The money-age distribution:

Empirical facts and economic modelling

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Abstract:
The money-age distribution is found to be hump-shaped for the US economy. The variation (inequality) of cash holdings within generations increases (declines) with age. Furthermore, cash holdings are found to be only weakly correlated with both income and wealth. We analyze three motives for money demand in an overlapping generations model in order to explain this effect: 1) money in the utility, 2) an economy with costly credit service, and 3) limited participation. Both the simple money-in-the-utility model and the economy with the cash-credit goods are able to replicate the hump-shape profiles of cash holdings and its variation, but not the decreasing inequality within generations over age. In addition, we discuss the optimality of the Friedman rule in heterogeneous-agent economies. In the first two models, the optimal inflation rate is above zero.

1 Introduction

The dynamic general equilibrium (DGE) literature has discussed the distribution of income and wealth to a large extent, but has basically ignored the distribution of cash. Díaz-Giménez, Quadrini and Ríos-Rull (1997) document the facts on the U.S. Distributions of earnings, income, and wealth. Earnings and income are much less concentrated than and only weakly correlated with wealth. Huggett (1996) shows that these facts can be replicated in a satisfactory manner in an OLG model where agents are characterized by heterogeneous productivity and receive social security. Huggett and Ventura (2000) also explain the consumption behavior over the life-cycle and explain why low-income households do not save.

To the best of our knowledge there is no comparable study on the money distribution over the life cycle. We use empirical evidence from the US to document that 1) cash holdings are hump-shaped over the life-cycle, 2) the variation of cash holdings increases with age, and 3) the variational coefficient of cash declines over the life-cycle. Furthermore, we find cash holdings to be only weakly correlated with income and wealth. Surprisingly, when we regress money holdings on age, income, and wealth, income is not a significant explanatory variable. The empirical evidence is found to be stable over time.

We develop a monetary general equilibrium model in order to explain the heterogeneity of cash holdings across individuals. Money is introduced in three different ways. First, we look at money in the utility. Households save in the form of money or capital. Second, we introduce costly credit. Households can consume a continuum of commodities that can be purchased with either cash or credit. Credit, however, is costly as in Dotsey and Ireland (1996). Again, money is a poor store of value since it is dominated in return by capital. Third, we look at limited participation. Firms need to finance wage expenditures with a loan, while households deposits part of their money at a bank. The central bank injects the money into the banking sector after the households have made the deposits, but before the firms ask for a loan. We find that the money-in-the-utility model falls short in the explanation of the inequality of cash holdings within generations. Contrary to the empirical observations the inequality generated by the model is low and increases over age.

The recent literature has emphasized the difference of the infinitely-lived representative agent model (ILRA) and the overlapping generations model (OLG) regarding the optimal-

ity of the Friedman rule. Freeman (1993) demonstrates that the Friedman rule is only optimal in an OLG model if the bequest motive is active. Bhattarchary, Haslag and Russell (2005) show that the key difference between the OLG and the ILRA model accounting for this divergent behavior are the intergenerational transfers that are caused by the redistribution of the seignorage. However, to the best of our knowledge, there is no paper that studies the optimal inflation rate in a computable general equilibrium OLG model for different specifications of money demand. We find that the optimal inflation rate crucially depends on the modelling of money. In our money-in-the-utility model, we find an optimal inflation rate equal to 4.60%. Reducing the inflation rate from the US postwar average rate of 4.32% to 0% would result in steady-state welfare losses equal to 1.15% of total consumption.

The remainder of the paper is structured as follows. Section 2 documents the empirical facts of the money-age distribution for the US economy. Section 3 introduces the overlapping-generations model with two assets, money and capital. The model is calibrated with regard to the characteristics of the US economy in section 4. Our numerical results are presented in section 5. Section 6 concludes.

2   Empirical observations

We use data from the 1994 PSID data and wealth file.[1] Money, m, is defined as money in checking or savings accounts, money market funds, certificates of deposit, government savings bonds, and treasury bills. Capital, k, consists of shares of stock in publicly held corporations, mutual funds, and investment funds and other savings or assets, such as bond funds and life insurance policies. Total family income is made of taxable and transfer income of head, wife, and other family unit members and Social Security Income. We observe the following regularities:

1. Money M is only weakly correlated with income Y (0.19) and wealth A (0.33).

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[1]We included only households with strictly positive cash holdings in our sample. To analyze the cash holding behavior depending on age in figures 1-3, we group the households in the following age categories: 20-24, 25-29, 30-34, 35-39, 40-44, 45-49, 50-54, 55-59, 60-64, 65-69, 70-74, 75-79, 80+.

Figure 1: Cash holdings over the life-cycle
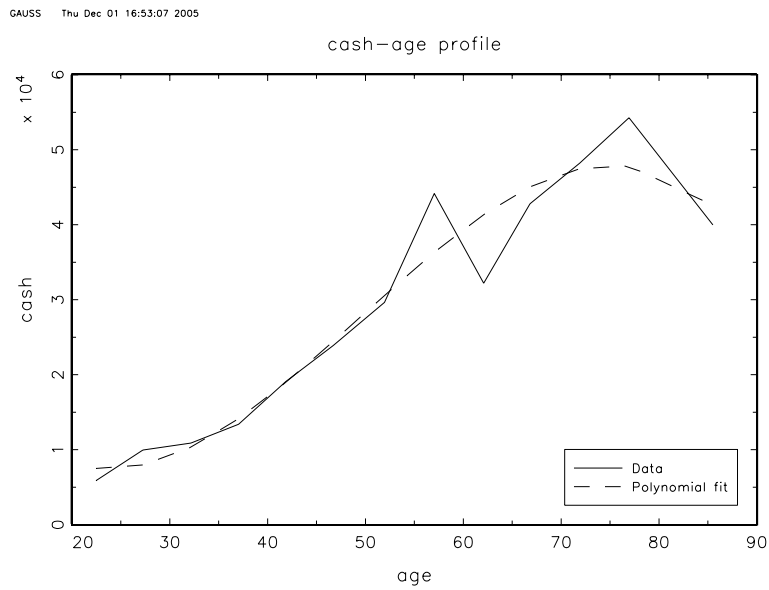
2. When we regress money on age, income and wealth, income is not a significant variable.[2]

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[2]This results is insensitive to the modelling of money and income in logs or level. However, we still have to study the sensitivity of these results with regard to the introduction of dummy terms on race and gender.

Table 1. Estimates of the following regression model model:

$$M_i = \beta_0 + \beta_1 Y_i + \beta_2 Y_i^2 + \beta_3 A_i + \beta_4 A_i^2 + \beta_5 \text{age} + \beta_6 \text{age}^2 + \varepsilon_t,$$

	Coefficient	Standard Deviation
β_0	2167	0.331
β_1	0.0280	0.573
β_2	-1.1E-08	9.3E-09
β_3	0.128	0.0157***
β_4	-1.8E-8	$4.5E-9$***
β_5	-183.9	289.9
β_6	6.233	2.811**
Adj. R^2		0.244
F-stat.		229.7

Note: *, **, *** $\equiv$ significance at 10, 5, 1% level of significance; Newey-West HAC standard errors.

3. Cash holdings over the life-cycle are hump-shaped. See Figure 1. We approximated the empirical money-age distribution (solid line) with the help of a 3rd degree polynomial (broken line).

4. The standard deviation of cash is hump-shaped as well. See Figure 2.

5. The dispersion as measured by the variational coefficient of cash holdings falls over the life-cycle, but the relationship is not monotonous as inequality of cash holdings

Figure 2: Standard deviation of cash over the life-cycle
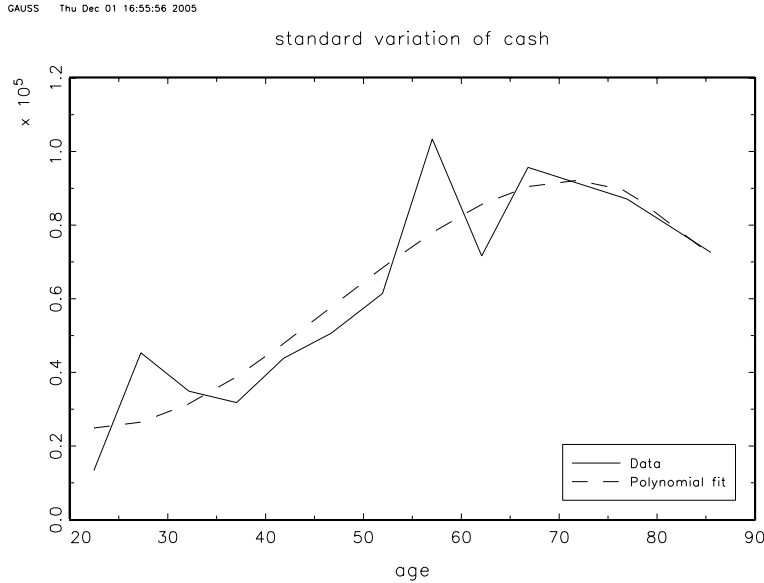
peak around age 25-29. See Figure 3. We still have to present evidence from various PSID surveys whether this observations is robust over time.

6. As illustrated in Figure 4, money is more unequally distributed than income and earnings, and almost as concentrated as wealth. The data for income, earnings and wealth is taken from the 1992 Survey of Consumer Finances.

3   The model

We study a general equilibrium overlapping generations model with three different motives for money demand: 1) money-in-the-utility, 2) a cash-credit good, and 3) limited participation. Four sectors can be charted: households, production, banking, and the government. Households maximize discounted life-time utility. Agents can save either with money or with capital. Individuals are heterogeneous with regard to their productivity and cannot insure against idiosyncratic income risk. Firms maximize profits. Output is produced with the help of labor and capital. The government collects taxes from labor and interest income in order to finance its expenditures on government consumption. The government

Figure 3: Cash holdings over the life-cycle
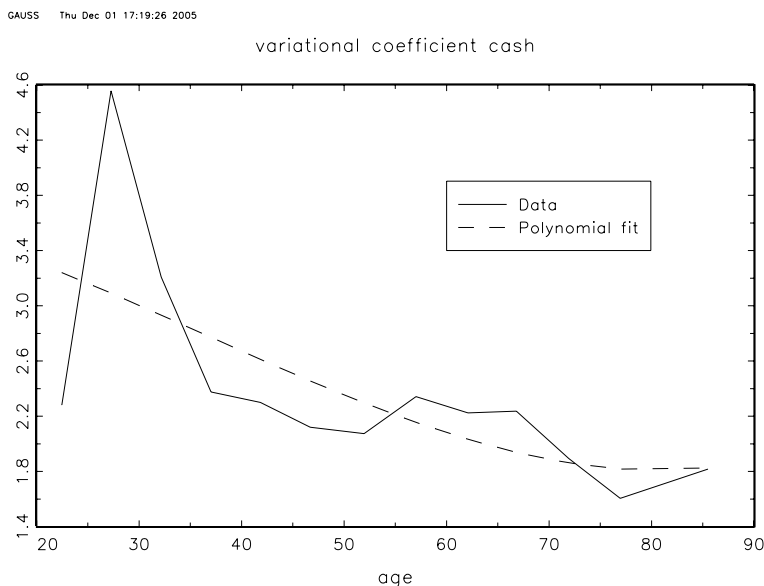
also provides social security and controls the money supply. In the limited-participation model, banks receive deposits from households and lend them to firms. We do restrict our analysis to steady-state behavior. For simplicity of notation we drop the time indices of the aggregate variables in our economy when appropriate.

3.1   Households

Every year, a generation of equal measure is born. The total measure of all generations is normalized to one. As we only study steady-state behavior, we concentrate on the behavior of an individual born in period 0. Their first period of life is period 1. The total measure of all households is normalized to one.

Households live a maximum of $T + T^R$ years. Lifetime is stochastic and agents face a probability s_j of surviving up to age j conditional on surviving up to age $j - 1$. During their first T years, agents supply one unit of labor inelastically. After T years, retirement is mandatory. Workers are heterogeneous with regard to their labor earnings. Labor earnings $e(z, j)w$ are stochastic and depend on individual age j, an idiosyncratic labor productivity shock z, and the wage rate w. Furthermore, agents hold two kinds of assets, real money

Figure 4: Cross-section distribution of earnings, income, wealth and money
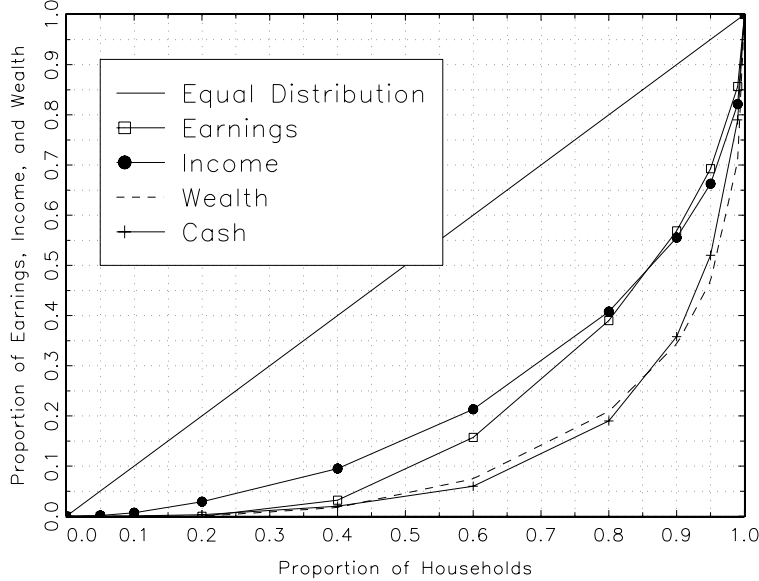
$m = M/P$ and capital k, where M and P denote nominal money and the price level, respectively. The household h is born without any capital in period t, $k_{1t}^h \equiv 0$. The first generation is endowed with a strictly positive amount of nominal money, $M_{1t}^h = \bar{M}_0^h$.[3] Capital or, equally, equity k earns a real interest rate r. Parents do not leave altruistic bequests to their children. All accidental bequests are confiscated by the state.

The household maximizes her life-time utility:

$$E_0 \left[\sum_{j=1}^{T+T^R} \beta^{j-1} \left(\Pi_{l=1}^j s_l \right) u_t, \right] \tag{1}$$

where β and u_t the discount factor and utility in period t, respectively. We will study various roles for money. In our case 1, we simply consider money in the utility:

$$\text{case 1:} \quad u(c,m) = \frac{\left(c^\gamma m^{1-\gamma}\right)^{1-\sigma}}{1-\sigma} \tag{2}$$

where c, m, and $\sigma > 0$ denote consumption, real-money balances, and the coefficient of relative risk aversion, respectively.

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[3] Otherwise, the level of utility at age 1 is not well-defined. The calibration of $\bar{M}_0^h$ is discussed in section 4.

In our second specification, consumers can purchase consumption with cash or credit as in Schreft (1992), Gillman (1993), or Dotsey and Ireland (1996). The consumption goods are indexed by $i \in [0, 1]$, and the consumption aggregator is given by $c = \inf_i\{c(i)\}$. Therefore, the individuals will consume the same amount of all goods as in Schreft (1992). The period utility u_t is of the form

$$\text{case 2 and 3:} \quad u(c) = \frac{c^{1-\sigma}}{1-\sigma}. \tag{3}$$

In order to buy an amount c of good i with credit, the household must purchase $\kappa(c, i)$ units of financial services. The function $\kappa(.,.)$ is weakly increasing in c, strictly increasing in i, and satisfies $\lim_{i \to 1} \gamma(c, i) = \infty$ for all $c \geq 0$. According to the latter assumption, some goods will be purchased with cash, and the demand for money is well defined. In particular, the transaction technology is given by the sum of a variable and a fixed costs term:

$$\kappa(c, i) = \kappa_0 \left(\frac{i}{1-i}\right)^\chi + \frac{\kappa_1}{c(i)}. \tag{4}$$

For $\kappa_1 = 0$, fixed costs are zero, and the technology displays constant returns to scale.[4] Intermediation of credit services is subject to perfect competition, and in order to produce one unit of service one efficiency unit of labor is used. In equilibrium, the financial service companies make zero profit, and the fees q per unit of financial service sold is equal to the wage rate w.

The household will purchase a fraction $\zeta \in [0, 1)$ of consumption goods with credit. He faces the following cash-in-advance constraint on his remaining purchases:

$$\text{case 2:} \quad c(1 - \zeta) \leq m. \tag{5}$$

In our third specification, we look at limited participation. In this case, household deposit part of the financial wealth at banks at the gross nominal interest R. The firms pay wages to the households before they sell their output. To finance the wage bill they borrow money from the banking sector. The government injects the money via the banking sector.

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[4]Erosa and Ventura (2002) have shown that inflation does not affect (increases) wealth inequality in the case of constant (decreasing) returns to scale.

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Crucially banks receive the monetary transfer after households have decided about the volume of their banking deposits.

Households hold financial wealth $M_t = D_t + X_t$ where D_t is the amount deposited at banks and X_t are cash balances kept for the purchase of consumption goods. Since households receive wages before they go shopping, their cash-in-advance constraint is

$$\text{case 3:} \quad c_t \leq \begin{cases} x_t + (1 - \tau_w - \theta)w_t e(z_t^h, j), & t \leq T \\ x_t + b(\bar{e}_{jt}^h), & t > T. \end{cases} \tag{6}$$

where x_t, τ_w, and θ denote real cash balances, labor income taxes, and social security contributions, respectively.

The j-year old agent h receives income from capital $k_{j,t}^h$ and labor $e(z_t^h, j)w$ in period t. After retirement agents do not work, $e(z, j) = 0$ for $j > T$. The budget constraint of the j-year old household h in period t, $j = 1, \ldots, T + T^R$, is given by:[5]

$$(1 - \tau_r)rk_{j,t}^h + (1 - \tau_w - \theta)we(z_t^h, j) + b(\bar{e}_{jt}^h) + tr + k_{j,t}^h + m_{j,t}^h \tag{7}$$

$$= \begin{cases} c_t^h + k_{j+1,t+1}^h + m_{j+1,t+1}^h(1 + \pi) - Seign & \text{case 1} \\ c_t^h + q \int_0^\zeta \kappa(c, i)\, di + k_{j+1,t+1}^h + m_{j+1,t+1}^h(1 + \pi) - Seign & \text{case 2} \\ c_t^h + (1 - \tau_r)(R - 1)d_t^h + \Omega^B + k_{j+1,t+1}^h + m_{j+1,t+1}^h(1 + \pi) & \text{case 3} \end{cases}$$

where $Seign$ and $\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$ denote seignorage and the inflation rate, respectively. In cases 1 and 2, households receive the seignorage. In the limited participation model, the central bank injects the increase in the money supply in the banking sector, while households also receive lump-sum profits from banks, Ω^B, and earn interest R on their real deposits d_t^h. Real interest income is taxed at the rate τ_r. In addition, the households receive transfers tr from the government. Social security benefits $b_t(\bar{e}^h, j)$ depend on the agent's age j as well as on an average of past earnings $\bar{e}^h$ of the household h. Following Huggett and Ventura (2000), social security benefits are composed of a lump-sum component and an earnings-related benefit:

$$b(\bar{e}_{j,t}) = \begin{cases} 0 & \text{for } j \leq T \\ b_0 + b_1(\bar{e}_{j,t}) & \text{for } j > T \end{cases} \tag{8}$$

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[5]At the end of the final period, $k_{T+T^R+1,t}^h = M_{T+T^R+1,t}^h \equiv 0$.

The function $b_1(\bar{e}_{j,t})$ is described in more detail in section 4.

3.2 Production

Firms are of measure one and produce output with effective labor N and capital K. Effective labor N is paid the wage w. In the case of the limited participation model, firms have to pay workers in advance and have to borrow wN at the nominal interest rate $R - 1$ in advance. In case 1 and 2, $R = 1$. Capital K is hired at rate r and depreciates at rate δ. Production Y is characterized by constant returns to scale and assumed to be Cobb-Douglas:

$$Y = F(K, N) = K^{\alpha} N^{1-\alpha}. \tag{9}$$

In a factor market equilibrium, factors are rewarded with their marginal product:

$$Rw = (1 - \alpha) K^{\alpha} N^{-\alpha}, \tag{10}$$

$$r = \alpha K^{\alpha-1} N^{1-\alpha} - \delta. \tag{11}$$

Consequently, profits are zero.

3.3 Banking Sector

In the limited participation model we also model a banking sector. At the beginning of period t banks receive deposits of size D_t from households. Government transfers the amount $M_{t+1} - M_t$ to the banks that are able to lend $D_t + M_{t+1} - M_t$ to firms. At the end of the period t they pay interest and principal RD_t to their creditors and distribute the remaining real profits Ω^B to the houesholds:

$$\Omega_t^B = \frac{R(D_t + M_{t+1} - M_t)}{P_t} - \frac{RD_t}{P_t} = R\frac{M_{t+1} - M_t}{P_t}. \tag{12}$$

3.4 Government

The government consists of the fiscal and monetary authority. Nominal money grows at the exogenous rate μ:

$$\frac{M_{t+1} - M_t}{M_t} = \mu. \tag{13}$$

In cases 1 and 2, seignorage $Seign = M_{t+1} - M_t$ is transferred lump-sum. In case 3, money is injected into the banking sector.

The government uses the revenues from taxing income and aggregate accidental bequests Beq in order to finance its expenditures on government consumption G, government transfers Tr, and transfers to the one-year old households $\tilde{m}$:[6]

$$PG + Tr + \tilde{m} = \tau_r rk + \tau wN + Beq. \tag{14}$$

Furthermore, the government provides social security benefits that are financed by taxes.

3.5 Stationary equilibrium

The concept of equilibrium applied in this paper uses a recursive representation of the consumer's problem following Stokey, Lucas, and Prescott (1989). Let $\phi_j(k, m, d, \bar{e}, z)$ and $V_j(k, m, d, \bar{e}, z)$ denote the measure and the value of the objective function of the j-year old agent with equity k, real money m, deposits d, average earnings $\bar{e}$, and idiosyncratic productivity level z, respectively. $V_j(k, m, d, \bar{e}, z)$ is defined as the solution to the dynamic program:

$$V_j(k, m, d, \bar{e}, z) = \max_{k', m', d', c} \{u + \beta s_{j+1} E\left[V_{j+1}(k', m', d', \bar{e}', z')\right]\} \tag{15}$$

subject to (7). k', m', d', $\bar{e}'$, and z' denote the next-period value of k, m, d, $\bar{e}$, and z, respectively. Optimal decision rules at age j are a function of k, m, d, $\bar{e}$, and z, i.e. consumption $c_j(k, m, d, \bar{e}, z)$, deposits $d_j(k, m, \bar{e}, z)$, next-period capital stock $k_{j+1}(k, m, d, \bar{e}, z)$, and next-period real money balances $m_{j+1}(k, m, d, \bar{e}, z)$. In case II, the optimal share of cash goods also depends on the individual state variables, $\zeta_j = \zeta_j(k, m, \bar{e}, z)$.

We will consider a stationary equilibrium where factor prices, aggregate capital, and labor are constant and the distribution of wealth is stationary.

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[6]We assume that the first-period money balances are financed by the government from, for example, accidental bequests. Alternatively, we could have modelled an intergenerational structure as in Laitner (1992,1993) and assume that the 50-year old agents (corresponding to $j = 30$) leave their 20-year old children (corresponding to $j = 1$) real money balances $\bar{m}_0^h$. In this case, we would have also made the assumption that agents do not die until age $j = 30$, which is in good accordance with empirical survival probabilities. Our results, however, are not affected by our modelling choice with regard to the financing of $\tilde{m}$.

4 Calibration

Periods correspond to years. We assume that agents are born at the real lifetime age 20 which corresponds to $j = 1$. Agents work $T = 40$ years corresponding to a real lifetime age of 60. They live a maximum life of 60 years ($T^R = 20$) so that agents do not become older than the real lifetime age 79. The sequence of conditional survival probabilities $\{s_j\}_{j=1}^{59}$ is set in accordance with the age-specific death rates in the US in the year 2000. The data is taken from the United States Life Tables 2000 provided by the National Center of Health.[7] The survival probabilities almost monotonously decrease with age. For the final period of our model, we set the survival probability s_{60} equal to zero.

The calibration of the production parameters α and δ and the Markov process $e(z, j)$ is chosen in accordance with existing general equilibrium studies: Following Prescott (1986), the capital income share α is set equal to 0.36. The annual rate of depreciation is set at $\delta = 0.08$. Earnings are the product of real wage per efficiency unit times the labor endowment $e(z, j)$. The labor endowment process is given by $e(z, j) = e^{z_j + \bar{y}_j}$, where $\bar{y}_j$ is the mean lognormal income of the j-year old. The mean efficiency index $\bar{y}_j$ of the j-year-old worker is taken from Hansen (1993) and interpolated to in-between years. As a consequence, we are able to replicate the cross-section age distribution of earnings of the US economy. We also normalize the average efficiency index to one. The age-productivity profile is hump-shaped and earnings peak at age 50. In our benchmark case, agents differ in log labor endowments at birth and there is no income mobility within an age cohort so that $z_j = z_{j-1}$ for all $j > 1$. In the appendix, we also study the case where the idiosyncratic productivity shock z_j follows a Markov process:

$$z_j = \rho z_{j-1} + \epsilon_j, \tag{16}$$

where $\epsilon_j \sim N(0, \sigma_\epsilon)$. Huggett (1996) uses $\rho = 0.96$ and $\sigma_\epsilon = 0.045$. Furthermore, we follow Huggett and choose a lognormal distribution of earnings for the 20-year old with $\sigma_{y_1} = 0.38$ and mean $\overline{y_1}$. As the log endowment of the initial generation of agents is normally distributed, the log efficiency of subsequent agents will continue to be normally distributed. This is a useful property of the earnings process, which has often been described as lognormal in the literature. With our earnings specification, we are also able to replicate the

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[7]See Table 1 in Arias (2002).

earnings heterogeneity that is observed in US data. Henle and Ryscavage (1980) compute an earnings Gini coefficient for men of 0.42 in the period 1958-77. In the model without (with) earnings mobility, the Gini coefficient also amounts to 0.424 (0.44).

The social security payment $b(\bar{e}, j)$ is calibrated and parameterized in order to match the US Social Security System and exactly follows Huggett and Ventura (2000).[8] Average earnings $\bar{e}_{j,t}$ of the j-year old in period t accumulate according to:

$$\bar{e}_{j,t} = \begin{cases} \left(\bar{e}_{j-1,t-1}(j-1) + \min \left\{ e(z_t^h, j) w_t, e_{max} \right\} \right) / j & \text{for } j \leq T \\ \bar{e}_{j-1,t-1} & \text{else.} \end{cases} \tag{17}$$

Notice that in the US benefits depend on mean earnings that are indexed so that later contributions in life are not discounted. Furthermore, average earnings are only calculated for up to some maximum earnings level e_{max} which amounts to 2.47 times average earnings $\bar{E}$.[9] Following Huggett and Ventura, we set the lump-sum benefit b_0 equal to 12.42% of GDP per capita in the model economy. Finally, benefits are regressive and a concave function of average earnings. Let $\bar{e}^h$ and $\bar{E}$ denote the average earnings of individual h and the average earnings of all workers, respectively. Depending on which earnings bracket the retired agent's average earnings $\bar{e}^h$ were situated, he received 90% of the first $20\% \cdot \bar{E}$, 32% of the next 104% of $\bar{E}$, and 15% of the remaining earnings $(\bar{e}^h - 1.24\bar{E})$ in 1994. Therefore, the marginal benefit rate declines with average earnings. The social security contribution rate θ is calibrated so that the budget of the social security balances. The remaining parameters of the government policy that we need to calibrate are the two tax rates τ_r and τ_w and government expenditures G. The two tax rates $\tau_r = 42.9\%$ and $\tau_w = 24.8\%$ are computed as the average values of the effective US tax rates over the time period 1965-88 that are reported by Mendoza, Razin, and Tesar (1994). The share of government consumption in GDP is $G/Y = 19.5\%$, which is equal to the average ratio of G/Y in the US during 1959-93 according to the Economic Report of the President (1994). The model parameters are presented in table 2.

We choose the coefficient of risk aversion $\sigma = 2$.[10] The discount factor $\beta = 1.011$ is set equal to the estimate of Hurd (1989). In case 1, the remaining parameter γ from the utility

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[8]For a more detailed description of this procedure please see Huggett and Ventura (2000).

[9]In the US Social Security System, only the 35 highest earnings payments are considered in the calculation of the average earnings. We simplify the analysis by using all 40 working years in our model.

[10]All our qualitative results also hold for the case $\sigma \in \{1, 4\}$.

Table 2: Calibration of parameter values for the US economy

Description	Function/Parameter	Parameter Value
utility function	$U = \dfrac{\left(c^{\gamma}m^{1-\gamma}\right)^{1-\sigma}}{1-\sigma}$	$\sigma = 2.0$, $\gamma = 0.9785$ (case 1) $\gamma = 1$ (cases 2 and 3)
discount factor	β	$\beta = 1.011$
production function	$Y = K^{\alpha}N^{1-\alpha}$	$\alpha = 0.36$
depreciation	δ	$\delta = 0.08$
financial services	$\kappa_0 \left(\frac{i}{1-i}\right)^{\chi} + \frac{\kappa_1}{c(i)}$	$\kappa_0 = 0.240$, $\chi = 0.3232$ $\kappa_1 \in \{0, 0.5\}$
money growth rate	μ	$\mu = 0.0432$
income tax rates	τ_r, τ_w	$\tau_r = 42.9\%$, $\tau_w = 24.8\%$
government consumption	G	$G/Y = 19.5\%$
social security benefits maximum earnings level lump-sum benefit $b_1(\bar{e})$	e_{max} b_0 earnings bracket $[0, 0.2\bar{E}]$ $(0.2\bar{E}, 1.24\bar{E}]$ $1.24\bar{E} < \bar{e} \leq e_{max}$	$e_{max} = 2.47\bar{E}$ $b_0 = 0.1241Y$ marginal benefit rate 0.90 0.32 0.15

function is chosen to match the average velocity of money PY/M. During 1960-2001, the average annual velocity of M1 amounted to 5.18, while the average inflation rate was equal to 4.32%. We set $\gamma = 0.9785$ implying a velocity of money in our benchmark model without productivity mobility equal to 5.18 (for $\pi = 4.32\%$). The initial endowment with money $\bar{M}_0^h$ is chosen to match the empirical distribution of money among the 20-year-old generation. For this reason, we computed the cash holdings (relative to the average cash holdings in the economy) of the income percentiles from the PSID 1994 data and wealth files and choose $\bar{M}_0^h/M$ in our model accordingly. In case 2, we follow Erosa and Ventura (2002) and choose the parameters κ_0 and χ so that i) 82% of all household transactions are made with cash and ii) the semi-interest elasticity of money demand for inflation rates between 0 and 10% amounts to 5.95.[11] In our benchmark, $\kappa_1 = 0$.

The computation of the model is briefly described in the appendix.

5 Findings

5.1 Money-age profile

5.1.1 Money in the utility

Figure 5 displays the age profiles of various variables for the money-in-the-utility model. As typically found in life-cycle models, agents build up capital until retirement, $T = 40$, and decumulate savings thereafter. Consumption is hump-shaped over the life-cycle (and accords well with empirical observations) as the net interest rate exceeds the inverse of the discount factor β in young years. Since the survival probability declines in old age, the discount factor increases and consumption declines again. Notice that our model is able to replicate the hump-shaped money holdings that we observe in the data. Both money holdings of each productivity class (upper right panel in figure 5) and average money holdings of each generation (lower right panel in figure 5) are hump-shaped over the life-cycle.

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[11] In the preliminary version, we set $\chi = 0.3232$ as in Erosa and Ventura (2002) and calibrate $\kappa_0 = 0.240$ implying a cash share equal to 82%.

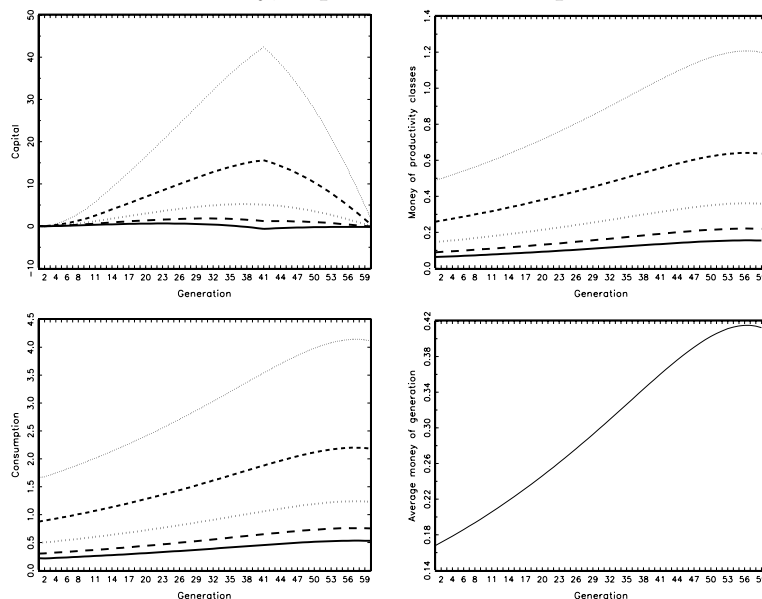Figure 5: Case 1: Money, capital and consumption over the life-cycle

Figure 6 displays the standard deviation and variational coefficient of money holdings within generations. While we are able to model the hump-shaped cash variance-age profile, the inequality of the cash distribution within generations increases with age in our model, while it declines in the data. In addition, the variational coefficient of the cash holdings within generations in our model falls short of those values observed empirically and is smaller by a factor 10. The Gini coefficient of money is equal to 0.498 in our model.

Money is also much more positively correlated with income (0.775) than in the data (0.19).

5.1.2   Costly credit

While money holdings increases over age even after retirement in the money-in the utility model, this observation does not hold in the economy with costly credit for the high-productivity households. As presented in Figure 7, the more productive agents build up high savings and increase the share of goods they buy with the help of credit. For the agents with the highest productivity, money holdings decrease after model period 18 (corresponding to real life age 49). In their final period of life, these agents finance less than 10% with the help of cash (please see lower right picture in Figure 7).[12]

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[12]In the final version of the paper, we are planning to confront this fact with the data.

Figure 6: Case 1: Dispersion and inequality of generational money holdings
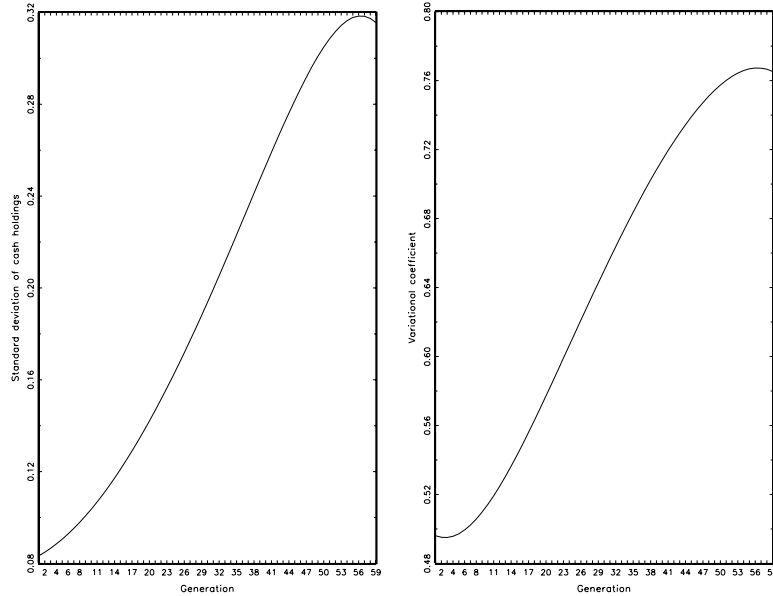
Similarly, the standard deviation of cash holdings is more hump-shaped as older agents finance a larger share of their consumption with the help of credit. The dispersion of money of age is illustrated in Figure 8. Compared to the empirical age distribution in Figure 2, the model with costly credit fits the data more accurately than the money-in-the-utility model. However, the inequality of generational money holdings, again, is hump-shaped and does not decline prior to retirement age.

The correlation of money with income is a little lower than in the money-in-the-utility model as richer agents finance a higher share of their consumption with costly credit. However, it is still much larger than in the data (0.65 versus 0.19). In addition, we find a stronger correlation of money and wealth (0.50) than in the data (0.35). Therefore, also the cash-credit economy does not proliferate a much more realistic description of the empirical observed portfolio allocation on money and wealth among households than the money-in-the-utility model.

5.1.3 Limited participation

In the limited participation model, consumption can be financed by both cash and the wage or wage replacement income (pensions). For this reason, cash holdings of the income-rich

Figure 7: Case 2: Money, capital and consumption over the life-cycle
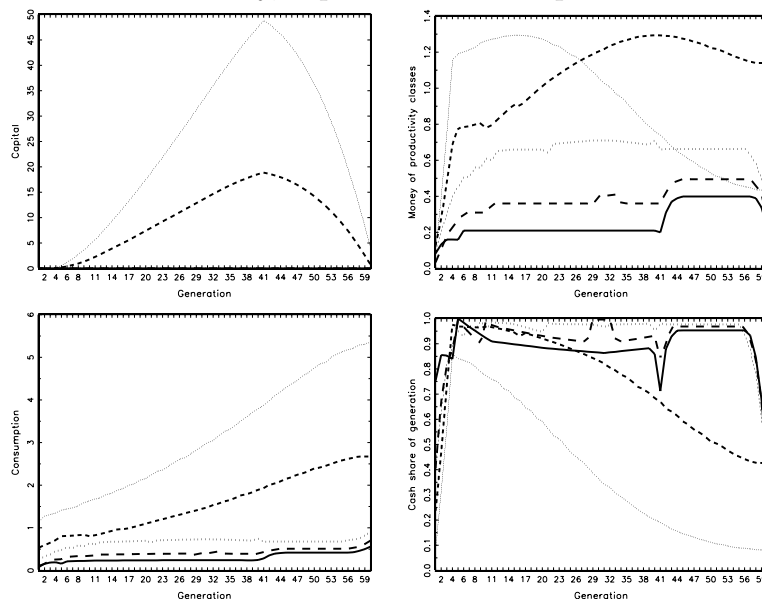
agents sharply increase at the age of retirement as pensions in period 41 are much lower than wage income in period 40 (see Figure 9). Likewise, the cash share, defined as the ratio of cash holdings to consumption also increases in period 41 when the agents retire. We do not observe this effect for the low-income households as pensions are almost equal to the wage income for these agents. Remember that, in the US, pensions also consist of a large lump-sum component. Since the behavior of the cash share and its change is different between the income groups we also observe an increase of the dispersion and inequality of cash holdings around retirement. The dispersion and the inequality of cash holdings among the generations is illustrated in fFigure 10.

Because of the sharp increase of money holdings during retirement we also observe a low correlation of money with income. In fact, the correlation of these two variables in the model (0.02) is even lower than the one observed empirically (0.19). Therefore, we carefully conclude that the introduction of a wage component in the cash-in-advance constraint helps to reconcile the implications of the model for the cross-section distribution with the empirical observations. Similarly, the correlation of money with wealth amounts to 0.27 (0.33 in the data), and the Gini coefficient of money is equal to 0.53.

Figure 8: Case 2: Dispersion and inequality of generational money holdings
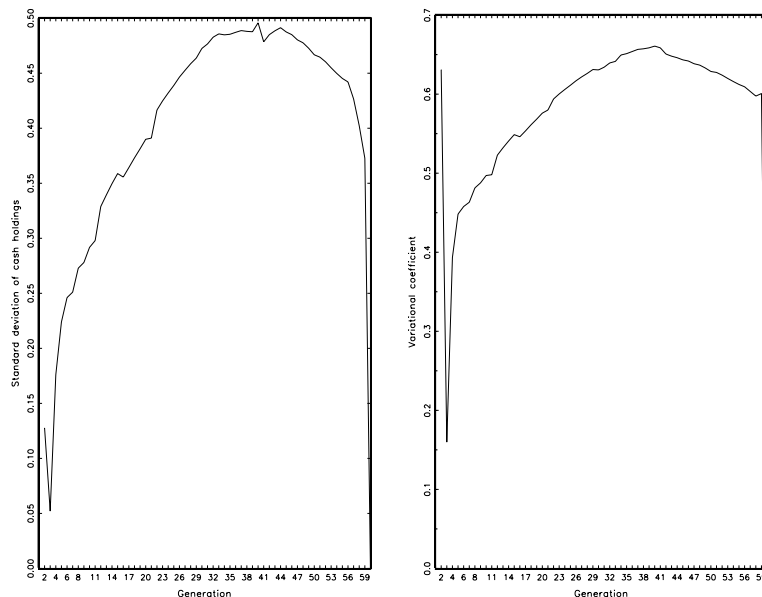

5.2 Optimal monetary policy

The optimality of the Friedman rule has been demonstrated to hold in many infinitely-lived representative agent models. However, recent literature on OLG models has shown that optimal inflation rates can also be positive. In the following, we will compute the actual numbers for optimal inflation rates for our three specifications of money demand.

Table 3: Optimal steady-state monetary policy

	optimal inflation rate	Welfare gain for $\pi = 0\%$
Money in the utility	4.60%	-1.15%
cash-credit purchases		-3.13%
		-4.20%
limited participation		

In our money-in-the-utility model the optimal inflation rate is equal to 4.6% (see table 3). Reducing the inflation rate from the average postwar US inflation rate of 4.32% to 0% entails steady-state welfare losses equivalent to 1.15% of total consumption. Welfare

19

Figure 9: Case 3: Cash and cash share over the life-cycle
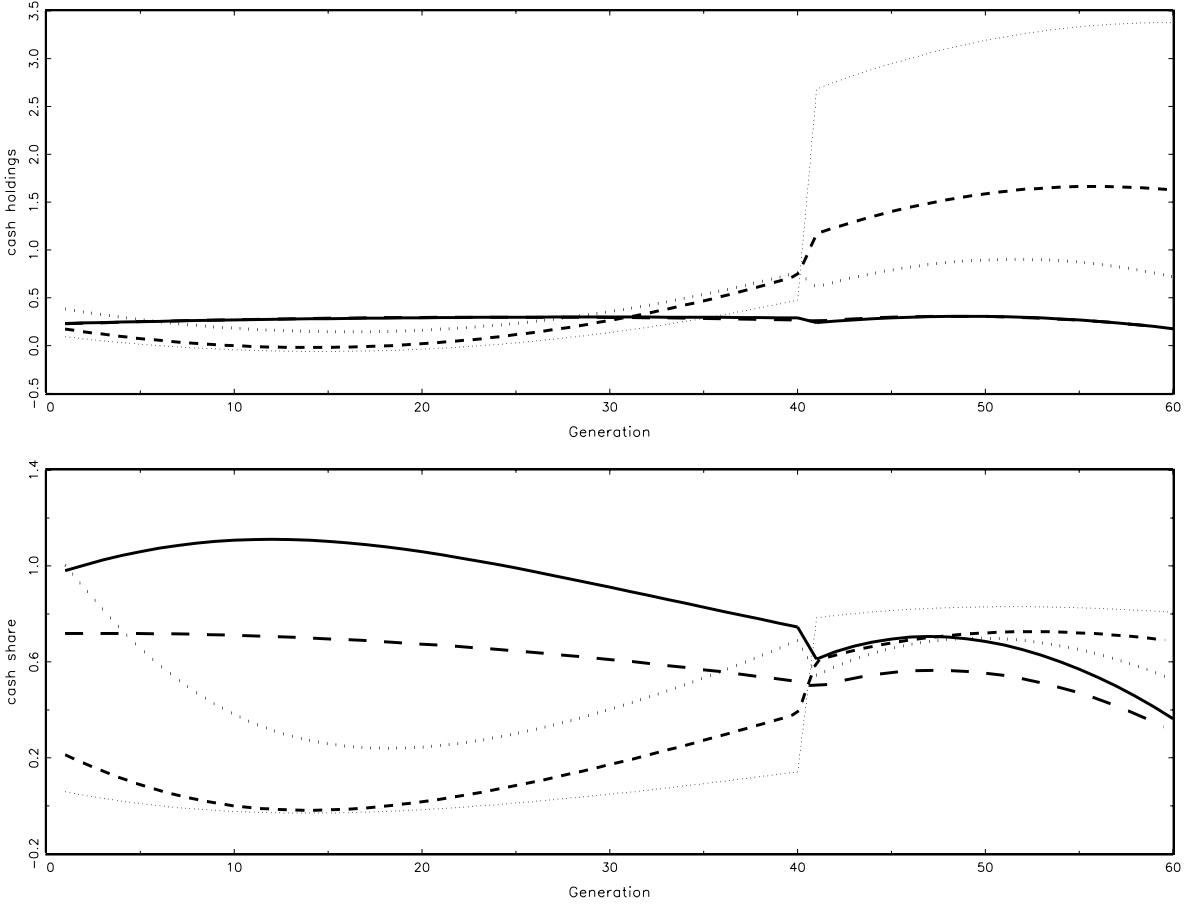
losses are even more significant in the case of the cash-credit economy and the limited-participation model. Clearly, the OLG model with income heterogeneity behaves different from the infinitely-lived representative-agent model.

6   Conclusion

When we extend the familiar infinitely-lived representative economy with either 1) money-in-the-utility, 2) costly credit or 3) limited participation to the overlapping-generations model with heterogeneous productivity types, we encounter many counterfactual implications for the money-age and cross-sectional money distribution. None of these economies with either of the three money demand motives is able to reconcile its implications for the

Figure 10: Case 3: Dispersion and inequality of generational money holdings
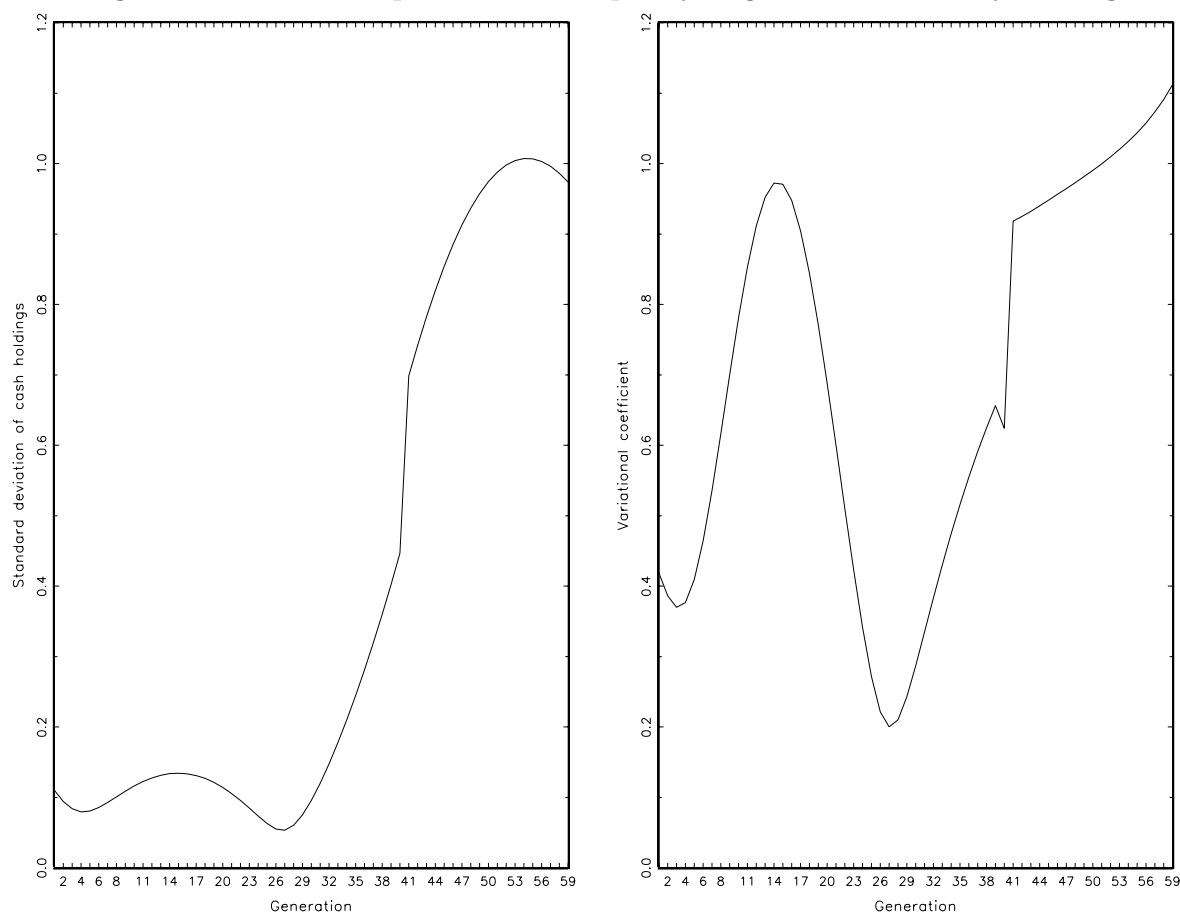
money distribution with the empirical facts with regard to the dispersion of money holdings and the cross-section correlation of money with income and wealth. Furthermore, all three specifications imply huge welfare losses from a reduction of the steady state inflation rate from the US postwar average of 4.32% to 0% amounting to more than one percentage point of consumption. We conclude that our knowledge of the cross-section distribution of money is limited. Further research is needed in order to understand the dispersion of cash holdings over the life-cycle.

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7 Appendix

7.1 First-order conditions: cash-credit economy (not to be published)

In steady state prices are constant, $q = w$, and the individual state only depends on age j and productivity ϵ.

The Lagrangian for the household at age 0:

$$
\begin{aligned}
\mathcal{L} = & \sum_{j=0}^{T+T^R} \beta^j \left(\Pi_{l=1}^j s_l \right) \left\{ \frac{\left(c_j^\epsilon \right)^{1-\sigma}}{1-\sigma} \right. \\
& + \lambda_j^\epsilon \left[(1-\tau_r)rk_j^\epsilon + (1-\tau_w-\theta)we(z,j) + b(\bar{e}_j) + tr + k_j^\epsilon + m_j^\epsilon \right. \\
& \left. - c_j^\epsilon - q \int_0^{\zeta_j^\epsilon} \kappa(i)\, di - \omega_{j+1}^\epsilon + Seign \right] \\
& \left. + \Psi_j^\epsilon \left[m_j^\epsilon - c_j^\epsilon(1-\zeta_j^\epsilon) \right] + \mu_j^\epsilon \left[\omega_j^\epsilon - k_j^\epsilon - m_j^\epsilon(1+\pi) \right] \right\}.
\end{aligned}
$$

Interior solution $0 < \zeta_j < 1$ if

$$
\left[1 + (1-\tau_r)r \right] k_j^\epsilon + (1-\tau_w-\theta)we(z,j) + b(\bar{e}_j) + tr + Seign < \omega_{t+1}
$$

In case of an interior solution $\underline{0 < \zeta_j < 1}$, the first order conditions are given by (dropping the index ϵ):[13]

$$
\begin{aligned}
c_j^{-\sigma} &= \lambda_j + \Psi_j(1-\zeta_j) & (18) \\
\lambda_j w \kappa(\zeta_j) &= \Psi_j c_j & (19) \\
\lambda_j &= \beta s_{j+1}\lambda_{j+1}\left[1 + (1-\tau_r)r \right] & (20) \\
\lambda_j &= \beta s_{j+1}\frac{\lambda_{j+1} + \Psi_{j+1}}{1+\pi} & (21) \\
m_j &= c_j(1-\zeta) & (22)
\end{aligned}
$$

---

[13]We used the Leibniz rule:

$$
\frac{d}{dx}\int_{a(x)}^{b(x)} F(t,x)\, dt = \int_{a(x)}^{b(x)} F_x(t,x)\, dt + F(b(x),x)\, b'(x) - F(a(x),x)\, a'(x).
$$

$$\Rightarrow$$

$$
\begin{align}
w\kappa(\zeta_j) &= Rc_j \tag{23}\\
c_j^{-\sigma} &= \lambda_j\left(1 + (1 - \zeta_j)R\right) \tag{24}\\
m_j &= (1 - \zeta_j)c_j, \tag{25}
\end{align}
$$

where the nominal interest rate R is given by

$$R = \left[1 + (1 - \tau_r)r\right](1 + \pi) - 1$$

In the case $\underline{\zeta_j = 0}$:

$$
\begin{align}
m_j &\geq c_j \tag{26}\\
c_j^{-\sigma} &= \lambda_j \tag{27}\\
c_j &= \left[1 + (1 - \tau_r)r\right]k_j^\epsilon + m_j + (1 - \tau_w - \theta)we(z,j) + b(\bar{e}_j) + tr + Seign - \omega_{j+1} \tag{28}\\
w\kappa(0) &> Rc_j. \tag{29}
\end{align}
$$

7.2 First-order conditions: economy with limited participation (not to be published)

The Lagrangian for the household at age 0:

$$
\begin{aligned}
\mathcal{L} \;=\; & \sum_{j=0}^{T+T^R} \beta^j \left(\Pi_{l=1}^j s_l \right) \left\{ \frac{\left(c_j^\epsilon \right)^{1-\sigma}}{1-\sigma} \right. \\
& + \lambda_j^\epsilon \left[(1-\tau_r) r k_j^\epsilon + (1-\tau_w - \theta) we(z,j) + b(\bar{e}_j) + tr + \Omega_B + k_j^\epsilon + (1 + (R-1)(1-\tau_r)) d_j^\epsilon + x_j^\epsilon \right. \\
& \left. - c_j^\epsilon - k_{j+1}^\epsilon - (d_{j+1}^\epsilon + x_{j+1}^\epsilon) \cdot (1+\pi) \right] \\
& \left. + \mu_j^\epsilon \left[(1-\tau_w - \theta) we(z,j) + b(\bar{e}_j) + x_j^\epsilon - c_j^\epsilon \right] \right\}.
\end{aligned}
$$

First-order conditions (we drop the index ϵ):

$$
\begin{aligned}
c_j^{-\sigma} &= \lambda_j + \mu_j \\
\lambda_j(1+\pi) &= \beta s_{j+1} \left[\lambda_{j+1} + \mu_{j+1} \right] \\
\lambda_j(1+\pi) &= \beta s_{j+1} \left[\left(\lambda_{j+1} \left(1 + (R-1)(1-\tau_r) \right) \right) \right] \\
\lambda_j &= \beta s_{j+1} \left[\left(\lambda_{j+1} \left(1 + (1-\tau_r)r \right) \right) \right].
\end{aligned}
$$

Furthermore, cash holdings are non-negative:

$$
x_j \geq 0
$$

If $x_j > 0$, the cash-in-advance constraint is binding if $R > 1$ (which is always the case for our calibration):

$$
c_j = (1 - \tau_w - \theta) we(z,j) + b(\bar{e}_j) + x_j^\epsilon
$$

The two first-order conditions with regard to the two interest-yielding assets imply equal returns:

$$
(1 + (1-\tau_r)r)(1+\pi) = (1 + (R-1)(1-\tau_r))
$$

In equilibrium, the portfolio allocation of the household is indeterminate and we consider the asset $a_j \equiv k_+ d_j$. Only the aggregate variables d and k are fixed by the equilibrium

conditions for the two interest rates R and r in the credit market and the factor market equilibrium for capital.

Of course, in the final period of the life, the household dissaves completely, $k_{T+T^R+1} = d_{T+T^R+1} = 0$. Also, the cash-in-advance constraint is binding. Consequently, after inserting the CIA in the budget constraint, we find:

$$(1 - \tau_r) r k_{T+T^R} + tr + \Omega_B + (1 + (R-1)(1-\tau_r)) \, d_{T+T^R} = 0$$

As $\Omega_B > 0$ and $tr > 0$ for our calibration, the household holds negative assets in the last period. This clearly is an awkward implication of our extension from the infinitely-lived representative agent model to the model with finite lifetime as it is in contradiction to empirical observations for the US economy. However, we can easily circumvent this problem if we introduce either 1) uncertain lifetime or 2) a bequest motive.

7.3 Computation

7.3.1 Money-in-the-utility

We solve a system of non-linear equations that is composed of the equilibrium conditions and the first-order conditions of the households. In order to use a modified Newton algorithm (that automatically scales down the step size if the array bounds are exceeded) we have to provide a good initial value. Therefore, we first solve the model without money that can be computed recursively starting in the last generation (see Heer/Maussner, 2005). In the model with money, we use the solution from the model without money and compute the initial guess for money with the help of the corresponding Ramsey model.

The Gauss programs *hs_olg1a.g, hs_olg1b.g, hs_olg1c.g* can be downloaded from the web.

7.3.2 Cash-credit economy

The computation is adapted from Erosa and Ventura (2002, Appendix A.1). Different from the authors, we compute an OLG model with earnings-dependent pensions.

We use value function iteration where the value function is a function of productivity, accumulated earnings, and wealth $\omega \equiv k + (1 + \pi)m$. The problem is solved in two stages. In the first stage, the optimal portfolio allocation is computed given the optimal next-period wealth $\omega' = \omega'(\omega, \bar{e}, z)$. The optimal capital stock, $k' = k'(\omega', \bar{e}, z)$, and the optimal money stock, $m' = m'(\omega', \bar{e}, z)$, are functions of next-period wealth ω', and present productivity and accumulated pension benefits. Therefore, we have to solve the system of two equations, the budget constraint and the first-order condition (23).

In the second stage, the optimal intertemporal allcoation problem is solved. We use discrete value function iteration and compute the optimal next-period wealth $\omega' = \omega'(\omega, \bar{e}', \bar{e}, z, z')$. Since we have deterministic productivity and exogenous labor supply, next-period average earnings and productivity are functions of this-period variables: $z' = z$, $\bar{e}' = \bar{e}'(\bar{e}, z)$. Therefore, we can simply compute $\omega' = \omega'(\omega, \bar{e}', z')$ (in case of productivity mobility, this is not possible).

The optimal policy functions are updated slowly so that we find a solution to the non-linear equation system.

The Fortran: *cashcred.dsw* can be downloaded from the web.

7.3.3   Limited participation

In the first step, we try to find an initial value for the capital stock and the cash holdings of generations $t = 2, \ldots, T + T^R$. For this reason, we use value function iteration. We choose a coarse grid over $\{x_t, k_t\}$, $t = 2, \ldots, T + T^R$. The Fortran program *limpart.dsw* computes the solution.

In the second step, we solve the system of non-linear equations that consists of the first-order conditions of the household, the budget constraint and the cash-in-advance constraint, the equation for the government budget balance, and the equilibrium condition in the credit market.

Given initial guess of $\{x_t, k_t\}$, $t = 2, \ldots, T + T^R$, we can compute the consumption from the budget constraint, c_t^1, and maximum consumption subject to the cash-in-advance constraint, c_t^0. Actual consumption is given by the minimum of the two, $c_t = \min(c_t^0, c_t^1)$. Given c_{T+T^R}, we can solve

$$c_j^{-\sigma} = \lambda_j + \mu_j$$

and

$$\lambda_j + \mu_j = (\lambda_j \left(1 + (1 - \tau_r)r\right)$$

in $j = T + T^R$ for λ_{T+T^R}. Solving

$$\lambda_j(1 + \pi) = \beta s_{j+1} \left[\lambda_{j+1} + \mu_{j+1}\right] = \beta s_{j+1} u'(c_{j+1})$$

for $j = T + T^R - 1, \ldots, 1$, we get $\{\lambda_j\}_{j=1}^{T+T^R}$.

Consequently, the first $T + T^R - 1 = 59$ conditions for k_j are given by the remaining first-order condition:

$$\lambda_j = \beta s_{j+1} \left[(\lambda_{j+1} \left(1 + (1 - \tau_r)r \right) \right]$$

The remaining 59 conditions for x_j follow from the following conditions: 1. If the cash-in-advance constraint is binding, $c_j^0 < c_j^1$, then

$$c_j^0 = c_j^1$$

If the cash-in-advance constraint is not binding, $c_j^0 > c_j^1$ and $\mu_j = 0$, there are two possibilities: first, we hold positive cash holdings. In this case, our equilibrium condition is given by:

$$u'(c_j) = \lambda_j.$$

Second, cash holdings are negative. In this case,

$$x_j = 0.$$

Two aggregate equilibrium conditions are given by the credit market equilibrium and the government budget. They determine the ratio of aggregate capital stock to savings and the government transfers.

The Gauss program *lp_olg1c.g* should solve this problem. However, it does not work yet.

7.4 Sensitivity Analysis

7.4.1 Fixed transaction costs $\kappa_1 > 0$

7.4.2 Productivity mobility

8 Anmerkungen fr Alfred

Lieber Alfred,

in der jetzigen Version treten noch folgende Ungereimtheiten und Schwächen auf:

1. Money-in-the-utility: In der Kürze der Zeit habe ich jetzt einfach meine alte Kalibrierung für die Produktivitäten aus unserem business-cycle paper mit money übernommen, die nicht in Übereinstimmung mit den empirischen Zahlen ist.

2. Cash-Credit economy:

   - In diesem Modell tritt noch Kurioses auf. Der ALgorithmus in cashcredit.dsw konvergiert zwar ganz hervorragend, aber die Reaktion der VOolkswirtschaft auf eine Absenkung der Inflationsrate ist kontraintuitiv: 1) der Kapitalstock sinkt, 2) die Wohlfahrt sinkt drastisch. Ich habe die Befürchtung, ich habe doch noch irgendwo einen Programmierfehler gemacht. Ich kann daher die Ergebnisse auch nicht richtig erklären. Oder ist es wirklich so einfach: Die HH finanzieren bei niedrigen Inflationsraten mehr und mehr Konsum mit Cash, halten deswegen weniger Kapital und das führt zu niedrigerem aggregiertem Sozialprodukt und Konsum?

     Im Augenblick steht übrigens noch viel redundanter Code in cashred.for, den ich oft eingefügt habe, um Zwischenergebnisse zu überprüfen.

   - Limited Participation

     Die value function iteration läuft, ist aber extrem langsam und ungenau, da ich ein weites grid benutzen musste.

     Die Lösung mit der direkten Berechnung der nicht-linearen Gleichungen (first-order conditions, constraints, equilibrium conditions) im Gauss program (siehe oben in section 7.3.3.) klappt noch nicht.