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Sticky Demand vs. Sticky Prices

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ABSTRACT

Sticky demand — sticky nominal spending — acts as a substitute for sticky prices. In an inventory-theoretic model of the demand for money, monetary injections are offset by endogenous movements in velocity that keep nominal spending flat. When embedded in a sticky price model, this reduces the model's reliance on implausibly large amounts of exogenous stickiness. For example: 6 months asset market segmentation plus 3 months stickiness gives as much persistence as a model with no segmentation and 12 months stickiness. Inventory-theoretic money demand gives rise to a monetary transmission mechanism whereby policy shocks induce slow moving changes in the distribution of money which are in turn translated into slow, cumulative movements in aggregate output and the distribution of consumption across households. Relative to a benchmark sticky price model, this reduces the initial impact effect of a monetary policy shock and leads to a delayed "hump-shaped" output response.

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1. Introduction

Inertia in nominal variables can translate monetary policy shocks into movements in real prices and quantities. Overwhelmingly, macroeconomists have focused on nominal inertia in the relative prices of producers and/or wages as a source of short-run monetary non-neutrality. In particular, if velocity is constant or nearly so and the price level responds less than proportionately to changes in the money supply, real output will be affected. However, if velocity is not approximately constant, this reasoning is incomplete. In a model where money demand is of the "inventory-theoretic" or Baumol (1952)-Tobin (1956) form, the distribution of money across households is itself a source of nominal inertia. For example, in the model developed by Alvarez, Atkeson and Edmond (2002), where asset market segmentation gives rise to inventory-theoretic money demand, a monetary injection leads to an endogenous offsetting movement in velocity. Since money and velocity move in opposite directions, nominal spending is kept relatively flat and does not respond in the way characteristic of a sticky price model with approximately constant velocity.

In order to focus attention on the dynamics of money demand, Alvarez et al limit their analysis to an endowment economy. When output is exogenously given, however, all of the inertia is attributed to sluggish *price* adjustment. In their paper, short-run monetary non-neutrality showed up through changes in real interest rates and changes in the distribution of consumption over households. This paper examines two questions: 1) what happens when the level of real output can also respond endogenously to a monetary shock, and 2) what are the interactions between inertia in nominal spending — *sticky demand* — and sticky prices.

More specifically, this paper extends the analysis of Alvarez et al by introducing elastic labor supply so that output is endogenous and by introducing a familiar mix of monopolistic competition as in Blanchard and Kiyotaki (1987) and sticky prices as in Calvo (1983). The model nests the benchmark "new Keynesian" sticky price model as a special case.

As documented extensively by Chari, Kehoe, and McGrattan (2000), however, sticky price models with static money demand lack an internal propagation mechanism. Monetary shocks are not translated into sustained movements in output over and above the amount of built-in stickiness. A sticky price economy with constant money demand has impulse response functions for output that essentially mimic the monetary shock fed into the economy. Also, in sticky price models of this kind, money demand does not play an important role. Typically, money demand is introduced in these models by giving households a preference for real balances that enters utility in a separable way. An important consequence of this is that money demand plays no role in determining the equilibrium behavior of any of the other variables.[1]

However, a sticky price model with the kind of dynamic, inventory-theoretic, money demand that comes from asset market segmentation turns out to have a rich monetary transmission mechanism whereby policy shocks are translated into slow, cumulative movements in both aggregate output and its distribution across households. The slow movements in the distribution of money across households that are characteristic of inventory-theoretic money demand can *substitute* for sticky prices in that moderate amounts of asset market segmentation can reduce the reliance of a sticky price model on what seem to be high amounts of exogenous stickiness. There are persistent liquidity effects even when prices are relatively

[1] If short-run money demand was unstable due to exogenous velocity shocks, then the practical irrelevance of money demand for the behavior of an economy and for the role of monetary policy is presumably a good thing. Alvarez, Atkeson and Edmond (2002) argue, however, that a large fraction of short-run movements in velocity can be accounted for by endogenous responses to policy shocks.

flexible.

As a brief summary of this paper's results, consider the following. Sbordone (2002) estimates price stickiness of slightly more than a year in order to minimize the distance between a Calvo-style sticky price model and the data. This amount of stickiness falls at about the 10th percentile in the distribution of price changes documented by Bils and Klenow (2002), so price changes of that infrequency are quite uncommon. However if modest amounts of asset market segmentation are introduced, so that households exchange money for interest-bearing securities once every six months, then the amount of price stickiness needed falls to three months, which is at about the 90th percentile of the Bils-Klenow distribution and so seems much more reasonable. In short, introducing inventory-theoretic money demand into a sticky price model has three important effects: given a monetary policy shock, it 1) reduces the impact effect on output, 2) leads to a delayed peak in the output response, and 3) increases the persistence of the output response.

Briefly, this paper is organized as follows. Section 2 develops a model that nests both a benchmark sticky price environment with static money demand and a model with inventory-theoretic money demand as special cases. The parameterization of this model is outlined in Section 3 and its steady state is analyzed in Section 4. I then consider equilibrium responses to monetary policy shocks by first summarizing the workings of a benchmark sticky price model with static money demand in Section 5, a flexible-price segmented asset markets economy in Section 6, and the interactions of the two effects in Section 7. I then offer some conclusions and suggestions for future research in the final section.

2. The benchmark economy

Consider a simple monetary economy populated by a mass (of measure 1) of infinitely-lived households. In this economy, time is discrete and denoted $t = 0, 1, 2, ...$ Each period t an event h_t is realized. The history of events up to and including t is denoted $h^t = (h_0, h_1, ...h_t)$ with given initial realization h_0. The probability density function for histories is denoted f_t.

There are four types of commodities in this economy: 1) a single, non-storable consumption good that is produced by a large number of identical perfectly competitive firms; 2) a continuum of differentiated intermediate goods which are produced by monopolistically competitive firms and which are inputs to the consumption good producing firms; 3) labor, which is supplied by households to a competitive spot market for its services; and 4) money. Money is introduced into the economy by government open market operations.

A. Households

As in a cash-in-advance model, each household consists of a worker and a shopper and there are distinct asset and goods markets. Each household has access to two financial intermediaries: 1) a *brokerage account* through which it manages its portfolio of assets and 2) a *bank account* through which it manages transactions in the goods market.

At the beginning of each period, an event h_t is realized and households use their brokerage accounts to trade in the asset market. The worker and shopper then split up with the worker supplying labor to a competitive spot market in return for the nominal wage $W_t(h^t) = P_t(h^t)w_t(h^t)$. Here, $P_t(h^t)$ denotes the price level in the current period. Firms produce goods with the labor of the workers and these goods are in turn bought by various shoppers using money held in their household's bank account. At the end of the period, each worker / shopper pair reunites. The worker brings home her wages which can then be spent next period while the shopper brings home the consumption goods that they both consume.

Unlike a standard cash-in-advance model, however, households do not have the opportunity to transfer money *between* the asset market and the goods market every period. Instead, as in Alvarez, Atkeson and Edmond (2002), I assume that each household has the opportunity to transfer money between its brokerage account and its bank account only once every $N \geq 1$ periods.[2] In other periods, the household can trade assets in its brokerage account and use money in its bank account to purchase goods — it simply cannot move money between these two accounts. Households that currently have the opportunity to transfer money between their brokerage and bank accounts are called *active households* while those households that currently are unable to transfer money between these accounts are *inactive households*. This form of market segmentation will imply that households manage their money holdings in a manner reminiscent of the inventory-theoretic models of money demand originally studied by Baumol (1952) and Tobin (1956).

For simplicity, the parameter N is taken to be exogenous and $1/N$ of the households are active in any period. Households differ according to how long it has been since they were active. The following notation keeps track of this heterogeneity: Each household is indexed by the number of time periods since it was last active, here denoted by $s = 0, 1, ..., N-1$. A household of type $s < N-1$ in the current period will be type $s+1$ in the next period. A household of type $s = N-1$ in the current period will be type $s = 0$ in the next period. Hence a household of type $s = 0$ is active in this period, a household of type $s = 1$ was active last period, and a household $s = N-1$ will be active next period. In period $t = 0$, each household

[2] If $N = 1$, then this model reduces to a standard cash-in-advance model.

has an initial type s_0 with fraction $1/N$ of the households of each type $s_0 = 0, 1, ..., N-1$. Let $S(t, s_0)$ denote the type in period t of a household that was initially of type s_0. For all s_0, $S(0, s_0) = s_0$. For all periods t and s_0 such that $S(t, s_0) = 0, 1, ..., N-2$, in period $t+1$, $S(t+1, s_0) = S(t, s_0) + 1$. For the s_0 such that $S(t, s_0) = N-1$ in period t, $S(t+1, s_0) = 0$.

Bank accounts and brokerage accounts

The households of type $s \geq 1$ are inactive in the current period. For an inactive household of type s, the quantity of money that it has on hand in its bank account at the beginning of goods market trading in state h^t is denoted $M_t(s, h^t)$. The shopper for this household spends some of this money on goods, $P_t(h^t)c_t(s, h^t)$, and the household carries the unspent balance in its bank account into next period, $Z_t(s, h^t)$. The balance that this household begins with is equal to the quantity of money that it held over in its bank account last period $Z_{t-1}(s-1, h^{t-1})$ plus the wage income of its mate which was brought home at the end of the last period. If the worker supplied $n_{t-1}(s-1, h^{t-1})$ units of labor in the previous period, this wage income is $W_{t-1}(h^{t-1})n_{t-1}(s-1, h^{t-1})$ units of account. Thus, for each date and state, the evolution of money holdings and consumption for these households is given by

$$M_t(s, h^t) \equiv Z_{t-1}(s-1, h^{t-1}) + W_{t-1}(h^{t-1})n_{t-1}(s-1, h^{t-1}) \tag{1}$$

$$M_t(s, h^t) \geq P_t(h^t)c_t(s, h^t) + Z_t(s, h^t) \tag{2}$$

and the non-negativity constraints $c_t(s, h^t) \geq 0$ and $Z_t(s, h^t) \geq 0$.

When a household is active, and hence of type $s = 0$, it chooses a transfer of money $D_t(h^t)$ from its brokerage account in the asset market into its bank account in the goods market. For each date and state, the money holdings and consumption of active households therefore satisfy

$$M_t(0, h^t) \equiv Z_{t-1}(N-1, h^{t-1}) + W_{t-1}(h^{t-1})n_{t-1}(N-1, h^{t-1}) + D_t(h^t) \tag{3}$$

$$M_t(0, h^t) \geq P_t(h^t)c_t(0, h^t) + Z_t(0, h^t) \tag{4}$$

with $c_t(0, h^t) \geq 0$, $Z_t(0, h^t) \geq 0$, and $D_t(h^t) \geq 0$. Notice that the constraint (4) is written as if $D_t(h^t)$ is a withdrawal from the brokerage account that supplements beginning-of-period money in the goods market. If $D_t(h^t)$ is negative, then the household is instead transferring money *to* the asset market.

In addition to the constraints on the household's bank account, equations (1)-(4) above, the household also faces a sequence of constraints on its brokerage account. In each period t, the household can trade a complete set of one-period state contingent bonds, each of which pays off one unit of account — one dollar, say — into the household's brokerage account next period if the relevant state of nature is in fact realized. Let $B_{t-1}(s-1, h^t)$ denote the stock of bonds held by inactive households of type $s \geq 1$ at the beginning of period t following history h^t and $B_t(s, h^t, h')$ denote the stock of bonds purchased by that household that will payoff next period if history $h^{t+1} = (h^t, h')$ is realized.[3] Let $M_t^A(s, h^t)$ denote money held by the household in its brokerage account at the end of period t. Since an inactive household cannot transfer money between its brokerage account and its bank

[3]There is a slight abuse of notation here; it would be more accurate to write the cumbersome $B_{t-1}(s-1, h^{t-1}, h_t)$ for the stock of bonds held at the beginning of period t, chosen when the history was h^{t-1} and conditional on the event h_t realized at the beginning of the period.

account, this household's bond and money holdings in its brokerage account must satisfy

$$B_{t-1}(s-1,h^t)+M^A_{t-1}(s-1,h^{t-1})+\Pi_t(h^t)-H_t(h^t) \tag{5}$$
$$\geq \int q_t(h^t,h')B_t(s,h^t,h')dh'+M^A_t(s,h^t)$$

where $q_t(h^t,h')$ is the price in period t given history h^t of a bond that will pay one dollar in period $t+1$ if h' is realized, $\Pi_t(h^t)$ denotes the nominal profits made by firms which are distributed in lump-sum fashion to households, and $H_t(h^t)$ denotes nominal lump-sum taxes levied by the government. Money held in the asset market must satisfy the non-negativity constraint $M^A_t(s,h^t)\geq 0$ and the choice of bond holdings must satisfy $B_t(s,h^t,h')\geq -\underline{B}$ for some arbitrarily large positive constant $\underline{B}$.

The analogous constraint for active households is

$$B_{t-1}(N-1,h^t)+M^A_{t-1}(N-1,h^{t-1})+\Pi_t(h^t)-H_t(h^t) \tag{6}$$
$$\geq \int q_t(h^t,h')B_t(0,h^t,h')dh'+D_t(h^t)+M^A_t(0,h^t)$$

The transfer $D_t(h^t)$ is written as a withdrawal; if $D_t(h^t)>0$, it is an expense in the household's brokerage account, but a source of income in its bank account.

Optimization problem

The preferences of the worker / shopper pair are given by a time- and state-seperable expected utility function over random sequences of consumption $c_t(s, h^t)$ and leisure $\ell_t(s, h^t)$

$$\sum_{t=0}^{\infty} \int \beta^t U[c_t(s, h^t), \ell_t(s, h^t)] f_t(h^t) dh^t, \qquad 0 < \beta < 1$$

with $s = S(t, s_0)$ and s_0 given. The period utility function U is assumed to be strictly increasing in each argument, strictly concave, and twice continuously differentiable on the interior of its domain. Households are endowed with one unit of labor time so that

$$1 \geq n_t(s, h^t) + \ell_t(s, h^t) \tag{7}$$

constrains the amount of time that can be supplied by the worker. For each date and state and taking as given the prices and aggregate variables, each household of type s_0 chooses transfers $D_t(h^t)$, consumption $c_t[S(t, s_0), h^t]$, labor supplies $n_t[S(t, s_0), h^t]$, cash and bond portfolios to hold over in the asset market, $M_t^A[S(t, s_0), h^t]$ and $B_t[S(t, s_0), h^t, h']$, and money holdings $Z_t[S(t, s_0), h^t]$, to maximize expected utility subject to the constraints (1), (2), and (5) in those periods t in which $S(t, s_0) > 0$, and constraints (3), (2), and (6) in those periods t in which $S(t, s_0) = 0$.

To simplify this optimization problem, use (1) and (3) to substitute out for money holdings $M_t(s, h^t)$ in constraints (2) and (4) and use (7) to substitute out for leisure in the objective function. Let $\psi_t[S(t, s_0), h^t] \geq 0$ denote the Lagrange multipliers on the constraints (2) and (4) of household $s = S(t, s_0)$ at (t, h^t), and let $\lambda_t[S(t, s_0), h^t] \geq 0$ denote the Lagrange multipliers for the constraints (5) and (6). Let $\delta_t^Z[S(t,s_0),h^t] \geq 0$ denote the multipliers on the non-negativity constraints for money held in the bank account, $Z_t[S(s_0,t),h^t]$, and let $\delta_t^A[S(t,s_0),h^t] \geq 0$ denote the multipliers on the non-negativity constraints for money held in the brokerage account, $M_t^A[S(t,s_0),h^t]$. The first order necessary conditions for the coalition's optimization problem include

$$D_t(h^t): \qquad \psi_t(0,h^t) = \lambda_t(0,h^t) \tag{8}$$

$$c_t(s,h^t): \qquad \beta^t \frac{U_{c,t}(s,h^t)}{P_t(h^t)} f_t(h^t) = \psi_t(s,h^t) \tag{9}$$

$$n_t(s,h^t): \qquad \beta^t \frac{U_{\ell,t}(s,h^t)}{P_t(h^t)} f_t(h^t) = w_t \int \psi_{t+1}(s+1,h^t,h')dh' \tag{10}$$

$$B_t(s,h^t,h'): \qquad q_t(h^t,h') = \frac{\lambda_{t+1}(s+1,h^t,h')}{\lambda_t(s,h^t)} \tag{11}$$

$$Z_t(s,h^t): \qquad \delta_t^Z(s,h^t) \leq \psi_t(s,h^t) - \int \psi_{t+1}(s+1,h^t,h')dh' \tag{12}$$

$$M_t^A(s,h^t): \qquad \delta_t^A(s,h^t) \leq \lambda_t(s,h^t) - \int \lambda_{t+1}(s+1,h^t,h')dh' \tag{13}$$

In these formulas, the notation $U_{c,t}(s,h^t)$ denotes the marginal utility of consumption for a household of type s at date t if h^t is realized. Similarly, $U_{\ell,t}(s,h^t)$ denotes the marginal utility of leisure. The optimality conditions (12) and (13) are Kuhn-Tucker complementary-slackness conditions that hold with equality, respectively, if $Z_t(s,h^t)$ or $M_t^A(s,h^t)$ is strictly positive.

A more complete discussion of the optimization problem including a derivation of these first order conditions is included in the Appendix to this paper. In what follows, I use these conditions to discuss some characteristics of optimal plans.

Asset prices

Let $Q_t(h^t)$ be the price of a dollar delivered in the asset market following history h^t. This is defined by the formula

$$Q_t(h^t) \equiv q_0(h_0, h_1) \times q_1(h^1, h_2) \times q_2(h^2, h_3) \times \cdots \times q_{t-1}(h^{t-1}, h_t)$$

Notice that the first order condition (11) implies that the marginal worth of a dollar available in the asset market, $\lambda_t(s, h^t)$, shrinks at a common rate — given by the price $q_t(h^t, h')$ — for all households. Therefore, the price of a dollar satisfies, for all s,

$$Q_t(h^t) = \frac{\lambda_t(s, h^t)}{\lambda_0(s_0)} = \frac{\lambda_t(0, h^t)}{\lambda_0(s_0)} \tag{14}$$

From the first order conditions for transfers, (8), and for consumption, (9), this means

$$\lambda_0(s_0) Q_t(h^t) = \beta^t \frac{U_{c,t}(0, h^t)}{P_t(h^t)} f_t(h^t) \tag{15}$$

This formula is completely standard except for the fact that it is the marginal utility of *active* households that determines asset prices, not the marginal utility of the average or typical household. In turn, equation (15) means that the price of a contingent bond satisfies

$$q_t(h^t, h') = \beta \frac{U_{c,t+1}(0, h^t, h')}{U_{c,t}(0, h^t)} \frac{P_t(h^t)}{P_{t+1}(h^t, h')} \frac{f_{t+1}(h^t, h')}{f_t(h^t)} \frac{\lambda_0(s_0 + 1)}{\lambda_0(s_0)} \tag{16}$$

By choosing the initial wealth distribution appropriately, it is always possible to ensure that the date zero multipliers are constant across all households, $\lambda_0(s_0) = \lambda_0(s_0 + 1)$ for all s_0. In what follows, I assume that this is indeed the case.

Money demand

Define the net nominal interest rate $i_t(h^t)$ paid on a one-period *sure* claim according to

$$\frac{1}{1+i_t(h^t)} = \int q_t(h^t, h')dh' \tag{17}$$

A bond that returns $i_t(h^t)$ is riskless in nominal terms, though it may be risky in real terms because of inflation. From the brokerage constraints (5)-(6), it should be clear that if the nominal interest rate is positive, so that the ratio on the left hand side of (17) is less than one, sure bonds dominate money as a store of value in the asset market. Using equations (13), (14) and (17), the multiplier $\delta_t^A(s, h^t)$ must satisfy

$$\frac{i_t(h^t)}{1+i_t(h^t)} \geq \frac{\delta_t^A(s, h^t)}{\lambda_t(s, h^t)}$$

with equality if and only if $M_t^A(s, h^t) > 0$. The quantity on the left hand side of this expression is the opportunity cost of holding money *in the asset market*. If the nominal interest rate is positive, money is not held by any household s in the asset market. In what follows, I will focus my attention on equilibria with positive nominal interest rates.

Money holdings *in the goods market* are governed by an Euler inequality. Combining (9) and (12) gives, for each $s = 0, 1, ..., N-1$,

$$1 \geq \int \beta \frac{U_{c,t+1}(s+1, h^t, h')}{U_{c,t}(s, h^t)} \frac{P_t(h^t)}{P_{t+1}(h^t, h')} \frac{f_{t+1}(h^t, h')}{f_t(h^t)} dh' \tag{18}$$

with equality if and only if $Z_t(s, h^t) > 0$. When a shopper holds money over in the goods market, the relevant rate of return is the return on money — the reciprocal of the inflation factor. The integral on the right hand side of this expression defines a conditional expectation with respect to the history h^t with transition density $f_{t+1}(h^t, h')/f_t(h^t)$. Typically, the money holding of the shoppers that will remain inactive next period will be positive while the households that will be active next period, the $s = N - 1$ households, will not have money left over at the end of their "trip". In this *regular* case, $Z_t(s, h^t) > 0$ for all $s < N - 1$ and $Z_t(N-1, h^t) = 0$. However, because households receive wage income each period, it is possible for households to choose not to hold any money over even before $s = N - 1$. As discussed in the Appendix, this is straightforward to check when equilibria are being computed.

Labor supply

Using the first order conditions (9) and (10), the labor supply of a household satisfies

$$U_{\ell,t}(s, h^t) = w_t \left[\int \beta U_{c,t+1}(s+1, h^t, h') \frac{P_t(h^t)}{P_{t+1}(h^t, h')} \frac{f_{t+1}(h^t, h')}{f_t(h^t)} dh' \right] \tag{19}$$

This is an intertemporal analogue to the usual static condition that requires the equality of the marginal rate of substitution between leisure and consumption to equal the real wage, the relative price of leisure in terms of consumption. Here, because the wage income from labor supplied this period is only able to be spent *next* period, the optimality condition requires that the household take into account discounting, inflation, and uncertainty as well.[4] However, if the household chooses to hold money over in the goods market, $Z_t(s, h^t) > 0$,

[4]An optimality condition of this kind is completely standard in a cash-in-advance model with endogenous labor supply. See Cooley and Hansen (1989) for a comprehensive analysis.

then (18) holds with equality and we can write (19) as the more familiar

$$\frac{U_{\ell,t}(s,h^t)}{U_{c,t}(s,h^t)} = w_t \tag{20}$$

B. Government

On the other side of the asset market is a government that levies lump-sum taxes and that conducts stochastic open market operations. Let $B_{t-1}(h^t)$ be the total stock of government bonds at the beginning of period t following history h^t. The government faces a sequence of budget constraints of the following form

$$M_t(h^t) - M_{t-1}(h^{t-1}) + H_t(h^t) + \int q_t(h^t,h')B_t(h^t,h')dh' \geq B_{t-1}(h^t)$$

so that a liability of $B_{t-1}(h^t)$ is covered by a mixture of money creation, taxes, and new issues of bonds. Just like the households, the government faces a constraint on its bond policy; $B_t(h^t,h') \geq -\underline{B}$ for some arbitrarily large positive constant $\underline{B}$.

The growth rate of the money supply is an exogenously specified stochastic process

$$\frac{M_t(h^t)}{M_{t-1}(h^{t-1})} = 1 + \mu_t(h^t)$$

At the beginning of period $t = 0$, the initial stock of government debt is B_0 [i.e., $B_0(h_0)$] and the initial money injection is $(\mu_0/(1+\mu_0))M_0$. This budget constraint implies that the government pays off its initial debt with a combination of lump-sum taxes and money injections achieved through open market operations. All households of type $s_0 \geq 1$ begin with money $M_0(s_0)$ in their bank accounts in the goods market. This quantity is the balance on the left side of (2) in period $t = 0$. For active households in period $t = 0$, those for whom $s_0 = 0$, the initial money stock $M_0(0)$ in (4) is composed of an initially given amount Z_{-1} and a transfer D_0 that they *choose*. Each household of type s_0 also begins period $t = 0$ with initial balance $B_0(s_0)$ in its brokerage account on the left side of constraints (5) and (6). The households initially have no money corresponding to M_{-1}^A in their brokerage accounts.

C. Firms

The market structure and price setting behavior of firms in this model is quite standard. The market structure follows Blanchard and Kiyotaki (1987) while the price setting model is a discrete time version of Calvo (1983). Since this setup is very standard in the literature, the exposition in this section will be brief.

Market structure

There is a large number of competitive final goods producers that assemble their product using differentiated inputs. The inputs are supplied by monopolistically competitive intermediate goods producers that are indexed by $\omega \in [0, 1]$. The supply of the final good is denoted $y_t(h^t)$; this is produced according to a concave CES aggregate of intermediate goods, $y_t(\omega, h^t)$,

$$y_t(h^t) = \left[\int_0^1 y_t(\omega, h^t)^\theta d\omega\right]^{1/\theta}, \qquad 0 < \theta < 1 \tag{21}$$

The parameter θ governs the degree of imperfect competition; as θ approaches one, the intermediate goods become closer substitutes.

Final good producers take as given the price level $P_t(h^t)$ and the prices of intermediates

$P_t(\omega, h^t)$ and maximize profits by choosing a schedule of demands, one for each intermediate input $\omega \in [0,1]$. That is

$$0 = \max_{y_t(\omega,h^t)} \left[P_t(h^t) y_t(h^t) - \int_0^1 P_t(\omega, h^t) y_t(\omega, h^t) d\omega \right], \tag{22}$$

subject to (21). The solution to this problem is the demand function:

$$y_t(\omega, h^t) = \left[\frac{P_t(\omega, h^t)}{P_t(h^t)} \right]^{-1/(1-\theta)} y_t(h^t) \tag{23}$$

with constant elasticity $\varepsilon = -1/(1-\theta)$. Thus the demand for intermediate ω's product is a simple decreasing function of its relative price. Substituting (23) into (22) gives the price index consistent with zero profits in the final good sector:

$$P_t(h^t) = \left[\int_0^1 P_t(\omega, h^t)^{\theta/(\theta-1)} d\omega \right]^{(\theta-1)/\theta} \tag{24}$$

Intermediate firms produce their goods with labor according to the constant returns technology

$$y_t(\omega, h^t) = A_t(h^t) n_t(\omega, h^t) \tag{25}$$

where $A_t(h^t)$ is an exogenously given technology index.

Aggregation

The aggregate supply of goods in this economy can be related to factor inputs in a simple way. Following Yun (1996), define an auxiliary price index $\bar{P}_t(h^t)$ according to

$$\bar{P}_t(h^t) = \left[\int_0^1 P_t(\omega, h^t)^{1/(1-\theta)} d\omega\right]^{1-\theta} \tag{26}$$

Then let $\bar{y}_t(h^t) = \int_0^1 y_t(\omega, h^t) d\omega$ and $\bar{n}_t(h^t) = \int_0^1 n_t(\omega, h^t) d\omega$ denote the simple sums of intermediate outputs and labor demands. According to the technology (25), these are related by $\bar{y}_t(h^t) = A_t(h^t)\bar{n}_t(h^t)$. Now integrate over the demand curve (23) and use (26) to get a relationship between the index $\bar{y}_t(h^t)$ and the supply of goods, $y_t(h^t)$,

$$\bar{y}_t(h^t) = A_t(h^t)\bar{n}_t(h^t) = \left[\frac{\bar{P}_t(h^t)}{P_t(h^t)}\right]^{-1/(1-\theta)} y_t(h^t) \tag{27}$$

When inflation is low, the two price indices will track each other closely so that the linear index $\bar{y}_t(h^t)$ and the true supply of goods $y_t(h^t)$ will also track each other closely.

Price setting

The model of price setting is a discrete time version of Calvo (1983). Each period t there is an exogenous signal in h_t that determines which firms will have the opportunity to change prices. Each period, a fraction η of intermediate goods prices remain unchanged, $0 < \eta < 1$, while the remaining $1-\eta$ fraction of intermediate firms get to choose their new price. Denote the new price chosen by active firms $P_t^*(h^t)$ and note that this will be the same for all ω intermediate firms that get this opportunity. The price index in (24) then evolves according to

$$P_t(h^t)^{\theta/(\theta-1)} = (1-\eta)P_t^*(h^t)^{\theta/(\theta-1)} + \eta P_{t-1}(h^{t-1})^{\theta/(\theta-1)} \tag{28}$$

Given the demand curve for their product, (23), an intermediate producer with the opportunity sets a price to maximize expected discounted profits taking into account the expected time until it will next be able to change its price

$$\max_p \left\{ \sum_{\tau=t}^{\infty} \int \eta^{\tau-t} \frac{Q_\tau(h^\tau)}{Q_t(h^t)} [p - P_\tau(h^\tau)\nu_\tau(h^\tau)] y_\tau(\omega, h^\tau) dh^\tau \right\}, \tag{29}$$

where $Q_t(h^t)$ again denotes the price as of date zero of a dollar delivered in h^t; thus $Q_\tau(h^\tau)/Q_t(h^t)$ is the price of a dollar at h^τ in units of dollars at h^t. Profits are given by the value of output less the cost of production, $P_t(h^t)\nu_t(h^t)\, y_t(\omega, h^t)$, with $\nu_t(h^t)$ denoting the firm's real marginal cost. As shown below, real marginal cost can be written independent of ω because of the constant returns nature of the technology in (25) and the competitive labor market. Any profits are distributed in lump-sum fashion to the households as payments, $\Pi_t(h^t)$, in their brokerage accounts.

The first order condition characterizing $P_t^*(h^t)$ is

$$0 = \sum_{\tau=t}^{\infty} \int \eta^{\tau-t} \frac{Q_\tau(h^\tau)}{Q_t(h^t)} [P_\tau(h^\tau)^{\frac{2-\theta}{1-\theta}} P_t^*(h^t)^{-1} \nu_\tau(h^\tau) - \theta P_\tau(h^\tau)^{\frac{1}{1-\theta}}] y_\tau(h^\tau) dh^\tau$$

which can be solved for

$$P_t^*(h^t) = \frac{1}{\theta}\frac{\sum_{\tau=t}^{\infty}\int \eta^\tau Q_\tau(h^\tau)P_\tau(h^\tau)^{\frac{2-\theta}{1-\theta}}\nu_\tau\,(h^\tau)\,y_\tau(h^\tau)dh^\tau}{\sum_{\tau=t}^{\infty}\int \eta^\tau Q_\tau(h^\tau)P_\tau(h^\tau)^{\frac{1}{1-\theta}}y_\tau(h^\tau)dh^\tau} \tag{30}$$

This is an intertemporal generalization of a constant $1/\theta$ markup over nominal marginal costs, $P_t(h^t)\nu_t\,(h^t)$. As θ approaches one, the markup vanishes so that price equals marginal cost. As η approaches zero, the probability that prices will remain unchanged vanishes and the model becomes one of perfectly flexible prices.

The real marginal cost of an intermediate producer ω is the minimum cost of producing a unit of the intermediate's output. Given a competitive real wage rate $w_t(h^t)$,

$$\nu_t\left(h^t\right) = \min_n \left[w_t(h^t)n \mid A_t(h^t)n \geq 1\right] \tag{31}$$

Optimization by intermediate producers requires

$$w_t(h^t) = \nu_t\left(h^t\right)A_t(h^t) \tag{32}$$

so that real marginal cost is simply the wedge between the real wage and labor productivity. The real marginal cost is the same for all ω.

D. Equilibrium

Given a government policy $\{\mu_t(h^t), B_t(h^t, h'), H_t(h^t)\}$, an *equilibrium* for this economy is a sequence of price functions, $\{q_t(h^t, h'), P_t(h^t), w_t(h^t)\}$, a sequence of allocation functions for households, $\{c_t(s, h^t), n_t(s, h^t), D_t(h^t), B_t(s, h^t, h'), Z_t(s, h^t), M_t(s, h^t)\}$, a sequence of price and allocation functions for intermediate firms, $\{P_t^*(h^t), n_t(\omega, h^t)\}$, and a sequence of allocation functions for final good firms, $\{y_t(h^t), y_t(\omega, h^t)\}$, such that: 1) taking as given the policy, the prices, and the behavior of firms, the households' allocation solves their decision problems for each s; 2) taking as given the policy, all prices but their own, and the behavior of households and final good firms, the intermediate firms' price and allocation solves their decision problems for each ω; 3) taking as given the policy, all prices, and the behavior of households and intermediate firms, the final good firms' allocation solves their decision problem; and 4) the goods, bonds, and money markets clear at each date and in each state.

Goods market clearing requires that the demand for goods equal supply

$$\frac{1}{N}\sum_{s=0}^{N-1} c_t(s, h^t) = y_t(h^t) = A_t(h^t)\bar{n}_t(h^t)\left[\frac{\bar{P}_t(h^t)}{P_t(h^t)}\right]^{1/(1-\theta)} \tag{33}$$

where $\bar{n}_t(h^t) = \int_0^1 n_t(\omega, h^t)d\omega$ and $\bar{P}_t(h^t)$ is the auxiliary index defined in (26). Similarly, the market for each contingent bond clears when

$$\frac{1}{N}\sum_{s=0}^{N-1} B_t(s, h^t, h') = B_t(h^t, h') \tag{34}$$

for each h'. Finally, the money market clears when

$$\frac{1}{N}\sum_{s=0}^{N-1}[M_t(s, h^t) + M_t^A(s, h^t)] = M_t(h^t) \tag{35}$$

The demand for money takes two forms, money held by households in the goods market, $M_t(s, h^t)$, and money held by households in the asset market, $M_t^A(s, h^t)$. The supply of money, $M_t(h^t)$, is generated by the shocks $\mu_t(h^t)$ and the given initial condition M_0.

A *symmetric equilibrium* for this economy is an equilibrium that also has the property that $n_t(\omega, h^t) = \bar{n}_t(h^t)$ for all ω. In what follows, I focus exclusively on symmetric equilibria.

Approximations to equilibria of this model are computed by first log-linearizing the equations that characterize an equilibrium around a non-stochastic steady state and then solving the resultant system of stochastic difference equations using the method of undetermined coefficients, as described by Blanchard and Kahn (1980) and Uhlig (1999).[5] To facilitate comparison with the rest of the literature, the log-linearization is taken around a *zero-inflation* steady state. Put differently, the model described in this section only makes sense if one is thinking of a relatively low inflation environment.

Upto this point, I have made explicit reference to uncertainty in the notation so as to give a clear characterization of state contingent asset prices, money demand, and labor supply. In what follows, I suppress dependence on histories h^t to conserve on notation, use an unadorned character (say, x) to denote steady state values and a circumflex (say, $\hat{x}_t$) to denote the log-deviation of a variable from its steady state.

Before discussing the properties of the model any further, it's worth briefly discussing how I assign values to the model's key structural parameters.

3. Parameterization

I assume an annual discount factor of 0.97 so that the annual real interest rate is approximately 3%. Since β is the discount factor per period, if the length of a period is one month, then $\beta = (0.97)^{1/12} = 0.9975$.

The period utility function is assumed to be of the form introduced by Greenwood,

[5] The Appendix gives more detail on the log-linear model and on computing approximate equilibria.

Hercowitz, and Huffman (1988):

$$U(c,\ell) = \frac{1}{1-\sigma}\left[\left(c - \frac{(1-\ell)^{1+\chi}}{1+\chi}\right)^{1-\sigma} - 1\right], \qquad \sigma > 0, \chi > 0 \tag{36}$$

The most important feature of these preferences is that for households that hold money over in the goods market, $Z_t(s) > 0$, so that (18) holds with equality and simplifies to

$$\log[n_t(s)] = \frac{1}{\chi}\log(w_t)$$

so that the marginal rate of substitution between leisure and consumption depends only on leisure.[6] All households that hold money over in the goods market supply the same amount of labor and for those households, the choice of labor supply is independent of their intertemporal consumption / savings choice. Typically, the only households for whom labor supply and savings behavior are intertwined are the $s = N-1$ shoppers that are about to return to the asset market. In practice, this means that the role of changes in the distribution of labor supply across households is minimized.

The elasticity $1/\chi$ gives the slope of the labor supply schedule for households that hold money over in the goods market. I use $\chi = 2/3$ so that the elasticity is 1.5 — which is the same as the Frisch elasticity[7] of labor supply used by Chari, Kehoe, and McGrattan (2000) and similar to the value of 1.7 used by Greenwood et al (1988).

Following Basu and Fernald (1997), I assume $\theta = 0.9$ so that the steady state markup

[6]Taken at face value, these preferences are inconsistent with balanced growth. However, by making the utility of leisure grow at a constant rate (equal to the rate of technical progress) these preferences can give a marginal rate of substitution between leisure and consumption that grows at the same rate as the real wage and is consistent with balanced growth.

[7]That is, the labor supply elasticity holding fixed the marginal utility of consumption.

is 11%.

The literature is somewhat divided as to the extent of price stickiness [compare Bils and Klenow (2002) with Sbordone (2002)], so I experiment with a number of settings for η. Since the expected duration of price stickiness is $(1-\eta)^{-1}$ model periods, I experiment with $\eta = 2/3, 5/6$ and $11/12$ so that in a monthly model, the expected duration of price stickiness is $3, 6$ or 12 months.

Finally, I use steady state information on average velocity v to pin down the parameter N that indexes the frequency of transactions. Since this can lead to quite large values for N, I also perform some sensitivity analysis by looking at $N = 3, 6, 12$ or 36 months.

4. Steady state

To develop some intuition for the workings of the model, I first examine the model's non-stochastic steady state. In a steady state, the inflation rate π equals the average growth rate of the money supply, μ. The price level and the money supply move in lock-step. Suppose that money growth is mean zero, $\pi = \mu = 0$. Then the price of a one-period zero-coupon bond is $q = \beta < 1$, the real interest rate is $\rho = \beta^{-1} - 1 > 0$, the real wage is given by $w = \nu A$, while real marginal cost is given by the steady state mark-up formula, $\nu = \theta$. To complete a solution for the steady state, I need to solve for the distributions of consumptions, real-balances, and labor supplies, $\{c(s), z(s), n(s)\}_{s=0}^{N-1}$. This is easily done by solving a system of non-linear equations. There are N bank account flow constraints, N sets of complementary slackness conditions for money holdings in the goods market, and N optimality conditions for labor supply to characterize these $3N$ variables. With these distributions in hand, aggregate output y, labor supply n, real balances m, real withdrawal[8] d, and velocity v can be computed.

Figures 1 and 2 illustrate the steady state. In particular, money holdings follow the "saw-toothed" pattern illustrated in Figure 1. The active $s = 0$ shopper acquires money for subsequent goods market purchases by making a withdrawal from the asset market. Over time, this initial withdrawal is run down and the money is exhausted when the shopper is of type $s = N - 1$. Typically, no money is taken back to the asset market. More precisely, if $U_c(N-1) > \beta U_c(0)$, the marginal utility of consumption towards the end of their trip is too high to warrant the abstention from consumption that bringing money back to the asset market requires. In steady state, this behavior is repeated continually so that the path of money holdings in the goods market has the saw-toothed shape illustrated. This shape is characteristic of inventory-theoretic models of the demand for money.

With preferences of the Greenwood-Hercowitz-Huffman form — as in (36) — the distributions of consumption, real balances and labor supplies are as shown in Figure 2. Consumption and real balances are highest for the $s = 0$ shopper in the asset market and decline steadily until the shopper is of type $s = N - 1$. Consumption is, however, much smoother than real balances. Real balances are use to smooth consumption so that the ratio $z(0)/c(0)$ is high relative to $z(N-1)/c(N-1)$. In fact, in the regular case where shoppers exhaust their money holdings before returning to the asset market, $z(N-1)/c(N-1) = 0$. This is illustrated in the first two panels of Figure 2.

Since these preferences also imply that the labor supply of all shoppers who hold money over are the same, the steady state distribution of labor supply has the form indicated

[8] By aggregating the brokerage account constraints of the households, imposing a zero inflation steady state and using the fact that the nominal interest rate is positive, it can be shown that the steady state withdrawal is equal to profits from the intermediate firms, $D = \Pi$.

in the third panel of Figure 2. For all but the last shopper type, labor supply is the same; $n(s) = w^{1/\chi}$ for all $s < N-1$. The shoppers about to return to the asset market supply less labor. Given the timing of this model, the income derived from labor supplied this period can only be spent on goods next period. Thus the labor supply of an $N-1$ worker drops dramatically relative to when they were $s = N-2$ because the income derived from labor supply at $s = N-1$ will become available next period when the worker is $s = 0$ — but when the household is active, it can withdraw money from the asset market and its consumption will jump up. Labor supply drops dramatically for the $s = N-1$ worker for the same reason that the $s = N-1$ shopper will typically not choose to bring money back to the asset market.

Another way to consider these expenditure patterns is to look at propensities to consume. Let $v(s) \equiv Pc(s)/M(s)$ denote the *average* propensity to consume out of steady state beginning-of-period money holdings

$$v(s) = \frac{c(s)}{c(s) + z(s)}$$

which for each s are numbers between zero and one, per period. Notice that the $v(s)$ correspond to *individual* velocities of money in the sense that aggregate velocity v can be written

$$v \equiv \frac{Py}{M} = \frac{1}{N}\sum_{s=0}^{N-1}\frac{Pc(s)}{M} = \frac{1}{N}\sum_{s=0}^{N-1} v(s)\frac{M(s)}{M}$$

Aggregate velocity is a weighted average of the individual velocities with weights given by shopper s's share of the total money supply, $M(s)/M$.

As illustrated in Figure 3, individual velocities are increasing in s. Since consumption, labor supply and real balances are all declining in s, the curve $v(s)$ can, in principle, have any shape. However, consumption and leisure are smoothed out by acquiring and then running down a "buffer-stock" of real balances. Put differently, $z(s)$ declines in s much more sharply than $c(s)$ or $n(s)$ do and so individual velocities *increase* in s. A shopper of type $s = 0$ has a lot of consumption and a lot of real balances. Towards the end of their cycle they have relatively little money left but are maintaining relatively flat consumption so that the average propensity to spend out of money has risen. The individual velocity of the last shopper, for whom $z(N-1) = 0$, is simply $v(N-1) = 1$ per period. Just as in an ordinary cash-in-advance model, these households spend all of their beginning-of-period money on goods and supply labor taking into account the delay between when labor effort is exerted and when the proceeds can be spent.

The implications of an inventory-theoretic model of money demand of this kind are discussed extensively in Alvarez, Atkeson, and Edmond (2002). In that paper, it was shown that in an *endowment* economy with inventory-theoretic money demand, a monetary injection leads to an offsetting endogenous fall in velocity so that if M increases, v falls and Py is relatively flat. Velocity falls following a monetary injection because in equilibrium new money is held by the households active in the asset market — the type $s = 0$ households — and so the distribution of money is tilted towards the households with the lowest individual velocity. Since aggregate velocity is a weighted sum of individual velocities with the weights given by the money shares $M(s)/M$, aggregate velocity falls. Money is not neutral in this setting because monetary shocks lead to changes in the real interest rate and to changes in the *distribution* of real consumption over households.

In an endowment economy, output is exogenous so the decomposition of movements in nominal spending Py into price and output components is not very interesting. However, in a model with elastic labor supply, such as the one outlined above, the level of output is endogenous and it will be possible for monetary shocks to change both the level and distribution of output in the short run.

Because a model with an inventory-theoretic money demand generates nominal inertia from a money distribution that moves sluggishly in response to a monetary shock, I will refer to such a model as one with *sticky demand* or sticky nominal spending. By contrast, a plain vanilla *sticky price* model — one with constant velocity[9] — will predict that following a monetary injection y will rise and P will be relatively constant so that Py rises.

I will study the interaction of sticky demand and sticky prices in three steps, by first looking at the response of the economy to a monetary policy shock when there are sticky prices in a model where money demand has the usual constant velocity form, then when money demand has an inventory-theoretic form but prices are flexible, and then when both effects are present.

5. Constant velocity (static money demand)

When $N = 1$, money demand is given by a textbook cash-in-advance constraint. To see this, note that when $N = 1$, all households are of type $s = 0$ (and there's no need to keep track of this index). Then if the nominal interest rate is positive, $i_t > 0$, it will be optimal

[9]While some authors — such as Dotsey, King and Wolman (1999) — simply append constant velocity money demand to their models, it should be noted that in many sticky price models that are derived from utility maximizing behavior, money demand arises from placing real balances in the utility function or from a cash-credit model where only some goods are subject to a cash-in-advance constraint. Either specification leads to an interest-elastic money demand and hence a model where velocity is endogenous and movements in velocity are given by movements in the nominal interest rate. However, the interest sensitivity of money demand in such models is usually thought to be so small that constant velocity is a reasonable working assumption — especially when such models are solved by log-linearizing around a zero inflation steady state and thus presume no trend in velocity.

for households to choose $Z_t = 0$ so that, from (4), $M_t = P_t c_t$ gives money demand. Money demand has an interest semi-elasticity of zero. Velocity is constant in and outside of steady state.

To simulate the effects of a shock to monetary policy in this model, I solve for a path of money growth that is consistent with a predetermined, persistent movement in the short-term nominal interest rate. This procedure raises a non-trivial technical issue: With policy of this form, there are many stochastic processes for money all consistent with the same exogenously specified path for nominal interest rates in equilibrium. In the experiments below, I choose one of the stochastic processes for the growth rate of the money supply that results in an equilibrium in which the short-term nominal interest rate follows the pre-specified stochastic process. The chosen process for money growth is the unique one that has the property that a shock to the nominal interest rate, on impact, is associated with *no movement in the current price level*. This choice is consistent with the schemes used to identify monetary policy shocks as discussed, for example, in Christiano, Eichenbaum, and Evans (1999). This issue and alternative specifications of policy are discussed further in the Appendix.[10] As a first exercise, I consider equilibrium impulse responses when the nominal interest rate $\hat{\imath}_t$ follows an exogenous AR(1) process.

[10]Other specifications of monetary policy that admit a determinate equilibrium could be used — for instance, a feedback rule for the nominal interest rate that is "active" in the sense of Clarida, Galí, and Gertler (2000). While a specification of this kind is fine if the issue at hand is *evaluating* the policy, it is unhelpful when trying to understand the workings of a model (because it convolutes the intrinsic workings of the private economy with the monetary authority's response to the state of the economy).

A. One-time nominal interest rate shock

The top two panels of Figure 4 show the responses of output $\hat{y}_t$, inflation $\hat{\pi}_t$, and the real interest rate $\hat{r}_t = \hat{\imath}_t - E_t\hat{\pi}_{t+1}$, when the monthly serial correlation of the nominal interest rate is 0.00 so that a monetary policy shock is not persistent. The second panel shows the associated money supply and price level paths. For this example $\eta = 11/12$ so that the expected duration of price stickiness is 12 months.

On impact, the nominal interest rate is dropped to one percent below steady state. By assumption, the price level does not respond[11] on impact and so the real interest rate drops below steady state exactly mirroring the fall in the nominal rate. Output blips up about 0.27 percent above steady state and is back to steady state one month later. The endogenous money supply mirrors the movement in output.

With no persistence in the monetary policy shock, the induced movements in real output are relatively small and very short-lived — even when nominal prices are expected to be sticky for as long as 12 months.

B. Persistent nominal interest rate shock

Things are a little different when the policy shock is persistent. The bottom panels of Figure 4 are shown the responses of output, inflation, and the real interest rate when the monthly serial correlation of the nominal interest rate is 0.80. The output response is essentially a mirror image of the AR(1) policy shock; the biggest deviation of $\hat{y}_t$ is on impact (0.98 percent above steady state) after which output moves smoothly back towards steady state, contracting after 8 or 9 months, and then converging monotonically back to steady state from

[11] As explained above, in identifying an equilibrium I have imposed that the response of the price level is in the first period. No constraint on the effect on price has been imposed for any subsequent periods.

below. On impact the money supply $\widehat{M_t}$ expands and then falls in tandem with real output. Since velocity is constant, nominal spending $\widehat{P_t y_t}$ is equal to $\widehat{M_t}$ and so has the same basic shape as output.

Since the price level moves only slowly, the real interest rate and the nominal interest rate move in tandem in the short run. Put differently, there is a persistent liquidity effect. Notice that in the long run, a persistent movement in the nominal interest rate induces a fall in the price level The long run is governed by Fisherian fundamentals in that the reduction in the nominal interest rate eventually reduces expected inflation and moves the nominal money supply and price level down to new steady state positions while the real variables are left unchanged.

Figure 5 shows that the impact effect of a reduction in the nominal interest rate is essentially determined by the amount of price stickiness. As η is reduced from 11/12 to 5/6 to 2/3 (expected price stickiness of 12, 6, and 3 months, respectively), the impact effect on output is reduced from 0.98 to 0.78 to 0.55. There are still some output effects, however, even when price stickiness is reduced to zero. Output still expands because the fall in the nominal interest rate reduces the opportunity cost of labor supply. In the terminology of Cooley and Hansen (1989), a fall in the nominal interest rate reduces the inflation tax on labor.

6. Endogenous velocity (inventory-theoretic money demand)

The results in the previous section are fairly familiar. Before moving on to look at the interactions between sticky prices and sticky nominal spending, I first highlight the implications of inventory-theoretic money demand when prices are *flexible*.

A. Inelastic labor supply

First I set $\chi = 0$ so that labor is supplied inelastically. In this case, the model is essentially the same as the one studied by Alvarez, Atkeson and Edmond (2002). Figure 6 shows the equilibrium impulse responses when the nominal interest $\hat{\imath}_t$ rate follows an exogenous AR(1) process and when $N = 12$ months so that households exchange money for interest-bearing assets only once every year.[12] The top two panels of Figure 6 show the impulse responses when the interest rate shock is serially uncorrelated while the bottom two panels show the responses when the monthly serial correlation is 0.80.

Since labor is inelastically supplied and is the only factor of production, output does not respond to the shock. However, even with flexible prices there are still persistent liquidity effects in this economy. These liquidity effects but persistent but small in magnitude when the policy shock is uncorrelated. Essentially, the real interest rate falls with the nominal interest rate on impact. If the shock is uncorrelated, the real interest rate the rises above its steady state value for a number of periods until blipping up 11 periods after the initial shock as the group of households that was active at the time of the initial shock prepares to return to the asset market (recall that the length of segmentation in the economy is $N = 12$ months). The liquidity effects work through the changes in the distribution of money across households.

For both policy experiments, a fall in the nominal interest rate induces a fall in velocity and a rise in the money supply — offsetting movements that keep the price level from responding after the initial period. In particular, if the policy shock is persistent, a fall in the

[12] Alvarez, Atkeson and Edmond (2002) argue for a much higher value for N, such as $N = 36$ months, in order to match consumption velocity in US data. I use $N = 12$ months here to parallel the discussion of the sticky price model with expected stickiness of 12 months.

nominal interest rate induces a relatively slow "hump-shaped" rise in the money supply — by way of contrast with the *immediate* jump in the money supply in the sticky price economy — which is offset by a fall in velocity. In the short run, $\widehat{M_t v_t}$ and $\widehat{P_t y_t}$ are flat. Since output is constant, this means that the price level responds slowly to the policy shock. However, Fisherian fundamentals again determine the long-run so that a fall in the nominal interest rate is associated with lower expected inflation and a fall in the price level to a new steady state position. Nominal spending is therefore flat in the short run before moving slowly to a lower level.

Notice that velocity and the nominal interest rate are positively correlated, just as in a model with real balances in the utility function. However, velocity is not simply proportional to the nominal interest rate (as such a model with a homothetic utility function would predict).[13] Instead, the fall in velocity takes time to reach its minimum (mirroring the rise in the money supply) while the nominal interest rate is at its minimum immediately on impact.

B. Elastic labor supply

When labor is inelastically supplied so that output is exogenous, nominal spending is flat in the short run because money and velocity move in opposite directions and keep $\widehat{P_t y_t}$ fairly constant. When output is endogenous, however, this flat movement in nominal spending still has to be decomposed into movements in the price level and movements in real output. Figure 7 shows the economy when $\chi = 1.5$ and prices are flexible. Again there are persistent liquidity effects with the real and nominal interest rates moving together in the short run. Without any price stickiness the effects on the *level* of aggregate output (as opposed to its distribution

[13]With real balances entering a homothetic utility function, money demand can be written $M_t/P_t = L(i_t)c_t$, $L'(i_t) \leq 0$. Consumption velocity is then given by $v_t = 1/L(i_t)$ which is increasing in i_t.

across households) are very small. Even with a persistent policy shock, the output effect is essentially zero on impact before rising to a maximum effect of 0.09 percent some 11 months after the policy innovation. Recall that in the benchmark sticky price model with 12 months stickiness, the output effect was an order of magnitude larger, 0.98 percent on impact, and that the response did not cumulate in any way.

The results from these two exercises suggest that inventory-theoretic money demand may provide a monetary transmission mechanism: 1) where there are persistent liquidity effects even with relatively flexible prices, and 2) which may lead to gradual "hump-shaped" responses of output to a monetary policy shock. The next section documents the interaction of sticky demand with sticky prices.

7. Sticky demand vs. sticky prices

To interact sticky demand with sticky prices, I compute impulse responses for the model with elastic labor supply ($\chi = 1.5$) for segmentation parameters $N = 1, 3, 6, 12$, and 36 months, each for expected durations of price stickiness of $0, 3, 6$ and 12 months. Table 1 reports the results for a serially uncorrelated policy shock while Table 2 reports the results when the monthly serial correlation of the nominal interest rate is 0.80. For visual information, I have plotted three representative impulse responses in Figures 9 through 11. Each of these figures is for a unit nominal interest rate reduction with a monthly serial correlation of 0.80, expected price stickiness of 6 months and varying amounts of asset market segmentation.

For each economy, Tables 1 and 2 report five numbers: 1) the size of the impact effect on output given the fall in the nominal interest rate; 2) the largest deviation of output from steady state; 3) the number of months after the shock at which output reaches its maximum deviation; 4) the mean number of periods taken for the impulse response to cross zero; and 5) an index of persistence. Actually, both of these last statistics are measures of persistence. In some models, persistence is a relatively straightforward trait to characterize. In these models, however, the equilibrium impulse responses can be quite rich — and are difficult to summarize in a single number. The mean lag provides one characterization of persistence.

Given an impulse response $\{\hat{y}_t\}_{t=0}^{\infty}$, the other measure of persistence I use is based on

$$p(\{\hat{y}_t\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \left| \frac{\hat{y}_t}{\hat{y}_0} \right| - 1$$

Since $\lim_{t \longrightarrow \infty} \hat{y}_t = 0$, this sum is well-defined. This statistic sums the total area under the impulse response relative to the size of the initial impact. To get some feel for it, suppose we had the AR(1) process $\hat{y}_t = \rho \ \hat{y}_{t-1} + \hat{\varepsilon}_t$ with $|\rho| < 1$, $\hat{y}_{-1} = 0$, $\hat{\varepsilon}_0$ given and $\hat{\varepsilon}_t = 0$ for $t \geq 1$. Then

$$p(\{\hat{y}_t\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \left| \frac{\rho^t \hat{y}_0}{\hat{y}_0} \right| - 1 = \frac{|\rho|}{1 - |\rho|}$$

so that the persistence measure approaches zero as the serial correlation $|\rho|$ goes to zero and approaches $+\infty$ as $|\rho|$ goes to unity.

Consider a model with static money demand, $N = 1$ month. Then if the policy shock is serially uncorrelated, increasing the amount of price stickiness has essentially no effect on either the impact effect or the persistence of the output response. If the policy shock is persistent (as in Table 2), increasing the amount of price stickiness directly increases the size of the impact effect on output, from 0.25 to 0.98, as prices are made less and less flexible. In all cases, the impact effect is the maximum effect. Notice that the persistence measure drops and then rises as prices become less flexible. The model with 3 months expected stickiness has only about $1.5/4.0 = 0.375$ of the amount of "endogenous" persistence as the flexible price model. This pattern is not specific to the $N = 1$ case; for $N = 3$ or 6, it is still true that going from a flexible price model to one with 3 months expected stickiness results in a fall in persistence.

Increasing asset market segmentation has three important effects; it 1) reduces the impact effect on output, 2) leads to a delayed peak in the output response, and 3) increases the persistence of the output response. For example, if $\eta = 5/6$ and $N = 3$ months, then the impact effect on output is 0.22 which reaches a maximum of 0.49 two months later with both persistence measures higher than for the model with static money demand and six months price stickiness. This is illustrated in Figure 8. Increasing to $N = 6$ reduces the impact effect to 0.13 and the maximal effect to 0.28 but increases the delayed peak to 10 months after the innovation and increases the persistence yet further. This is illustrated in Figure 9.

Now consider a model where expected price stickiness is 12 months, which is on the order of that estimated by Sbordone (2002) but which would be in around the 10th percentile of the distribution of price changes documented by Bils and Klenow (2002). For this case, going to large values of N generates considerably more persistent movements in output. If $N = 36$ months, then the output response is more than six times as persistent as in the benchmark case.

On the other hand, consider a model where expected price stickiness is 3 months, which would be around the 90th percentile of the distribution of price changes in Bils and Klenow (2002). How much asset market segmentation is required for the model with 3 months stickiness to match the persistence of the benchmark model with 12 months stickiness? About $N = 6$ months is all that is required for the model with inventory-theoretic money demand to generate essentially the same persistence. This is a low value of N relative to the numbers that Alvarez, Atkeson, and Edmond (2002) argue for.

Loosely speaking, sticky demand can be a *substitute* for sticky prices in that moderate asset market segmentation can generate an additional source of nominal inertia. The entire burden of nominal rigidity need not be borne exclusively by prices; the money demand patterns induced by asset market segmentation lead naturally to offsetting velocity movements that have the effect of diminishing the price level response to a monetary policy shock. There can be persistent liquidity effects even when prices are relatively flexible. Moreover, sticky demand leads to more reasonable output dynamics in the sense that the initial impact effect of a fall in the nominal interest rate is lower while the cumulative effect is higher. Sticky price models with static money demand result in output dynamics that essentially mimic the exogenous shock. Sticky price models with dynamic, inventory-theoretic, money demand have a rich monetary transmission mechanism whereby policy shocks are translated into slow, cumulative movements in both aggregate output and its distribution across households.

8. Conclusions

This paper introduces money demand with an inventory-theoretic flavor into an otherwise standard sticky price model. Households are constrained from exchanging money for interest-bearing securities every period. Given this segmentation, the money holdings of households have the "saw-toothed" shape that is characteristic of Baumol-Tobin models of money demand. As discussed by Alvarez, Atkeson and Edmond (2002), money demand of this form has the implication that, following a monetary injection, there is an endogenous offsetting velocity response that serves to keep aggregate nominal spending relatively flat. In particular, an increase in the money supply tilts the distribution of money towards households with a low individual average propensity to spend money which in turn makes aggregate velocity fall. Changes in the distribution of money give rise to sticky nominal spending — "sticky demand" — that can interact with sticky nominal prices.

Sticky demand acts as a substitute for sticky prices. That is, introducing asset market segmentation reduces a sticky price model's reliance on implausibly large amounts of exogenous stickiness. Six months segmentation plus three months stickiness gives as much persistence as a model with no segmentation and one year of stickiness. Moreover, introducing asset market segmentation reduces the initial impact effect of a monetary policy shock on output and leads to a delayed "hump-shaped" output response. In short, dynamic inventory-theoretic money demand gives rise to a rich theory of the monetary transmission mechanism whereby policy shocks induce slow moving changes in the distribution of money which are in turn translated into slow, cumulative movements in both aggregate output and the distribution of consumption across households.

One limitation of the analysis in this paper is that the only factor of production is labor. There is no capital accumulation and hence no investment response to a monetary shock. All of the delayed response in output comes from movements in the labor supply that are intertwined with the households' consumption smoothing and money holding decisions. Consider, however, a version of the model in this paper where households supply labor but where firms own and manage the capital stock. In this case, a monetary policy shock may have a larger impact effect on output because of the investment response. Chari, Kehoe and McGrattan (2000) show that capital accumulation tends to systematically undermine the persistence generated by sticky price models. More work is required to determine if that conclusion remains true for a model, like the one presented in this paper, that has a non-trivial monetary transmission mechanism.

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