

On the Coexistence of Money and Bonds*

David Andolfatto
Simon Fraser University

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Abstract

Preliminary

1 Introduction

The question addressed in this paper concerns a phenomenon that, on the surface at least, appears to defy a simple no-arbitrage principle. That is, consider two securities, each of which is issued by the government. One security represents a risk-free claim against itself, called ‘money.’ The other security represents a risk-free claim against money, called ‘bonds.’ In an unfettered financial market, theory suggests that one of two things should happen: [1] that the rates of return on these two securities should be equated, if they are to be willingly held in the wealth portfolios of individuals; or [2] that the higher return asset should drive the lower return asset out of circulation. As is well known, this stark prediction appears to be inconsistent with the rate of return differential that normally exists between money and bonds. This discrepancy between theory and facts is known as the *coexistence puzzle*.

The common explanation for the coexistence puzzle is that money is a ‘special’ type of asset. In particular, money provides nonpecuniary ‘liquidity’ services that makes it a preferred method of payment relative to other types of assets. This is the basic idea behind any model that generates a demand for money by assuming that money enters into the utility (or shopping time) function or by assuming that some purchases are subject to a cash-in-advance constraint. While these ‘short-cut’ methods have their uses, they are ill-suited for the question at hand since they basically assume away the coexistence puzzle. Here, I take the view that any satisfactory explanation should be couched in terms of a model with a physical environment that renders money *essential* for trade; e.g., see Kocherlakota (1998).

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To date, there have been relatively few attempts to address the coexistence puzzle within the context of a model that features suitable microfoundations. Perhaps the leading explanation continues to be the ‘legal restrictions’ hypothesis, which I critically evaluate in Section 2. In Section 3, I lay out the basic framework of my own model without government bonds to highlight the role played by money in this environment. Section 4 then introduces government bonds and examines the conditions under which coexistence prevails. Section 5 provides a brief review of some competing theories that have recently been offered. Section 6 provides a summary and offers suggestions for future research.

2 The Legal Restrictions Hypothesis

One of the first and still leading explanations for the coexistence puzzle is the ‘legal restrictions’ hypothesis put forth by Bryant and Wallace (1980) and Wallace (1983). In the overlapping generations model that they consider, money and bonds naturally earn the same rate of return. However, the authors highlight two features of the environment that may prevent this from happening. First, negotiable bearer bonds tend to be issued in very large denominations. Second, legal restrictions prevent banks from issuing low denomination banknotes, which prevents the intermediation of large denomination government debt.

The large denomination property of government debt is only a necessary and not sufficient condition. In particular, a profit maximizing bank could exploit an existing arbitrage opportunity by issuing its own small denomination notes that are fully backed by large denomination government bonds. Competition among banks would presumably drive the return on private banknotes in line with the return on bonds (with perhaps a small differential to cover the costs of intermediation). For any strictly positive interest rate, the demand for non-interest bearing money should then fall to zero. The legal restriction on intermediation is what prevents this from happening.

The legal restrictions hypothesis has been challenged by those who question the empirical validity of its two underlying assumptions. Because these challenges are relevant for my own hypothesis, I take some time here to review them and evaluate their legitimacy.

Let me consider first the assumption that interest-bearing government debt is issued in large denominations. Some authors have pointed out that there are a number of historical episodes in which governments have issued ‘low’ denomination interest bearing treasury debt. Makinen and Woodward (1986), for example, report that during 1915–27, the government of France issued treasury bills (called *bons*) in denominations as low as 100 Francs. The *bons* were issued with terms of 1 month, 3 months, 6 months, and 1 year. Unlike other interest-bearing bearer debt sold by the French government, *bons* were continuously available (i.e., they were not sold as periodic subscription issues). The *bons* could be obtained at all banks (including branches of the Bank of France),

post offices, and numerous local offices of the Finance Ministry. To facilitate their circulation, *bons* were made bearer notes and no mechanism was created to assist the government in collecting taxes on any interest earned. Furthermore, the Bank of France stood ready to redeem any paper (presumably for Bank of France notes) due within 3 months at a discount equal to the prevailing bank rate. In most of their years of existence, the quantity of *bons* outstanding was comparable with the note circulation. But despite the attractive physical and contractual attributes of *bons*, and the fact that they earned interest, they did not drive Bank of France notes out of circulation. On the contrary, *bons* were evidently much less liquid than notes.

Makinen and Woodward argue that the *bons* episode in France provides evidence that overturns one of the underlying assumptions of the legal restrictions hypothesis. But there is reason to doubt this claim. Evidently, the lowest denomination *bon* was 100 Francs, which by my own calculation is equivalent to roughly 100 USD today. Other indirect evidence suggesting that the *bon* was a larger denomination note is the fact that it was readily used in larger transactions (e.g., real estate). Thus, I do not believe that one can conclude, as Makinen and Woodward claim, that *bons* were suitable for ‘everyday exchange.’¹ The same can be said of the ‘low’ denomination interest bearing treasury bills issued by both the Northern and Southern governments in the United States during the civil war; see Burdekin and Weidenmier (2002). During this episode, the minimum denomination interest bearing note issued was \$5, which again is approximately the equivalent of 100 USD today.

At this stage, I conclude that there is no compelling evidence to suggest that governments have ever issued interest bearing currency in the range of denominations of their non-interest bearing counterparts. Thus, the assumption of large denomination bonds appears to be a valid one.

Let me now turn attention to the assumption that banks are legally prohibited from issuing low denomination notes. We know of historical episodes in which this assumption appears to be invalid. For example, during the so-called ‘free-banking’ era in the United States (1836–63), most states passed laws making it relatively easy to establish a state bank and issue low denomination notes.² While banks did intermediate government bonds, the notes they issued (redeemable in specie) generally did not pay interest. Similarly, during the Scottish free banking system (1716–1864), while Scottish banks were not prohibited from issuing low denomination interest bearing notes (again, redeemable in specie), they evidently chose not to (although they did pay interest on demand deposit accounts); e.g., see White (1987). The fact that zero interest notes (and specie) coexisted with higher yielding securities in these eras of

¹There were at least two other potentially important legal discrepancies between *bons* and notes. First, *bons* were not legal tender. Second, *bons* were not redeemable on demand (whereas large denomination notes could presumably be exchanged for smaller denominations on demand at the Bank of France).

²Shortly after the National Banking Act of 1863, a 10% tax was imposed on banknotes and they quickly disappeared from circulation.

relatively free banking casts some doubt on the legal restrictions hypothesis.

In my view, however, perhaps the clearest evidence that contradicts the second assumption of the legal restrictions hypothesis is to be found in most modern economies with well-developed electronic payment systems. While banks do not issue low denomination payment instruments in paper form (either willingly or by legal restriction), they can and do issue highly divisible interest bearing demand deposits that these days are essentially the electronic equivalent of privately issued interest bearing paper notes. Unlike the banknotes of old, which were made redeemable for specie, the electronic bank money of today is made redeemable for government issued zero interest paper notes (cash). Thus, in the context of present day economies, the coexistence puzzle may be framed as asking why cash continues to coexist with what appears to be a dominant monetary instrument (that is backed, in part, by government bonds).

In the model I develop below, I maintain the assumption that government interest bearing debt is issued in large denominations, making it unsuitable for ‘everyday’ payments. But I dispense with the assumption that legal restrictions prevent banks from intermediating large denomination bonds. In the model below, banks can and do intermediate bonds by issuing a highly divisible ‘electronic’ interest bearing payment instrument. I replace the legal restriction with what is arguably a more plausible assumption: That there is a less than complete public record of individual trading histories. This assumption implies that privately-issued debt instruments are not universally accepted for all types of payments—a restriction that makes fiat money essential for trade. Efficiency in this environment entails the emergence of banks whose assets constitute cash, bonds and loans; and whose interest-bearing liabilities must be made redeemable on demand for cash. Bonds earn interest because they must compete with capital in the wealth portfolios of individuals. Non-interest bearing cash is valued for its ability to facilitate exchange in trading opportunities where the seller cannot easily verify the legitimacy of the buyer’s bank instrument.

3 The Basic Model

The model developed here is similar to one developed by Smith (2002). Consider an economy consisting of two separate ‘locations’ A and B . Of course, one need not interpret ‘locations’ literally as being spatially separated. The key assumption is that the trading histories of an agent (or agency) belonging to A are not observable by those agents (or agencies) belonging to B ; and vice-versa. This restriction on the environment implies that private liabilities issued in location A are not recognized in location B (and vice-versa). Each location is populated by N_t young agents at date $t = 1, 2, \dots, \infty$ who live for two periods. There is an initial old generation N_0 in each location. Let n denote the (gross) rate of population growth.

Each young agent is relocated to the ‘other’ location with probability $0 <$

$\pi < 1$. One can interpret this event as the probability of being confronted with a trading opportunity in which the buyer and seller (and their respective banks) are anonymous to each other (so that privately-issued liabilities are not an acceptable form of payment). In what follows, I will refer to π as the probability of a ‘liquidity shock.’ Since there is no aggregate uncertainty, π also represents the fraction of young agents making a transition to a ‘foreign’ location. Hence, the young (in location A) have an expected utility function:

$$U = (1 - \pi)u(c_A) + \pi u(c_B), \quad (1)$$

where $u' > 0 > u''$ and c_j denotes consumption in location $j = A, B$ (the young in location B have similar preferences).

Young individuals are endowed with $y > 0$ units of output. There is an investment technology that takes k units of current output and delivers xk units of future output, where $x > n$. The investment choice must be made before the realization of the liquidity shock. Assume that capital is too costly to scrap so that it cannot be transported across locations. As well, assume that capital depreciates fully after yielding its return. Young agents only care about consumption when old (so that they save their entire endowment).

Since capital cannot be scrapped and since private liabilities are not accepted in foreign locations, there is a role for fiat money. Assume that the initial old of each location are endowed with a stock of fiat money M . The supply of money is held constant over time, so that it earns a real (gross) return equal to n . Note that money will be valued (for its insurance properties) despite being dominated in rate of return.

Since the young save their entire endowment, the only relevant choice is over the composition of their savings; i.e., money (q) or capital (k). Since the portfolio decision must be made before an agent experiences a liquidity shock, the young have incentive to form a coalition (which I will call a *bank*). The bank takes deposits y , which it uses to purchase money (from the old), investing the remainder in the location specific capital project. Formally, the bank’s choice problem is to maximize (1) subject to:

$$\begin{aligned} (1 - \pi)c_A + \pi c_B &\leq xk + nq; \\ \pi c_B &\leq nq; \end{aligned}$$

where $q + k = y$.

Since $x > n$, the second constraint will bind. In this case, the equilibrium real demand for money balances q^* is characterized by:

$$xu' \left(\frac{x(y - q^*)}{1 - \pi} \right) = nu' \left(\frac{nq^*}{\pi} \right).$$

Note that the nominal interest rate is positive; i.e. $(x/n) > 1$. Andolfatto (2004) demonstrates that holding the stock of money constant is an optimal monetary

policy (from the perspective of a representative young generation). In other words, the Friedman rule is not optimal in this environment. The intuition is straightforward. While generating a deflation to equate the return on capital and money has the benefit of providing full insurance, this gain is more than offset by the implied contraction in capital spending (as banks divert deposits away from business loans and into fiat money).

4 Adding Government Bonds

There is an outstanding stock of nominal government bonds B that are in the hands of the initial old. Assume that these bonds have no maturity date, but that they are transferable (across generations) and pay a nominal (gross) interest rate R . The government's policy is to maintain a fixed bond/money ratio $\theta \equiv (B/M)$. Interest payments on the debt are financed with a lump-sum tax on bond-holders.

Let b denote the real bond holdings per young agent. Then the bank's choice problem is to maximize (1) subject to:

$$\begin{aligned} (1 - \pi)c_A + \pi c_B &\leq xk + Rnb + nq - \tau; \\ \pi c_B &\leq nq. \end{aligned}$$

where τ is a lump-sum tax. Note that this specification assumes that government bonds are issued in sufficiently large denominations that prohibit bank money from being redeemable in government bonds (when I go to the bank machine, the ATM spits out non-interesting bearing cash, not small denomination notes of interest-bearing government debt).³

If bonds are to compete with capital, the nominal interest rate must be such that $R^* = (x/n) > 1$. Note that since the equilibrium price level falls at rate n , both bonds and capital earn the same real return x . In this case, the bank is indifferent between bonds and capital, so that $s \equiv b + k = y - q$. The bank's choice problem is therefore given by:

$$\max_q (1 - \pi)u \left(\frac{x(y - q) - \tau}{1 - \pi} \right) + \pi u \left(\frac{nq}{\pi} \right).$$

The demand for real money balances is characterized by:

$$xu' \left(\frac{x(y - q^D) - \tau}{1 - \pi} \right) = nu' \left(\frac{nq^D}{\pi} \right). \quad (2)$$

³If bonds are issued in small denominations, then the second constraint would be given by:

$$\pi c_B \leq xb + nq.$$

In this case, money would be driven out of circulation.

From the government's budget constraint:

$$\tau = (x - 1)b.$$

Since $\theta \equiv b/q$, we can alternatively write this constraint as:

$$\tau = (x - 1)\theta q.$$

Substituting this constraint into condition (2) yields a condition that characterizes the equilibrium real money balances:

$$xu' \left(\frac{xy - [x + (x - 1)\theta]q^*}{1 - \pi} \right) = nu' \left(\frac{nq^*}{\pi} \right).$$

Note that if $\theta = 0$, then the model reduces to the earlier specification.

Let me now summarize the pattern of trades in this economy. The 'initial' old are endowed with M dollars of currency and B dollars of interest bearing debt. After paying the lump sum tax (that finances the interest cost of outstanding government bonds), the initial old are left with $M + B$ dollars. Banks collect deposits from each of the initial young (consisting of claims against y). A portion of these claims are used to purchase the $M + B$ dollars of government debt instruments from the old. The remainder of these claims are used to finance capital investment.

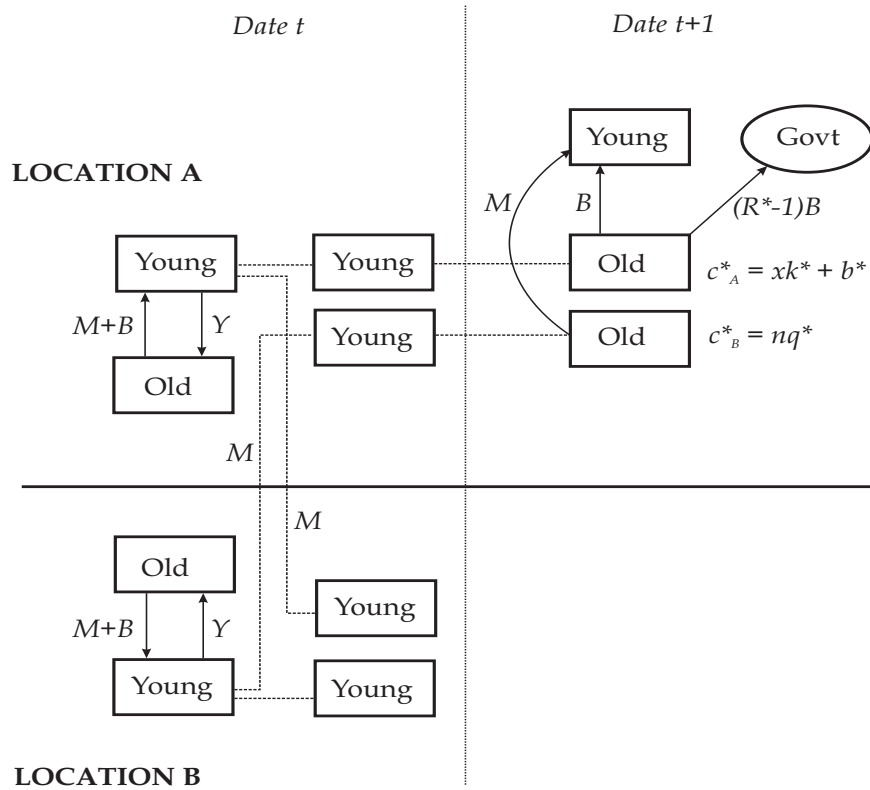
Thus, on the asset side of its balance sheet, the bank holds government cash, government bonds, and private loans (claims against the economy's capital stock). Efficiency dictates that the liability side of the bank's balance sheet consist of demandable debt (demand deposits). One can think of demandable debt consisting of electronic transactions balances that can be redeemed on demand for government cash. These demandable debt instruments serve as a private money instrument. If depositors choose to exercise the redemption option (i.e., withdraw funds early), then they receive government cash but earn no interest on their savings. Depositors who carry their savings at the bank into the future period use the principal and interest to purchase future output.

After the realization of individual liquidity shocks in the current period, the bank's cash balances are drained entirely by those who need to exercise the redemption option on their demand deposits. This cash is taken to the 'foreign' location where it is used to purchase output in the future period. Of course, there is an equal amount of cash arriving in the 'domestic' location from the young on the 'foreign' location who exercised the redemption option on their demand deposits. Thus, the total supply of government cash remains constant in each location.

As time unfolds, the young become old. Those that have cash, purchase claims to output (issued by the new bank that arises in that period). Those that still have their bank money use it to purchase output. Some of this output consists of the return to capital investment and some of this output consists of the endowment brought into the period by the new generation of young agents.

This latter output is purchased by selling B dollars of government bonds (the amount remaining after satisfying the tax obligation) to the bank representing the new generation of young agents. The pattern of trades is depicted in Figure 1.

FIGURE 1
Pattern of Trades



5 Discussion

In this model, interest-bearing government debt coexists with non-interest-bearing government money, despite the fact that private intermediaries are not legally restricted from issuing small denomination banknotes. The model does not explain why the government would want to issue large denomination interest-bearing debt. But given that it does, the model here explains the interest premium as follows:

1. Government bonds are issued in large denominations, so that ATMs cannot spit out low-denomination government notes that pay interest;
2. While banks can and do intermediate large denomination government notes by issuing private liabilities that are backed by government bonds, these private liabilities are not an acceptable form of payment in some transactions (e.g., when I try to pay for groceries in New York with a check drawn on my bank in San Francisco).

Given that history appears to provide no examples of governments that have issued interest bearing notes in the range of denominations available in their non-interest bearing counterparts, the coexistence of money and bonds should not be surprising. However, there is a subtler dimension to the coexistence puzzle that concerns the coexistence of money and bonds of identical denominations (e.g., a 100 Franc Bank of France note and a 100 Franc *bon*). To explain this phenomenon, one needs to dispense with Fact 1.

As it turns out, I think that a plausible reinterpretation of the model developed above can explain the coexistence puzzle even in the absence of Fact 1. In order to do so, however, we need to take seriously a dimension that has been largely neglected in modern theories of money; namely the legal/contractual properties of different monetary instruments. An almost universal property of all government money is that it constitutes *legal (or lawful) tender*; see Breckenridge (1903). The phenomenon is too pervasive to be interpreted as mere coincidence or irrelevant custom. It seems more likely to suppose that this legal provision has at least some bite; i.e., at least *some* payments at some level need to be settled in terms of the economy's legal tender.⁴

Unlike the notes issued by the Bank of France, the *bon* was not made legal tender (why not?). In the context of the model developed above, we might imagine that the 'liquidity shock' corresponds to a transaction opportunity that requires payment in legal tender. Note that the probability of this liquidity shock can be made arbitrarily small (as long as it remains strictly positive). With this simple reinterpretation, the coexistence of similarly denominated bills with different pecuniary returns can be explained.

6 Alternative Explanations

1. White (1987).
2. Aiyagari, Wallace, Wright (1996).

⁴To the extent that the legal tender status of government money has bite, one might question whether 'unbacked' government money should in fact be interpreted as fiat, since a legal tender note has the power to discharge a real tax obligation; see Goldberg (2004). On the issue of whether a purely fiat money has ever existed at all, see Goldberg (2003).

Shi (2003) considers a model with the following features. The (search) environment is such that fiat money is essential. In addition to fiat money, the government issues one-period treasury bills (bonds) that are redeemable upon maturity at face value. Bonds are assumed to be divisible, so that Facts 1 and 2 play no role. There is a temporary spatial separation between the period's goods market and the market for newly issued bonds, precluding the latter from being used in payment for goods, so that newly issued bonds sell at a discount. At maturity, bonds may be redeemed at face value or held as a monetary instrument (precluding future redemption). The main question asked is under what conditions do matured bonds circulate along with money.

Since matured bonds share precisely the same physical and legal characteristics of fiat money, Shi finds that money and (matured) bonds coexist as media of exchange (with identical rates of return). However, by introducing an exogenous (arbitrarily small but positive) probability that bonds are not acceptable as payment in all goods transactions, bonds are driven out of circulation. This probability is interpreted by Shi as an individual's chance encounter with a government agency that is instructed to accept only government 'cash' as payment for services rendered (or tax obligation discharged). Another way to think about this is as I have done above; i.e., that some payments have to be made in the economy's legal tender (which does not include bonds). In this equilibrium, bonds do not circulate in equilibrium and newly issued bonds sell at a discount because an exogenous market separation that makes bonds less liquid than money.

Shi's explanation for the coexistence puzzle is somewhat different than my own. In my model, government bonds trade at a discount (earn interest) because they must compete with private capital if they are to be willingly held in individual wealth portfolios. As well, the bonds in my model *do* circulate in the same sense as fiat money does (i.e., by passing from generation to generation). There is also another sense in which the bonds in my model serve as money. In particular, while the bonds themselves are not held directly by any individual, an intermediated private money instrument backed in part by these bonds is. In other words, intermediation converts government bonds into a form of money. However, unlike government money, this private money is not acceptable in all types of exchanges, which accounts for the rate of return differential. Furthermore, my model suggests that if the government was to issue bonds in denominations similar to cash (and render such bonds legal tender), the demand for government non-interest bearing debt would fall to zero.

Zhu and Wallace (2003) view coexistence as one of many possible equilibrium outcomes that may occur in bilateral exchanges involving goods and two 'outside' assets (money and bonds). I do not understand this paper. An equilibrium with coexistence appears to rely on the idea that there may be a 'convention' (an equilibrium selection) such that a buyer's bargaining power is increased with the amount of cash (as opposed to bonds) brought into an exchange. In this equilibrium, individuals are willing to hold cash (which is dominated in rate of

return) since it allows them to extract a greater fraction of the surplus associated in any exchange opportunity. However, this same environment also generates outcomes that do not distinguish between money and bonds and outcomes that reverse the role of bonds and money. Wallace (2003) claims not to be bothered by such multiplicity (or the fact that the unique equilibrium in a large economy does not feature coexistence). I am not bothered by multiplicity either, but I find their interpretation of the coexistence puzzle somewhat of a stretch.

7 Policy Implications

optimal policy: small denomination bonds – Friedman rule (suboptimal).

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