STOCHASTIC DYNAMIC PROGRAMMING IN SPACE: AN APPLICATION TO BRITISH COLUMBIA FORESTRY

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Abstract

We construct an intertemporal model of rent-maximizing behaviour on the part of a single seller of timber under multi-dimensional risk as well as geographical heterogeneity. Subsequently, we use recursive methods (in particular, the method of dynamic programming) to characterize the optimal policy function, the rent-maximizing timber-harvesting profile. One noteworthy feature of our empirical application is the unique and detailed information we have organized in the form of a dynamic geographical information system to account for site-specific cost heterogeneity in harvesting and transportation as well as uneven-aged stand dynamics in timber growth and yield across space and time in the presence of stochastic lumber prices and timber volumes. Our model is a powerful tool with which to conduct policy analysis.

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We cannot overstate the contribution of our research assistant, Mark Weldon. A mechanical engineer by training, a forester in a previous profession, and currently a Ph.D. student in environmental engineering, Mark toiled quietly and efficiently for two years to build us an extraordinary data set; we are in his debt.

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1. Motivation and Introduction

The application of recursive methods and, in particular, the method of dynamic programming to structure economic-decision problems involving both risk and time is by now quite standard, especially in the natural-resource economics literature. This paper goes beyond what is contained in those that build on Faustmann (1849): *viz.*, Kaya and Buongiorno (1987); Brazee and Mendelsohn (1988); Morck, Schwartz, and Strangeland (1989); Reed and Clarke (1990); Haight and Holmes (1991); Thomson (1992); Reed (1993); Provencher (1995); or Reed and Haight (1996). First, we take geography seriously, both in the planar sense and in the three-dimensional sense. Second, we take site-specific heterogeneity seriously both on the cost side in terms of harvesting and transportation and on the growth and yield side in terms of heterogeneous stands of timber. Third, we model initial conditions. In particular, we do not take as a starting point a steady-state allocation, or even an optimal allocation. Instead, we take the existing, potentially uneven, age distribution of timber as given and derive the optimal policy function — the optimal timber-harvesting profile — in terms of this age distribution. Fourth, we use best-practice biological methods to model the dynamics of uneven-aged forest growth and yield. Fifth, in the past, economists have typically demonstrated their methods by solving simple examples in closed-form or they have imposed conditions sufficient to sign comparative static predictions. Below, we harness recent developments in computational methods to solve numerically for the optimal policy function.

We are able to accomplish these advances because we have had access to information from unique and elaborate databases maintained by the Ministry of Sustainable Resource Managment, Terrestial Information Branch, and the Ministry of Forests, Forest Analysis Branch (formerly, the Timber Supply Branch), in the province of British Columbia, Canada. From these different databases, we have constructed a dynamic geographical information system whose different relations we have then exploited to develop estimates of site-specific cost heterogeneity in harvesting and transportation as well as uneven-aged stand dynamics in timber growth and yield across space and time.

Our empirical framework is quite rich and allows us to conduct a variety of different policy experiments that previous researchers could not. For example, we can simulate what the optimal economic response to a Spruce beetle infestation would be. In addition, we can also investigate the implications of differential productivity improvements across the sawmills in our study area. Furthermore, we can compare our estimates of the optimal harvesting policy with the harvests that have occurred during the last decade as well as those harvests that are planned over the next decade. Finally, from the perspective of industrial organization, we can investigate how the the province of British Columbia might behave as a "big player" in the lumber market.[1]

2. Previous Theoretical Structure

In order to place our research in an historical context, we first develop a notation and then outline a simple theoretical framework which some previous researchers have used to investigate the optimal harvesting of timber. This work allows us to isolate a variety of different and important features of the timber-harvesting problem. Subsequently, we go on to extend the existing research in section 3.

2.1. Biological Environment

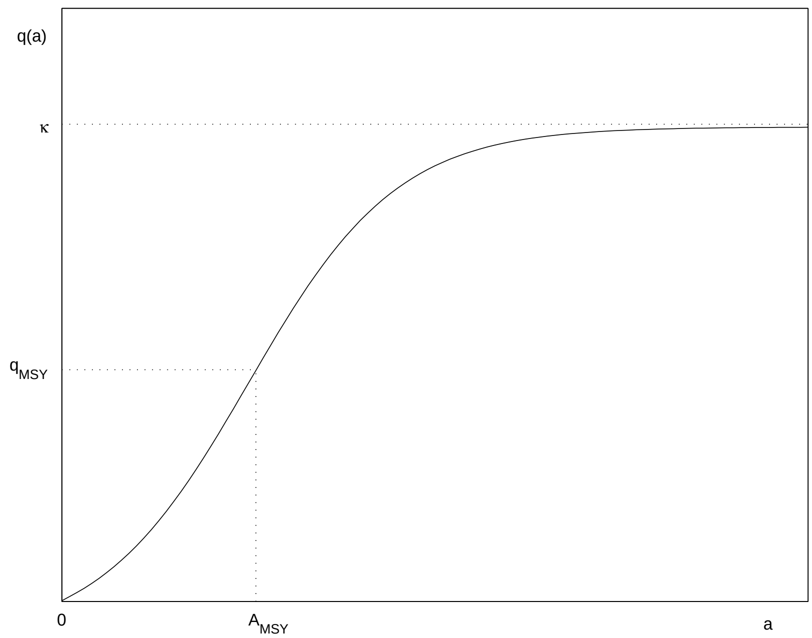
Forests are biological assets whose net returns typically depend on, among other things, the age of the standing timber. In Figure 1, we present a stylistic graph of the relationship between the age a of an even-aged stand of timber on a harvest block of a particular area and its average (mean) volume $q(a)$. Note that with aging the mean volume rises, initially quite quickly, but subsequently at a slower rate. In the absence of disaster or disease, the mean volume will approach an asymptote κ. Both the rate-of-change of mean volume $\Delta q(a)$ and the asymptote of mean volume κ are species dependent and can be influenced by a variety of environmental factors, some of which are under the direct control of the forester; *e.g.*, the density of stems, whether the stand is old or new growth, and so forth.

2.2. Maximum Sustainable Yield

One commonly-used criterion for determining the harvest of timber as well as other biological resources, such as fish, involves the concept of *maximum sustainable yield.*

[1] Sawmills in British Columbia produce about twenty-five percent of the softwood lumber supply in North America.

Figure 1
Age Path of Timber Volume



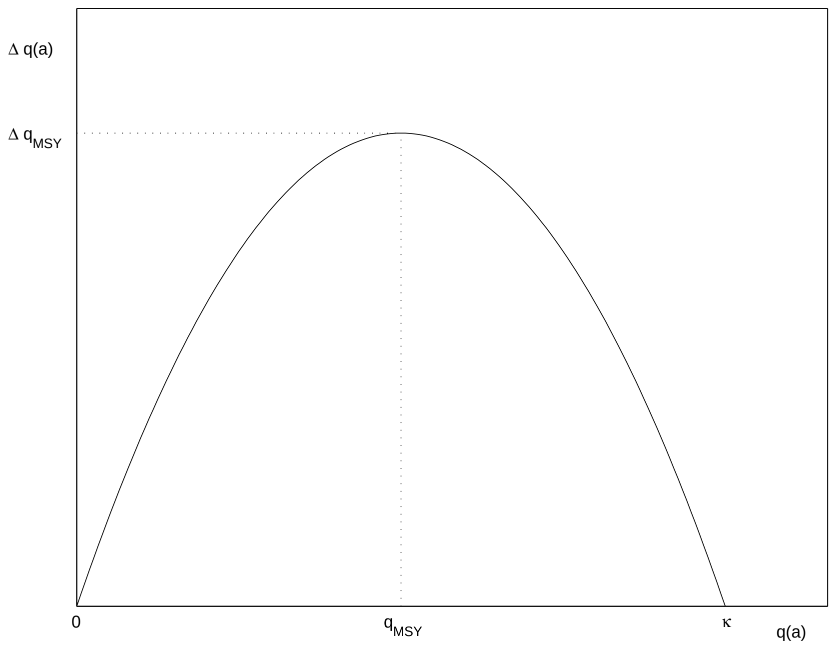
In Figure 2, q_{MSY} denotes that volume at which a stand's rate-of-change in mean volume is at its maximum Δq_{MSY}. Under the maximum-sustainable-yield management criterion, an age A_{MSY} exists at which to harvest the stand, yielding q_{MSY} units of timber.

Suppose there are a total of N harvest blocks. Consider dividing all of these blocks into A_{MSY} different types of plantations, each of a different age, but each with the same species and stem density. Under the maximum-sustainable-yield criterion, in a steady state, a uniform age distribution $f(a)$, often referred to as a *normal forest*, will obtain. Such an age distribution is depicted in Figure 3. Given the N harvest blocks, this implies that (N/A_{MSY}) sites will be harvested in each period, yielding an average total volume of timber $[Nq(A_{MSY})/A_{MSY}]$, which we shall denote $\bar{t}$.

2.3. Lumber Production

A useful approximation to the milling process of timber is the Leontief production function. For this technology, hours of labour input h and cubic metres of timber t are combined according to a fixed-coefficient production function to yield an output,

Figure 2
Biological Rate-of-Change



lumber ℓ, in thousands of board according to the following:

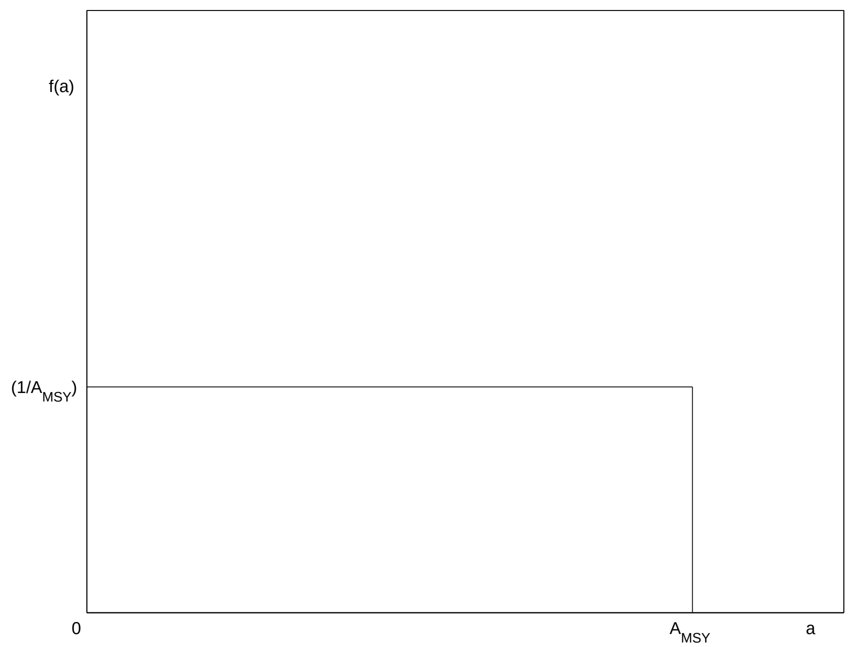
$$\ell = \min(\alpha h, \beta t) \quad 0 < \alpha,\ 0 < \beta.$$

The parameter β is often referred to by lumbermen as the *lumber recovery factor* (LRF). The unit isoquant for this production is depicted in Figure 4 by the right-angled curve with 1 beside it; in this case $(1/\alpha)$ units of h are combined with $(1/\beta)$ units of t to produce one unit of ℓ. Another representative isoquant, which exceeds that for 1, is depicted to the northeast of 1 and denoted ℓ_1. A key feature of this production function is that along an isoquant no scope exists to substitute the factor input labour h for timber t to get more output lumber ℓ, so economically-efficient production obtains along the ray where t equals $(\alpha h/\beta)$; *i.e.*, a constant factor-input ratio is maintained regardless of factor-input prices.

Given the Leontief technology and assuming that producers take input prices as given one can derive the following total- (C) and marginal- (c) cost functions:

$$C(\ell; w, s) = \left(\frac{w}{\alpha} + \frac{s}{\beta}\right)\ell$$

Figure 3
Age Distribution, Normal Forest



and

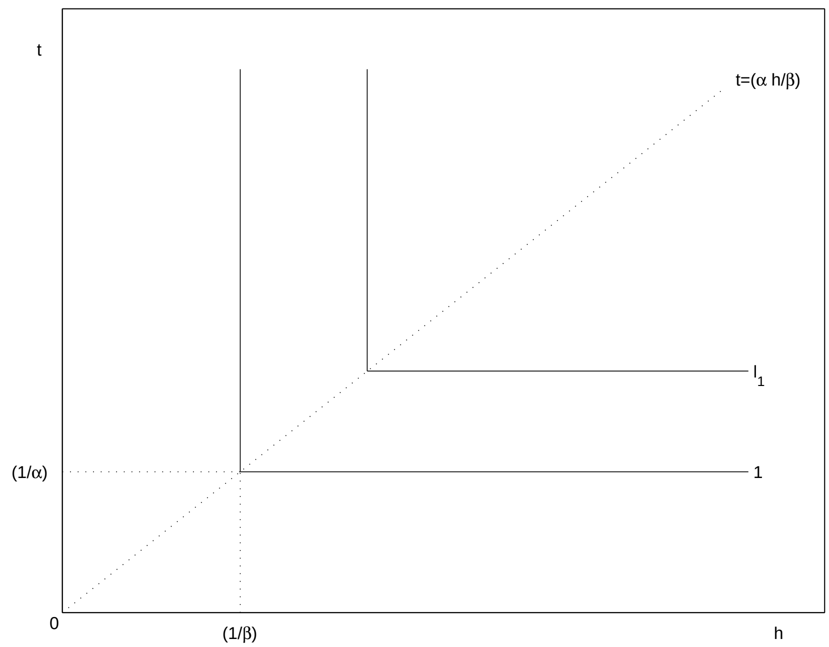
$$c(\ell; w, s) = \left(\frac{w}{\alpha} + \frac{s}{\beta}\right)$$

where w is the wage of labour and s is the price of timber, often referred to as the *stumpage rate*. Let c^0 denote $c(\ell; w, 0)$, the marginal cost when s is zero. With a Leontief technology, c^0 is the marginal cost of all factor inputs, excluding the natural-resource input timber.

2.4. Some Institutional Features

In many jurisdictions, government agencies often dispose of publicly-owned timber using administratively-set prices. In such situations, questions of what to charge for such publicly-owned assets arise naturally. Conditional on a maximum-sustainable-yield volume $\bar{t}$, for example, the principle behind determining the optimal stumpage rate $\bar{s}^*$ dates back to Rothery (1936) and involves rent maximization. Basically, absent formal markets for timber, the price that a "central planner" should charge

Figure 4
Leontief Production Function



for each cubic metre of timber is that resource's residual value, its rent; *viz.*

$$\bar{s}^* = \beta(p - c^0) = \beta\left(p - \frac{w}{\alpha}\right).$$

This can be deduced from the graph in Figure 5. Here, the parameter β in front of $(p - c^0)$ simply ensures that the units match; p and c^0 are in dollars per thousands of board feet of lumber while $\bar{s}^*$ is in dollars per cubic metre of timber. Thus, the units of β are thousands of board feet per cubic metre, those of the LRFs.

2.5. Site Heterogeneity

Different stands of timber often have different LRFs. In the absence of other information, one convenient way to model stand-specific differences in LRFs is as random draws from a probability density function $g(\beta)$. An example of $g(\beta)$ is depicted in Figure 6. Random differences in LRFs then mean that the rent-maximizing stumpage rates depend on stand-specific factors, so the stumpage rate $\bar{s}^*(\beta)$ will be a function of the stand-specific LRF β. Thus, stumpage rates will reflect differential factor rents in the sense of David Ricardo (1817).

Different stands of timber are often located at different distances d (in kilometres) from timber-processing facilities. Typically, transportation costs per unit volume γd are significant, where γ is the cost of transporting the timber equivalent of one thousand board feet of lumber one kilometre. In this case, the rent-maximizing stumpage rate is determined according to the following:

$$\bar{s}^*(\beta, d) = \beta(p - c^0 - \gamma d) = \beta\left(p - \frac{w}{\alpha} - \gamma d\right).$$

Thus, the rent-maximizing stumpage rate has a location-specific rental component in the sense of Johann von Thünen (1826) as well as a Ricardian component.

2.6. Rent-Maximizing Solution

Most of the analysis considered above has been couched in terms of a steady-state, maximum-sustainable-yield harvest $\bar{t}$. An unusual feature of this solution, at least from the perspective of an economist, is that $\bar{t}$, the volume of timber brought to market in each period j, is independent of economic variables and determined solely by biological parameters.

The German forester Martin Faustman (1849) introduced the rent-maximizing way in which to rotate a forest, which is often referred to as the *Faustmann solution*. To begin, we shall introduce the Faustmann solution in terms of the management of an even-aged stand of timber on a single harvest block.

Assume that k, the costs of planting a harvest block at a particular stem density, are incurred in period 0, while in period A a net revenue of $\beta(p - c^0 - \gamma d)q(A)$ is realized from the sale of the harvested timber as lumber. When the discount rate is δ, the present-discounted profit π from the sale of a single rotation of the timber is

$$\pi(A) = -k + \beta(p - c^0 - \gamma d)q(A)\exp(-\delta A).$$

In a multi-rotation setting, after the harvest of the first stand, another stand will be planted and then harvested and, after that, another, and so forth. Thus, the rent to

Figure 5
Demand and Supply of Lumber

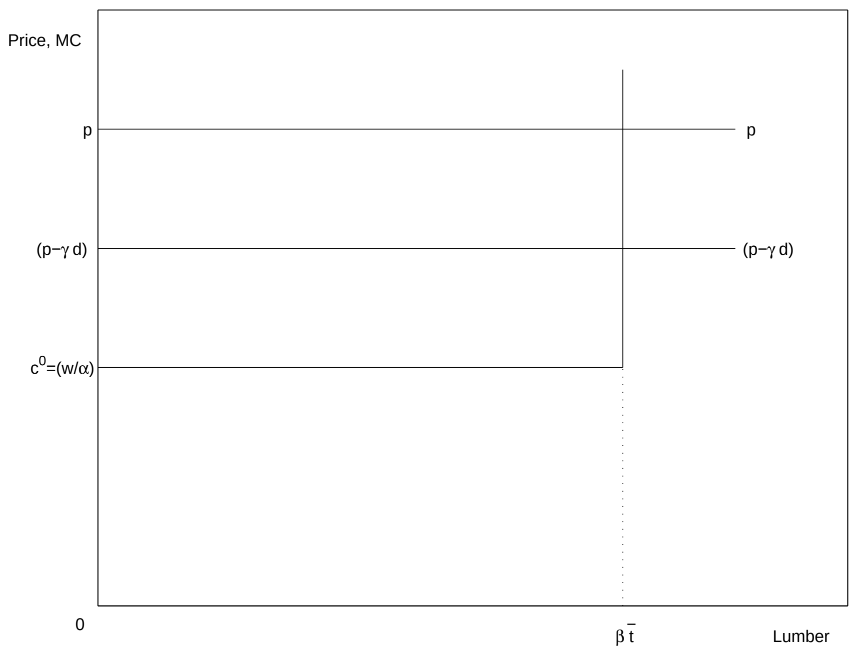
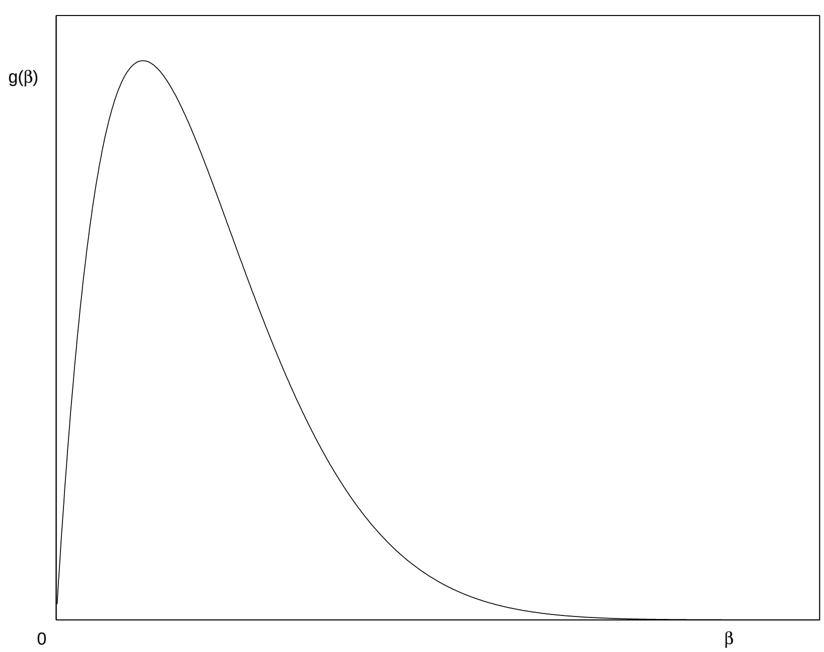


Figure 6
Site-Specific Heterogeneity in Lumber Recover Factors



the scarce land of an infinite number of rotations of the same species of stand is

$$\begin{aligned} V(A) &= \pi(A) + \pi(A)\exp(-\delta A) + \pi(A)\exp(-2\delta A) + \ldots \\ &= \pi(A)\sum_{i=0}^{\infty}\exp(-i\delta A) \\ &= \frac{\pi(A)}{[1-\exp(-\delta A)]} \\ &= \frac{[\beta(p-c^0-\gamma d)q(A)\exp(-\delta A)-k]}{[1-\exp(-\delta A)]}. \end{aligned}$$

For this block of land, the rent-maximizing harvest date A^* is characterized by the following first-order condition:

$$\beta(p-c^0-\gamma d)q'(A^*) = \delta\beta(p-c^0-\gamma d)q(A^*) + \delta V(A^*).$$

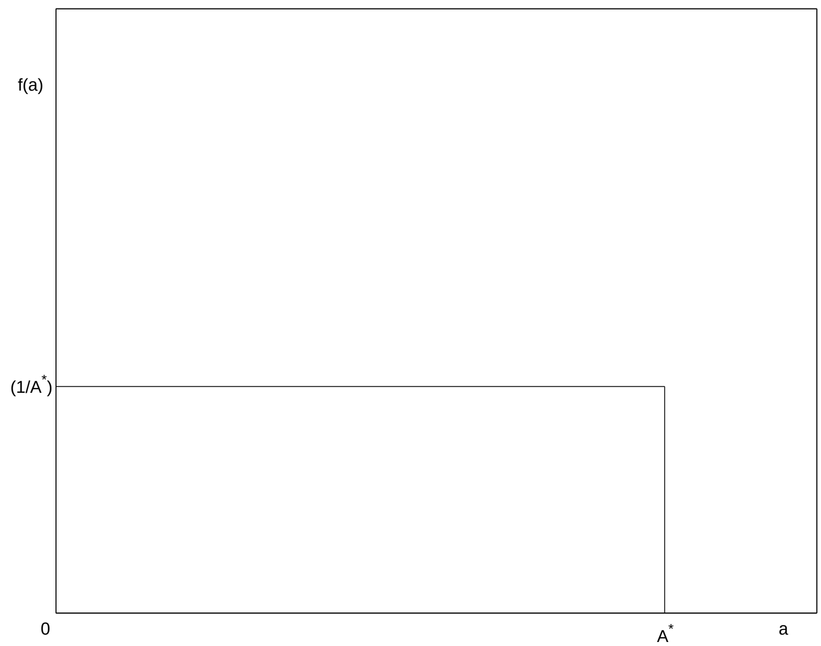
The term on the left represents the marginal benefit from holding the tree an extra "period," while the two terms in the sum on the right represent the marginal cost. The marginal benefit is the rent-maximizing value of the timber multiplied by the change in volume, while the first term of marginal cost is the opportunity cost of interest on the net revenue and the second term is the rent on the land.

Again, consider dividing the N harvest blocks into a number of different types of plantations, each of a different age, but the same species and stem density. This time, however, let there be A^* different ages instead of A_{MSY}. In a steady-state, a normal forest will obtain which is depicted in Figure 7. One can replace $\bar{t}$ with t^*, which equals $[Nq(A^*)/A^*]$, and much of the economic analysis of the maximum-sustainable-yield case presented above carries through without any major modifications. Now, however, A^* depends on the biological parameters (κ for example) as well as p and w (through c_0), and also α, β, γ, δ, and d. Thus, the optimal volume of timber harvested t^* (the supply curve) as well as the optimal stumpage rate s^* depends on these too: *viz.*, $t^*(p; w, \alpha, \beta, \gamma, \delta, d)$ and $s^*(p; w, \alpha, \beta, \gamma, \delta, d)$. Also, s^* is potentially quite different from $\bar{s}^*$.

2.7. Common Criticisms of the Faustmann Framework

Practioners attempting to implement the Faustmann solution often complain that the framework provides little guidance concerning what to do when a forest has

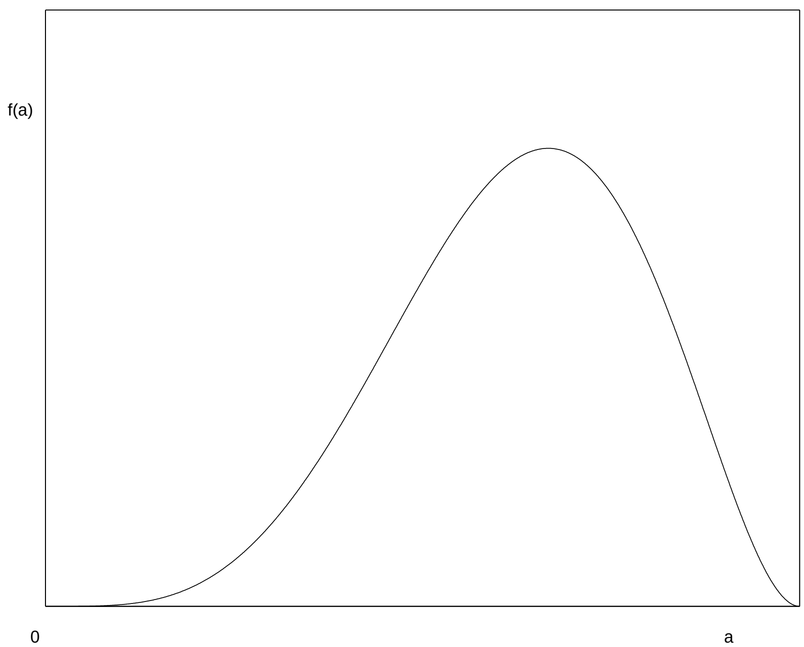
Figure 7
Faustmann Normal Forest



previously been unmanaged, so the current age distribution is uneven. For, in many jurisdictions, some stands of timber have never been previously harvested, while others have been harvested, but allowed to regenerate naturally, so the age distributions in such stands are not those of a normal forest in either the maximum-sustainable-yield or the Faustmann sense. This is perhaps the most common criticism of Faustmann's work. Moreover, this criticism does not go away once an optimal, steady-state obtains. For even if the initial forest were in a steady-state, even-aged Faustmann distribution, a change in the economic environment (for example, because of an increase in lumber prices p), would induce an uneven-age distribution. After several shocks, a representative age distribution might look something like that sketched in Figure 8.

Another criticism of the Faustmann framework is that selective harvesting is implicitly assumed. The economic and engineering reality of harvesting timber implies that selective harvesting a particular strata of the age distribution is often impossible to do effectively. Thus, unlike in even-aged plantation tree farming, where selectively cutting a particular part of the age distribution is feasible, in old-growth forests, clear-cutting the entire heterogeneous age distribution in a stand of timber is a fact

Figure 8
Age Distribution of Old-Growth Forest



of life.

A third criticism of the Faustmann framework is that forests exist in space: they are in different planar locations as well as at different elevations. This heterogeneity implies that the their growth and yield functions as well as their harvesting and transportation costs are heterogeneous.

A final criticism of the Faustmann framework is that economic variables, such as lumber prices, and biological variables, such as volumes of merchantable timber, are typically subject to stochastic variation over time. These features change markedly the decision problem.

3. A Geographical, Intertemporal, and Stochastic Model

Below, we develop a theoretical model which admits the four features of either natural forests or the economic environment discussed in the previous section: first, initial conditions; second, engineering and physical constraints; third, geography; and fourth, stochastic variation in both biological and economic variables.

3.1. Recursive Solution via the Method of Dynamic Programming

To provide a solution to the problem having the above features, we adopt a recursive modelling strategy and, in particular, the method of dynamic programming. What we want to do is take the infinite horizon faced by the decision-maker, and break it into a decision to be made this period, and then a continuation into the future. All of our assumptions are made to ensure that such a recursive decomposition can be constructed in a computationally-tractable way.

3.1.1. Assumptions concerning the Economic and Physical Environment

We begin by assuming that the central planner (the government, also referred to as the Crown below) has timber-bearing land which is divided into individual harvest blocks. We shall formulate a dynamic-programming problem whose solution will determine the optimal time at which to clear-cut the timber on a particular block. The objective is to maximize the expected discounted value of rents earned from managing a portfolio of blocks over an infinite horizon. Below, we shall refer to the decision to clear-cut any particular block as the decision "to harvest" the timber on that block. We assume that clear-cutting is optimal because the costs of selectively harvesting individual stems on a block are prohibitively high.

We assume that the timber growing on each block is relatively homogeneous in terms of the age, biomass, and species of trees. In fact, when we come to implement our framework, a harvest block will be defined in terms of a GIS grid which is homogeneous in this sense as well as in terms of harvesting costs.

We assume that no capacity constraints exist on the resources needed to harvest different blocks; *i.e.*, any particular harvester in a Timber Supply Area (TSA) or any particular Tree Farm Licensee (TFL) can procure additional harvesting capacity at a constant marginal cost.[2] This assumption allows us to treat different blocks

[2] In British Columbia, nearly 90 percent of all timber is on government-owned (Crown) land. Basically, the Crown, through the Minister of Forests, sells the right to harvest the timber on this land in two different ways. During our sample period, the most common way was charging administratively-set prices to a small number of firms who held Tree Farm Licenses or other similar agreements. The terms of these agreements were negotiated over the last three-quarter century, and require that the licensee adopt specific harvesting as well as reforestation plans. About ninety percent of all Crown timber is harvested by firms holding Tree Farm Licenses

separately, greatly simplifying the dynamic-programming problem. Otherwise, we would have to keep track of the available harvesting capacity and consider how to ration this available capacity during periods when the number of blocks scheduled for harvesting exceeds the havesting capacity.

A related assumption is that the cost of harvesting timber is independent of the number of blocks to be harvested or the volume of timber harvested on a given block, although the cost of harvesting could depend on the particular characteristics of each block.[3] This is equivalent to the assumption that harvesters face a perfectly elastic supply of firms willing to fell trees on any particular block, and that a decision to cut a larger number of blocks will not significantly bid up the prices these firms charge.

We also assume that British Columbia is a small player in the international market for lumber as well as pulp and paper, so that at any particular time the Crown faces a perfectly elastic demand for timber at the current market-determined spot price.[4]

Finally, we assume that any given block will always be used for growing and harvesting timber and that the block has no alternative use. Later, we shall show how the problem can be modified to allow for a decision to convert permanently the block to a best alternative use, such as conversion into parkland, or sale of a block for a housing or industrial development project, and so forth.

3.1.2. Solution Heuristic

In our model, harvesting decisions are made at discrete points in time, such as the beginning of each month, quarter, or year. The periodicity of the model can be changed easily, assuming sufficient data exist to estimate transition probabilities of the

or similar agreements. The second, and less common way, to sell timber is at public auction through the Small Business Forest Enterprise Program.

[3] In fact, we shall make use of elaborate and unique site-specific data gleaned from a GIS to control for this sort of heterogeneity.

[4] Our framework is sufficiently general that it can be used later to investigate the case where the Crown is a "big player" in the timber market, so its harvesting decisions have an impact on the spot price of lumber. As in the case of capacity constraints on harvesting decisions, this will create an inter-dependency in the harvesting decisions since, on the margin, increasing the volume of timber harvested can depress current spot prices and this may make it optimal to delay harvesting timber on certain plots to avoid unduly depressing the current spot price of lumber.

state variables at sufficiently fine time intervals. The basic decision in the dynamic-programming model is binary: to harvest a particular block or to delay harvesting to a future period.

The dynamic-programming problem we analyze is a considerable generalization of the simple Faustman timber-harvesting problem outlined above because we specify much richer and more realistic stochastic models of both timber volume growth and future lumber prices. Our model is often referred to as a *regenerative, optimal-stopping problem*. Similar problems have been analyzed and solved previously by Rust (1987) and Provencher (1995).[5] The *stopping* decision is equivalent to the harvest decision. The problem is referred to as *regenerative* because, once a harvest occurs, we assume that the new seedlings to be planted are identical genetically to those stems that came before them. As in the Faustman problem, an infinite sequence of harvests over an infinite horizon occurs. However, unlike in the Faustman problem, the optimal harvests in the stochastic version of the problem will occur at random intervals of time depending on the expected rate of change in the spot price of lumber and the condition of timber on the block.

Thus, the harvest decision at time j will depend on a vector $\boldsymbol{x}_j$ of *state variables* that describe the state of a block, the price of lumber, and other macroeconomic variables useful in forecasting future lumber prices and harvesting costs. In our initial analysis, the vector $\boldsymbol{x}_j$ will consist of just two variables (q_j, p_j) where q_j denotes the current volume of merchantable timber on the block, measured in cubic metres, and p_j denotes the current spot price of lumber. Later, we shall incorporate z_j, an indicator of disease in the timber on the block, as well as $\boldsymbol{m}_j$, a vector of macroeconomic indicators (such as the unemployment rate, industrial production, *etc.*), useful in predicting future lumber prices.

We plan to include a disease indicator as a state variable because recently the Spruce beetle, *Dendroctonus rufipennis*, has become an important problem in British Columbia. Disease can spread to a block and damage the timber or slow its growth. The presence of a disease may be an important reason to accelerate the decision to harvest a particular block. Disease may cause external effects too; *e.g.*, disease

[5] Provide a description of how these solutions are different from those contained in: Kaya and Buongiorno (1987); Brazee and Mendelsohn (1988); Morck, Schwartz, and Strangeland (1989); Reed and Clarke (1990); Haight and Holmes (1991); Thomson (1992); Reed (1993); or Reed and Haight (1996).

could spread to neighbouring blocks of timber, so even if a given block is not yet diseased, the presence of disease in nearby blocks may be a good reason to accelerate the decision to harvest. A complete analysis will require analyzing all blocks in a given region simultaneously. For simplicity, initially, we shall ignore potential interdependencies and only consider indicators of disease in the block under consideration, although z_j could easily be interpreted as an indicator of the presence of disease in a nearby block and the subsequent analysis would proceed in the same fashion.

Initially, we have ignored other natural disasters (such as fires, windstorms, landslides, or avalanches). On any particular block, these disasters are approximately serially independent. Thus, we shall assume a transition probability $\tau_1(q_{j+1}|q_j, z_{j+1})$ that accounts for such disasters in the sense that there is some probability that the total volume of timber in the next period is less than the current volume ($q_{j+1} < q_j$) as a result of a fire or some other natural disaster that occurs in period $(j + 1)$, as indicated by z_{j+1}. Otherwise, under ordinary conditions, q_{j+1} will exceed q_j reflecting the usual growth of timber on the block. The rate of growth may be uncertain because of random fluctuations in rainfall, the amount of sunlight, and so forth, and it can also be affected by diseases. We do not believe it is reasonable to treat diseases as being serially independent. For this reason, we plan to account for the presence of the disease indicator z_{j+1} in addition to q_j in our stochastic predictions of next period quanity q_{j+1}. We also plan to account for stochastic disease progression via a separate transition probability $\tau_3(z_{j+1}|z_j)$. We have ignored weather as a state variable because we believe that, except for seaonal patterns, variations in weather are, to a first approximation, serially independent events. However, subtle longer-term dynamic changes in climate may exist, such as those induced by global warming. If these longer-term patterns are important, then we can include weather and/or climate state variables in future versions of the model.

Randomness in the evolution of the spot price of lumber is reflected in the transition probability $\tau_2(p_{j+1}|p_j, \boldsymbol{m}_{j+1})$. Next period's spot price p_{j+1} is random, but its probability distribution can depend on this period's spot price p_j as well as a vector of macroeconomic indicators $\boldsymbol{m}_{j+1}$ that are useful in predicting future spot prices. As mentioned above, these might include the unemployment rate, and various indices of industrial production and home building that affect the overall demand for lumber. Since these macroeconomic indicatorss are certainly serially correlated, we shall also include a final transition probability $\tau_4(\boldsymbol{m}_{j+1}|\boldsymbol{m}_j)$ that reflects the stochastic evolution of these variables.

3.1.3. Bellman's Equation

Let $V(\boldsymbol{x}_j)$ denote the expected present discounted value of the profits earned from optimally harvesting and selling timber on the tract. $V(\boldsymbol{x}_j)$ is the solution to the following *Bellman equation* of dynamic programming, where for notational simplicity we drop the j and $(j+1)$ subscripts and let p denote a current period variable and p' denote its (random) value next period.

$$V(q,p) = \max\left[p\beta q - C + \delta\int V(0,p')\tau_2(p'|p),\ \delta\int V(q',p')\tau_1(q'|q)\tau_2(p'|p)\right]. \quad (3.1)$$

In the above Bellman equation, the current value of the block $V(q,p)$ is the maximum of two options: 1) to harvest or 2) not to harvest. If the Crown elects to let a TFL harvest the block, then we assume that the quantity harvested q is sold at the current spot price for lumber p resulting in revenue $p\beta q$ where β is the LRF. However, this revenue is reduced by the amount of harvesting costs C that depend on the amount harvested as well as its location and, possibly, also on current macroeconomic conditions (in good times, prices charged by firms for harvesting timber may be higher than in bad times). We also assume that replanting costs are included in C.

The first term inside the "max" operator in the Bellman equation is the current net profits from harvesting a block plus the expected discounted profits from future harvests. Since the harvest is assumed to reduce the effective volume of timber on the block to 0, the expected value function has q' equal 0, reflecting the fact that the harvest has occurred. If the Crown decides not to harvest, then we assume that there are no revenues or costs associated with allowing the block of land to remain untouched another period, so the value of this option is simply the expected discounted value of profits from some future harvest (and sequence of subsequent harvests). This option has a q' not equal to 0, reflecting the fact that a harvest has not occurred. However, as noted above, disease or disaster implies a positive probability that q' is less than q. Under normal conditions, however, expected growth is positive, so $\mathcal{E}(q'|q,z)$ is greater than q.

The solution to the dynamic-programming problem partitions the state space into two regions: 1) a *continuation region* in which harvests do not occur, and 2) a *stopping region* in which it is optimal to harvest. The stopping region will be a subset of the space (q, p) in which the value of harvesting now exceeds the value of waiting to harvest later. We can describe general properties of the stopping region, but its precise characterization will depend on the numerical solution of the dynamic-programming problem (3.1). For example, the stopping region will generally have the property that if it is optimal to harvest at quantity q it will also be optimal to harvest at all values $\hat{q}$ which are greater than q. Similarly, when the stochastic process for lumber prices is sufficiently mean-reverting then, if it is optimal to harvest for a particular spot price p, it will also be optimal to harvest for all higher spot prices $\hat{p}$ which exceed p.[6]

However, these basic predictions will not necessarily hold in the presence of the additional state variables $(z, \boldsymbol{m})$. *Ceteris paribus*, the presence of disease should make it more likely that harvesting is optimal; *i.e.*, if we define a locus of points (q, p) at which the Crown is indifferent between harvesting and not harvesting — and thus representing the optimal stopping or harvesting threshold for any given values of $(z, \boldsymbol{m})$ — the presence of disease will shift this optimal threshold downward, making it optimal to harvest in a larger set of (q, p) values. The impact of macroeconomic shocks is unclear and will depend on how these shocks affect beliefs about future lumber prices. For example, if the macroeconomic variables indicate a current recession and the likelihood of a protracted period of low lumber prices, we would ordinarily expect that this would shift the optimal harvesting threshold downward, making it optimal to harvest the block for a larger set of (q, p) values. But if $\boldsymbol{m}$ indicates a strong economy, then the effect may not be symmetric: an improvement in economic conditions may signal a likelihood of rising lumber prices in the future, in which case it might be optimal to delay harvesting as long as possible to take advantage of the run up in prices.

The only way to calculate detailed predictions of the optimal harvesting strategy is to solve the dynamic-programming problem numerically. In previous work, Rust (1996,1997) has developed efficient computational algorithms that make it feasible to solve dynamic-programming problems of the type outlined above. The key to an accurate solution to the problem is access to good data on the volume of timber on particular blocks and data on diseases so that we can estimate the transition

[6] With mean reversion, the higher the current spot price, the greater the likelihood that the spot price in the next period will fall.

probabilities τ_1 and τ_3. It will also be important to obtain data concerning the spot price of lumber to estimate the transition probability τ_2 and site-specific harvesting and transportation data to estimate C.

3.1.4. Policy Experiments

Once the dynamic-programming problem has been solved, it can be used to determine the *sale value* of a particular block. The predicted value of a block will, in general, be $V(q, z, p, \boldsymbol{m})$, which equals the expected present discounted value of the stream of harvesting profits. We can generalize the dynamic-programming problem to consider the case where the block of land might have alternative uses that might have a potentially higher value, such as using the land for parks or for commercial, housing, or resort developments. In this case, the Crown has two basic choices: 1) to sell outright the block to the highest bidder or to devote it to its highest-value alternative use, such as conversion to park land; or 2) to continue to use the land for harvesting timber. Of course, the land might be used for park/recreational activities in periods where timber is not being harvested, but in this analysis we have assumed there is no rental value for these uses. It would be easy to adapt the analysis above to include the case where there is a *shadow rental value* representing alternative use of the block in periods where harvesting does not occur.

We can introduce an additional state variable v_j representing the value of the best alternative use of the block of land. Accounting for the possibility of optimally converting or selling the block for its best alternative use, the Bellman equation would be modified to

$$V(v, q, z, p, \boldsymbol{m}) =$$

$$\max \Big[v, R(q, p, \boldsymbol{m}) + \delta \int V(v', 0, z', p', \boldsymbol{m}')\tau_0(v'|v, \boldsymbol{m}')\tau_3(z'|z)\tau_2(p'|p, \boldsymbol{m}')\tau_4(\boldsymbol{m}'|\boldsymbol{m}),$$

$$r(q, p, \boldsymbol{m}) + \delta \int V(v', q', z', p', \boldsymbol{m}')\tau_0(v'|v, \boldsymbol{m}')\tau_1(q'|q, z')\tau_3(z'|z)\tau_2(p'|p, \boldsymbol{m}')\tau_4(\boldsymbol{m}'|\boldsymbol{m}) \Big].$$

In this version of the Bellman equation, we let $R(q, p, \boldsymbol{m})$ denote the expected revenue from auctioning the right to harvest the block in the current period, $r(q, p, \boldsymbol{m})$ represents the shadow rental value of alternative uses of the block when harvesting does not occur, and v denotes the expected present discounted value of the best alternative use of the block. The transition probability $\tau_0(v'|v, \boldsymbol{m}')$ represents the stochastic evolution of this best alternative value, and it reflects the fact that the next period value v' may depend on current macroeconomic conditions $\boldsymbol{m}'$. Ordinarily, $V(v, q, z, p, \boldsymbol{m})$ will exceed v, in which case it is better to use the block for harvesting timber since the expected present value of the profits from continued use of the block for timber harvesting exceed its next best use. However, if $V(v, q, z, p, \boldsymbol{m})$ equals v, then it will be optimal to sell or to convert the block to its next best use. We have assumed this sale or conversion is irreversible for simplicity, although it would be possible to consider cases where we allow the Crown to lease the block or to buy back the block after a previous sale.

Our empirical framework is also quite rich and allows us to conduct a variety of different policy experiments that previous researchers could not. For example, as alluded to above, we can simulate what the optimal economic response to a Spruce beetle infestation would be. In addition, we can also investigate the implications of differential productivity improvements across the sawmills in our study area. Furthermore, we can compare our estimates of the optimal harvesting policy with the harvests that have occurred during the last decade as well as those harvests that are planned over the next decade.

4. Geographic Information System

How will the rent map of a particular region be calculated? The analytic device we chose to organize our data is a geographic information system (GIS). A GIS is a computer system capable of assembling, storing, manipulating, and displaying geographically-referenced information; *i.e.*, data identified according to their locations.

Our GIS contains several types of information. For example, first we have political maps which define the boundaries of the TSA in terms of planar coördinates. Second, we have maps in which those areas of the TSA that are on Crown land and which are not protected against harvest are listed; protected areas include federal and provincial parks as well as ecologically sensitive areas. Third, we have maps of elevations as well as maps of natural creeks, lakes, and rivers as well as man-made roads. Fourth, we have maps showing soil characteristics and vegetation types along with age distributions and stem densities. We exploit the different relations in this GIS to construct measures of different economic concepts.

To illustrate how we use the GIS, consider the following simple example: in Figure 8 the map of the political boundaries which determine in our case a TSA available for harvest; we abstract from protected areas for presentational parsimony. Clearly, these can be introduced easily. In constructing a final raster map of costs, we shall impose regulations, such as the prohibition of harvesting near streams and watersheds.

In Figure 9, we present a digital elevation model (DEM) of the contours of elevation. This information will be important in estimating harvesting costs as these vary considerably by elevation and slope as well as transportation cost since steep grades are difficult to drive.

In Figure 10, a map of the creeks, lakes, and rivers is presented, while in Figure 11 a map of the roads and highways is presented. This information will be important in determining whether timber harvesting will affect watersheds and thus the ecology of the region as well as transportation costs.

By layering the maps in Figure 8 to 11 one atop the other, one can put together a physical map of the TSA which we depict in Figure 12. Using engineering information as well as harvesting regulations (such as prohibitions against harvesting near streams), one can breakup each raster of the map in Figure 12 into an estimate of harvesting costs as well as one of transportation costs. The estimates are derived from the topography of the land as well as site-specific information contained in the GIS relations. In Figure 13, the number in each square (raster) represents some measure concerning how much each cubic metre of timber will cost to harvest and to transport to a sawmill. Higher numbers represent higher costs, while the "x"s represent rasters which cannot be harvested under any circumstance; *e.g.*, because of harvest regulations.

5. Modelling Growth and Yield in Stands of Timber

From a capital-theoretic perspective, one of the most important features of this controlled stochastic, dynamic decision problem is the growth and yield of merchantable timber from the forest. Basically, two types of forests exist: old-growth and newly-planted. Typically, old-growth forests, and even some second-growth forests which have regenerated naturally, are heterogeneous in species, age, and density. Such heterogeneity is difficult to model. Foresters use a variety of different methods to

Figure 8
Political Map of Timber Supply Area

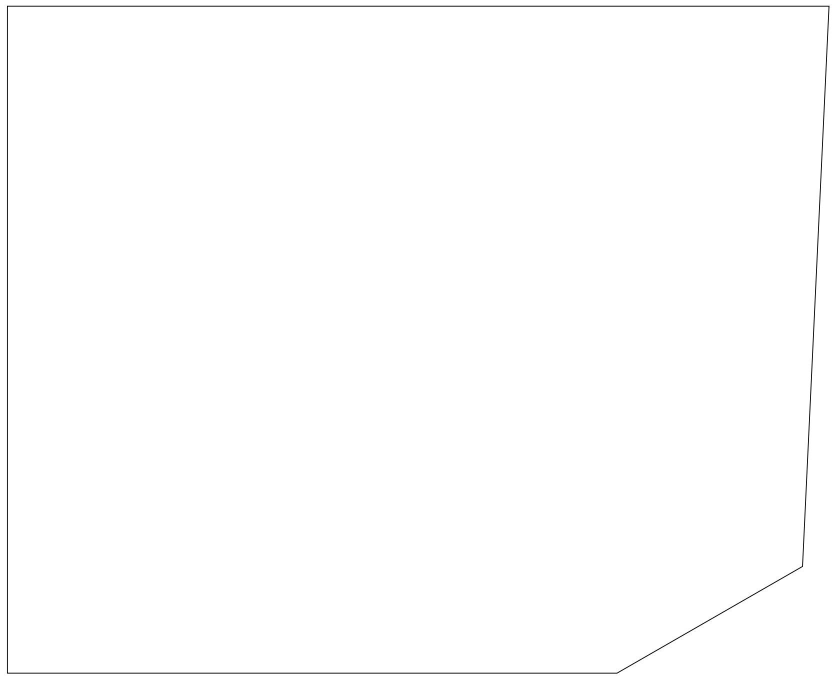
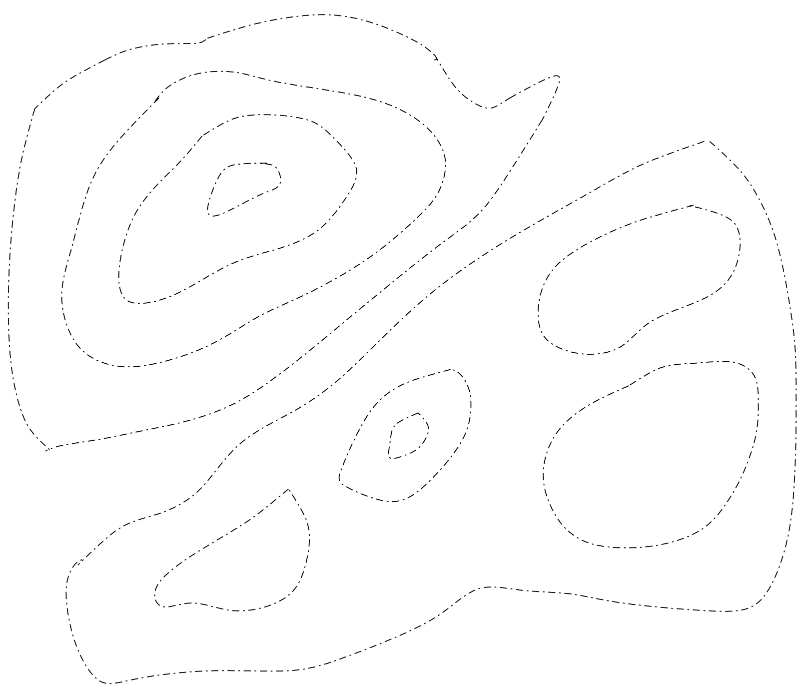


Figure 9
Digital Elevation Model



estimate the growth and yield of uneven-aged forests; these have been summarized by Peng (2000). On the other hand, newly-planted forests are typically homogeneous with respect to species as well as stem age and density. Modelling these forests is straightforward, relatively speaking, of course.

5.1. Predicting Growth and Yield in Planted Forests: *TASS*

The Ministry of Forests in British Columbia has devoted considerable time and resources to investigating the growth of seedlings of the same species planted according to a particular stem density on sites having different productivity. Foresters in the Research Branch have a computer programme, *TASS* (Timber and Stand Simulator), that can be used to simulate the growth and yield of a particular species, planted according to a pre-specified stem density, on a block of a particular site index, productivity.[7] To run *TASS*, one must provide a species (or species) as well as a site index and an initial stem density. Based on experiments done by foresters, *TASS* will simulate a future forest, and then estimate the volume of merchantable timber at any age. We used *TASS* to simulate a variety of different forest yields for different combinations of single species as well as site indices and stem densities. In particular, we simulated 100 forests over a 150-year life-span for the following 64 combination of species, site index, and stem density: (Fir,Spruce)$\times(10, 15, 20, \ldots, 40, 45)\times(1200, 1400, 1600, 1800)$.[8] We then used the output to estimate $\tau_1(q_{j+1}|q_j)$ for different species as well as different site indices and stem densities.

5.2. Predicting Growth and Yield in Old-Growth Forests: *VDYP*

VDYP (Variable Density Yield Prediction) is a computer programme designed to implement a prediction system to estimate average yields and provide forest inventory updates over large areas. It is intended for use in unmanaged natural stands of pure or mixed species composition. Basically, field observations on the yields from

[7] A site index is a summary statistic concerning the productivity of a particular block. It is derived by foresters during surveys. Our site index is the diameter at breast height of a twenty-five year-old stem. Historically, this measure has correlated quite well with the height and, consequently, the volume of a stem. For a particular stem density, one can then estimate relatively accurately the volume of merchantable timber on an hectare of land.

[8] We are grateful to Ken Polsson of the Ministry of Forests, Research Branch, for running the simulations with *TASS* and providing us the output.

Figure 10
Map of Creeks, Lakes, and Rivers

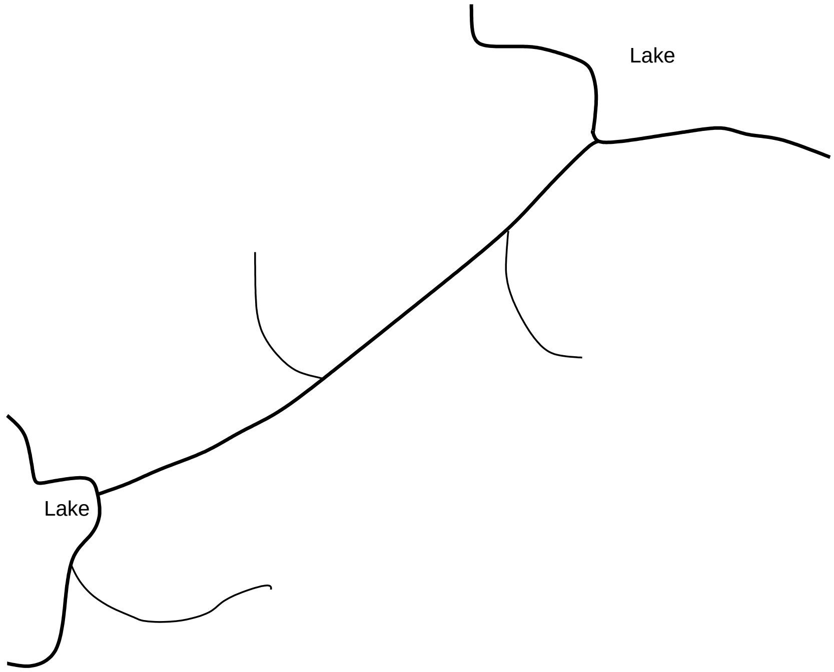


Figure 11
Map of Roads and Highways

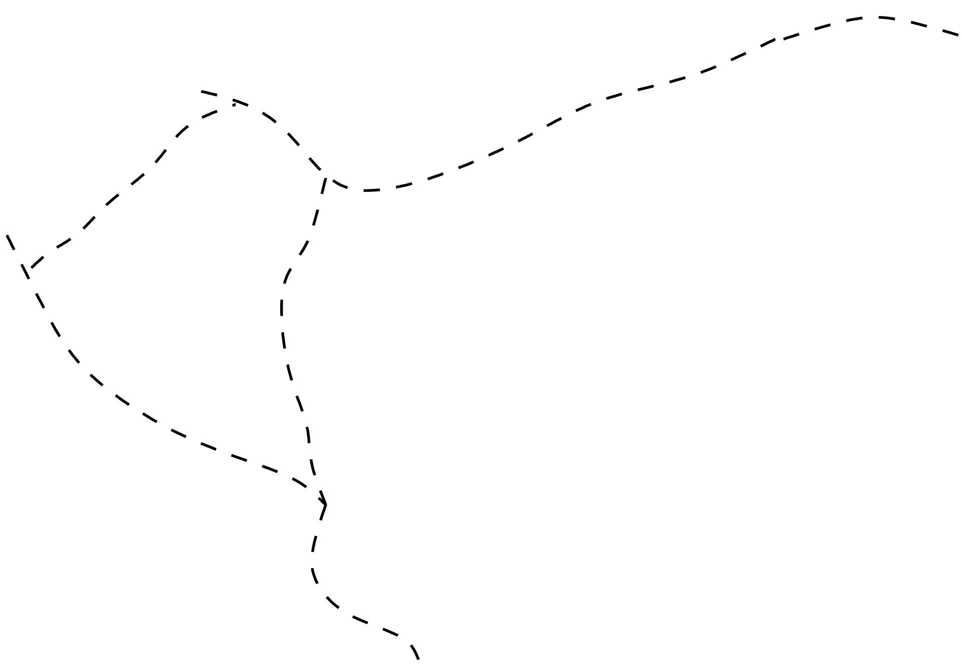
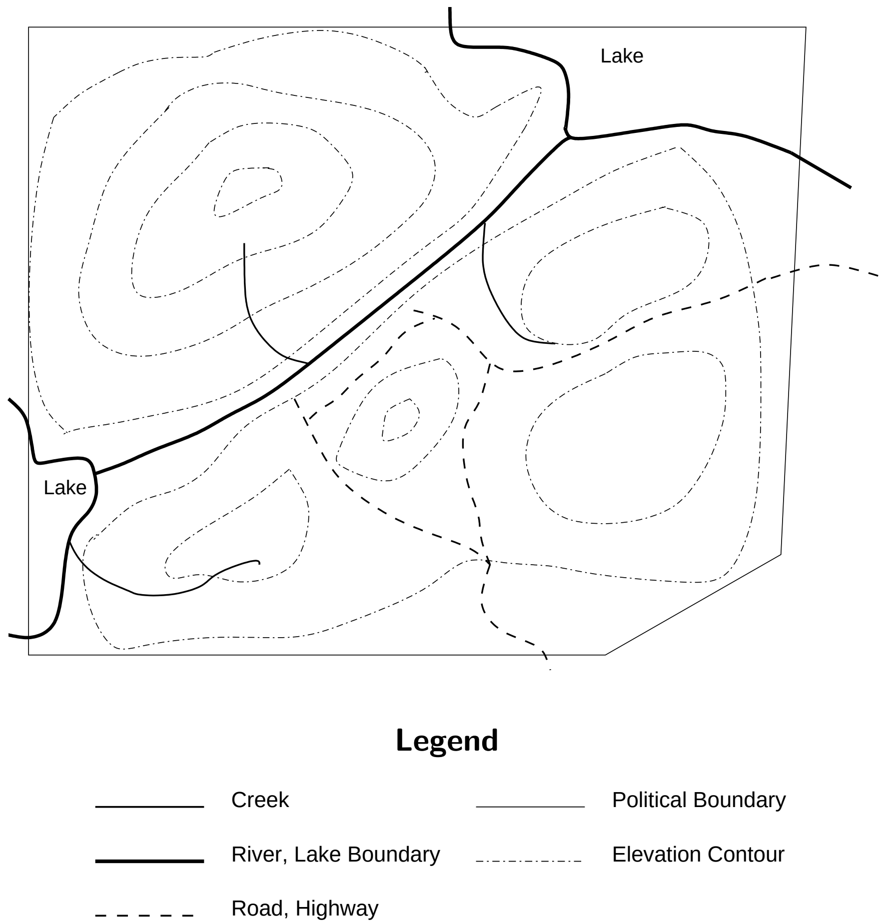
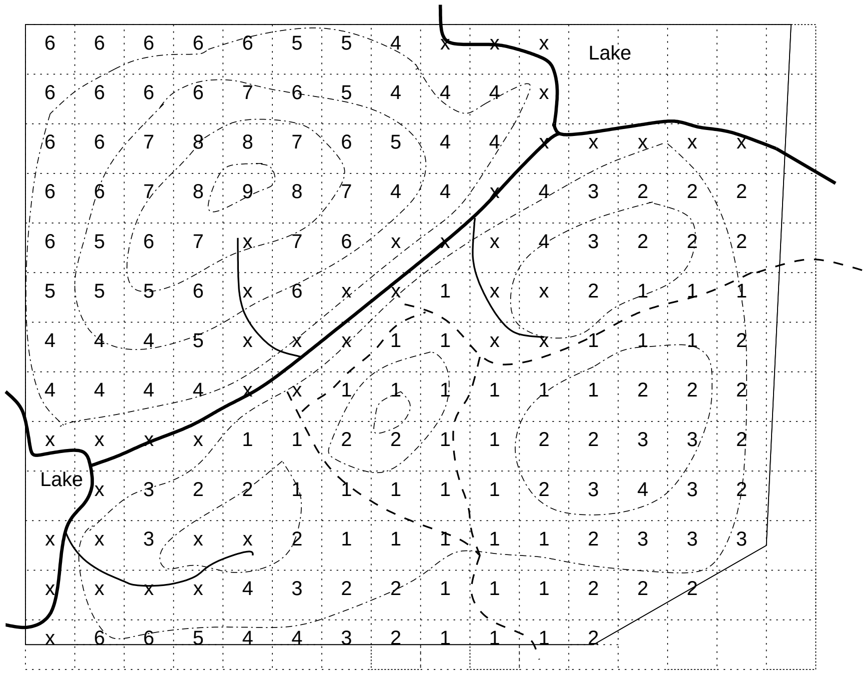


Figure 12
Physical Map



forests of different species compositions and ages under different site indices were used to develop a regression model, the output of which is an estimated merchantable volume of timber at some point in the future. Because natural forests can be quite heterogeneous, *VDYP* is not as "accurate" as *TIPSY* (Table Interpolation Program for Stand Yields), which uses the average of several *TASS* runs to estimate average timber yields. Of course, the prediction problem under the conditions assumed for *VDYP* is much more difficult than the one for *TASS* and *TIPSY*, so the comparison is somewhat unfair. We used the output of *VDYP* to estimate $\tau_1(q_{j+1}|q_j)$ for different species compositions and ages as well as different site indices and stem densities.

Figure 13
Rasters of Costs



6. Empirical Implementation

We applied the framework discussed in the previous sections to the Fraser Timber Supply Area (FTSA) in British Columbia. In Figure 14, we depict the location of the FTSA in the province, the southwest corner of the mainland. The FTSA is often referred to as the Chilliwack Forest District because the district office of the Ministry of Forests is located in Chilliwack, a small town about an hour by automobile from Vancouver.

6.1. Some Relevant Features of the Data Set

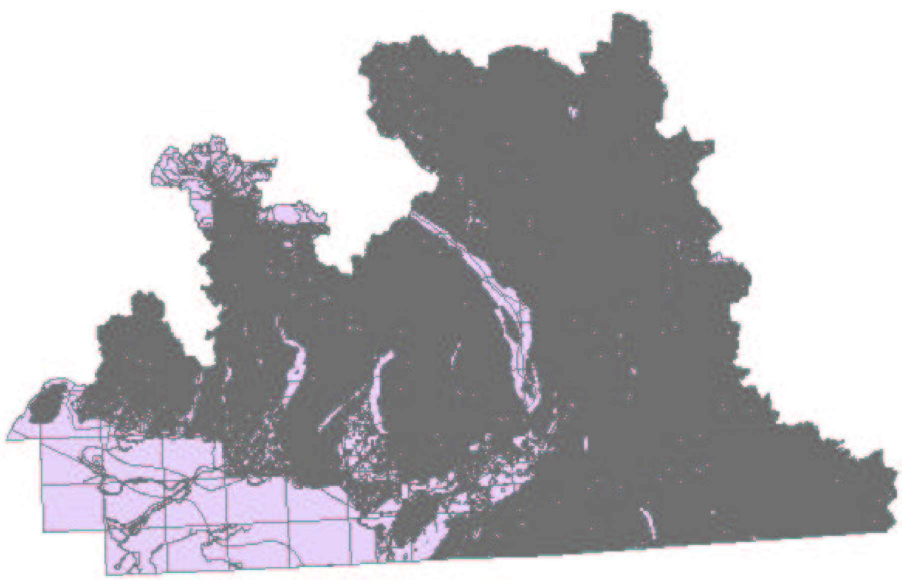
Below, we describe briefly the main features of our data set, while in an appendix to the paper we describe in detail the mechanics of how we built the data set.

The Chilliwack Forest District is about 1.4 million hectares in area, around 5,400 square miles.[9] Not all of this land, however, is under the jurisdiction of the Ministry

[9] An hectare is 100 metres square or 10,000 square metres or, approximately, 2.4711 acres.

Figure 14
British Columbia

Figure 15
Fraser Timber Supply Area



of Forest. In Figure 15, we depict all Crown land, the darkly-shaded areas. The lightly-shaded areas are bodies of water, inhabited areas, or private land. Having imposed this screen left us an area of about 706,603 hectares, which we represented as grid squares, hectares of land.

On these sites, we estimated harvesting costs based on which harvesting technology could be used. For relatively flat sites, which we defined as a slope less than 75 percent, standard yarding technology can be used, while on very steep sites at high elevations, only helicopter logging can be pursued.[10] We deduced the topography of the land from a DEM of the FTSA.

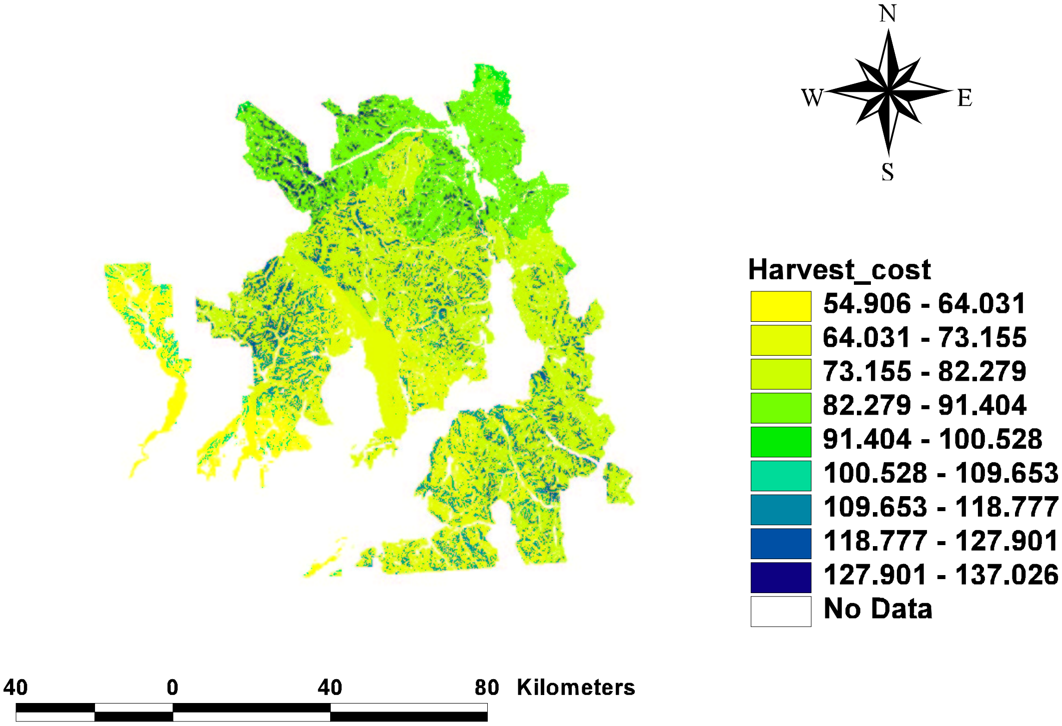
We then used the *Coast Appraisal Manual*, which is published by the Revenue Branch of the Ministry of Forests and which is used to determine stumpage rates throughout the coastal region of British Columbia, to estimate site-specific harvesting costs. We also used the extant road network in the FTSA, and contained in our GIS, to estimate the distance to the nearest sawmill. In the FTSA, several sawmills exist, but the bulk of these are located near Chilliwack. We chose the centre of gravity of these mills, weighting each mill by the volume of lumber it could produce, as the destination of all timber.[11] From these distance estimates, we then formed estimates of transportation costs. Thus, for each site, we have both harvesting- and transportation-cost estimates. In Figure 16, we depict our map of costs using different colours or, sometimes in printed copy, different shades of gray.

Our next task was to determine growth and volume for each block. On each block, we were able to obtain some 134 pieces of biological information; *e.g.*, the species composition, a site index, and so forth. However, not all of the grids were suitable for harvesting. In fact, some 113,667 had no species codes for trees, presumably because these were comprised mostly of rock. For another 4,937 hectares, the site indices were incredibly low, so that no merchantable timber was predicted to grow, while another 4,750 hectares had no volumes for other reasons, and 2,173 hectares had missing parameters for our growth and volume programmes, *VDYP* and *TASS*. In the end, we were left with 581,076 viable hectare blocks; these made up our analysis unit. The reader should note that this area is subtantially larger than the 206,910 hectares

[10] A slope of 100 percent is defined to be a 45°-degree pitch, so 75 percent would be a pitch of about 33.3°-degrees.

[11] We are grateful to Steven Fletcher, of the Revenue Branch, for providing the GIS locations and the volumes of each sawmill in the FTSA.

Figure 16
Estimated Cost Map



reported by Larry Pedersen, the chief forester of British Columbia, in the December 2003 Fraser Timber Analysis.[12] Presumably, our larger area obtained because we did not constrain ourselves by the rules contained in the *Forest Practices Code* of British Columbia.[13]

6.2. Computational Issues

Solving nearly 600,000 stochastic dynamic programmes is extremely time consuming, even under the best of circumstances. We broke up our computations into two parts. In the first part, we solved for the optimal value functions assuming that a particular block had just been harvested. We broke up the sum of harvesting- and transportation-costs into intervals of \$5.00 CAD per cubic metre from a minimum of

12 *Fraser Timber Supply Area Analysis Report*. Victoria, Canada: British Columbia Ministry of Forests, Forest Analysis Branch, 2003.

13 *Forest Practices Code of British Columbia Act* of 2002. Victoria, Canada: Queen's Printer, 2002.

\$55.00 to a maximum of \$140.00; *i.e.*, eighteen different cost regions. For each of these different regions, we entertained 64 different combinations of replanted sites. In total, we solved (18×64), or $1,152$ continuation stochastic dynamic programmes. Given that our software can solve a stochastic dynamic programme in under 13 seconds by discrete policy iteration, the $1,152$ optimal continuation value functions took just under five hours to solve on a desktop computer.

Solving for the optimal policy functions for the unmanaged natural stands is much more time-consuming than in the newly-planted case. Basically, in the FTSA, for our $581,076$ available harvest blocks, there are $46,360$ different covariate combinations; *i.e.*, combinations of site indices, species compositions, age compositions, and so forth. Given our computational technology, this would involve around 200 hours of computer time to solve them all. Because we needed to get some results to make a conference deadline, we adopted an alternative strategy. For each block i, we approximated the volume profiles generated by *VDYP* by the following three-parameter function:

$$q_{j+1} = a_0^i + a_1^i q_j + a_2^i q_j^2 + U_i.$$

We estimated a_0^i, a_1^i and a_2^i by the method of least squares for all $706,603$ blocks. The majority of the R^2 for these models was above 0.99. Those that were not were flagged as anomalies. The anomalies were then examined. Some $113,667$ had no species codes for trees, presumably because these were comprised mostly of rock. For another $4,937$ hectares, the site indices were incredibly low, so that no merchantable timber was predicted to grow, while another $4,750$ hectares had no volumes for other reasons, and $2,173$ hectares had missing parameters for our growth and volume programmes, *VDYP* and *TASS*. In the end, $581,076$ blocks could be dimesion-reduced in the above way. We then took the triplets of estimated parameters $(\hat{a}_0^i, \hat{a}_1^i, \hat{a}_2^i)$ and used cluster analysis to assign them to particular sets. We chose 64 sets. Again, we broke up the sum of harvesting- and transportation-costs into eighteen different cost regions. For each of these different regions, we entertained the 64 different triplets, and solved another $1,152$ stochastic dynamic programmes, where we conditioned on the appropriate continuation optimal value function discussed above.

6.3. Preliminary Results

In Figure 17, we present the optimal value function for the initial distribution on a

Figure 17
Optimal Value Function for the Initial Distribution

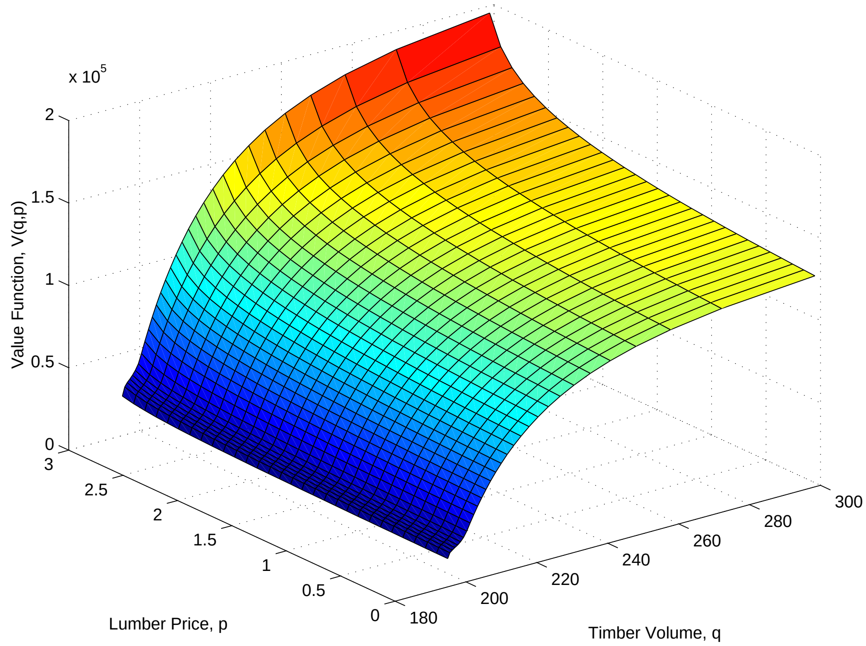


Figure 18
Optimal, Decision Rule for the Initial Distribution

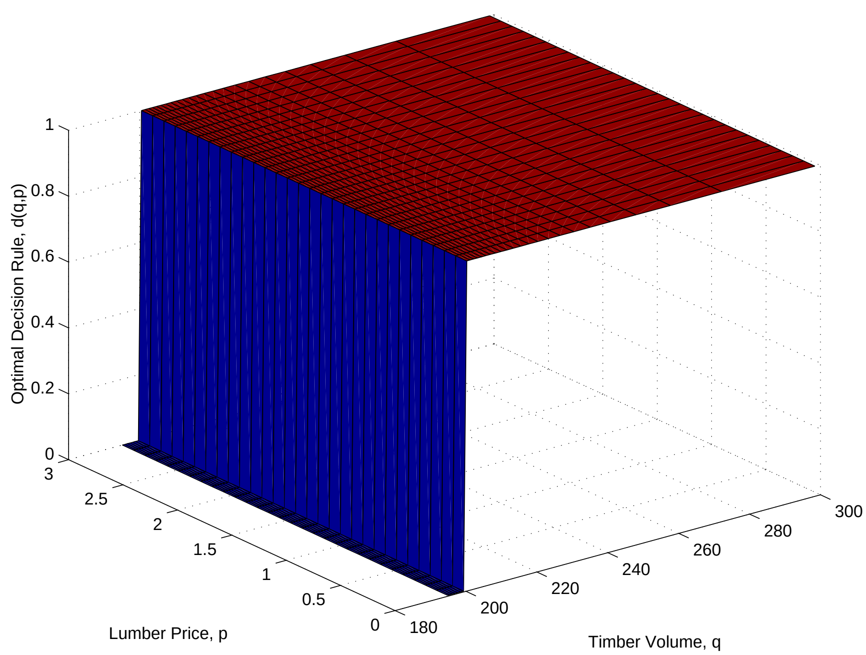


Figure 19

Optimal Decision Rule, Initial Distribution: Harvest when $q > q(p)$

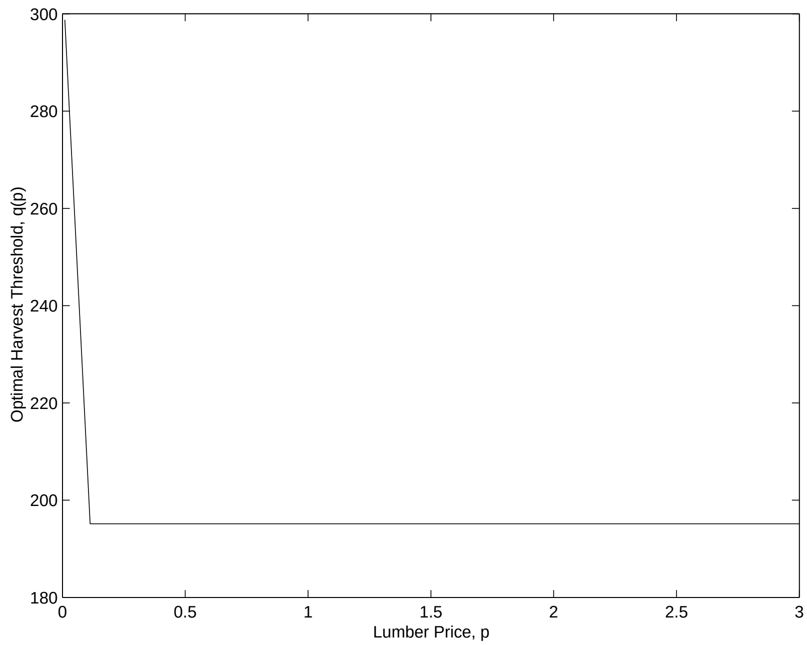


Figure 20

Optimal, Steady-State, Value Function

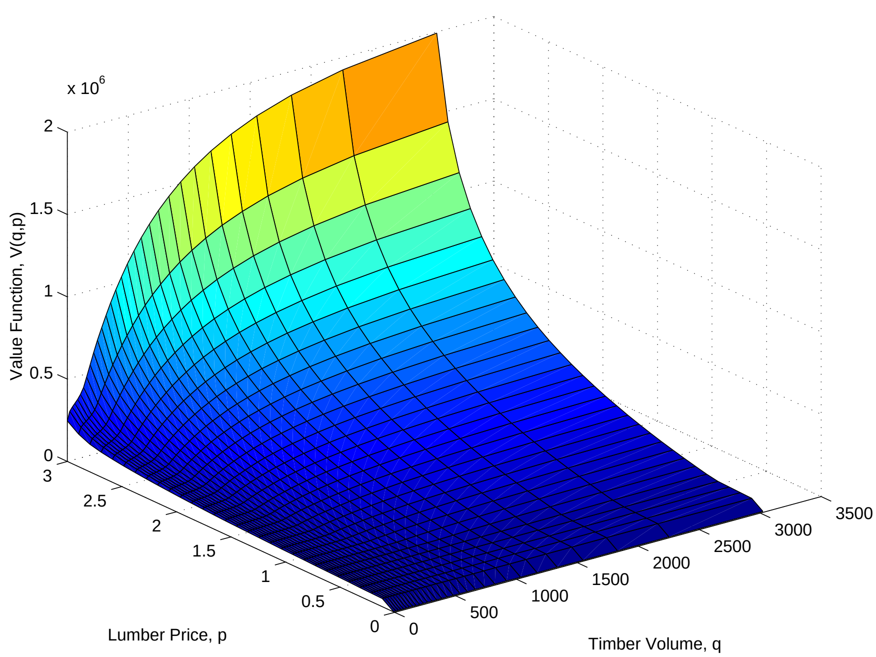


Figure 21
Optimal, Steady-State, Decision Rule

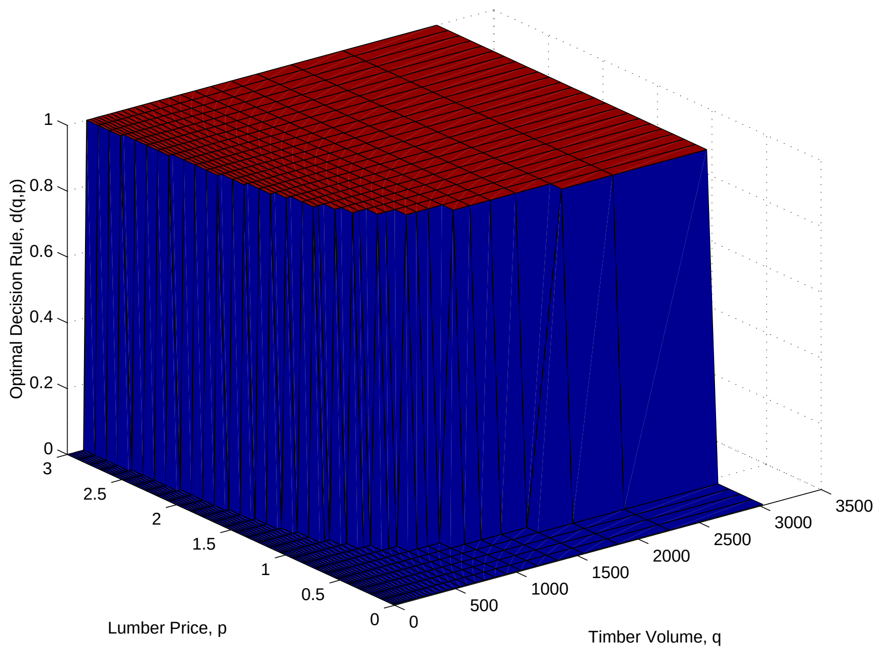
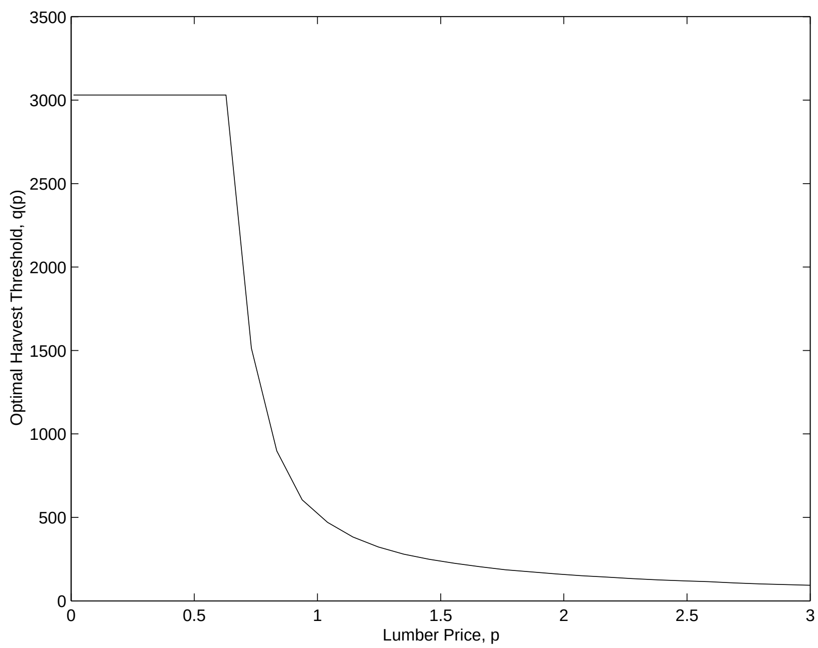


Figure 22
Optimal, Steady-State Decision Rule: Harvest when $q > q(p)$



representative block, while in Figure 18 we present the optimal harvesting rule for the same block, and in Figure 19 we present the locus of points in (p, q) space, again for the initial distribution. In Figures 20, we present the optimal, steady-state value function on the same representative block, while in Figure 21 we present the optimal, steady-state harvesting rule for the initial distribution on that block, and in Figure 22 we present the locus of points in (p, q) space, again in a steady-state.

As one might expect, it is very difficult to describe the outcome on each and every one of the $581,076$ harvest blocks, in each month for the next 100 years. Even presenting a snapshot at a point in time, as we did for the static costs depicted in Figure 16, is not particularly illuminating. Because our empirical results are dynamic, we have chosen to present the output from solving the $1,152$ different stochastic dynamic programmes in the form of an animated presentation using *MacroMedia*. Essentially, each grid in the FTSA is given a colour: just-harvested blocks are white, while newly-planted blocks are deep red because the fixed costs of replanting have just been incurred. As the timber on the block becomes more valuable, the deep red begins to turn to pink, then yellow, then green, and in the period just before harvest it becomes black.

7. Summary and Conclusions

In this paper, we have constructed an intertemporal model of rent-maximizing behaviour on the part of a single seller of timber under multi-dimensional risk as well as geographical heterogeneity. Subsequently, we have used the method of dynamic programming to characterize the optimal policy function, the rent-maximizing timber-harvesting profile. We then applied our theoretical framework to analyze unique and detailed information from the databases of the British Columbia Ministry of Forest concerning the Fraser Timber Supply Area. We have organized these data in the form of a dynamic geographical information system to account for site-specific cost heterogeneity in harvesting and transportation as well as uneven-aged stand dynamics in timber growth and yield across space and time in the presence of stochastic lumber prices and timber volumes. Our model is a powerful tool with which to conduct policy analysis for a number of reasons. First, we take geography seriously, both in the planar sense and in the three-dimensional sense. Second, we take site-specific heterogeneity seriously both on the cost side in terms of harvesting and transportation and on the growth and yield side in terms of heterogeneous stands of timber.

Third, we model initial conditions. In particular, we do not take as the starting point a steady-state allocation, or even an optimal allocation. Instead, we take the existing uneven-aged timber stand as given and derive the optimal policy function — the optimal timber-harvesting profile — in terms of this age distribution. Fourth, we use best-practice biological methods to model the dynamics of uneven-aged forest growth and yield. Fifth, in the past economists have typically demonstrated their methods by solving simple examples in closed-form or they have imposed conditions sufficient to sign comparative static predictions. We have harnessed recent developments in computational methods to solve numerically for the optimal policy function. Our empirical framework is quite rich and allows us to conduct a variety of different policy experiments that previous researchers could not. For example, we can simulate what the optimal economic response to a Spruce beetle infestation would be. In addition, we can also investigate the implications of differential productivity improvements across the sawmills in our study area. Furthermore, we can compare our estimates of the optimal harvesting policy with the harvests that have occurred during the last decade as well as those harvests that are planned over the next decade. Finally, from the perspective of industrial organization, we can investigate how the the province of British Columbia might behave as a "big player" in the lumber market.

A. Appendix

In this appendix, we summarize the geographic work we undertook to estimate harvesting and transportation costs as well as timber growth and yields for the Fraser Timber Supply Area of British Columbia, Canada. Our calculations required detailed geographic data because, for example, mountainous terrain led to significant spatial variation in timber growth as well as harvesting costs, while the clustering of sawmills in specific locations and the relative sparseness of suitable roadways influenced transportation costs. Such factors suggest that any economic analyses should be for sites of small area (*e.g.*, by the hectare which is an area 100 by 100 metres or 10,000 square metres). Also, a geographic analysis is essential to provide the site-specific data needed. Consequently, a geographic information system (GIS) becomes very useful when analyzing, processing, storing, and presenting information at the scale necessary to support the economic analyses.

A.1. GIS Software

The GIS software used was the ESRI suite of programmes. This included the following:

1) *ArcInfo Workstation* – analysis using command line interface;
2) *ArcView GIS 3.3* – analysis using *Windows*-based interface;
3) *ArcMap 8.0* – analysis using Windows based interface;
4) *ArcCatalog* – file management functions;
5) *ArcToolbox* – collection of subroutines.

ArcInfo is the oldest of the listed programmes and, generally, the most powerful; *i.e.*, it has the most functionality and allows geographic analyses that cannot be performed using the other programmes. It, however, requires an understanding of the function and format of numerous commands.

ArcView and *ArcMap* perform geographic analyses in a *Windows* environment. *ArcView* is an earlier implementation, but both programmes are still used and supported. *ArcMap* can be more complex, so at times *ArcView* is a quicker and simpler way to complete some tasks. Both programmes are menu driven and more intuitive than *ArcInfo*.

ArcCatalog is a specialized interface used to manage files. Numerous files can be created during a geographic analysis, often without the user's knowing that a specific file was created. Thus, copying, moving, and deleting files can be tricky without an "intelligent" file manager that recognizes the whole range of geographic data files and their association with other files.

ArcToolbox is a collection of subroutines from within *ArcInfo*. It uses a simpler and *Windows*-based interface to perform specific tasks. Some of this functionality exists within *ArcView* and *ArcMap*.

In general, GIS software has a specific vocabulary used to describe geographic data. Some of the terms in this vocabulary are discussed here; a glossary is presented in Table A.1. Geographic data are usually represented by *areas*, *lines*, or *points*. Areas can be represented by *polygons* or *grid cells*. A polygon is simply a set of lines that encompasses an area and can be of virtually any shape or size as long as the area is completely bounded. On the other hand, grid cells are typically squares of a common dimension and reference point. Within the ESRI system, these types of geographic data can be represented in *coverage*, *shapefile*, or *raster* formats. Coverages and shapefiles apply to polygons, lines, and points; rasters are another name for grid-cell formats. Coverages and shapefiles differ in the amount of information available concerning neighbouring features (topology). Coverages are more complex and include more information concerning neighbouring features than do shapefiles.

A.2. Data Sources

Our data were supplied by the Ministry of Forests of the province of British Columbia, Canada. Data files were provided in ESRI exchange format (E00) and ported to a local workstation via CD.

A.2.1. Method

For this research, the geographical analysis can be broadly described by the following:

1) define study area (Figures 14 and 15 in paper);

Table A.1
Glossary

Term	Definition
arc	a line or curve within a GIS coverage or shapefile.
coverage	a GIS format that contains information about topology; *i.e.*, the explicit spatial relationship among data.
GIS	geographic information system, a computer-based mapping technology.
grid	a GIS format based on square cells, with one unique value for each cell.
hectare	an area represented by a square having sides of 100 metres.
join	a specific GIS technique to associate data from different files or data bases based on a common variable.
point	a position in space, applies to coverages and shapefiles.
polygon	an area represented by a continuous closed boundary, applies to coverages and shapefiles.
raster	another term for grid.
shapefile	a GIS format that contains does not contain specific information about topology, spatial relationships are visible but not explicitly coded within the data.
slope break point	the value (percent) that distinguishes between conventional logging and helicopter logging methods.
timber mark	an identifying number for timber harvested at a particular time and place.
TASS	Timber and Stand Simulator. It simulates the yields of highly-managed stands under pest free conditions. It is based on observed growth trends or research plots and represents the potential of a specific site, species, and management routine. For more information, go to `http://www.for.gov.bc.ca/hre/gymodels/tass/`
TIPSY	Table Interpolation Program for Stand Yields. It predicts the yields of highly managed stands under pest free conditions. It is based on observed growth trends or research plots and represents the potential of a specific site, species, and management routine. For more information, go to `http://www.for.gov.bc.ca/hre/gymodels/tipsy/`
VDYP	Variable Density Yield Prediction programme. It predicts the average yield of naturally regenerated forests. For more information see `http://www.for.gov.bc.ca/hre/gymodels/vdyp/`

2) estimate harvesting costs (Figure 16 in paper);

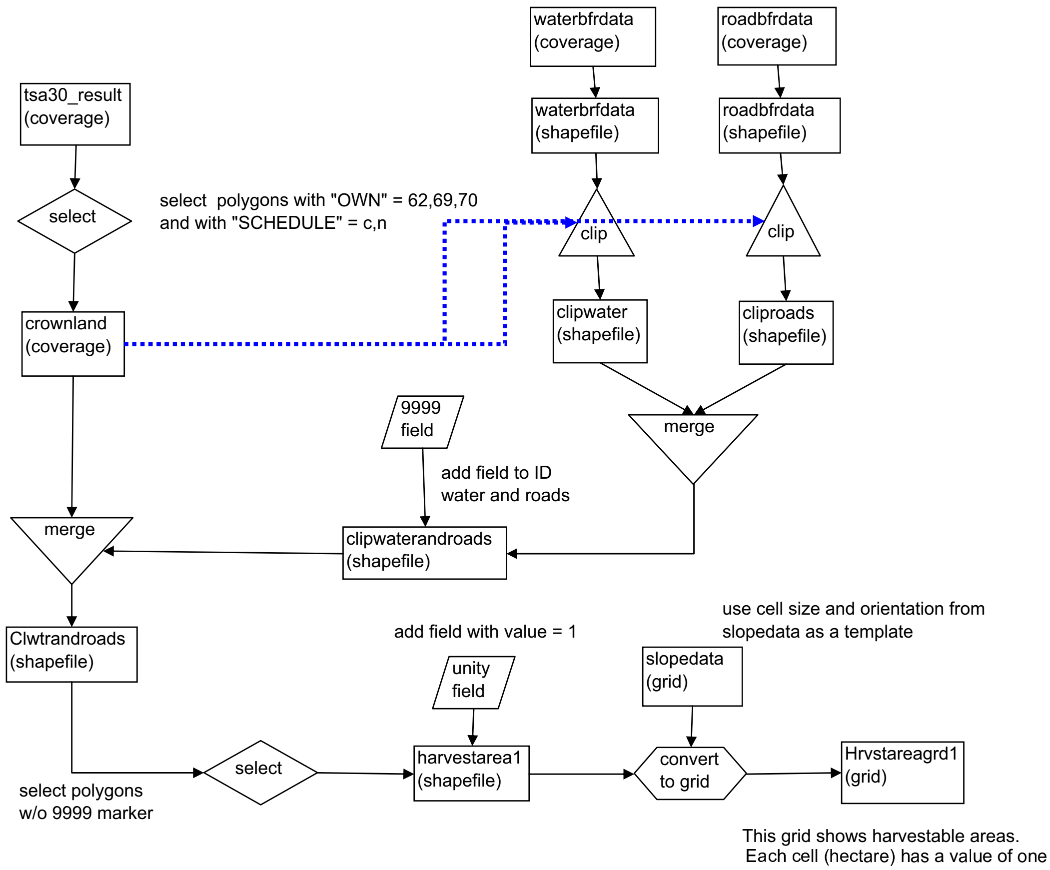
3) import vegetation data;

4) create point coverage of vegetation and cost data;

5) export data in ASCII format.

The study area was defined preliminarily as the Chilliwack Forest District, the FTSA, with geographic data (`tsa30_result`) supplied by the Ministry of Forests. The FTSA

Figure A.1
Flow Chart of GIS Definition



is located in the far southwestern portion of British Columbia and borders the United States on the south and the Strait of Georgia on the west, which connects with the Pacific Ocean. The FTSA also includes the city of Vancouver. In Figure A.1, we present a flow chart in which refinements of `tsa30_result` are described.

The study area was refined by identifying lands within the FTSA that are owned by the provincial government; *i.e.*, Crown lands. This was done via a selection process where the data file was queried for records that matched the criteria of:

- `OWN` equals 62 or 69 or 70 and i `SCHEDULE = c` or `n`.

Thus, from an ownership perspective, Crown lands became areas where harvesting is feasible. From within this set, it was necessary to identify areas where harvesting is infeasible for technical, cultural, or aesthetic reasons. For our research, we excluded areas near roads and streams. Starting with line coverages of roads and water bodies, we applied a 100 metre buffer around each feature, added an identifying code (`9999`) and merged these files with the file representing Crown lands. The merged file was then queried for all areas without the `9999` marker, with these areas representing land where harvesting is permitted. This information was saved in a raster format to produce a grid showing permissible harvesting areas. Each grid cell measures 100 metres by 100 metres, one hectare.

In the 2002 *Coast Appraisal Manual*, which is published by the Revenue Branch of Ministry of Forests, two primary factors affect harvesting and transportation costs: the slope of the harvest block and the distance from the harvest block to the sawmill. The Ministry of Forests uses the following equation:

$$\text{Harvesting Costs } (\$/m^3) = \text{Slope Factor} + \text{Distance}(m) \times 0.000153 + 32.05.$$

to estimate harvesting costs. Here, the "Slope Factor" was either $21.55(\$/m^3)$ or $66.60(\$/m^3)$, depending on whether conventional or helicopter logging was used. The smaller number was applied when a slope was below 75 percent, while the larger number was applied to slopes above 75 percent. Note that a slope of 100 percent represents a 45-degree angle from horizontal. A breakpoint of 75 percent was chosen based on conversations with Ministry of Forests. It was also compared with historical harvest-cost data by examining $2,172$ records from the FTSA and computing the error between the historical and predicted harvesting costs at the same location. Increasing the break-point slope resulted a slight decrease in the standard deviation of the error between the two data sets (*i.e.*, 24 rather than 22), but it was not considered significant enough to change the base break-point value. Historical costs were adjusted by the Canadian Consumer Price Index (CPI) to be comparable to predicted costs. The land's slope was derived from a digital elevation model (DEM) and grid files of both slope and elevation were provided by Ministry of Forests. The slope grid file was trimmed to the extents of the permissible harvesting areas to minimize data processing and storage requirements.

Computing the distance between each hectare of harvestable land and a sawmill involved three steps. First, we had to identify an appropriate point to represent a sawmill receiving wood for each hectare of forest land. Next, we had to compute the road distance from the sawmill location to all points on the road network. Finally, we had to compute the off-road distance from the hectare of forest to the nearest road plus the on-road distance from that point to the sawmill location.

We chose to represent all sawmills at one central location. This simplified calculating transportation costs because we did not have to assign timber to specific sawmills. We computed the geographic centre of gravity for all sawmills within the FTSA and used this as a reference point for calculating distances. We accounted for production volumes in this procedure, so that larger mills were weighted more heavily than smaller ones and, thus, had more influence on the location of the reference point. Most of the sawmills in the FTSA are clustered in the Chilliwack area, while areas to be harvested are primarily located to the north and northeast of Chilliwack. Thus, errors associated with representing many sawmills by one central or average location is less severe than it might be otherwise.

Computing distances required a data file of the road network; this was provided by Ministry of Forests. This network was converted to a grid and then the grid cells were counted from the sawmill point to each other point on the network. This provided an on-road distance to each part of the forest within the study area. As part of this process, it was necessary to adjust the sawmill reference point so that it was actually on the road network. This was a minor adjustment of only a couple of hundred metres, at most, and was accomplished using a shift function.

The road network grid was also used as the source grid for calculating off-road distances between each forest hectare and its nearest road. The *ArcInfo* function `EUCALLOCATION` was used to compute total distance from each forest hectare to the central sawmill location. `EUCALLOCATION` produces two output grids, `OUT_ALLOC` and `OUT_DIST`. The first of these contained the value of the nearest road cell (on-road distance), and the second output grid contained the distance from each forest hectare to the nearest road. Summing these two output grids produced a new grid containing the total distance from each forest hectare to the central sawmill location. This new grid was converted to a point format for later joining with timber-growth parameters.

The next part of the analysis was to import data concerning vegetation. These data served as inputs to the timber-growth programmes *VDYP* and *TASS*. Vegetation data were also supplied by Ministry of Forests. This file consisted of over 168,000 polygons in a coverage format, with each polygon having of 134 parameters. Of these, 20 parameters were required for the timber-growth programmes. These parameters are listed in Table A.2, along with their column number in the original data file, their

Table A.2
***VDYP* Input Variables**

Variable	Label	Column	Type	Row
Forest Inventory Zone	`FIZ`	65	`character(1)`	28
Volume Adjustment Factors	`adj_volume_factor`	75	`real*8`	38
Stand Crown Closure	`crown_closure`	76	`integer`	39
Stocking Class	`class_cd`	78	`real*8`	41
Adjusted Site Index	`site_index`	79	`real*8`	42
Species 1	`sp1`	87	`character(4)`	50
Species 2	`sp2`	88	`character(4)`	51
Species 3	`sp3`	89	`character(4)`	52
Species 4	`sp4`	90	`character(4)`	53
Species 5	`sp5`	91	`character(4)`	54
Species 6	`sp6`	92	`character(4)`	55
Percentage Species 1	`pct1`	93	`integer`	56
Percentage Species 2	`pct2`	94	`integer`	57
Percentage Species 3	`pct3`	95	`integer`	58
Percentage Species 4	`pct4`	96	`integer`	59
Percentage Species 5	`pct5`	97	`integer`	60
Percentage Species 6	`pct6`	98	`integer`	61
Projected Adjusted Age	`proj_adj_age`	102	`integer`	65
Projected Adjusted Height	`proj_adj_height`	106	`integer`	69

variable type, and their position in a data equivalency table.

The above data were joined to a raster representation of harvestable areas within the study area using the original polygon `ID` numbers as references. The resulting file was converted to a point coverage in preparation for combining vegetation data with cost data.

Once cost and vegetation data were available in point coverages, they were combined into a single file in which they were geo-referenced to each other. Joining these files required an exact match, a one-to-one mapping, between the point coverages. However, the coverages contained some mismatches at the edges of the study area where data existed for vegetation data (`vdyppnt1`), but not for harvest costs (`costrnpnt1`). Edge effects are common in these sorts of analyses because the data are from two different sources and a number of data transformations have been undertaken. These edge errors were identified visually and removed from the data. The function `JOINITEM` in *ArcInfo* was then used to combine cost and vegetation data into a single file for later export.

The combined data set (`polyidpnt1`) was exported in ASCII format, having filename `siteind4b.asc`. In addition to the 20 timber-growth variables discussed above, four other variables were included in the exported file. They included an identification code, an x-coördinate, and a y-coördinate as well as the total of harvesting- and transportation-costs.

A.2.2. Output

The file `siteind4b.asc` contains 706,603 records. Each record represents one hectare of potentially harvestable forest area within our study area and has the 24 parameters listed above associated with it.

Harvesting and transportation costs are calculated from a geographic analysis of the study area at a scale of one hectare. The results of the geographic analysis are discussed below. Timber-growth information was not created in this work, but rather transferred into a form appropriate for analysis.

Harvesting and transportation costs for the study area are depicted in Figure 16. Calculated values range from \$55 to \$137 CAD per cubic metre. In general, costs increase from the southwest to the northeast sections of the study area. This is expected and in keeping with the clustering of sawmill locations in the Chilliwack area which is in the southwest part of the study area. Superimposed over this general trend are local variations in costs due to changes in the slope of the land and due to the remoteness of some sections from the local road network.

From the Revenue Branch of the Ministry of Forests, we also obtained historical data concerning over 2,172 timber marks that were recently harvested in the FTSA.[14] Among other things, these historical data contained information concerning harvesting and transportation costs as well as fixed costs associated with such things as road construction. By and large, our estimated harvesting and transportation costs correlate quite highly with the historical data. Fixed costs, however, were impossible to predict in our model. In addition, these fixed costs were sometimes high. We chose to deal with this by subtracting the average of the fixed costs of our sample from the total costs for each site. In essences, we assume that a harvester gets a draw from an urn of fixed costs for each block.

A.3. Stochastic Process of Lumber Prices

Our data set concerning lumber prices contains 250 monthly time-series observations

[14] A timber mark is like a tax identification number. It maps the timber harvested from a particular geographical area into a stumpage rate that must be paid to the Crown for timber harvested. Historically, a timber mark was made with a hammer on the end of a log. Today, it is simply a name for tax record.

from January 1979 to October 1999. We have the real, average-monthly price per thousand board feet (1MBF) for one box car of Western, Kiln-Dried (KD), Spruce-Pine-Fir (SPF), 2x4s, Standard and Better (Std&Btr), Random Lengths (R/L). The series was contructed from weekly reports in the trade publication *Madison's Canadian Lumber Reporter*, weekly issues, January 1979 to October 1999. This price series is listed as "less 5&2 percent" discounts, and is free-on-board (FOB) mill. Moreover, it is quoted in nominal U.S. dollars. To convert the series into Canadian dollars we used the Canadian/U.S. spot exchange rate from the CANSIM database, matrix 933, series B40001.[15] We converted nominal price data into real terms by dividing by the Canadian Consumer Price Index (CPI) setting January 2002 to one.

We specified $\tau_2(p_{j+1}|p_j)$, the the stochastic process of p_j, according to the following continuous-state, Markov process:

$$\log p_{j+1} = \rho_0 + \rho_1 \log p_j + \sigma\varepsilon_{j+1}$$

where ε_j is an independent and identically-distributed Gaussian error term having mean zero and variance one. For numerical stability, when solving the dynamic-programming model, we re-scaled the units of prices from dollars per thousand board feet to dollars per board feet. We also imposed reflecting lower and upper bounds on the level of the price process, denoted $\underline{p}$ and $\bar{p}$, respectively. These were 0.100 and 0.750, respectively. To wit, the lowest price imaginable is \$100 CAD per thousand board feet, in 2002 dollars, and the highest is \$750 CAD per thousand board feet. Our estimates of ρ_0 and ρ_1 are 0.05 and 0.98, respectively.

[15] Statistics Canada information is used with the permission of Statistics Canada. Users are forbidden to copy the data and redisseminate them, in an orginal or modified form, for commercial purposes, without the expressed permission of Statistics Canada. Information on the availability of a wide range of data from Statistics Canada can be obtained from Statistics Canada's regional offices, from its website at `http://www.statcan.ca` or from its toll-free number 1 (800) 263-1136.

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