effects on the economy as whole. In particular, in case of predation, there may be significant gains from large R&D investments, which are often disregarded by competition policies. We therefore start the analysis by comparing the R&D investments.

6.1 R&D investment

Compared to the accommodation strategy, strategic predation requires a significantly higher investment in R&D in the first phase. This way Firm 1 rapidly decreases its own and due to spillovers also rival's costs. Therefore, the quantities the firms produce increase and the price decreases. However, as spillovers are imperfect, Firm 2's decrease in costs is less rapid and at time T its costs become equal to the equilibrium price. Firm 2 is not able to compete with Firm 1 further, but it remains as a threat. The shorter the targeted

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[27]The area of region $\mathcal{R}_m$ is $\int_{\beta_0}^{1/2} \left[\frac{3\rho}{(1-\beta)(2-\beta)} - 2\rho \right] d\beta = \left(2 - \sqrt{7} + 3\log(4 - \sqrt{7})\right)\rho.$

[28]The area of region $\mathcal{R}_{cm}$ is $\int_0^{\beta_0} \frac{3\rho}{(1-\beta)(2-\beta)} d\beta + \int_{\beta_0}^{1/2} 2\rho \, d\beta = \left(-2 + \sqrt{7} + 3\log\frac{4+\sqrt{7}}{6}\right)\rho.$

[29]This area can be computed as difference of the area of region $\mathcal{R}_{cm}$ and $\int_0^{1/2} g_\infty \, d\beta = \frac{1}{4}(3 - 4\sqrt{5} + \sqrt{17}) - 2\arcsin\frac{2}{3} + 2\arcsin\frac{8}{9}.$

predation time T is, the more rapid decrease in unit costs and hence the higher R&D investment is necessary.

As shown in Section 3.2.3, when Firm 1 follows the accommodation strategy, its optimal R&D investment continuously increases over time and converges to the value $x^* = (z^*)^2$, where z^* is given by (33). On the other hand, when Firm 1 follows strategic predation strategy, the R&D investment profile is rather different. In predation phase, the R&D investment is constant over time and is equal to $(z^u(T))^2$, whereas in the second phase, it is x_{cm} in constrained and x_m in unconstrained monopoly.[30]

Comparing the investment levels in predation phase to the level for accommodation, we obtain

$$\frac{z^u(T)}{z^*} = \frac{9\rho - g(2-\beta)^2}{g(2-\beta)(1-2\beta)} \cdot \frac{1}{1-e^{-\mu T}} > \frac{1}{1-e^{-\mu T}} > 1.$$

The inequality holds everywhere in region $\mathcal{R}$. As we can see, the R&D investment in predation phase is higher than the steady-state investment in duopoly. Moreover, the gap is larger when targeted elimination time T is smaller. Since the R&D investment in duopoly is increasing over time, then

$$z^u(T) > z_{opt}(t), \qquad \text{for } t \in (0, T]. \tag{71}$$

On the other hand, the relation between the investment in duopoly and constrained or unconstrained monopoly phase is ambiguous. A direct computation reveals that for $g \to g_d^-$ the steady-state of R&D in duopoly is higher than the ones in constrained and unconstrained monopoly (whatever is feasible). However, this relation becomes weaker, when g becomes smaller. In particular $z^* < z_{cm}^*$, when $g \to 0^+$.

As the investment level in the second phase of predation may be lower than the steady-state investment level in duopoly, we cannot make a clear-cut comparison of R&D investments between strategic predation and accommodation. Therefore, we compare their present values. The present value of R&D investments for the accommodation strategy, is equal to $\int_0^\infty x(t)e^{-rt}dt$. The present value of R&D investments for the predation strategy is defined analogously, but consists of two parts: the present value in the predation phase and the present value in the constrained or unconstrained monopoly phase. As the expressions are rather complicated (we omit them here), for comparison we used numerical simulations. We perform the simulations in the same way and for the same parameter values as we do for comparison of profits. The simulations show that for small values of

---

[30]Following previous notation, we define $z_{cm} = (x_{cm})^2$ and $z_m = (x_m)^2$.

μ, the present value of R&D investment is always (i.e., in region $\mathcal{R}$) larger when Firm 1 follows the predation strategy. This result is rather intuitive, since in the predation phase, Firm 1 makes huge R&D investments in order eliminate the rival. Moreover, the result conforms to the result by Etro (2004) that a leader may have greater R&D incentives in order to defend his position. However, as the speed of adjustment μ increases, the present value of R&D investments for accommodation strategy may be higher above the value in predation. This occurs in a small region near the upper boundary, where significant investments are profitable due to high R&D efficiency g; see Figures 12 and 13.[31] As we can see, this region does not increase significantly with increasing μ. Figure 14 shows this region when $\mu \to \infty$.[32]

| Insert | *Figure 12* | HERE |

| Insert | *Figure 13* | HERE |

| Insert | *Figure 14* | HERE |

6.2 Consumer surplus

Now we turn our attention to the effects on consumers. As already mentioned above, the intuition suggests that since investment in R&D decreases the cost of production, it increases the quantity produced and hence lowers the market price. Therefore, intuitively we can predict that the effect of higher investment in the predation phase on consumers should be positive. When Firm 1 follows the predation strategy, the R&D investment lowers the price rapidly in the first phase. If it becomes constrained monopolist in the second phase, the price will increase and converge to c_0. On the other hand, if Firm 1 becomes unconstrained monopolist in the second phase, the price will be monotone but may be both increasing or decreasing, depending on parameters. Figures 15 and 16 show the comparison of this time patterns to the price pattern for the accommodation strategy.

---

[31]The shaded area represent those values of parameters where the present value of R&D investments in accommodation is higher than the one in predation.

[32]When $\mu \to \infty$, the present values of R&D investments have rather simple forms: $\frac{(A-c_0)^2(2-\beta)^2 g}{[9-(2-\beta)^2 g]^2 r}$ for accommodation strategy, $\frac{(A-c_0)^2 g}{4r}$ for predation with constrained monopoly, and $\frac{(A-c_0)^2 g}{(4-g)^2 r}$ for predation with unconstrained monopoly (recall that the optimal predation time is $T = 0$ in this case). Figure 14 was obtained directly by comparing those values.

Insert   *Figure 15*   HERE

Insert   *Figure 16*   HERE

For a better exposition and in order to be able to evaluate the effects on social welfare, we compare the present value of consumer surplus. First consider the instantaneous consumer surplus which can be in any point in time t evaluated as

$$CS(t) = \int_{p(t)}^{A} (v - p(t))dv = \frac{1}{2}(A - p(t))^2 = \frac{1}{2}(q_1(t) + q_2(t))^2.$$

This is a classical form of consumer surplus for linear demand. Then the present value of the consumer surplus for accommodation strategy is $CS^* = \int_0^\infty CS(t)dt = \int_0^\infty (q_1(t) + q_2(t))^2 e^{-rt}dt$. The above formula shows that consumers surplus is negatively related to the price. According to our intuition, we might expect the consumer surplus to be higher, when Firm 1 follows the predation strategy.

Insert   *Figure 17*   HERE

Insert   *Figure 18*   HERE

Insert   *Figure 19*   HERE

Indeed, strategic predation yields a lower price than accommodation at any (positive) time in the predation phase. Moreover, if $g \leq 3\rho/(2 - \beta)$, then strategic predation also yields a lower price at any time in the constrained monopoly phase. The proofs of both these statements can be found in Appendix. As a consequence, if $g \leq 3\rho/(2-\beta)$, then the consumer surplus from strategic predation is higher that the one from accommodation. Conversely, if $g > 3\rho/(2 - \beta)$ then $p^* < c_0$, which means that the price in strategic predation raises the constrained monopoly price at some point in time. In this case we cannot make a direct inference about consumer surplus.

The above results suggest that a higher level of R&D investments in predation phase benefits consumers. It is necessary to point out that this is a consequence of how the predation works. Firm 1 needs to significantly lower the price so that it is equal to opponent's marginal costs, which in result benefits the consumers.

In order to complete the analysis, we again perform numerical simulations. They show that for low values of μ, consumer surplus from predation is always larger than the one from accommodation. However, similarly as for the R&D investments, as the speed of adjustment μ increases, there is a small region where consumer surplus from accommodation is higher, as shown on Figures 17 and 18.[33] When $\mu\infty$, this region covers a substantial part of region $\mathcal{R}_m$, but only a small part of region $\mathcal{R}_{cm}$, as shown on Figure 19.[34]

6.3   Social welfare

The instantaneous social welfare is defined as the sum of consumers surplus and firms' profits (by Firm 1's profits we mean the net profit, i.e., after subtracting the R&D costs). Drawing on previous results, it is not possible (with exception of small domains) to make a clear comparison of social welfare for accommodation and predation strategies.

Hence similarly as in previous cases, we compare the social welfare using simulation technique and perform the simulations in the same way and for the same parameter values as we did for comparison of profits. The results for different values of μ are shown on Figures 20, 21, and 22.[35] The result when $\mu \to \infty$ is shown on Figure 23 (obtained partially by numerical simulations).

The simulations show that the relation between the strategy chosen by Firm 1 and the socially optimal strategy is ambiguous. There are regions where Firm 1 chooses its strategy efficiently, i.e., where Firm 1's choice and social welfare are higher for the same strategy (accommodation or predation). However, there is a region, where accommodation is the more desirable outcome from social point of view, but Firm 1 prefers predation. This creates a need for appropriate anti-competitive remedies.

Surprisingly, there is also a region where Firm 1 chooses accommodation, but predation is more efficient. This region is described by either small spillovers or high efficiency of R&D and it shrinks with increasing speed of adjustment μ. Note that it disappears

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[33]The shaded area represent those values of parameters where consumer surplus in accommodation is higher than the one in predation.

[34]Again, when $\mu \to \infty$, the present values of consumer surpluses have rather simple forms: $\frac{(A-c_0)^2[6-(1-\beta)(2-\beta)g]^2}{2[9-(2-\beta)^2g]^2r}$ for accommodation strategy, $\frac{(A-c_0)^2}{2r}$ for predation with constrained monopoly, and $\frac{2(A-c_0)^2}{(4-g)^2r}$ for predation with unconstrained monopoly (again recall that the optimal predation time is $T = 0$ in this case). Figure 19 was obtained directly by comparing those values.

[35]The shaded area represent those values of parameters where social welfare in accommodation is higher than the one in predation.

34

when $\mu \to \infty$. However, this is an important remedy for competition policy. Although predatory behavior is considered as anticompetitive act, from economic point of view, it can lead to an increase in social welfare. The appropriate policy should then support Firm 1's predatory behavior (for example using subsidies; see also Footnote 7).

Insert   *Figure 20*   HERE

Insert   *Figure 21*   HERE

Insert   *Figure 22*   HERE

Insert   *Figure 23*   HERE

7   Conclusion

The empirical findings and stylized facts concerning the relation between innovativeness, leadership and market power, have motivated out paper in that we aim to describe and analyze a particular setup in which the persistence of monopoly is likely to arise in the long run. More specifically, we study the situation in which the market leader undertakes pre-emptive R&D investment ("strategic predation") that eventually leads to exit of the follower firm. It is also important to stress that we allow the follower to benefit via R&D spillovers from the R&D activity of the leader. We then, within the same basic setup, contrast the outcomes of strategy predation with the ones in which the leader "accommodates' the follower in a duopoly market structure. This comparison enables us to study positive aspects of the two main strategies of the leader — accommodation and strategic predation- as well as social welfare implications of the two resulting market structures — duopoly and constrained monopoly.

In studying the above phenomenon, we first start with static analysis and then subsequently develop a corresponding dynamic model. While the very comparison between the static model and its dynamic counterpart is an insightful exercise *per se*, we argue that, due to inherently dynamic phenomenon of innovation activity, a dynamic model is better suited at capturing both accommodating and the pre-emptive or predatory behavior of the leader, and consequently, it fits better with observed empirical findings about persistence of monopoly and its high intensity of innovative activity.

While monopoly appears marginal market structure in a static environment and is therefore often excluded from the analysis by assumptions (e.g., by restrictions on parameters), in the dynamic setup when there is a fast adoption of the new technology, on the contrary, this market structure becomes the prevalent one and strategic predation turns out to be the dominant market strategy in general. The quicker the time of the innovation adoption, the larger the range, in which the predation becomes the optimal strategy. Both R&D intensity and R&D stock are likely to be larger in predation strategy compared to accommodation strategy. Put together, these two facts yield a testable prediction in that the most propulsive innovative firms, that commercialize their investment in innovation quickly, are the ones likely to use the strategy predation through investing large sums of money into innovative activity (cf. AT&T, Microsoft, etc.).

As for the social welfare considerations, strategic predation as dominant market strategy may be socially preferable as well since it might lead to both higher consumer surplus, and (even more often) to higher social welfare generated despite the fact that only one firm (the leader) remains in the market in the long run. This all bears important competition policy implications. First, the size of market share per se might not be sufficient condition for a legal offence and, second, abuses of dominant positions may not even be an issue in dynamic markets where competition takes place through investments in R&D rather than through static pricing and where the very presence of competitors constrain the behavior of market leaders. The challenge for the design of antitrust policy against predation is related to the ability of the antitrust authority to distinguish between the price that is low for other predatory purposes from a price that might be set very low as part of an efficiency enhancing process that in turn results in enhanced competition leading in the end to the exit of the competitors but also in the enhancement in the both consumer surplus and social welfare. For instance, in the presence of network effects or learning effects it would be legitimate and consistent with vigorous competition that firms set very low prices when they are introducing new products, when they are targeting new customer segments or rivals, installed bases, or when they are in the first phase of the learning curve. Thus, the competition authority with limited knowledge of industry- and firm-specific data faces a complex problem when attempting to identify those circumstances under which loss-inducing predatory prices cause harm to competition. For that reason the antitrust authorities have to be fully aware of the risks of misclassification when approaching a predation case. Nevertheless, our model clearly favors still controversial proposition of the *efficiency defenses* of the dominant firms that allows for otherwise abusive strategy for the dominant firm if it creates a net efficiency gains which benefits

consumers.[36]

However, our results concerning both private and social optimality of strategic predation are obtained under the assumption of goods homogeneity. As we know (at least from Dixit, 1979), product differentiation makes the strategic predation more difficult and more costly for the leader. Moreover, strategic predation in this case leads to less product varieties in the market, and this in turn harms consumers. However, by the continuity argument, it is pretty safe to claim that all our findings would also hold in the situation when the degree of product differentiation is not "large," that is, when the goods are "close" substitutes.

Another policy concern that can arise in the above setup might be that the policy makers worry of having only one firm in the market as was recently the case with the General Electric and Honeywell banned merger. This could be an issue if there is no credible threat from entry of any other firm because the size of the entry barriers is high but this again leads to Etro's remark (2006) that the barriers to entry should be targeted rather than market leaders. In the technical sense, our analysis could be further extended in a several directions. The speed of adjustment could be "endogenized" as a function of the R&D intensity or R&D stock, for instance. Furthermore, we could model the last, quantity competition stage between the leader and follower explicitly by relying on the concepts of state dependent strategies and Markov perfect equilibria. This approach can make the game even more "dynamic" with possibly additional insights.

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[36]Etro (2006) gives a nice summery of the controversy. See also Rey et al. (2005), and the discussion paper on the reform of Art. 82 of the Treaty on exclusionary abuses by the European Commission, 2005.

A   APPENDIX: Proofs and Derivations

Proof of Lemma 1:

Introducing the new time scale $d\tau = \mu dt$, in matrix notation system (31,32) reads as

$$\frac{dY}{d\tau} = \Gamma Y + \Delta, \tag{72}$$

$$\text{where} \quad Y = \begin{pmatrix} z \\ q \end{pmatrix}, \quad \Gamma = \begin{pmatrix} \rho & -\gamma \\ \gamma & -1 \end{pmatrix}, \quad \Delta = \begin{pmatrix} 0 \\ B \end{pmatrix}.$$

$Tr(\Gamma) = \rho - 1 = \frac{r}{\mu} > 0$ – which implies that the sum of eigenvalues λ_1, λ_2 is always positive.

$Det(\Gamma) = -\rho + \gamma^2 < 0$ – which means that the product of the eigenvalues is negative.

$D = (Tr(\Gamma))^2 - 4Det(\Gamma) = (\rho + 1)^2 - 4\gamma^2 > (\rho + 1)^2 - 4\rho = (\rho - 1)^2 \geq 0$ – which yields that the eigenvalues are real.

Therefore, if $\gamma^2 < \rho$ then the eigenvalues are real and of opposite sign (i.e. the equilibrium is a saddle). The latter proves the existence of a unique trajectory converging to the steady state (a stable arm of a saddle).[37]

Q.E.D.

Proof of Lemma 2:

(i) Since β is always positive, (35) holds true when the right-hand side of (35) is negative, i.e. for $g < \frac{9}{4}\rho$.

Moreover, by construction the generalized discount factor $\rho \geq 1$. As μ (the speed of adjustment) tends to infinity, ρ monotonically declines to 1, and $\min \rho = 1$. Therefore, with $g < \frac{9}{4} \min \rho = \frac{9}{4}$ the RHS of (35) always remains negative for any constellation of β, μ and r.

(ii) Since $\max \gamma = \frac{4}{3}$, the condition $\mu < \frac{9}{7}r$ guarantees that $\rho = \frac{r}{\mu} + 1 > \frac{16}{9} = \max \gamma^2$. Therefore, for any γ, $\rho > \gamma^2$.

Q.E.D.

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[37]Loosely speaking, an equilibrium (z^*, q^*) is

1. unstable focus if $D < 0$ ($\lambda_{1,2}$ are complex) and $Tr(\Gamma) > 0$ ($Re(\lambda_{1,2}) > 0$); holds true for $\gamma > \frac{\rho+1}{2}$;

2. unstable node, if $D \geq 0$ ($\lambda_{1,2}$ are real) and $Det(\Gamma) \geq 0$ ($\lambda_1, \lambda_2 \geq 0$); holds true for $\sqrt{\rho} \leq \gamma \leq \frac{\rho+1}{2}$;

3. saddle, if $D \geq 0$ and $Det(\Gamma) < 0$ ($\lambda_1 < 0 < \lambda_2$); holds true for $\gamma^2 < \rho$.

However, in our model the first two options are ruled out due to non-negativity of z.

Derivation of the analytical solution to system (31, 32).

The eigenvalues of Γ are

$$\lambda_{1,2} = \frac{\rho - 1 \pm \sqrt{(\rho+1)^2 - 4\gamma^2}}{2} = \frac{1}{2\mu}\left(r \pm \sqrt{(r+2\mu)^2 - 4\gamma^2\mu^2}\right), \quad \lambda_1 < 0 < \lambda_2,$$

and the eigenvectors corresponding to λ_1 and λ_2 are evaluated as

$$U_1 = \begin{pmatrix} \frac{2\gamma}{\rho+1+\sqrt{(\rho+1)^2-4\gamma^2}} \\ 1 \end{pmatrix}, \qquad U_2 = \begin{pmatrix} \frac{2\gamma}{\rho+1-\sqrt{(\rho+1)^2-4\gamma^2}} \\ 1 \end{pmatrix}.$$

Therefore, the general solution to (72) becomes

$$Y(\tau) = C_1 U_1 e^{\lambda_1 \tau} + C_2 U_2 e^{\lambda_2 \tau} + Y_p,$$

where C_1, C_2 are arbitrary constants and the particular solution Y_p is a constant solution satisfying the equation $\Gamma Y_p + \Delta = 0$, i.e.

$$Y_p = \begin{pmatrix} z^* \\ q^* \end{pmatrix}.$$

The transversality condition demands for $C_2 = 0$ (in other words, the optimal solution must be bounded).

The constant C_1 is determined from the initial condition $q(0) = B$, which implies $C_1 = -\frac{B\gamma^2}{\rho - \gamma^2}$.

Finally, we find the optimal control path as

$$q_{opt}(\tau) = -\frac{B\gamma^2}{\rho - \gamma^2} e^{\lambda_1 \tau} + q^*,$$

$$z_{opt}(\tau) = -\frac{B\gamma^2}{\rho - \gamma^2} \cdot \frac{2\gamma}{\rho + 1 + \sqrt{(\rho+1)^2 - 4\gamma^2}} e^{\lambda_1 \tau} + z^*,$$

or, restoring the original time scale,

$$q_{opt}(t) = -\frac{B\gamma^2}{\rho - \gamma^2} e^{\frac{\rho - 1 - \sqrt{(\rho+1)^2 - 4\gamma^2}}{2}\mu t} + q^*,$$

$$z_{opt}(t) = -\frac{B\gamma^2}{\rho - \gamma^2} \cdot \frac{2\gamma}{\rho + 1 + \sqrt{(\rho+1)^2 - 4\gamma^2}} e^{\frac{\rho - 1 - \sqrt{(\rho+1)^2 - 4\gamma^2}}{2}\mu t} + z^*.$$

Q.E.D.

<u>Optimal solution to (23), (24) with $\mu \to \infty$ by applying Euler's equation.</u>

Under instantaneous adjustment scenario, as $\mu \to \infty$, equation (24) becomes

$$q(t) = B + \gamma z,$$

and the optimization problem (23) degenerates to

$$\max_{z^2(t)} \int_0^\infty ((B + \gamma z(t))^2 - z^2(t))e^{-rt}dt.$$

Euler's equation associated with this problem reduces simply to

$$\frac{\partial}{\partial z}((B + \gamma z(t))^2 - z^2(t))e^{-rt} = 0$$

which has to be satisfied at each point in time. It yields

$$z_{opt} = \frac{B\gamma}{1 - \gamma^2}.$$

Therefore, in the limiting case the optimal level of R&D expenditures is constant over time and coincides with the equilibrium value of z^* in (33) evaluated at $\mu \to \infty$.

<u>Proof of Lemma 3:</u>

$\dot{q} = \mu(B - q + \gamma z).$

Therefore $\dot{q} + \mu q = f(t)$ with $f(t) = \mu B + \mu \gamma z(t) > 0$ for all $t \geq 0$.

The solution to the latter equation is

$$q(t) = e^{-\mu t}\left(B + \int_0^t f(\tau)e^{\mu\tau}d\tau\right),$$

which is always positive.

Q.E.D.

<u>Proof of Lemma 5:</u>

Necessity follows from Lemma 4. To prove sufficiency, first note that $q_2^* = B + \gamma_2 z^*$. Thus non-positivity of q_2^* implies $B + \gamma_2 z^* \leq 0$, that is equivalent to

$$B \leq \frac{1}{3}(1 - 2\beta)\sqrt{g} \cdot \frac{B \cdot \frac{1}{3}(2 - \beta)\sqrt{g}}{\rho - \frac{1}{9}(2 - \beta)^2 g}.$$

The latter inequality simplifies to

$$1 \leq \frac{(1 - 2\beta)(2 - \beta)}{9\frac{\rho}{g} - (2 - \beta)^2}$$

40

and can be further re-arranged as

$$\frac{3\rho}{g} \le (2-\beta)(1-\beta), \quad \text{or } g \ge \frac{3\rho}{(2-\beta)(1-\beta)} \equiv g_d. \tag{73}$$

Given that $\rho \ge \gamma^2 = \frac{1}{9}(2-\beta)^2 g$, the latter inequality implies that

$$\frac{1}{9}(2-\beta)^2 g \cdot \frac{3}{g} < \frac{3\rho}{g} \le (2-\beta)(1-\beta).$$

Therefore

$$\frac{1}{3}(2-\beta)^2 < (2-\beta)(1-\beta),$$

i.e., $\beta < \frac{1}{2}$ is necessary for q_2^* to be non-positive.

Q.E.D.

An alternative approach to the optimal predation problem.

In order to eliminate Firm 2, Firm 1 must lower its price below the costs of production of Firm 2, i.e. the following condition must hold:

$$A - q_1(t) \le c_2(t).$$

In other words, at the very moment the price of Firm 1 falls below the costs of Firm 2, Firm 1 becomes a constrained monopolist.

Therefore, the corresponding optimization problem for Firm 1 is very similar to that elaborated earlier: the condition for becoming a constrained monopolist reads as

$$A - q_1(T) = c_2(T),$$

or, in other words,

$$q_1(T) + c_2(T) = A.$$

With two equations of motion of q_1 and c_2 in hand

$$\dot{q}_1 = \mu(B - q_1 + \gamma_1 z)$$

and

$$\dot{c}_2 = \mu(c_0 - c_2 - \beta\sqrt{g}z)$$

we can introduce the new variable (say, ξ), $\xi = q_1 + c_2$, which must satisfy the boundary conditions

$$\xi(0) = q_1(0) + c_2(0) = B + c_0, \qquad \xi(T) = A,$$

and is subject to the following equation of motion:

$$\dot{\xi} = \mu(B + c_0 - \xi + (\gamma_1 - \beta\sqrt{g})z).$$

Therefore – with a few modification – the optimization problem under consideration replicates the problem (44):

$$\max_{z(t)} \int_0^T -1 dt \tag{74}$$

s.t. $\quad \dot{\xi} = \mu(B + c_0 - \xi + (\gamma_1 - \beta\sqrt{g})z),$

$\qquad \xi(0) = B + c_0, \quad \xi(T) = A, \quad T \text{ is free},$

$\qquad\qquad \text{and } z \in [0, z^u].$

And there is little wonder that the solution of (74) will be identical to (47) and (48).

Derivation of optimal solution in constrained monopoly.

With $c_2(t)$ given by (55), the objective functional in (56) expands to the sum

$$\int_T^{+\infty} [c_{cm}(t)(c_2(t) - A) - x(t)]e^{-rt}dt + \int_T^{\infty} [c_2(t)(A - c_2(t)]e^{-rt}dt$$

in which the second term integrates to a constant, say, $\mathcal{K}$, that does not depend on the control variable and thus can be discarded. For that reason the optimization problem (56) reduces to

$$\max_{x(t)} I(x(t)) = \int_T^{+\infty} [c_{cm}(t)(Ke^{-\mu t} + c_0 - A) - x(t)]e^{-rt}dt, \tag{75}$$

$$\text{subject to}$$

$$\frac{dc_{cm}}{dt} = \mu(c_0 - c_{cm}(t) - \sqrt{gx(t)}).$$

In order to solve the above problem, we form Hamiltonian $\mathcal{H} = [c_{cm}(t)(Ke^{-\mu t} + c_0 - A) - x(t)]e^{-rt} + \lambda(t)\mu(c_0 - c_{cm}(t) - \sqrt{gx(t)})$ with first order conditions $\mathcal{H}_x = 0$ and $\mathcal{H}_{c_{cm}} = -\dot{\lambda}$ and transversality condition $\lim_{t\to\infty} \lambda(t)c_{cm}(t) = 0$. These yield the equation of motion of optimal control variable:

$$\dot{x}(t) = 2(r + \mu)x(t) + (Ke^{-\mu t} + c_0 - A)\mu\sqrt{g}\sqrt{x(t)}$$

or, in new variable $z(t) = \sqrt{x(t)}$,

$$\dot{z} = (r + \mu)z + \frac{\mu\sqrt{g}}{2}(Ke^{-\mu t} + c_0 - A) \tag{76}$$

Equation (76) has general solution

$$z_{cm}(t) = Ce^{(r+\mu)t} + \frac{(A - c_0)\mu\sqrt{g}}{2(r + \mu)} - \frac{K\mu\sqrt{g}}{2(r + 2\mu)}e^{-\mu t}.$$

42

The transversality condition demands for boundedness of the optimal solution. Therefore $C = 0$ and

$$z_{cm}(t) = \frac{(A - c_0)\mu\sqrt{g}}{2} \left(\frac{1}{r + \mu} + \frac{\beta e^{-\mu(t-T)}}{(1 - 2\beta)(r + 2\mu)} \right). \tag{77}$$

Derivation of optimal predation time when $\mu \to \infty$.

First note that $\rho \to 1$ as $\mu \to \infty$. Therefore, the feasibility condition $g < 4\rho$ becomes $g < 4$ and sustainability condition $g < 2\rho$ becomes $g < 2$.

In case of constrained monopoly, Firm 1's profit from strategic predation becomes

$$\frac{(A - c_0)^2}{4(1 - 2\beta)^2 rg} \left[4 \left((1 - \beta)^2 g - 1 \right) + \left((1 - 2\beta)^2 g^2 - 4(1 - \beta)^2 g + 4 \right) e^{-rT} \right],$$

when $\mu \to \infty$. The coefficient at e^{-rT} is positive if and only if

$$(1 - 2\beta)^2 g^2 - 4(1 - \beta)^2 g + 4 > 0.$$

We will show that this inequality holds for all $\beta \in [0, \frac{1}{2}]$ and $g \in [0, 2]$. Obviously it holds for $g \leq 1$, since then $(1-2\beta)^2 g^2 - 4(1-\beta)^2 g + 4 \geq -4(1-\beta)^2 + 4 \geq 0$. On the other hand, for $g > 1$, we rewrite the inequality in an equivalent form $(2-g)^2 + 4g(2-g)\beta - 4g(1-g)\beta^2 > 0$, which clearly holds when $1 < g < 2$.

With unconstrained monopoly in the second phase Firm 1's profit from strategic predation becomes

$$\frac{(A - c_0)^2}{(1 - 2\beta)^2 rg} \left[(1 - \beta)^2 g - 1 + \frac{(2 - (1 - \beta)g)^2}{4 - g} e^{-rT} \right]^2,$$

when $\mu \to \infty$. As $g < 4$, the coefficient at e^{-rT} is positive. Hence the profit is decreasing in T.

Comparison of prices in Section 6.2.

First we show that predation strategy yields a lower price in predation phase, i.e., for $t \in (0, T]$. Obviously, both prices are the same and equal to $\frac{1}{3}(A + 2c_0)$ at time $t = 0$. For $t > 0$, both are described by a differential equation of the form (22), or

$$\dot{p} = \mu \left((B + c_0) + p(t) - (\gamma_1 + \gamma_2)\sqrt{g}z(t) \right),$$

where $z(t) = z_{opt}(t)$ for the accommodation strategy and $z(t) = z^u(T)$ for predation strategy. According to (71), the latter is higher, yielding a lower price. This is easy do see, when we consider the difference d between the duopoly price and the predation price, which follows the differential equation $\dot{d} = \mu[d(t) - (\gamma_1 + \gamma_2)(z_{opt}(t) - z^u(T))]$,

43

with initial condition $d(0) = 0$. Obviously $d(t) > 0$, since $\gamma_1 + \gamma_2 = \frac{1}{3}(1 + \beta) > 0$ and $z_{opt}(t) - z^u(T) < 0$.

Now it remains to show that predation yields a lower price in the constrained monopoly phase. Since the price is decreasing in duopoly and increasing in constrained monopoly, it is sufficient to show the inequality for their limits, i.e., $p^* \geq c_0$. Substitution of (39) yields an equivalent form $g \leq 3\rho/(2 - \beta)$.

B APPENDIX: Figures
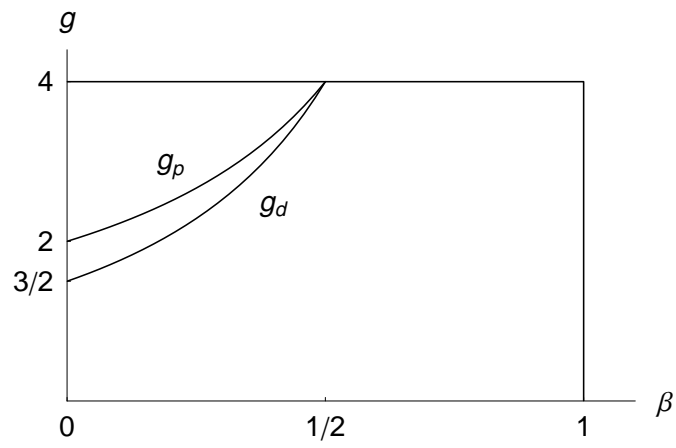
Figure 1: *Region of "Strategic Predation"*
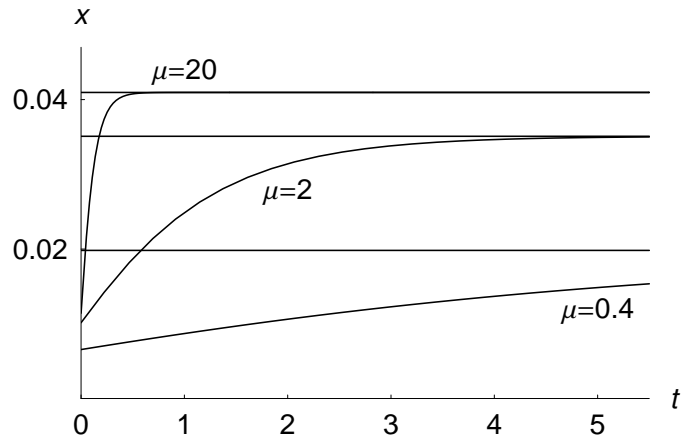
Figure 2: *R&D pattern for different values of μ: μ = 0.4, 2, 20*
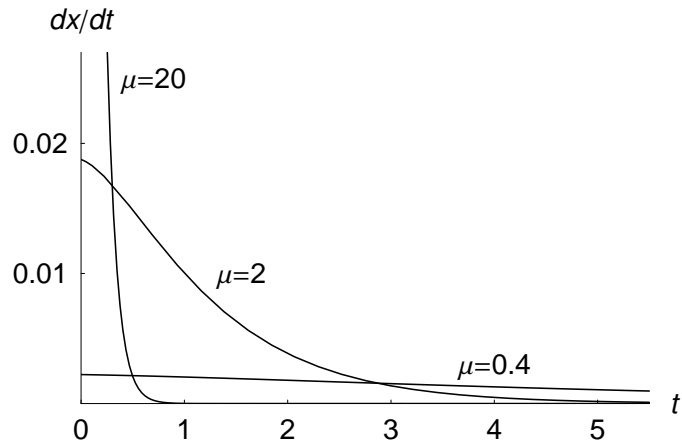
Figure 3: *Growth rates of R&D for different values of μ: μ = 0.4, 2, 20*
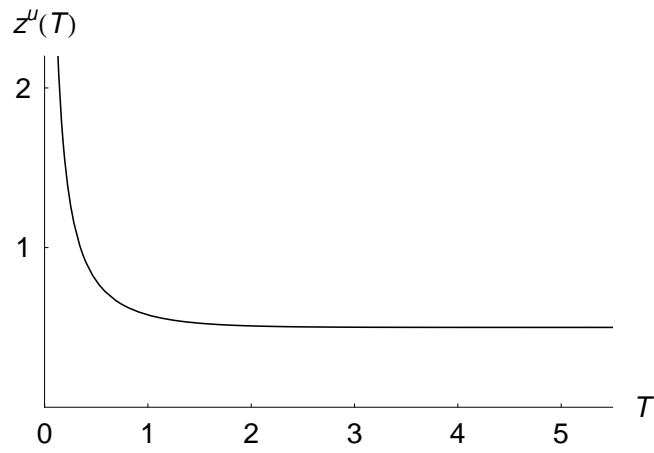
Figure 4: *The $z^u - T$ schedule*
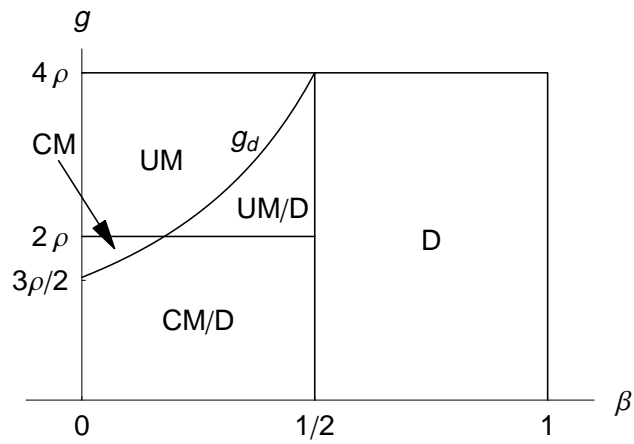
Figure 5: *Feasibility and sustainability regions*
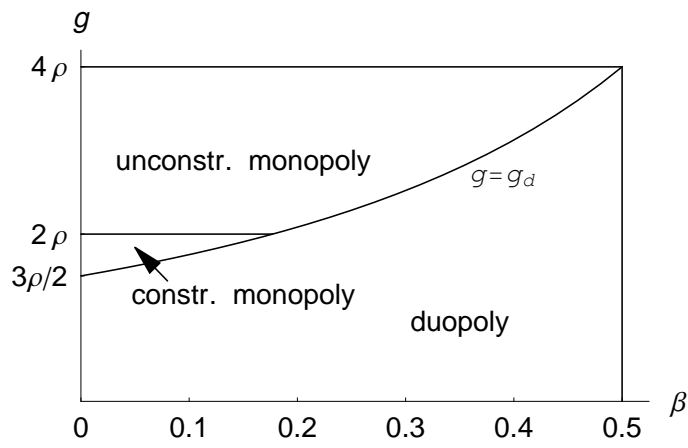
Figure 6: *Strategic predation (simulation results for $\mu = 0.2$)*
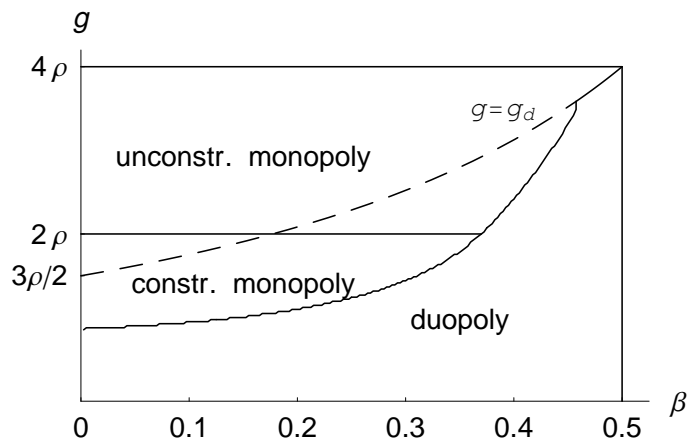
Figure 7: *Strategic predation (simulation results for $\mu = 2$)*
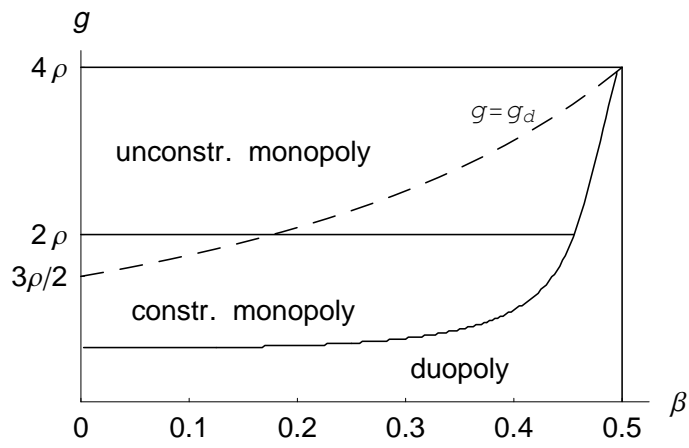
Figure 8: *Strategic predation (simulation results for $\mu = 20$)*
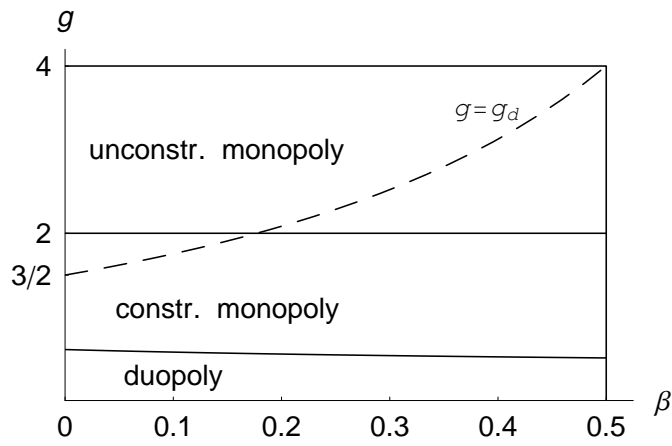
Figure 9: *Strategic predation when $\mu \to \infty$*

49
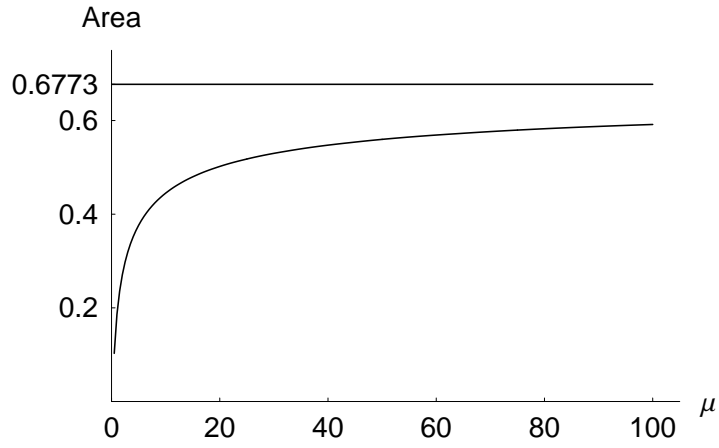
Figure 10: *Dependence of area where predation is preferred to duopoly on μ, in region* $\mathcal{R}_{cm}$ *(simulation results)*
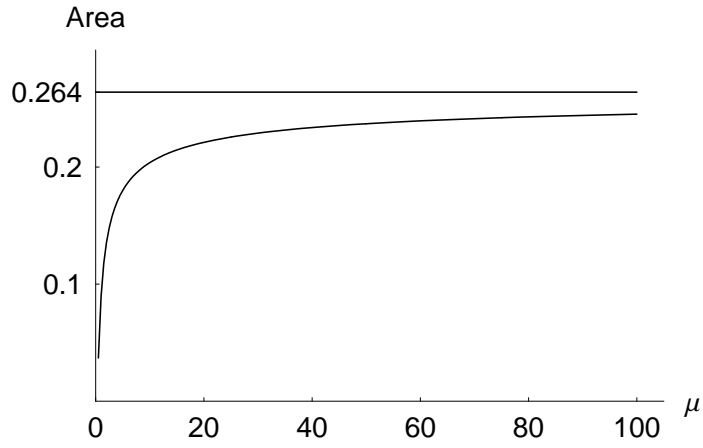
Figure 11: *Dependence of area where predation is preferred to duopoly on μ, in region* $\mathcal{R}_{m}$ *(simulation results)*
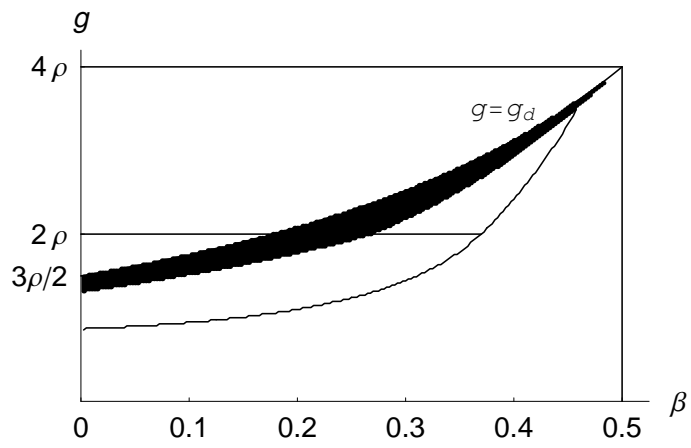
Figure 12: *R&D investment comparison (simulation results for $\mu = 2$)*
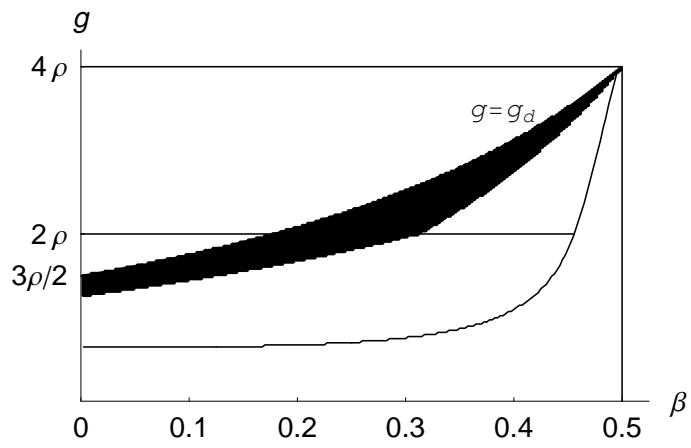
Figure 13: *R&D investment comparison (simulation results for $\mu = 20$)*

51
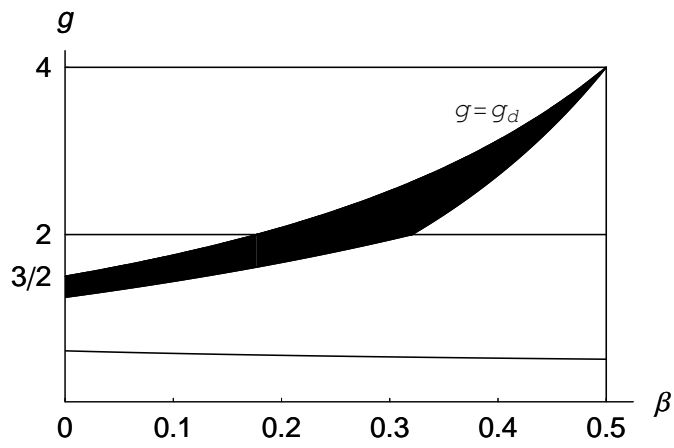
Figure 14: *R&D investment comparison when $\mu \to \infty$*
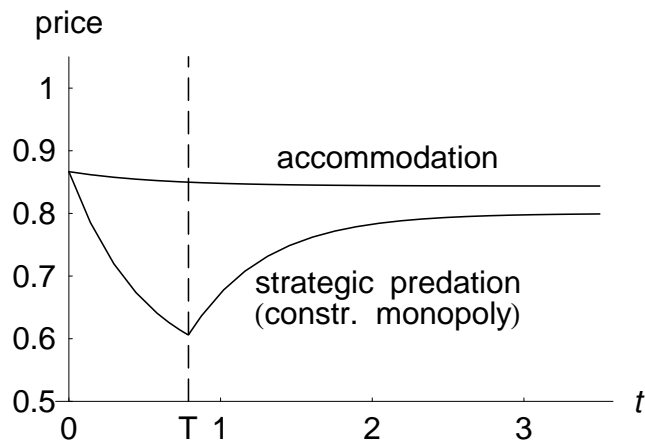
Figure 15: *Pattern of price over time with constrained monopoly ($\mu = 2$, $\beta = 0.33$, $g = 1$)*
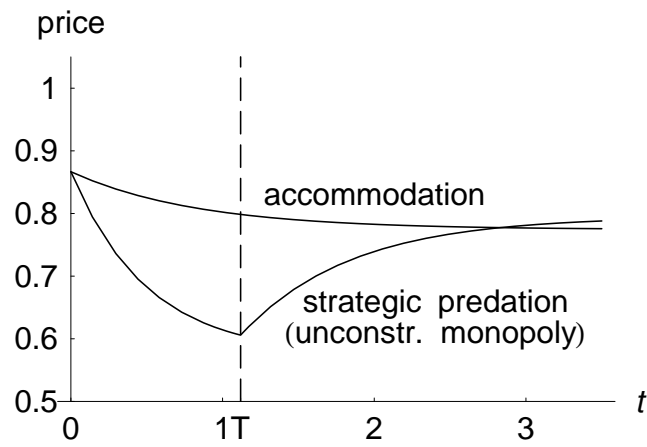
Figure 16: *Pattern of price over time with unconstrained monopoly ($\mu = 2$, $\beta = 0.33$, $g = 2.1$)*
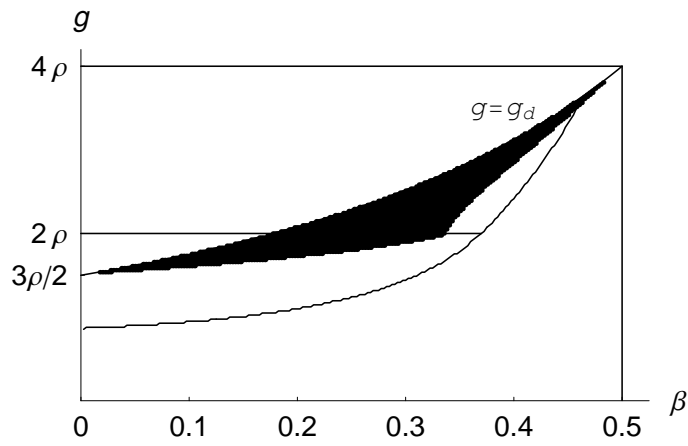
Figure 17: *Consumer surplus comparison (simulation results for $\mu = 2$)*
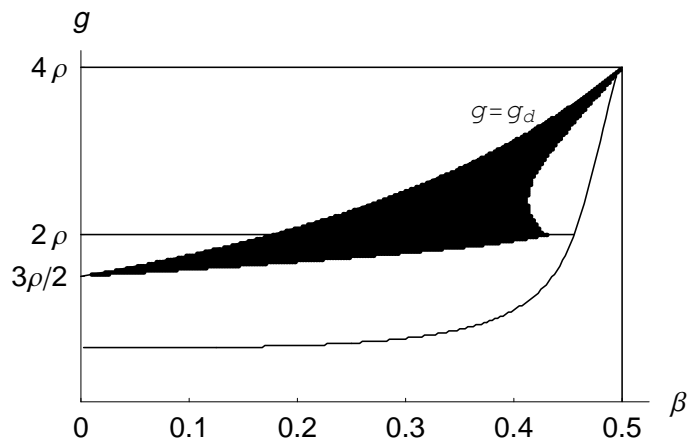
Figure 18: *Consumer surplus comparison (simulation results for $\mu = 20$)*
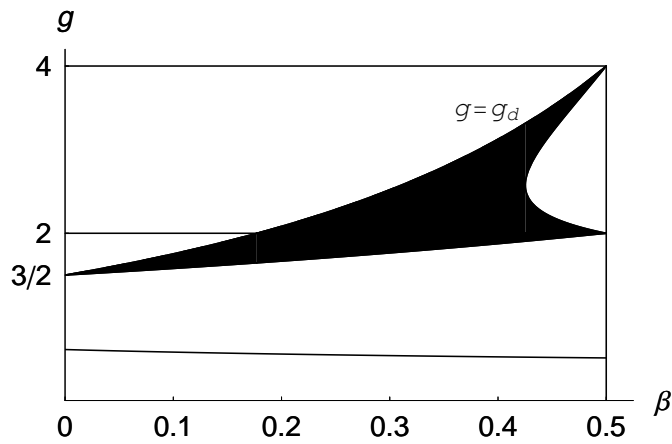
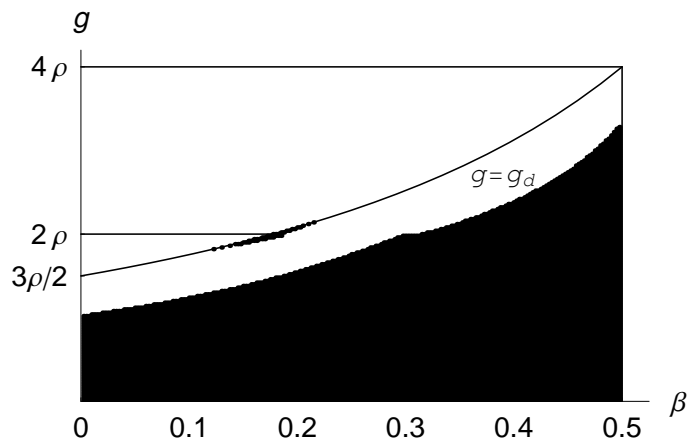Figure 19: *Consumer surplus comparison when $\mu \to \infty$*

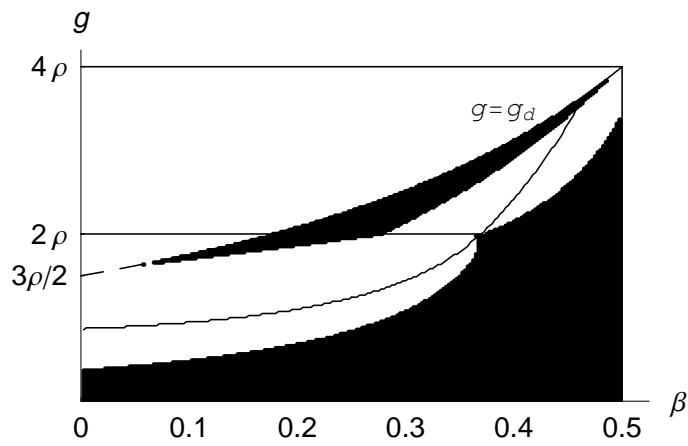Figure 20: *Welfare comparison (simulation results for $\mu = 0.2$)*

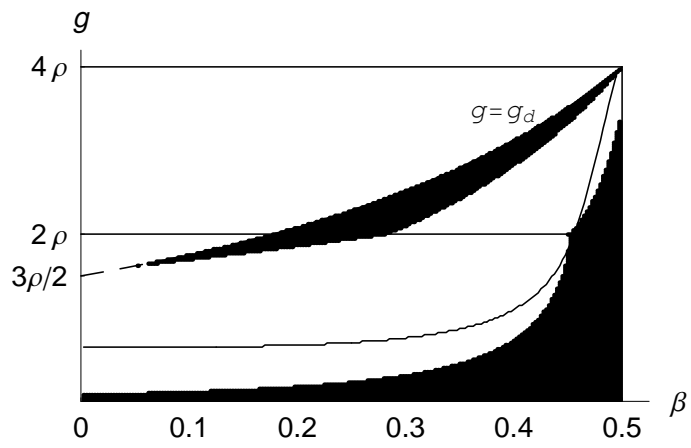Figure 21: *Welfare comparison (simulation results for $\mu = 2$)*

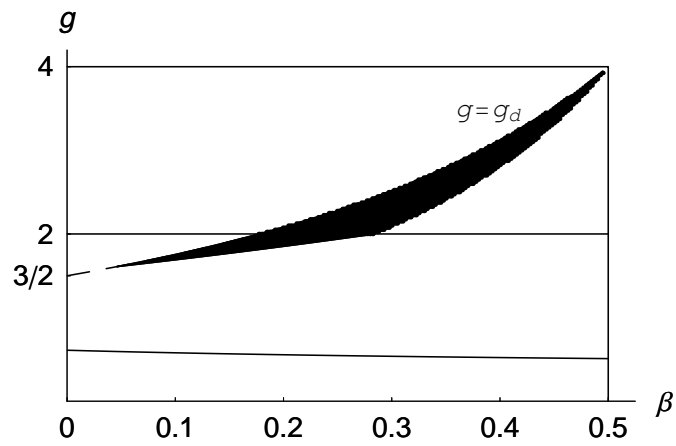Figure 22: *Welfare comparison (simulation results for $\mu = 20$)*

Figure 23: *Welfare comparison when $\mu \to \infty$ (partially simulation results)*

56

References

[1] Aghion, P. and R. Griffith (2004). *Competition and Growth*, Cambridge, MA: The MIT Press.

[2] D'Aspremont, C. and A. Jacquemin (1988). "Cooperative and Noncooperative R&D in Duopoly with Spillovers," *American Economic Review, 78*, 113–137.

[3] Baumol, W.J., J.C. Panzar, and R.D. Willig (1982). *Contestable Markets and the Theory of Industry Structure*, New York: Harcourt Brace Jovanovich, Inc.

[4] Blundell, R., R. Griffin, and J. van Reenen (1999). "Market Share, Market Value and Innovation in a Panel pf British Manufacturing Firms," *Review of Economic Studies, 66* (3), 529–554.

[5] Bolton, P. and D.S. Scharfstein (1990). "A Theory of Predation Based on Agency Problems in Financial Contracting," *American Economic Review, 80* (1), 93–106.

[6] Carlin, W., M.E. Schaffer, and P. Seabright (2004). "A Minimum of Rivalry: Evidence from Transition Economies on the Importance of Competition on Innovation and Growth," *CEPR Discussion Paper, No. 4343*.

[7] Chin, J.C. and G.M. Grossman (1990). "Intellectual Property Rights and North-South Trade," in R. W. Jones and A. O. Krueger, eds., *The Political Economy of International Trade: Essays in Honor of Robert E. Baldwin*, Cambridge, MA: Basil Blackwell.

[8] Cohen, W.M. and D.A. Levinthal (1989). "Innovation and learning: The two faces of R&D," *The Economic Journal, 99*, 569–596.

[9] Dixit, A. (1979). "A Model of Duopoly Suggesting a Theory of Entry Barriers," *The Bell Journal of Economics, 10* (1), 20–32.

[10] Economist (2004). "Slackers or Pace-Setters: Monopolies may have more incentives to innovate than economist have thought," *The Economist*, May 20, 2004.

[11] Etro, F. (2004). "Innovation by Leaders," *The Economic Journal, 114* (485), 281–303.

[12] Etro, F. (2006). "Aggressive Leaders," *The Rand Journal of Economics*, forthcoming.

[13] Etro, F. (2006), "Industrial Policy: A New Approach," Executive Summary, Available: `www.intertic.org/Strategic%20Trade%20Papers/CompetitionPolicy.pdf`.

[14] European Commission (2005). "DG Competition discussion paper on the application of Article 82 of the Treaty to exclusionary abuses," Bruxelles.

[15] Fershtman, C. and M. Kamien (1987). "Dynamic Duopolistic Competition with Sticky Prices," *Econometrica, 55* (5), 1151–1164.

[16] Griliches, Z. (1992). "The Search for R&D Spillovers," *Scandinavian Journal of Economics, 94*, S29–S47.

[17] Haucap, J. and J. Kruse (2002)."Predatory Pricing in Liberalized Telecommunications Market," *Universitaet der Bundeswehr Hamburg Discussion Papers in Economic Policy Research, No. 118*.

[18] Kobayashi, S. (2001). "A Model of R and D in Oligopoly with Spillovers," Mimeo, Department of Economics, Nihon University, Tokyo, Japan.

[19] McGee, J. (1980). "Predatory Pricing Revisited," *Journal of Law and Economics, 23*, 289–330.

[20] Nolan, D. (1998). "Predatory Pricing in an Oligopoly Framework," Mimeo, Department of Economics, Royal Holloway University of London.

[21] Ordover, J., and G. Saloner (1989). "Predation, Monopolization, and Antitrus," Ch. 9 in the *Handbook of Industrial Organization, Vol. 1*, North-Holland.

[22] Rey, P., J.M. Gual, M. Hellwig, A. Perrot, M. Polo, K. Schmidt, and R. Stenbacka (2005). "An Economic Analysis to Article 82," Report of the Economic Advisory Group for Competition Policy, European Union.

[23] Scherrer, F.M. and D. Ross (1990). *Industrial Market Structure and Economic Performance*, 3rd edition, Boston: Houghton Mifflin Company.

[24] Shapiro, C. (1989). "Theories of Oligopoly Behaviour," in R. Schmalensee and R. Willig, eds., *Handbook of Industrial Organization*, pp. 329–410, Amsterdam: North-Holland.

[25] Stenbacka, R. and M.M. Tombak (1993). "Commitment and Efficiency in Reach Joint Ventures," Mimeo.

[26] Telser, L. (1966). "Cutthroat Competition and Long Purse," *Journal of Law and Economics, 9*, 259–277.

[27] Tirole, J. (1991). *The Theory of Industrial Organization*, Cambridge, MA: The MIT Press.

[28] Vinogradov, V. and K. Žigič (1999). "R&D Spillovers in Duopoly: Static *versus* Dynamic Models," *CERGE-EI Discussion Paper, No. 1999–34*.

[29] Wiethaus, L. (2005). "Excess Absorptive Capacity and the Persistence of Monopoly," paper presented at *6. Symposium zur Ökonomischen Analyse der Unternehmung*, also available at `www.intertic.org/Theory/Intertic-theory.htm`.

[30] Žigič, K. (1998). "Intellectual Property Rights Violations and Spillovers in North-South Trade," *European Economic Review 42*, 1779–1799.

[31] Žigič, K. (2000). "Strategic trade Policy, Intellectual Property Rights Protection, and North South Trade," *Journal of Development Economics, 61*, 27–60.