Preliminary and incomplete

The European Unemployment Experience: Theoretical Robustness

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Abstract

1 Introduction

This paper puts three models of the labor market through their paces as a way to evaluate the robustness of what we believe is an important interaction between microeconomic shocks and labor market institutions. The exercise 'looks under the hoods' of the three models and illuminates the economic forces at work in each of them.

We study the same microeconomic shocks and unemployment-benefit institutions that we have used to unravel mysterious behavior of post World War II U.S. and European unemployment rates. During the 1950s and 60s, unemployment rates in Europe were lower than in the U.S. During the 1980s and 1990s, they became persistently higher. In Ljungqvist and Sargent (2002), we explained that outcome with an equilibrium search model in which people and technologies are identical in 'Europe' and 'the U.S.', but in which two labor market institutions differ: in Europe, there is a tax on job destruction and unemployment benefits are longer and more generous. Holding these different labor market institutions constant across time in both Europe and the U.S., we attributed the different outcomes across the 50s-60s and the 80s-90s to a change in the physical economic environment. In particular, our model imputed *both* the lower European unemployment of the 50s and 60s *and*the higher European unemployment of the 80s and 90s to the ways that those labor market institutions induce workers respond to an increase in uncertainties about workers'

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labor market prospects that we expressed as increased 'turbulence'. We modelled 'turbulence' as a change in a single parameter that governs an instantaneous loss in human capital that visits workers who suffer an 'involuntary' (but not a 'voluntary') job dissolution. We showed that our representation of increased turbulence produced labor market outcomes that are consistent with studies starting with Gottschalk and Moffit (1994) that have documented increased volatility of permanent and transitory components of earnings.[1]

Like any quantitative model, ours has special features and is as distinguished for what it ignores as by what it includes. By ignoring the benefits of insurance, we designed our model to focus on adverse incentives: our workers are risk-neutral and care only about present values of after-tax earnings and benefits. There are no firms in the model and no bargaining over wages or any other theory of endogenous wage determination. Instead, in the spirit of John McCall's (1970) search model, our model takes as exogenous a fixed distribution of wages. We add dynamics for skills to McCall's model and take earnings to be a wage draw times a skill level. Skills accrue during periods of employment and deteriorate during periods of unemployment.

Den Haan, Haefke, and Ramey (2001), referees, and discussants at several conferences have all questioned whether an interaction between our turbulence shock and labor market institutions would have produced similar outcomes had we adopted some other model that incorporated at least some of the features missing from our model, e.g., firms that post vacancies, equilibrium wages that emerge from bargaining, a Beveridge curve, adverse congestion effects captured in a matching function, risk-averse workers who engage in precautionary saving because of incomplete markets, physical capital, or perhaps a competitively determined market wage instead of bargaining. Others of our friends (e.g., Prescott (2002), Rogerson (2003)) have questioned our findings by claiming that a model of a frictionless labor market with comprehensive insurance arrangements can explain the important differences between Europe and U.S. labor market outcomes once cross-country differences in tax wedges are recognized.

One model cannot include all of these features simultaneously because some contradict others, but the literature has provided several alternative models that capture some of these features in coherent ways. With an eye to the question at hand, this paper constructs quantitative versions of three such models. We designed the models so that common features in their physical environments allow us to represent the kind of interactions between skill dynamics and both unemployment benefit levels and the job destruction taxes that Ljungqvist and Sargent (2002) focussed on. The models are (1) a matching model (really a suite of matching models) inspired by Den Haan, Haefke, and Ramey (2001) and Mortensen and Pissarides (1999); (2) an adaptation of a search-islands model of Alvarez and Veracierto (2001); and (3) a 'one-big-happy-family' model with comprehensive insurance arrangements in the style of Hansen (1985), Rogerson (1988) and Prescott (2004).

[1]For a survey of the empirical evidence on increased earnings volatility, see Katz and Autor (1999).

2 Three economic environments

2.1 Common features of the environments

There is a continuum of potential workers. In the matching model, each worker faces a constant probability ρ of dying. In the other two frameworks, ρ is the probability that a worker will retire and become unable to work, and σ is the probability that a retired worker dies. Agents who exit a model are replaced by newborn workers, keeping the total population and the shares of workers and retirees constant over time.

Besides stochastic retirement and death, there are three other sources of exogenous uncertainty. First, an employed worker faces a probability π^o that his job is "terminated." Second, workers experience stochastic accumulation or deterioration of skills conditional on employment status and instances of exogenous job terminations. Third, idiosyncratic shocks impinge on employed workers' productivity.[2]

Two possible skill levels of a worker are indexed by $h \in \{0, H\}$. All newborn workers enter the labor force with the lowest skill level, $h = 0$. An employed worker with skill level h faces a probability $p^n(h, h')$ that his skill level at the beginning of next period is h', conditional on no exogenous job termination. In the event of an exogenous job termination, a laid off worker with last period's skill level h faces a probability $p^o(h, h')$ that his skill level becomes h'. A worker's skill level remains unchanged during an unemployment spell. We will consider the following parameterization of the skill technology,

$$p^n = \begin{bmatrix} 1-\pi^u & \pi^u \\ 0 & 1 \end{bmatrix}, \tag{1}$$

$$p^o = \begin{bmatrix} 1 & 0 \\ \pi^d & 1-\pi^d \end{bmatrix}. \tag{2}$$

That is, conditional on no exogenous job termination, an employed worker faces a probability π^u of experiencing an upgrade in skills next period unless he has already attained the high skill level $h = H$. After an exogenous job termination, a laid off worker faces a probability π^d of seeing an immediate downgrade in skills unless he is already at the lowest skill level $h = 0$.

The process of uniting firms and workers differs across the three frameworks but has several common features. Firms face a cost μ associated with posting a vacancy in the matching model or of creating a job that can be immediately filled in the other two frameworks. We model a new job opportunity as a draw of productivity z from a distribution $Q_h^o(z)$. The productivity of an ongoing job is governed by a Markov process: $Q_h(z, z')$ is the probability that next period's productivity is z', given current productivity z. The probability distributions, $Q_h^o(z)$ and $Q_h(z, z')$, will depend on the worker's skills h in the matching model, but not in the other two frameworks. The conditional probability distribution $Q_h(z, z')$ first-order

[2] There will also be sources of *endogenous* uncertainty in each model. For example, any endogenous job separations will impose additional uncertainty on individual agents beyond that associated with exogenous terminations. Whether the agents can insure against such risks varies across our models.

stochastically dominates the conditional distribution associated with a lower value of z, i.e., for any two productivity levels z and $\hat{z} < z$, the probability distribution satisfies

$$\sum_{z' \leq \bar{z}} Q_h(z, z') < \sum_{z' \leq \bar{z}} Q_h(\hat{z}, z'), \qquad \text{for all } \bar{z}. \tag{3}$$

The government levies layoff taxes on job destruction and provides benefits to the unemployed. In particular, it imposes a layoff tax Ω on each exogenous job termination or endogenous job separation, except when a separation is due to a retirement. It pays unemployment benefits as a replacement rate η on a measure of past income that differs somewhat across models. Newborn workers are assumed to be entitled to the lowest benefit level in the economy. The government runs a balanced budget policy where unemployment benefits are financed with layoff tax revenues and some model-specific tax instruments.

2.2 Matching model

Our formulation of the matching model is inspired by Den Haan, Haefke and Ramey (2001), who include skill dynamics in one version of their matching framework.[3] Low-skilled ($h = 0$) and high-skilled ($h = H$) workers are distinguished by the productivity distributions from which they draw. In particular, the probability distributions of high-skilled workers stochastically dominate the corresponding probability distributions of low-skilled workers, i.e.,

$$\sum_{z' \leq \bar{z}} Q_H^o(z') < \sum_{z' \leq \bar{z}} Q_0^o(z') \qquad \text{and} \qquad \sum_{z' \leq \bar{z}} Q_H(z, z') < \sum_{z' \leq \bar{z}} Q_0(z, z'), \tag{4}$$

for all $\bar{z}$, and given that z is a permissible productivity level for both low-skilled and high-skilled workers. We follow Den Haan, Haefke and Ramey (2001) and assume that benefits are determined by a replacement rate η on the average after-tax labor income in the worker's skill category of his last employment.[4] Hence, we can index a worker's benefit entitlement to his skill level in his last employment, $b \in \{0, H\}$, with a benefit level given by some function $\tilde{b}(b)$. Let $u(h, b)$ be the number of unemployed workers with skill level h and benefit entitlement b. The total number of unemployed $\bar{u}$ is then given by

$$\bar{u} = \sum_{h,b} u(h, b). \tag{5}$$

We replace Den Haan, Haefke and Ramey's (2001) assumption of an exogenous number of firms by the common assumption of free entry and a zero-profit condition. Let v be

[3] We are thankful to Wouter Den Haan, Christian Haefke and Garey Ramey for generously sharing their computer code, which we have adapted to our model specification.

[4] We make two simplifications to Den Haan, Haefke and Ramey's (2001) specification of the benefit system. First, newborn workers are entitled to the lowest benefit level without having to work one period. Second, workers who experience an upgrade in skills are immediately entitled to the higher benefit level even if the match breaks up immediately. These assumptions greatly simplify solving the model. We believe that they do not unduly affect aggregate outcomes.

the endogenous number of vacancies and $M(v, \bar{u})$ is the exogenous matching function that determines the number of matches for given values of v and $\bar{u}$. The matching function is increasing, concave, and linearly homogeneous;

$$M(v, \bar{u}) = \bar{u}\, M\left(\frac{v}{\bar{u}}, 1\right) \equiv \bar{u}\, m(\theta), \tag{6}$$

where the ratio $\theta \equiv v/\bar{u}$ is the endogenously determined degree of "market tightness." Under the assumption of random matching, the job finding probability, $M/\bar{u} = m(\theta)$, is an increasing function of market tightness, and the probability of filling a vacancy, $M/v = m(\theta)/\theta$, is a decreasing function of market tightness. At first we assume a single matching function for all vacancies and all unemployed workers. Later we will also consider multiple matching functions.

We keep Den Haan, Haefke and Ramey's (2001) specification that postulates risk neutral agents who care only about consumption c_t, i.e., the utility function is

$$E_0 \sum_{t=0}^{\infty} \beta^t (1-\rho)^t c_t, \tag{7}$$

where the discounting of future utility is determined by the agent's subjective discount factor $\beta \in (0,1)$ and the survival probability $(1-\rho)$. We also keep their assumption that the government finances the unemployment compensation scheme by levying a flat-rate tax τ on workers' output, in addition to the layoff tax revenues in our current setting.

2.3 Search model with capital and incomplete markets

We adopt and modify the model of Alvarez and Veracierto (2001) who specify the following preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t (1-o_t)^t \left[\log c_t + A\frac{(1-s_t)^{\gamma}-1}{\gamma}\right], \qquad \text{with } A>0,\ \gamma>-1, \tag{8}$$

where $(1-o_t)$ is the survival probability with $o_t = \rho$ if the agent is of working age and $o_t = \sigma$ if the agent is retired; and $s_t \in [0,1)$ is the agent's choice of search intensity if unemployed and of working age. The search intensity s_t determines an unemployed worker's probability s_t^{ξ} of finding a centralized labor market in the next period, where $0 \le \xi \le 1$. Workers who find the labor market are offered a single market-clearing wage rate. To accommodate our new feature that workers differ in terms of their skills, we let w denote the wage rate per unit of skill, where the skill of a low-skilled worker is normalized to one and the skill of a high-skilled worker is $1+H$. Hence, a low-skilled worker earns w and a high-skilled worker earns $(1+H)w$ and the after-tax wage rate is w.

In the spirit of our matching model, we abstract from Alvarez and Veracierto's firm size dynamics and let each firm employ only one worker. The firm also employs physical capital. The production function of a firm is

$$z_t k_t^{\alpha} (1+h_t)^{1-\alpha}, \qquad \text{with } \alpha \in (0,1), \tag{9}$$

where z_t is the current productivity level, h_t is the skill of the firm's worker and k_t is physical capital, which depreciates at the rate δ. Output can be devoted to consumption, investment in physical capital and startup costs associated with new jobs. The rest of Alvarez and Veracierto's analysis of firms enters our framework as follows. After incurring a startup cost μ, the firm creates a job opportunity in the following period by drawing a productivity level from the distribution $Q^o(z)$. After seeing the realization z, the firm decides whether to hire a worker from the centralized labor market. We retain Alvarez and Veracierto's key assumption that the matching of firms and workers in the labor market takes place under the veil of ignorance about their partner's state vector. Thus, the firm employs a worker drawn randomly from the pool of unemployed without observing any of his characteristics. After joining the worker, the firm must pay the wage rate w per unit of skill and can hire the profit-maximizing amount of physical capital conditional on the worker's skill level. The firm must retain the worker for at least one period.

Besides markets for goods, labor, and capital rentals, there is a market in which agents can acquire non-negative holdings of risk-free assets. Following Alvarez and Veracierto, we postulate financial intermediation through a competitive banking sector that accepts deposit from agents that are invested in physical capital and the ownership to firms and that earn the net interest rate i. The banking sector rents physical capital to firms at the competitive rental rate $i + \delta$. The banks that invest in the ownership to firms will hold a fully diversified portfolio of all firms so that there is no exposure to individual firms' idiosyncratic fortunes.

Due to the non-negativity constraint on agents' asset holdings, retired agents die with accidental bequest to an offspring to whom they are indifferent. In the spirit of Alvarez and Veracierto, we assume that an agent who dies is replaced by a newborn unemployed worker who inherits the assets of his predecessor. We specify that this newborn worker has the low skill level, $h = 0$.

The government pays unemployment compensation equal to a replacement rate η times an unemployed worker's last labor earnings. Newborn workers are entitled to the lowest benefit level in the economy. Together with revenues from collecting layoff taxes, the government's balances its budget by setting an income tax rate encoded in the difference between before-tax and after-tax wage rates, $(w^* - w)$.

2.4 Representative family model

The representative family model makes three changes to our search model:

1. Agents belong to a "representative family."

2. The labor market is frictionless.

3. A disutility of working replaces the disutility of search.

The representative family consists of a continuum of "lineages," indexed on the unit interval. Each agent retires with probability ρ then dies with probability σ. The new agent who replaces the deceased agent is his descendant, i.e., the next member of the lineage. The representative family has a dynastic utility function over consumption and disutility of work,

$$\int_0^1 \sum_{t=0}^{\infty} \beta^t u(c_t^j,\, n_t^j) dj \;=\; \int_0^1 \sum_{t=0}^{\infty} \beta^t \Big[\log(c_t^j) - n_t^j A\Big] dj\;, \tag{10}$$

where c_t^j is lineage $j's$ consumption at time t and n_t^j equals one if the current member of lineage j is working and equals zero otherwise. The parameter $A > 0$ captures the disutility of working.

Otherwise the assumptions are the same as in our search model. For example, consider the timing of shocks. In each period, the order of shocks are 1) retirement, 2) exogenous job termination, and 3) skill evolution. Thereafter, firms and families take actions, and job seekers are frictionlessly matched with vacancies. As before, a firm's hiring is done under the veil of ignorance about the job seeker's skill and renting physical capital can be chosen after seeing the worker's skill level.

The representative family with a continuum of members can provide insurance within the family.

3 Matching model

We again let v and $\bar{u}$ denote aggregate numbers of vacancies and unemployed workers. Aggregate unemployment $\bar{u}$ is the sum of unemployed workers with different skills h and benefit entitlements b, as shown in equation (5). Given the matching function in equation (6), we let $\lambda^f(h,b)$ denote the probability that a firm meets a worker with skills h and benefit entitlement b,

$$\lambda^f(h,b) = \frac{M(v,\bar{u})}{v}\,\frac{u(h,b)}{\bar{u}} = m(\theta)\,\frac{u(h,b)}{v}. \tag{11}$$

Analogously, we let $\lambda^w(h,b)$ denote the probability that a worker with skills h and benefit entitlement b is matched with a vacancy. When there is a single matching function, this probability is the same value

$$\lambda^w(h,b) = \frac{M(v,\bar{u})}{\bar{u}} = m(\theta), \tag{12}$$

but it will differ across workers when we introduce multiple matching functions.

3.1 Match surplus

When an unemployed worker with skills h and benefit entitlement b meets a firm with a vacancy, the firm-worker pair draws productivity z from a distribution $Q_h^o(z)$ that depends on the worker's skill level h. Whether the firm and the worker form a match depends on the match surplus $S^o(h,z,b)$ defined by

$$\begin{aligned} S^o(h,z,b) &= \max\Big\{(1-\tau)z - [1-\beta(1-\rho)]\,W(h,b) \\ &\quad +\beta(1-\rho)\Big[-\pi^o\Omega + (1-\pi^o)\sum_{h',z'} p^n(h,h')Q_{h'}(z,z')S(h',z')\Big]\,,\ 0\Big\}, \end{aligned} \tag{13}$$

where τ is a tax rate on the firm's output, $W(h,b)$ is the worker's outside value, and $S(h,z)$ is the match surplus associated with a continuing match. If the surplus is positive, a match is formed. The assumption of free entry makes the firm's outside value zero. We assume that the firm and worker split the match surplus $S^o(h,z,b)$ through Nash bargaining, with the outside values as threat points. Let $\psi \in (0,1)$ denote the worker's share of the match surplus. Because both parties want a positive match surplus, there is mutual agreement on whether to form a match. The reservation productivity $\bar{z}^o(h,b)$ satisfies

$$S^o(h,\bar{z}^o(h,b),b) = 0. \tag{14}$$

The match surplus of a continuing match is

$$\begin{aligned} S(h,z) &= \max\Big\{(1-\tau)z - [1-\beta(1-\rho)]\,W(h,h) \\ &\quad +\beta(1-\rho)\Big[-\pi^o\Omega + (1-\pi^o)\sum_{h',z'} p^n(h,h')Q_{h'}(z,z')S(h',z')\Big]\,,\ -\Omega\Big\}. \end{aligned} \tag{15}$$

The government's policy of imposing a layoff tax Ω on matches that break makes (15) differ from expression (13) because of [5] A reservation productivity, $\bar{z}(h)$ characterizes whether a match is dissolved. That reservation productivity satisfies

$$S\left(h,\bar{z}(h)\right) = -\Omega. \tag{16}$$

A worker with skills h and benefit entitlement b has an outside option $W(h,b)$ given by

$$W(h,b) = \tilde{b}(b) + \beta(1-\rho)\Big[W(h,b) + \lambda^w(h,b)\sum_z \psi S^o(h,z,b)Q^o_h(z)\Big]. \tag{17}$$

3.2 Equilibrium condition

In equilibrium, firms must expect to break even when posting a vacancy, i.e., the following zero-profit condition must hold,

$$\mu = \beta(1-\psi)\sum_{h,z,b}\lambda^f(h,b)S^o(h,z,b)Q^o_h(z). \tag{18}$$

This condition will pin down the equilibrium value of market tightness θ.

[5] Another difference between expressions (13) and (15) is that an employed worker's benefit entitlement is encoded in his skill level h, so there is one less state variable in surplus expression (15).

3.3 Wage determination

Different wage structures can support the same equilibrium allocation. We follow Mortensen and Pissarides (1999) and work with a two-tier wage system.[6] In particular, when a firm with a vacancy meets an unemployed worker with skill h and benefit entitlement b, they bargain. The worker's outside value is $W(h,b)$ and the firm's outside value is zero. The layoff tax does not directly affect bargaining, since if the firm and worker do not reach an agreement, they do not incur a layoff tax. But if they do form a match, the firm must pay the layoff tax after any future breakup. This is captured in the Nash bargaining by setting the firm's threat point equal to $-\Omega$ in future negotiations.

As in Mortensen and Pissarides (1999), these assumptions give rise to a two-tier wage system. There is one wage function $\tilde{w}^o(h,z,b)$ for the initial round of negotiations between a newly matched firm and a worker and another wage function $\tilde{w}(h,z)$ associated with renegotiations in an ongoing match. These wage functions satisfy

$$\begin{aligned}\tilde{w}^o(h,z,b) &= W(h,b) + \psi S^o(h,z,b) - \beta(1-\rho)\Big\{\pi^o \sum_{h'} p^o(h,h')W(h',h) \\ &\quad +(1-\pi^o)\sum_{h',z'} p^n(h,h')Q_{h'}(z,z')\Big(\psi\big[S(h',z')+\Omega\big] + W(h',h')\Big)\Big\}, \quad (19)\\ \tilde{w}(h,z) &= W(h,h) + \psi\big[S(h,z)+\Omega\big] - \beta(1-\rho)\Big\{\pi^o \sum_{h'} p^o(h,h')W(h',h) \\ &\quad +(1-\pi^o)\sum_{h',z'} p^n(h,h')Q_{h'}(z,z')\Big(\psi\big[S(h',z')+\Omega\big] + W(h',h')\Big)\Big\}, \quad (20)\end{aligned}$$

3.4 Multiple matching functions

We will also consider multiple matching functions. Mortensen and Pissarides (1999) postulated that low-skilled and high-skilled workers get matched with vacancies in separate matching functions. They assumed that workers are permanently endowed with either low or high skills.

We shall consider three specifications:

1. Separate matching functions for unemployed workers based on their skills, i.e., there is an equilibrium quantity of vacancies $v(h)$ for each value of the unemployed worker's skills, $h \in \{0, H\}$.

[6] The risk neutral firm and worker would be indifferent between adhering to this two-tier wage system or the one implied by workers receiving a fraction ψ of the match surplus $S(h,z)$ in every period (which would include the worker paying a share ψ of any future layoff tax). As emphasized by Ljungqvist (2002), it is only the wage profile that is affected by the two-tier wage system. Optimal reservation productivities are unaffected. Hence, under the two-tier wage system, a newly hired worker is effectively posting a bond that equals his share of the future layoff tax.

2. Separate matching functions for unemployed based on their benefit entitlements which in turn are based on the skill category that they belonged to in their last employment. Here there is an equilibrium quantity of vacancies $v(b)$ for each value of the unemployed worker's skills in their last employment, $b \in \{0, H\}$.

3. Separate matching functions for unemployed workers based on both their current skill h and their skill b in their last employment. Here there is an equilibrium quantity of vacancies $v(h, b)$ for each pair of values $(h, b) \in \{0, H\} \times \{0, H\}$.

Case 1: When workers are sorted according to their current skills h, market tightness in each separate market is given by

$$\theta(h) = \frac{v(h)}{\sum_b u(h, b)}. \tag{21}$$

The probabilities that an unemployed worker finds a vacancy and that a firm with a vacancy finds a worker, respectively, equal

$$\lambda^w(h, b) = \frac{M\Big(v(h),\, \sum_b u(h, b)\Big)}{\sum_b u(h, b)} = m(\theta(h))\,, \tag{22}$$

$$\lambda^f(h, b) = \frac{M\Big(v(h),\, \sum_b u(h, b)\Big)}{v(h)} \frac{u(h, b)}{\sum_b u(h, b)} = m(\theta(h))\, \frac{u(h, b)}{v(h)}. \tag{23}$$

The zero-profit condition for posting a vacancy in the market for unemployed workers with skill h, becomes

$$\mu = \beta(1 - \psi) \sum_{z,b} \lambda^f(h, b) S^o(h, z, b) Q_h^o(z). \tag{24}$$

Case 2: When workers are sorted according to their skills b in their last employment, the market tightness in each separate market is given by

$$\theta(b) = \frac{v(b)}{\sum_h u(h, b)}. \tag{25}$$

The probabilities that an unemployed worker finds a vacancy and that a firm with a vacancy finds a worker, respectively, equal

$$\lambda^w(h, b) = \frac{M\Big(v(b),\, \sum_h u(h, b)\Big)}{\sum_h u(h, b)} = m(\theta(b))\,, \tag{26}$$

$$\lambda^f(h, b) = \frac{M\Big(v(b),\, \sum_h u(h, b)\Big)}{v(b)} \frac{u(h, b)}{\sum_h u(h, b)} = m(\theta(b))\, \frac{u(h, b)}{v(b)}. \tag{27}$$

The zero-profit condition for posting a vacancy in the market for unemployed workers whose skills were b in their last employment becomes

$$\mu \;=\; \beta(1-\psi)\sum_{h,z}\lambda^f(h,b)S^o(h,z,b)Q_h^o(z). \tag{28}$$

Case 3: When workers are sorted both according to their skills h and their skills b in their last employment, the market tightness in each separate market, indexed by (h,b), is given by

$$\theta(h,b) = \frac{v(h,b)}{u(h,b)}. \tag{29}$$

The probabilities that an unemployed worker finds a vacancy and that a firm with a vacancy finds a worker, respectively, equal

$$\lambda^w(h,b) \;=\; \frac{M\Big(v(h,b),\, u(h,b)\Big)}{u(h,b)} = m(\theta(h,b))\,, \tag{30}$$

$$\lambda^f(h,b) \;=\; \frac{M\Big(v(h,b),\, u(h,b)\Big)}{v(h,b)} = m(\theta(h,b))\,\frac{1}{\theta(h,b)}. \tag{31}$$

The zero-profit condition for posting a vacancy for unemployed workers with current skill h and skill b in their last employment becomes

$$\mu \;=\; \beta(1-\psi)\lambda^f(h,b)\sum_{z}S^o(h,z,b)Q_h^o(z). \tag{32}$$

4 Search model with capital and incomplete markets

4.1 Firm's problem

The Bellman equations of an existing firm are

$$V^f(h,z) \;=\; \max\Big\{\tilde{V}^f(h,z)\,,-\Omega\Big\}\,, \tag{33}$$

$$\tilde{V}^f(h,z) \;=\; \max_k\Big\{zk^\alpha\,(1+h)^{1-\alpha} - w^*\,(1+h) - (i+\delta)\,k\Big\} + \frac{1-\rho}{1+i}\Big[-\pi^o\Omega + (1-\pi^o)\sum_{h',z'}p^e(h,h')\,V^f(h',z')\,Q\,(z,z')\Big]. \tag{34}$$

The first-order condition for capital in problem (34) is

$$z\alpha k^{\alpha-1}\,(1+h)^{1-\alpha} = (i+\delta)\,, \tag{35}$$

which can be solved for k to obtain the firm's policy function for the optimal choice of capital,

$$k(h,z) = \left[\frac{z\,\alpha}{i+\delta}\right]^{\frac{1}{1-\alpha}} (1+h)\,. \tag{36}$$

Associated with the solution to an existing firm's optimization problem is a reservation productivity $\bar{z}(h)$ that satisfies

$$\tilde{V}^f(h,\bar{z}(h)) = -\Omega. \tag{37}$$

For later use we define the following indicator function,

$$\Lambda(h,z) = \begin{cases} 1, & \text{if } z \geq \bar{z}(h); \\ 0, & \text{otherwise.} \end{cases} \tag{38}$$

The break-even condition for starting a new firm is

$$\mu = \frac{1}{1+i}\sum_z \max\left\{(1-\phi)\tilde{V}^f(0,z) + \phi\tilde{V}^f(H,z)\,,\,0\right\} Q^o(z)\,, \tag{39}$$

where μ is the start-up cost and ϕ is the fraction of high-skilled workers among all new hires. The maximization operator in the above expression defines a reservation productivity $\bar{z}^o$ that determines whether a new firm hires a worker after it observes its productivity level. The reservation productivity satisfies

$$(1-\phi)\tilde{V}^f(0,\bar{z}^o) + \phi\tilde{V}^f(H,\bar{z}^o) = 0. \tag{40}$$

For later use we define the following indicator function,

$$\Lambda^o(z) = \begin{cases} 1, & \text{if } z \geq \bar{z}^o; \\ 0, & \text{otherwise.} \end{cases} \tag{41}$$

The productivity distribution of new firms that hire workers is

$$\Gamma(z) = \frac{\Lambda^o(z)\,Q^o(z)}{\sum_{z'}\Lambda^o(z')\,Q^o(z')}. \tag{42}$$

4.2 Household's problem

We define three value functions $V^n(a,h,z)$, $V^u(a,h,b)$, and $V^r(a)$ for an employed worker, an unemployed worker and a retired worker, respectively. The state variables are last period's assets (a), skill level (h), the firm's current productivity level if employed (z), and the worker's benefit entitlement if unemployed (b). The benefit entitlement is determined by the worker's last earnings, so here we index b by the worker's skill level when he last worked. Newborn workers are entitled to the lowest benefits level of ηw, so that unemployed newborn workers and laid off unskilled workers have the same benefit entitlement indexed by $b = 0$.

The problem of an employed agent is to maximize

$$
\begin{aligned}
V^n(a,h,z) \quad = \quad \max_{c,a'} \Bigg[& \log c + \beta\rho V^r(a') + \beta(1-\rho)\Bigg(\pi^o \sum_{h'} p^o\left(h,h'\right) V^u(a',h',h) \\
& +(1-\pi^o) \sum_{h',z'} p^e(h,h') \Big\{ V^n(a',h',z')\,\Lambda\left(h',z'\right) \\
& +V^u(a',h',h)\left[1-\Lambda\left(h',z'\right)\right)\Big\} Q\left(z,z'\right]\Bigg)\Bigg] \qquad (43)
\end{aligned}
$$

subject to

$$
\begin{aligned}
c + a' &\leq (1+i)\,a + (1+h)\,w\,, \\
c,\ a' &\geq 0\,.
\end{aligned}
$$

Two policy functions $\bar{c}^n(a,h,z)$ and $\bar{a}^n(a,h,z)$ give optimal levels of consumption and savings, respectively.

The problem of the unemployed agent is to maximize

$$
\begin{aligned}
V^u(a,h,b) \quad = \quad \max_{c,a',s} \Bigg[& \log c + A\frac{(1-s)^\gamma - 1}{\gamma} + \beta\rho V^r(a') + \beta(1-\rho) \\
& \cdot\Bigg(\left(1-s^\xi\right) V^u(a',h,b) + s^\xi \sum_{z'} V^n(a',h,z')\,\Gamma\left(z'\right)\Bigg)\Bigg] \qquad (44)
\end{aligned}
$$

subject to

$$
\begin{aligned}
c + a' &\leq (1+i)\,a + \eta\,(1+b)\,w\,, \\
c,\ a',\ s &\geq 0\,.
\end{aligned}
$$

Three policy functions $\bar{c}^u(a,h,b)$, $\bar{a}^u(a,h,b)$ and $\bar{s}\,(a,h,b)$ give optimal levels of consumption, savings, and search effort, respectively.

The problem of a retired agent is to maximize

$$
V^r(a) \quad = \quad \max_{c,a'} \Big[\log c + \beta(1-\sigma) V^r(a') \Big] \qquad (45)
$$

subject to

$$
\begin{aligned}
c + a' &\leq (1+i)\,a\,, \\
c,\ a' &\geq 0\,.
\end{aligned}
$$

Two policy functions, $\bar{c}^r(a)$ and $\bar{a}^r(a)$, give optimal consumption and savings, respectively.

4.3 Steady state

In a steady state, a time-invariant measure $N(h,z)$ describes the number of firms operating with workers of skill level $h \in \{0, H\}$ and productivity level z. This measure must be consistent with the stochastic process for idiosyncratic shocks and the employment decisions of firms. If v is the number of new firms being created, then $N(\cdot\,,\cdot)$ must satisfy

$$\begin{aligned} N(0,z') &= vQ^o(z')\Lambda^o(z')(1-\phi) + (1-\rho)(1-\pi^o)\Lambda(0,z') \\ &\quad \cdot \sum_{h,z} p^n(h,0) N(h,z) Q(z,z'), \end{aligned} \tag{46}$$

$$\begin{aligned} N(H,z') &= vQ^o(z')\Lambda^o(z')\phi + (1-\rho)(1-\pi^o)\Lambda(H,z') \\ &\quad \cdot \sum_{h,z} p^n(h,H) N(h,z) Q(z,z'). \end{aligned} \tag{47}$$

The cross-sectional distribution of households is characterized by time-invariant measures $y^n(a,h,z)$, $y^u(a,h,b)$ and $y^r(a)$ that describe, respectively, the number of employed, unemployed, and retired households across individual states. These measures are implied by the optimal decision rules by firms and households. In particular, these measures satisfy

$$\begin{aligned} y^n(a',h',z') &= (1-\rho)\Bigg[(1-\pi^o)\Lambda(h',z') \sum_{a,h,z:\bar{a}^n(a,h,z)=a'} p^n(h,h')\, y^n(a,h,z)\, Q(z,z') \\ &\quad + \Gamma(z') \sum_{a,b:\bar{a}^u(a,h',b)=a'} \bar{s}(a,h',b)^{\xi}\, y^u(a,h',b)\Bigg]; \end{aligned} \tag{48}$$

$$\begin{aligned} y^u(a',h,b) &= (1-\rho)\,\pi^o \sum_{a,z:\bar{a}^n(a,b,z)=a'} p^o(b,h)\, y^n(a,b,z) + (1-\rho)(1-\pi^o) \\ &\quad \cdot \Bigg\{ \sum_{a,z,z':\bar{a}^n(a,b,z)=a'} p^n(b,h)\, y^n(a,b,z) \Big[1-\Lambda(h,z')\Big] Q(z,z') \\ &\quad + \sum_{a:\bar{a}^u(a,h,b)=a'} y^u(a,h,b) \Big[1-\bar{s}(a,h,b)^{\xi}\Big] \Bigg\} \\ &\quad + I(h,b)\,\sigma \sum_{a:\bar{a}^r(a)=a'} y^r(a)\,, \end{aligned} \tag{49}$$

where $I(h,b)$ is an indicator function that is equal to one if $h=b=0$ and is equal to zero otherwise;

$$\begin{aligned} y^r(a') &= (1-\sigma) \sum_{a:\bar{a}^r(a)=a'} y^r(a) \\ &\quad + \rho \Bigg[\sum_{a,h,z:\bar{a}^n(a,h,z)=a'} y^n(a,h,z) + \sum_{a,h,b:\bar{a}^u(a,h,b)=a'} y^u(a,h,b) \Bigg]. \end{aligned} \tag{50}$$

Following Alvarez and Veracierto (2001), we consider steady-state equilibria without public debt. Hence, the government balances its budget every period and satisfies the following stationary budget constraint,

$$0 = (w^* - w)\sum_{h,z}(1+h)N(h,z) + \Omega D - \eta w \sum_{a,h,b}(1+b)y^u(a,h,b)\,, \tag{51}$$

where the amount of job destruction D is given by

$$D = (1-\rho)\left\{\pi^o \sum_{h,z} N(h,z) + (1-\pi^o)\sum_{h,h',z,z'} p^n(h,h')\left[1-\Lambda(h',z')\right]N(h,z)\,Q(z,z')\right\}. \tag{52}$$

The market-clearing condition in the goods market is

$$\bar{c} + \delta\bar{k} + \mu v = \sum_{h,z} N(h,z)\,z\,k(h,z)^\alpha (1+h)^{1-\alpha}, \tag{53}$$

where $\bar{c}$ and $\bar{k}$ are aggregate consumption and the aggregate capital stock, respectively, as given by

$$\bar{c} = \sum_{a,h,z}\bar{c}^n(a,h,z)\,y^n(a,h,z) + \sum_{a,h,b}\bar{c}^u(a,h,b)\,y^u(a,h,b) + \sum_a \bar{c}^r(a)\,y^r(a)\,, \tag{54}$$

$$\bar{k} = \sum_{h,z} N(h,z)\,k(h,z). \tag{55}$$

There are two equilibrium conditions in the labor market. First, the measure of new firms that hire workers, $v\sum_z \Lambda^o(z)Q^o(z)$, must equal the measure of unemployed workers who accept employment. Second, the skill ratio ϕ among new hires that the firm takes as given must equal the actual ratio of skilled workers among all new hires. We can use the time-invariant population measures to express these equilibrium conditions as follows:

$$v = \frac{(1-\rho)\sum_{a,h,b}\bar{s}(a,h,b)^\xi\,y^u(a,h,b)}{\sum_z \Lambda^o(z)\,Q^o(z)}\,, \tag{56}$$

$$\phi = \frac{\sum_{a,b}\bar{s}(a,H,b)^\xi\,y^u(a,H,b)}{\sum_{a,h,b}\bar{s}(a,h,b)^\xi\,y^u(a,h,b)}. \tag{57}$$

In the market for savings, households' aggregate assets equal

$$\bar{a} = \sum_{a,h,z} a\,y^n(a,h,z) + \sum_{a,h,b} a\,y^u(a,h,b) + \sum_a a\,y^r(a)\ . \tag{58}$$

This demand for assets should equal the supply of assets, which consist of the aggregate capital stock $\bar{k}$ and the ownership to the economy's firms. Thus, the market-clearing condition in the asset market is

$$\bar{a} = \bar{k} + \frac{\sum_{h,z}\Big[z\,k(h,z)^{\alpha}(1+h)^{1-\alpha} - w^{*}(1+h) - (i+\delta)k(h,z)\Big]N(h,z) - \mu v - \Omega D}{i}. \quad (59)$$

Risk averse households will deposit their savings in the banking sector, which in turn holds the market portfolio of firms. The households earn a deterministic rate of return equal to $(1+i)$ and are not exposed to risky payoff streams associated with individual firms.

5 Representative family model

5.1 Permissible benefit policies

We confine our study to benefit policies that are not generous enough to induce families to accumulate human capital simply to furlough high-skilled workers into unemployment, thereby forfeiting their higher market earnings to get for benefits from the public sector. The pertinent restriction on benefit policies can be derived by considering a steady state in which the family is initially satisfying its preference for leisure by keeping some of its low-skilled workers unemployed, then asking how the family's wealth would change were it to send an unemployed low-skilled agent to work with the intention of later furloughing him into unemployment after he has attained the higher skill level. We impose that during the skill accumulation phase, the family keeps its leisure unchanged by temporarily furloughing an already high-skilled worker into unemployment. This strategy gives rise to stochastic streams of costs during the accumulation phase and payoffs after attaining the higher skill level. These can be exchanged for their expected present value evaluated at a stationary interest rate equal to $(1+i) = \beta^{-1}$.

During the accumulation phase when the low-skilled agent replaces the high-skilled agent in the labor market, the family gains net of benefits an amount $(1-\eta)w$ from sending the low-skilled agent to work but loses net of benefits an amount $(1-\eta)(1+H)w$ from furloughing the high-skilled agent into unemployment. Thus, the impact on the family's disposable earnings during the accumulation phase is $-(1-\eta)Hw$ per period. This loss continues with probability $(1-\pi^{u})$ in the following period, i.e., as long as the low-skilled agent does not experience an upgrade in skills.[7] But with probability π^{u}, the low-skilled agent does attain the higher skill level. When that happens, the family recalls the originally high-skilled agent to employment and furloughs the originally low-skilled but newly high-skilled agent into unemployment. That originally low-skilled agent is now entitled to benefits that exceed his earlier benefit level by ηHw. The family keeps this stream of a higher disposable income until the agent with the upgraded skill level retires.

[7]Note that the retirement probability ρ does not enter these calculations since if either the low-skilled or the high-skilled agent is retired while the strategy is being executed, the family will just replace that agent by another agent from his category.

Let κ_0^H be the capital value of this whole strategy on its inception, and let κ_H^H be the capital value of the higher benefit stream at the time when the low-skilled agent gains the higher skill level and is furloughed into unemployment. These capitalized values satisfy the following expressions

$$\kappa_0^H = -(1-\eta)Hw + \beta\Big[\pi^u \kappa_H^H + (1-\pi^u)\kappa_0^H\Big], \tag{60}$$

$$\kappa_H^H = \eta Hw + \beta(1-\rho)\kappa_H^H. \tag{61}$$

After solving for κ_H^H from equation (61) and substituting into equation (60), we can solve for the capital value associated with this strategy,

$$\kappa_0^H = \frac{-(1-\eta) + \dfrac{\beta\pi^u\eta}{1-\beta(1-\rho)}}{1-\beta(1-\pi^u)}\, Hw. \tag{62}$$

We require that a permissible benefit policy should make this strategy unprofitable, i.e., $\kappa_0^H \leq 0$. This implies the restriction that

$$\beta\pi^u\eta \leq [1-\beta(1-\rho)](1-\eta). \tag{63}$$

This condition implies an upper bound on the generosity of the replacement rate η. Alternatively, for a given replacement rate η, expression (63) states that the probability π^u of experiencing an upgrade and the subjective discount factor β must be sufficiently low that it is not worthwhile to accumulate skills just in order to collect benefits at the higher skill level. It should be in the worker's interest to reap the returns from any skill accumulation by seeking employment in the labor market.

5.2 Steady-state employment and population dynamics

We will study an economy in a stochastic steady state. The representative family runs the household sector. In a steady state, the household's optimal policy is characterised by two flow rates into unemployment: a fraction e_0 of newborns that enter into life-time unemployment, and a fraction $e_\triangle$ of all laid off workers with human capital losses who enter into unemployment for the rest of their lives.[8] When the benefit policy satisifes restriction (63), there will be no unemployment among high-skilled workers in a steady state.

[8] Note that the optimal policy is not unique with respect to the identity of low-skilled unemployed workers who are entitled to the lowest benefit level. For example, agents would be indifferent between the proposed strategy of randomly furloughing newborn workers into life-time unemployment and other strategies that repeatedly randomize employment status among low-skilled agents who are entitled to the lowest benefit level. So long as the strategies result in identical aggregate employment outcomes, agents would derive the same *ex ante* expected life-time utility. Similarly, there is also nonuniqueness with respect to the identity of unemployed workers with human capital losses whenever the optimal allocation requires some of these agents to work. For example, agents would be indifferent between the proposed strategy of randomly furloughing a fraction of laid off workers with human capital losses into unemployment for the rest of their lives and alternative strategies with higher inflow rates but correspondingly shorter unemployment spells among laid off workers who experience human capital losses.

At time t, let R_t be the fraction of a family's members who are retired. The remaining working-age members are divided into four categories as follows. Let U_{0t}, $U_{\triangle t}$, N_{0t}, and N_{Ht} be the fractions of a family's members who are unemployed from birth, unemployed after suffering skill loss, employed with low skills, and employed with high skills, respectively. These fractions satisfy

$$R_t + U_{0t} + U_{\triangle t} + N_{0t} + N_{Ht} = 1. \tag{64}$$

For given flow rates $(e_0, e_\triangle)$, the laws of motion are given by

$$\begin{aligned}
R_t &= (1-\sigma)R_{t-1} + \rho\Big[U_{0t-1} + U_{\triangle t-1} + N_{0t-1} + N_{Ht-1}\Big], && (65)\\
U_{0t} &= (1-\rho)U_{0t-1} + e_0\sigma R_{t-1}, && (66)\\
U_{\triangle t} &= (1-\rho)\Big[U_{\triangle t-1} + \pi^o\pi^d e_\triangle N_{Ht-1}\Big], && (67)\\
N_{0t} &= (1-\rho)\Big\{\Big[1-(1-\pi^o)\pi^u\Big]N_{0t-1} + \pi^o\pi^d(1-e_\triangle)N_{Ht-1}\Big\} + (1-e_0)\sigma R_{t-1}, && (68)\\
N_{Ht} &= (1-\rho)\Big\{\Big[1-\pi^o\pi^d\Big]N_{Ht-1} + (1-\pi^o)\pi^u N_{0t-1}\Big\}. && (69)
\end{aligned}$$

We can use equations (64) and (65) to solve for the stationary fraction of retired members

$$R = \frac{\rho}{\sigma+\rho}, \tag{70}$$

which can be substituted into equation (66) to obtain the stationary fraction of family members who have been unemployed since birth

$$U_0 = \frac{e_0\,\sigma}{\sigma+\rho}. \tag{71}$$

To compute the stationary labor allocation, we start with equation (69) and express N_H in terms of N_0,

$$N_H = \frac{(1-\rho)(1-\pi^o)\pi^u}{1-(1-\rho)(1-\pi^o\pi^d)}N_0, \tag{72}$$

which can be substituted together with equation (70) into equation (68) and then solved for

$$N_0 = \frac{\Big[1-(1-\rho)(1-\pi^o\pi^d)\Big](1-e_0)\sigma\rho}{\chi^e(\sigma+\rho)}, \tag{73}$$

where

$$\chi^e \equiv 1-(1-\rho)\Big\{1+\rho\Big[1-\pi^o\pi^d-(1-\pi^o)\pi^u\Big] - (1-\rho)\pi^o\pi^d e_\triangle(1-\pi^o)\pi^u\Big\} > 0; \tag{74}$$

χ^e is strictly positive since

$$\chi^e \geq 1-(1-\rho)\Big\{1+\rho\Big[1-\pi^o\pi^d-(1-\pi^o)\pi^u\Big]\Big\} \geq 1-(1-\rho)(1+\rho) = \rho^2 > 0.$$

By using equations (72) and (73), we can solve for $U_\triangle$ from equation (67),

$$U_\triangle = \frac{(1-\rho)^2 \pi^o \pi^d e_\triangle (1-\pi^o)\pi^u (1-e_0)\sigma}{\chi^e(\sigma+\rho)}. \tag{75}$$

It is interesting to note that the skill composition of employed workers is a function only of exogenous parameters and does not depend on the choice of flow rates $(e_0, e_\triangle)$. Use equation (72) to compute

$$\phi_N = \frac{N_H}{N_0+N_H} = \frac{(1-\rho)(1-\pi^o)\pi^u}{\rho+(1-\rho)\pi^o\pi^d+(1-\rho)(1-\pi^o)\pi^u} \in (0,1). \tag{76}$$

5.3 A perturbation of employment

Before turning to equilibrium labor dynamics in a steady state, we examine two perturbations from a steady-state labor allocation. We will use these perturbations below to compute a steady state.

Suppose that the steady state is such that the representative family has a positive measure of unemployed workers who have suffered skill loss. We can then ask : how would the family's wealth change if the set of unemployed workers who have suffered skill loss is permanently reduced by one agent? That is, the family considers sending one such worker to the labor market and whenever he retires replacing him with another unemployed worker who has suffered skill loss. Such a succession of workers will give rise to a stochastic stream of labor income that the family can immediately exchange for the present value of the stream's expected value discounted at the stationary interest rate $(1+i)=\beta^{-1}$.

Let $\kappa_0^\triangle$ be the capital value of the labor income associated with this strategy of reducing unemployment among workers who have suffered skill loss. Moroever, let $\kappa_h^\triangle$ be the capitalized value of the stream of labor income at a future point in time when this worker (or one of his successors) has attained high skills. These capitalized values satisfy

$$\begin{aligned} \kappa_0^\triangle &= w+\beta(1-\rho)\Big\{\Big[1-(1-\pi^o)\pi^u\Big]\kappa_0^\triangle+(1-\pi^o)\pi^u\kappa_H^\triangle\Big\}+\beta\rho\kappa_0^\triangle, & (77)\\ \kappa_H^\triangle &= (1+H)w+\beta(1-\rho)\Big\{(1-\pi^o\pi^d)\kappa_H^\triangle+\pi^o\pi^d\kappa_0^\triangle\Big\}+\beta\rho\kappa_0^\triangle, & (78) \end{aligned}$$

where w is the market-clearing after-tax wage rate.

We can use equation (78) to solve for $\kappa_H^\triangle$,

$$\kappa_H^\triangle = \frac{(1+H)w+\beta\Big[(1-\rho)\pi^o\pi^d+\rho\Big]\kappa_0^\triangle}{1-\beta(1-\rho)(1-\pi^o\pi^d)}, \tag{79}$$

which can be substituted into equation (77),

$$\kappa_0^\triangle = \frac{1-\beta(1-\rho)(1-\pi^o\pi^d)+\beta(1-\rho)(1-\pi^o)\pi^u(1+H)}{\chi^0}\,w>0, \tag{80}$$

where

$$\begin{aligned}\chi^0 &\equiv \Big[1-\beta(1-\rho)(1-\pi^o\pi^d)\Big]\Big\{1-\beta(1-\rho)\Big[1-(1-\pi^o)\pi^u\Big]-\beta\rho\Big\} \\ &\quad -\beta(1-\rho)(1-\pi^o)\pi^u\beta\Big[(1-\rho)\pi^o\pi^d+\rho\Big] \\ &= (1-\beta)\Big\{1-\beta(1-\rho)\Big[1-(1-\pi^o)\pi^u-\pi^o\pi^d\Big]\Big\} > 0. \end{aligned} \tag{81}$$

5.4 A second perturbation of employment

Suppose in the steady state that the representative family has a positive measure of unemployed workers who have never been employed. We then ask: how would the family's wealth change if the set of unemployed workers who have never worked is permanently reduced by one agent? That is, the family considers sending one such worker to the labor market and when he retires replacing him with an unemployed worker who has never worked. Once again, such a succession of workers will give rise to a stochastic stream of labor income that the family can immediately exchange for the present value of the stream's expected value discounted at the stationary interest rate $(1+i)=\beta^{-1}$.

We add a twist to this strategy. Whenever the worker (or one of his successors) has become high-skilled and then loses those skills at a layoff triggered by an absorbing productivity level of zero, the strategy dictates that the worker is furloughed into unemployment indefinitely. He is replaced in the work force by another unemployed family member who has never worked. This switch of agents yields a gain to the family because a stream of low unemployment benefits becomes a stream of high unemployment benefits. The uncertainty associated with retirement makes the gain of benefits stochastic, but the associated stochastic stream of gains can be sold immediately for its expected present value, as given by κ_H^H in expression (61),

$$\kappa_H^H = \frac{\eta H w}{1-\beta(1-\rho)} \tag{82}$$

where η is the replacement. (Recall that newborn workers are also entitled to the lower benefit level ηw.)

Let κ_0^0 be the capital value of the labor income associated with this strategy of reducing unemployment among the workers who have never been employed. Moroever, let κ_H^0 be the capitalized value of the stream of labor income at a future point in time when this worker (or one of his successors) has attained high skills. These capitalized values satisfy

$$\kappa_0^0 = w+\beta(1-\rho)\Big\{\Big[1-(1-\pi^o)\pi^u\Big]\kappa_0^0+(1-\pi^o)\pi^u\kappa_H^0\Big\}+\beta\rho\kappa_0^0, \tag{83}$$

$$\kappa_H^0 = (1+H)w+\beta(1-\rho)\Big\{(1-\pi^o\pi^d)\kappa_H^0+\pi^o\pi^d(\kappa_0^0+\kappa_H^H)\Big\}+\beta\rho\kappa_0^0. \tag{84}$$

Equation (84) implies that

$$\kappa_H^0 = \frac{(1+H)w+\beta(1-\rho)\pi^o\pi^d\kappa_H^H+\beta\Big[(1-\rho)\pi^o\pi^d+\rho\Big]\kappa_0^0}{1-\beta(1-\rho)(1-\pi^o\pi^d)}, \tag{85}$$

which can be substituted into equation (83),

$$\kappa_0^0 = \frac{\Big[1-\beta(1-\rho)(1-\pi^o\pi^d)\Big]w + \beta(1-\rho)(1-\pi^o)\pi^u\Big[(1+H)w + \beta(1-\rho)\pi^o\pi^d\kappa_H^H\Big]}{\chi^0}$$
$$= \kappa_0^{\triangle} + \frac{\beta^2(1-\rho)^2(1-\pi^o)\pi^o\pi^u\pi^d\eta H w}{\Big[1-\beta(1-\rho)\Big]\chi^0}, \tag{86}$$

where χ^0 is given by equation (81).

5.5 Steady-state consumption

The representative family takes wages and interest rates as given. Since the utility function is additively separable in consumption and leisure, it is optimal for the family to dispense the same consumption to each of its members. In a steady state with constant consumption, the stationary interest rate must equal $1+i=\beta^{-1}$ and the family must be content to hold a constant level of wealth in the form of physical capital and ownership of firms.

Given the family's optimal labor decisions as codified in flow rates into unemployment $(e_0, e_\triangle)$, the representative family has fractions N_0 and N_H of its members employed with low skills and high skills, respectively, as detemined by equations (72) and (73). The stationary production of consumption goods per worker c can be converted into per-capita consumption $\bar{c}$ by

$$\bar{c} = \bar{n}c \tag{87}$$

where $\bar{n}$ is the fraction of employed agents among all members of the family,

$$\bar{n} = N_0 + N_H = \frac{\Big\{1-(1-\rho)\Big[(1-\pi^o\pi^d)-(1-\pi^o)\pi^u\Big]\Big\}(1-e_0)\rho\sigma}{\chi^e(\sigma+\rho)}. \tag{88}$$

The representative family's utility in a steady state is

$$\int_0^1 \sum_{t=0}^{\infty} \beta^t u(c_t^j,\, n_t^j)dj = \frac{\log(\bar{c}) - \bar{n}A}{1-\beta}. \tag{89}$$

It remains to describe optimal labor decisions in a steady state.

5.6 Steady-state labor dynamics

When the benefit policy satisfies restriction (63), ll high-skilled workers will be employed in a steady state. Unemployment will be reserved for workers who currently have low skills, and there are two possibilities concerning steady-state outcomes:

1. $e_0 = 0$ and $e_\triangle \in [0,1]$;

2. $e_0 \in (0, 1]$ and $e_\triangle = 1$.

If there is any unemployment among low-skilled workers with low benefits, all high-skilled workers who suffer skill losses must flow into unemployment, i.e., $e_\triangle = 1$. If that were not true, the family would be better off working low-skilled worker with low benefits instead of a laid off high-skilled worker who has just suffered a skill loss. Both workers are equally productive in the work place, but the latter is entitled to higher unemployment compensation. Hence, the steady-state labor dynamics must fall into either class 1 or 2 above.

What is the optimal setting of the two flow rates into unemployment, $(e_0, e_\triangle)$? To check whether a candidate pair of flow rates constitute a steady state, we examine the welfare effects of the perturbations to employment that were decribed above. If the candidate $(e_0, e_\triangle)$ falls into class 1, we examine the first type of perturbation in which the set of unemployed workers who have suffered skill loss is permantently reduced by one agent. That increases the family's labor income by a capitalized value equal to $\kappa_0^\triangle$. In a steady state with equilibrium interest rate β^{-1}, it would be optimal for the family to convert this capitalized value into an annuity flow of $(1-\beta)\kappa_0^\triangle$ and permanently increase consumption by that amount. The utility derived from this extra flow of consumption should be compared to the loss of benefits $\eta(1 + H)w$ and the loss of leisure. The condition for an interior optimum is

$$u_c(\bar{c},\, \bar{n})\Big[(1-\beta)\kappa_0^\triangle - \eta(1+H)w\Big] + u_n(\bar{c},\, \bar{n}) = 0 \tag{90}$$

where the marginal utilities of consumption and leisure are evaluated at the candidate steady-state allocation. Given our particular utility function, which is additively separable in the logarithm of consumption and a linear disutility term for labor, the condition for an interior optimum in class 1 becomes

$$\frac{1}{\bar{c}}\Big[(1-\beta)\kappa_0^\triangle - \eta(1+H)w\Big] = A. \tag{91}$$

If the candidate $(e_0, e_\triangle)$ falls into class 2, we examine the second type of perturbation in which the set of unemployed workers who have never worked is permanently reduced by one worker. Analyzing this perturbation leads to the following condition for an interior optimum:

$$u_c(\bar{c},\, \bar{n})\Big[(1-\beta)\kappa_0^0 - \eta w\Big] + u_n(\bar{c},\, \bar{n}) = 0, \tag{92}$$

which with our preference specification implies

$$\frac{1}{\bar{c}}\Big[(1-\beta)\kappa_0^0 - \eta w\Big] = A. \tag{93}$$

6 Calibration

A worker keeps his productivity from last period with probability $(1 - \pi)$ and draws a new productivity with probability π from the distribution $Q_h^o(z')$, so that new productivities on existing jobs are drawn from the same distribution as the productivities at the time of job creation. Recall from above that this probability distribution will depend on the worker's current skills h in the matching model, but not so in the other two frameworks.

6.1 Parameter values common to all models

Following Alvarez and Veracierto (2001), we set the model period equal to half a quarter, and specify a discount factor, $\beta = 0.99425$, and a probability of retiring, $\rho = 0.0031$, which are assumed to be the same in all three frameworks. Hence, agents in working age have an annualized subjective discount rate of 4.7%, and the average time spent in the labor force is 40 years.

Table 1 shows that the parameterization of the skill accumulation process is also kept the same across the models. The transition probabilities are motivated by our original analysis of the European unemployment puzzle (Ljungqvist and Sargent, 1998, 2002). One key feature is that it takes a long time to build up the highest skill level. This motivates our choice of a semiquarterly probability of upgrading skills $\pi^u = 0.0125$ so that it takes on average 10 years to move from low skills to high skills, conditional on no job loss. Exogenous layoffs occur with probability $\pi^o = 0.005$, i.e., on average once every 25 years. The probability of a productivity switch on the job equals $\pi = 0.05$, so that a worker on average keeps his productivity 2.5 years.

Another common assumption is that productivities are drawn from a truncated normal distribution with mean 1.0 and standard deviation 1.0. However, some model-specific assumptions dictate how these productivity draws enter the production technology.

6.2 Matching model

Here we adopt most of the parameter values of Ljungqvist and Sargent (2004a) who study the matching framework by Den Haan, Haefke and Ramey (2001). The calibration is reported in Table 1. The only substantial departures from our earlier study is that

1. the earlier uniform productivity distributions are replaced by normal distributions;
2. the earlier fixed number of firms is replaced by the assumption of free entry and the introduction of a Cobb-Douglas matching function and a vacancy cost (μ).

The truncated normal distribution described above forms the basis for workers' potential productivities. In particular, low-skilled workers draw their productivities from that exact distribution, while high-skilled workers draw productivities from a distribution with the same standard deviation but a mean that is twice as high as that for low-skilled workers.

Table 1 shows that our parameterization of the matching technology and the Nash bargaining between workers and firms is fairly standard. Workers' bargaining weight is assumed to be equal to $\psi = 0.5$, which in turn is equal to the matching elasticity of the Cobb-Douglas matching function.

The semiquarterly vacancy cost $\mu = 0.5$ can be put in perspective by computing the expected cost of filling a vacancy, as given by $\theta\mu/m(\theta)$. In the laissez-faire economy, that average recruitment cost is equal to 3.2, which can be compared to the average semiquarterly output of 2.3 goods per worker. Our calibration of the matching model yields a laissez-faire unemployment rate of 4.8%.

6.3 Search model with capital and incomplete markets

In addition to the discount factor and the probability of retiring, we adopt several other parameter values from Alvarez and Veracierto (2001), see our Table 1. In particular, the following survival, technology and preference parameters are the same: $\{\sigma, \delta, \xi, \gamma\}$. Given that the model period is equal to half a quarter and the survival probability in retirement is assumed equal to $\sigma = 0.0083$, the average duration of retirement becomes 15 years. The semiquarterly depreciation rate is $\delta = 0.011$. Settings of the exponent on the search technology ($\xi = 0.98$) and on the disutility of search ($\gamma = 0.98$), respectively, make these close to linear.

In contrast to Alvarez and Veracierto, we abstract from firm size dynamics and decreasing returns to scale by postulating that one-worker firms operate a constant-returns-to-scale Cobb-Douglas production technology with a capital share parameter $\alpha = 0.333$. Each firm has an idiosyncratic multiplicative productivity that is drawn from the truncated normal distribution as described above. Low-skilled workers have one unit of human capital while high-skilled workers have twice that amount, $(1 + H) = 2$.

The cost of starting a firm, i.e., making a fresh draw from the distribution of productivities equals 5. This can be measured against the laissez-faire outcome that only around 20% of all such draws exceed the optimally chosen reservation productivity where the firm hires a worker at a semiquarterly equilibrium wage rate equal to 6.4 for low-skilled workers. Hence, the average cost of recruiting a worker is around 6 months of wages of a low-skilled worker.

The disutility parameter A for job search is set equal to 5 which generate a laissez-faire unemployment rate of 4.4%.

6.4 Representative family model

As can be seen in Table 1, the representative family model and the search model are calibrated in the same way, except for parameters pertaining to job search. Since the representative family model is a frictionless environment, there is neither any search technology nor any disutility of searching. Instead, the new parameter A in the representative family model represents disutility of working. By setting the disutility of working equal to $A = 1.01$, the laissez-faire unemployment rate becomes 4.7%.

7 Numerical analysis

7.1 Matching model

See figures 1–10.

7.2 Search model with capital and incomplete markets

See figures 11–15.

7.3 Representative family model

See figures 16–18.

8 Conclusion

We have examined the theoretical robustness of Ljungqvist and Sargent's (1998) hypothesis that the persistent increase in European unemployment since the 1980s can be explained by the interaction of labor market institutions and microeconomic turbulence. Our conclusion is that the proposed mechanism seems robust to the choice of theoretical framework. In the matching model, the search-islands model and the representative family model, high unemployment erupts in a welfare state with generous benefits when laid off workers are subject to increased turbulence with respect to their earnings potential while the unemployment rate in a laissez-faire economy remains unchanged or decreases. The higher unemployment in a welfare state is mainly made up of workers who have suffered losses of earning potential at the time of their layoffs. The fact that welfare benefits are based on past earnings has the effect of "marginalizing" these workers in the labor market so that they end up unemployed for long periods of time. In the matching model, these unemployed workers with high benefits relative to their current earnings potential encounter fewer acceptable matches since a vacancy's idiosyncratic productivity must be so much higher to yield an attractive wage rate relative to the benefit level. In the search-islands model, these workers with generous benefits but poor labor market prospects choose to invest less in the job search process, i.e., they choose lower search intensities. In the representative family model, the welfare of the family is maximized by furloughing laid off workers with high benefits and low earnings potential into idleness because this is the most cost-effective allocation of the family's leisure.

Ljungqvist and Sargent (2002) extend the analysis of the European unemployment experience by showing that the inclusion of an additional labor market institution in the form of layoff taxes can explain why unemployment rates were actually much *lower* in Europe than in the United States until the 1970s. This additional mechanism is here shown to be robust in two of our three alternative frameworks. In the matching model and the search-islands model, layoff taxes slow down the reallocation of labor so that workers get "locked into" their current employment and frictional unemployment falls. In contrast, the unemployment rate increases in the representative family model at the introduction of a layoff tax. Layoff taxes also diminish labor turnover in the representative family model but in this frictionless framework there is no frictional unemployment to be suppressed and the explanation to the opposite unemployment outcome can instead be sought in the fact that layoff taxes reduce the equilibrium wage rate. In response to a lower private return to work, the representative family substitutes away from consumption towards leisure by sending a smaller fraction of its members to work.[9]

[9] For a detailed comparison of the employment implications of layoff taxes in different frameworks, see Ljungqvist (2002), and for a critical discussion of the aggregation theory based on a representative family, see Ljungqvist and Sargent (2004b).

Alvarez, Fernando and Marcelo Veracierto (2001) "Severance Payments in an Economy with Frictions," *Journal of Monetary Economics*, **47**, 477–498.

Den Haan, Wouter, Christian Haefke and Garey Ramey (2001) "Shocks and Institutions in a Job Matching Model," NBER Working Paper 8463.

Gottschalk, Peter and Robert Moffitt (1994) "The Growth of Earnings Instability in the U.S. Labor Market," *Brookings Papers on Economic Activity*, 2, 217–272.

Hansen, Gary D. (1985) "Indivisible Labor and the Business Cycle," *Journal of Monetary Economics*, **16**, 309–327.

Katz, Lawrence F. and David H. Autor (1999) "Changes in the Wage Structure and Earnings Inequality," in *Handbook of Labor Economics* **3A**, ed. Orley Ashenfelter and David Card. Amsterdam: Elsevier.

Layard, Richard, Stephen Nickell and Richard Jackman (1991) *Unemployment: Macroeconomic Performance and the Labour Market*. Oxford: Oxford University Press.

Ljungqvist, Lars (2002) "How Do Layoff Costs Affect Employment?," *Economic Journal*, **112**, 829–853.

Ljungqvist, Lars and Thomas J. Sargent (1998) "The European Unemployment Dilemma," *Journal of Political Economy*, **106**, pp. 514–550.

Ljungqvist, Lars and Thomas J. Sargent (2002) "The European Employment Experience," CEPR Discussion Paper 3543.

Ljungqvist, Lars and Thomas J. Sargent (2004a) "European Unemployment and Turbulence Revisited in a Matching Model," *Journal of the European Economic Association*, **2**, 456–468.

Ljungqvist, Lars and Thomas J. Sargent (2004b) "Lotteries for Consumers Versus Lotteries for Firms," in *Frontiers in Applied General Equilibrium Modeling*, ed. Timothy J. Kehoe, T.N. Srinivasan and John Whalley. Cambridge, UK: Cambridge University Press.

McCall, John J. (1970) "Economics of Information and Job Search," *Quarterly Journal of Economics*, **84**, 113–126.

Mortensen, Dale T. and Christopher A. Pissarides (1999) "Unemployment Responses to 'Skill-Biased' Technology Shocks: The Role of Labor Market Policy," *Economic Journal*, **109**, 242–265.

Prescott, Edward C. (2002) "Richard T. Ely Lecture: Prosperity and Depression," *American Economic Review*, **92**, 1–15.

Prescott, Edward C. (2004) "Non-Convexities in Quantitative General Equilibrium Studies of Business Cycles," in *Frontiers in Applied General Equilibrium Modeling*, ed. Timothy J. Kehoe, T.N. Srinivasan and John Whalley. Cambridge, UK: Cambridge University Press.

Rogerson, Richard (1988) "Indivisible Labor, Lotteries, and Equilibrium," *Journal of Monetary Economics*, **21**, 3–16.

Rogerson, Richard (2003) "Structural Transformation and the Deterioration of European Labor Market Outcomes," mimeo, Arizona State University.

Table 1: Parameter values (one period is half a quarter)

Parameters common to all models	
Discount factor, β	0.99425
Retirement probability, ρ	0.0031
Probability of upgrading skills, π^u	0.0125
Probability of exogenous breakup, π^o	0.005
Probability of productivity change, π	0.05
Productivity distribution	
truncated normal with mean	1.0
standard deviation	1.0
Additional parameters in matching model	
Matching function, $M(v, u)$	$0.5\, v^{0.5} u^{0.5}$
Vacancy cost, μ	0.5
Worker's bargaining weight, ψ	0.5
Low-skilled workers' potential productivity,	
distribution as above with mean	1.0
High-skilled workers' potential productivity,	
distribution as above but with mean	2.0
Parameters common to search / representative family model	
Probability of dying, σ	0.0083
Capital share parameter, α	0.333
Depreciation rate, δ	0.011
Job creation cost, μ	5.0
Low skill level	1.0
High skill level, $(1 + H)$	2.0
Additional parameters in search model	
Disutility of search, A	5.0
γ	0.98
Search technology, ξ	0.98
Additional parameter in representative family model	
Disutility of working, A	1.01

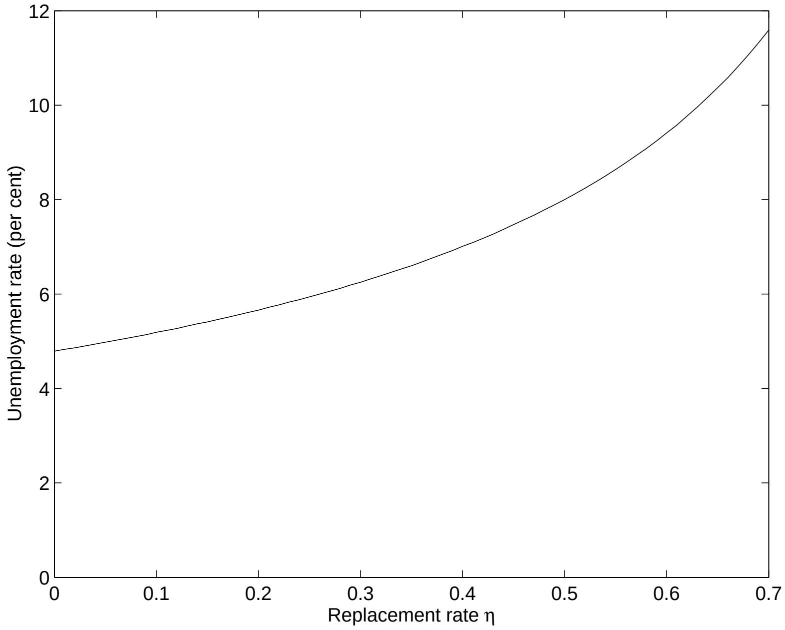


Figure 1. (Matching model) Unemployment rates for different replacement rates η, given tranquil economic times and no layoff taxes.

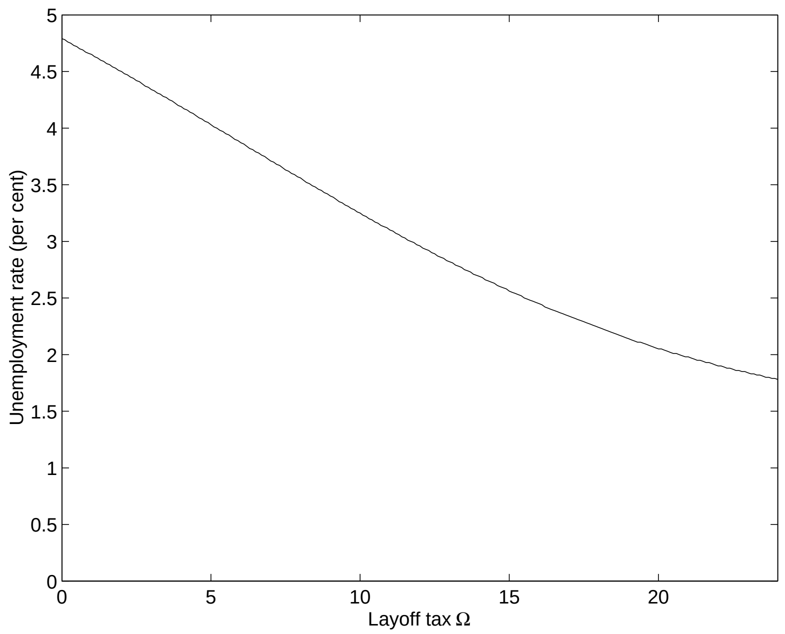


Figure 2. (Matching model) Unemployment rates for different layoff taxes Ω, given tranquil economic times and no benefits. The magnitude of the layoff tax can be compared to an average semiquarterly output of 2.3 goods per worker in the laissez-faire economy, i.e., a layoff tax equal to 19 corresponds to approximately one year's of a worker's output.

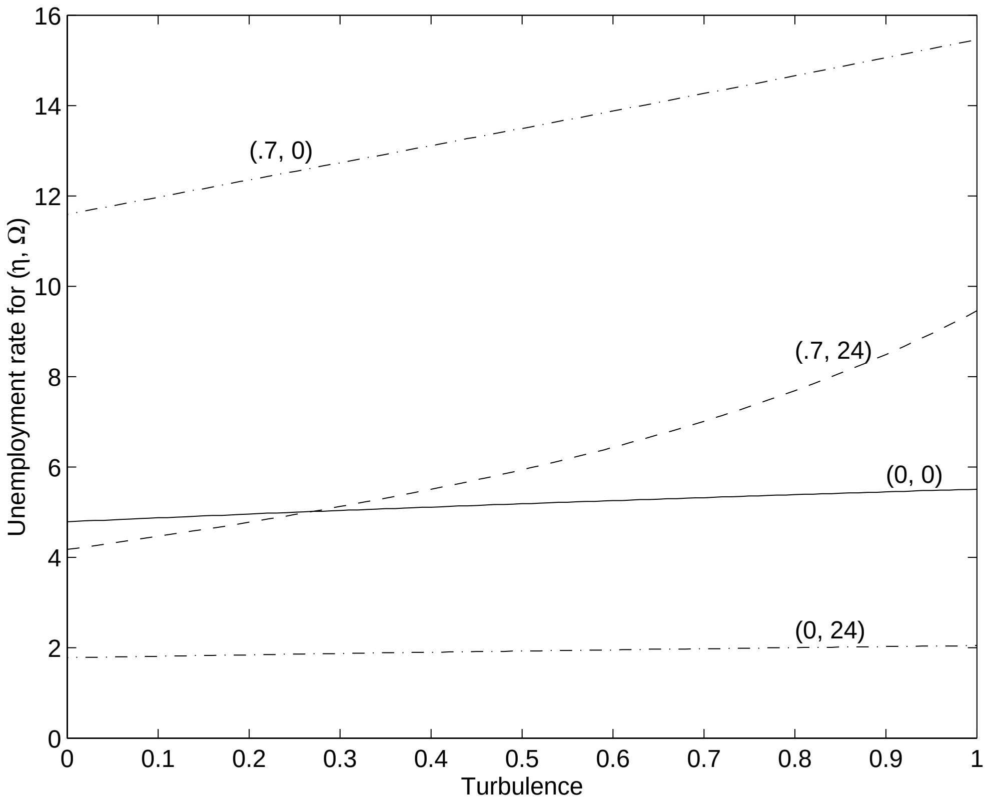


Figure 3.(Matching model) Unemployment rates for different government policies as indexed by $(\eta,\ \Omega)$ where η is the replacement rate and Ω is the layoff tax. The solid line indexed by $(0, 0)$ refers to the laissez-faire economy without government intervention.

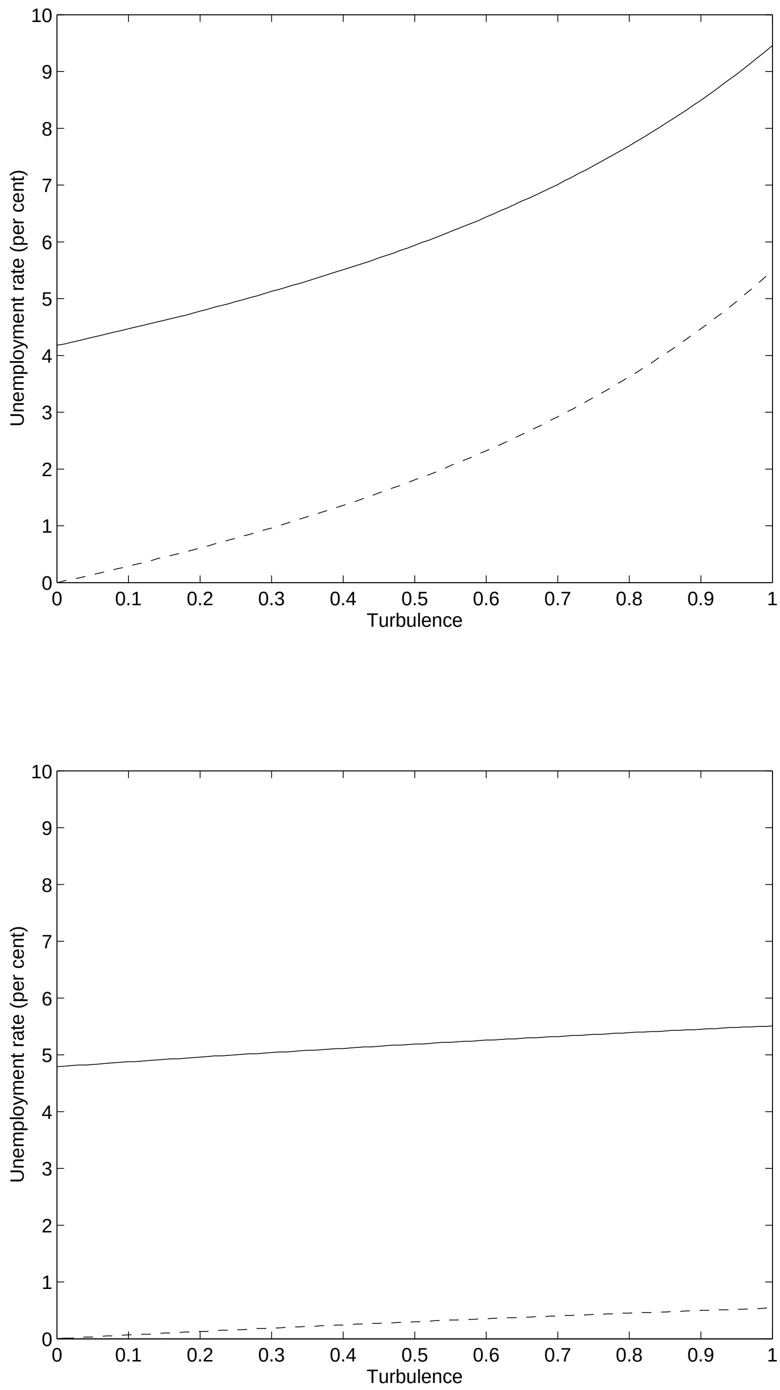


Figure 4. (Matching model) Unemployment rates in the welfare state (upper panel) and the laissez-faire economy (lower panel). The solid line is total unemployment. The dashed line shows the unemployed who have suffered skill loss. The policy of the welfare state is given by $(\eta, \Omega) = (0.7, 24)$.

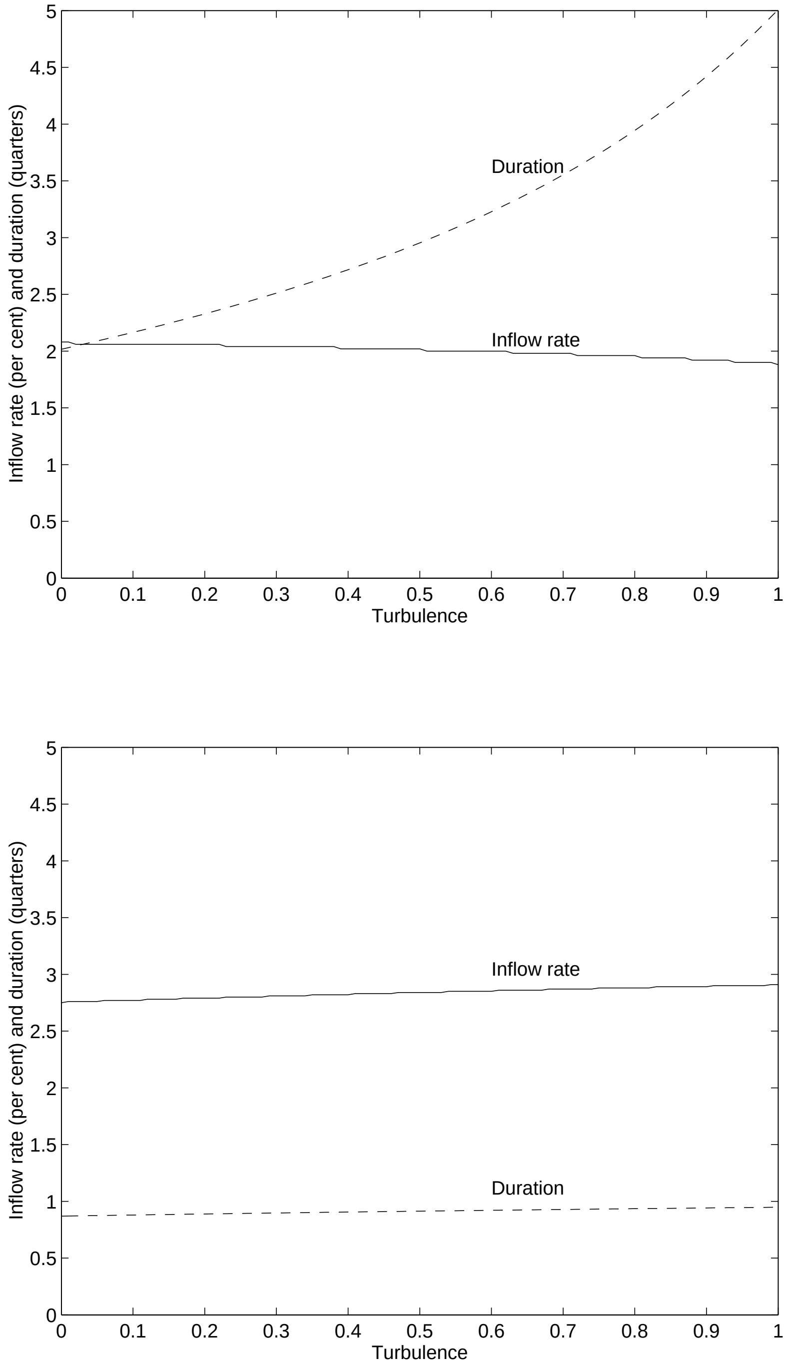


Figure 5. (Matching model) Inflow rate and average duration of unemployment in the welfare state (upper panel) and the laissez-faire economy (lower panel). The dashed line is the average duration of unemployment in quarters. The solid line depicts the quarterly inflow rate into unemployment as a per cent of the labor force. The policy of the welfare state is given by $(\eta, \Omega) = (0.7, 24)$.

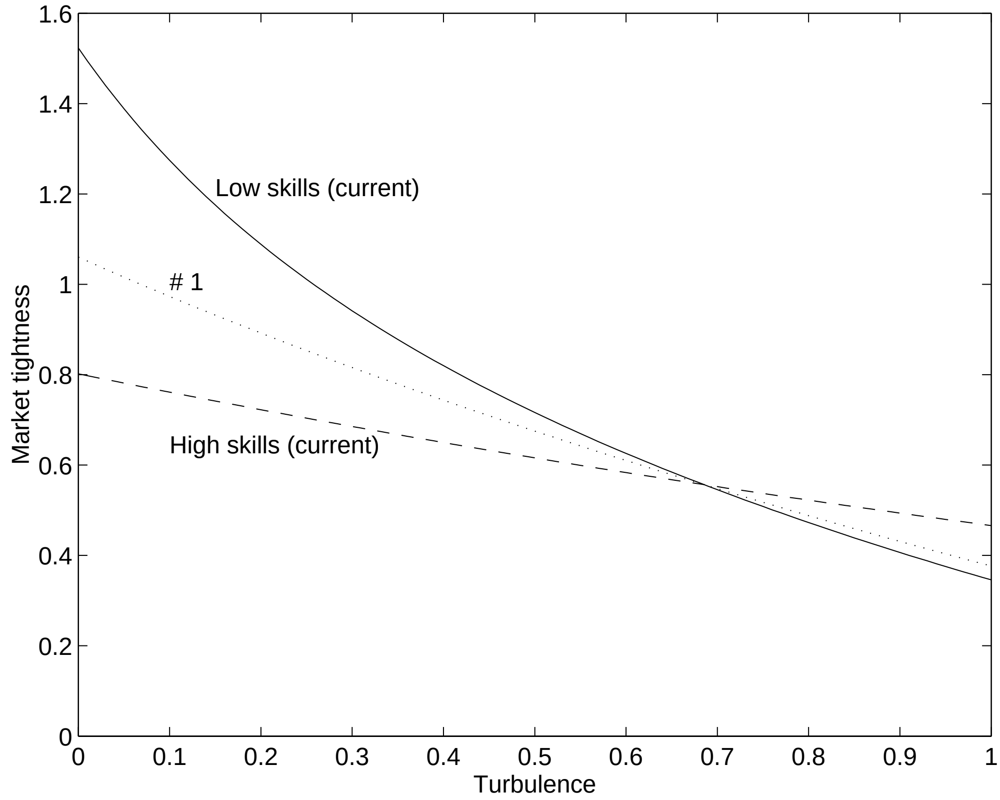


Figure 6. (Matching model) Market tightness θ when there are separate matching functions for unemployed based upon their current skills; low skills (solid line) and high skills (dashed line). As a benchmark, the dotted line depicts market tightness in the economy with a single matching function. The goverment's policy is given by $(\eta, \Omega) = (0.7, 24)$.

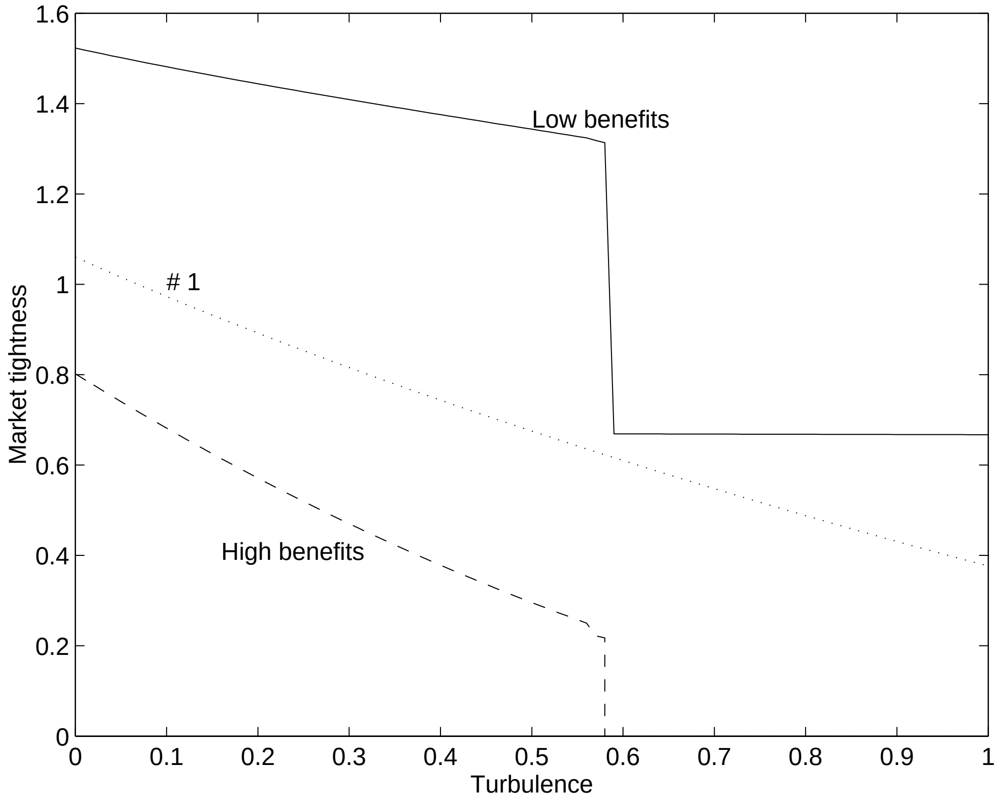


Figure 7. (Matching model) Market tightness θ when there are separate matching functions for unemployed based upon their benefits; low benefits (solid line) and high benefits (dashed line). As a benchmark, the dotted line depicts market tightness in the economy with a single matching function. The goverment's policy is given by $(\eta, \Omega) = (0.7, 24)$.

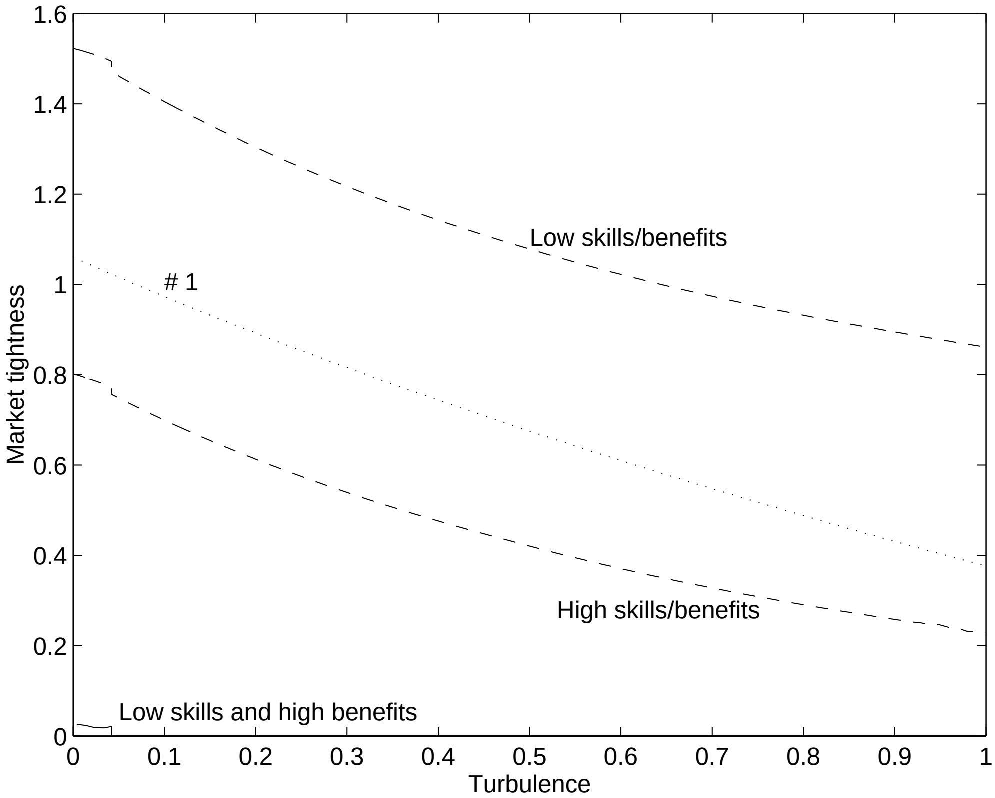


Figure 8. (Matching model) Market tightness θ when there are separate matching functions for unemployed based upon both their current skills and benefits; low skills/benefits (solid line), high skills/benefits (dashed line) and low skills but high benefits (dash-dotted line). As a benchmark, the dotted line depicts market tightness in the economy with a single matching function. The goverment's policy is given by $(\eta, \Omega) = (0.7, 24)$.

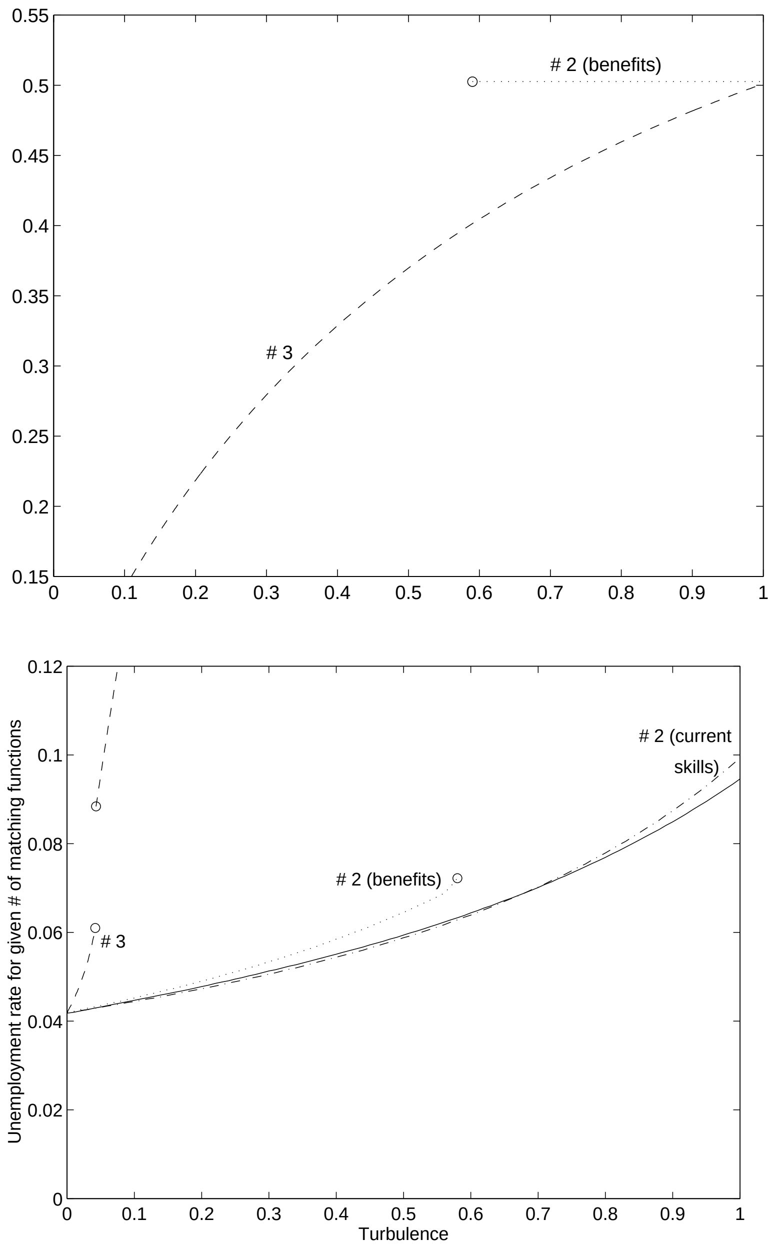


Figure 9. (Matching model) Unemployment rates for different number of matching functions. The solid line depicts the benchmark model with one matching function. The dash-dotted and the dotted line refer to the models with two matching functions where the unemployed are sorted by their current skills and the skills in their last job, respectively. An unemployed worker's skills in the last job determine her current benefit level. The dashed line depicts the model with three matching functions, i.e., the unemployed are perfectly sorted along all of their attributes. The goverment's policy is given by $(\eta, \Omega) = (0.7, 24)$.

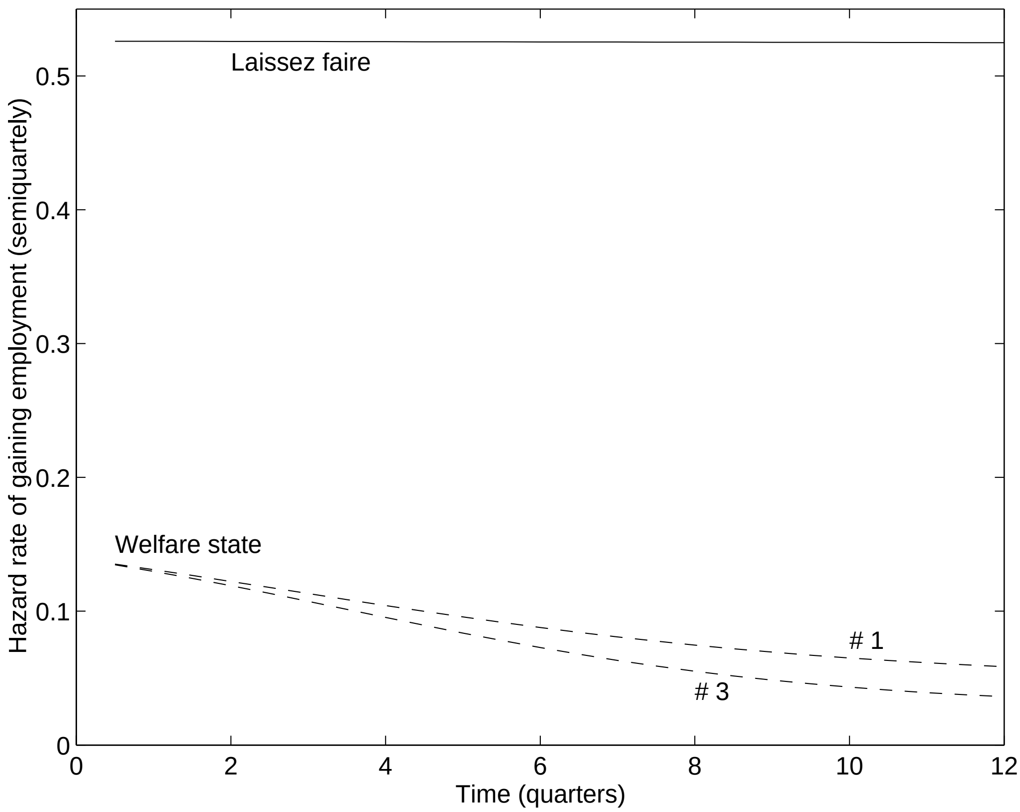


Figure 10. (Matching model) Semiquarterly hazard rates of gaining employment in turbulent economic times, $\pi^d = 1.0$. The solid line refers to the laissez-faire economy. The two dashed lines indexed by #1 and #3 depict the welfare state with one and three matching functions, respectively. The policy of the welfare state is given by $(\eta, \Omega) = (0.7, 24)$.

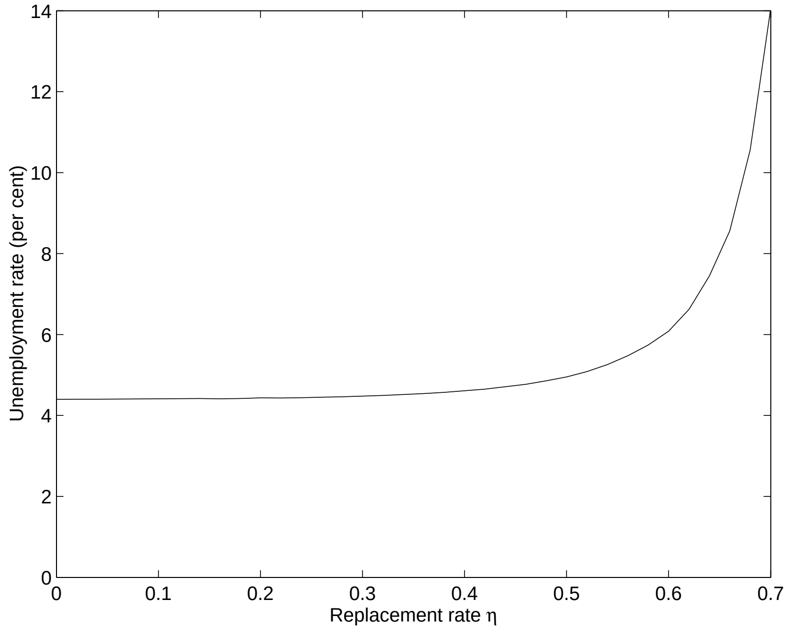


Figure 11. (Search model) Unemployment rates for different replacement rates η, given tranquil economic times and no layoff taxes.

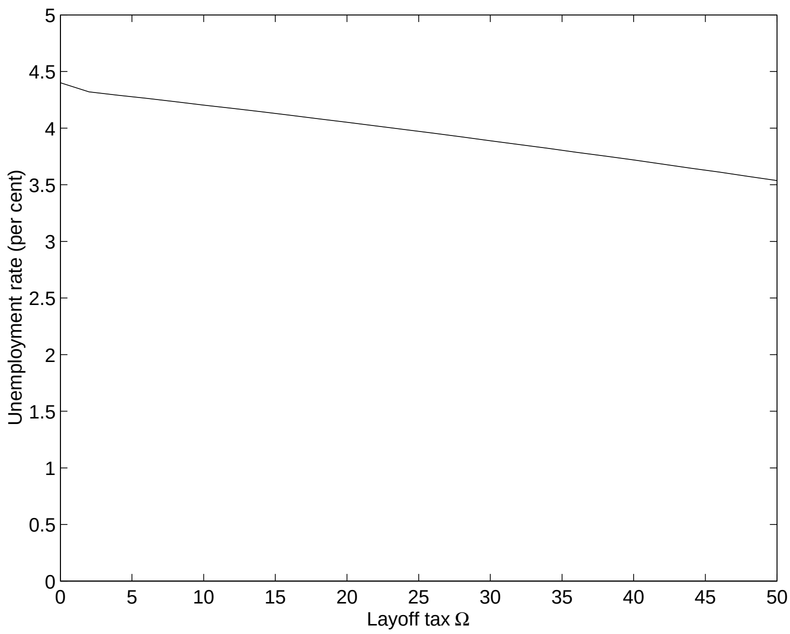
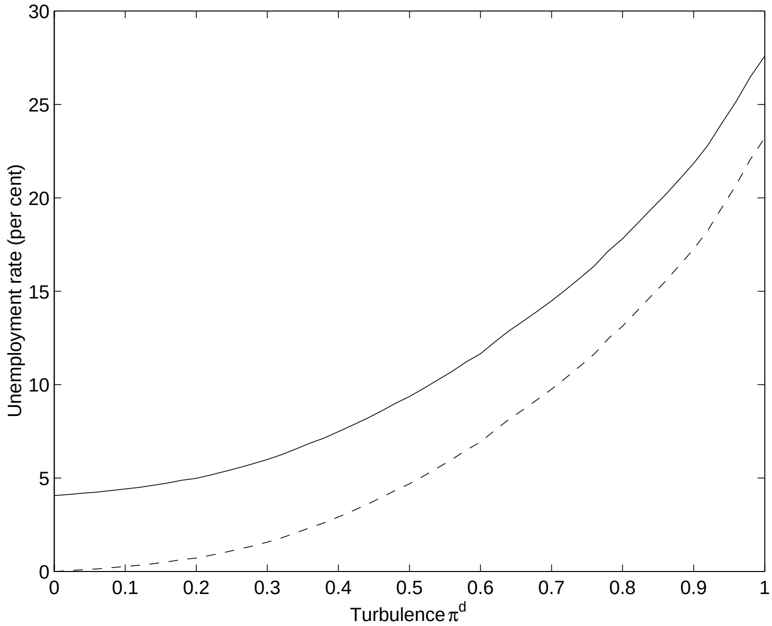


Figure 12. (Search model) Unemployment rates for different layoff taxes Ω, given tranquil times and no benefits. The magnitude of the layoff tax can be compared to a semiquarterly equilibrium wage of 6.4 per unit of skill in the laissez-faire economy, i.e., a layoff tax equal to 50 corresponds to roughly one year of wage income for a low-skilled worker.



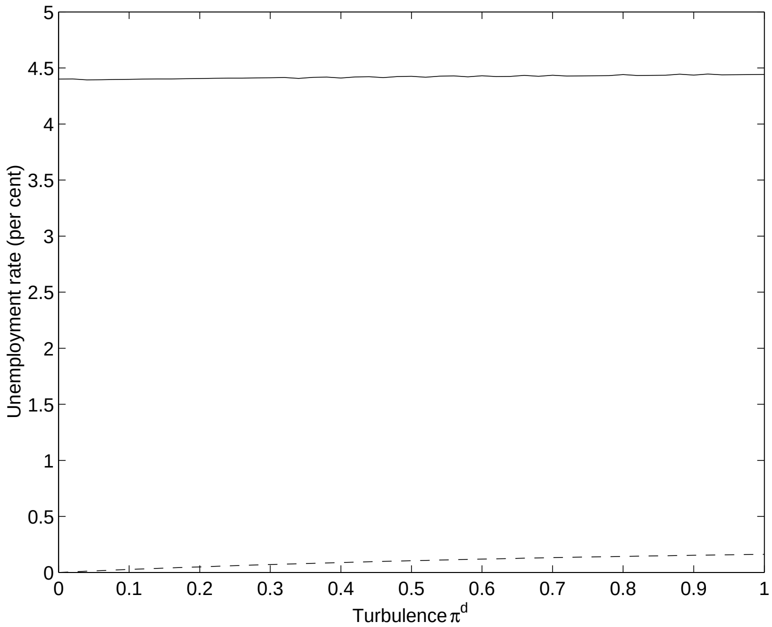


Figure 13. (Search model) Unemployment rates in the welfare state (upper panel) and the laissez-faire economy (lower panel). The solid line is total unemployment. The dashed line shows the unemployed who have suffered skill loss. The policy of the welfare state is given by $(\eta, \Omega) = (0.55, 50)$.

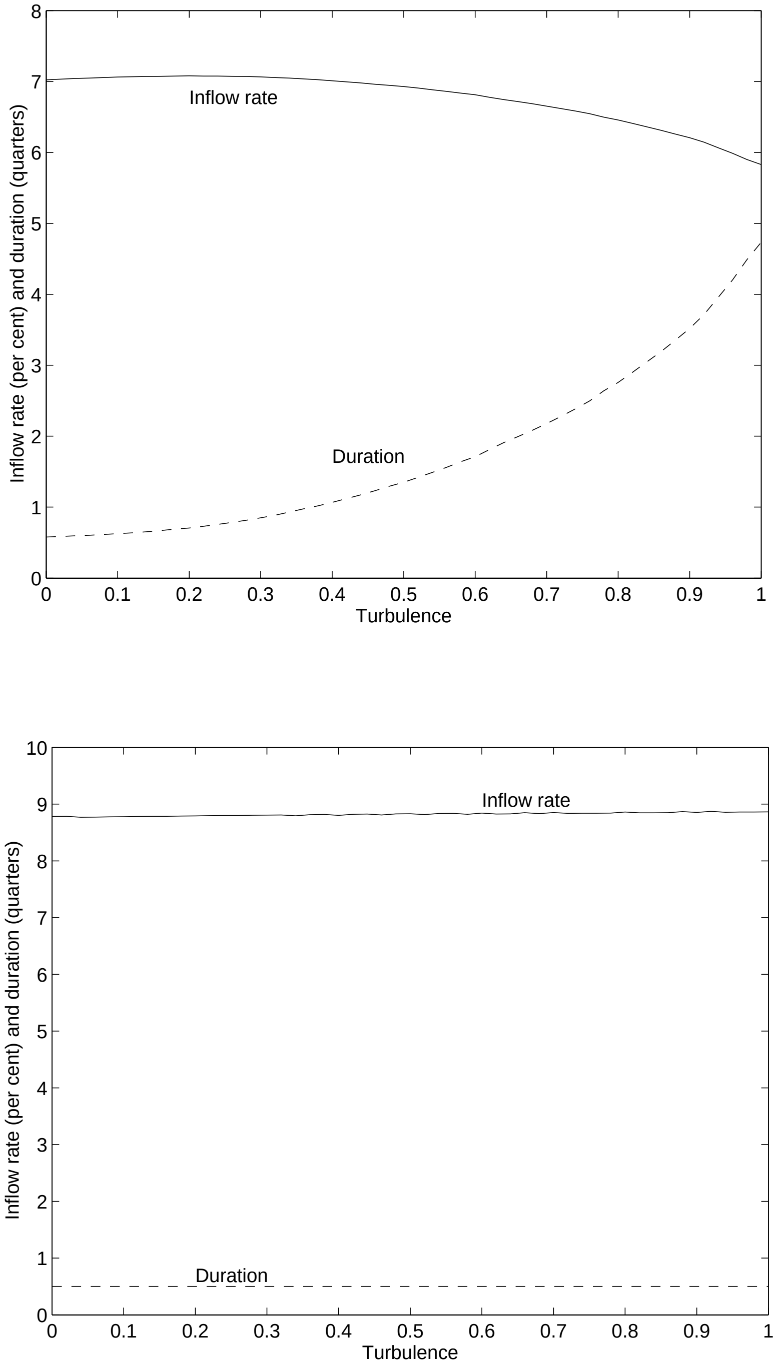


Figure 14. (Search model) Inflow rate and average duration of unemployment in the welfare state (upper panel) and the laissez-faire economy (lower panel). The dashed line is the average duration of unemployment in quarters. The solid line depicts the quarterly inflow rate into unemployment as a per cent of the labor force. The policy of the welfare state is given by $(\eta, \Omega) = (0.55, 50)$.

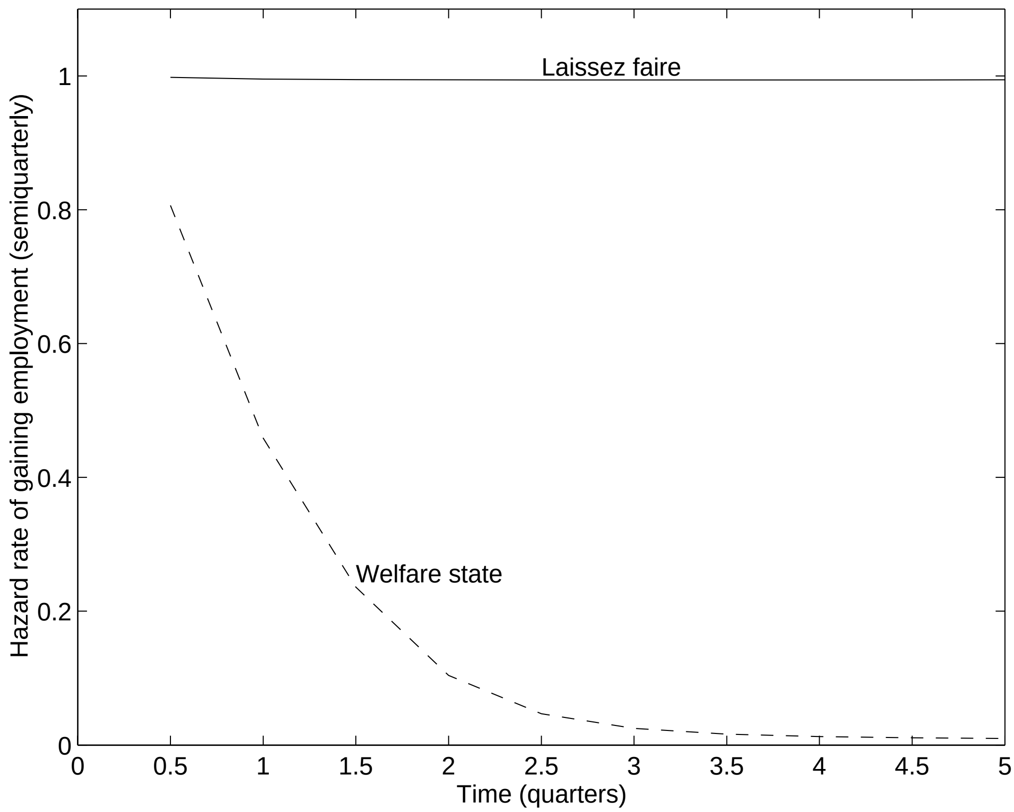


Figure 15. (Search model) Semiquarterly hazard rates of gaining employment in turbulent economic times, $\pi^d = 1.0$, in the welfare state (dashed line) and in the laissez-faire economy (solid line). The policy of the welfare state is given by $(\eta, \Omega) = (0.55, 50)$.

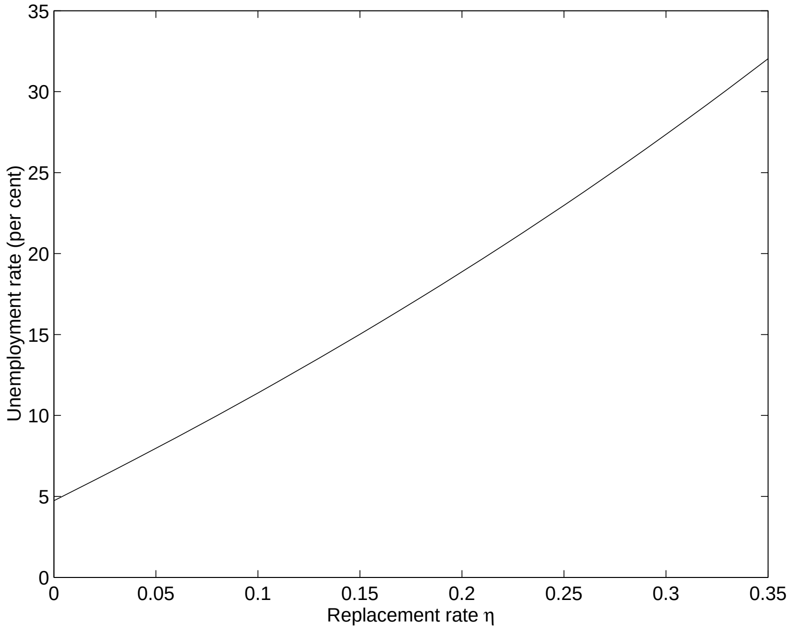


Figure 16. (Representative family) Unemployment rates for different replacement rates η, given tranquil economic times and no layoff taxes.

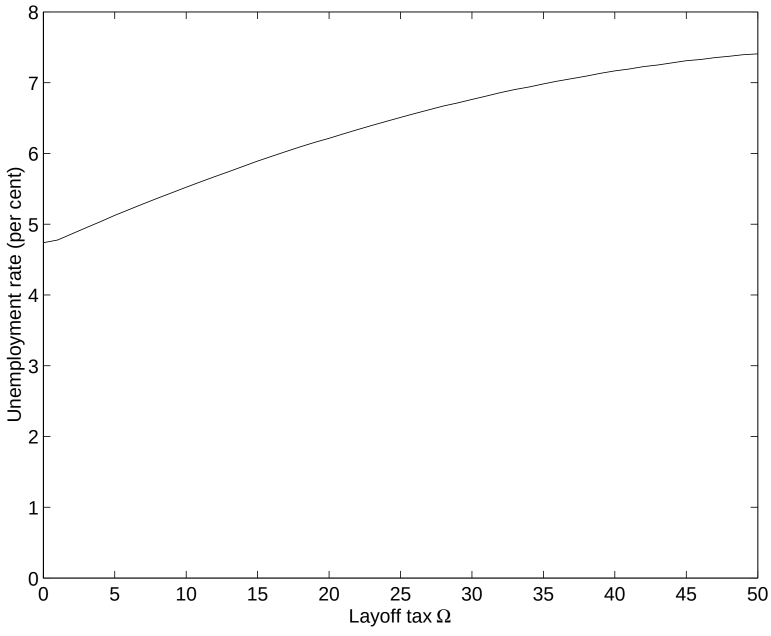


Figure 17. (Representative family) Unemployment rates for different layoff taxes Ω, given tranquil times and no benefits. The magnitude of the layoff tax can be compared to a semiquarterly equilibrium wage of 6 per unit of skill in the laissez-faire economy, i.e., a layoff tax equal to 48 corresponds to one year of wage income for a low-skilled worker.

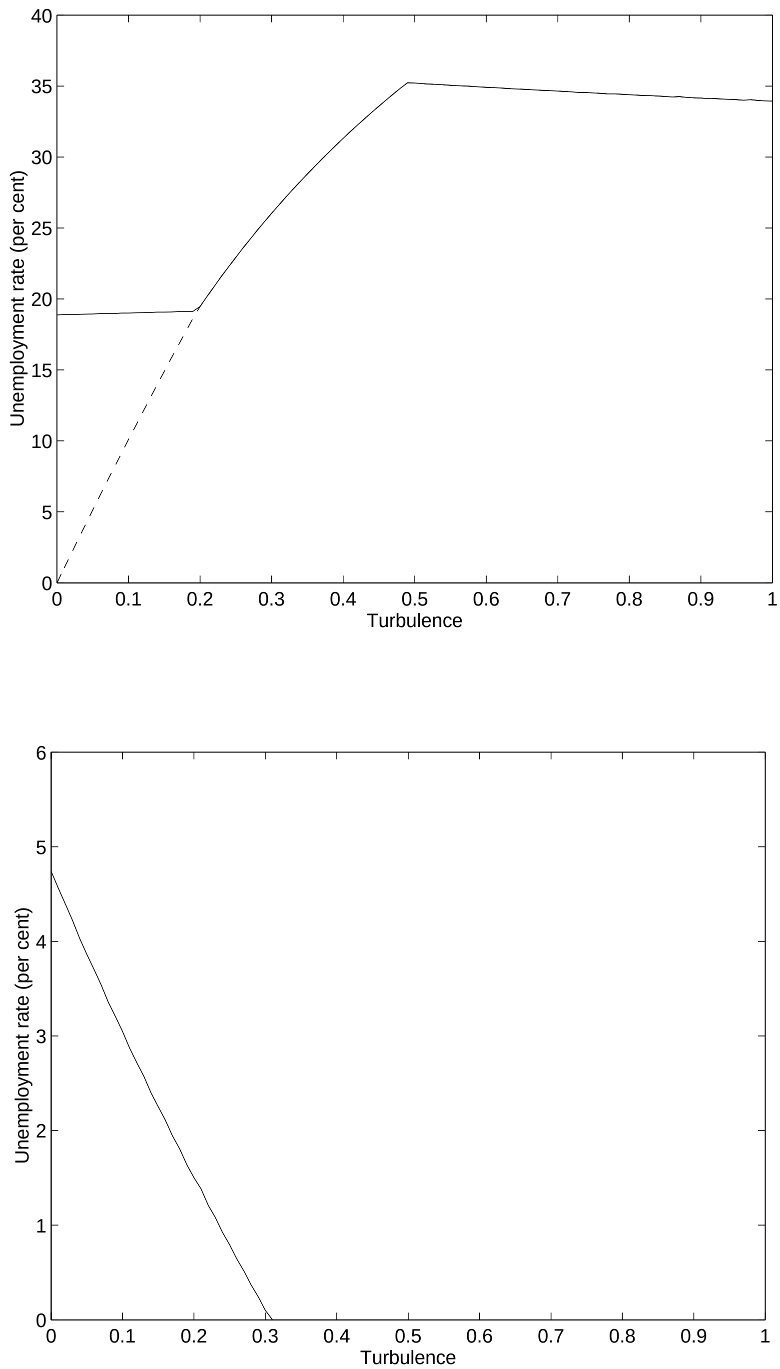


Figure 18. (Representative family) Unemployment rates in the welfare state (upper panel) and the laissez-faire economy (lower panel). The solid line is total unemployment. In the welfare state, the policy is given by $(\eta, \Omega) = (0.2, 0)$ and the dashed line shows the unemployed who have suffered skill loss (which is not a uniquely determined quantity in the laissez-faire economy and is therefore left out from the lower panel).