Structural Estimation and Evaluation of Calvo-Style Inflation Models*

Jean-Marie Dufour† Université de Montréal

Lynda Khalaf‡ Université Laval

Maral Kichian§ Bank of Canada

February 2006

Preliminary, Please Do Not Quote

*We would like to thank Yash Mehra for useful comments, and Wendy Chan and Amanda Armstrong for research assistance. This work was supported by the Canada Research Chair Program (Econometrics, Université de Montréal, and Environment, Université Laval), the Institut de Finance Mathématique de Montréal (IFM2), the Alexander-von-Humboldt Foundation (Germany), the Canadian Network of Centres of Excellence (program on Mathematics of Information Technology and Complex Systems [MITACS]), the Canada Council for the Arts (Killam Fellowship), the Natural Sciences and Engineering Research Council of Canada, the Social Sciences and Humanities Research Council of Canada, the Fonds de Recherche sur la Société et la Culture (Québec), the Fonds de Recherche sur la Nature et les Technologies (Québec), and the Chair on the Economics of Electric Energy (Université Laval).

†Canada Research Chair Holder (Econometrics). Centre interuniversitaire de recherche en analyse des organisations (CIRANO), Centre Interuniversitaire de Recherche en Économie Quantitative (CIREQ), and Département de Sciences Économiques, Université de Montréal. Mailing address: Département de sciences economiques, Université de Montréal, C.P. 6128 succursale Centre-ville, Montréal, Québec, Canada H3C 3J7. TEL: (514) 343 2400; FAX: (514) 343 5831; email: jean.marie.dufour@umontreal.ca.

‡Canada Research Chair Holder (Environment). Département d'economique and groupe de recherche en economie de l'energie de l'Environement et des Ressources Naturelles (GREEN), Université Laval, and CIREQ. Mailing address: GREEN, Université Laval, Pavillon J.-A.-De Sève, St. Foy, Québec, Canada, G1K 7P4. TEL: (418) 656 2131-2409; FAX: (418) 656 7412; email: lynda.khalaf@ecn.ulaval.ca

§Research Department, Bank of Canada, Ottawa, Canada. TEL: (613) 782 7460; FAX: (613) 782 7163; email: mkichian@bankofcanada.ca.

Abstract

The authors structurally estimate and evaluate, for the U.S., the E.U., and Canada, various classes of recently-proposed Calvo-type models, using identification-robust methods. The models differ in their assumptions regarding price indexation (when firms cannot re-optimize their pices), in the way capital is used (homogenous or firm-specific), and in the elasicity of intermediate goods demand facing firms. Our approach is to obtain point and confidence-set structural parameter estimates, based on inverting identification-robust test statistics. Importantly, we maintain the focus on the structural aspect of the model, and formally impose the restrictions that map the theoretical model into the econometric one. In addition, we propose a test statistic that is invariant to the considered delay between the time firms re-optimize their prices and the time they implement these new prices.

Results are as follows. For the U.S., we find no statistical support for the standard Calvo model. Instead, there is evidence in favour of a dynamic indexation model with firm-specific capital and an increasing elasicity of intermediate goods demand facing firms. For Canada, there is some support for a dynamic indexation Calvo model regardless of whether capital is assumed to be firm-specific or not, but only if no price implementation delays are present. For the E.U., the results are mixed. Overall, we also find that, in all cases, when firm-specific capital is assumed, results are almost identical whether adjustment costs are assumed to be zero or not. Second, outcomes are very different depending on the considered implementation delay. Third, except when allowing for uncertainty in the considered implementation delay, the uncertainty about the average frequency of price adjustment in the economy is dramatically large.

JEL classification: C13, C52, E31

1. Introduction

Calvo-type sticky price models have received extensive interest in economic research circles, specially in the analysis of the effects of monetary policy. One reason for the popularity of these models is the so-called Calvo assumption—the number of firms that can change their prices at any given time is determined exogenously—which can make working with these time-dependent models substantially easier than working with (more intuitive) state-dependent models. Another reason is the statistical support found for the inflation dynamics equation based on such models (see, for example, Galí and Gertler (1999); Galí, Gertler, and Lopez-Salido (2001), Sbordone (2002)). Despite criticisms raised about specification bias, the use of limited-information setups, and the appropriateness of instrumental-variables-based inference (see, for example, Rudd and Whelan (2005); Linde (2005); Dufour, Khalaf, and Kichian (2005)), recent studies continue to claim support for Calvo-type models, and, in particular, for more generalized versions of such models. Christiano, Eichenbaum, and Evans (2005) allow for frictions in the labor market, calibrate one group of parameters, and estimate another group of parameters by matching moments.[1] Based on these estimates, they find inertial response in inflation and a hump-shaped response in output following a monetary policy shock. Eichenbaum and Fisher (2005), using generalized method of moments (GMM), claim statistical support for Calvo-style sticky price models, in general, and add that these models imply a plausible average frequency of firm price re-optimization if capital is allowed to be firm-specific and the elasicity of intermediate goods demand facing firms is allowed to be variable. Finally, Galí, Gertler, and Lopez-Salido (2005) re-affirm the validity of their original statistical findings (and, in particular, of the importance of the forward-looking component of aggregate inflation) by conducting GMM estimation of the closed form of their model, and by estimations with nonlinear instrumental variables (NLIV) methods. Their analysis emphasizes the fact that any econometric method should formally take into account the constraints on the parameters and/or error terms, as implied by the underlying theoretical model. This should hold true whether inference is based on a single structural equation, on the closed form, or on a structural system.

Clearly, the usefulness of the existing variants of the Calvo-type models for empirical or policy analysis depends importantly on their statistical identifiability, *i.e.* whether reliable econometric methods allow to estimate their underlying parameters with measurable precision. The theoretical framework typically yields Euler equations which lead to orthogonality conditions amenable to estimation via instrumental variables [IV] or GMM. So, when taken to the data, these models are often confronted with two central concerns: (i) endogeneity [which stems, in particular, from the presence of expectations-based regressors and from errors-in-variables issues], and (ii) parameter nonlinearity [which results, of course, from the connection between the key parameters of the underlying theoretical model and the parameters of the estimated econometric model].[2]

Both, endogeneity, and nonlinear parameter constraints, complicate statistical analysis in a non-trivial way. Nonlinearities can impose discontinuous parameter restrictions. In many

[1] More precisely, they minimize a measure of the distance between the model's impulse response functions and the empirical impulse response functions.

[2] For a discussion on both problems, see, for example, Galí, Gertler, and Lopez-Salido (2005).

cases, models are heavily parametrized so that some of the parameters are calibrated, and direct estimation is typically feasible for transforms of parameters of interest. Estimates of the latter are then "backed-out", and confidence intervals are constructed using the *delta*-method or alternative projection techniques; see, for instance, Eichenbaum and Fisher (2005). Such difficulties, in conjunction with possibly-weak instruments, lead to the eventuality of *weak identification*. Weak-identification causes the breakdown of standard asymptotic procedures based on estimated standard errors [including IV-based t-tests and Wald-type confidence intervals of the form: estimate $\pm$ (asymptotic standard error) $\times$ (asymptotic critical point)], and a heavy dependence on unknown nuisance parameters. As a result, standard, and even bootstrap-based, tests and confidence intervals can be unreliable. Spurious model rejections thus occur frequently, even with large data sets.[3]

It is important to understand the fundamental reason that leads to such failures. When parameters are not identifiable on a subset of the parameter space, or when the admissible set of parameter values is unbounded (as occurs, for example, with nonlinear parameter constraints such as ratios), then any valid method for the construction of confidence sets should allow for possibly unbounded outcomes [Dufour (1997)]. Wald-type intervals are "bounded" by construction, and are thus inappropriate in a fundamental way. They cannot be saved nor improved. Therefore, one has to rely on a different method which, by construction, allows for unbounded outcomes.[4] So even if MLE is used, resorting to usual t-type significance tests, or reliance on the *delta*-method, will lead to the same problems which plague GMM and linear or nonlinear IV. On recalling that identifying restrictions typically imply nonlinearity, we thus see that weak identification is inherent to the definition of structural models. In fact, this holds true even with a single linear simultaneous equation which is identified via "exclusion" restriction; this is easy to see when one derives the reduced-form or the structural likelihood function. Despite the huge associated theoretical literature, such problems remain somewhat misunderstood, and confused with issues such as very large estimated standard errors or poorly approximated cut-off points. We thus emphasize that usual point and interval estimation methods (whether based on MLE or on IV, and whether one considers a single structural equation or a multi-equation structural system), are flawed and should not be used.

The works of Christiano, Eichenbaum, and Evans (2005), Eichenbaum and Fisher (2005), and Galí, Gertler, and Lopez-Salido (2005) all rely on standard approaches, and thus are prone to the danger of drawing wrong conclusions because of the concerns mentioned above. The pitfalls of weak instruments are quite subtle, as demonstrated by Dufour, Khalaf, and Kichian (2005). These authors re-examine the Galí and Gertler (1999) model with essentially the original dataset and instruments, but use methods that are robust to weak instruments. They find clear evidence of identification difficulty in that model. In particular, although the Hodges-Lehmann point estimates of the deep parameters (see explanations below) yield a fairly large forward-looking component for inflation, the identification-robust confidence set associated with the parameter estimates is quite large, and includes the case where the backward-looking component of inflation is more important than the forward-looking part.

[3]The so-called *weak instruments* literature is now considerable; see Dufour and Jasiak (2001), Moreira (2003), Kleibergen (2002) and the surveys by Stock et al. (2002) and Dufour (2003).

[4]See Dufour (1997) for further analysis of the bounded parameter case.

Furthermore, when survey expectations are used instead of rational expectations, identification difficulties remain, and both the Hodges-Lehmann point estimates and the identification-robust confidence set point to a larger role for the backward-looking component for inflation.

With this backdrop in mind, in this paper we re-examine various classes of models associated with Calvo-type models, and for three different data sets, using identification-robust methods. The models that we estimate and test are generalizations of the standard Calvo sticky-price models and were judged to be statistically acceptable according to GMM-based criteria in Eichenbaum and Fisher (2004); Eichenbaum and Fisher (2005). The models differ in their assumptions regarding price indexation (when firms cannot re-optimize their pices), in the way capital is used (homogenous or firm-specific), and in the extent of the elasicity of intermediate goods demand facing firms.

Identification-robust methods make use of inference procedures where error probabilities can be controlled in the presence of endogeneity and nonlinear parameter constraints, even in the presence of identification difficulties. Our approach differs from the usual IV-based one in the fact that it avoids: (i) standard t-type confidence intervals, and (ii) reliance on the *delta*-method. Rather, we propose confidence set (CS) parameter estimates based on "inverting" identification-robust test statistics. Inverting a test yields the set of parameter values that are not rejected by this test. In addition, least-rejected parameters yield so-called Hodges-Lehmann point estimates [see Hodges and Lehmann (1963), Hodges and Lehmann (1983), and Dufour, Khalaf, and Kichian (2005). In contrast to the usual t-type confidence intervals, confidence sets formed by inverting a test lead (by construction) to possibly unbounded solutions, a prerequisite for ensuring reliable coverage [see Dufour (1997)].

In sum, the tests we invert ensure identification-robustness, maintain the focus on the structural aspect of the model, and formally impose the restrictions that map the theoretical model into the econometric one. This is done respecting the calibration exercise, the moving-average error structure, and avoiding the *delta*-method altogether. In other words, we do not need to "back-out" the structural parameters of interest from estimated transforms, in contrast to Eichenbaum and Fisher (2005).[5] We deal with all such irregularities via the use of simple F-type procedures (with or without standard autocorrelation-robust corrections), for which standard finite-sample and asymptotic distributional theory applies. This exercise is extremely simple, despite the complexity of the nonlinear model under consideration. Our procedure has two further "built-in" advantages. First, extremely wide confidence sets provably reveal identification difficulties. Second, if all economically-sound values of the model's deep parameters are rejected at some chosen significance level, then the confidence set will be empty and we can infer that the model is soundly rejected. This provides an identification-robust alternative the standard GMM-based J-test.

In the next section we present Eichenbaum and Fisher's NKPC model under static and dynamic indexation. Section 3 discusses our methodology. Section 4 presents the empirical results, and section 5 concludes.

[5] The projection technique used by Eichenbaum and Fisher (2005) is valid, in principle, when the underling transformation is monotonic; since the model at hand is highly nonlinear, monotonicity is not granted.

2. Eichenbaum and Fisher's NKPC Models

Firms evolve in a competitive environment but face constraints on the adjustment of their prices. A Calvo-type assumption is used for this purpose: at time t, any given firm faces a $(1-\theta)$ probability of adjusting its price. When it can adjust the price, it re-optimizes. However, when the firm is not allowed to adjust, its price is indexed to some measure of aggregate inflation: dynamic indexation is the case where prices are indexed to previous period's aggregate inflation level, and static indexation is the case where prices are indexed to average aggregate inflation.

For the dynamic indexation case, the aggregate inflation process, π_t, evolves as:

$$\Delta\hat{\pi}_t = \beta E_{t-\tau}\Delta\hat{\pi_{t+1}} + \lambda E_{t-\tau}\hat{s}_t, \tag{1}$$

and for the static indexation case, the model is:

$$\hat{\pi}_t = \beta E_{t-\tau}\hat{\pi_{t+1}} + \lambda E_{t-\tau}\hat{s}_t, \tag{2}$$

with

$$\lambda = \frac{A.D.(1-\theta)(1-\beta\theta)}{\theta}. \tag{3}$$

In the above, Δ is the first difference operator, $\hat{x}$ is the variable x in deviation from its steady-state value, s_t is real marginal costs and υ_{t+1} is the inflation surprise term at $t+1$, τ refers to the implementation delay (that is, the number of periods between the time the re-optimization decision is taken and the actual implementation of the changes) and β is the subjective discount rate. The parameters A and D are defined following various assumptions regarding the price elasticity of intermediate good demand that firms face, and the type of capital market, respectively. For the case where capital is firm-specific, adjustments costs may intervene. The assumptions in question are:

H_1: the standard version of sticky price Calvo models, where capital is homogeneous and firms face a constant price elasticity of demand;

H_2: capital is homogeneous ($D = 1$), but firms face a variable price elasticity of demand ($A < 1$).

H_3: Firms face a variable price elasticity of demand ($A < 1$), and capital is firm-specific ($D < 1$). In the latter case, adjustment costs are either zero or positive.

In the case of H_1,

$$A = D = 1. \tag{4}$$

Instead, if capital is assumed to be firm-specific, and the elasticity of demand increasing in price, then $D < 1$ and $A < 1$. The latter parameters have fundamental structural implications; A governs the degree of pass-through from a rise of marginal cost to prices, or alternatively

$$A = \frac{1}{\zeta\epsilon + 1} \tag{5}$$

where ϵ is the % change in the elasticity of demand for a given intermediate good due to a one % change in the relative price of the good at steady state, and ζ denotes the firm's steady state mark-up. D is a nonlinear function of β, θ and A, and further deep parameters as follows:

$$D = \frac{(1-\beta\theta\kappa_1)}{(1+\overline{\eta}\xi A)(1-\beta\theta\kappa_1)+\xi A\beta\theta\kappa_2} \tag{6}$$

where $\overline{\eta}$ is the steady state elasticity of demand which relates to ζ as follows

$$\zeta = \overline{\eta}/(\overline{\eta}-1)-1, \tag{7}$$

[in steady state, an intermediate good firm sets price as a markup over marginal cost]

$$\xi = \overline{\alpha}/(1-\overline{\alpha}), \tag{8}$$

$\overline{\alpha}$ is the share of capital in the production function, and κ_1 and κ_2 are the solutions of the 3-equations system (in κ_1 and κ_2 and ν) subject to $|\kappa_1| < 1$:

$$\begin{aligned}
1-\left[\phi+(1-\theta\nu)\left(\beta\kappa_2-\Xi\right)\right]\kappa_1+\beta\kappa_1^2 &= 0 && (9)\\
\Xi\theta+\left[\phi-\beta\left(\theta+\kappa_1\right)-(1-\theta)\,\Xi\nu\right]\kappa_2+\beta\left(1-\theta\right)\nu\kappa_2^2 &= 0 && (10)\\
\frac{\xi A(1-\beta\theta)}{(1+\overline{\eta}\xi A)(1-\beta\theta\kappa_1)+\xi A\beta\theta\kappa_2}-\nu &= 0 && (11)
\end{aligned}$$

with

$$\begin{aligned}
\Xi &= \left(1-\beta\left(1-\delta\right)\right)\overline{\eta}\frac{1}{1-\overline{\alpha}}\frac{1}{\psi}, && (12)\\
\phi &= 1+\beta+\left(1-\beta\left(1-\delta\right)\right)\frac{1}{1-\overline{\alpha}}\frac{1}{\psi}; && (13)
\end{aligned}$$

ψ is the adjustment cost parameter; and δ is defined accordingly such that the elasticity of the investment-to-capital ratio with respect to Tobin's q (evaluated at steady state) is given by $1/(\delta\psi)$. When $\psi = 0$,

$$\begin{aligned}
\kappa_1 &= 0, \quad \kappa_2 = -\widetilde{\Xi}/\widetilde{\phi}, \quad \nu = \xi A(1-\beta\theta)/\left[(1+\overline{\eta}\xi A)+\xi A\beta\theta\kappa_2\right],\\
\widetilde{\Xi} &= \left(1-\beta\left(1-\delta\right)\right)\overline{\eta}/(1-\overline{\alpha}), \quad \widetilde{\phi} = 1+\beta+\left(1-\beta\left(1-\delta\right)\right)/(1-\overline{\alpha}),
\end{aligned}$$

in which case

$$D = \frac{1}{(1+\overline{\eta}\xi A)-\xi A\beta\theta\widetilde{\Xi}/\widetilde{\phi}}. \tag{14}$$

The authors estimate θ calibrating all other parameters. We prefer to estimate θ and β (constraining β to fall between 0.90 and 0.99), maintaining the same calibrated values used by Eichenbaum and Fisher (2005) for all other parameters. For clarity of presentation, we set

$$\omega = \begin{pmatrix} \overline{\alpha} & \psi & \delta & \zeta & \epsilon \end{pmatrix}' \tag{15}$$

and express the parameters A and D as functions of ω:

$$A = a(\omega),\ D = d(\beta,\theta,\omega) \tag{16}$$

where the function $a(.)$ is given by (4) under assumption A_1 and by (5) under assumptions A_2 and A_3, and the function $d(.)$ is given by (4), (6) and (14) under assumptions A_1, A_2 and A_3 respectively.

3. Methodology

In this section, we describe the methodology as it applies to the dynamic indexation case.[6] The econometric model used to assess (1) sets is given by:

$$\Delta\widehat{\pi}_t = \beta\Delta\widehat{\pi_{t+1}} + \lambda\ \hat{s}_t + u_{t+1}, \qquad t = 1, ..., T. \tag{17}$$

Alternatively, we can write

$$y_t = Y_t'\gamma + u_{t+1},$$

where $y_t \equiv \Delta\hat{\pi}_t$, $Y_t = (\Delta\widehat{\pi_{t+1}}, \hat{s}_t)'$, $\gamma = (\beta, \lambda)'$, the error term reflects the rational expectations hypothesis and steady state values are proxied by the variables' sample means respectively. An instrument set is also available; we denote the $k \times 1$ vector of instruments at time t by X_t. Eichenbaum and Fisher (2005)'s theoretical model implies that u_{t+1} follows an MA(τ) structure. To simplify presentation, we further adopt the following notation: y is the T dimensional vector of observations on $\Delta\widehat{\pi}_t$, Y is the $T \times 2$ matrix of observations on $\Delta\widehat{\pi_{t+1}}$ and $\hat{s}_t$, X is the $T \times k$ matrix of the instruments, and u is the T dimensional vector of error terms. We also denote by $M[V]$ the projection matrix $I - V(V'V)^{-1}V'$.

The methodology we consider can be summarized as follows. To obtain a confidence set with level $1 - \alpha$ for the deep parameters, we invert an identification-robust test (presented in what follows) associated with the null hypothesis

$$H_0 : \beta = \beta_0,\ \theta = \theta_0 \text{ and } \omega = \omega_0, \tag{18}$$

where ω_0, β_0, and θ_0 are known values. Formally, this implies collecting the values β_0 and θ_0 [given the calibrated ω_0] that are not rejected by the test (i.e. the values for which the test is not significant at level α). To do so, we proceed as follows; further discussion and references are provided in Dufour, Khalaf, and Kichian (2005).

Using a grid search over the economically meaningful set of values for ω, β, and θ, we sweep the economically relevant choices for ω_0, β_0, and θ_0.[7] For each parameter combination considered, we compute test statistics (described below) and their respective p-values. The parameter vectors for which the p-values are greater than the level α constitute a confidence set with level $1 - \alpha$. In addition, the values of ω^0, β^0, and θ^0 that lead to the largest p-value formally yield the set of "least rejected" models, i.e., models that are most compatible with the data. This method underlies the principles of the Hodges-Lehmann estimation method; see Hodges and Lehmann (1963); Hodges and Lehmann (1983). Whereas uniqueness (as expected via a usual point estimation approach) is not granted, analyzing the economic information content of these least rejected models provides very useful model diagnostics.

So let us now present the tests we conduct for each choice for β_0, θ_0 and ω_0.

1. Solve (16) for values of A and D associated with ω_0, β_0, and θ_0; using (3), obtain the corresponding value for λ; we denote the latter by λ_0.

[6] The static indexation case is similar.

[7] The parameter space considered for β, and θ is the (0,1) interval, and the search increment is 0.01.

2. Consider the regression [which we will denote the AR-regression, in reference to Anderson and Rubin (1949)] of

$$\{\Delta\widehat{\pi}_t - \beta_0 \Delta\widehat{\pi_{t+1}} - \lambda_0\ \hat{s_t}\} \text{ on } \{\text{the } \textit{instruments } X_t\}. \tag{19}$$

Under the null hypothesis [specifically (17)-(18)], the coefficients of the latter regression should be zero. Hence testing for a zero null hypothesis on the coefficients of X_t in (19) provides a test of (18).

Our approach maps the structural equation (17) [which faces identification difficulties] into the standard regression (19). The latter provides a regular framework (because the right-hand side regressors are not "endogenous") where identification constraints are no longer needed, so the usual statistics which test the exclusion of $\{$the *instruments* $X_t\}$ can conveniently be applied. For instance, under the *i.i.d.* error assumption for (19) [i.e. the case $\tau = 0$], the usual F statistic can be used:

$$AR\,(\omega_0, \beta_0, \theta_0) = \frac{(y - Y\gamma_0)'\,(I - M[X])\,(y - Y\gamma_0)\,/(k)}{(y - Y\gamma_0)'\,M[X]\,(y - Y\gamma_0)\,/\,(T-k)}, \tag{20}$$

with the $\mathrm{F}(k, T-k)$ or $\chi^2(k)$ null distribution. To correct for departures from the *i.i.d.* error hypothesis [the case where $\tau > 0$], we use a Wald-type statistic with Newey-West autocorrelation consistent covariance estimator for the coefficient of the AR regression (19):

$$\begin{aligned} AR\text{-HAC}\,(\omega_0, \beta_0, \theta_0) &= (y - Y\gamma_0)'\,X(X'X)^{-1}\widehat{Q}^{-1}(X'X)^{-1}X'\,(y - Y\gamma_0) \\ \widehat{Q} &= \frac{1}{T}\sum_{t=1}^{T}\widehat{u}_t^2 X_t X_t' + \frac{1}{T}\sum_{l=1}^{L}\sum_{t=l+1}^{T} w_l \widehat{u}_t \widehat{u}_{t-l}\left(X_t X_{t-l}' + X_{t-l}X_t'\right) \\ w_l &= 1 - \frac{l}{L+1} \end{aligned} \tag{21}$$

where $\widehat{u}_t$ is the OLS residual associated with (19) and L is the number of allowed lags; we use $L = 3$ and 12, and the $\chi^2(k)$ null distribution. We also consider an alternative correction specifically targeted for the MA(1) error case [the case with $\tau = 1$]. This method involves augmenting the regression (19) by an extra term $\widehat{e}_{t-1}$, where $\widehat{e}_t$ are residuals from a long autoregression of $y_t - Y_t'\gamma_0$ on its lags (up to $T^{1/4}$). Denote the latter vector $\widehat{e}_{(-1)}$ and the augmented regressor matrix X_1. We test whether the coefficients of all response variables except $\widehat{e}_{t-1}$ are zero, using the usual F-statistic

$$AR\text{-HR}\,(\omega_0, \beta_0, \theta_0) = \frac{(y - Y\gamma_0)'\left(M[\widehat{e}_{(-1)}] - M[X_1]\right)(y - Y\gamma_0)\,/(k)}{(y - Y\gamma_0)'\,M[X_1]\,(y - Y\gamma_0)\,/\,(T-k-1)} \tag{22}$$

with the $\mathrm{F}(k, T-k-1)$ or $\chi^2(k)$ null distribution.

Despite the complex underlying nonlinearities (recall the definition of A and D and λ), our approach is extremely simple, and is identification-robust, in the sense that it is statistically valid whether the model is identified or not.

To conclude, we note that if the instrument sets underlying the above procedures are chosen conformably, inverting the AR-HAC$(\omega_0, \beta_0, \theta_0)$ statistic provides confidence sets that are invariant to τ. This useful property is not shared by the usual GMM-based t-type confidence intervals.

4. Empirical Results

We estimate the static and dynamic indexation models for the U.S., Canada, and the E.U., and under the alternative assumptions described above, using the Anderson-Rubin methods. The instrument set contains the same variables as the set used by Eichenbaum and Fisher (2005), except that we do not include lags of inflation in our set. More precisely, our instrument set includes a constant, the second lag of each of real marginal cost, quadratically-detrended real GDP, and the growth rate of nominal wages, as well as the third lag of the Euler error.[8] The lag structure for our instruments supposes $\tau \leq 1$; our aim here is to maintain the same overall setup as in Eichenbaum and Fisher (2005), to allow useful comparisons of outcomes.

The results are reported in Tables 1–6c below. For all the data sets, we report the case where β is estimated but constrained to lie between 0.90 and 0.99. For the homogeneous capital market case, the results are given in Tables 1, 3, and 5, for the U.S., Canada, and the E.U., respectively. For the firm-specific market case, the corresponding tables are Tables 2, 4, and 6, respectively. We also estimate, for all the data sets, the case where β is calibrated at 0.99. The outcomes for these are found in Tables 1c, 3c, and 5c, for the U.S., Canada, and the E.U., respectively, and for the homogeneous capital market case, and in Tables 2c, 4c, and 6c, for the corresponding firm-specific capital market models.

Let us first consider the U.S. results for the homogeneous capital market case (Tables 1 and 1c). In these Tables, $D = 1$, but the elasticity of intermediate goods' demand facing the firms is allowed to change (so that $A = 1$, $A = 0.5$, and $A = 0.23$). The first thing we notice is that, as A decreases, the Hodges-Lehmann point estimate of θ also decreases. This is the case whether the model assumes static indexation, or dynamic, and whether β is estimated, or constrained to equal 0.99.

From Table 1 (the unconstrained estimation case), and based on the $\tau = 0$ and HR $\tau = 1$ results, the static model yields more economically-reasonable (i.e., lower) point estimates for θ than the corresponding dynamic model. Furthermore, when β is constrained to equal 0.99 (Table 1c), when there is no implementation lag (i.e., $\tau = 0$), the static model, again, yields lower θ values than the corresponding dynamic model.

The same is true when capital is assumed to be firm-specific. The results in Tables 2 and 2c show that, the point estimate for θ decreases as A decreases, and that the static model yields lower θ values than the corresponding dynamic model when $\tau = 0$.

Despite the favourable results for the static models discussed above, an examination of the HAC $\tau = 1$ outcomes leads us to prefer the dynamic indexation models over the static ones unequivocably. The reason is that, only for these cases are the confidence sets for θ tightly bounded. Contrast this with the obtained confidence sets with all other statistics, and the outcome is a CS that is bounded away from 0.01 at the lower end, but that hits the uppermost value allowed in our grid search (namely, 0.99). This is a sign that the models in those cases are not identified at high values of θ.

Having decided that the data prefers dynamic indexation, the next step is to examine which assumption for the type of capital market is supported by the data. By estimating

[8] Note, also, that our output gap measure is real-time, in the sense that the gap value at time t does not use information beyond that date. See Dufour, Khalaf, and Kichian (2005) for the details.

β along with θ and obtaining the relevant confidence sets, it is clear that the firm-specific assumption is more favoured by the data than the homogeneous capital market case. The reason is that the confidence sets for β in the latter case exclude the $\beta = 0.99$ value. In particular, the preferred specification is the dynamic model with firm-specific capital assumed, and for $A = 0.5$. Furthermore, the point estimate for θ is 0.39, so that the estimate for the average frequency of price adjustment is about 1 and a half quarters.

Our identification-robust results are in contrast with Eichenbaum and Fisher (2005) in that the standard Calvo model with dynamic indexation is not supported by the data (since confidence sets for the θ esimate hit the boundary at the upper end, so that the models are not well-identified in those cases). In fact, we find that only one specific model among the class of generalized models is upheld by the data: the dynamic model with $A = 0.5$ and $D < 1$.

Turning to the case of Canada, and in contrast to the U.S. case, there is some support for the dynamic model when $\tau = 0$ (i.e., when there is no price implementation lag), and regardless of whether capital is assumed to be homogeneous or firm-specific. In particular, constrained estimation results yields non-empty confidence sets, and very economically-reasonable point estimates for θ. However, p-values are generally lower than in the U.S. case. In addition, the confidence set for θ is empty when the HAC $\tau = 1$ statistic is applied. Furthermore, there is evidence of weak identification because the confidence sets for θ are not bounded at the upper end.

Finally, we turn to the European Union case. Based on the three statistics taken together, there is more support for the static indexation model than the dynamic one, specially when capital is assumed to be homogeneous (see the cases for $A = 0.5$ and $A = 0.23$). However, even though the confidence set for θ is tightly bounded with the HAC statistic, the confidence set for β does not include the 0.99 value. In contrast, through the CS are empty for $\tau = 0$ and HR $\tau = 1$, it is very tightly bounded for the dynamic model with $A = 0.5$, and for firm-specific capital. Of course, in this case, the rest of the statistics yield empty confidence sets.

In sum, based on our identification-robust methods, we find no statistical support for the standard Calvo dynamic model for the U.S., but find good support for a specific version of the dynamic model, namely the model with $A = 0.5$ and $D < 1$. There is also tentative support for the dynamic model for Canada, for the case where there is no implementation lag. In this case, it matters little whether capital is assumed to be firm-specific or not. For the E.U., the results are mixed, with some support for the static indexation model and some for the dynamic model. Overall, only with the HAC $\tau = 1$ statistic do we find bounded confidence sets for θ, suggesting that constraining the implementation lag to 0 or to 1 specifically works against the model. Finally, though not reported (to save space), results are almost identical whether adjustment costs are zero or non-zero.

References

Anderson, T.W. and H. Rubin. 1949. "Estimation of the Parameters of a Single Equation in a Complete System of Stochastic Equations." *Annals of Mathematical Statistics* 20: 46–63.

Christiano, L., M. Eichenbaum, and M. Evans. 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economics* 113: 1–45.

Dufour, J.M. 1997. "Some Impossibility Theorems in Econometrics, with Applications to Structural and Dynamic Models." *Econometrica* 65: 1365–89.

———. 2003. "Identification, Weak Instruments and Statistical Inference in Econometrics." *Canadian Journal of Economics* 36(4): 767–808.

Dufour, J.M. and J. Jasiak. 2001. "Finite Sample Limited Information Inference Methods for Structural Equations and Models with Generated Regressors." *International Economic Review* 42: 815–43.

Dufour, J.M., L. Khalaf, and M. Kichian. 2005. "Inflation dynamics and the New Keynesian Phillips Curve: An Identification-Robust Econometric Analysis." *Journal of Economic Dynamics and Control* forthcoming.

Eichenbaum, M. and J. Fisher. 2004. "Evaluating the Calvo Model of Sticky Prices." Working Paper. Federal Reserve Bank of Chicago.

———. 2005. "Evaluating Calvo-Style Sticky Price Models." Working Paper. Federal Reserve Bank of Chicago.

Galí, J. and M. Gertler. 1999. "Inflation Dynamics: A Structural Econometric Analysis." *Journal of Monetary Economics* 44: 195–222.

Galí, J., M. Gertler, and J.D. Lopez-Salido. 2001. "European Inflation Dynamics." *European Economic Review* 45: 1237–70.

———. 2005. "Robustness of the Estimates of the Hybrid New Keynesian Phillips Curve." *Journal of Monetary Economics* 52: 1107–18.

Hodges, J.L., Jr. and E.L. Lehmann. 1963. "Estimates of Location Based on Rank Tests." *Annals of Mathematical Statistics* 34: 598–611.

———. 1983. "Hodges-Lehmann Estimators." In *Encyclopedia of Statistical Sciences, Volume 3*, edited by N.L. Johnson, S. Kotz, and C. Read, 642–45. New York: John Wiley & Sons.

Kleibergen, F. 2002. "Pivotal Statistics for Testing Structural Parameters in Instrumental Variables Regression." *Econometrica* 70(5): 1781–1803.

Linde, J. 2005. "Estimating New Keynesian Phillips Curves: A Full Information Maximum Likelihood Approach." *Journal of Monetary Economics* 52: 1135–49.

Moreira, M.J. 2003. "A Conditional Likelihood Ratio Test for Structural Models." *Econometrica* 71(4): 1027–48.

Rudd, J. and K. Whelan. 2005. "New Tests of the New Keynesian Phillips Curve." *Journal of Monetary Economics* 52: 1167–81.

Sbordone, A. 2002. "Prices and Unit Labor Costs: A New test for Price Stickiness." *Journal of Monetary Economics* 49: 265–92.

Stock, J.H., J.H. Wright, and M. Yogo. 2002. "A Survey of Weak Instruments and Weak Identification in Generalized Method of Moments." *Journal of Business and Economic Statistics* 20(4): 518–29.

Appendix: Tables

Table 1: U.S. Unconstrained Estimation Results
Homogeneous Capital Market

	A=1.00				A=0.50				A=0.23			
Test	$\hat{\beta}$	$\hat{\theta}$	$\hat{Fq}$	p-val	$\hat{\beta}$	$\hat{\theta}$	$\hat{Fq}$	p-val	$\hat{\beta}$	$\hat{\theta}$	$\hat{Fq}$	p-
					Dynamic Model							
$\tau = 0$	0.99	0.95	20.00	0.70	0.99	0.93	14.29	0.70	0.99	0.90	10.00	0
	(0.90,0.99)	(0.77,0.99)	(4.35,100)		(0.90,0.99)	(0.69,0.99)	(3.22,100)		(0.90,0.99)	(0.58,0.99)	(2.38,100)	
$\tau = 1$	0.90	0.97	33.33	0.85	0.90	0.95	20.00	0.85	0.90	0.92	12.50	0
HR	(0.90,0.99)	(0.89,0.99)	(9.09,100)		(0.90,0.99)	(0.85,0.99)	(6.67,100)		(0.90,0.99)	(0.79,0.99)	(4.76,100)	
$\tau = 1$	0.92	0.58	2.38	0.81	0.92	0.46	1.85	0.92	0.93	0.32	1.47	0
HAC	(0.92,0.97)	(0.56,0.58)	(2.27,2.38)		(0.90,0.95)	(0.45,0.60)	(1.82,2.50)		(0.90,0.98)	(0.32,0.48)	(1.47,1.92)	
					Static Model							
$\tau = 0$	0.90	0.87	7.69	0.95	0.90	0.81	5.26	0.95	0.90	0.72	3.57	0
	(0.90,0.99)	(0.79,0.99)	(4.76,100)		(0.90,0.99)	(0.70,0.99)	(3.33,100)		(0.90,0.99)	(0.58,0.99)	(2.38,100)	
$\tau = 1$	0.97	0.92	12.50	0.71	0.97	0.89	9.09	0.71	0.97	0.83	5.88	0
HR	(0.90,0.99)	(0.83,0.99)	(5.88,100)		(0.90,0.99)	(0.75,0.99)	(4.00,100)		(0.90,0.99)	(0.65,0.99)	(2.86,100)	
$\tau = 1$ HAC	empty				empty				empty			

The above are the Anderson-Rubin test results applied to the econometric models of the dynamic and static models found in equations (1) and (2), respectively. The delay between the time prices are re-optimized and actually changed is the implementation lag, τ. For $\tau = 0$, the AR statistic is applied, while for$\tau = 1$, two variants of the AR are used: the Hannan-Risannen statistic (denoted in the tables as HR), and a Wald-type statistic with Newey-West autocorrelation-robust standard covariance estimator (denoted in the tables as HAC). The instruments include a constant, the third lag of the Euler error, and the second lags of each of $\hat{s_t}$, wage inflation, and the one-sided quadratically-detrended output gap. Point estimates are Hodges-Lehmann point estimates, and which correspond to the least rejected set of parameter values. Fq is the average frequency of price adjustment, measured in quarters, and p-val denotes p-values. The parameter A is defined as in equation (5) and is calibrated according to different values of ϵ taken from the set (0,10,33).

Table 1c: U.S. Constrained Estimation Results
Homogeneous Capital Market

Test	A=1.00				A=0.50				A=0.23			
	β	$\hat{\theta}$	$\hat{Fq}$	p-val	β	$\hat{\theta}$	$\hat{Fq}$	p-val	β	$\hat{\theta}$	$\hat{Fq}$	p-val
						Dynamic Model						
$\tau = 0$	0.99	0.95	20.00	0.70	0.99	0.93	14.29	0.70	0.99	0.90	10.00	0.70
		(0.77,0.99)	(4.35,100)	(0.07,0.70)		(0.69,0.99)	(3.23,100)	(0.07,0.70)		(0.58,0.99)	(2.38,100)	(0.07,0.70)
$\tau = 1$ HR	0.99	0.95	20.00	0.78	0.99	0.93	14.29	0.78	0.99	0.89	12.50	0.78
		(0.89,0.99)	(9.09,100)	(0.05,0.67)		(0.85,0.99)	(6.67,100)	(0.07,0.66)		(0.79,0.99)	(4.76,100)	(0.10,0.66)
$\tau = 1$ HAC	0.99		empty		0.99		empty		0.99		empty	
						Static Model						
$\tau = 0$	0.99	0.90	10.00	0.89	0.99	0.87	7.69	0.89	0.99	0.81	5.26	0.89
		(0.79,0.99)	(4.76,100)	(0.07,0.82)		(0.72,0.99)	(3.57,100)	(0.09,0.82)		(0.61,0.99)	(2.56,100)	(0.07,0.82)
$\tau = 1$ HR	0.99	0.94	16.67	0.69	0.99	0.92	12.50	0.69	0.99	0.88	8.33	0.69
		(0.84,0.99)	(6.25,100)	(0.06,0.66)		(0.78,0.99)	(4.55,100)	(0.06,0.66)		(0.69,0.99)	(3.23,100)	(0.06,0.66)
$\tau = 1$ HAC	0.99		empty		0.99		empty		0.99		empty	

Note - See notes in Table 1.

Table 2: U.S. Unconstrained Estimation Results
Firm-Specific Capital Market

	A=1.00				A=0.50				A=0.23			
Test	$\hat{\beta}$	$\hat{\theta}$	$\hat{Fq}$	p-val	$\hat{\beta}$	$\hat{\theta}$	$\hat{Fq}$	p-val	$\hat{\beta}$	$\hat{\theta}$	$\hat{Fq}$	p
					Dynamic Model							
$\tau = 0$	0.99	0.88	8.33	0.70	0.99	0.87	7.69	0.70	0.99	0.86	7.14	0
	(0.90,0.99)	(0.51,0.99)	(2.04,100)		(0.90,0.99)	(0.49,0.99)	(1.96,100)		(0.90,0.99)	(0.44,0.99)	(1.79,100)	
$\tau = 1$	0.90	0.89	9.09	0.85	0.90	0.88	8.33	0.854	0.90	0.86	7.14	0
HR	(0.90,0.99)	(0.74,0.99)	(3.85,100)		(0.90,0.99)	(0.73,0.99)	(3.70,100)		(0.90,0.99)	(0.69,0.99)	(3.23,100)	
$\tau = 1$	0.91	0.26	1.35	0.79	0.95	0.38	1.61	0.43	0.98	0.34	1.52	0
HAC	(0.90,0.97)	(0.25,0.41)	(1.33,1.69)		(0.90,0.99)	(0.22,0.39)	(1.28,1.64)		(0.90,0.98)	(0.19,0.34)	(1.23,1.52)	
					Static Model							
$\tau = 0$	0.90	0.66	2.94	0.95	0.90	0.64	2.78	0.95	0.90	0.60	2.50	0
	(0.90,0.99)	(0.51,0.99)	(2.04,100)		(0.90,0.99)	(0.49,0.99)	(1.96,100)		(0.90,0.99)	(0.44,0.99)	(1.79,100)	
$\tau = 1$	0.97	0.80	5.00	0.71	0.97	0.78	4.55	0.71	0.97	0.75	4.00	0
HR	(0.90,0.99)	(0.58,0.99)	(2.38,100)		(0.90,0.99)	(0.56,0.99)	(2.27,100)		(0.90,0.99)	(0.51,0.99)	(2.04,100)	
$\tau = 1$ HAC	empty				empty				empty			

Note - See notes in Table 1.

Table 2c: U.S. Constrained Estimation Results
Firm-Specific Capital Market

	A=1.00				A=0.50				A=0.23			
Test	β	$\hat{\theta}$	$\hat{Fq}$	p-val	β	$\hat{\theta}$	$\hat{Fq}$	p-val	β	$\hat{\theta}$	$\hat{Fq}$	p-val
					Dynamic Model							
$\tau = 0$	0.99	0.88	8.33	0.70	0.99	0.87	7.69	0.70	0.99	0.86	7.14	0.70
		(0.51,0.99)	(2.04,100)	(0.06,0.70)		(0.49,0.99)	(1.96,100)	(0.07,0.70)		(0.44,0.99)	(1.79,100)	(0.06,0.70)
$\tau = 1$ HR	0.99	0.87	7.69	0.78	0.99	0.86	7.14	0.78	0.99	0.84	6.25	0.78
		(0.74,0.99)	(3.85,100)	(0.06,0.66)		(0.73,0.99)	(3.70,100)	(0.08,0.66)		(0.69,0.99)	(3.23,100)	(0.06,0.65)
$\tau = 1$ HAC	0.99		empty		0.99	0.39	1.64	0.35	0.99		empty	
						(0.22,0.39)	(1.28,1.64)	(0.18,0.35)				
					Static Model							
$\tau = 0$	0.99	0.77	4.35	0.89	0.99	0.75	4.00	0.89	0.99	0.72	3.57	0.89
		(0.55,0.99)	(2.22,100)	(0.08,0.82)		(0.52,0.99)	(2.08,100)	(0.06,0.82)		(0.48,0.99)	(1.92,100)	(0.07,0.82)
$\tau = 1$ HR	0.99	0.85	6.67	0.69	0.99	0.84	6.25	0.69	0.99	0.82	5.56	0.69
		(0.64,0.99)	(2.78,100)	(0.07,0.66)		(0.62,0.99)	(2.63,100)	(0.07,0.66)		(0.57,0.99)	(2.33,100)	(0.05,0.66)
$\tau = 1$ HAC	0.99		empty		0.99		empty		0.99		empty	

Note - See notes in Table 1.

Table 3: Canadian Unconstrained Estimation Results
Homogeneous Capital Market

	A=1.00				A=0.50				A=0.23			
Test	$\hat{\beta}$	$\hat{\theta}$	$\hat{F}q$	p-val	$\hat{\beta}$	$\hat{\theta}$	$\hat{F}q$	p-val	$\hat{\beta}$	$\hat{\theta}$	$\hat{F}q$	p-va
					Dynamic Model							
$\tau = 0$	0.90 (0.90,0.99)	0.62 (0.34,0.99)	(1.52,100)	0.39	0.90 (0.90,0.99)	0.50 (0.23,0.99)	(1.30,100)	0.39	0.90 (0.90,0.99)	0.36 (0.13,0.99)	(1.15,100)	0.3
$\tau = 1$ HR	0.99 (0.90,0.99)	0.99 (0.77,0.99)	(4.35,100)	0.15	0.99 (0.90,0.99)	0.99 (0.69,0.99)	(3.23,100)	0.15	0.90 (0.90,0.99)	0.99 (0.58,0.99)	(2.38,100)	0.1
$\tau = 1$ HAC	empty				empty				empty			
					Static Model							
$\tau = 0$	empty				empty				empty			
$\tau = 1$ HR	0.99 (0.98,0.99)	0.99 (0.80,0.99)	(5.00,100)	0.06	0.99 (0.98,0.99)	0.99 (0.73,0.99)	(3.70,100)	0.06	0.99 (0.98,0.99)	0.99 (0.62,0.99)	(2.63,100)	0.0
$\tau = 1$ HAC	empty				empty				empty			

Table 3c: Canadian Constrained Estimation Results
Homogeneous Capital Market

	A=1.00				A=0.50				A=0.23			
Test	β	$\hat{\theta}$	$\hat{F}q$	p-val	β	$\hat{\theta}$	$\hat{F}q$	p-val	β	$\hat{\theta}$	$\hat{F}q$	p-val
						Dynamic Model						
$\tau = 0$	0.99	0.60 (0.34,0.99)	2.50 (1.52,100)	0.36 (0.06,0.32)	0.99	0.48 (0.23,0.99)	1.92 (1.30,100)	0.36 (0.06,0.32)	0.99	0.35 (0.13,0.99)	1.54 (1.15,100)	0.36 (0.05,0.32)
$\tau = 1$ HR	0.99	0.99 (0.77,0.99)	100.00 (4.35,100)	0.15 (0.05,0.15)	0.99	0.99 (0.69,0.99)	100.00 (3.23,100)	0.15 (0.05,0.15)	0.99	0.99 (0.58,0.99)	100.00 (2.38,100)	0.15 (0.05,0.15)
$\tau = 1$ HAC	0.99		empty		0.99		empty		0.99		empty	
						Static Model						
$\tau = 0$	0.99		empty		0.99		empty		0.99		empty	
$\tau = 1$ HR	0.99	0.99 (0.80,0.99)	100.00 (5.00,100)	0.06 (0.05,0.06)	0.99	0.99 (0.73,0.99)	100.00 (3.70,100)	0.06 (0.05,0.06)	0.99	0.99 (0.62,0.99)	100.00 (2.63,100)	0.06 (0.05,0.06)
$\tau = 1$ HAC	0.99		empty		0.99		empty		0.99		empty	

Note - See notes in Table 1.

Table 4: Canadian Unconstrained Estimation Results
Firm-Specific Capital Market

	A=1.00				A=0.50				A=0.23			
Test	$\hat{\beta}$	$\hat{\theta}$	$\hat{Fq}$	p-val	$\hat{\beta}$	$\hat{\theta}$	$\hat{Fq}$	p-val	$\hat{\beta}$	$\hat{\theta}$	$\hat{Fq}$	p-va
					Dynamic Model							
$\tau = 0$	0.90 (0.90,0.99)	0.30 (0.10,0.99)	(1.11,100)	0.39	0.90 (0.90,0.99)	0.27 (0.09,0.99)	(1.10,100)	0.39	0.90 (0.90,0.99)	0.23 (0.07,0.99)	(1.08,100)	0.3
$\tau = 1$ HR	0.90 (0.90,0.99)	0.99 (0.51,0.99)	(2.04,100)	0.15	0.90 (0.90,0.99)	0.99 (0.49,0.99)	(1.96,100)	0.15	0.90 (0.90,0.99)	0.99 (0.44,0.99)	(1.79,100)	0.1
$\tau = 1$ HAC	empty				empty				empty			
					Static Model							
$\tau = 0$	empty				empty				empty			
$\tau = 1$ HR	0.99 (0.98,0.99)	0.99 (0.56,0.99)	(2.27,100)	0.06	0.99 (0.98,0.99)	0.99 (0.54,0.99)	(2.17,100)	0.06	0.99 (0.98,0.99)	0.99 (0.49,0.99)	(1.96,100)	0.0
$\tau = 1$ HAC	empty				empty				empty			

Table 4c: Canadian Constrained Estimation Results
Firm-Specific Capital Market

	A=1.00				A=0.50				A=0.23			
Test	β	$\hat{\theta}$	$\hat{Fq}$	p-val	β	$\hat{\theta}$	$\hat{Fq}$	p-val	β	$\hat{\theta}$	$\hat{Fq}$	p-val
					Dynamic Model							
$\tau = 0$	0.99	0.28 (0.10,0.99)	1.39 (1.11,100)	0.36 (0.07,0.32)	0.99	0.26 (0.09,0.99)	1.35 (1.10,100)	0.36 (0.08,0.32)	0.99	0.22 (0.07,0.99)	1.28 (1.08,100)	0.36 (0.07,0.32)
$\tau = 1$ HR	0.99	0.99 (0.51,0.99)	100.00 (2.04,100)	0.15 (0.05,0.15)	0.99	0.99 (0.49,0.99)	100.00 (1.96,100)	0.15 (0.05,0.15)	0.99	0.99 (0.44,0.99)	100.00 (1.79,100)	0.15 (0.05,0.15)
$\tau = 1$ HAC	0.99		empty		0.99		empty		0.99		empty	
					Static Model							
$\tau = 0$	0.99		empty		0.99		empty		0.99		empty	
$\tau = 1$ HR	0.99	0.99 (0.56,0.99)	100.00 (2.27,100)	0.06 (0.05,0.06)	0.99	0.99 (0.54,0.99)	100.00 (2.17,100)	0.06 (0.05,0.06)	0.99	0.99 (0.49,0.99)	100.00 (1.96,100)	0.06 (0.05,0.06)
$\tau = 1$ HAC	0.99		empty		0.99		empty		0.99		empty	

Note - See notes in Table 1.

Table 5: E.U. Unconstrained Estimation Results
Homogeneous Capital Market

	A=1.00				A=0.50				A=0.23			
Test	$\hat{\beta}$	$\hat{\theta}$	$\hat{F}q$	p-val	$\hat{\beta}$	$\hat{\theta}$	$\hat{F}q$	p-val	$\hat{\beta}$	$\hat{\theta}$	$\hat{F}q$	p
					Dynamic Model							
$\tau = 0$	empty				empty				empty			
$\tau = 1$ HR	empty				empty				empty			
$\tau = 1$ HAC	0.91	0.61	(2.56,2.56)	0.32	0.94	0.20	(1.25,1.25)	0.47	empty			
	(0.91,0.91)	(0.61,0.61)			(0.94,0.94)	(0.20,0.20)						
					Static Model							
$\tau = 0$	0.90	0.92	(7.69,100)	0.96	0.90	0.88	(5.26,100)	0.97	0.90	0.82	(3.45,100)	0
	(0.90,0.99)	(0.87,0.99)			(0.90,0.99)	(0.81,0.99)			(0.90,0.99)	(0.71,0.99)		
$\tau = 1$ HR	0.93	0.93	(10.00,100)	0.79	0.93	0.89	(6.25,100)	0.79	0.94	0.84	(4.17,100)	0
	(0.90,0.99)	(0.90,0.99)			(0.90,0.99)	(0.84,0.99)			(0.90,0.99)	(0.76,0.99)		
$\tau = 1$ HAC	empty				0.95	0.17	(1.20,1.20)	0.28	0.92	0.09	(1.10,1.10)	0
					(0.95,0.95)	(0.17,0.17)			(0.92,0.92)	(0.09,0.09)		

Table 5c: E.U. Constrained Estimation Results
Homogeneous Capital Market

	A=1.00				A=0.50				A=0.23			
Test	β	$\hat{\theta}$	$\hat{F}q$	p-val	β	$\hat{\theta}$	$\hat{F}q$	p-val	β	$\hat{\theta}$	$\hat{F}q$	p-val
					Dynamic Model							
$\tau = 0$	0.99		empty		0.99		empty		0.99		empty	
$\tau = 1$ HR	0.99		empty		0.99		empty		0.99		empty	
$\tau = 1$ HAC	0.99		empty		0.99		empty		0.99		empty	
					Static Model							
$\tau = 0$	0.99	0.96 (0.87,0.99)	25.00 (7.69,100)	0.82 (0.07,0.81)	0.99	0.94 (0.82,0.99)	16.67 (5.56,100)	0.82 (0.07,0.81)	0.99	0.91 (0.74,0.99)	11.11 (3.85,100)	0.82 (0.05,0.81)
$\tau = 1$ HR	0.99	0.96 (0.90,0.99)	25.00 (10.00,100)	0.65 (0.08,0.58)	0.99	0.94 (0.86,0.99)	16.67 (7.14,100)	0.65 (0.08,0.58)	0.99	0.90 (0.80,0.99)	10.00 (5.00,100)	0.65 (0.09,0.58)
$\tau = 1$ HAC	0.99		empty		0.99		empty		0.99		empty	

Note - See notes in Table 1.

Table 6: E.U. Unconstrained Estimation Results
Firm-Specific Capital Market

	A=1.00				A=0.50				A=0.23			
Test	$\hat{\beta}$	$\hat{\theta}$	$\hat{F}q$	p-val	$\hat{\beta}$	$\hat{\theta}$	$\hat{F}q$	p-val	$\hat{\beta}$	$\hat{\theta}$	$\hat{F}q$	p-
					Dynamic Model							
$\tau = 0$	empty				empty				empty			
$\tau = 1$ HR	empty				empty				empty			
$\tau = 1$ HAC	0.96 (0.96,0.97)	0.29 (0.29,0.29)	(1.41,1.41)	0.15	0.96 (0.96,0.99)	0.07 (0.07,0.27)	(1.08,1.37)	0.37	empty			
					Static Model							
$\tau = 0$	0.90 (0.90,0.99)	0.77 (0.66,0.99)	(2.94,100)	0.97	0.90 (0.90,0.99)	0.75 (0.63,0.99)	(2.70,100)	0.97	0.90 (0.90,0.99)	0.72 (0.59,0.99)	(2.44,100)	0.
$\tau = 1$ HR	0.93 (0.90,0.99)	0.79 (0.70,0.99)	(3.33,100)	0.79	0.93 (0.90,0.99)	0.78 (0.68,0.99)	(3.13,100)	0.80	0.93 (0.90,0.99)	0.75 (0.64,0.99)	(2.78,100)	0.
$\tau = 1$ HAC	empty				empty				empty			

Table 6c: E.U. Constrained Estimation Results
Firm-Specific Capital Market

	A=1.00				A=0.50				A=0.23			
Test	β	$\hat{\theta}$	$\hat{Fq}$	p-val	β	$\hat{\theta}$	$\hat{Fq}$	p-val	β	$\hat{\theta}$	$\hat{Fq}$	p-val
						Dynamic Model						
$\tau = 0$		empty				empty				empty		
$\tau = 1$ HR		empty				empty				empty		
$\tau = 1$ HAC		empty			0.99	0.27 (0.27,0.27)	1.37 (1.37,1.37)	0.18 (0.18,0.18)		empty		
						Static Model						
$\tau = 0$	0.99	0.89 (0.70,0.99)	9.09 (3.33,100)	0.82 (0.08,0.81)	0.99	0.89 (0.68,0.99)	9.09 (3.13,100)	0.82 (0.08,0.81)	0.99	0.87 (0.64,0.99)	7.69 (2.78,100)	0.82 (0.07,0.81)
$\tau = 1$ HR	0.99	0.88 (0.75,0.99)	8.33 (4.00,100)	0.65 (0.05,0.58)	0.99	0.88 (0.74,0.99)	8.33 (3.85,100)	0.65 (0.07,0.58)	0.99	0.86 (0.71,0.99)	7.14 (3.45,100)	0.65 (0.08,0.57)
$\tau = 1$ HAC		empty				empty				empty		

Note - See notes in Table 1.