Operating Leverage, Stock Market Cyclicality, and the Cross-Section of Returns

Job Market Paper

François Gourio*

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Abstract

I use a putty-clay technology to explain several asset market facts. The key mechanism is as follows: a one percent increase in sales leads to a more-than-one percent increase in profits, since labor costs don't move one-for-one. This amplification is greater for plants with low productivity for which the average profit margin (sales minus costs) is small. This "operating leverage" effect implies that low productivity plants benefit disproportionately from business cycle booms. These plants have thus higher systematic risk and higher average returns. This model can help explain the empirical findings of Fama and French (1992), and more generally the sources of differences in market betas across firms. I obtain supporting evidence for the mechanism using firm- and industry-level data. The aggregate effect follows from trend growth: low-productivity plants outnumber high-productivity plants, making the aggregate stock market procyclical. I examine these aggregate implications and find that this model generates a volatile stock market return that predicts the business cycle.

*Graduate Student, Department of Economics, University of Chicago. Email: francois@uchicago.edu. An appendix is available on my web page, http://home.uchicago.edu/~francois. I thank the members of my committee Fernando Alvarez, John Cochrane, and Anil Kashyap for their support and guidance, and I owe a special debt to my chairman Lars Hansen for his advice and his encouragement. I received useful comments from participants in several University of Chicago workshops, and from many people, especially Jeff Campbell, Jonas Fisher, Boyan Jovanovic, Hanno Lustig, Pierre-Alexandre Noual, Monika Piazzesi, Adrien Verdelhan and Pierre-Olivier Weill. All errors are mine. First draft of this project: June 2003.

1 Introduction

Ever since Fama and French (1992) showed that firms with high ratios of book value to market value have high average returns, the economic interpretation of their finding has remained elusive. Is book-to-market really an indicator of firm riskiness, if so why? In this paper, I consider the asset pricing implications of a putty-clay technology: capital and labor are substitutable only ex-ante (i.e., before capital investment is done). I show that under this technology, book-to-market differences reflect differences in labor productivity, which result in different exposures to business cycle risk. From a more general standpoint, the paper offers a theory of why different firms have different expected returns by linking a firm's financial risk with its real, observable attributes (or characteristics). This technology-based interpretation of stock returns also has some interesting aggregate implications: it is consistent with the fact that the return on the stock market is volatile and forecasts the business cycle.

The key idea that I formalize is the "operating leverage". Since costs move less than one-for-one with output, an increase in output will result in a more than one-for-one increase in profits. This amplification effect makes aggregate profits more cyclical than GDP; but the strength of the amplification differs across firms. It follows that *high productivity firms are less responsive to aggregate shocks*, i.e. less cyclical. This will be reflected in prices: *the relative value of productive firms falls in booms.* The notion of productivity here is labor productivity. Having a relatively productive plant is more valuable in recessions than in booms, because in recessions aggregate productivity declines are larger than wages declines, making labor relatively expensive as compared to capital. Conversely, in booms capital is scarce and labor is relatively cheap, so that labor productivity is less advantageous. This explanation relies on the well-known fact that wages don't move as much as labor productivity over the business cycle. The theory is also consistent with the Schumpeterian view that the least productive suffer more from recessions.

The logic of asset pricing based on macroeconomic risk implies that low productivity firms, which are more procyclical, are more risky, and thus earn higher average returns. The model rationalizes the correlation between book-to-market ratios and return by noting that productive firms will tend to have a lower book-to-market, so that returns and book-to-market will be positively correlated. The return differentials across assets are exactly justified by corresponding differences in betas.

After developing the model and analyzing its asset pricing implications, I proceed to test this theoretical explanation. I show that the book-to-market ratio is systematically related across firms to productivity and operating leverage. I also show that the pattern of cyclicality of operating income is close to the one predicted by the model: high book-to-market firms are more sensitive to GDP changes and to labor compensation changes than low book-to-market firms, and these estimates are of the order of magnitude predicted by the model. I also provide some supportive patterns at the industry level.

When the relative value of productive firms falls, the value of the aggregate stock market changes as well. How can a relative price effect change the aggregate value of the capital stock? This is because trend growth in the economy makes installed capital on average less productive than new investment, and thus the stock of capital resides predominantly with less productive firms. Hence, when the value of low productivity firms increases relative to the value of high productivity firms, the overall stock market rises. This revaluation mechanism is an alternative to the widely used adjustment cost formulation (Cochrane 1991, Hall 2001). I show that the mechanism can account relatively well for the business-cycle frequency movements in the stock market: at the onset of recessions when capital is relatively abundant, the relative value of productive units increases, which drives the stock market down. When expansions begin and labor is relatively abundant, the low-productivity units become more valuable, which drives the stock market up. The stock market return is a leading indicator of macroeconomic growth in the model and in the data.

Organization

The next section is an overview of the main ideas and results of the paper. Section 3 presents the model and Section 4 derives the asset pricing implications. Section 5 looks at firm- and industry-level evidence, and Section 6 discusses macroeconomic evidence. Section 7 concludes.

2 Overview and main results

Although I develop a full dynamic stochastic general equilibrium model, the basic mechanism is easily demonstrated in a static example. Consider the operating income of a firm with fixed capital K and a standard constant-return-to-scale (CRS) production function F, facing aggregate total factor productivity (TFP) A and wage w. Labor can be varied and profit is $\pi(K, A, w) = \max_{N \geq 0} \{AF(K, N) - wN\}$. This profit is the value of the firm (its capital stock and its technology) in a one-period world. The percentage increase in value in response to a one-percent increase in aggregate TFP A is obtained using the envelope theorem, assuming for now that the wage does not respond to a change in A :

$$\varepsilon_{\pi,A} = \frac{d\log\pi}{d\log A} = \frac{1}{s_K} \tag{2.1}$$

where s_K is the capital share of output. The value of a firm with low s_K is more sensitive to shocks to A. This is because firms translate a one-percent increase in sales into a more than one-percent increase in their profits, and this amplification is bigger for firms with a lower capital share.[1] For more productive firms, the profit $AF(K,N) - wN$ is a larger share of sales, and thus the amplification is smaller.

This mechanism captures the notion of "operating leverage": since operating costs do not move as much as sales, profits are more volatile than GDP. This amplification is larger for firms with lower productivities, since in this case the leverage is bigger.[2] Hence, the model gives a theory of why different firms have different exposures to aggregate risk.

I first use this theory to understand the correlation between average returns and book-to-market. Book-to-market is computed as the ratio of book value (assets minus liabilities, including debt) to the market value of equity. This measure is close to the inverse of Tobin's q, the ratio of the market value to the replacement value.[3] Fama and French (1992) and many others have noted that the high book-to-market firms tend to have large average returns, which are apparently not justified by their risk (as measured by the CAPM). This finding has attracted a lot of attention, and is widely perceived as a puzzle. Hence, understanding if these firms are risky investments, and if so why, is an important research question.

My model explains this correlation, because both in the data and in the model, productivity is negatively correlated with book-to-market. In the model, firms that draw good idiosyncratic productivity shocks have a high market value relative to their book value, which records past investment. Hence, high productivity firms will have a low book-to-market. As a consequence, book-to-market and expected return will be positively correlated through productivity. Model

[1] An operating leverage effect could also generated by fixed costs, but as I show it is enough that costs react less than one-for-one to an increase in sales. Some applied economists emphasize this effect: "operating leverage [is] at work: as sales have accelerated and have covered fixed costs, much of the top-line improvement has gone straight to the bottom line" (Richard Berner, Morgan Stanley Economic Forum, 12/1/03; www.msdw.com/gef).

[2] The idea of operating leverage has not been much studied in the academic literature, though there is some related work on the cyclicality of the capital share (Gomme and Greenwood (1995), G. Hansen and Prescott (2000)).

[3] The difference is that Tobin's q adds the market value of debt to the market value of equity on the numerator, and correspondingly takes out the debt liabilities on the denominator. Previous empirical work suggests that this change in construction does not matter very much, because the two measures are highly correlated.

simulations reveal that the effect can be significant.

Next, I test empirically the proposed explanation for these cross-sectional facts. I examine the model's prediction that high book-to-market firms have a higher operating leverage, using as a proxy for operating leverage the inverse capital share; this prediction holds in the data. I then show that the cash flows (the operating income) of the high book-to-market firms is more cyclical. More precisely, in the model, one can decompose changes in firm-level operating income into changes in aggregate TFP and changes in the aggregate wage; denoting x the labor productivity of the firm, we have:

$$\Delta OI_t(x) = \underbrace{\frac{A_t x}{A_t x - w_t}}_{\frac{1}{s_K(x)}} \Delta A_t + \underbrace{\frac{-w_t}{A_t x - w_t}}_{1-\frac{1}{s_K(x)}} \Delta w_t, \tag{2.2}$$

where ΔOI_t is the % change in operating income, ΔA_t the % change in aggregate TFP, and Δw_t is the % change in the aggregate wage, and $s_K(x)$ is the capital share of a firm with productivity x. Equation (2.2) makes the following predictions: (i) if we take the average capital share to be 1/3, the coefficient on TFP growth should be on average 1/(1/3)=3; (ii) high book-to-market firms, which have low productivity x, should have a higher coefficient in front of TFP growth; (iii) and inversely, the coefficient on wage growth should be decreasing in book-to-market. I run this regression for each portfolio (i.e. groups of firms sorted by book-to-market; with GDP instead of TFP, see the details in Section 5). Perhaps surprisingly in light of the previous literature, the estimates actually give some support to the model: a 1% increase in GDP leads to a 3% increase of operating income for the whole sample of firms, but the effect differs across firms. The operating income of the the high book-to-market firms rises by 6% whereas it rises by only 1.5% for the low book-to-market firms. Finally, the coefficient on wage growth, though not monotonic, is more negative for higher book-to-market portfolios. These results are illustrated in figure 1 below, which plots the coefficient estimates on GDP growth, for each portfolio. Hence, the model captures an economic reason why high book-to-market firms are more risky.

Finally, I examine the aggregate, time-series implications of the model. Low-productivity plants outnumber high-productivity plants: trend growth in the economy makes installed capital on average less productive than new investment, and thus the stock of capital resides predominantly with less productive firms. (To put it another way, low-productivity here means a productivity lower than marginal plants, which are the ones we are building today - the only margin of action possible in this simple setup.) Hence, when the value of low productivity firms increases relative to the value of high productivity firms, the overall stock market rises. The

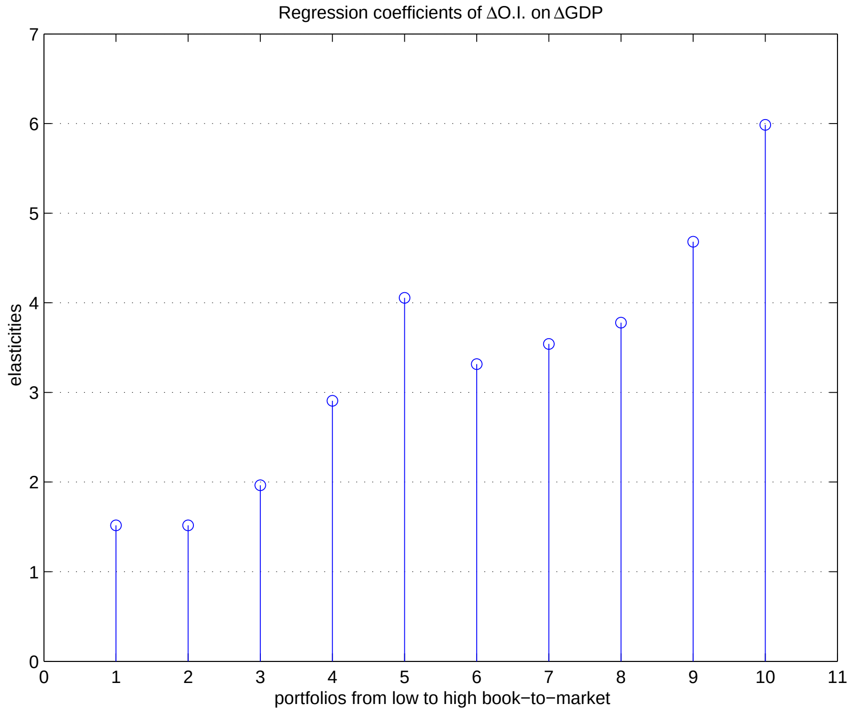

Figure 1: Regression coefficient of Operating Income Growth on GDP Growth for each of 10 portfolios, from low to high book-to-market.

model gives thus a theory of return volatility, and can also generate the fact that the stock market return leads the business cycle. Empirically, I use aggregate data to infer the value of the stock market. More precisely, the model gives a simple formula for the value of the aggregate stock of capital: $V_t = \left(Y_t/(A_t i_t^{\alpha}) - (1-\alpha)N_t\right) i_t/\alpha$, where N_t is hours worked, Y_t output, A_t is TFP, α the average capital share, and i_t is the capital intensity of new investment. This formula is akin to the one which the adjustment cost model (the standard $q-$theory) delivers: $V_t = q_t K_t$ and $q_t = 1 + c'(I_t/K_t)$. I use my new formula to extract the stock market value from some macroeconomic time series. The result, shown in figure 2, suggests that the model captures the business cycle frequency movements in returns. The correlation of these two series is 0.64. On this dimension the model performs better than a standard adjustment cost model.

3 A Putty-Clay model

The model is a simplification of the model in Gilchrist and Williams (2000). The exact relation to their model is explained in a subsection below. Gilchrist and Williams used this specific technology to address some failures of real-business-cycle models. While they looked only

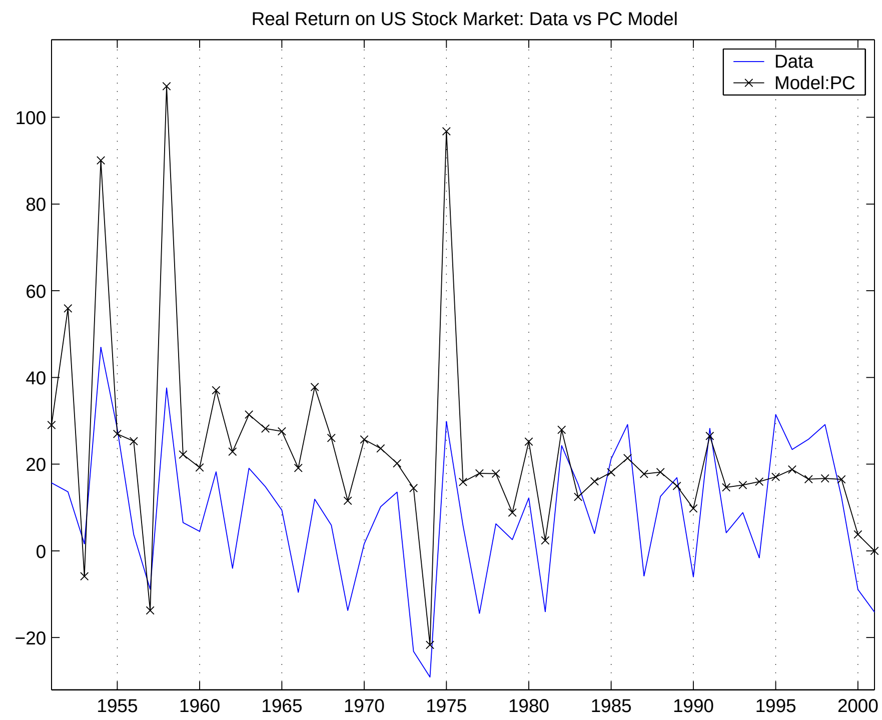

Figure 2: Real return on U.S. Stock Market, Data and Putty-Clay (PC) model

at quantities, I will concern myself primarily with asset prices. The simplification I use serves two main purposes. First, it highlights the key mechanism of the model, and allows to derive some asset pricing results analytically in Section 4A. Second, it simplifies the numerical computation of the equilibrium, by reducing the size of the state space.

A. Technology

Production occurs in a continuum of plants, which will turn out to have different productivities. I assume constant return to scale at the plant level; hence, there is no loss of generality in setting the size of the labor force of each plant to one worker. There is an ex-ante standard Cobb-Douglas production function, with plant output equal to $y_t = A_t \mu i^{\alpha}$, where A_t is an aggregate TFP process with a trend and some persistent shocks, which will be specified below; μ is a permanent idiosyncratic shock; and i is the capital per worker. (This is a standard Cobb-Douglas expressed in per capita terms.) When the plant is created, the capital per worker i is chosen first, then μ is drawn randomly, but these variables remain fixed thereafter. As a consequence, the production function is ex-post Leontief, since capital and labor cannot be substituted - there is a fixed amount of capital for one worker. Let $x = i^{\alpha}\mu$ be the labor productivity of the plant (divided by TFP); we have $y_t(x) = A_t x$.

More precisely, new plants are built each period; but in contrast to the standard neoclassical model where new investment is simply a matter of choosing the quantity of capital to build, this model introduces an extra choice, the capital intensity of the new investment. Each period, each new plant decides once and for all its capital per worker, or capital intensity, i_t. (Since we normalize the size to be one worker, i_t is also the total investment required to build the plant.) Once this choice is made, each plant draws a permanent idiosyncratic productivity shock μ with c.d.f. $H(.)$ and mean $\mathbb{E}\mu = 1 = \int_0^\infty t dH(t)dt$. This yields a plant of productivity[4] $i_t^\alpha \mu$. The average productivity of new plants built at time t is thus $x_t = \mathbb{E}_\mu (i_t^\alpha \mu) = i_t^\alpha$. Investment is irreversible. Moreover, it is not possible to increase ex-post the capital of a given plant: hence, one cannot take advantage of a good draw of μ to increase the size of the plant. Rather, an increase in investment must take the form of new plants creation, which will draw their own new μ.[5]

Let h_t be the number of new plants built at time t. These plants come into operation in the following period. Moreover, plants die at rate δ: hence, depreciation is a matter of plants disappearing, not a shrinkage of capital within each plant. Hence, we obtain the following law of motion for the measure G_t of plants with productivity less than x:

$$G_{t+1}(x) = (1-\delta)G_t(x) + h_t H\left(\frac{x}{x_t}\right), \tag{3.1}$$

i.e. plants of productivity less than x at time $t+1$ are plants of productivity less than x at time t that did not depreciate, plus new plants: these were designed in quantity h_t with average productivity x_t but $H(x/x_t)$ happened to draw $\mu \leq x/x_t$ and end up with a productivity μx_t, less than x. In the end, the distribution of productivity G_t will exhibit dispersion for three reasons: first, the idiosyncratic shock μ; second, the trend imparted by growth to the average productivity x_t: capital deepening (or embodied progress, see footnote 4) makes recently built units more productive; and third, stochastic variations around the trend in x_t. Since the heterogeneity in

[4] It is natural in this setup to introduce embodied technology: the productivity of new plants is then $i_t^\alpha \mu B_t$ where $\log B_t = \mu_b t$. This does not prove very useful for my asset pricing work, so for simplicity I withdraw this source of growth and business cycles. See Christiano and Fisher (2003), Fisher (2002), Gilchrist and Williams (2000) and Greenwood, Hercowitz and Krusell (2001) for evaluations of the business cycle consequences of embodied technology shocks.

[5] As can be deducted from the results of section 3, it is possible to obtain similar results without the productivity shock μ. However, in this case: (1) cohort effects will explain all of productivity variation, and of book-to-market variation, and (2) the cross-sectional distribution over productivity is degenerate. To avoid these weaknesses, and since adding μ leads to no loss in tractability, I consider μ to be important. (See the discussion on the empirical evidence for μ in the text.)

productivity is central to the model, it is noteworthy that a large recent literature in industrial organization has measured large differentials of productivity between establishments, even within 4-digit industries (Bartelsman and Doms (2000)). As an example, Syverson (2004) computes that within such an industry, a plant in the 25th percentile of labor productivity is nearly twice as productive as a plant in the 75th percentile.

There are no adjustment costs along the extensive margin, i.e. the number of new plants opening: hence, a free-entry condition determines the quantity h_t of new plants. The key variable in the model is i_t, the capital intensity of new investment, so let us pause to understand how it is chosen. This capital intensity choice is dictated by the following trade-off: a higher capital intensity i_t requires a higher initial investment, but decreases future costs per unit of output - that is, future labor costs will absorb a lower share of output. Hence, depending on expected future interest rates, TFP and wages, firm will choose the cost-minimizing i_t. Since plants are identical ex-ante, and make the same forecasts regarding future wages, TFP and interest rates, they have no reason to choose different i_t, and thus all end up with the same choice. Of course, because of the idiosyncratic shock μ, some plants get a higher productivity, and some a lower productivity, than $x_t = i_t^{\alpha}$.

Aggregating across plants yields total output Y_t and total hours worked N_t :

$$Y_t = \int_0^{\infty} A_t x \, dG_t(x) \tag{3.2}$$

$$N_t = \int_0^{\infty} dG_t(x). \tag{3.3}$$

Since $G_t(.)$, the quantity of capital of each productivity, is predetermined, output is predetermined, up to the TFP shock to A_t, and so are hours. Within the period, labor demand is fully inelastic. As usual, I assume that the labor market clears, with the wage moving to induce supply to adjust to the fixed demand.

Define $\widetilde{Y}_t = \int_0^{\infty} x dG_t(x) = Y_t/A_t$ the total productive capacity of the economy, not taking into account TFP. Using the law of motion (3.1) of the distribution $G_t(.)$, we can find the law of motion for $\widetilde{Y}_t$:

$$\begin{aligned}
\widetilde{Y}_t &= \frac{Y_t}{A_t} = \int_0^{\infty} x dG_t(x) \\
&= \int_0^{\infty} x \left((1-\delta) dG_{t-1}(x) + h_{t-1} dH\left(\frac{x}{x_{t-1}}\right)\right) \\
&= (1-\delta) \int_0^{\infty} x dG_{t-1}(x) + h_{t-1} \int_0^{\infty} x dH\left(\frac{x}{x_{t-1}}\right)
\end{aligned}$$

$$= \quad (1-\delta)\widetilde{Y}_{t-1} + h_{t-1}x_{t-1}, \tag{3.4}$$

where the last line used that $\mathbb{E}\mu = \int_0^\infty t dH(t) = 1$. Similarly for N_t:

$$\begin{aligned} N_t &= \int_0^\infty dG_t(x) \\ &= \int_0^\infty \left((1-\delta)dG_{t-1}(x) + h_{t-1}dH\left(\frac{x}{x_{t-1}}\right)\right) \\ &= (1-\delta)\int_0^\infty dG_{t-1}(x) + h_{t-1}\int_0^\infty dH\left(\frac{x}{x_{t-1}}\right) \\ &= (1-\delta)N_{t-1} + h_{t-1}, \end{aligned} \tag{3.5}$$

where the last line uses that $dH(.)$ is a density.

The distribution $G_t(.)$ disappears from our problem since all its effects are summarized in the state variables $\widetilde{Y}_t$ and N_t: these two variables together tell us the total production capacity and the total number of plants (Here and everywhere, the number of plants is the number of their employees). This is a payoff for the simplification I make to the setup of Gilchrist and Williams.

B. Preferences, Resource Constraint, Planner Problem

The model is closed with a representative household who has preferences $\mathbb{E}_0\left(\sum_{t\geq 0}\beta^t U\left(c_t, 1-N_t\right)\right)$. The resource constraint is $c_t + h_t i_t \leq Y_t$. Note that $h_t i_t$ is the aggregate investment: there are h_t new plants with i_t units of capital per new job. Finally, growth occurs through growth in TFP A_t, i.e., ways of using existing plants more efficiently. I assume a deterministic trend, and AR(1) deviations from the trend:

$$\begin{aligned} \log A_t &= \mu_a t + \varepsilon_t^a \\ \varepsilon_t^a &= \rho_a \varepsilon_{t-1}^a + u_t^a, \end{aligned}$$

with the innovation u_t^a iid $N(0, \sigma_a^2)$.

In the end, the model can be solved using the following planning problem:

$$\max_{\left\{c_t, i_t, h_t, \widetilde{Y}_{t+1}, N_{t+1}\right\}_{t=0}^\infty} \mathbb{E}\left[\sum_{t=0}^\infty \beta^t U(c_t, 1-N_t)\right]$$

$$s.t.: c_t + i_t h_t \leq A_t \widetilde{Y}_t$$

$$\begin{aligned} \widetilde{Y}_{t+1} &= (1-\delta)\widetilde{Y}_t + i_t^\alpha h_t \\ N_{t+1} &= (1-\delta)N_t + h_t \end{aligned}$$

$$\widetilde{Y}_0, N_0 \text{ given}$$

First-order conditions are easily obtained (see appendix 2). The numerical techniques used to find the equilibrium and to compute asset prices are discussed in appendix 4.

Comparison with Gilchrist and Williams and Business Cycle Implications

In their model, Gilchrist and Williams (2000) consider the possible decision to shut down a plant, i.e. not hire a worker, if it becomes unprofitable, whereas I do not take this into account. As a result, the mechanisms I emphasize do not rely on varying utilization.[6] On the other hand, one might worry that some plants are operating despite being unprofitable in my version. (firms that draw a very low μ will still operate despite making losses.) In some cases, there are no such plants: my simplification is exactly true. In some other cases, it will be a good approximation to the Gilchrist and Williams model where plants choose to open or close each period.[7] Alternatively, my model can be viewed as building on the opposite assumption than Gilchrist and Williams: no closures are allowed, whereas in their model switching between opening and closing is instantaneous and costless. As a result from the similarity between the models, business-cycle results close to those of Gilchrist and Williams (2000) are obtained in this model. These findings are documented in appendix 3, where I also show that the number of unprofitable plants in my model is small for the parameter values I choose.

4 Asset Pricing

This section first gives some analytical results that develop the intuition given in the introduction, then offers some numerical simulations.

A. Analytical results

Since each plant is a particular capital good, characterized by its labor productivity, I start by pricing each one separately. I then obtain the aggregate implications - the return on a diversified portfolio of plants - by summing over the existing stock of plants. A plant of productivity x will yield cash-flows $D_t(x) = A_t x - w_t$ where w_t is the wage and A_t is TFP. The ex-dividend price is

[6] In the model with variable utilization, because units are kept indefinitely and switched on/off at no cost, the value of a plant has an important option component to it. (Indeed, the price as a function of productivity is convex, whereas it will be linear here.) I do not study these effects, which may be interesting.

[7] The simplification is the exact solution if the technology growth rates are zero, the depreciation rate is large enough, the aggregate shocks have a small variance, and if the distribution h has a narrow support around its unit mean; the simplification is a good approximation for not-too-big changes in the parameters from these values.

the present-value of cash-flows, discounted using a *stochastic discount factor*[8] $m_{t,t+j}$:

$$\begin{aligned}
P_t(x) &= \mathbb{E}_t\left(\sum_{j\geq 1} m_{t,t+j}\,(1-\delta)^{j-1}\,D_{t+j}(x)\right) \\
P_t(x) &= x\cdot\mathbb{E}_t\left(\sum_{j\geq 1} m_{t,t+j}\,(1-\delta)^{j-1}\,A_{t+j}\right) - \mathbb{E}_t\left(\sum_{j\geq 1} m_{t,t+j}\,(1-\delta)^{j-1}\,w_{t+j}\right) \\
P_t(x) &= xv_{1t} - v_{2t}, \qquad (4.1)
\end{aligned}$$

where the last equation defines v_{1t} and v_{2t} as the present discounted values of TFP and the wage. This formula reveals that plants with different productivities x will have different sensitivities to aggregate shocks: these prices are all driven by the same aggregate variables v_{1t} and v_{2t}, but plants with different x will react differently to changes in v_{1t} and v_{2t}. I show below that v_{1t} and v_{2t} can both be expressed solely as a function of the variable i_t, the capital intensity of new plants. In the terminology of asset pricing, i_t is thus an asset pricing factor.[9]

Comparison to other technologies and related literature

Before developing in more detail the implications of (4.1), it may be useful to see which assumptions drive this representation, and how it relates to other possible technological assumptions (but some readers may prefer to skip this section on first reading and proceed directly to the analysis of my model). First, notice that in a model with fixed capital, i.e. $k_{t+j} = k(1-\delta)^{j-1}$, and fixed idiosyncratic shock x, but a fully adjustable capital-labor ratio (i.e., labor) in a Cobb-Douglas production function, the price would be

$$\begin{aligned}
P_t(x,k) &= \mathbb{E}_t\left(\sum_{j\geq 1} m_{t,t+j}\left\{\max_{n_{t+j}} A_{t+j}xk_{t+j}^{\alpha}n_{t+j}^{1-\alpha} - w_{t+j}n_{t+j}\right\}\right) \\
&= \mathbb{E}_t\left(\sum_{j\geq 1} m_{t,t+j}\zeta k(1-\delta)^{j-1}\left(A_{t+j}x\right)^{\frac{1}{\alpha}} w_{t+j}^{\frac{\alpha-1}{\alpha}}\right),
\end{aligned}$$

where ζ is a constant; we can write $P_t(x,k) = x^{1/\alpha}v_t$ and firms with different x would have the same sensitivities to aggregate shocks. Hence, the representation (4.1) breaks down with a simple Cobb-Douglas production function. However, this will not be true under more general

[8] A stochastic discount factor is a variable that adjusts payoffs for the times and states in which the payoffs are paid, to reflect impatience and risk-aversion. In complete markets, the stochastic discount factor equals the ratio of the state-contingent Arrow-Debreu price to the probability of the state. In our case, the stochastic discount factor will also equal the ratio of marginal utilities of consumption: $m_{t,t+j} = \beta^j U_c(c_{t+j}, 1-N_{t+j})/U_c(c_t, 1-N_t)$.

[9] This result breaks down when embodied-technology shocks are added; in this case, v_{1t} and v_{2t} are two non-redundant factors.

CRS production functions. Thus, my result and central argument does not *require* putty-clay, and it seems that the argument will hold with a low, positive substitutability between capital and labor. The putty-clay formulation however is simpler and delivers sharper results.[10] The model features thus constant capital and labor at the plant level. While the full structure of the model will be exploited for the aggregate results, it seems that the cross-sectional results do not rely on the putty-clay assumption per se: the fact that labor is fixed in particular is not important. What is key is that the cost of labor moves less than the average productivity in response to a shock. More generally, one could envision having fixed as well as variable costs: what really matters is that costs do not change much *over time* (for a marginal shock). The fixed capital assumption is more important, but I conjecture that similar results would hold under strong adjustment costs. (The literature, up to now, relies on strong adjustment costs to prevent the reallocation of capital towards the most efficient units.)

Adjustment costs

In my model, firms cannot add capital by extending productive plants: they must set up new plants which will draw new productivity shocks; hence it is not really possible to take advantage of a good productivity. This is a strong form of adjustment costs - but on the other hand, there are *no* adjustment costs along the new plants margin. To go beyond this fixed capital formulation, the natural idea is to introduce adjustment costs, which will relate book-to–market to risk through investment patterns. First, note that heterogeneity in capital, without heterogeneity in productivity, will not help, since in this case again all firms' values move in step. To see this, write the Bellman equation

$$V(K,z) = \max_I \left\{ Kh(z) - I\phi\left(\frac{I}{K}\right) + \mathbb{E}_{z'/z}\left(m(z,z')V(K',z')\right) \right\},$$

where z is an aggregate Markov state; K is capital; m is a stochastic discount factor; and the profit function $Kh(z)$ is linear in capital under constant returns and perfect competition. Hayashi's theorem (1982) implies that we can write $V(K,z) = Kg(z)$, and thus $d\log V(K,z)/d\log z$ is independent of K. An obvious remedy is to add idiosyncratic shocks, and write the problem (still assuming a fixed x) as

$$V(K,z,x) = \max_I \left\{ Kh(z,x) - I\phi\left(\frac{I}{K}\right) + \mathbb{E}_{z'/z}\left(m(z,z')V(K',z',x)\right) \right\}.$$

Again $V(K,z,x) = Kg(z,x)$, but now unless g is multiplicatively separable, i.e. unless g can be written as $g(z,x) = g_1(z)g_2(x)$, firms with different productivities will have different exposures

[10] Note that the aggregate effects that will be derived below rely on the whole structure of the model and are less robust to model extensions.

to aggregate states z. This model can potentially generate the book-to-market effect: high productivity firms have higher price and lower book-to-market, and may be more risky. A variant of this model, in partial equilibrium, has been studied by Zhang (2004). One hindrance to studying this model in general equilibrium is a "curse of dimensionality" that arises since it is necessary to keep track of the whole cross-sectional distribution. (Also, the limited success of the firm-level q-theory embedded in this model may have discouraged researchers.) Kogan, Gomes and Zhang (2003) and Gala (2004) have studied related models with adjustment costs where additional assumptions allow to break the curse of dimensionality and to justify the size effect (the tendency for smaller firms to have larger average returns than a CAPM risk adjustment predicts). Carlson et al. (2004) emphasized option values, and Cooper (2003) studied a model with fixed costs to adjusting the capital stock.

Cross-sectional implications of my model

I now return to the analysis of (4.1). To examine the sensitivities of the different plants to aggregate shocks, I compute the effect of a shock on the price, i.e. for a TFP shock: $\varepsilon_{P(x),A} = d\log P_t(x)/d\log A_t$, which gives the percentage change in price[11] in response to a one-percent shock to A_t. (Thereafter, I use systematically $\varepsilon_{x,y}$ to denote the elasticity of x with respect to y). Clearly,

$$\begin{aligned} \varepsilon_{P(x),A} &= \frac{xv_{1t}}{xv_{1t}-v_{2t}}\varepsilon_{v1,A} + \frac{-v_{2t}}{xv_{1t}-v_{2t}}\varepsilon_{v2,A} \\ &= s_t(x)\varepsilon_{v1,A} + (1-s_t(x))\varepsilon_{v2,A}. \end{aligned} \tag{4.2}$$

The number $s_t(x)$ gives the sensitivity of a plant of productivity x to a change in the present value of TFP v_{1t}, and $1 - s_t(x)$ is its sensitivity to a change in the present value of wages v_{2t}. This number measures the operating amplification effect discussed in the introduction, i.e. the amount by which the increase in the present value of TFP is amplified in an increase in profits. It is immediate that $s_t(x) > 1$, $s_t'(x) < 0$, and $s_t(x) \to +\infty$ if the price of the plant is near 0. Clearly plants with high productivity x will not be sensitive to wage movements, since they do not require much labor; their profits will move approximately one-for-one with TFP, and their prices one-for-one with the present value of TFP v_{1t}. On the other hand, low x plants will be much exposed to changes in the cost of labor, with $s_t(x) >> 1$, and will thus amplify considerably the effects of a change in TFP. In appendix 6, I show that with constant risk premia and under some

[11] The return on holding a plant of productivity x is $R_{t,t+1}(x) = ((1-\delta)P_{t+1}(x) + D_{t+1}(x))/P_t(x)$. The price change / capital gain is the main part of the return empirically. Hence $d\log P_t(x)/d\log A_t$ is a good approximation to $dR_{t,t+1}(x)/d\log A_t$.

simplifying assumptions, $s_t(x) \simeq A_t x/(A_t x - w_t)$ is the inverse of the capital share: we obtain thus again approximately the measure of operating leverage presented in the static model discussed in the overview section. (In Section 5, I will use this as an empirical measure of operating leverage.)

To obtain a more precise characterization of how the prices of plants of different productivities react to aggregate shocks, one needs to use the two conditions that govern the creation of new plants.[12]

- First, when building a new plant, each investor chooses the capital intensity i_t of his plant, to obtain the highest expected value of the plant, net of the investment cost; the expectation is taken over μ, which is unknown at the time of investment:

$$i_t \in \arg\max_i \{\mathbb{E}_\mu P_t (i^\alpha \mu) - i\}$$

$$\Rightarrow i_t \in \arg\max_i \{i^\alpha v_{1t} - v_{2t} - i\},$$

  where the second line uses our expression (4.1) for the price of a project, the linearity in x of the price, and the fact that $\mathbb{E}_\mu \mu = 1$. The first-order condition yields

$$v_{1t} = \frac{i_t^{1-\alpha}}{\alpha}. \tag{4.3}$$

- Next, we use the free-entry condition that the (expected) price of the plants that are built today equals their cost:

$$\begin{aligned} \mathbb{E}_\mu P_t (i_t^\alpha \mu) &= i_t \\ i_t^\alpha v_{1t} - v_{2t} &= i_t. \end{aligned}$$

  To put it another way, Tobin's q is one for the plants which we are adding today to our capital stock, since there are no adjustment costs along this margin. Combining this equation with (4.3) yields

$$v_{2t} = \frac{1-\alpha}{\alpha} i_t. \tag{4.4}$$

These relations agree with intuition. According to (4.3), when future discounted TFP v_{1t} is high, plants choose a high capital intensity i_t, using capital deepening to take advantage of the future good productivity. Similarly, according to (4.4), when the future discounted wages v_{2t} are high, plants economize on labor by increasing the capital intensity i_t.

[12] These conditions can be found directly by solving the social planner problem mentioned above (see appendix 2). Here I justify these conditions using a market interpretation.

Taking into account these two conditions simplifies our computation of the price sensitivity (4.2):

$$\begin{aligned}
\varepsilon_{P(x),A} &= s_t(x)\varepsilon_{v1,A} + (1 - s_t(x))\varepsilon_{v2,A} \\
&= s_t(x)(1-\alpha)\varepsilon_{i,A} + (1 - s_t(x))\varepsilon_{i,A} \\
&= (1 - \alpha s_t(x))\,\varepsilon_{i,A}. \qquad (4.5)
\end{aligned}$$

Hence we see that plants with different x react proportionately to $\varepsilon_{i,A}$, but the sign and magnitude of the response depends on x. Simple algebra using (4.3) and (4.4) allows us to rewrite (4.5) as the:

- **Result 1:** The sensitivity of the price of a plant with productivity x to a TFP shock is

$$\varepsilon_{P(x),A} = (1 - \alpha s_t(x))\,\varepsilon_{i,A} = \frac{1-\alpha}{\alpha}\frac{x - x_t}{P_t(x)}\varepsilon_{i,A} \qquad (4.6)$$

  where $x_t = i_t^{\alpha}$ is the average productivity of plants built today (today's "optimal productivity"), $\varepsilon_{i,A}$ is the elasticity of i_t with respect to A_t, and $s_t(x) = xv_{1t}/(xv_{1t} - v_{2t})$.

In response to a shock to TFP, the price of plants with a productivity lower than x_t goes up if i_t goes down. The reason for this relation is clear: when the capital intensity i_t of new investment falls, it reflects that low productivity firms becomes relatively more efficient. (This is because investors that have a choice, those who are building new plants, prefer a lower capital intensity and thus a lower productivity - their choice is the only margin of adjustment of the economy, and thus reveals us the value of the existing stock.) As a consequence, the price of the low-productivity plants goes up by more than the high-productivity plants if and only if $\varepsilon_{i,A} < 0$ (since $\partial\varepsilon_{P(x),A}/\partial x = -\alpha s_t'(x)(x)\varepsilon_{i,A}$ from (4.5), and $s_t'(x) < 0$). Note that in this case, the present value of TFP and the wage fall when a TFP shock hits the economy: though TFP is higher, interest rates move up even more.[13]

Of course, the capital intensity of new investment i is itself an endogenous variable, chosen to maximize profits: it balances higher expected discounted wages with higher expected discounted TFP (as explained in the discussion of 4.3-4.4). An increase in i signals that labor is relatively expensive, since new investment takes the form of capital-intensive plants that economize on

[13] This suggests that I will obtain this result for a low IES in my simulation, which is actually true. I also require a high substitability of labor and a nonpermanent shock to get this key condition $\partial i/\partial A$ numerically. Of course, other mechanisms, such as rigid wages, or labor market frictions, could explain why the present value of wages moves less than the present value of TFP.

labor. On the other hand, a low i signals that labor is relatively cheap. And cheap labor makes low productivity plants relatively more attractive: variations in the value of high-productivity vs. low-productivity plants can be traced down to variations in the relative value of labor.

I will concentrate my studies to the case where $\varepsilon_{i,A} < 0$: the optimal capital intensity of new investment falls in booms; this requires some explanations. First, note that this condition does not hold for all parameter values in the model,[14] but I will choose a parametrization such that it does. Why am I drawn to consider this condition? Since $\varepsilon_{v2,A} = \varepsilon_{i,A}$ and $\varepsilon_{v1,A} = (1-\alpha)\varepsilon_{i,A}$, it is equivalent to having $\varepsilon_{v1,A} > \varepsilon_{v2,A}$ i.e. the present-value of TFP changes by more than the present-value of wages. I believe this is the empirically relevant case for business-cycle movements, since the wage is not strongly procyclical: the value of production moves by more than the cost of labor. Another way to see it is to interpret $\varepsilon_{i,A} < 0$ as follows: in booms, the capital intensity of new investment falls, i.e. we try to economize on capital by using more labor, which reflects that labor is relatively cheap. It may seem surprising at first that labor is relatively cheap in booms; however, this makes perfect sense: since the wage does not move as much as average labor productivity, using labor is more attractive in booms than in recessions.[15] In Section 6, I show that an empirical measure of i is indeed strongly countercyclical.

With $\varepsilon_{i,A} < 0$, we obtain the intuition given in the introduction: plants with a low operating income $A_t x - w_t$, i.e. low-productivity plants, are more sensitive to aggregate shocks: $\varepsilon_{P(x),A}$ is decreasing in x. As in the introduction, this results from the relative smoothness of wages. The Consumption Asset Pricing Model that is embedded in this model then implies that low x plants have higher returns. Note two extra predictions from (4.6): the very high x could have a negative risk premium, and the pattern of volatility should be U- shaped, with low x and very high x having higher volatilities.

Taking a step back, the idea that less productive firms suffer more from recessions is intuitive and empirically supported, and it has a long history, associated with Schumpeter (1942). Caballero and Hammour (1994) develop the idea that recessions "clean" the economy of the less productive units. Baily, Bartelsman and Haltiwanger (2001) provide evidence that less productive plants are more cyclical. Bresnahan and Raff (1991) discuss the vivid example of car plants during the Great Depression.

[14] Indeed, there is a tendency for the opposite to occur, since we know that the standard response to a permanent increase in TFP is capital deepening, i.e. an increase in the capital-labor ratio.

[15] Another way to state it, is to look at the complementary factor: capital is scarcer in booms and almost marginally useless in recessions.

Relation with Book-to-market and Tobin's q

I will apply the model to book-to-market sorted portfolios. This is because book-to-market will be strongly negatively correlated with productivity in this model. Book-to-market is defined as $i_{t-j}/P_t(x)$ where i_{t-j} is the capital intensity chosen at the time of investment, which is the investment cost, and $x = i_{t-j}^{\alpha}\mu$ is the productivity obtained as a result. Plants that draw good idiosyncratic shocks μ will have a higher productivity x and thus a higher value $P_t(x)$. (Remember that $P_t(x) = v_{1t}x - v_{2t}$ is always increasing in x.) Since this good idiosyncratic shock is not reflected in their book value i_{t-j}, they will have a low book-to-market ratio.

There is another reason why book-to-market will negatively correlate with productivity, which has to do with the age of the plant. Since productivity, up to TFP, is fixed once and for all in each plant at the construction stage, whereas productivity increases over time in the economy (due to capital-deepening (and embodied technology, if any)), a plant's price will on average fall over time. This occurs because wages grow faster than the plant's productivity. Since the book value is fixed, and the price declines over time, old firms will have a higher book-to-market, in as much as their price fell because they became less productive than the most recent ones. This gives another source of negative correlation between productivity and book-to-market. Because of stochastic fluctuations around the balanced growth path, all these correlations are imperfect.

It is interesting to note that these correlations are also found in the data. Using the LRD plant-level manufacturing data set, Dwyer (2001) found that plants with low TFP or low labor productivity have high book-to-market. Jovanovic and Rousseau (2002) found that old firms have higher book-to-market, where age is measured as time being listed.

Notice that my definition draws a distinction between Tobin's q and the book-to-market ratio: the book-to-market ratio measures the ratio of price to the *investment* cost, not the *replacement* cost.[16] Still, it will prove instructive to compute Tobin's average q for any plant of productivity x as the ratio of the market value $P_t(x)$ to the replacement cost.[17] The replacement cost is

[16] A firm that draws a good productivity shock will be identical ex-post to a firm that chose a high capital intensity. Hence, the replacement cost counts the efficiency units of capital to be installed to replicate this firm, and it will be higher than the book value. Perfect and Wiles (1994) perform comparisons of various measures of Tobin's q and find some differences; in particular, the measure that uses the book value, as opposed to an estimate of the replacement value, on the denominator, leads to different results in some regressions often used in corporate finance. It seems hard, however, to generalize from their results.

[17] Note that the book value measure that Fama and French use is the book value of stockholder's equity, roughly assets minus liabilities. Since liabilities include the debt book value (not market value) this could create a measurement problem. (Debt market value is hard to find.) Previous empirical work however suggests that debt

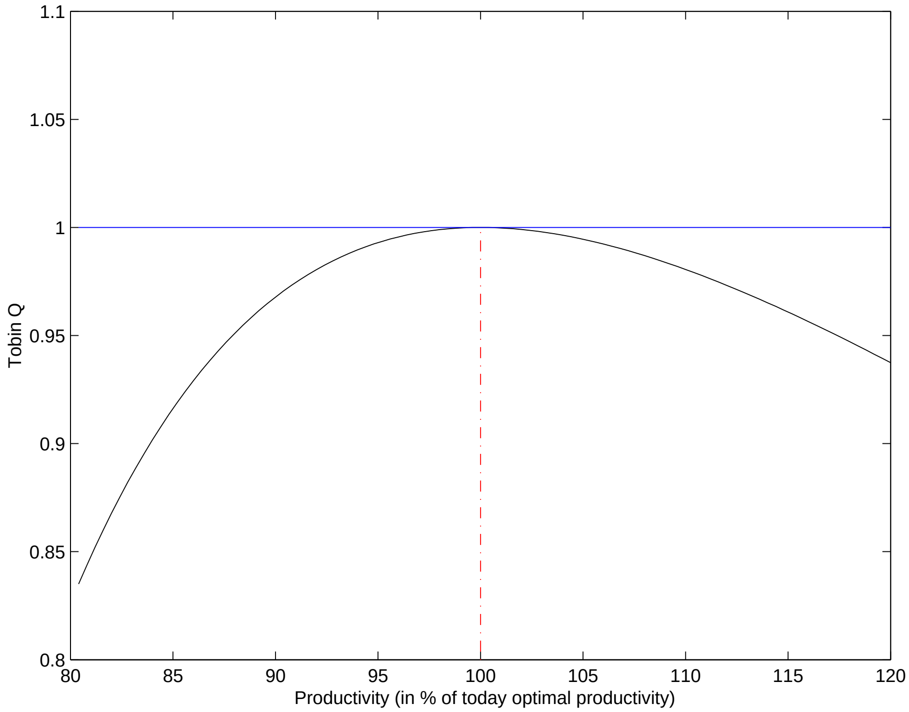

Figure 3: Tobin's q as a function of productivity. Tobin's q is one for the productivity of capital built today.

$c(x) = x^{1/\alpha}$ since to obtain a plant of productivity x, one needs to put up $i = x^{1/\alpha}$ units of capital. Thus,

$$q_t(x) = \frac{P_t(x)}{c(x)} = \frac{v_{1t}x - v_{2t}}{x^{\frac{1}{\alpha}}}.$$

This function q_t satisfies $q_t(x_t) = 1$, $q_t'(x_t) = 0$, and $q_t(x) \leq 1$ for all $x \geq 0$: Tobin's q is below one for all productivities, except the one which is optimal today. The irreversibility constraint binds for all productivities except the one that is optimal today; hence the market value falls below the replacement cost for all productivities but $x = x_t$, as shown in figure 3.

The effect of a TFP shock is depicted in figure 4: because the optimal productivity -the point where $q = 1$- falls, under the maintained assumption that $\varepsilon_{i,A} < 0$, the curve shifts to the left; hence, the q of the low-productivity plants rises whereas the q of the high-productivity falls: this corresponds to a rise in the price of low-productivity plants, and a fall in the price of high-productivity plants, which is an illustration of result 1.

Aggregate Implications: aggregate consequences from the relative price effect

I now turn to the aggregate implications. The (ex-dividend) value of the aggregate stock

does not explain the correlation between returns and book-to-market.

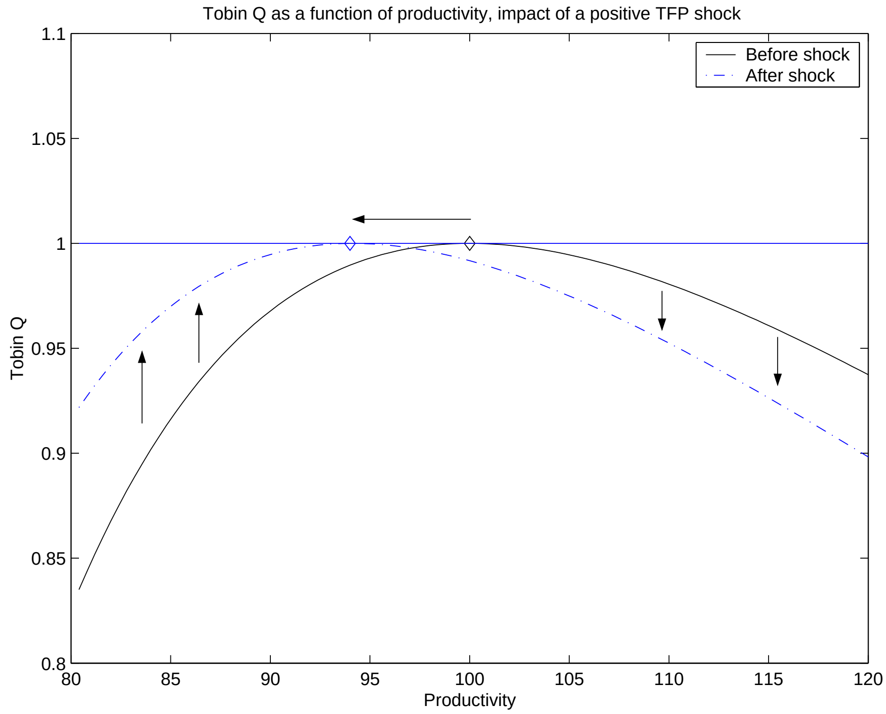

Figure 4: The shift in the Tobin's q curve of figure 3, following a positive TFP shock: the optimal productivity falls; low productivity plants gain value and high productivity plants lose value.

market V_t is found by summing the values of all the plants in the economy:

$$\begin{aligned} V_t &= \int_0^\infty P_t(x) dG_t(x) dx \\ &= \int_0^\infty (v_{1t}x - v_{2t})\, dG_t(x) dx \\ &= v_{1t}\widetilde{Y}_t - v_{2t}N_t, \end{aligned}$$

where the third line uses the fact that $\widetilde{Y}_t = \int_0^\infty x dG_t(x)$ and $N_t = \int_0^\infty dG_t(x)$. Using the expressions (4.3-4.4) found above for v_{1t} and v_{2t} delivers the:

- **Result 2:** The ex-dividend value of capital installed at the beginning of time t is:

$$V_t = \frac{i_t}{\alpha}\left(\frac{\widetilde{Y}_t}{i_t^\alpha} - (1-\alpha)N_t\right). \tag{4.7}$$

This formula (4.7) should be contrasted with what arises in the neoclassical growth model with no adjustment costs: $V_t = K_t$. In the neoclassical model, the value of the stock market is predetermined and moves only in as much as the real quantity of capital changes. In our case, since i_t is a jump variable, the stock market value is not predetermined, even though the quantities of capital - the $G_t(x)$ - are. Hence, this theory generates some volatility in the price of capital, even though there are no adjustment costs to the creation of new plants.[18]

Since $\widetilde{Y}_t$ and N_t are predetermined (state) variables, any instantaneous impact on the value of capital must go through i_t, the only jump (control) variable in (4.7). In appendix, a simple evaluation of derivatives yields the following:

- **Result 3:** around the nonstochastic steady-state, V_t falls when i_t rises if and only if there is trend growth.

We already know that the price of individual plants will move only if i_t changes (see equation (4.6)). This result shows that to obtain a change in the aggregate price, one needs on top that there is trend growth. The intuition is clear: with trend growth, there are much more low-productivity plants than high-productivity: in the formula (4.6), most x are below the current optimal one $x_t = i_t^{\alpha}$. This is depicted in figure 5 below, where the current optimal productivity is clearly above the median of the distribution of x. Hence, the twist of the relative price of low-productivity vs. high-productivity capital does lead to changes in the value of the aggregate capital stock.[19]

The interpretation is the same as the one given for the pricing of the individual plants, with the two related angles:

- With non-responsive wages, the present-value of TFP increases by more than the present-value of wages, which increases the price of low-productivity plants and decreases the price of high-productivity plants. With many low-productivity plants in the economy, this increases the aggregate value of capital.

- A change in i reflects a change in the relative price of labor, with a high i reflecting expensive labor, which hurts capital, thus $\partial V/\partial i < 0$; in turn, if we choose parameters such that $\varepsilon_{i,A} < 0$, we obtain $dV/dA = \partial V/\partial i.\partial i/\partial A > 0$: the stock market is procyclical.

B. Numerical Results

This section gives some results from numerical simulations of the model. These numerical simulations confirm the qualitative results discussed in the previous section. They do not, at the

[18] In a standard adjustment cost model, $V_t = q_t K_t$ where q_t is Tobin's q, an increasing function of the investment/capital ratio. In this case, variation in q leads to price variation.

[19] The result can also be glimpsed in figure 2 above: Tobin's q increase for low productivity plants, and decreases for high productivity, but since there are many more low-productivity units than high productivity units, in the aggregate Tobin's q rise: the stock market jumps up.

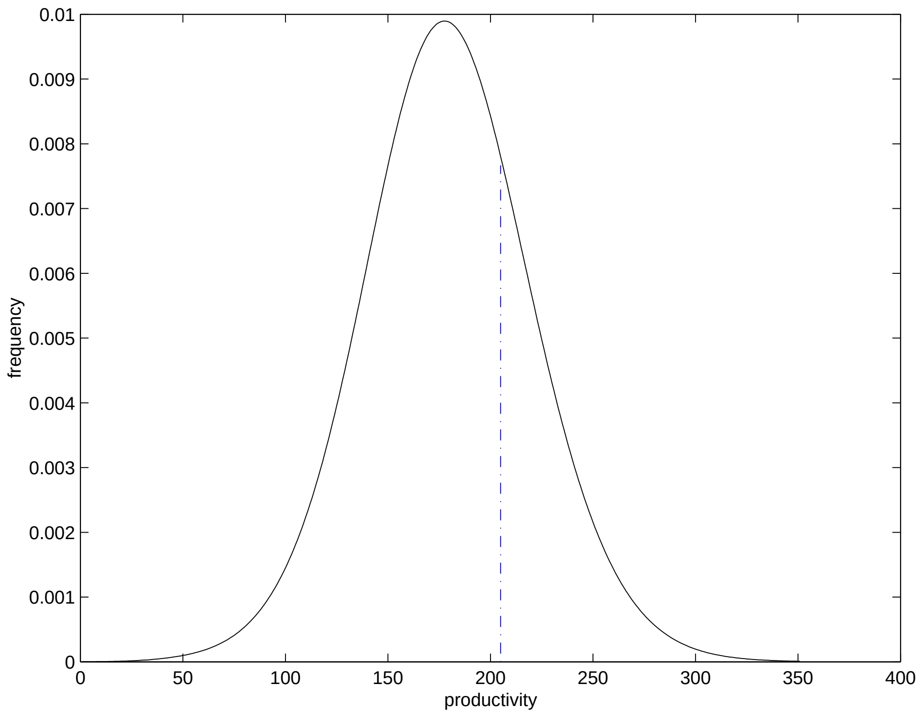

Figure 5: Cross-sectional distribution of productivity $G_t(x)$. The dashed line indicates the average productivity of capital built today, which is above the median.

current stage, provide a good quantitative match of the data, for reasons that are discussed in appendix 5. (One main reason is the difficulty of raising equity premia in production economies, as studied by Boldrin, Christiano and Fisher (2001) and Jermann (1998)). These simulations should thus be taken more as numerical illustrations than as fully quantitative exercises.

I discuss in appendix 5 the details of the calibration. The key ingredients are to imposes a high risk aversion to increase risk premia, and a very elastic labor supply, to obtain the key condition that $\partial i_t / \partial A_t < 0$. Finally, to ensure that this condition holds, the TFP shock need to transitory ($\rho_a < 1$). The aggregate equity premia generated by the model are small, of the order of 0.5% to 0.8% per year, depending on the exact calibration; partly this is because marginal utility is not extremely volatile (the market price of risk is around 0.25), and partly because the return of capital is not very volatile (less than 6% per year, as compared to 16% in the data). It should be noted, however, that the return volatility that we obtain is still an order or magnitude bigger than the one generated by the standard RBC model (less than 0.5% per year). There are several fixes that one could impose to the model, which would help increase overall risk premia, the most obvious being to impose an adjustment cost for the creation of new plants. Another issue that needs to be tackled is the definition of firms in this model; I have thus far looked only at groups of plants, without taking into account the future investment done by these plants. These issues are discussed in more detail in appendix 5, where I also display the impulse responses to a TFP shock.

I also examine the consequences of a second shock in this model, a labor tax (or labor supply) shock. A strand of the business cycle literature has documented that more sources of shocks may be required, notably because the first-order condition governing labor supply doesn't hold well in the data.[20] In terms of the cross-sectional differences in returns, this shock may be quantitatively important since an increase in the labor cost will affect differently low and high productivity firms; as a result it will also affect the aggregate stock market. Impulse responses to this shock, shown in the appendix, confirm this; moreover, because output doesn't rise immediately, the model delivers the stylized fact that the stock market forecasts (or leads) output.[21] This fact is not matched by adjustment cost models (e.g., Jermann 1998), or by the preferred two-sector model of Boldrin, Christiano and Fisher (2001), though Lamont's work (2000) suggests that a model with planning lags might capture it. We return to this fact in more detail in Section 6.

I next replicate Fama and French's empirical approach on simulated data from my model. But first, table 1 gives empirical results similar to those of Fama and French: these are summary statistics of 10 portfolios sorted by book-to-market, for two different samples. These portfolios are constructed by sorting firms,at the beginning of each year, by the ratio of their book value to their market value, and then assigning the 10% with the lowest book-to-market ratio into one portfolio, the next 10% into a second portfolio, and so on up to the top 10%. (Each year, firms are sorted anew and assigned to a potentially different portfolio.) Portfolio 1 is a "growth" portfolio (i.e., low book-to-market) and portfolio 10 is a "value" portfolio (high book-to-market). The lower table is close to the original results of Fama and French (1992), which go strongly against a CAPM interpretation. The upper table shows that the early part of the sample is actually more favorable to the CAPM, though in this case too it fails to account for the cross-section. The

[20] On the need for more shocks, see e.g. Chari, Kehoe and McGrattan (2004), Hall (1997), Ingram et al. (1991), and the remarks of Ljungqvist and Sargent (2004, chap. 11, p. 334), and on the difficulties with the labor supply equation, see Hall (1997) and Mulligan (2001). Gilchrist and Williams (2001) also considered a tax shock, which was quantitatively important in some of their moment-matching exercises. In my dissertation I consider the possibility of embodied-technology shocks, as in Krusell, Hercowitz and Greenwood (2000), Christiano and Fisher (2004), but I do not find them very useful to understand asset prices.

[21] I thank Jonas Fisher for emphasizing this. The fact that the stock market forecasts output growth has been noted at least since Fischer and Merton (1984), though Stock and Watson (2003) consider it unreliable for out-of-sample forecasting. See section 6 for some empirical documentation.

empirical results summarized in these tables are the source of a large literature in finance. Fama and French (1992, 1996) propose a three-factor model that explains well this cross-section, but fails to explain other asset prices. Lettau and Ludvigson (2001) show that incorporating conditioning information can improve the performance of the CAPM considerably. Recently, many papers similarly introduce some discount factors incorporating cyclical macroeconomics variables with some empirical success (See, among many others, Santos and Veronesi (2002) for labor income, Lustig and Van Nieuwerburgh (2004) and Piazzesi, Schneider and Tuzel (2004) for housing, Pakos (2004) and Yogo (2004) for durable goods). Finally and most recently, properties of the long-run risk of the high book-to-market portfolios have been shown to be quite different, which in recursive utility setups can rationalize their excess returns (Bansal, Dittmar and Lundblad (2004); Campbell and Vuolteenaho (2004); Hansen, Heaton and Li (2004); Julliard and Parker (2004)). Finally, it should be noted that the consumption CAPM actually outperforms the market CAPM on this cross-section, and some argue that with proper measurement, it can account well for this cross-section (Jagannathan and Zhang 2004). This result would, of course, be consistent with my model.

1927-2003	1	2	3	4	5	6	7	8	9	10
mean return	8.34	9.38	8.97	8.74	10.80	10.43	11.48	13.10	14.10	14.21
std. dev.	22.97	19.41	19.92	22.48	22.67	23.05	24.74	26.90	28.72	32.53
market beta	1.01	0.98	0.95	1.06	0.97	1.06	1.12	1.13	1.31	1.41
sharpe ratio	0.49	0.63	0.60	0.52	0.61	0.58	0.58	0.60	0.60	0.53
mean B/M	0.26	0.45	0.60	0.76	0.90	1.09	1.32	1.66	2.29	4.92

Sample A: 1927-2003

1963-2003	1	2	3	4	5	6	7	8	9	10
mean return	6.22	7.82	7.69	7.50	8.14	9.35	10.80	11.09	12.16	14.09
std. dev.	21.19	16.64	16.48	17.44	16.03	15.44	17.99	17.151	17.38	20.40
market beta	1.09	1.03	1.02	0.97	0.89	0.90	0.85	0.84	0.90	0.98
sharpe ratio	0.50	0.73	0.73	0.68	0.78	0.89	0.84	0.90	0.95	0.90
mean B/M	0.20	0.37	0.49	0.61	0.71	0.82	0.94	1.10	1.34	1.97

Sample B: 1963-2003

Table 1: Financial statistics for ten book/market sorted portfolios

The statistics are in yearly, real terms. The market betas are computed with monthly data.

Value-Weighted portfolios from Prof. French's website. CPI from Prof. Shiller's website.

Table 2 gives the results obtained from a simulated panel data set from my model.[22] Qualitatively, the sign of the relation is reproduced: high book-to-market portfolios have higher average returns. The magnitude of the relation is however not matched. This failure is in part attributable to the relatively small aggregate risk premia in the model (another part is due to the fact that these plants do not reinvest). The pattern that high book-to-market portfolios are more volatile is stronger in the model than in the data. (However, note that the U-shaped pattern of volatilities is also evident in the data for the 1963-2003 sample at least.) Of course, in the model, these return differentials are fully explained by differences in β. More work remains to be done to obtain a full quantitative match of the data, and to understand why the market βs do not account for return differentials.

Calibration C	1	2	3	4	5	6	7	8	9	10
$\mathbf{E}R$	0.39	0.44	0.49	0.54	0.59	0.64	0.71	0.82	0.99	1.88
$\sigma(R)$	5.23	4.95	4.74	4.55	4.35	4.16	3.97	3.83	4.00	7.78
BM	0.19	0.41	0.47	0.52	0.58	0.64	0.73	0.85	1.09	2.12

Table 2: Simulated panel data from model:
B/M sorted portfolios (TFP+Labor tax shocks)
The data construction follows Fama and French (1992).

5 Evidence on the Cross-Section of Returns and the Book-to-market effect

I now examine the empirical relevance of the mechanism that I developed in the last section. The model predicts some correlations between the financial and real characteristics of firms. A key prediction is that firms which have higher labor productivity x or a higher capital share will have a lower "operating leverage". Their operating income will be less sensitive to aggregate shocks. This property of their cash flows will lead them to have lower βs and lower average returns. In the model, the low x firms also have more volatile returns than the high x firms;[23] and finally the low x firms have on average a higher book-to-market ratio (since part of productivity shows up in the market value, but not in the book value). More precisely, equation (4.5) $\varepsilon_{P(x),A} = (1 - \alpha s_t(x))\, \varepsilon_{i,A}$ implies that $\beta(x)$, which is proportional to $\partial \mathbb{E}R(x)/\partial A \simeq \varepsilon_{P(x),A}$, is positively related to $s_t(x)$

[22] I follow the portfolio formation rules of Fama and French (1992): firms are sorted each year by book-to-market; then, with a 2 quarter lag they are assigned to porfolios based on the decile of B/M; and they are reassigned next year to a new portfolio.

[23] The exception is that the $x > x_t$ will be volatile and negatively correlated with the market, see equation (3.6).

and thus negatively to x. (Remember that $\varepsilon_{i,A} < 0$ for our preferred parameter values, and that s_t is a decreasing function of x.) A quantitative prediction is that differences in β and average returns across stocks should be proportional to differences in inverse capital share s_t.[24]

In Section A I examine whether these real characteristics can explain differences of returns across book-to-market sorted portfolios, and in Section B I look at differences in average returns across industries.

A. Book-to-market sorted portfolios

In this section, I examine the model's implications by looking at 10 portfolios of firms, created by sorting listed firms by book-to-market.[25] As noted by Fama and French (1992) and others, the portfolios with higher book-to-market equity have higher average returns, and the spread in returns is large: the difference between the highest and lowest return is around 7% annually (See Section 4B for some statistics and a quick literature review). The interpretation for these return differences that I propose is that the sort on book-to-market is effectively a sort on productivity x. High book-to-market are low productivity firms which have a high operating leverage, are more sensitive to aggregate shocks, and have thus higher average returns. I proceed to check all these associations, by measuring directly productivity, operating leverage, sensitivity to aggregate shocks and mean returns for these portfolios.

Of course, in my theory, the differences in sensitivities should show up in differences in market βs; we know in advance from the finance literature that they won't. In this section, I will discard the evidence on β and relate directly real attributes that determine riskiness in my model with mean average returns.[26]

[24] However, the proportionality factor, related to the sensitivity of consumption to shocks and the market price of risk, requires to impose the specification of the full model (including preferences and the structure of shocks), which I do not want to do.

[25] Each year, firms are ranked by their book-to-market equity, and assigned by deciles to one of the 10 portfolios. The lowest B/M portfolio is 1 (also called "growth") and the highest B/M is 10 ("value"). Source: Compustat annual data, 1963-2002.

[26] There are two reasons why despite this, the predictions are interesting. First, even if the return differentials across book-to-market were explained by differences in betas, it would be interesting to understand why some firms bear a disproportionate share of the cyclical variation (i.e. where the heterogeneity in book-to-market comes from). Second, we can look directly at the correlation between the characteristic and average return, implied by the model, and discard the information in β, for instance because it is ill-measured. In a sense, this is just a matter of "fixing" the utility function that closes the model (but of course, it remains to check if with this modification, the betas would still be correlated with productivity).

High Book-to-Market have a low capital share and a low productivity/profitability

As explained above and proved in the appendix, I can approximate the measure of operating leverage $s_t(x)$ by the inverse of the capital share of a firm. In practice, constructing firm-level value added is a hard task. I thus proxy the capital share by the ratio of operating income to sales, and the first line of table 3 gives, for each portfolio, the average (over time) of the ratio sales/operating income.[27] This ratio is strongly upward sloping: high book-to-market firms tend indeed to have a small capital share. This is a first success for our proposed interpretation: high book-to-market firms have higher operating leverage.

I now examine whether we can trace this back to differences in labor productivity. Previous work has shown that high book-to-market firms have low profitability (Fama and French (1995)). In table 3, I reproduce their main finding: the ratio of operating income to book equity is decreasing in book-to-market. There are several ways in which one can measure more precisely labor productivity; one would like to have the value added per employee. As noted above, Dwyer (2001) found a negative correlation of -0.3 in plant-level data between book-to-market and productivity. In my data set, I can proxy labor productivity by the sales/employee ratio or the operating income per employee. The second ratio is indeed strongly downward sloping, but the first one is hump-shaped. Overall, I conclude that high book-to-market firms tend to have lower productivity/profitability per employee. This is not really surprising in the light of the finance literature which views these firms as "distressed", i.e. experiencing (temporary or permanent) difficulties.[28]

Portfolios $i=$	1	2	3	4	5	6	7	8	9	10
$\frac{Sales}{Op.\,Inc.}$	5.42	6.26	6.89	7.36	7.59	7.82	7.94	8.68	9.44	12.39
$\frac{Op.Inc.}{Employees}$	1.25	1.03	0.98	1.00	1.04	1.08	1.03	0.93	0.83	0.62
$\frac{Sales}{Employees}$	0.89	0.86	0.90	1.00	1.06	1.15	1.08	1.04	1.02	0.95
$\frac{Op.\,Inc.}{Book}$	0.50	0.42	0.37	0.35	0.32	0.30	0.28	0.26	0.24	0.20

Table 3: Time Means of variables, for each portfolio

Source: Compustat, 1963-2002. Portfolios sorted by increasing book-to-market each year.

Lines 2&3: the mean refers to the mean of series: Portfolio variable/Aggregate variable

High book-to-market firms have more cyclical operating income

[27] The variables here are portfolio aggregates: for instance, the first line is total sales of the portfolio divided by the total operating income. This is akin to considering value-weighted portfolios.

[28] It is possible to add transitory idiosyncratic productivity shocks to the model, without changing its implications.

In the model, operating leverage is associated with risk because operating income is more cyclical. (As a consequence, the cash-flows correlate more strongly with consumption growth.) In the model the operating income is $OI_t(x) = A_t x - w_t$. This implies that the growth rate of operating income is a weighted average of the growth-rate of TFP and the wage:

$$\Delta OI_t(x) = \underbrace{\frac{A_t x}{A_t x - w_t}}_{\frac{1}{s_K(x)}} \Delta A_t + \underbrace{\frac{-w_t}{A_t x - w_t}}_{1-\frac{1}{s_K(x)}} \Delta w_t, \tag{5.1}$$

where the coefficient on TFP is again the inverse of the capital share, and the coefficient on the wage is one minus the coefficient on TFP.

In table 4, I report for each portfolio the result from this regression. First, notice that TFP in my model cannot be computed as a simple Solow residual; for this reason I replace A_t with GDP: in the theory, the two are closely correlated since changes in GDP are changes in TFP plus a capital accumulation term. (Alternatively this can be thought as a simple way of measuring the cyclicality of operating income.) Hence, I run for each portfolio $i = 1...10$ an OLS regression

$$\Delta OI_{i,t} = a_i + b_i \Delta GDP_t + c_i \Delta w_t + \varepsilon_{i,t}, \tag{5.2}$$

where Δ denotes a growth rate. I also run this regression with the restriction implied by (5.1) that $c_i = 1 - b_i$. Note that in the model, there is no error term; we can appeal to measurement error to justify this regression. Neglecting Jensen's inequality, we expect that the coefficient b_i should be on average the inverse of the capital share, or 1/(1/3)=3 for the whole economy, and that the high book-to-market firms have larger b_i since they have lower capital shares; moreover, the c_i's should be negative and decreasing in book-to-market. The coefficient estimates are reported in table 4 and displayed in figure 6 (the heights of the bars are the coefficient estimates, which have the dimension of elasticities).[29]

It appears that the coefficients b_i vary almost monotonically across portfolios. The order of magnitude of the coefficients is economically meaningful: the average estimate is 3, as expected,

[29] OLS equation-by-equation here is equivalent to OLS on the whole system. There is some evidence of time correlation in the residual for some of these equations (see the Durbin-Watson statistics), and there is also some correlation across portfolios in the residuals. Given the short sample I prefer to use OLS rather than a GLS estimator, but the standard errors are adjusted to correct for the time-series heteroskedasticity using the Newey-West formula. In the appendix, I give a principal component analysis of the residuals to justify the lack of correction for cross-section correlation, and I also give the OLS standard errors, which are not very different from the Newey-West ones.

and there is a fair amount of dispersion.[30] Some of these coefficients are not statistically significant though, because portfolio-level income growth is quite volatile. The c_i are negative (except c_7) and tend to decrease when we move from low book-to-market to high book-to-market, as predicted, but the pattern is not fully monotonic. It is however supportive that the high book-to-market have a much larger sensitivity to labor compensation than the low book-to-market. To my knowledge, neither empirical pattern had been noted previously. When we impose the restriction, we see a clear increasing pattern of coefficients.

One way to summarize the strength of these correlations is to show that they account for a large share of the variance of returns. Figure 7 gives the relation between our empirical estimate of $s_t(x)$, the mean of the sales/operating income ratio (the mean inverse of the "capital share") and average returns, and figure 8 displays similarly the relation between the regression coefficient on GDP (from table 4a) and average returns. The fit is relatively good, which suggests that these variables account for the underlying risk of these portfolios. Note that the quantitative prediction underlined above implies that figure 7 and 8 should be straight lines (not any kind of monotonic function), since the differences in returns are proportional to differences in inverse capital share; hence the high R^2 are supportive.

[30]If we were measuring capital shares in table 3, we could check that the estimates for b_i are indeed the inverse of the capital shares of each portfolio. However, we divide by sales, which are greater than value added, hence this direct test is not possible.

$$4a: \Delta OI_{it} = a_i + b_i \Delta GDP_t + c_i \Delta w_t + \varepsilon_{it}$$

Portfolios $i =$	1	2	3	4	5	6	7	8	9	10
b_i	**1.52**	**1.52**	**1.96**	**2.90**	**4.05**	**3.31**	**3.54**	**3.78**	**4.68**	**5.99**
$\sigma(b_i)$ NW	0.62	0.49	0.73	0.59	0.73	0.53	0.91	0.86	0.89	1.24
c_i	**-0.47**	**-1.11**	**-1.84**	**-0.40**	**-2.45**	**-2.05**	**0.66**	**-1.09**	**-2.30**	**-3.24**
$\sigma(c_i)$ NW	1.24	0.88	1.07	0.79	1.13	1.40	1.45	0.98	1.07	1.66
R^2 (%)	10.6	16.2	14.2	38.0	39.6	36.7	44.8	31.0	47.0	30.2
DW	1.60	2.10	1.64	1.57	1.63	1.61	1.65	1.94	2.10	2.01

$$4b: \Delta OI_{it} = a_i + b_i \Delta GDP_t + (1 - b_i) \Delta w_t + \varepsilon_{it}$$

b_i	**1.51**	**1.63**	**2.13**	**2.62**	**3.94**	**3.27**	**2.94**	**3.46**	**4.42**	**5.66**
$\sigma(b_i)$ NW	0.61	0.48	0.62	0.52	0.65	0.54	0.92	0.70	0.84	1.21
R^2 (%)	10.6	15.2	13.1	34.2	39.2	36.6	33.0	28.6	45.4	27.4
DW	1.60	2.01	1.66	1.62	1.64	1.60	1.55	1.86	2.09	2.09

TABLES 4A,4B: POINT ESTIMATES AND STANDARD ERRORS OF (5.2)

Table 4b reports results imposing the restriction $c_i = 1 - b_i$, table 4a reports results without it.

(Compustat, BLS and BEA, 1963-2002). Portfolios sorted each year by increasing book-to-market.

B. Industry-Level Evidence

In this section, I examine the model's implications by looking at 17 portfolios of firms, sorted by 2-digit SIC industries.[31] One motivation for using industry-level data is that better data (e.g., value added) is available. On the other hand, the model has no explicit industries, which make the interpretation of the findings less straightforward, as will be discussed below.

I follow the same methodology that I followed for book-to-market portfolios, and examine whether productivity, capital shares (aka operating leverage), and the cyclicality of profits, are related to betas and mean returns. In the book-to-market section, I discarded the evidence regarding beta, and showed that the model's variables explained well average returns. In this section, it will appear that the converse holds for industries, and the model matches betas and volatilities, but not returns.

[31] Data from Prof. French's website (Portfolios) and the BEA (Industry accounts and Fixed asset tables). Details in appendix.

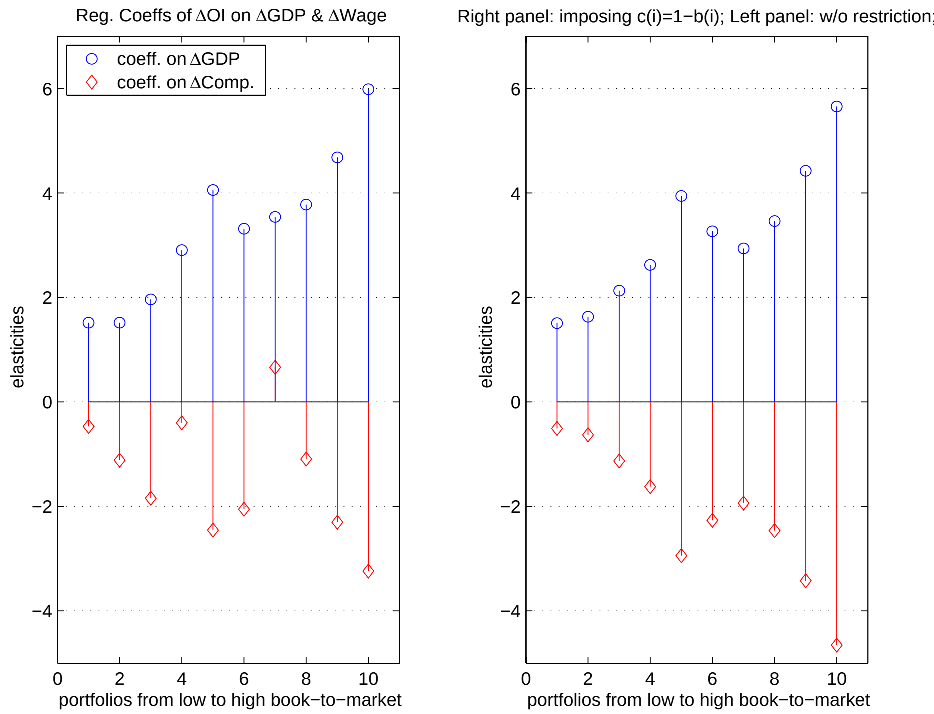

Figure 6: Point estimates of (5.2), for each portfolio. Left panel: results of the uncontrained regression; right panel: imposing the restriction $c_i = 1 - b_i$. Porfolios sorted by increasing book-to-market. Data from Compustat, BLS and BEA, 1963-2002.

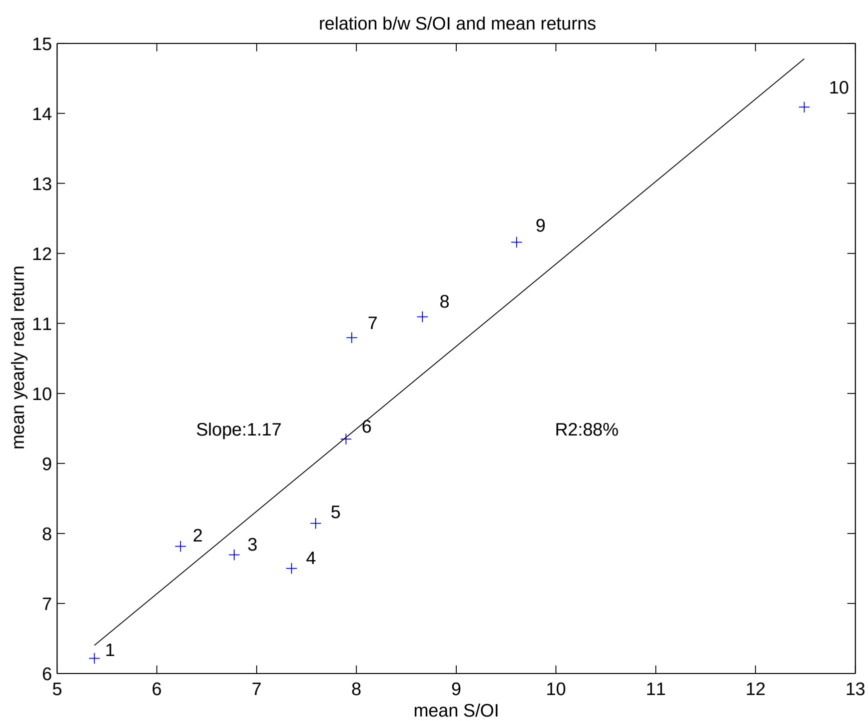

Figure 7: Scatter plot of the average ratio sales/operating income, and the average yearly real return.

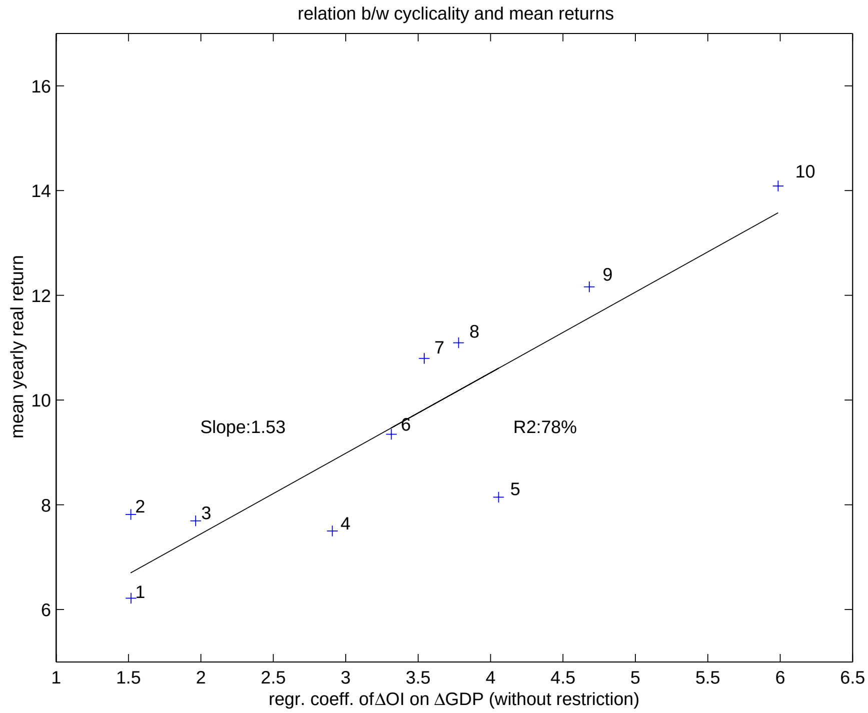

Figure 8: Scatter plot of the point estimate of b_i in equation (5.2), and the average real yearly return. The point estimate comes from the unconstrained regression.

First, in the same spirit as table 3, I examine whether book-to-market, betas, volatilities or mean of returns are related to labor productivity and the inverse capital share.[32] The first two columns reveal that, as predicted by the model, low capital shares or low productivity industries have higher betas and more volatile returns, but do not have higher returns. The positive correlations are displayed in figures 10 and 11. Moreover, the correlation between productivity and book-to-market is positive, not negative as predicted. Hence, it appears that the real attributes of industries account well for a different set of financial characteristics than the real attributes of book-to-market portfolios.

[32] A table in appendix gives all these statistics for each industry.

	1/Capital Share	VA/Empl (rel.)	$\beta_{\Delta OI,\Delta GDP}$	$\beta^{\text{restricted}}_{\Delta OI,\Delta GDP}$
mean real return	-0.14	-0.02	0.02	0.00
std real return	0.80	-0.57	0.58	0.58
market β	0.79	-0.65	0.41	0.43
book/market (beg. of period)	0.02	0.47	0.40	0.25

Table 5: Correlations between productivity, financial and economic variables across industries

Sources: see appendix 8.

As I did for book-to-market, I estimate the equation (5.2) for the cyclical sensitivity of profits, again with and without imposing the restriction; the point estimates are displayed in figure 9 (the full table of results is available in appendix 8). I examine whether these point estimates relate to either book-to-market, betas, volatilities or mean returns, in the last two columns of table 5. As the theory predicts, more cyclical operating income is positively correlated with betas and volatilities, but again the correlation with mean returns is nil, and the correlation with book-to-market has the wrong sign.

These results can be attributed to the failure of the CAPM for the industry portfolios, as documented by Fama and French (1997): the market betas explain only a small share of the variance of stock returns, hence though my model explains the betas well, it fails to explain the returns.

Difficulties with Cross-Industry Correlations

There are at least two particularities of industries which the model does not capture. First, real-life industries have different elasticities of demand and some are more cyclical, for the simple reason that they produce goods which demand vary more (e.g., luxuries or durables). Second, there are different production technologies, which in the model can be captured simply as different α. This second fact does not really preclude the model from making predictions: changes in the aggregate variable i will still drive all industries returns, but now their reactions will also differ because they have different α. Further work will examine these issues in more detail. In ongoing work, I use the NBER manufacturing productivity database and the Compustat data set to continue the above analysis at a finer level. I also plan to look at the two dimensions of the data set, i.e. examining how the risk of an industry evolve over time, in relation to the evolution of the real variables. This is motivated by Fama and French (1997), who show that the estimates of risk at the industry level vary over time.

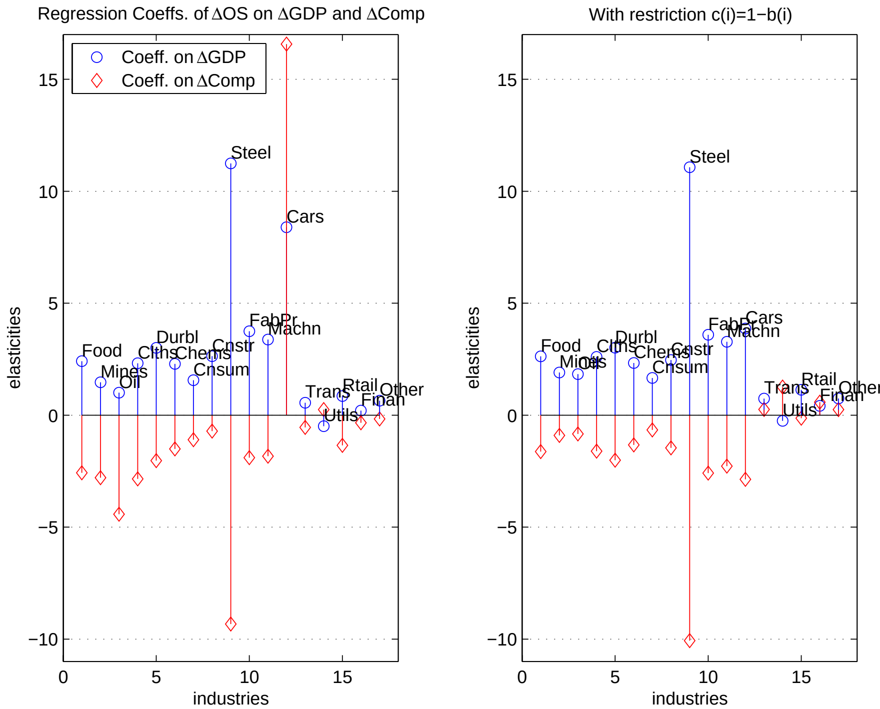

Figure 9: Point estimates of (5.2), for each portfolio. Left panel: results of the uncontrained regression; right panel: imposing the restriction $c_i = 1 - b_i$. Porfolios sorted 2-digit SIC. Data from prof. French, BLS and BEA, 1963-2002.

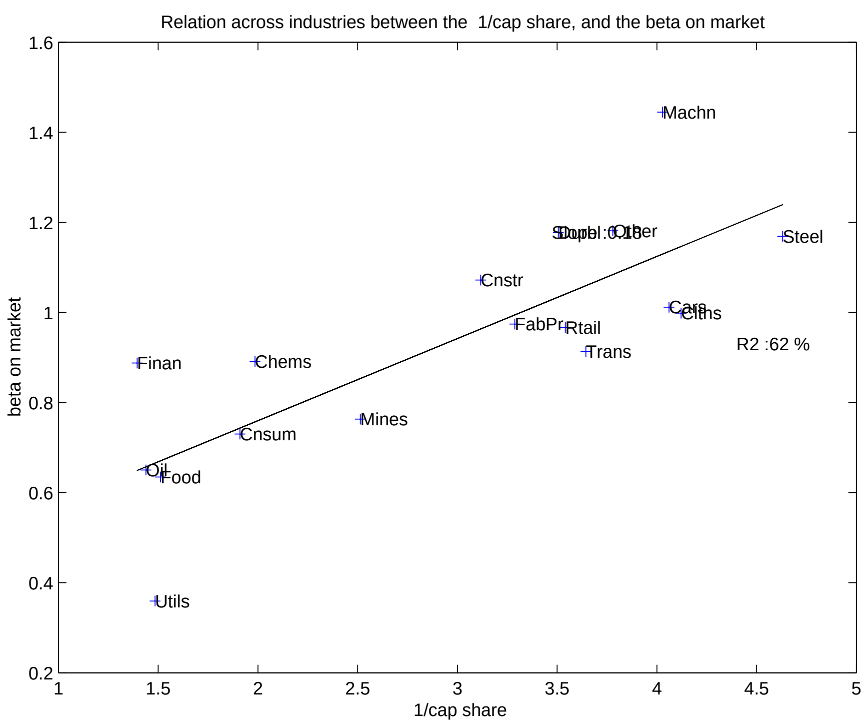

Figure 10: Scatter plot of average inverse capital share and beta on market. 17 industries. Data from Prof. French's website and BEA; 1947-2002.

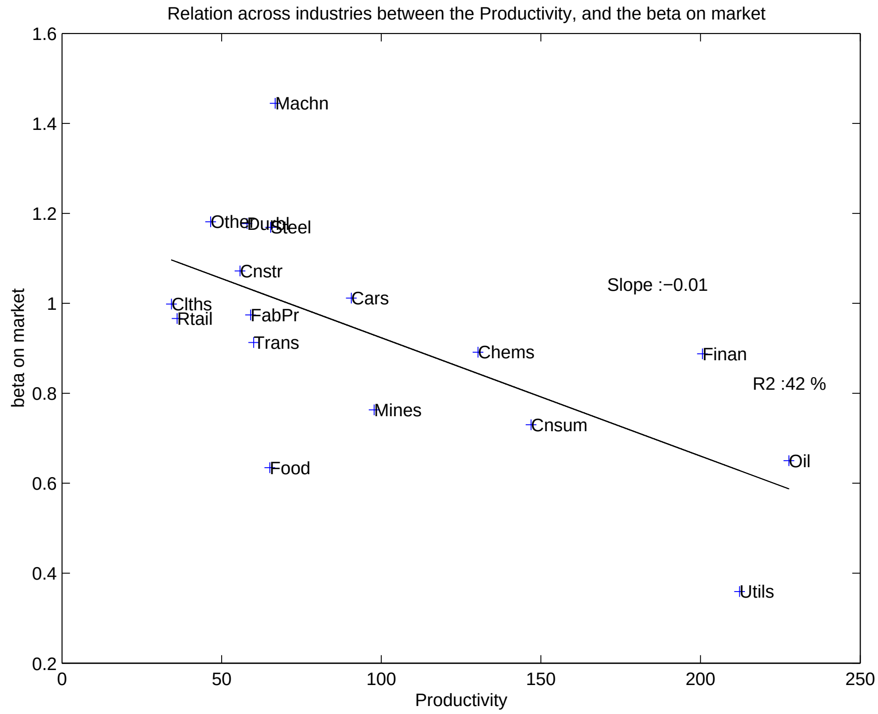

Figure 11: Scatter plot of average labor productivity (relative to the average) and beta on market. 17 industries. Data from Prof. French's website and BEA; 1947-2002.

6 Evidence using Aggregate Data: Stock Market Cyclicality

To consider the model's aggregate implications, I follow Cochrane (1991) and Hall (2001) who derive implications without specifying a full general equilibrium model. Instead I use only the production side of the model, backing out the value of the stock market from *some* model relationships and macroeconomic time series. Using this approach, the model captures quantitatively the cyclical movements of stock market value. In the data, the ratio of gross investment to net jobs created is countercyclical. In the model, this means that the capital intensity of new plants i_t is high in recessions, and the key condition $\partial i_t / \partial A_t < 0$ that gives a procyclical stock market is thus verified in the data: variations in the relative scarcity of capital and labor generate large movements in capital value. In what follows, I contrast the results obtained from my putty-clay formulation with those implied by a standard adjustment cost model and find that it does better at capturing the cyclicality of the stock returns: the stock market return forecasts the business cycle.

Concretely, I use the result 3:

$$V_t = \frac{i_t}{\alpha} \left(\frac{\widetilde{Y}_t}{i_t^{\alpha}} - (1-\alpha) N_t \right),$$

to construct V_t, the ex-dividend value of aggregate capital, given data from macroeconomic time series, and compare it to the aggregate stock market value. (i_t is the capital intensity of new plants; N_t is total hours worked; $\widetilde{Y}_t$ is the productive capacity, or GDP divided by TFP.) To use this formula and find V_t, I need to compute on i_t, N_t, and $\widetilde{Y}_t$. The following paragraph explains how I extract my model's series from the data.

I take N_t directly as an index of total hours worked, and I use the model law of motion $N_{t+1} = (1-\delta)N_t + h_t$ to compute h_t given an assumed value for δ. Then, I use the model identity $i_t = I_t/h_t$ an NIPA data on aggregate investment I_t to construct i_t, the capital intensity of new jobs. Finally, to obtain $\widetilde{Y}_t$, I iterate on the second law of motion $\widetilde{Y}_{t+1} = (1-\delta)\widetilde{Y}_t + i_t^{\alpha} h_t$. I need for this to assume a value for α and $\widetilde{Y}_0$. Finally, I use data on GDP Y_t to display V_t/Y_t, the stock-market / GDP ratio, implied by the model. I also compute the return on aggregate capital $R_{t,t+1} = (V_{t+1} + D_{t+1})/V_t$, where the "dividend" is, from the model, $D_{t+1} = Y_{t+1} - w_{t+1}N_{t+1} = Y_{t+1}\left(1 - s_{L,t+1}\right)$. The series for the labor share $s_{L,t+1}$ is taken directly from the data. This dividend component turns out to be quantitatively unimportant relative to the capital gain part.

There are few degrees of freedom implicit in this analysis: δ, α and $\widetilde{Y}_0$. The following results set $\alpha = 0.3$ and $\delta = 0.12$, and use annual data. (The data sources are listed in the appendix 9, with a discussion of the sensitivity of these results.) Figure 12 gives the path for h_t, inferred as described above: h_t, the quasi-difference in hours, trends upward because of employment growth, but it is volatile, falling a lot in recessions. Figure 13 shows i_t, the investment per new job (or capital intensity of new jobs); this variable also has a long-run trend, reflecting capital deepening, but the spikes, due to the troughs of h_t, are substantial. These spikes in i_t create downturns in the stock-market, as explained in Section 3; the stock market value is displayed in figure 14. This graph deserves some comments. First, note that there is little point in fitting the average value of the stock market, because the model gives the value of the total stock of capital - but we know that a big part of the stock of capital is not publicly traded, and another part of it is actually owned by bondholders (not included here).[33] Second, some recent work (e.g., Hall (2001), Greenwood and Jovanovic (1999), Hobijn and Jovanovic (2001), Laitner and Stolyarov (2003)) examines the low-frequency movements in the stock-market: why was the market so low in the late 70s to mid 80s, with Tobin's q well below 1, and why was it so high in the 90s? The model is not designed to look at this question, since its medium-run implications are the same as the standard neoclassical model, with the stock market reflecting the quantity of capital. What the

[33] However, I did not rescale the series of V_t, since its level turns out to be consistent with the data series.

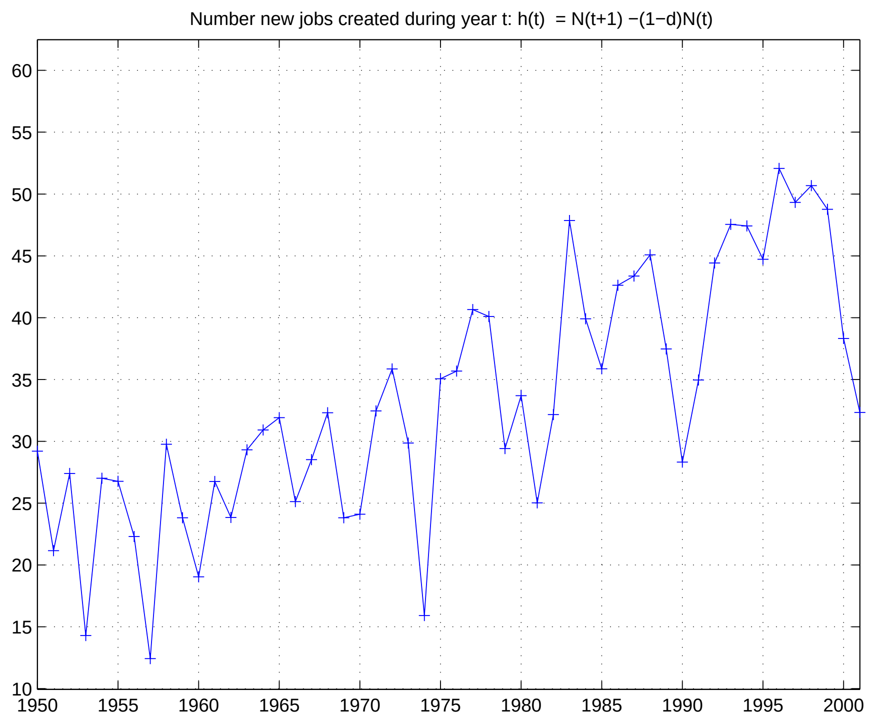

Figure 12: h_t, as inferred from employment data

model does relatively well is accounting for business-cycle movements in the stock market: the year-to-year ups and downs are well tracked, for instance in the 60s and 80s. (1974, according to the model, was just one big recession, leading to a huge recovery in the stock market, much bigger than the - already sizeable - one in the data.) To emphasize the business-cycle correlations, figure 15 gives the HP-filtered graph of the two series. (The return series is displayed in the overview section.)

The correlation between the unfiltered stock-market/GDP series is 0.41, and 0.50 for the filtered series. The correlation of the return series is 0.62. To understand mechanically how the model correlates with the data, one needs to be careful about the timing. As in the standard model, there is a one-period lag between the time when investment takes place and the time when capital becomes usable. Hence, the number of new jobs created during year t equals jobs in year $t+1$ minus $(1-\delta)$ jobs in year t. When few jobs are created between year t and year $t+1$, relative to the current flow of investment, i_t goes up: this means that the value of the stock market at time t falls (since i_t reflects a decision based on the time t information set). Economically, the fact that new jobs have a high capital intensity (and thus a high labor productivity) reflects that capital is relatively plentiful, and thus the marginal product of capital is low. The irreversibility of investment at the plant level implies that these movements in marginal product lead to large

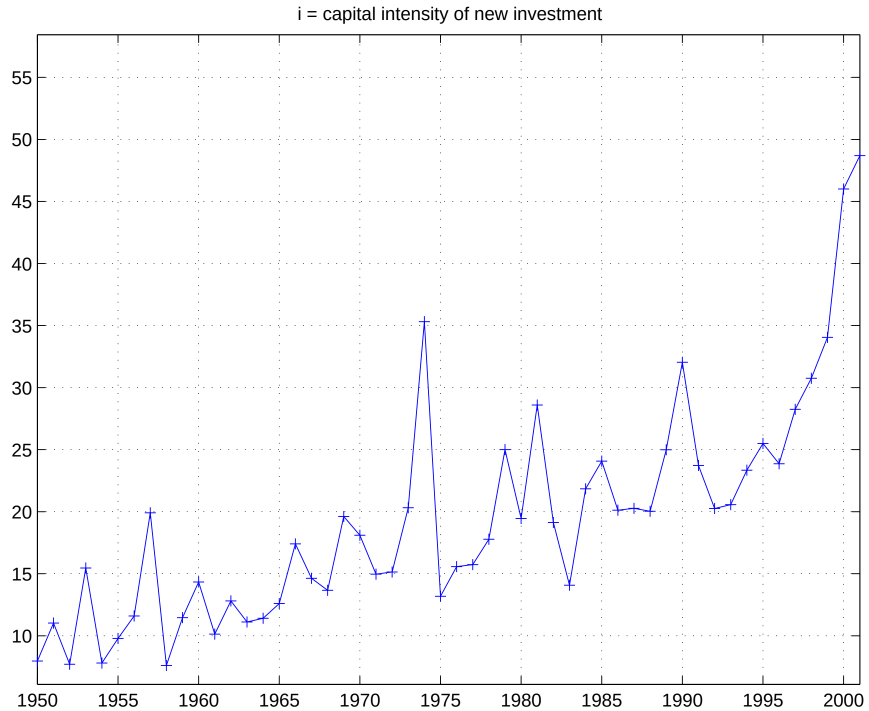

Figure 13: $i_t = I_t/h_t$, the capital intensity of new investment

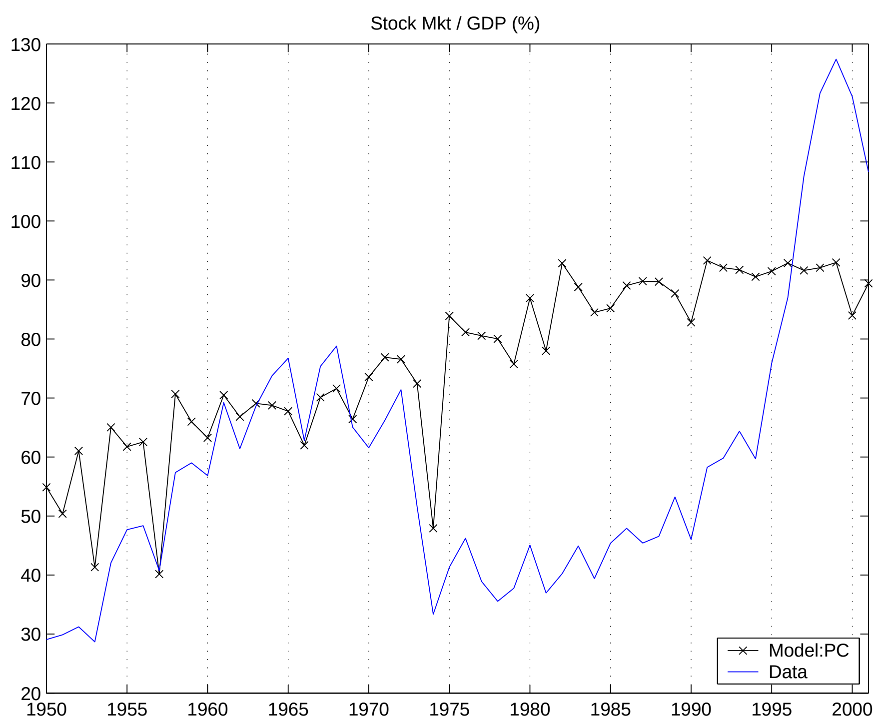

Figure 14: U.S. Stock Market Value divided by GDP, Data and Putty-Clay (PC) Model

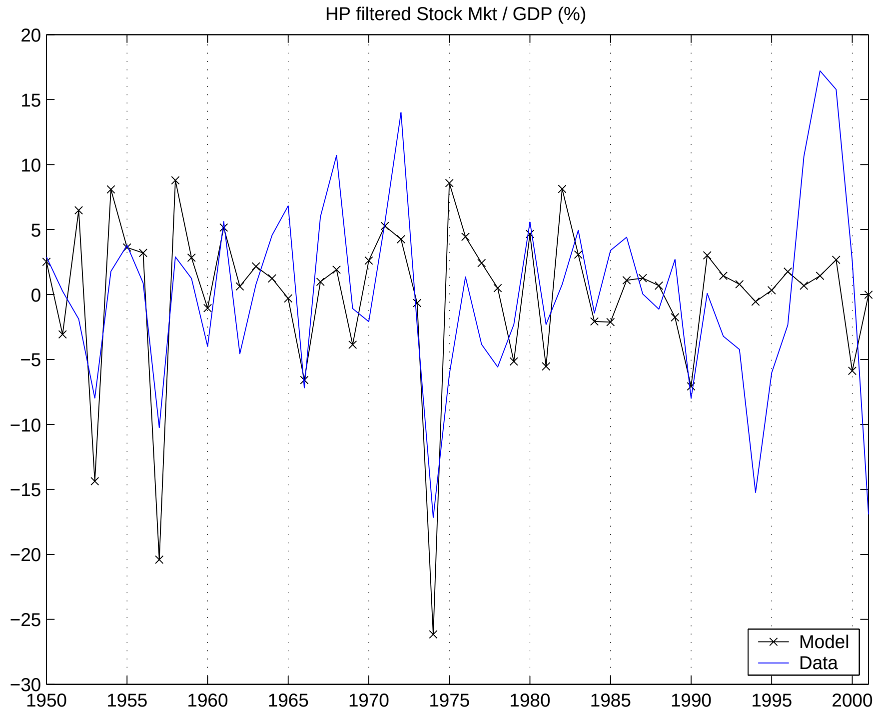

Figure 15: HP-Filtered Stock Market Value, Data and Putty-Clay Model

movements in prices.[34]

Caveats

These results are supportive, but there are some caveats. First, the results are sensitive to the choice of δ. If δ is too low, the spikes in i become huge, leading to tremendous movements in the market value. If it is too big, the spikes are too small, and the market value movements are too small. A related difficulty is that the model fits worse at quarterly frequencies. This is because net job growth is too volatile at the quarterly frequency (See appendix 9.) Finally, direct microeconomic measures of the capital intensity of new jobs might not be as volatile as the measure we obtain here by applying the model to aggregate data.

Comparison with an adjustment cost model, and the leading indicator fact

A similar computation can be done for the familiar adjustment cost model of investment.

[34] That is, in a standard model where capital is homogeneneous, the price of capital is 1 as long as the aggregate irreversibility constraint does not bind, so that variations in the current period marginal product lead to tiny movements in returns: $R_{t,t+1} = 1 - \delta + A_{t+1} F_K(K_{t+1}, N_{t+1})$ is (for a standard Cobb-Douglas production function) at least 10 times *less* volatile than output. The economics are simply that since we're adding to the capital stock, the current capital is still worth its consumption opportunity cost. In my model, the plant-level irreversibility constraint binds (for all but one productivity level) and thus the changes in the future marginal products move the price substantially.

That model serves as a natural benchmark against which to evaluate the results of figure 14 and 15. Adjustment cost theory posits that the marginal cost of investment is an increasing function of the investment-capital ratio. Assume that the cost of investing I_t is $I_t\left(1+\phi\left(\frac{I_t}{K_t}-\delta\right)\right)$ with $\phi(x)=\kappa x^{\beta}$ and $\beta>0$. If the profit function is linear in capital, we obtain from Hayashi's theorem (1982) the equality of average q and marginal q:

$$
\begin{aligned}
V_t &= K_t q_t \\
q_t &= 1+\phi\left(\frac{I_t}{K_t}-\delta\right)+\frac{I_t}{K_t}\phi'\left(\frac{I_t}{K_t}-\delta\right)\stackrel{def}{=} g\left(\frac{I_t}{K_t}-\delta\right)
\end{aligned}
$$

Hence for a choice for κ and β, the second equation yields q_t given data on investment and capital, and the first equation delivers V_t. For simplicity I focus on the quadratic specification: $\beta=1$. I set κ to 10, which leads to a time-series average for marginal q of 1.44. I adjust the resulting series for V_t by a multiplying factor to fit the level of the stock market (the rationale for this being, again, that a lot of capital is privately held and not publicly listed).

Figure 16 shows the results. The predictions of the adjustment cost model miss the long-run swings in the market for the very same reason the putty-clay model does: the medium-run movements are driven in both models by changes in the quantity of capital, which doesn't move as much as the stock market does. The short-run movements can be relatively important, so that the adjustment cost also predicts a volatile return on capital. The series does slightly better than the putty-clay one for the 90s. However, the adjustment cost model has a systematic failure: its predicted return systematically lags the stock market. (See e.g. 1973-1975,1965-1967, 2001, and the figure 17 for the HP-filtered stock-market). This failure has been noted before, for instance by Cochrane (1991), and it is easy to understand the empirical facts that drive it. On the one hand, the stock market return predicts output systematically, as I document below. On the other hand, the adjustment cost model predicts that the value of the stock market is high when the investment-capital ratio is high, which is contemporaneous with high output growth. Thus, it appears that in this dimension, the putty-clay model does better than the adjustment cost model. This shows up in the correlations of table 6: the predicted return is actually negatively correlated with the data return. (Note that changing the adjustment cost parameter can increase the volatility of the adjustment cost return, but does not alter the lag problem. This lag problem also persists, though somewhat weakly, in quarterly data - see the appendix.)

To quantify this fact, I run regressions of current GDP growth on either the current or lagged return, and I run similar regressions with the returns predicted by each model. The results are shown in table 5. While the current return of the Adjustment cost model is the one that is

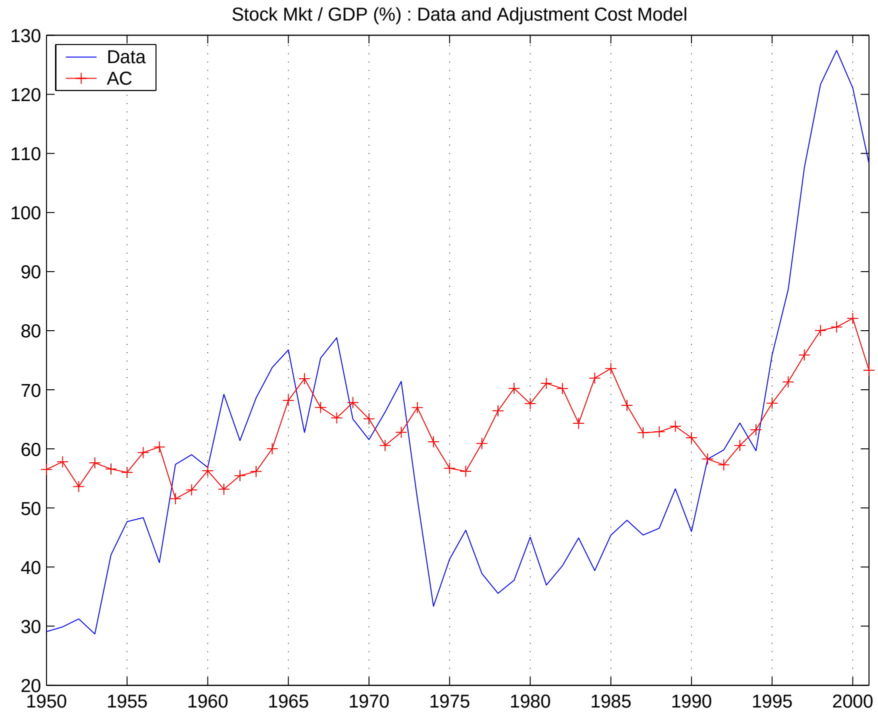

Figure 16: Stock Market Value, Data and Adjustment cost (AC) model.

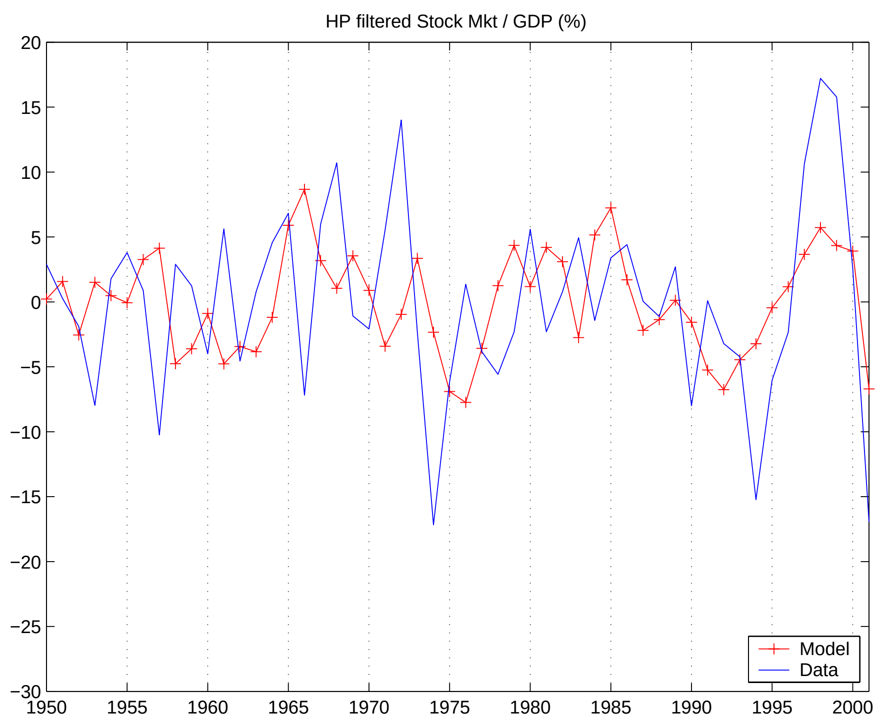

Figure 17: HP-Filtered Stock Market Value, Data and AC model.

most correlated with GDP growth, both the data and the Putty-clay model yield that the lagged return is the one that is most important. The order of magnitude of the coefficient also favors the putty-clay model. (Broadly similar results are obtained with quarterly data.)

The fact that the stock market leads output is documented in figures 18 and 19. Figure 18 gives the raw series, where careful observation allows to eyeball the lagged correlation. Figure 19 gives the impulse response of GDP growth to a return shock, estimated from a quarterly VAR, under the two possible orthogonalizations. One finds a large positive response between one and four quarters after the shock. Most simply, a look at the yearly correlogram $Corr(R_{t,t+1}, \Delta GDP_{t+k,t+k+1})$ reveals that the only lag or lead that is significant is the $k = 1$ year lead, where the correlation is 0.55.

Lamont (2000) uses survey data on investment plans to show that investment plans, which lead investment, correlate better with the stock market. This suggests one may improve the fit of the adjustment cost model by adding a time-to-plan feature, on top of the one-period time-to-build that is present in this model, to account for the cyclical pattern.

$$\Delta GDP_{t\to t+1} = \alpha + \beta R_{\left\{\substack{t\to t+1 \\ t-1\to t}\right.} + \varepsilon_{t+1}$$

	$R_{t\to t+1}$					$R_{t-1\to t}$			
	coeff.	$\sigma(\beta)$ NW	$\sigma(\beta)$ OLS	R^2 (in %)		coeff.	$\sigma(\beta)$ NW	$\sigma(\beta)$ OLS	R^2 (in %)
data	-0.013	0.019	0.021	0.72	data	0.092	0.019	0.016	39.7
PC	-0.032*	0.004	0.014	9.45	PC	0.062	0.007	0.011	38.4
AC	0.171*	0.019	0.020	58.7	AC	-0.002	0.034	0.032	0.01%

TABLE 5: OLS regressions of stock–market return on contemporaneous or lagged GDP growth. Yearly Return from the data, from the Putty-Clay (PC), and from the Adjustment Cost (AC) model. Sample 1951-2000. I display Newey-West corrected standard errors (3 lags) and the OLS standard errors. $*$ = significant at the 5% level. See text for model descriptions.

	Data	PC	AC
$\sigma(R)$	16.3	22.7	10.5
Corr($R^{\text{model}}, R^{data}$)	1	0.64	-0.19

TABLE 6: Statistics on the returns

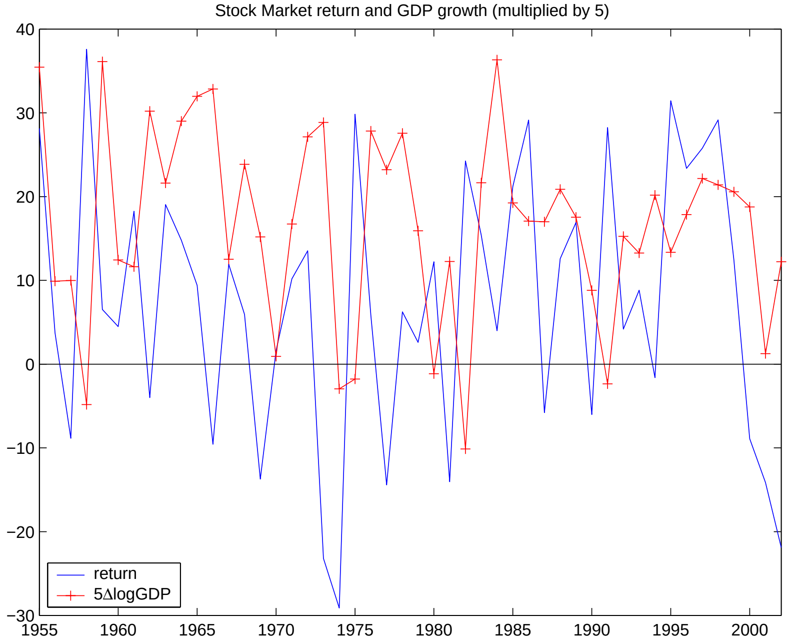

Figure 18: GDP growth (crosses) and US Stock Market Return. 1955-2002, yearly data from BEA and CRSP. Real GDP growth is multiplied by 5 for comparable scale. Note the leading indicator fact, e.g. 2000, 1981;1975;1969;1957.

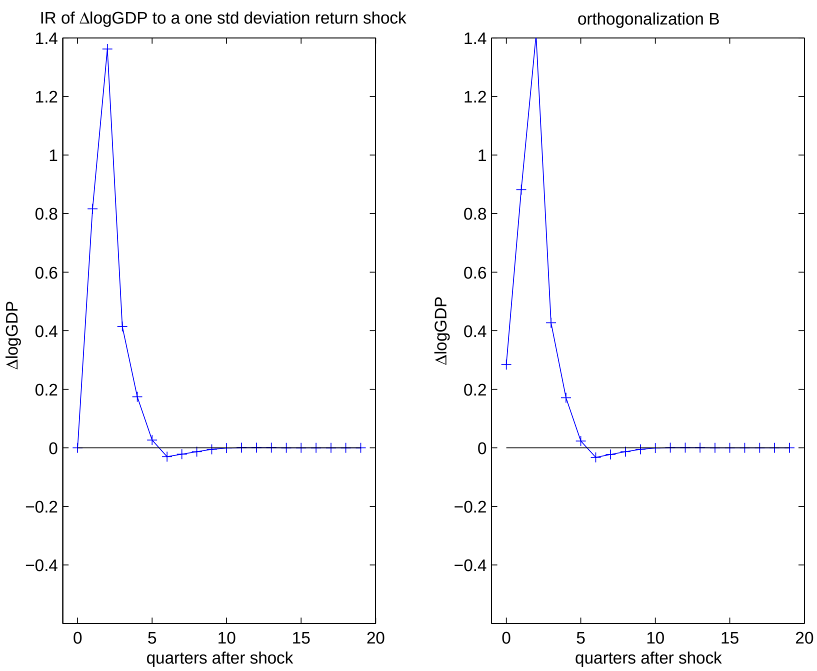

Figure 19: Impulse response of GDP growth to a return shock. Derived from quarterly VAR with 8 lags (1947-2003). Left panel: orthogonalization with return=0 at t=0 if GDP shock; right panel: gdp=0 at t=0 if return shock.

7 Conclusions

This paper showed that introducing a putty-clay technology in a stochastic growth model leads to interesting new asset pricing implications which help us understand both the time series movements in the aggregate stock market, and the differences of expected returns across assets. This work complements Gilchrist and Williams (2000) who found that the putty-clay technology also delivers interesting business cycle implications. Many of these business cycle implications are maintained in the simple variant I develop, which may prove useful for other applications.

The use of this putty-clay technology allowed me to link real sources of risk with financial measures of risk. This approach stands in contrast to the standard practice in finance of estimating the assets' betas as free parameters. Little research in empirical finance and macroeconomics addresses the question, why are some firms more risky and more cyclical? This topic is important for macroeconomics, since firm heterogeneity in response to aggregate shocks implies that some firms account for a large fraction of macroeconomic volatility and play an important role in aggregate fluctuations. However, a limitation of my approach is that it does not explain why return differentials cannot be traced to differences in consumption or market betas, which is the subject of an already very large literature in empirical finance.

The substantive issues that this paper tackles are still not fully resolved, suggesting fruitful directions for future research. A few facts may help us distinguish among the competing explanations of the correlation between book-to-market ratio and returns. First, at the firm level, productivity, inverse capital shares and cyclicality are related to book-to-market ratios and to average returns. Second, at the industry level, productivity, inverse capital shares and cyclicality are related to the market beta and volatility of stocks. Moreover, The book-to-market effect is completely a within-industry effect: between industries, expected returns do not correlate with book-to-market. Explicitly modeling industries, taking into account differences in technology or in the cyclicality of demand, might help. One may also want to incorporate firms size dynamics in more detail. Finally, recent research in empirical finance suggests that theories that emphasize time-varying risk fare much better in explaining the return differentials across portfolios. In turn, the new betas thus obtained will need to be linked to real characteristics.

The aggregate results, while supportive, may seem to require too much rigidity in terms of capital-labor substitutability. But the alternative adjustment cost model has also unattractive features and implications. I believe my results give promise for related models that will link labor movements to firm valuation; labor is a more important input than capital in most industries, and one reason why firms earn quasi-rents is that assembling a team of workers takes time and is costly. Microfoundations such as these for the valuation of capital deserve more study.

References

1. Antras, Pol. 2003. "Is the U.S. Aggregate Production Function Cobb-Douglas? New Estimates of the Elasticity of Substitution". Working paper.

2. Alvarez, Fernando, and Urban Jermann. 2004. "Using Asset Prices to Measure the Persistence of the Marginal Utility of Wealth". Forthcoming, Econometrica.

3. Atkeson, Andrew and Patrick Kehoe. 1999. "Models of Energy Use: Putty-Putty versus Putty-Clay", American Economic Review, 89:4, 1028-1043.

4. Baily, Martin, Eric Bartelsman and John Haltiwanger. 2001. "Labor Productivity: Structural Change and Cyclical Dynamics". Review of Economics and Statistics, August.

5. Bansal, Ravi, Bob Dittmar and Chris Lundblad. 2004. "Consumption, Dividends, and the Cross-Section of Equity Returns". Journal of Finance, forthcoming.

6. Bansal, Ravi, and Amir Yaron. 2004. "Risks for the Long Run". Journal of Finance, forthcoming.

7. Bartelsman, Eric, and Dunne. 2001. "Understanding Productivity: Lessons from Longitudinal Microdata" Journal of Economic Literature.

8. Boldrin, Michele, Lawrence Christiano and Jonas Fisher. 2001. "Habit Persistence, Asset Returns and Business Cycle". American Economic Review 91:1, 149-166.

9. Bresnahan, Timothy, and Daniel Raff. 1991. "Intra-Industry Heterogeneity and the Great Depression: The American Motor Vehicles Industry, 1929-1935". The Journal of Economic History, 51:2, 317-331.

10. Caballero, Ricardo, and Mohamed Hammour. 1994. "The Cleansing Effect of Recessions". American Economic Review, 84:5, 1350-1368.

11. Campbell, John and Tuomo Vuolteenaho. 2004. "Bad Beta, Good Beta". American Economic Review, forthcoming.

12. Carlson, Murray, Adlai Fisher and Ron Giammarino. 2004. "Corporate Investment and Asset Price Dynamics: Implications for the Cross-Section of Returns ". Forthcoming, Journal of Finance.

13. Christiano, Lawrence and Jonas Fisher. 2003. "Stock Market and Investment Goods Prices: Implications for Macroeconomics ". NBER Working Paper #10031.

14. Cochrane, John. 1991. "Production-Based Asset Pricing and the Link Between Stock Returns and Economic Fluctuations", Journal of Finance, 46:1, 209:237.

15. Cogley Timothy, and James Nason. 1995. "Output Dynamics in Real-Business-Cycle Models". American Economic Review, 85:3 492-511.

16. Cooper, Ian. 2003. "Asset Pricing Implications of Non-Convex Adjustment Costs of Investment". Norwegian School of Management Working Paper.

17. Dwyer, Douglas. 2001. “Plant-level productivity and the Market Value of a Firm”. Census Bureau CES working paper.

18. Fisher, Jonas. 2003. “Technology Shocks Matter”. Federal Reserve Bank of Chicago working paper.

19. Fama, Eugene and Kenneth French. 1992. “The Cross-Section of Expected Stocks Returns”. Journal of Finance, 47:2, 427-465.

20. Fama, Eugene and Kenneth French. 1995. “Size and Book-to-Market Factors in Earnings and Returns”. Journal of Finance, 50:1., 131-155.

21. Fama, Eugene and Kenneth French. 1996. “Multifactor Explanations of Asset Pricing Anomalies”. Journal of Finance, 51:1, 55-84.

22. Fama, Eugene and Kenneth French. 1997. “Industry costs of equity”. Journal of Financial Economics, 43:2, 153-193.

23. Fama, Eugene and Kenneth French. 1998. “Value versus Growth: The International Evidence”. Journal of Finance, 53:6, 1975-1999.

24. Fischer, Stanley and Robert Merton. 1984. “Macroeconomics and Finance: The Role of the Stock Market”, Carnegie-Rochester Conference Series on Public Policy,21, pp.57-108.

25. Gala, Vito. 2004. “Investment and Stock Returns”. University of Chicago GSB working paper.

26. Gilchrist, Simon, and John Williams. 2000. “Putty-Clay and Investment: A Business Cycle Analysis”, Journal of Political Economy, 108:5, 928-960.

27. Gomes Joao, Leonid Kogan, and Lu Zhang. 2003. “Equilibrium Cross Section of Returns”. Journal of Political Economy, 114:4, 693-732.

28. Gomme, Paul and Jeremy Greenwood. “On the cyclical allocation of risk”. JEDC, 19:1-2, 91-124.

29. Greenwood, Jeremy, Zvi Hercowitz and Per Krusell. 1997. “Long-Run Implications of Investment-Specific Technological Change”. American Economic Review, 87:3, 342-362.

30. Greenwood, Jeremy, Zvi Hercowitz and Per Krusell. 2001. “The role of investment-specific technological change in the business cycle”. European Economic Review, 40:1, 91-115.

31. Hayashi, Fumio. 1982. “Tobin’s Marginal q and Average q: A Neoclassical Interpretation”. Econometrica, 50:1, 213-224.

32. Hall, Robert. 2001. “The Stock Market and Capital Accumulation”. American Economic Review, 91:6, 1185-1202.

33. Hansen, Gary and Edward Prescott. 2000. “Capacity Constraints, Asymmetries, and the Business Cycle”. Working paper.

34. Hansen, Lars, John Heaton and Nan Li. 2004. “Consumption Strikes Back”. Working paper.

35. Jermann, Urban.1998: “Asset Pricing in Production Economies”, Journal of Monetary Economics, 41:2, 257-275.

36. Johansen, Leif. 1959. "Substitution versus Fixed Production Coefficients in the Theory of Economic Growth: A Synthesis". Econometrica, 27:2, 157-176.

37. Jovanovic, Boyan and Peter Rousseau. 2002. "Moore's law and Learning-by-doing". Review of Economic Dynamics 5, 346-375.

38. Julliard, Christian and Jonathan Parker. 2004. "Consumption Risk and the Cross-Section of Expected Returns". Journal of Political Economy, forthcoming.

39. Lamont, Owen. 2000. "Investment Plans and Stock Returns", Journal of Finance, 55:6, 2719-2745.

40. Lustig, Hanno, and Stijn Van Nieuwerburgh, 2004. "Housing Collateral, Consumption Insurance and Risk Premia: an Empirical Perspective". Journal of Finance, forthcoming.

41. Lettau Martin and Sydney Ludvigson. 2001. "Resurrecting the (C)CAPM: A Cross-Sectional Test When Risk Premia Are Time-Varying". Journal of Political Economy. 109:6, 1238:1287.

42. Lettau Martin and Sydney Ludvigson. 2003. "Modelling and measuring the risk-return trade-off". Working paper.

43. Pakos, Michal. 2004. "Asset Pricing with Durable Goods and Non-Homothetic Preferences". Working paper, University of Chicago GSB.

44. Perfect, Steven and Kenneth Wiles. 1994. "Alternative constructions of Tobin's q: An empirical comparison". Journal of Empirical Finance, 1,313:341.

45. Piazzesi, Monika, Martin Schneider and Selale Tuzel. 2004. "Housing, consumption, and asset pricing". Working paper.

46. Rouwenhorst, G. 1995. "Asset Pricing Implications of Equilibrium Business Cycle Models," in T. Cooley, ed., Frontiers of Business Cycle Research, Princeton University Press.

47. Santos, Tanos and Pietro Veronesi. "Labor Income and Predictable Stock Returns". Working paper, 2004.

48. Schumpeter, Joseph. 1942. "Capitalism, Socialism and Democracy". New York: Harper, 1942.

49. Stock, James and Mark Watson. 2003. "Forecasting Output and Inflation: The Role of Asset Prices". Journal of Economic Literature, 41, 788-829.

50. Syverson, Chad. 2004. "Product Substitutability and Productivity Dispersion". Review of Economics and Statistics, May.

51. Tallarini, Thomas. 2000. "Risk-sensitive real business cycles". Journal of Monetary Economics, 45:3, 507-532.

52. Wei, Chao. 2003. "Energy, the stock market, and the putty-clay investment model". American Economic Review. 93:1, 311-323.

53. Yogo, Motohiro. 2004. "A Consumption-Based Explanation of Expected Stock Returns". Working Paper.

54. Zhang, Lu. 2004. "The Value Premium". Journal of Finance, forthcoming.