The Benefits of Low Inflation: Financial Development within Endogenous Growth *

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Abstract

The paper presents a cash-in-advance, endogenous growth, economy with financial development in which the benefits of moving towards low inflation depend on the degree of financial development. The model is calibrated and simulated to focus on two different types of financial development that have opposite predictions for the effect on growth. The type of development related to economies of scale in the production of the credit sector is consistent with the extensive panel date evidence presented. Both instrumental variables and the use of multiple splines that extends the threshold literature add robustness to the results. The implication is that more financially developed economies have a greater growth gain in moving marginally towards zero inflation.

JEL: C23, E44, O16, O42

Keywords: cash-in-advance, inflation, endogenous growth, panel data, multiple thresholds

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1. Introduction

The effect of inflation on growth at low levels of inflation affects how desireable it is for developed countries to approach price stability, when EU accession countries should adopt the Euro with its mandatory low inflation rate, and whether developing countries should follow the worldwide phenomenon of low-inflation rate targeting. If inflation reductions have significant long-term benefits in terms of higher growth, and these benefits result from a reduction in the inflation rate even from low levels, then moving towards price stability across regions has a greater imperative.

The effect on growth of moving to low inflation may depend on the level of development, in particular the level of financial development. Since greater financial development can give a country a better means to avoid the inflation tax, there is for example a logic that the more developed is the economy the less costly should be inflation avoidance and so the lower should be the effect of inflation on growth. This logic is consistent with conventional wisdom as based on the influential work on financial development and growth, for example in Levine, Loayza, and Beck (2000).

A novel alternative hypothesis is that with greater economies of scale, as we would expect in more financially developed countries, at low levels of inflation the marginal cost of producing the means to avoid inflation is actually higher. And this can cause the growth rate to be lower in the more financially developed country for a given low inflation rate. The paper here presents an endogenous growth monetary model through which both both alternatives are presented analytically and numerically at low levels of the inflation rate: a higher financial sector productivity level that causes higher growth and in constrast a greater financial sector economies of scale that causes a lower growth rate. The paper then presents extensive panel data evidence that bears upon which hypothesis may be supported. The results suggest that the increase in the growth rate, from moving for example from a 3% inflation rate towards a zero inflation rate, are greater for the more financially developed economy.

1.1. Inflation and Growth Evidence

There is a significant amount of empirical evidence to suggest a negative, non-linear effect of inflation on growth (see, for example, Gillman, Harris, and Mátyás 2004). Judson and Orphanides (1996), Ghosh and Phillips (1998) Khan and Senhadji (2001) and Drukker, Gomis-Porqueras, and Hernandez-Verme (2004) all find a significant negative inflation-growth effect above a certain low "threshold" value of the inflation rate, and no significant effect below the threshold value. Further the threshold appears to vary depending on the level of development. For example, in Khan and Senhadji (2001), the threshold is estimated to be 1% for developed countries and 11% for developing countries. Using instrumental variables, Ghosh and Phillips (1998) and Gillman, Harris, Matyas (2004), find that a negative effect of inflation re-emerges at all inflation rates. While this research deals with whether there is one threshold value of inflation and whether the inflation effect is positive or negative, Gonzalo and Pitarakis (2002) provide theory for extending such research to unknown multiple thresholds and Drukker, Gomis-Porqueras, and Hernandez-Verme (2004) apply this to the inflation and growth relation with (one-way) fixed panel effects.

1.2. Financial Development, Investment Ratios, and Inflation Splines

Including inflation, but without thresholds or the investment ratio, Levine et al (2000) and Rousseau and Wachtel (2001) find a significant positive effect on growth for financial development. However, following Kormendi and Meguire (1985), Gillman, Harris and Matyas (2004) also add the investment to GDP ratio for samples with different levels of development, an OECD and an APEC region, while including a single threshold in the inflation-growth relation and the use of instrumental variables. The results suggest that financial development causes the negative inflation-growth effect to be even larger, and question the findings of a positive effect of financial development on growth.

Khan and Senhadji (2000) and Dawson (2003) also include the investment ratio, although not the inflation rate, and find a negative or insignificant effect for financial development. A simple hypothesis emerges: financial development may be acting as a proxy for the real rate of return on capital, which theoretically determines the growth rate in any Solow-Cass-Koopmans model. The investment ratio may be a better proxy for this return, causing financial development to affect growth only residually, and possibly negatively, when the investment ratio is included.

The paper provides extensive new empirical evidence on growth that includes the inflation rate and the investment ratio and also extends the new multiple threshold, or "spline", literature that tests for endogenous breaks in the inflation-growth relation. Empirical panel evidence using OLS, fixed effects and random effects, without splines and with splines, including endogeneity testing and dynamic estimation, provide robust results of a negative effect from financial development. This supports the hypothesis that greater economies of scale in financial development, rather than greater productivity, explains the empirical effect that financial development is found to have on growth.

2. Representative Agent Model

The economy here is an extension of Gillman and Kejak (2005). The exchange means, which are money and credit, must be used not only for consumption goods, as in Lucas (1980) and Gomme (1993), but also for investment goods, as in Stockman (1981) and Ireland (1994). This means that the velocity of money is now in terms of the total income velocity of money and not just the consumption velocity of money. The second change is an extension of the credit technology specification so that both capital and labor are included as inputs to production instead of only labor. This moves away from the time-only shopping-time specifications of McCallum and Goodfriend (1987) and Lucas (2000) towards a calibration of the model using sectoral production parameters. Through these parameters, both productivity and economies of scale in the credit sector are investigated analytically and numerically.

2.1. Consumer Problem

The representative agent's utility depends on goods c_t and leisure x_t:

$$\int_{t=0}^{\infty} e^{-\rho t} \frac{c_t^{1-\theta} x_t^{\alpha(1-\theta)}}{(1-\theta)}. \tag{2.1}$$

The income constraint can be expressed in terms of the change in assets, these being nominal money and bond stocks, or M_t and B_t. Money grows at a constant rate of σ through a lump sum transfer of $V_t = \sigma M_t$. With the shares of capital across the finance (F), goods (G) and human capital (H) sectors adding to one, as in $1 = s_{Ft} + s_{Gt} + s_{Ht,}$ and of labor adding to $1 - x_t = l_{Ft} + l_{Gt} + l_{Ht}$, with r_t and w_t the real rental and wage rates, and R_t the nominal interest rate, the continous time change in money and bond stocks is equal to the nominal capital income $P_t r_t s_{Gt} k_t$, labor income $P_t w_t l_{Gt} h_t$, the bond income $R_t B_t$, and the transfer V_t minus nominal expenditure on consumption $P_t c_t$ and investment $P_t i_t$; or

$$\dot{M}_t + \dot{B}_t = P_t r_t s_{Gt} k_t + P_t w_t l_{Gt} h_t + R_t B_t + V_t - P_t c_t - P_t i_t. \tag{2.2}$$

With $A_H > 0$,and $\epsilon \in [0, 1]$, human capital investment uses both effective labor and capital:

$$\dot{h}_t = A_H[(1 - s_{Gt} - s_{Ft})k_t]^{1-\epsilon}[(1 - l_{Gt} - l_{Ft} - x_t)h_t]^{\epsilon} - \delta_h h_t; \tag{2.3}$$

while physical capital changes according to

$$\dot{k}_t = i_t - \delta_k k_t.$$

The agent purchases a fraction a_t of output, nominal consumption plus investment, $P_t c_t + P_t i_t$, with money:

$$M_t = a_t P_t (c_t + i_t). \tag{2.4}$$

The rest is bought with credit, the real quantity of which equals the quantity of real units purchased, or $(1-a_t)y_t$. This quantity is produced by effective labor, capital and given output in a CRS fashion. Put differently, the share of purchases made by credit is produced using effective labor per unit of output and capital per unit of output in a diminishing returns fashion. Given that $1-\gamma_1-\gamma_2>0$; $\gamma_1, \gamma_2 \in [0,1)$:[1]

$$(1-a_t)y_t = A_F(l_{Ft}h_t/y_t)^{\gamma_1}(s_{Ft}k_t/y_t)^{\gamma_2}y_t; \tag{2.5}$$

or solving for a_t this gives $a_t = 1 - A_F(l_{Ft}h_t)^{\gamma_1}(s_{Ft}k_t)^{\gamma_2}(c_t+i_t)^{-(\gamma_1+\gamma_2)}$. Combining together these conditions by substituting for a_t gives a single exchange constraint that can be thought of as a generalization of a shopping time constraint, now with capital and investment along with real money, consumption and time, the traditional shopping time arguments:

$$M_t = [1 - A_F(l_{Ft}h_t)^{\gamma_1}(s_{Ft}k_t)^{\gamma_2}(c_t+i_t)^{-(\gamma_1+\gamma_2)}]P_t(c_t+i_t). \tag{2.6}$$

Also rather than having a constant interest elasticity of money demand, as in the typical shopping time specification, here the the interest elasticity has an increasing magnitude with inflation as in Cagan (1956), resulting from the diminishing returns production specification.

2.2. Goods Producer Problem

The goods producer competitively hires labor and capital for use in its Cobb-Douglas production function. Given $A_G > 0$, $\beta \in [0,1]$,

$$y_t = A_G(l_{Gt}h_t)^{\beta}(s_{Gt}k_t)^{1-\beta},$$

with the first-order conditions of

$$\begin{aligned} w_t &= \beta A_G(l_{Gt}h_t)^{\beta-1}(s_{Gt}k_t)^{1-\beta}, \\ r_t &= (1-\beta)A_G(l_{Gt}h_t)^{\beta}(s_{Gt}k_t)^{-\beta}. \end{aligned}$$

[1] If decentralized, there would be profit from the return to the y_t factor that is returned lump sum to the consumer. This production function also implies an implicit interest differential in a way related to Berk and Green (2004) and Canzoneri and Diba (2005).

2.3. Balanced-Growth Path Equilibrium

The optimization problem and balanced-growth conditions are given in Appendix A.1. The effects of both financial development and inflation on the balanced-growth path come primarily through changes in the goods to leisure tradeoff. An increase in inflation causes substitution towards leisure, and increased credit use. The increase in leisure causes the growth rate to decline. But as the inflation rate continues to rise, the growth rate declines at a decreasing rate as more credit and less leisure is used as the substitute for the inflation-taxed good (see the explanation in Gillman and Kejak, 2005). Financial development enters through the credit production technology parameters and affects the substitution towards leisure and how many resources are used up avoiding the inflation tax (see Lucas, 2000, for a discussion of such use of resources).

At the Friedman optimum, the nominal interest R equals zero and no credit is used. But as inflation rises, the agent substitutes from goods towards leisure while equalizing the margin of the ratio of the shadow price of goods to leisure, $x/(\alpha c) = (1+\tilde{R})/wh$. Here $\tilde{R} = aR + (\gamma_1 + \gamma_2)R(1-a)$ is the average exchange cost per unit of output. It is a weighted average, with weights of a and $1-a$, of the average cost of using cash, R, and the average cost of using credit, $(\gamma_1+\gamma_2)R$. That $(\gamma_1+\gamma_2)R$ is the average cost can be computed by dividing the total cost of credit production by the total output of credit production. Substitution towards leisure causes a fall in the human capital return of $r_H \equiv \varepsilon A_H (s_H k/l_H h)^{(1-\varepsilon)}(1-x)$. The marginal product of physical capital r_K also then falls, as a result of a Tobin-type substitution from labor to capital across all sectors in response to the higher real wage rate; the rise in $s_H k/l_H h$ mitigates but does not reverse the fall in the return to human capital caused by the increase in leisure. The growth rate is given by $g = r_H - \delta_H - \rho = [r_K/(1+\tilde{R})] - \delta_K - \rho$, and it falls as R rises since both r_H and r_K fall.

The role of financial development can be shown analytically within a special no-physical capital case of the economy. In this case, with a linear goods and human capital production of $c = A_G l_G h$ and $\dot{h}_t = A_H l_{Ht} - \delta_h h_t$ and a credit production of $(1 - a_t) = A_F(l_{Ft}h_t/c_t)^{\gamma_1}$, and assuming that $\delta_h = 0$, the solutions for leisure and the growth rate are given by $x = (\alpha\rho/A_H)[1+a^*R+(1-a^*)\gamma_1 R]/[1+(1-a^*)\gamma_1 R]$,and $g = A_H(1-x) - \rho$. An increase in leisure directly reduces the growth rate. The terms comprising leisure supply certain intuition about what is happening when there is inflation, and how financial development affects this. Essentially leisure is a constant times the ratio of the private shadow average cost of goods, $1 + a^*R + (1-a^*)\gamma_1 R$, divided by the social average cost of goods, $1 + (1-a^*)\gamma_1 R$. The private shadow average cost tells about the substitution towards leisure as inflation rises while the social average cost tells about the income effect of wasted resources that can decrease leisure use as inflation rises. The term $(1-a^*)\gamma_1 R$ is indeed how much of resources are used up avoiding the inflation tax, per unit of consumption, and not returned lump sum to the consumer as are the inflation tax proceeds.

Two contrasting effects of financial development on leisure use and growth, for a given inflation rate, can be clearly delineated in this no-physical capital case.

Proposition 1. An increase in the credit sector productivity level, A_F, causes an unambiguous decrease in leisure and increase in growth.

Proof: The solution for $1 - a^*$ is $(\gamma_1 R/A_G)^{\gamma_1/(1-\gamma_1)} A_F^{1/(1-\gamma_1)}$ and by rewriting the solution for leisure as $x = (\alpha\rho/A_H)[1 + R - (1-\gamma_1)(1-a^*)R]/[1 + (1-a^*)\gamma_1 R]$ it can be seen directly that $\partial x^*/\partial A_F$ depends negatively on the sign of $\partial(1-a^*)/\partial A_F$. The latter equals $(1-a^*)/[(1-\gamma_1)A_F]$ and is positive, thereby making $\partial x^*/\partial A_F < 0$ and $\partial g^*/\partial A_F > 0$; see Appendix A.2 for details.

Intuitively, the substitution and income effects both go in the same direction of decreasing leisure use.

Proposition 2. An increase in the parameter γ_1, which acts to decrease the degree of the diminishing returns to labor, causes an increase in leisure and a decrease in the growth rate for sufficiently low levels of the nominal interest rate R.

Proof: Similarly it can be shown that $\partial x^*/\partial \gamma_1$ depends negatively on $\partial(1-a^*)/\partial\gamma_1$, which equals $[(1-a^*)/(1-\gamma_1)][1 + (\ln R\gamma_1)/(1-\gamma_1)]$. The latter approaches negative infinity as R approaches zero; this makes $\partial x^*/\partial\gamma_1 > 0$ and $\partial g^*/\partial\gamma_1 < 0$ for sufficiently small R; see Appendix A.2 for details.

Intuitively, the substitution and income effects both go in the same direction of increasing leisure for small R. What determines the substitution and income effects ultimately is the effect of changes in the credit technology parameters on the marginal cost (MC) of credit production, since if the MC pivots up, while R is constant, then the credit share $1-a$ falls. While if the MC pivots down, with R constant, then the credit share $1-a$ rises. And both Propositions 1 and 2 depend on what happens to $1-a$ when the credit technology parameters are changed.

The marginal costs of money and credit are equal at the margin to the nominal interest rate; with no capital this equilibrium equation is $R = wl_F h/[\gamma_1(1-a)c]$ (see Appendix A.1 for the case with capital). Defining the MC per unit of c as $wl_F h/[\gamma_1(1-a)]$, consider that in Figure 1 the calibrated function for the MC of credit unambigously pivots down as the productivity factor A_F increases from 2 (thick line) to 2.5 (dashed line) to 3 (dot-dash line). The decrease in the MC for a given nominal interest rate (the horizontal line) explains why credit output $1-a$ increases and why leisure use decreases by both substitution (exchange goods are less costly) and income (average resource cost is higher) effects.

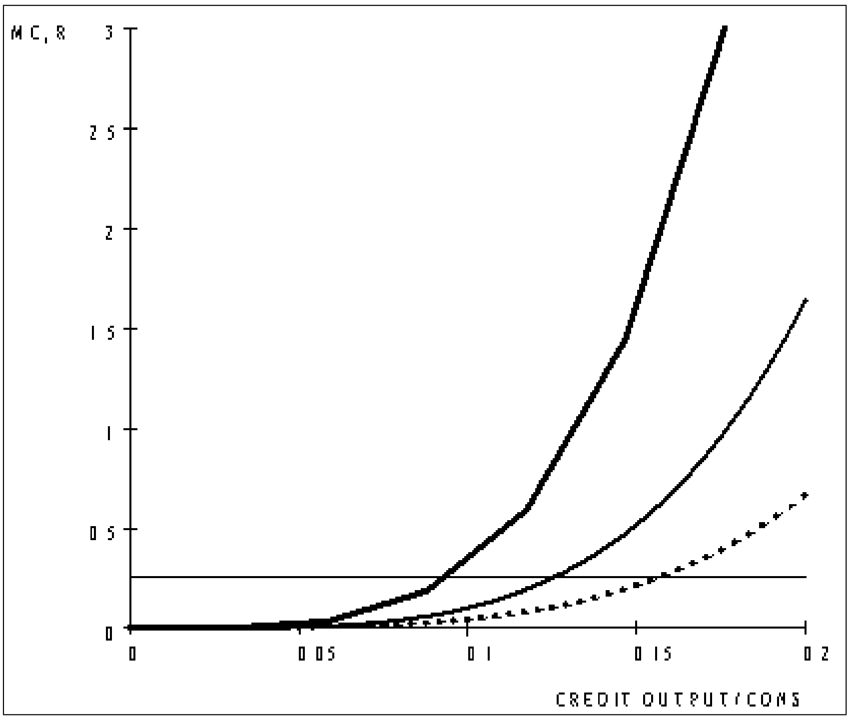


Figure 1. Marginal Cost of Credit With A_F Change

However the opposite is found in the calibrated functions for increases in γ_1 for low nominal interest rates. Figure 2 shows that the MC increases as γ_1 increases from 0.2 (thick line) to 0.3 (dashed line) to 0.4 (dot-dash line), for low nominal interest rates. This is true even though the MC becomes a flatter one-sided U-shape that involves lower marginal costs at high nominal interest rates: the cost curve exhibits increasing economies of scale at high production rates and approaches constant returns to scale as γ_1 approaches 1. But at low production rates the marginal costs are higher even as the economies of scale are bigger. This causes the substitution and income effects to go towards increasing leisure use and decreasing the growth rate.

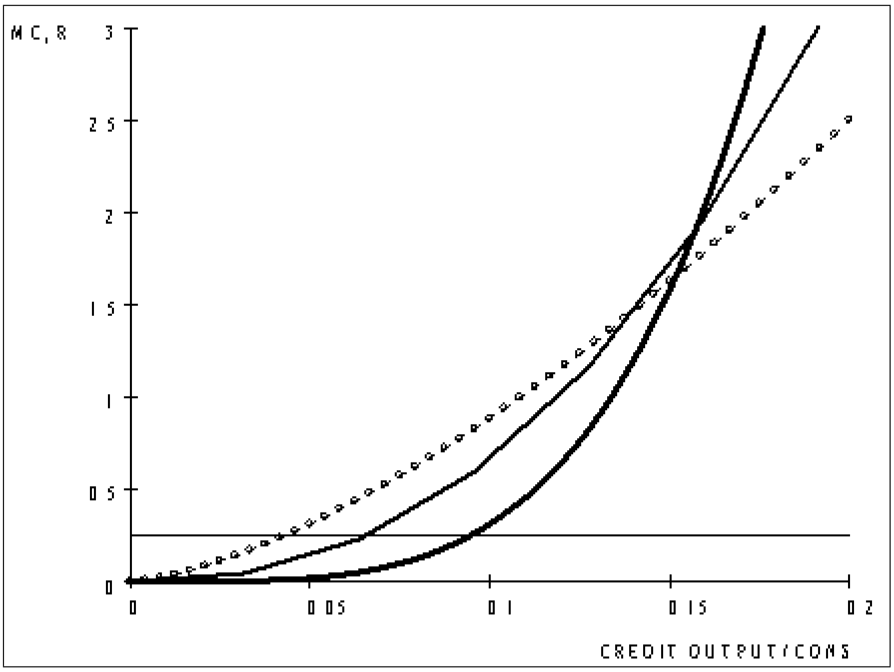
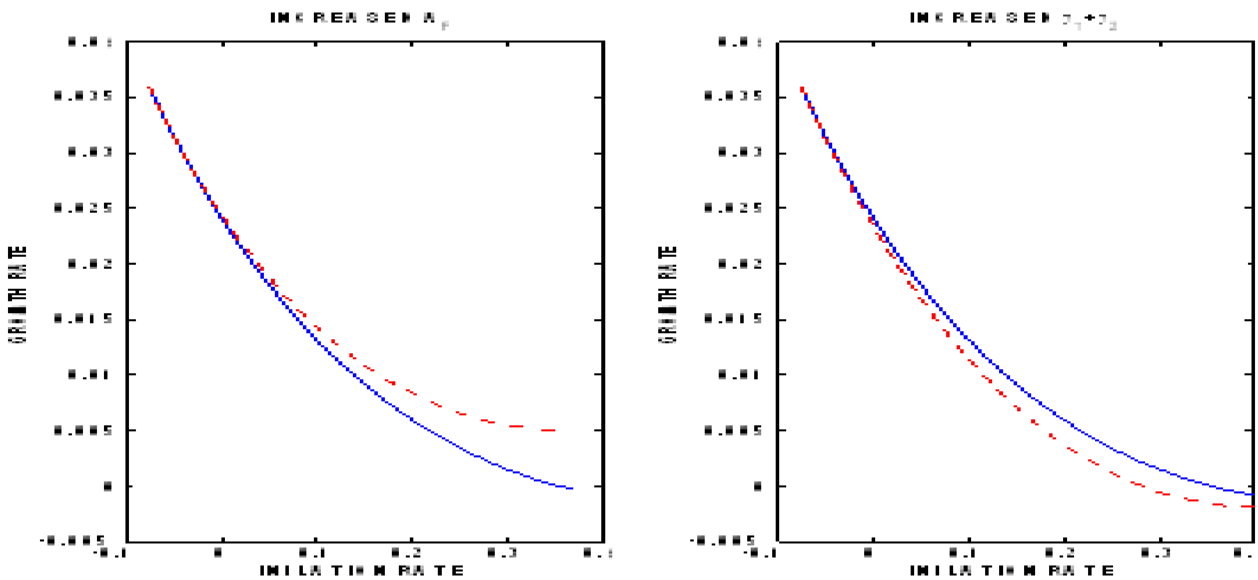


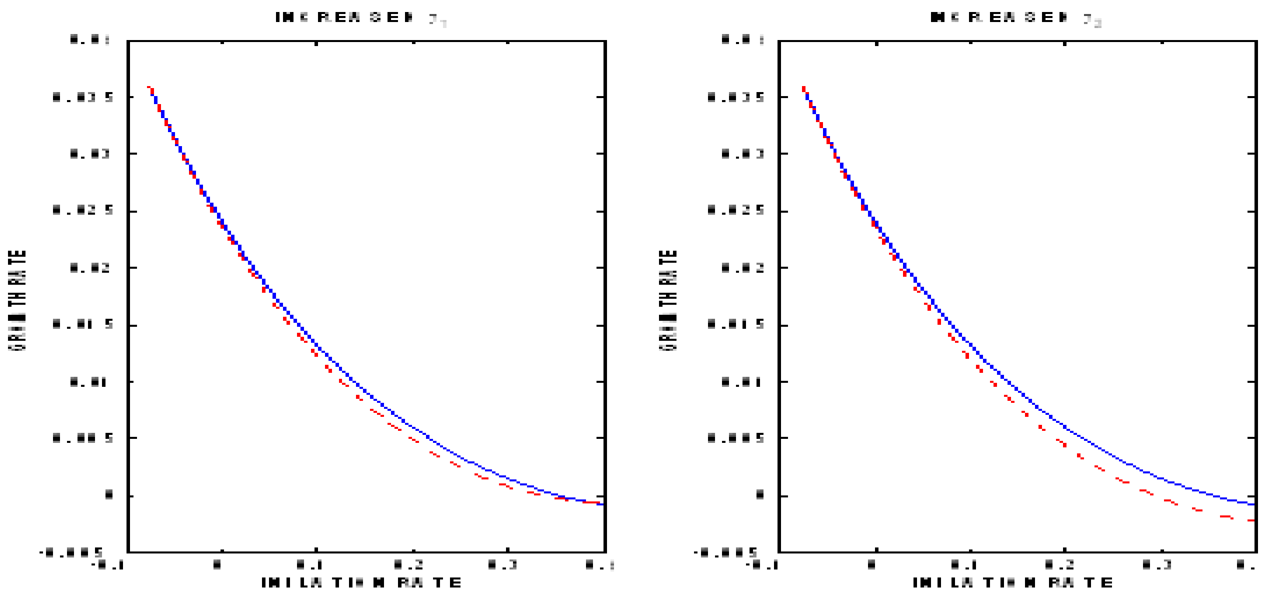
Figure 2. Marginal Cost of Credit with γ Change

2.4. Calibration and Simulation

For the more general economy, simulations are presented for the effects of changing A_F (Figure 3a), changing γ_1 and γ_2 at once (Figure 3b) or changing γ_1 (Figure 4a), and γ_2 (Figure 4b) individually. The results are similar to the simpler no-physical capital case with marginal cost changes as in Figures 1 and 2. Here, instead of the MC, the effects of the parameter changes on the inflation-growth profile are graphed.



Figures 3a, 3b: Baseline Simulation and Change in A_F; and in both γ_1 and γ_2



Figures 4a and 4b: Changes in γ_1; and in γ_2.

For this benchmark calibration, represented by the solid lines in Figure 3 and 4, the parameter values are $\rho = 0.04$, $\delta_K = \delta_H = 0.1$, $\theta = 1$, $\beta = \varepsilon = 0.64$, $\gamma_1 = \gamma_2 = 0.2$, $\alpha = 5$, $A_G = 0.292$, $A_H = 0.95$, $A_F = 1$; variable values are $a = 0.73$, $x = 0.66$, $g = 0,018$, $\pi = 0.052$, $l_G = 0.12$, $l_F = 0.0011$, $s_G = 0.36$, $s_F = 0.0059$. In the changes to the baseline simulation, represented by the dotted lines, $A_F = 1.1$ instead of 1 in Figure 3a; $\gamma_1 = \gamma_2 = 0.25$ instead of 0.2 in Figure 3b; $\gamma_1 = 0.25$, instead of 0.2 in Figure 4a; $\gamma_2 = 0.25$ instead of 0.2 in Figure 4b.

3. The Data

The financial development data is that of Levine, Loayza, and Beck (2000), as is the data for output, prices, government expenditure, trade and black market premiums.[2] While the original sample consists of 74 countries over the period 1961-1995, supplementing this data with the investment to output ratio (*Econ-Data*) and the money supply (*IFS*), as used in Gillman, Harris, and Mátyás (2004), results in reducing the sample to 27 countries with full information on all required.[3]

Five-yearly, non-overlapping, data averages are used such that are seven observations per country. The variables are defined as g, the real per capita growth in GDP; $\dot{p}$, the natural log of one plus the CPI rate of inflation; I, the ratio of gross domestic investment to GDP; y_0, the natural log of the real per capita GDP in the initial period; gov, the natural log of the share of government expenditure in GDP; $trade$, the natural log of the share of total international trade in GDP; and bmp, the natural log of one plus a black market premium.

For the three financial development variables, the notation is $private$, which is the natural log of the ratio of the value of credits by financial intermediaries

[2] We are very grateful to those authors for kindly supplying their data.

[3] These countries are Australia, Austria, Belgium, Canada, Chile, Denmark, Finland, France, Greece, Ireland, Italy, Japan, the Republic of Korea, Malaysia, Mexico, Netherlands, New Zealand, Norway, Peru, Philippines, Portugal, Spain, Sweden, Switzerland, Thailand, the United Kingdom, and the United States. Note that for the inflation rate data, 4 data points of the 186 are above 50%, three for Peru and one for Mexico, and there are no negative rates of inflation.

to the private sector relative to GDP; lly, which is the natural log of the ratio of liquid liabilities of the financial system to GDP; $\dot{p}private$, which is the product of $\dot{p}$ and $private$ (an "interaction term"); $\dot{p}lly$,which is the product of $\dot{p}$ and lly (an alternative interaction term). The third financial development variable is the ratio of commercial assets to total banking assets.

4. The Econometric Model

The econometric model is specified as

$$g_{it} = \mu_i + \lambda_t + \beta_1 \dot{p}_{it} + \beta_2 I_{it} + \alpha_{it}^k F_{it}^k + [Other]_{it}' \boldsymbol{\eta} + \varepsilon_{it}, \qquad (4.1)$$

where: g_{it} is the real per capita annual rate of growth of GDP of country i in period t; μ_i an unobservable effect (or "individual effect") for country i; λ_t an unobservable effect for time period t; and F_{it}^k is the level of financial intermediary development as proxied by the $k = 3$ variables, with unknown weights α_{it}^k. However since the ratio of commercial assets to total banking assets is found to be generally insignificant, results for this variable are not reported, and only $private$ and lly results are presented. The variables in the vector $Other_{it}$ with unknown weights $\boldsymbol{\eta}$ include the initial income, trade, and black market variables; the disturbance term is ε_{it}.

The model further proposes that the financial intermediary effect, α_{it}^k, can be a function of the inflation rate:

$$\alpha_{it}^k = \beta_3^k + \beta_4^k \dot{p}_{it}, \qquad (4.2)$$

where β_3^k and β_4^k are parameters for the k number of financial development variables. Substituting in for α_{it}^k gives

$$g_{it} = \mu_i + \lambda_t + \beta_1 \dot{p}_{it} + \beta_2 I_{it} + \beta_3^k F_{it}^k + \beta_4^k (\dot{p}_{it} F_{it}^k) + [Other]_{it}' \boldsymbol{\eta} + \varepsilon_{it}, \qquad (4.3)$$

where $\dot{p}_{it} F_{it}^k$ represents the interaction between inflation and financial development.

4.1. Estimation Results

A panel data estimation with two-way fixed effects techniques is used since Hausman tests indicate the presence of correlations between the observed and unobserved effects. With such correlations both fixed and random effects specifications can yield consistent parameter estimates. In the fixed effects approach the effects are treated as constants. In a random effects approach, instrumental variable (IV) techniques can be applied. Table 4.1 presents first the fixed effects approach. Inflation and the investment ratio are highly significant with the expected negative and positive signs respectively. The level of financial development is consistently statistically insignificant; results are presented with it removed from the model except as it enters through the interaction term. The interaction term of financial development and inflation for both the $\dot{p}private$ and $\dot{p}lly$ variables is significant and negative. Endogeneity tests indicate that the inflation rate enters exogenously in models 1 to 3.

Because the Hausman specification tests suggest possible correlation between the observed and unobserved effects, as a further robustness check, this correlation is accounted for by using a random effects framework as in Hausman and Taylor (1981) and Amemiya and MaCurdy (1986). These results indicate a significant and negative effect of the level of financial intermediation. The interaction term between inflation and financial intermediation generally remains significant and negative. However a problem is that there is a rejection of the null hypothesis of valid instruments using the Sargan criteria, and so the results are not reported here (see Gillman and Harris, 2004, for these details). Thus the baseline model apparently finds only negative effects of financial development, assuming a linear relation between inflation and growth.

4.2. Threshold Effect Extensions

Non-linear effects can be estimated by breaking the regression into segments, or splines, and then looking for threshold levels of inflation with differentiated mar-

Table 4.1: Growth Regression Results

	Model 1	Model 2	Model 3	Model 4
Constant	0.425	0.458	0.388	0.446
	(0.05)**	(0.06)**	(0.06)**	(0.06)**
$\dot{p}$	-0.250	-0.187	-0.189	-0.206
	(0.05)**	(0.04)**	(0.04)**	(0.04)**
I	0.003	0.002	0.002	0.002
	(0.00)**	(0.00)**	(0.00)**	(0.00)**
y_0	-0.052	-0.056	-0.053	-0.054
	(0.01)**	(0.01)**	(0.01)**	(0.01)**
gov	-	-	-0.020	-
			(0.01)**	
$trade$	-	-	-0.016	-
			(0.01)**	
bmp	-	-	-	-0.028
				(0.02)*
$\dot{p}private$	-	-0.043	-0.043	-0.058
		(0.02)**	(0.01)**	(0.02)**
$\dot{p}lly$	-0.088	-	-	-
	(0.02)**			
$\overline{R}^2$	0.735	0.722	0.737	0.727
$LR \sim \chi^2_{33}$	169.697	163.91	169.244	167.578
$Hausman \sim \chi^2_{df}$	47.34 (4)	50.19 (4)	56.83 (7)	49.03 (5)
$Endogeneity \sim N(0,1)$	0.224	1.723	1.056	2.173
NT	186	186	186	186

**Significant at 5% size (2-sided). *Significant at 10% size (1-sided). LR refers to Likelihood Ratio tests of $\alpha_i = \lambda_t = 0, \forall i, t$. *Hausman* tests are of fixed *versus* random specifications. *Endogeneity* tests the null-hypothesis that the inflation variable is exogenous; the critical value is 1.96.

ginal effects for either side of the threshold value. Some advances in such estimated thresholds include Hansen (1999), who provides procedures for estimating multiple unknown breakpoints within the context of a one-way fixed effects panel model. Hansen (2000) presents distribution theory for the estimation of multiple threshold effects for either cross-section or time series data; Gonzalo and Pitarakis (2002) introduce a model selection based procedure which simultaneously estimates the unknown threshold parameters and their optimal number; and Drukker, Gomis-Porqueras, and Hernandez-Verme (2004) combine the model selection procedures of Gonzalo and Pitarakis (2002) with the panel data aspects of Hansen (1999) in estimating one-way fixed effects for the non-linear effect of inflation on growth.

Here a multiple threshold approach is developed and applied to panel data for the inflation-growth relation as an extension of the model in equation (4.3). The threshold results come from several novel econometric extensions. These are that two-way country (individual) and time fixed effects are used in the panel's endogenously determined splines, as compared to one-way (individual) fixed effects in Drukker, Gomis-Porqueras, and Hernandez-Verme (2004); the use of instrumental variables where the splined variable is potentially endogenous; the application of the model selections criteria to choose simultaneously across both the model type (OLS, one-way panel, two-way panel) and the number of breakpoints (thresholds); and the estimation of the multiple endogenous splines when they are forced to be piecewise continuous. The methodology of these extensions is presented in the Appendix A.3.

The econometric model is estimated without any unobserved effects (labelled OLS), with fixed unobserved country effects ($1 - Way$), and with fixed unobserved country and time effects ($2 - Way$). Table 4.2 presents the results for the estimated breakpoints in terms of the inflation rate using each method, and for the optimal number of thresholds for each method and overall as based on the Information Criterion (IC) procedure detailed in Appendix A.3.1-A.3.2. The procedure is undertaken for both *tied* and *untied* spline functions; see Appendix A.3.3. Although IC methods are used to ascertain the optimal number of breakpoints, the estimation procedure works sequentially, implying in a sense that the first breakpoint reported in Table 4.2, which is the first found in the estimation, is the "strongest" one, the second reported in the table is less strong, and the third the least strong.

Table 4.2: Estimated Threshold Effects

Breaks	OLS	$1-Way$	$2-Way$	OLS	$1-Way$	$2-Way$
	$lly : untied$			$private : untied$		
0	*	-	-			
1	0.07	0.16	0.23	0.07*	0.23	0.23
2	0.23	0.23*	0.16**	0.23	0.16*	0.16
3	0.16	0.03	0.03	0.16	0.03	0.04**
	$lly : tied$			$private : tied$		
0	*	-	-			
1	0.23	0.03	0.03**	0.23*	0.23	0.03**
2	0.07	0.05*	0.04	0.07	0.05*	0.04
3	0.10	0.04	0.03	0.10	0.02	0.23

**Preferred model overall (based on minimum IC). *Preferred model for each estimation procedure (based on minimum IC).

When lly is used, the optimal combination of number of *untied* breakpoints and estimation method is $2-Way$ with two splines; with the breakpoints occurring at inflation rates of 16% and 23% (the optimal number is zero for OLS and two for $1-Way$). Note that both the $1-Way$ and $2-Way$ select the same number and value of breakpoints. If *private* is used as the proxy for financial development, again $2-Way$ is preferred. Here there are three thresholds at 4, 16 and 23% rates of inflation, similar to the lly results in that the 16% and 23% breakpoints coincide. Forcing the spline function to be piecewise continuous, the optimal model for both lly and *private* is $2-Way$. For both there is now only one threshold effect at a 3% rate of inflation.

Table 4.3 contains the estimation results corresponding to the estimated threshold values of Table 4.2. All of the estimations presented are undertaken using

$2 - Way$ fixed effects and correspond to the optimally chosen model and number of breakpoints. With regard to the variables I, y_0, and the interaction term between inflation and financial development, results vary little across specifications and in comparison to Table 4.1. Investment has a positive and significant effect on growth; initial GDP has a significantly negative effect, as does the interaction term, while the level of financial development has a insignificant negative effect. For the inflation rate, the lly proxy shows a significant negative effect at all levels; the $private$ proxy shows an insignificant positive effect at low levels and a significant negative effect at all other levels. Forcing the inflation-growth splines to be piecewise continuous, the effect of inflation at low levels up to 3% is significant and positive for both proxies. However these later results with a positive effect at low levels are not robust to using instrumental variables, as the next subsection indicates.

Another way to compare Models 1 to 4 in Table 4.3 is using the Information Criterion numbers. These are -3.1646 for lly not tied, -3.1362 for lly tied, -3.1287 for $private$ not tied, and -3.0831 for $private$ tied. With a lower IC value being a better one, this indicates that the models using lly are the preferred ones, and that the results not forcing the continuity of the splines are preferred over the tied spline results for both financial development proxies. Model 1 is the preferred model.

4.3. Simultaneity Bias

Using splines it is difficult to test for endogeneity in the splined variable, in this case the inflation rate. However an instrumental variables (IV) estimation can be made and the results can then be compared to the Table 4.3. For the estimation with IVs, a procedure similar to two-stage least squares is used. First fitted values of inflation are constructed by regressing it against all of the exogenous variables in the model plus a money supply instruments. The observed inflation rate is then replaced by its fitted value $\widehat{\dot{p}}_{it}$ and the spline procedure as described above is then implemented on $\widehat{\dot{p}}_{it}$ as opposed to $\dot{p}_{it}$. To take into account the issue of generated

Table 4.3: Threshold Growth Results: Standard Errors in Paranetheses.

	lly		*private*	
	untied	*tied*	*untied*	*tied*
Constant	0.511	0.513	0.542	0.559
	(0.06)**	(0.06)**	(0.06)**	(0.06)**
I	0.002	0.002	0.002	0.002
	(0.00)**	(0.00)**	(0.00)**	(0.00)**
y_0	-0.061	-0.062	-0.064	-0.066
	(0.01)**	(0.01)**	(0.01)**	(0.01)**
$\dot{p}FD$	-0.049	-0.074	-0.011	-0.034
	(0.02)**	(0.02)**	(0.02)	(0.02)**
FD	-0.005	-0.005	-0.002	-0.002
	(0.00)	(0.00)	(0.00)	(0.00)
$\dot{p}_{low}$	-0.214	0.222	0.093	0.252
	(0.05)**	(0.10)**	(0.13)	(0.11)**
$\dot{p}_{medium}$	-0.278	-	-0.131	
	(0.05)**	-	(0.05)**	
$\dot{p}_{medium-high}$	-0.180		-0.215	
	(0.05)**		(0.04)**	
$\dot{p}_{high}$		-0.228	-0.110	-0.169**
		(0.05)**	(0.04)**	(0.04)
$\overline{R}^2$	0.736	0.723	0.733	0.708
NT	175	175	175	175

**Significant at 5% size (2-sided). *Significant at 10% size (1-sided). FD refers to the appropriate measure of financial development.

regressors, coefficient standard errors are estimated by bootstrap methods. Note that the sample loses one time period and one country due to missing observations on the money supply and this means that the procedure searches over somewhat different ranges of inflation.

Table 4.4 presents the IV results for the optimal number and position of the breakpoints. Using *lly* and a piecewise discontinuous function, the optimal model now is OLS with two breakpoints at 8% and 17% rates of inflation, as compared to the optimal $2-Way$ with two breakpoints at 3% and 16% in Table 4.2. However the test statistic that choses OLS as optimal is very close to the test statistics for the $1-Way$ and $2-Way$. For *private untied*, the optimal choice is OLS with 3 breakpoints as compared to $2-way$ with 3 breakpoints in Table 4.2.

Table 4.4: Estimated Threshold Effects using IVs

Breaks	OLS	$1-Way$	$2-Way$	OLS	$1-Way$	$2-Way$
	$lly : untied$			$private : untied$		
0	-	-	-			
1	0.17	0.28*	0.18	0.10	0.29	0.05
2	0.08**	0.06	0.05	0.14	0.13	0.09
3	0.11	0.04	0.15*	0.11**	0.11*	0.03*
	$lly : tied$			$private : tied$		
0	-	-	-			
1	0.08	0.04*	0.05	0.11**	0.11	0.15
2	0.29**	0.05	0.04*	0.10	0.13	0.05
3	0.07	0.05	0.18	0.06	0.15*	0.09*

**Preferred model overall (based on minimum IC). *Preferred model for each estimation procedure (based on minimum IC).

Table 4.5 presents the regression results corresponding to the optimal model as indicated in Table 4.4. The results that pass the $Sargan$ test are those using *lly untied*. As in previous findings, I, y_0 and the interaction of financial development and inflation are all significant, and once more the level of financial development is insignificant. All levels of inflation have a significant negative effect, although

Table 4.5: IV Threshold Growth Results: Standard Errors in Paranetheses.

	lly		*private*	
	untied	*tied*	*untied*	*tied*
Constant	0.045	0.015	0.095	0.024
	(0.03)*	(0.02)	(0.10)	(0.02)
I	0.003	0.002	0.002	0.002
	(0.00)***	(0.00)	(0.00)**	(0.00)***
y_0	-0.007	-0.006	-0.010	-0.005
	(0.00)***	(0.00)	(0.01)*	(0.00)***
$\dot{p}FD$	-0.142	0.053	-0.209	0.217
	(0.08)**	(0.05)	(0.40)	(0.05)***
FD	-0.008	-0.015	-0.001	-0.014
	(0.01)	(0.01)	(0.01)	(0.00)***
$\dot{p}_{low}$	-0.397	0.170	-0.647	0.163
	(0.26)*	(0.07)	(1.18)	(0.07)***
$\dot{p}_{low/medium}$	-	-	-0.748	-
	-	-	(1.15)	-
$\dot{p}_{medium/high}$	-0.535	-0.148	-0.900	-
	(0.20)***	(0.88)	(1.13)	-
$\dot{p}_{high}$	-0.379	0.046	-0.640	0.532
	(0.16)***	(0.11)	(1.08)	(0.14)***
$\overline{R}^2$	0.457	0.434	0.456	0.457
$Sargan$	0.430	0.078	0.022	0.072
NT	144	144	144	144

***Significant at 5% size (2-sided). **Significant at 5% size (1-sided);*Significant at 10% size (1-sided); $Sargan$ refers to the bootstrapped empirical $p-$value of the Sargan statistic for instrument validity, accept H_0 of valid instruments for $p > 0.05$.

weaker at lower levels. For *private untied*, the model fails the *Sargan* test. Forcing the continuity of the splines, the *tied* results for *lly* and *private* suggest that inflation positively affects growth in ranges, but the *Sargan* statistic that indicate instruments with borderline validity.[4]

4.4. A Dynamic Growth Approach

Finally, consider for robustness dynamic growth equations. Here the basic model is extended by including lagged growth, $g_{i,t-1}$. For the dynamic panel model the usual estimation techniques are inconsistent. To allow for growth to follow an autoregressive process while removing the unobserved effects, it is common to write the model in terms of first differences and including a lagged dependent variable

$$\Delta g_{it} = \delta \Delta g_{i,t-1} + \Delta \mathbf{x}'_{it}\boldsymbol{\beta} + \Delta\varepsilon_{it}. \tag{4.4}$$

Following Arellano and Bond (1991) it is possible to consistently estimate the model by GMM estimation based upon the moment conditions,

$$E\left(\Delta\varepsilon_{it} g_{i,t-j}\right) = 0,\ j = 2, \ldots, t-1;\ t = 3, \ldots, T. \tag{4.5}$$

The moment conditions imply that the $\Delta\varepsilon_{it}$ do not follow a second-order serial correlation process, a condition that is tested here.

The results of the instrumental variables estimation of the previous subsection indicate a greater validity for the *lly* proxy of financial development, with discontinuous splines. Here this model is used (Model 1 in Table 4.3) and it is assumed that the true threshold model has two breakpoints as found in both Table 4.2 and Table 4.4. Re-estimating the model with the inclusion of the lagged dependent variable as in equation (4.4), Table 4.6 presents the results.

The model passes the *Sargan* test for instrument validity and indicates results consistent with Tables 4.3 and 4.5 in terms of the significance and signs of the explanatory variables. The growth process is autoregressive with the lagged dependent variable being strongly significant. This variable's negative sign indicates

[4] These are bootstrapped empirical values of the standard *Sargan* test.

Table 4.6: Dynamic Growth Results

Variable	Coefficient	Standard Error
$g_{i,t-1}$	-0.488	$(0.06)^{**}$
I	0.002	$(0.00)^{**}$
y_0	-0.046	$(0.01)^{**}$
$\dot{p}lly$	-0.058	$(0.02)^{**}$
lly	0.000	(0.01)
$\dot{p} \times 1\,(\dot{p} < 16\%)$	-0.166	$(0.04)^{**}$
$\dot{p} \times 1\,(16\% \leq \dot{p} < 23\%)$	-0.244	$(0.04)^{**}$
$\dot{p} \times 1\,(23\% \leq \dot{p})$	-0.1943	$(0.04)^{**}$
$\overline{R}^2$	0.694	
$Sargan$	0.1	
m_2	-2.4	
NT	125	

**Significant at 5% size (2-sided). $Sargan$ refers to the $p-$value of the Sargan statistic for instrument validity, accept H_0 of valid instruments for $p > 0.05$. m_2 tests for second-order serial correlation and is $\tilde{a}N\,(0,1)$ under the null hypothesis.

a cyclical return to the equilibrium growth path following a shock. The strong significance of the lagged growth term while the remaining variables have similar effects to the static estimations suggests that the potential omitted variable bias arising from the previous exclusion of $g_{i,t-1}$ is small. Note that there is some evidence that the $\Delta\varepsilon_{it}$ follow a second-order serial correlation process.

5. Discussion

The estimation results are robust with respect to one of the three measures of financial development, the ratio of liquid financial assets to GDP. This measure showed that inflation has a negative effect on growth at all levels of inflation that was robust across multiple threshold testing, instrumental variable estimation and dynamic panel estimation. The other two measures examined were either insignificant or yielded potentially inconsistent results. Use of the private credit variable in the model for example indicates a positive effect of inflation at low levels of the inflation rate that is not robust to considerations of endogeneity of the inflation rate, in that the inflation rate is insignificant at all levels for the IV results. The IV results are important to consider since such endogeneity can be suspected *a priori* at low inflation rates because of the interaction of the business cycle with the price level. The price level has been found to comove with output in the short run (Den Haan 2000), which is a manifestation of how low levels of the measured inflation rate and the output level could be simultaneously determined. However this can be more of a relative price change involving changes in the aggregate price level due to real output changes over the business cycle than to monetary, inflation-type, effects and so it should be controlled for with instruments. Use of the money supply as an instrument is also found in Gillman, Harris and Matyas (2004).

The other way in which a positive effect of inflation at low levels is replicated is through a procedure to force the multiple splines to be piecewise continuous. But this approach yields consistently worse results using the Information Criterion and so is found to involve a nontrivial assumption that can yield misleading results. The models without this assumption perform better and are preferred.

For the other variables, the level of financial development robustly has a negative effect on growth through its interaction with the inflation rate. The investment ratio has a robust postive effect; the initial value of GDP has a robust negative effect. By itself the level of financial development is robustly insignificant. But note that the standard results of a positive financial development effect can be replicated with the data set. These results are not reported because the models giving such results lack significant missing variables. For example, using the private credit measure of financial development and the black market variable, but excluding the investment ratio, the results show a significant financial development variable at the 1% level (two-sided). And here a significant negative inflation effect is replicated at a 5% level of significance (one-sided). Or, excluding only the investment ratio from the Model 1 in Table 4.1, while using the liquid liabilities measure of financial development, finds that the level of financial development is positive while the interaction term is negative, although both with weak levels of significance ($t-$statistics respectively of 1.296 and -1.182).

6. Conclusion

The paper presents a general equilibrium monetary model with an exchange constraint that is determined by the production of credit and the substitution between money and credit to buy the consumption good. The model is used to analyse how financial development affects the inflation-growth profile, or similarly, how it affects the growth rate for a given inflation rate. Changes in the two structural parameters of the credit technology cause contrasting endogenous effects on the growth rate, and on the nature of the inflation-growth relation. Extensive empirical results find support for a negative effect of financial development, which is consistent with the model's explanation that increased economies of scale in the financial sector interact with inflation avoidance activity to cause a lower growth rate. These results also are robust at all levels of the inflation rate, including low levels, for the one measure of financial development found to give robust results.

The implied intuition from the general equilibrium model is that having to produce at low levels of credit output because of a low inflation rate, while having a large economies of scale in credit production, is more costly than having lower such economies of scale. Less developed countries with more limited markets in using developed financial technology and with less such specialization would tend to have lower economies of scale than developed countries. This implies, as the simulations of Figures 3 and 4 illustrate and as the evidence supports, that as more developed countries decrease their inflation rate from already low levels, they will have bigger increases in growth than less developed countries.

A. Appendix

A.1. Equilibrium

$$
\begin{aligned}
&\mathcal{H}(k,h,M,B;c,x,l_G,l_F,s_G,s_F;\lambda,\mu,\nu,\xi,\eta,t) = \\
&\quad = \ e^{-\rho t}\frac{c_t^{1-\theta}x_t^{\alpha(1-\theta)}}{(1-\theta)} \\
&\quad + \ \lambda_t\left\{P_t r_t s_{Gt} k_t + P_t w_t l_{Gt} h_t - P_t c_t - P_t i_t + R_t B_t\right\} \\
&\quad + \ \mu_t\{M_t - [1 - A_F(l_{Ft}h_t)^{\gamma_1}(s_{Ft}k_t)^{\gamma_2}(c_t+i_t)^{-(\gamma_1+\gamma_2)}]P_t(c_t+i_t)\} \\
&\quad + \ \nu_t\{Q - M - B\} \\
&\quad + \ \xi_t\{i_t - \delta_k k_t\} \\
&\quad + \ \eta_t\{A_H[(1-s_{Gt}-s_{Ft})k_t]^{1-\epsilon}[(1-l_{Gt}-l_{Ft}-x_t)h_t]^{\epsilon} - \delta_h h_t\}
\end{aligned}
$$

The stationary equilibrium along the balanced growth path is expressed in the equations below as a function of the leisure variable x and given parameters; for simplicity it is assumed that $\theta = 1$:

$$R = \sigma + \rho;$$

$$[\Omega(1-x)(1+\tilde{R})]^{\frac{1}{1-\varepsilon+\beta}} = \left[\frac{(1-a)^{1-\gamma_1-\gamma_2}}{A_F}\left(\frac{A_G}{R}\right)^{\gamma_1+\gamma_2}\left(\frac{\beta}{\gamma_1}\right)^{\gamma_1}\left(\frac{1-\beta}{\gamma_2}\right)^{\gamma_2}\right]^{\frac{1}{(1-\beta)\gamma_1-\beta\gamma_2}};$$

$$\tilde{R} = aR + (\gamma_1+\gamma_2)R(1-a);$$

$$\Omega = \frac{A_H}{A_G}\varepsilon^{\varepsilon}(1-\varepsilon)^{1-\varepsilon}\beta^{1-\varepsilon}(1-\beta)^{\beta-2};$$

$$\frac{\gamma_2}{\gamma_1}\frac{\beta}{1-\beta}\left(\frac{l_F h}{s_F k}\right) = \frac{1-\varepsilon}{\varepsilon}\frac{\beta}{1-\beta}\left(\frac{l_H h}{s_H k}\right) = \left(\frac{l_G h}{s_G k}\right) = \left(\Omega(1-x)(1+\tilde{R})\right)^{\frac{1}{1-\varepsilon+\beta}};$$

$$l_H = \varepsilon(1-x);$$

$$r_H = \varepsilon A_H\left(\frac{l_H h}{s_H k}\right)^{-(1-\varepsilon)}(1-x);$$

$$g = r_H - \delta_H - \rho = \frac{r_K}{1+\tilde{R}} - \delta_K - \rho = A_G\left(\frac{l_G h}{s_G k}\right)s_G - \frac{c}{k} - \delta_k;$$

$$\pi = \sigma - g;$$

$$R = wl_F h/[\gamma_1(1-a)y] = rs_F k/[\gamma_2 R(1-a)y];$$

$$\frac{x}{\alpha c} = \frac{1+\tilde{R}}{wh}.$$

A.2. Proofs of Propositions 1 and 2

For the no physical capital, log-utility. case of the economy, $x = (\alpha\rho/A_H)[1 + a^*R + (1-a^*)\gamma_1 R]/[1+(1-a^*)\gamma_1 R]$, and $g = A_H(1-x) - \rho$. Then proof of Proposition 1 and 2 requires showing what happens to x, which in turn directly determines g, when the parameters A_F and γ_1 change, respectively. $\partial x/\partial A_F = \{(\alpha\rho/A_H)/[1+\gamma_1 R(1-a^*)]^2\}\{[1+\gamma_1 R(1-a^*)][-(1-\gamma_1)R(\partial[1-a^*]/\partial A_F)]$
$-[1+R-(1-\gamma_1)R(1-a^*)][\gamma_1 R(\partial[1-a^*]/\partial A_F)]\}$. Because $[1+R-(1-\gamma_1)R(1-a^*)] > 0$, then $\partial x/\partial A_F$ depends on two terms that are negative if $\partial[1-a^*]/\partial A_F$ is positive, or that are positive if $\partial[1-a^*]/\partial A_F$ is negative. The solution gives that $1-a^* = (\gamma_1 R/A_G)^{[\gamma_1/(1-\gamma_1)]} A_F^{1/(1-\gamma_1)}$, so that $\partial[1-a^*]/\partial A_F > 0$. This implies $\partial x/\partial A_F < 0$, and $\partial g/\partial A_F > 0$.

For the second proposition $\partial x/\partial\gamma_1 = \{(\alpha\rho/A_H)/[1+\gamma_1 R(1-a^*)]^2\}\{[1+\gamma_1 R(1-a^*)][R(1-a^*) - (1-\gamma_1)R(\partial[1-a^*]/\partial\gamma_1)]$
$-[1+R-(1-\gamma_1)R(1-a^*)][R(1-a^*) + \gamma_1 R(\partial[1-a^*]/\partial\gamma_1)]\}$. Again the sign depends crucially on $\partial[1-a^*]/\partial\gamma_1$. And this can be found to be given by $\partial[1-a^*]/\partial\gamma_1 = [(1-a^*)/(1-\gamma_1)][1+(\ln[\gamma_1 R/A_G])/(1-\gamma_1)]$. As $R \longrightarrow 0$, $\partial[1-a^*]/\partial\gamma_1 \longrightarrow -\infty$. Thus for small enough R, $\partial x/\partial\gamma_1 > 0$, and $\partial g/\partial\gamma_1 < 0$.

A.3. Multiple Threshold Effects

The threshold model, for two regimes, considered in Hansen (1999) and Drukker, Gomis-Porqueras, and Hernandez-Verme (2004) is of the form $g_{it} = \alpha_i + \mathbf{x}'_{it}\boldsymbol{\beta} + \gamma_1 \dot{p}_{it} \times 1(\dot{p}_{it} \leq \gamma_1^*) + \gamma_2 \dot{p}_{it} \times 1(\dot{p}_{it} > \gamma_1^*) + \varepsilon_{it}$.This can be written more compactly; by defining $\dot{\mathbf{p}}_{it}(\gamma_1^*) \equiv [\dot{p}_{it} \times 1(\dot{p}_{it} \leq \gamma_1^*), \dot{p}_{it} \times 1(\dot{p}_{it} > \gamma_1^*)]$, $\boldsymbol{\gamma} \equiv (\gamma_1, \gamma_2)'$:

$$g_{it} = \alpha_i + \mathbf{x}'_{it}\boldsymbol{\beta} + \dot{\mathbf{p}}_{it}(\gamma_1^*)\boldsymbol{\gamma} + \varepsilon_{it}. \tag{A.1}$$

Here $\mathbf{x}_{it}$ is the vector of explanatory variables net of the splined variable. Thus if inflation is less than or equal to the (unknown) threshold value γ_1^*, its marginal effect on growth is given by γ_1 and by γ_2 otherwise. For identification, $\mathbf{x}_{it}$ cannot contain any time-invariant variables; it is also assumed that the threshold effects are time-invariant. The error term, ε_{it}, is *iid* with zero mean and finite variance, σ_ε^2.

The usual approach to estimating one-way panel models, is to use the $Within$ operator to transform the variables into differences from time means for each i and then to apply ordinary least squares (OLS) to the transformed model (Mátyás and Sevestre 2005). For the unsplined variables, the transformation is such that for typical element of $\mathbf{x}$ we have $x_{it}^* = x_{it} - \overline{x}$, with $\overline{x}_{i.} = T^{-1}\sum_{t=1}^{T} x_{it}$. For the inflation variable the relevant transformation is $\overline{\dot{\mathbf{p}}}_{it}(\gamma_1^*) = T^{-1}\sum_{t=1}^{T}\dot{\mathbf{p}}_{it}(\gamma_1^*) = \left[T^{-1}\sum_{t=1}^{T}\dot{p}_{it}\times 1(\dot{p}_{it}\leq\gamma_1^*), T^{-1}\sum_{t=1}^{T}\dot{p}_{it}\times 1(\dot{p}_{it}>\gamma_1^*)\right]$. With $\mathbf{G}^*, \mathbf{X}^*$ and $\dot{\mathbf{P}}^*$ defined as the matrix stacked versions of $g_{it}^*, \mathbf{x}_{it}^*$ and $\dot{p}_{it}^*$ respectively, the estimating equation is $\mathbf{G}^* = \mathbf{X}^*\boldsymbol{\beta} + \dot{\mathbf{P}}^*(\gamma_1^*)\boldsymbol{\gamma} + \boldsymbol{\varepsilon}^*$.With $\mathbf{Z}^*(\boldsymbol{\gamma}_1^*) = \left[\mathbf{X}^*, \dot{\mathbf{P}}^*(\gamma_1^*)\right]$ and $\boldsymbol{\phi} = (\boldsymbol{\beta}', \boldsymbol{\gamma}')'$, this rewrites as $\mathbf{G}^* = \mathbf{Z}^*(\gamma_1^*)\boldsymbol{\phi} + \boldsymbol{\varepsilon}^*$.For any given value of γ_1^*, the matrix $\boldsymbol{\phi}$ can be estimated by $\widehat{\boldsymbol{\phi}} = \left[\mathbf{Z}^*(\boldsymbol{\gamma}_1^*)'\mathbf{Z}^*(\boldsymbol{\gamma}_1^*)\right]^{-1}\mathbf{Z}^*(\boldsymbol{\gamma}_1^*)'\mathbf{G}^*$, with covariance matrix $V\left(\widehat{\boldsymbol{\phi}}\right) = \sigma_\varepsilon^2\left[\mathbf{Z}^*(\boldsymbol{\gamma}_1^*)'\mathbf{Z}^*(\boldsymbol{\gamma}_1^*)\right]^{-1}$. However, γ_1^* is unknown. The estimation procedure (Chan 1993, Hansen 1999, Hansen 2000, Gonzalo and Pitarakis 2002) involves a grid search over all possible values of γ_1^*, while ensuring that a sufficiently large number of observations ($\eta\%$) lie in each regime (η is set equal to 5%). The optimal value of γ_1^* is obtained by minimising the concentrated sum of squared errors, which means choosing the value of γ_1^* that yields the smallest sum of squared errors (SSE) over the grid-searched possible values of γ_1^*. In practice, the sorting is on the observed $\dot{p}_{it}$ with search between the $\eta\%$ and $(1-\eta)\%$ quantile.

It is possible that there may be several such threshold effects. A convenient result is that sequential estimation of the breakpoints is consistent (see Chong 1994, Bai 1997, Bai and Perron 1998, Hansen 1999, Hansen 2000, Gonzalo and Pitarakis 2002). This suggests a procedure to estimate multiple breakpoints:

estimate the single threshold point; fix the first stage estimate at $\widehat{\gamma}_1^*$; conditional on this estimate, repeat the procedure to find $\widehat{\gamma}_2^*$; with both $\widehat{\gamma}_1^*$ and $\widehat{\gamma}_2^*$ treated as fixed, repeat the procedure to find $\widehat{\gamma}_3^*$; continue for $m = 1, \ldots, M$ possible breakpoints. In subsequent grid searches, the range over which to search is reduced so as to ensure a minimum number of observations ($\eta\%$) in each regime.

A.3.1. Model Selection Criteria

Hansen (1999) and Hansen (2000) suggest using bootstrapped versions of likelihood ratio statistics to determine the optimal number of breakpoints. Gonzalo and Pitarakis (2002) alternatively offer an appealing approach of choosing the model which minimises the information criterion (IC) function: $IC(\gamma_1^*, \gamma_2^*, \ldots, \gamma_m^*) = \ln SSE(\gamma_1^*, \gamma_2^*, \ldots, \gamma_m^*) + \frac{\omega_S}{S}[k^*(m)]$, where S is the sample size, k^* is the number of freely estimated response parameters that in turn are functions of m, and ω_S is a penalty term, typically a function of the sample size.[5] Gonzalo and Pitarakis (2002) suggest that $\omega_S = \ln(S)$, which corresponds to a Bayesian Information Criteria, performs the best.

This procedure can be adapted to the panel data by letting $S = NT$, with $k^*(m)$ reflecting the reduction in degrees of freedom involved in the panel estimation. This involves a loss of $N-1$ degrees of freedom for a fixed effects one-way model and of $(N-1)(T-1)$ degrees of freedom for a fixed effects two-way model.

A.3.2. Time effects

As it currently stands, equation (A.1), or its multiple regime counterpart, is inconsistent with equation (4.3) due the former's omission of the time, or business cycle, effects of λ_t.[6] Time effects can be incorporated into the threshold procedure described above. The relevant data transformations for a typical element of $\mathbf{x}$ are $x_{it}^* = x_{it} - \overline{x}_{i.} - \overline{x}_{.t} + \overline{x}$, with the appropriate definition of the time, individual and

[5] For example, if the total number of explanatory variables, including the inflation variable is denoted k, and $m = 0$, then $k^*(m) = k$, for $m = 1$, $k^*(m) = k+1$ and for $m = 2, k^*(m) = k+2$.

[6] In the baseline model, which is an unsplined specification, there is a clear rejection of both of the null hypotheses: $H_0 : \lambda_t = 0$ and $\alpha_i = 0$, for all t, i and $H_0 : \lambda_t = 0$, for all t.

overall mean variables. Applied to the splined variable ($\dot{p}_{it}$), it needs to be determined if there are unobserved time and/or country effects present in the data, and what are the optimal number of breakpoints in the inflation-growth profile. Devising such a testing procedure is complicated since for example a two-way fixed effects panel model can yield a different optimal value of m as compared to a simple OLS model. An alternative approach is to use the Information Criteria procedure suggested by Gonzalo and Pitarakis (2002) to choose both across m and among estimation technique (OLS, one- and two-way models), once appropriate degrees of freedom corrections have been made to $k^*(m)$. That is, fix M; estimate for $m = 0, \ldots, M$ the model by each of the three estimation procedures; and for each model estimation calculate the $IC(\gamma_1^*, \gamma_2^*, \ldots, \gamma_m^*)$. Finally, choose the optimal model with regard to the m number of breakpoints and the estimation procedure that yields the smallest value of the $IC(\gamma_1^*, \gamma_2^*, \ldots, \gamma_m^*)$.

A.3.3. Tied versus Untied Splines

The inflation-growth relation with thresholds may be assumed to be piecewise continuous or allowed to be discontinuous at the spline knot. To force the relationship to be continuous, as is made explicit in Tables 4.2-4.5, it is possible to follow Greene (2003), p.122, and re-define $\dot{\mathbf{p}}_{it}(\gamma_1^*)$ as $\dot{\mathbf{p}}_{it}(\gamma_1^*) = [\dot{p}_{it}, (\dot{p}_{it} - \gamma_1^*) \times 1(\dot{p}_{it} > \gamma_1^*)]$.

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