# Competition, Innovation and Growth with Limited Commitment*

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**Abstract**

We study how limited enforcement of contracts and barriers to business start-up affect the investment in knowledge capital and the adoption of new technologies. We show that barriers to business start-up, *i.e.* limited competition, is an important obstacle to growth. Limited enforceability of contracts is detrimental to growth if there are barriers to business start-up. Our results are consistent with cross-country evidence showing that the cost of business start-up is negatively correlated with the level and growth of per-capita income.

## 1 Introduction

It is widely recognized that sustained economic growth—especially, in advanced societies—requires investment in R&D, adoption of advanced technologies and innovation. A distinguished feature of modern technologies

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such as information and communication technologies, biotechnologies and nanotechnologies, is the importance of *knowledge* capital, which is highly complementary to skilled human capital. The ability of a society to innovate and grow is then dependent on how knowledge capital is accumulated, organized and the returns shared among the participants in the innovation process. This is especially important when the parties who provide the financial needs differ from those who acquire the innovation skills. For instance, whether a certain innovation project is funded depends on the ability of the investors to recover at least some of the returns of the project. Similarly, workers and managers must have the right incentives to invest in knowledge and enhance their innovation skills. Of course, this depends on the contractual arrangements that are feasible and enforceable. The goal of this paper is to study how the accumulation of knowledge capital—which is distinct from physical capital—affects the rate of innovation and growth when contracts are not fully enforceable.

The limited enforceability of contracts is double-sided. On the one hand, the entrepreneur can engage in innovation activities that are not optimal for the investor. On the other, the investor can replace the entrepreneur and renege promises of payments. While the limited commitment of the entrepreneur may hold-up the investor from investing in physical capital, the limited commitment of the investor may hold-up the entrepreneur from accumulating knowledge. A major finding of this paper is that the hold-up of entrepreneurs, induced by the lack of commitment of investors, depends crucially on the presence of barriers to business start-up or more generally barriers to alternative uses of knowledge capital.

Without barriers, the entrepreneur can always quit the firm and start a new business. This implies that the entrepreneur can rely on the outside value of his or her knowledge capital as a threat against the investor's attempt to renegotiate. In this case, the entrepreneur may even over-accumulate knowledge in order to keep the threat value high. In contrast, when there are barriers to business entry, knowledge capital does not have an independent value outside the firm. The limited commitment of the investor then implies that the entrepreneur will not be remunerated for the knowledge investment and, as a result, he or she will not accumulate any knowledge. Hence, barriers to entry or lack of competition are detrimental to innovation and growth.

Our result differs from other models with hold-up problems. In some of these models the firm has full control over the accumulation of human capital. For example, in Acemoglu & Shimer (1999), is the employer that decides the amount of training. In this environment, greater mobility or outside opportunities worsen the hold-up problem because the workers capture a larger share of the firm's rents. In other models, such as the one studied in Acemoglu (1997), workers do control the accumulation of skills, but the main conclusion does not change: greater mobility worsens the hold-up problem because workers are less likely to benefit from their accumulation of skills. In contrast to these studies, we show that mobility and competition increase human capital (knowledge) investment. The key factor leading to this result is the limited enforceability of long-term contracts from investors.

In our framework, investors prefer a lower rate of innovation than entrepreneurs because of the creative destruction of physical capital. Without business entry or competition, it is the firm that holds up the entrepreneur. This conflict could be resolved if long-term contracts were enforceable. In this case the investor would retain the entrepreneur and adopt a slower pace of innovations by promising higher payments. However, once the innovation skills have been accumulated, the investor would renege these payments. As a result, the only way to retain the entrepreneur is by adopting a faster rate of innovation. But for this to be the outcome, it is crucial that the entrepreneur has the option to quit and start a new firm: it is the threat of quitting that induces the investor to accept a faster rate of innovation. Because lower barriers generate greater potential *mobility* and greater *competition*, we will refer to an environment with no artificial barriers as the 'competitive economy'.

Whether competition enhances innovation has been a major topic of research and debate since Schumpeter's claim that, while product market competition could be detrimental for innovations, competition in the innovation sector enhances the rate of innovation. See, for example, Aghion & Howitt (1999). Most of the following literature has focused on market structure and product market competition. In particular, on the ability to appropriate the returns to R&D and to gain market shares by introducing new products, as in Aghion, Bloom, Blundell, Griffith, & Howitt (2002). More closely related to our work is Aghion, Blundell, Griffith, Howitt, & Prantl (2004). They show—both, theoretically and empirically—that 'firm entry' spurs innovation in technological advanced sectors as firms try to 'escape competition'. In contrast, we focus on the less studied dimension of 'knowledge capital competition'. There is also an escape competition effect in our model; but of a very different nature. Competition for knowledge capital can spur innovation as a mechanism for retaining entrepreneurs: When (and only when) investors are unable to commit to the promises of future payments, the only way to retain entrepreneurs is by agreeing on a faster pace of innovation.

Our results are consistent with the technological advances of the U.S. economy. For example, Bresnahan & Malerba (2002) argue that the lead of the US in the computer industry was possible thanks to a highly competitive environment, more prompt to stimulate innovations. For example, they claim that "The most important U.S. National institutions and policies supporting the emergence at this time [of the PC industry] were entirely non-directive: the existence of a large body of technical expertise in universities and the generally supportive environment for new firm formation in the United States". In the next section we also provide cross-country evidence that growth is positively associated with the degree of contract enforcement and negatively associated with the cost of business start-up.

The paper relates to several strands of literature. First, the labor literature that studies the hold-up problem (e.g., Acemoglu (1997), Acemoglu & Pischke (1999), Acemoglu & Shimer (1999)). Second, the literature that studies the linkages between competition and innovation (e.g., Aghion et al. (2002) and Aghion et al. (2004)). Third, the endogenous growth literature that studies the economics of ideas and its impact on economic growth, starting with the pioneering work of Romer (1990, 1993). As related is the literature that studies R&D based models of growth such as Jones (1995). Fourth, the recent growth literature that, building on the work of economic historians (e.g., Mokyr (1990)), emphasizes the role of barriers to riches in slowing growth (Parente & Prescott (1990)). Fifth, and foremost, the literature on dynamic contracts with enforcement constraints such as Marcet & Marimon (1992). In the contest of knowledge capital, however, limited enforcement is not caused by the ability of the entrepreneur to 'grab the money and run'. Rather, it is the ability to engage in different innovation projects relative to the ones preferred by the investor. Most of the models with limited enforcement ignore the issue of technology adoption and innovation. One exception is Kocherlachota (2001) who shows that limited enforcement may result in a lower rate of technological adoption when the division of the social surplus is sufficiently unequal. Another exception is Cooley, Marimon, & Quadrini (2004). In that paper, however, the arrival of new technologies is exogenous and limited enforcement affects only the propagation of new technologies, not the rate of innovation. Our paper also differs from the literature on the limited enforcement of contracts in the consideration of a double-sided commitment problem, which is crucial for some of the main results.

The plan of the paper is as follows. In Section 2 we provide cross-country evidence about contract enforcement, barriers to business start-up and macroeconomic performance. Section 3 describes the model. To facilitate the intuition for the theoretical results, Section 4 studies a simplified version of the model with only two periods. Sections 5 and 6 generalize it to the infinite horizon. The analysis of the infinite horizon model allows us to capture additional properties that are not captured by the two period model. Section 8 concludes.

## 2 Cross-country evidence

A recent publication from the World Bank[1] provides data on the quality of the business environment for a cross-section of countries. Especially important for this study is the *Cost of Starting a Business* and the *Degree of Contract Enforcement*.

Figure 1 plots the level of per-capita income against the cost of starting a business. This is the 'average pecuniary cost' needed to set-up a corporation in the country, expressed in percentage of the country per-capita income. Both variables are logged. The normalization of the cost of business start-up by the level of per-capita income better captures the importance of barriers to business start-up than the absolute dollar cost. What is relevant for the decision to start a business is the comparison between the cost of business start-up and the value of creating a business. Although the dollar cost of creating a business is on average higher in developed economies, the value of a new business is also higher in these economies.

As can be seen from Figure 1, there is a strong negative correlation between the cost of starting a business and the development of the country. A similar figure would result if we use alternative measures of the cost of business start-up—also reported by the World Bank—such as the 'number of bureaucratic procedures' or the 'number of days' needed to start a new business.

Figure 2 plots the per-capita income against the cost of contract enforcement. This is the average pecuniary cost sustained to resolve a legal dispute, as a percentage of the original debt. Both variables are logged. The figure shows that there is a negative correlation between the cost of contract enforcement and the level of development. A similar pattern is obtained if we use alternative measures of the cost of contract enforcement, such as the

[1] *Doing Business in 2005: Removing Obstacles to Growth.* World Bank, Washington.

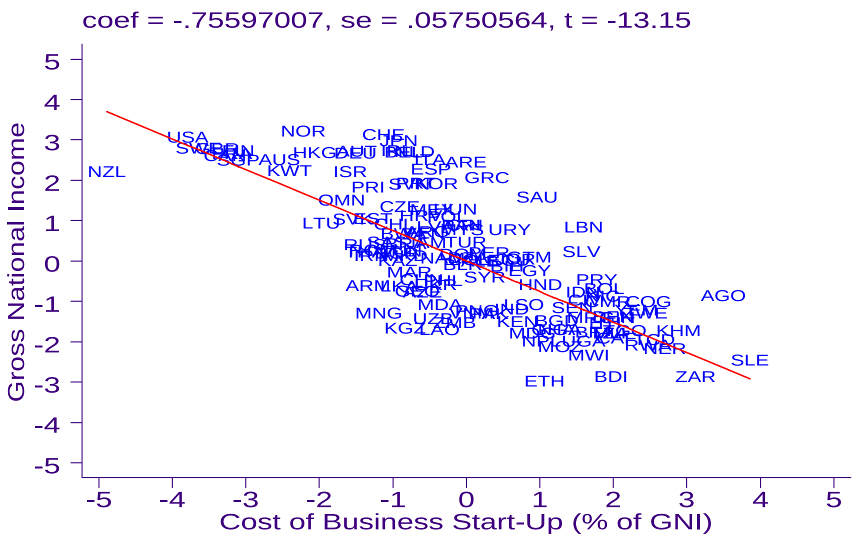


Figure 1: Cost of starting a business and level of development.

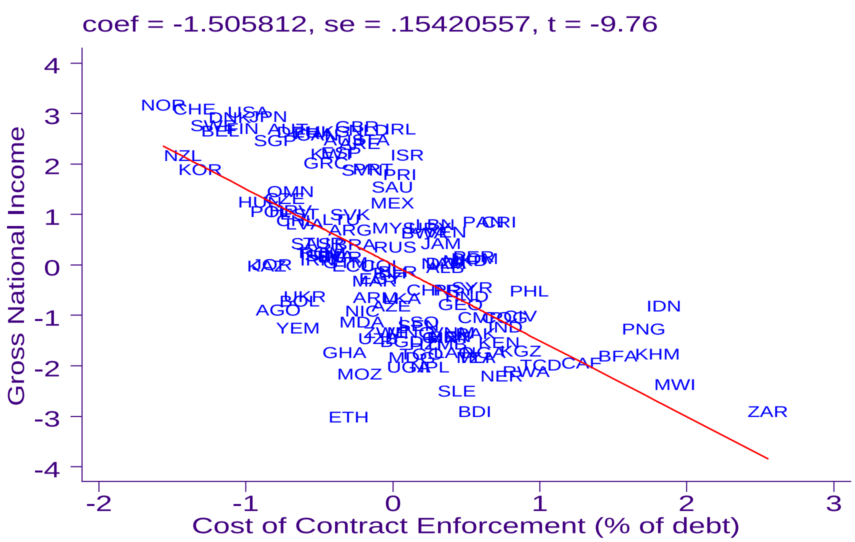


Figure 2: Cost of contract enforcement and level of development.

'number of bureaucratic procedures' that need to be filed or the 'number of days' elapsed before a legal dispute is resolved.

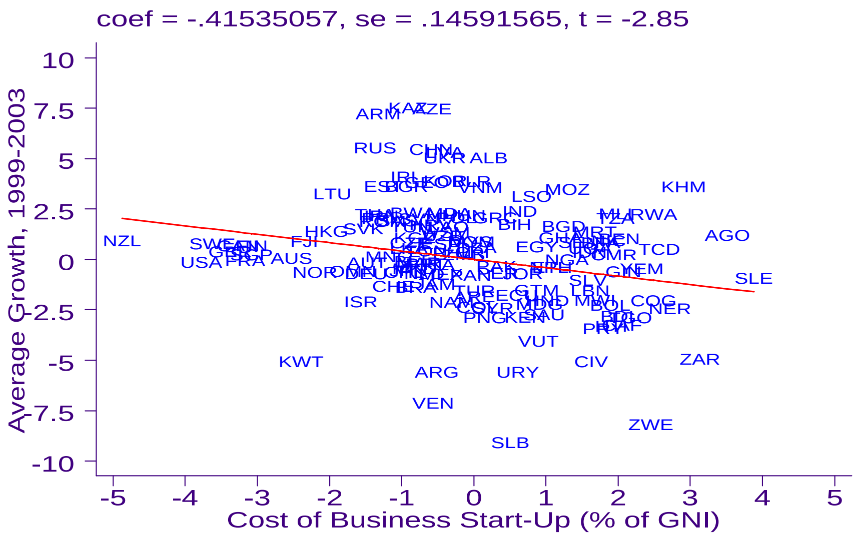


Figure 3: Cost of starting a business and economic growth.

The costs of business start-up and contract enforcement are also negatively related to economic growth. Figure 3 correlates the average growth rate in per-capita GDP in the last five years of available data (1999-2003) to the cost of starting a business. The correlation is negative and statistically significant, showing that countries with lower barriers to entry tend to experience faster growth. This finding is robust to the choice of alternative years to compute the average growth rate. Figure 4 correlates the average growth rate and the cost of contract enforcement. The correlation is negative and statistically significant. Therefore, countries with better enforcement of contracts are also countries that on average experience faster growth.

Because the cost of starting a business and the cost of contract enforcement are positively correlated, it is possible that the correlation with economic growth is not independent of each other. To investigate this possibility we regress the five years average growth in per-capita GDP to the cost of business start-up and the cost of contract enforcement. We also include the 1998 per-capita GDP to control for the initial level of development. The estimation results, with $t$-statistics in parenthesis, are reported in the first section of Table 1.

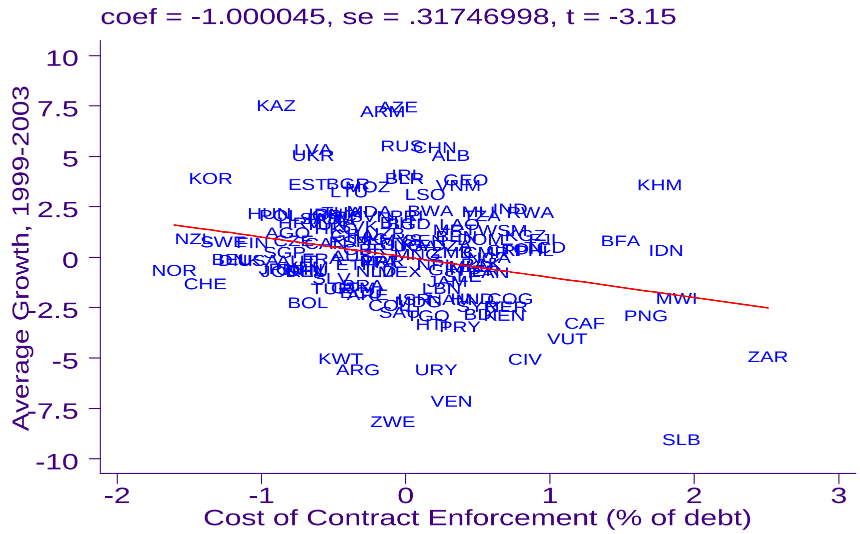


Figure 4: Cost of contract enforcement and economic growth.

The regression results show that both the cost of business start-up and the cost of contract enforcement are negatively associated with the five-year average growth in per-capita GDP. Furthermore, the statistical significance of the coefficients is not affected by the inclusion of the initial per-capita income. The results are robust to the choice of alternative periods to compute the average growth rate.

To show that these findings are not an artifact of normalizing the cost of business start-up by the level of per-capita income, the bottom section of Table 1 repeats the same regression estimation but using the dollar value, in log, of the cost of business start-up. Again, the cost of business start-up is statistical significant with a negative sign. The estimate for the cost of contract enforcement does not change significantly, although the initial per-capita GDP is no longer significant.

To summarize, the general picture portrayed by the analysis of this section is that the economic development and growth of a country is negatively associated with the cost of starting a business and the cost of enforcing contracts. In the following sections we present a model that rationalizes these findings.

Table 1: Business environment variables and growth.

| | | Constant | Initial Per-Capita GDP | Cost of Business Start-Up | Cost of Contract Enforcement |
|---|---|---|---|---|---|
| (a) | Coefficients | 18.25 | -1.21 | -0.75 | -1.25 |
| | $t$-Statistics | *(5.11)* | *(-3.86)* | *(-3.58)* | *(-3.26)* |
| | *R-square* | 0.177 | | | |
| | *N. of countries* | 136 | | | |
| (b) | Coefficients | 11.00 | -0.17 | -1.08 | -1.08 |
| | $t$-Statistics | *(3.69)* | *(-0.65)* | *(-3.33)* | *(-2.67)* |
| | *R-square* | 0.141 | | | |
| | *N. of countries* | 136 | | | |

*NOTES:* Dependent variable is the average annual growth rate in per-capita GDP for the five year period 1999-2003. Initial Per-Capita GDP is the log of per-capita GDP in 1998. The cost of business enforcement is in percentage of the disputed debt. In panel (a) the costs of business start-up is in percentage of the per-capital Gross National Income as reported in *Doing Business in 2004*. In panel (b) is the dollar value of this cost. Both measures of the cost of business start-up and the cost of contract enforcement enter the regressions in logs.

## 3 The model

There are two types of agents in the economy: 'investors' and 'entrepreneurs'. The lifetime utility is $\sum_{t=0}^{\infty} \beta^t c_t$ for investors and $\sum_{t=0}^{\infty} \beta^t (c_t - e_t)$ for entrepreneurs, where $c_t$ is consumption and $e_t$ is the effort to accumulate knowledge capital as specified below.

To generate firm turnover, we assume that entrepreneurs survive with probability $p$. The dead of an entrepreneur implies the exit of the firm and the entrance of a new firm managed by a newborn entrepreneur.[2]

Entrepreneurs do not save. This simplifying assumption should be interpreted as an approximation to the case in which entrepreneurs discount more heavily than investors. The risk neutrality of investors implies that the equilibrium interest rate is equal to their intertemporal discount rate, that is, $r = 1/\beta - 1$.

[2]Alternatively we could assume that each entrepreneur exogenously separates from the current investor with probability $1 - p$ and rematches with new a investor. The properties of the model are similar but the characterization of the optimal contract more cumbersome.

A firm produces output according to:

$$y_t = z_t k_t^\alpha$$

where $z_t$ is the level of technology and $k_t$ is the input of capital chosen at time $t-1$.

The variable $z_t$ changes over time as the firm adopts new technologies. The key assumption is that more advanced technologies require higher knowledge embodied in the skills of the entrepreneur. Given $h_t$ the knowledge capital of the entrepreneur, the level of technology available to the firm is:

$$z_t = A h_t^{1-\alpha}$$

This assumption formalizes the idea that innovations are complementary to knowledge capital. Therefore, even though the investor has the control of the firm, innovations are ultimately controlled by the entrepreneur.

The accumulation of knowledge requires effort from the entrepreneur: Higher is the accumulation of knowledge, $h_{t+1} - h_t$, and higher is the required effort. We also assume that the effort cost depends on the economy-wide level of knowledge, $H_t$, due to leakage or spillover effects. This is consistent with a widespread view about the importance of externalities for economic growth. See, for example, Klenow & Rodríguez-Clare (2004). We denote the effort cost as:

$$e_t = \varphi(H_t; h_t, h_{t+1})$$

The function $\varphi$ is homogeneous of degree 1, strictly decreasing in $H_t$ and $h_t$, strictly increasing in $h_{t+1}$ and satisfies $\varphi(H_t; h_t, h_t) = 0$.

Physical capital is technology-specific. Therefore, when the firm innovates, only part of the physical capital can be used with the new technology. Furthermore, the obsolescence of the existing capital increases with the degree of innovation. This is formalized by denoting the depreciation rate by:

$$\delta_t = \psi\left(\frac{z_{t+1}}{z_t}\right)$$

The function $\psi$ is strictly increasing, strictly convex and satisfies $\psi(1) = 0$. Because of capital obsolescence, *incumbent* firms have less incentives to innovate than *new* firms, still uncommitted to any previous investment.

The set-up of a new firm requires an initial fixed investment $\kappa_t = \bar{\kappa} H_t$, which is sunk. We assume that this cost is proportional to the aggregate level of knowledge to insure the existence of a balanced growth path.

The initial knowledge capital of a newborn entrepreneur is proportional to the economy-wide knowledge, that is, $h_t^N = \xi H_t$. A reasonable assumption is that $\xi < 1$. However, as we will see later, the qualitative results of the paper are not affected by the value of $\xi$.

The final assumption is that the entrepreneur has a reservation utility $R(\mathbf{s}_t)$, where $\mathbf{s}_t$ denotes the aggregate state variables as defined below. This imposes a lower bound to the value that the entrepreneur receives from innovating and managing a firm. Although we assume that the reservation utility is exogenous, we could have made it endogenous if labor was also an input of production. In this case $R(\mathbf{s}_t)$ would be the value of being a worker.

In characterizing the optimal contracting problem, we distinguish the case in which the entrepreneur can leave the firm and start a new business from the case in which this is not feasible or allowed. We refer to the first case as the *competitive* economy and to the second as the *non-competitive* economy. For each environment we will separately consider the case in which the investor commits to the long-term contract from the case of limited commitment.

## 4 Equilibrium in the two-period model

To gauge some intuitions about the key properties of the model, it would be convenient to consider first a simplified version of the model with only two periods: period zero and period one. The state variables of the firm at the beginning of period zero are $h_0$ and $k_0$. After making the investment decisions $h_1$ and $k_1$, the firm generates output $y_1 = z_1 k_1^\alpha$ in period one. Because $z_1 = A h_1^{1-\alpha}$, the output can also be written as $y_1 = A h_1^{1-\alpha} k_1^\alpha$. In this simple version of the model we assume that knowledge and physical capital fully depreciate after production. The entrepreneur receives a payment from the firm (compensation) at the end of period zero, after the choice of $h_1$. Allowing for additional payments before the choice of $h_1$ and/or in period one does not change the results as we will point out below. For the analysis of this section we also assume that there is no discounting and the effort cost does not depend on the economy-wide knowledge $H$. The leakage or spillover effect is not relevant when there are only two periods. This is also the case for the probability of survival, which we also ignore here.

The timing of the model can be summarized as follows: The firm starts period zero with initial states $h_0$ and $k_0$. At this stage the entrepreneur decides whether to stay or quit the firm. If he quits and there are no barriers to entry, he will start a new business funded by a new investor. The repudiation value is then given by the entrepreneur's share of the surplus generated by the new firm. If there are barriers to business entry, that is, we are in a non-competitive economy, the repudiation value is the reservation utility $R$. We assume that $R$ is sufficiently small that the value of starting a new business is bigger than $R$. If the entrepreneur decides to stay, he will choose the new level of knowledge capital $h_1$ and the investor provides the funds to accumulate the new physical capital $k_1$. After the investment decision has been made, the investor pays the entrepreneur $d_0$. At this stage the entrepreneur can still quit, but he cannot change the knowledge investment $h_1$. The investor is the residual claimant of the firm's profits.

### 4.1 Competitive economy with investor commitment

When the investor commits to the long-term contract, all variables are chosen at the beginning of the first period to maximize the total surplus. Because the economy is competitive, the repudiation value for the entrepreneur is the value of starting a new business. Let $D(h_0)$ be the repudiation value before choosing $h_1$ and $\widehat{D}(h_1)$ the repudiation value after choosing $h_1$. From now on, we will use the *hat* sign to denote all functions that are defined *after* the investment in knowledge. The participation of the entrepreneur requires that the value of staying with the firm is greater than the repudiation value before and after the knowledge investment, that is,

$$\begin{aligned} d_0 - \varphi(h_0, h_1) &\geq D(h_0) \\ d_0 &\geq \widehat{D}(h_1) \end{aligned}$$

The first is the participation constraint before choosing $h_1$ and the second is the participation constraint after the choice of $h_1$. For the moment we assume that the repudiation values are known. We will derive them after writing the optimization problem. At that point we will also show that, if the first constraint is satisfied, the second is also satisfied. Using this result, the optimization problem can be written as:

$$\max_{h_1,k_1,d_0} \left\{ -\varphi(h_0,h_1) - k_1 + \left[1 - \psi\left(\frac{h_1^{1-\alpha}}{h_0^{1-\alpha}}\right)\right] k_0 + A h_1^{1-\alpha} k_1^{\alpha} \right\} \tag{1}$$

$$\textbf{s.t.} \quad d_0 - \varphi(h_0, h_1) \geq D(h_0)$$

where we have substituted $z_1 = Ah_1^{1-\alpha}$ in the production and depreciation function. From the optimization problem it is clear that the choice of the entrepreneur's payment $d_0$ is independent of the investment choices (in knowledge and physical capital). This follows from the fact that $d_0$ does not enter the objective function. The only constraint is that this payment, net of the effort cost, is not smaller than the value that the entrepreneur would get from quitting.

To determine the repudiation value before the choice of $h_1$, we have to solve for the optimal investment when the entrepreneur quits the current firm and rematches with a new investor. Also in this case the optimal contract maximizes the total surplus, that is:

$$S(h_0) = \max_{h_1,k_1,d_0} \Big\{ -\varphi(h_0,h_1) - \kappa - k_1 + Ah_1^{1-\alpha}k_1^{\alpha} \Big\} \tag{2}$$

$$\textbf{s.t.} \quad d_0 - \varphi(h_0,h_1) \geq D(h_0)$$

Notice that this problem differs from the previous problem only because a new firm does not have any physical capital to start with and must pay the step-up cost $\kappa$. But it is still the case that the choice of $h_1$ and $k_1$ is independent of $d_0$.

The surplus $S(h_0)$ will be split between the entrepreneur and the investor according to their relative bargaining powers. Without loss of generality we assume that the entrepreneur gets the whole surplus, which is the outcome if financial markets are competitive.[3] This implies that $D(h_0) = S(h_0) = d_0 - \varphi(h_0,h_1)$, where $h_1$ solves problem (2). Therefore, if the entrepreneur stays with the incumbent firm, the payment $d_0$, net of the effort cost $\varphi(h_0,h_1)$, must be at least as large as $S(h_0)$. Formally,

$$d_0 \geq S(h_0) + \varphi(h_0,h_1) \tag{3}$$

Problems (1) and (2) show the different incentive to invest for an incumbent versus a new firm. New firms do not have any physical capital and innovations do not generate capital obsolescence. Hence, they have a greater incentive to innovate than incumbent firms. This is clearly shown by the first order conditions with respect to $h_1$ in problems (1) and (2):

$$\psi_{h_1}\left(\frac{h_1^{1-\alpha}}{h_0^{1-\alpha}}\right) \cdot k_0 + \varphi_{h_1}(h_0,h_1) = (1-\alpha)\left(\frac{k_1}{h_1}\right)^{\alpha} \tag{4}$$

[3]The alternative assumption that the entrepreneur gets only a fraction of the surplus does not change the results.

$$\varphi_{h_1}(h_0, h_1) \;=\; (1-\alpha)\left(\frac{k_1}{h_1}\right)^{\alpha} \tag{5}$$

where the subscripts in the functions $\varphi$ and $\psi$ denote the derivatives.

The first condition is for problem (1), that is, for an incumbent firm, while the second is for problem (2), which is the problem solved by a new firm. The left-hand-side terms are the marginal costs of investing in knowledge and the right-hand-side terms are the marginal benefits, captured by the marginal productivity of knowledge. For a new firm the marginal cost of investing in knowledge is given by the effort cost incurred by the entrepreneur. For an incumbent firm there is an additional cost due to the obsolescence of physical capital induced by innovations. This is the term $\psi_{h_1} > 0$. Because the marginal cost of accumulating knowledge capital is higher for incumbent firms, they choose a lower value of $h_1$.

This result will be used below to study the equilibrium when the investor does not commit to the contract. Before proceeding, however, we have to show that, if the participation constraint for the entrepreneur is satisfied before investing in knowledge, it will also be satisfied after the choice of $h_1$. We implicitly used this result in the analysis above. To show this, we have to consider the problem solved by a new firm started after the entrepreneur has chosen $h_1$. This problem can be written as:

$$\widehat{S}(h_1) \;=\; \max_{k_1,d_0}\left\{ -\kappa - k_1 + A h_1^{1-\alpha} k_1^{\alpha} \right\} \tag{6}$$

$$\textbf{s.t.} \quad d_0 \geq \widehat{D}(h_1)$$

Assuming that the entrepreneur gets the whole surplus, we have that $\widehat{D}(h_1) = \widehat{S}(h_1) = d_0$. Therefore, the participation constraint, after the investment in knowledge, can be written as:

$$d_0 \geq \widehat{S}(h_1) \tag{7}$$

It is now easy to show that, if constraint (3) is satisfied, then constraint (7) is also satisfied. This follows from the fact that $S(h_0) + \varphi(h_0, h_1) \geq \widehat{S}(h_1)$, as can be verified from problems (2) and (6).[4]

[4] Given problems (2) and (6), we can write $S(h_0) = \max_h\{-\varphi(h_0, h) + \widehat{S}(h)\} \geq -\varphi(h_0, h_1) + \widehat{S}(h_1)$, for any $h_1$. Therefore, $S(h_0) + \varphi(h_0, h_1) \geq \widehat{S}(h_1)$.

### 4.2 Competitive economy without investor commitment

Does the limited commitment from the investor lead to lower accumulation of knowledge? We show in this section that, as long as there is competition (free entry), the opposite holds true.

We want to show first that, after the entrepreneur has accumulated the knowledge capital, the investor has an incentive to renegotiate the optimal contract studied above. From the previous analysis we know that $S(h_0) + \varphi(h_0, h_1) \geq \widehat{S}(h_1)$. But, as long as the depreciation of physical capital increases with $h_1$, this inequality is strict. In fact, from problems (2) and (6) we can write:

$$S(h_0) = \max_h \Big\{ -\varphi(h_0, h) + \widehat{S}(h) \Big\} \geq -\varphi(h_0, h_1) + \widehat{S}(h_1)$$

where $h_1$ is the knowledge investment chosen by an incumbent firm. Let $h_{max}$ be the knowledge investment chosen by a new firm. This is the solution to the above maximization problem. As we have shown in the previous section, $h_{max} > h_1$ (see conditions (4) and (5)). This implies that the above inequality is strict and the participation constraint after the choice of knowledge is not binding. In fact,

$$d_0 = S(h_0) + \varphi(h_0, h_{max}) > \widehat{S}(h_1)$$

Because, $d_0 > \widehat{S}(h_1)$, the investor has an incentive to renegotiate down the payment promised to the entrepreneur. The ability to renegotiate can be justified by assuming that the investor can replace the current entrepreneur by poaching other entrepreneurs (currently managing other firms and have the same level of knowledge). Of course, the entrepreneur anticipates that the promised payments will be renegotiated after the knowledge investment. Therefore, he will quit the firm at the beginning of the period unless the investor agrees to the same knowledge investment chosen by a new firm, that is, $h_1 = h_{max}$. This implies that $S(h_0) + \varphi(h_0, h_{max}) = \widehat{S}(h_{max})$ and $d_0 = \widehat{S}(h_{max})$. In this way the entrepreneur keeps the repudiation value high and prevents the investor from renegotiating.

The analysis above shows that the lack of commitment from the investor does not lead to lower investment in knowledge. It is important to emphasize, however, that this is true only if there is competition. As we will see below, limited commitment does lead to lower accumulation of knowledge if there is not competition.

### 4.3 Non-competitive economy with investor commitment

When the investor commits, the optimal contract chooses $h_1$ and $k_1$ to maximize the total surplus as in (1) and (2). The only difference is that now the repudiation values, $D(h_0)$ and $\widehat{D}(h_1)$, are equal to the reservation utility $R$. But we have seen that, when the investor commits, the investment in knowledge does not depend on the repudiation values. They only affect the entrepreneur's payment $d_0$. Therefore, the lack of competition does not affect the rate of innovation as long as there is commitment from the investor.

### 4.4 Non-competitive economy without investor commitment

As in the previous case, the repudiation value for the entrepreneur is the reservation utility $R$. Therefore, the payment to the entrepreneur must satisfy the following constraints:

$$\begin{aligned} d_0 &\geq \varphi(h_0, h_1) + R \\ d_0 &\geq R \end{aligned}$$

The first is the participation constraint at the beginning of the period, before investing in knowledge. The second is the participation constraint after investing in knowledge.

Because the investor does not commit to the optimal contract, he will renegotiate down any payment that exceeds the reservation utility after the investment in knowledge. This implies that the second participation constraint will be satisfied with equality, that is, $d_0 = R$. But then the first constraint will not be satisfied unless $h_1 = h_0$. This implies that there will be no investment in knowledge and the economy stagnates.

Intuitively, the entrepreneur anticipates that any promise of payments above $R$ will be renegotiated. Because knowledge does not have any value outside the firm, there is no ways in which the entrepreneur can benefit from investing in knowledge. It is important to point out that allowing for payments before the investment does not solve the problem: the investor would not make any payment because the entrepreneur could quit the firm and enjoy the reservation utility $R$ after receiving the payment. The limited commitment is for both, the investor and the entrepreneur.

### 4.5 Summary results

We summarize the properties of the two-period version of the model in Table 2. We denote with $g^* = h_1/h_0 - 1 > 0$ the growth rate of knowledge capital in the competitive economy with investor's commitment. This economy acts as a reference of comparison. The key finding is that limited enforcement of contracts is not a cause of stagnation as long as there is competition. On the contrary, limited enforcement may even enhance growth if there is competition. At the same time, the lack of competition is not a cause of stagnation if there is commitment from the investor. What is harmful for growth is the lack of both commitment and competition.

Table 2: Summary results

| | *Competitive Economy* | *Non-competitive Economy* |
|---|---|---|
| *Commitment* | Growth=$g^*$ | Growth=$g^*$ |
| *No commitment* | Growth>$g^*$ | Growth=0 |

In the next section we study the general model with an infinite number of periods and with knowledge spillovers. The analysis of the infinite horizon model provides some further insights that are not captured by the simple two-period model. In particular, while in the two period model barriers to business start-up do not affect growth as long as there is commitment from the investor, this is no longer the case in the general model. In particular, we will show that, even if the investor commits to the contract, the equilibrium growth rate may be smaller when there are barriers to business start-up. The key ingredients leading to this result is the continuous formation of new businesses and the economy-wide spillovers that introduce important general equilibrium effects.

## 5 The infinite horizon model

In this section we study the general model with infinitely lived agents. We first characterize the equilibrium for the competitive economy with investor's commitment. We will turn then to the study of the other economies.

For the analysis that follows, it will be convenient to define the gross output function, inclusive of undepreciated capital, as follows :

$$\pi(h_t, k_t, h_{t+1}) = Ah_t^{1-\alpha}k_t^{\alpha} + \left[1 - \psi\left(\frac{h_{t+1}^{1-\alpha}}{h_t^{1-\alpha}}\right)\right]k_t$$

In writing this expression, we have substituted $z_t = Ah_t^{1-\alpha}$ in the production and depreciation function.

We start characterizing the optimization problem solved by a new firm created at the beginning of period $t$ by an entrepreneur with knowledge capital $h_t$. This is before the current investment in knowledge. Because we assume that the entrepreneur gets the whole initial surplus, it will be convenient to characterize the optimal contract by maximizing the value for the entrepreneur, subject to the enforceability and participation constraints.[5]

Let $D(\mathbf{s}_t; h_t)$ be the repudiation value for the entrepreneur at the beginning of the period, before investing in knowledge, where $\mathbf{s}_t$ denotes the aggregate states. This is the value that the entrepreneur with knowledge $h_t$ would get from starting a new firm. Furthermore, let $\widehat{D}(\mathbf{s}_t; h_{t+1})$ be the value of quitting after choosing the knowledge investment, and therefore, after the effort. At this point the stock of knowledge is $h_{t+1}$. For the moment we take these two functions as given.

In the general model entrepreneurs and firms survive to the next period with probability $p$. This implies that the discount factor becomes $\bar{\beta} = p\beta$. The optimization problem with investor's commitment can be written as:

$$V(\mathbf{s}_t; h_t) = \max_{\{d_\tau, k_{\tau+1}, h_{\tau+1}\}_{\tau=t}^{\infty}} \sum_{\tau=t}^{\infty} \bar{\beta}^{\tau-t}\Big[d_\tau - \varphi(\mathbf{s}_\tau; h_\tau, h_{\tau+1})\Big] \tag{8}$$

subject to

[5] Alternatively, we could maximize the whole surplus as we did in the analysis of the two-period model. Of course, this would give the same results.

$$\sum_{j=\tau}^{\infty}\bar{\beta}^{j-\tau}\Big[d_j-\varphi(\mathbf{s}_j;h_j,h_{j+1})\Big]\geq D(\mathbf{s}_\tau;h_\tau),\qquad \text{for } \tau\geq t \tag{9}$$

$$d_\tau+\sum_{j=\tau+1}^{\infty}\bar{\beta}^{j-\tau}\Big[d_j-\varphi(\mathbf{s}_j;h_j,h_{j+1})\Big]\geq \widehat{D}(\mathbf{s}_\tau;h_{\tau+1}),\quad \text{for } \tau\geq t \tag{10}$$

$$-\kappa_t-d_t-k_{t+1}+\sum_{\tau=t+1}^{\infty}\bar{\beta}^{\tau-t}\Big[\pi(h_\tau,k_\tau,h_{\tau+1})-d_\tau-k_{\tau+1}\Big]\geq 0 \tag{11}$$

The objective is the discounted flow of utilities for the entrepreneur. In each period the entrepreneur receives the payment $d_\tau$, which is subject to a non-negativity constraint, and faces the disutility from effort $\varphi(\mathbf{s}_\tau;h_\tau,h_{\tau+1})$. Constraints (9) and (10) are the enforcement constraints. Starting at time $t+1$, the entrepreneur could quit at the beginning of the period, before choosing the investment in knowledge. In this case the repudiation value is $D(\mathbf{s}_\tau;h_\tau)$. After choosing the knowledge investment, the value of quitting becomes $\widehat{D}(\mathbf{s}_\tau;h_{\tau+1})$. The last constraint is the participation constraint for the investor or break-even condition. This simply says that the value of the contract for the investor cannot be negative.

For an entrepreneur who starts a new firm *after* investing in knowledge, the value of the new contract is:

$$\widehat{V}(\mathbf{s}_t;h_{t+1}) = \max_{\{d_\tau,k_{\tau+1},h_{\tau+2}\}_{\tau=t}^{\infty}}\left\{d_t+\sum_{\tau=t+1}^{\infty}\bar{\beta}^{\tau-t}\Big[d_\tau-\varphi(\mathbf{s}_\tau;h_\tau,h_{\tau+1})\Big]\right\}$$

subject to (9), (10) and (11)

The key difference respect to problem (8) is that the current effort cost has already been sustained by the entrepreneur and $h_{t+1}$ is given. Therefore, the current return for the entrepreneur is only $d_t$. This also explains why the choice of knowledge starts at $t+2$.

Given the definitions of $V(\mathbf{s}_t;h_t)$ and $\widehat{V}(\mathbf{s}_t;h_{t+1})$, it is easy to see that these two functions are related as follows:

$$V(\mathbf{s}_t;h_t)=\max_{h_{t+1}}\Big\{-\varphi(H_t;h_t,h_{t+1})+\widehat{V}(\mathbf{s}_t;h_{t+1})\Big\} \tag{12}$$

The optimization problems above assume that we know the repudiation functions $D(\mathbf{s}_t;h_t)$ and $\widehat{D}(\mathbf{s}_t;h_{t+1})$. But these functions are unknown because they dependent on the value functions $V(\mathbf{s}_t;h_t)$ and $\widehat{V}(\mathbf{s}_t;h_{t+1})$. More specifically, they are given by:

$$D(\mathbf{s}_\tau;h_\tau) = \max\Big\{R(\mathbf{s}_\tau)\,,\,V(\mathbf{s}_\tau;h_\tau)\Big\} \tag{13}$$

$$\widehat{D}(\mathbf{s}_\tau;h_{\tau+1}) = \max\Big\{R(\mathbf{s}_\tau)\,,\,\widehat{V}(\mathbf{s}_\tau;h_{\tau+1})\Big\} \tag{14}$$

Because the entrepreneur has always the option to the reservation utility $R(\mathbf{s}_\tau)$, the repudiation value is the maximum between $R(\mathbf{s}_\tau)$ and the value of starting a new business. However, in the analysis that follows we assume that the reservation utility $R(\mathbf{s}_\tau)$ is sufficiently small that managing a firm is always preferable. Therefore, the repudiation value is simply equal to the value of starting a new business, that is, $D(\mathbf{s}_\tau;h_\tau) = V(\mathbf{s}_\tau;h_\tau)$ and $\widehat{D}(\mathbf{s}_\tau;h_{\tau+1}) = \widehat{V}(\mathbf{s}_\tau;h_{\tau+1})$.

Before proceeding we state a property that simplifies the characterization of the optimal contracting problem.

**Lemma 1** *Constraint (10) is always satisfied if constraint (9) is satisfied.*

**Proof 1** *See Appendix A.*

Hence, in characterizing the solution of the problem with investor's commitment, we can ignore the enforcement constraint (10). This constraint becomes relevant when the investor does not commit, as we will see later.

### 5.1 Recursive formulation and equilibrium

The aggregate state of the economy, denoted by $\mathbf{s}_t$, is the economy-wide distribution of knowledge and physical capital. Because there is persistent growth, the optimization problem is not stationary. It is then convenient to adopt a normalization of variables. Define $g_t = H_{t+1}/H_t$, $\tilde{h}_t = h_t/H_t$, $\tilde{k}_t = k_t/H_t$, $\tilde{d}_t = d_t/H_t$. Furthermore, define $\tilde{\mathbf{s}}_t$ as the distribution of firms over the normalized knowledge and physical capital $(\tilde{h},\tilde{k})$. This is the aggregate states in the normalized problem. Because the effort cost $\varphi(H_t;h_t,h_{t+1})$ and the gross output $\pi(h_t,k_t,h_{t+1})$ are homogeneous of degree 1, these two functions can be rewritten as:

$$\begin{aligned}\varphi(H_t;h_t,h_{t+1}) &= \varphi(\tilde{h}_t,g_t\tilde{h}_{t+1})\cdot H_t\\ \pi(h_t,k_t,h_{t+1}) &= \pi(\tilde{h}_t,\tilde{k}_t,g_t\tilde{h}_{t+1})\cdot H_t\end{aligned}$$

We then have the following lemma:

**Lemma 2** *The functions $V(H_t;h_t)$, $D(H_t;h_t))$, $\widehat{D}(H_t;h_{t+1})$ take the form:*

$$\begin{aligned} V(\mathbf{s}_t;h_t) &= V(\tilde{\mathbf{s}}_t;\tilde{h}_t)\cdot H_t \\ D(\mathbf{s}_t;h_t) &= D(\tilde{\mathbf{s}}_t;\tilde{h}_t)\cdot H_t \\ \widehat{D}(\mathbf{s}_t;h_{t+1}) &= \widehat{D}(\tilde{\mathbf{s}}_t;g_t\tilde{h}_{t+1})\cdot H_t \end{aligned}$$

**Proof 2** *See Appendix B.*

Given this lemma, we can use the aggregate stock of knowledge $H_t$—which grows over time—as a scaling factor for all growing variables. The optimization problem can be rewritten as:

$$V(\tilde{\mathbf{s}}_t,\tilde{h}_t) = \max_{\{\tilde{d}_\tau,\tilde{k}_{\tau+1},\tilde{h}_{\tau+1}\}_{\tau=t}^{\infty}} \left\{\sum_{\tau=t}^{\infty}\bar{\beta}_{t,\tau}\Big[\tilde{d}_\tau-\varphi(\tilde{h}_\tau,g_\tau\tilde{h}_{\tau+1})\Big]\right\} \tag{15}$$

subject to

$$\sum_{j=\tau}^{\infty}\bar{\beta}_{\tau,j}\Big[\tilde{d}_j-\varphi(\tilde{h}_j,g_j\tilde{h}_{j+1})\Big]\geq D(\tilde{\mathbf{s}}_\tau,\tilde{h}_\tau), \qquad \text{for } \tau\geq t \tag{16}$$

$$-\bar{\kappa}-\tilde{d}_t-g_t\tilde{k}_{t+1}+\sum_{\tau=t+1}^{\infty}\bar{\beta}_{t,\tau}\Big[\pi(\tilde{h}_\tau,\tilde{k}_\tau,g_\tau\tilde{h}_{\tau+1})-\tilde{d}_\tau-g_\tau\tilde{k}_{\tau+1}\Big]\geq 0 \tag{17}$$

where $\bar{\beta}_{t,\tau}=\Big(\Pi_{j=t}^{\tau-1}\bar{\beta}g_j\Big)$.

The optimization problem solved by the firm depends on the future evolution of the economy-wide knowledge $H_t$, which in turn depends on the distribution of firms over the normalized knowledge and physical capital, $\tilde{\mathbf{s}}_t$. In the analysis that follows we concentrate on the balanced growth path equilibrium where the distribution of firms remains constant and the average knowledge grows at the constant rate $g$. Under these conditions, the discount factor becomes $\bar{\beta}_{t,\tau}=(g\bar{\beta})^{\tau-t}$ and we can ignore the distribution $\tilde{\mathbf{s}}$ as an explicit argument in the value functions. Following is a formal definition of a balanced growth equilibrium.

**Definition 1** *A Balanced Growth Equilibrium with investor's commitment is defined as (i) A value function $V(\tilde{h})$; (ii) A repudiation function $D(\tilde{h})$; (iii)*

*Decision rules $d_j(\tilde{h}_0)$, $h_j(\tilde{h}_0)$ and $k_j(\tilde{h}_0)$, for $j = 0, 1, ...$; and (iv) A distribution of normalized knowledge and physical capital $\tilde{\mathbf{s}}$. Such that: (i) The decision rules solve problem (15) and $V(\tilde{h})$ is the associated value function; (ii) The repudiation function is the value of starting a new firm, $D(\tilde{h}) = V(\tilde{h})$; (iii) The distribution $\tilde{\mathbf{s}}$ remains constant over time.*

In the above definition we have denoted by $d_j(\tilde{h}_0)$ the entrepreneur's payment at time $j$ for a firm started at time zero with initial knowledge $\tilde{h}_0$. Similarly, $h_j(\tilde{h}_0)$ and $k_j(\tilde{h}_0)$ are the normalized knowledge and physical capital chosen at time $j$ by the same firm.

Even though we limit the analysis to the balanced growth path, the transition experienced by new firms affects the long-term growth. To characterize the equilibrium, it will be convenient to use a transformation of problem (15). Appendix C shows that in the balanced growth path the contractual problem can be reformulated as follows:

$$\min_{\mu' \geq \mu_0} \max_{\tilde{d} \geq 0, \tilde{h}', \tilde{k}'} \Bigg\{ -\bar{\kappa} - \tilde{d} - g\tilde{k}' + \mu' \Big[\tilde{d} - \varphi(\tilde{h}_0, g\tilde{h}')\Big] - (\mu' - \mu_0)D(\tilde{h}_0) + g\bar{\beta}W(\mu', \tilde{h}', \tilde{k}')\Bigg\} \quad (18)$$

where $\mu_0$ is the inverse of the Lagrange multiplier associated with the participation constraint (17) and the function $W$ is defined recursively as:

$$W(\mu, \tilde{h}, \tilde{k}) = \min_{\mu' \geq \mu} \max_{\tilde{d} \geq 0, \tilde{h}', \tilde{k}'} \Bigg\{ \pi(\tilde{h}, \tilde{k}, g\tilde{h}') - \tilde{d} - g\tilde{k}' + \mu' \Big[\tilde{d} - \varphi(\tilde{h}, g\tilde{h}')\Big] - (\mu' - \mu)D(\tilde{h}) + g\bar{\beta}W(\mu', \tilde{h}', \tilde{k}')\Bigg\} \quad (19)$$

Problem (18) is the problem solved by a new firm started by an entrepreneur with (normalized) knowledge $\tilde{h}_0$. After starting the firm and choosing the first period investment, the problem becomes recursive as written in (19). Therefore, (19) is the problem solved by an incumbent firm. The variable $\mu$ can be interpreted as the weight that a hypothetical planner gives to the entrepreneur. The weight given to the investor is 1. Over time the planner increases the weight $\mu$ to make sure that the entrepreneur does not quit the firm, until $\mu = 1$. Larger is the initial weight $\mu_0$ and higher is the initial value of the contract for the entrepreneur, and therefore, lower is the value for the investor. The value of $\mu_0$ is determined such that the investor breaks even, that is, constraint (17) is satisfied with equality. See Marcet & Marimon (1997) for details about the use of the saddle-point formulation for the recursive formulation of the optimization problem.

### 5.2 First order conditions

The first order conditions for problem (18) are:

$$D(\tilde{h}_0) \leq \tilde{d} - \varphi(\tilde{h}_0, g\tilde{h}') + g\bar{\beta}D(\tilde{h}') \tag{20}$$
$$\mu' \leq 1 \tag{21}$$
$$\mu'\varphi_2(\tilde{h}_0, g\tilde{h}') = \bar{\beta}W_2(\mu', \tilde{h}', \tilde{k}') \tag{22}$$
$$\bar{\beta}\pi_2(\tilde{h}', \tilde{k}', g\tilde{h}') = 1 \tag{23}$$

The first order conditions for problem (19) are:

$$D(\tilde{h}) \leq \tilde{d} - \varphi(\tilde{h}, g\tilde{h}') + g\bar{\beta}D(\tilde{h}') \tag{24}$$
$$\mu' \leq 1 \tag{25}$$
$$-\pi_3(\tilde{h}, \tilde{k}, g\tilde{h}') + \mu'\varphi_2(\tilde{h}, g\tilde{h}') = \bar{\beta}W_2(\mu', \tilde{h}', \tilde{k}') \tag{26}$$
$$\bar{\beta}\pi_2(\tilde{h}', \tilde{k}', g'\tilde{h}'') = 1 \tag{27}$$

Here we use subscripts to denote the derivative with respect to the particular argument. In conditions (20) and (24) the inequality constraints are strict if $\mu' > \mu$, while in conditions (21) and (25) the inequalities are strict if the entrepreneur's payment is zero, that is, $\tilde{d} = 0$. The envelope term $W_2$ is equal to:

$$W_2(\mu, \tilde{h}, \tilde{k}) = \pi_1(\tilde{h}, \tilde{k}, g\tilde{h}') - \mu'\varphi_1(\tilde{h}, g\tilde{h}') - (\mu' - \mu)D_1(\tilde{h})$$

The first order conditions characterize the solution of the optimal contract problem. Firms that survive for a sufficient long period of time will have $\mu = \mu' = 1$. Furthermore, their knowledge and physical capital will converge to some constant values $\tilde{h}^*$ and $\tilde{k}^*$. These values satisfy:

$$-\pi_3(\tilde{h}^*, \tilde{k}^*, g) + \varphi_2(\tilde{h}^*, g\tilde{h}^*) = \bar{\beta}\Big[\pi_1(\tilde{h}^*, \tilde{k}^*, g\tilde{h}^*) - \varphi_1(\tilde{h}^*, g\tilde{h}^*)\Big]$$
$$\bar{\beta}\pi_2(\tilde{h}^*, \tilde{k}^*, g\tilde{h}^*) = 1$$

where $g$ is the steady state growth rate of aggregate knowledge. These two conditions are functions of three variables: $g$, $\tilde{h}^*$ and $\tilde{k}^*$. Therefore, to solve for these variables we need an extra condition. This is given by $\int_{\tilde{h},\tilde{k}} \tilde{h}\tilde{\mathbf{s}}(\tilde{h},\tilde{k}) = 1$, that is, the aggregation of normalized knowledge must be equal to 1. To verify this condition we need to know the steady state distribution $\tilde{\mathbf{s}}$, which requires us to solve for the whole transition dynamics of new entrant firms.

## 6 The other economies

We now turn to the study of the other environments. The analysis conducted in the previous section will facilitate the characterization of the equilibria in these alternative environments.

### 6.1 Competitive economy without investor's commitment

Without commitment, the investor could renegotiate the payments promised to the entrepreneur by the long-term contract. The renegotiation of these payments is made credible by the ability of the investor to replace the current entrepreneur with other entrepreneurs, currently running other firms. In particular, the investor will renegotiate payments whose present value exceeds the repudiation value of the entrepreneur.

We have shown in the previous section that in the long-term contract with investor's commitment, constraint (10) is never binding. This implies that, after the entrepreneur has chosen the knowledge investment, the investor has an incentive to renegotiate down the payments up to the point in which the value of staying with the firm is equal to the value of quitting. Therefore, without investor's commitment, the optimization problem can still be written as in (8) but with the enforcement constraint (10) satisfied with equality. For convenience, we rewrite below the enforcement constraints for the entrepreneur:

$$\sum_{j=\tau}^{\infty} \bar{\beta}^{j-\tau}\Big[d_j - \varphi(H_j; h_j, h_{j+1})\Big] \geq D(\mathbf{s}_\tau, h_\tau)$$

$$d_\tau + \sum_{j=\tau+1}^{\infty} \bar{\beta}^{j-\tau}\Big[d_j - \varphi(H_j; h_j, h_{j+1})\Big] = \widehat{D}(\mathbf{s}_\tau, h_{\tau+1})$$

which must hold for any $\tau \geq t$. Remembering that $D(\mathbf{s}_\tau, h_\tau) = V(\mathbf{s}_\tau, h_\tau)$ and $\widehat{D}(\mathbf{s}_\tau, h_{\tau+1}) = \widehat{V}(\mathbf{s}_\tau, h_{\tau+1})$, we can rewrite these two constraints as follows:

$$\sum_{j=\tau}^{\infty} \bar{\beta}^{j-\tau}\Big[d_j - \varphi(H_j; h_j, h_{j+1})\Big] \geq V(\mathbf{s}_\tau, h_\tau)$$

$$\sum_{j=\tau}^{\infty}\bar{\beta}^{j-\tau}\Big[d_j-\varphi(H_j;h_j,h_{j+1})\Big]=-\varphi(H_\tau;h_\tau,h_{\tau+1})+\widehat{V}(\mathbf{s}_\tau,h_{\tau+1})$$

In the previous section we have shown that the two value functions are related as follows (see equation (12)):

$$V(\mathbf{s}_t;h_t)=\max_{h}\Big\{-\varphi(H_t;h_t,h)+\widehat{V}(\mathbf{s}_t;h)\Big\}$$

The solution to this problem is the investment in knowledge chosen by a new firm, that we denote by $h_{max}$. But the knowledge investment chosen by an incumbent firm, $h_{t+1}$, may be different from $h_{max}$. If this is the case, we will have that $V(\mathbf{s}_t;h_t)>-\varphi(H_t;h_t,h_{t+1})+\widehat{V}(\mathbf{s}_t;h_{t+1})$. But then one of the two enforcement constraints will be violated. Therefore, the only feasible solution when the investor does not commit to the long-term contract is the one for which incumbent firms choose the same investment level as new firms, that is, $h_{t+1}=h_{max}$. We summarize this in the following proposition:

**Proposition 1** *Without investor's commitment, the knowledge investment $h_{\tau+1}$ chosen by an incumbent firm is equal to the knowledge investment chosen by a newly created firm.*

**Proof 1** *Implicit in the analysis above.*

This result has a simple intuition. Because the investor can renegotiate the promised payments after the investment in knowledge, the entrepreneur would not stay with the firm unless the investor agrees to the same knowledge investment chosen by a new firm. In this way, the entrepreneur keeps his outside value high, avoiding the risk of renegotiation.

Proposition 1 implies that the knowledge investment of an incumbent firm is not under the control of the firm. Denote by $\tilde{h}_{\tau+1}=f(\tilde{\mathbf{s}}_\tau;\tilde{h}_\tau)$ the normalized knowledge investment of a new firm started at time $\tau\geq t$. The optimization problem for a new firm created at time $t$ can be written as:

$$V(\tilde{\mathbf{s}}_t;\tilde{h}_t)\;=\max_{\tilde{h}_{t+1},\{\tilde{d}_\tau,\tilde{k}_{\tau+1}\}_{\tau=t}^{\infty}}\sum_{\tau=t}^{\infty}\bar{\beta}^{\tau-t}\Big[\tilde{d}_\tau-\varphi(\tilde{h}_\tau,g_\tau\tilde{h}_{\tau+1})\Big]\tag{28}$$

subject to

$$\sum_{j=\tau}^{\infty}\bar{\beta}^{j-\tau}\Big[\tilde{d}_j-\varphi(\tilde{h}_j,g_j\tilde{h}_{j+1})\Big]\geq D(\tilde{\mathbf{s}}_\tau;\tilde{h}_\tau), \qquad \text{for } \tau\geq t$$

$$-\bar{\kappa}-\tilde{d}_t-g_t\tilde{k}_{t+1}+\sum_{\tau=t+1}^{\infty}\bar{\beta}^{\tau-t}\Big[\pi(\tilde{h}_\tau,\tilde{k}_\tau,g_\tau\tilde{h}_{\tau+1})-\tilde{d}_\tau-g_\tau\tilde{k}_{\tau+1}\Big]\geq 0$$

$$\tilde{h}_{\tau+1}=f(\tilde{\mathbf{s}}_\tau;\tilde{h}_\tau), \qquad \text{for } \tau\geq t+1$$

The difference with the case in which the investor commits to the long-term contract is that the investment in knowledge, after the first period, is determined by the function $f(\tilde{\mathbf{s}}_\tau;\tilde{h}_\tau)$, which is taken as given by the firm. We can now define the equilibrium in the balanced growth path.

**Definition 2** *A Balanced Growth Equilibrium without investor's commitment is defined by (i) A value function $V(\tilde{h})$; (ii) Decision rules $h_0(\tilde{h}_0)$, $d_j(\tilde{h}_0)$, and $k_j(\tilde{h}_0)$, for $j=0,1,...$; (iii) An investment function for incumbent firms $f(\tilde{h})$; (iv) A repudiation function $D(\tilde{h})$; and (v) A distribution of normalized knowledge and physical capital $\tilde{\mathbf{s}}$. Such that: (a) The decision rules solve problem (28) and $V(\tilde{h})$ is the associated value function; (b) The investment policy of incumbent firms is equal to the policy of new firms, $f(\tilde{h})=h_0(\tilde{h})$ for all $\tilde{h}$; (c) The repudiation function is the value of starting a new firm, $D(\tilde{h})=V(\tilde{h})$; (d) The distribution $\tilde{\mathbf{s}}$ remains constant over time.*

The definition is similar to the one provided for the case of commitment with the exception of condition (b). This condition imposes that the knowledge investment chosen by an incumbent firm is equal to the investment chosen by a new firm with the same states.

The next step is to show that, because of capital obsolescence for incumbent firms, the knowledge investment chosen by a new firm is higher than the value preferred by an incumbent firm and this leads to faster growth. Following the same steps of Appendix C, we can show that problem 28 can be reformulated as follows:

$$\min_{\mu'\geq\mu_0}\max_{\tilde{d}\geq 0,\tilde{h}',\tilde{k}'}\bigg\{-\bar{\kappa}-\tilde{d}-g\tilde{k}'+\mu'\Big[\tilde{d}-\varphi(\tilde{h}_0,g\tilde{h}')\Big] -(\mu'-\mu_0)D(\tilde{h}_0)+g\bar{\beta}W(\mu',\tilde{h}',\tilde{k}')\bigg\} \quad (29)$$

where $\mu_0$ is the inverse of the Lagrange multiplier associated with the participation constraint for the investor and the function $W$ is defined as:

$$W(\mu,\tilde{h},\tilde{k}) = \min_{\mu'\geq\mu}\max_{\tilde{d}\geq 0,\tilde{k}'}\Bigg\{\pi\Big(\tilde{h},\tilde{k},gf(\tilde{h})\Big)-\tilde{d}-g\tilde{k}'+\mu'\Big[\tilde{d}-\varphi\Big(\tilde{h},gf(\tilde{h})\Big)\Big] -(\mu'-\mu)D(\tilde{h})+g\bar{\beta}W\Big(\mu',f(\tilde{h}),\tilde{k}'\Big)\Bigg\} \quad (30)$$

As in the case with investor's commitment, the problem can be divided in two parts: the problem solved by new firms and the problem solved by incumbent firms. For new firms the problem is equivalent to the case of commitment. For incumbent firms, instead, the investment in knowledge is not chosen optimally but it is given by the policy function $f(\tilde{h})$. Obviously, the optimal value of $\tilde{h}'$ chosen by a new firm in problem (29) also depends on the investment policy that the firm will follow in the future, that is, $f(\tilde{h})$. Therefore, we have to solve for a non-trivial fixed point problem. This is in addition to the fixed point problem in the determination of the repudiation function $D(\tilde{h})$.

For problem (29), the first order conditions are given by (20)-(23) and for problem (30) they are given by (24), (25) and (27). Notice that condition (26) is no longer relevant because incumbent firms take the investment policy in knowledge as given. As a result, the envelope term $W_2$ is now equal to:

$$W_2(\mu,\tilde{h},\tilde{k}) = \pi_1\Big(\tilde{h},\tilde{k},gf(\tilde{h})\Big)-\mu'\varphi_1\Big(\tilde{h},gf(\tilde{h})\Big)-(\mu'-\mu)D_1(\tilde{h}) + gf_1(\tilde{h})\pi_3\Big(\tilde{h},\tilde{k},gf(\tilde{h})\Big)$$

Substituting the envelope term in condition (22), the first order condition for the investment in knowledge of new firms is:

$$\mu'\varphi_2(\tilde{h},g\tilde{h}') = \bar{\beta}\Big[\pi_1\Big(\tilde{h}',\tilde{k}',gf(\tilde{h}')\Big)-\mu''\varphi_1\Big(\tilde{h}',gf(\tilde{h}')\Big) -(\mu''-\mu')D_1(\tilde{h}')+gf_1(\tilde{h}')\pi_3\Big(\tilde{h}',\tilde{k}',gf(\tilde{h}')\Big)\Big] \quad (31)$$

Because incumbent firms innovate at the same rate as new firms, this is also the condition that determines the investment in knowledge of incumbent firms. Therefore, in order to determine whether the lack of commitment from the investor leads to faster growth, we have to compare this condition with the condition that determines the optimal investment in knowledge when the investor is able to commit to the long-term contract. This is condition (26). We have the following proposition.

**Proposition 2** *Suppose that $p = 1$ and $\mu_0 = 1$ in the balanced growth path. Then the steady state growth rate in the competitive economy without investor's commitment is higher than in the economy with commitment.*

**Proof 2** *See Appendix D.*

Although it is difficult to prove this result analytically for the general model in which $p < 1$ and $\mu_0 < 1$, we believe that this result applies more generally. The numerical solution of the model, for given parameter values, supports this view as we will show in Section 7.

### 6.2 Non-competitive economy with investor's commitment

In this case the entrepreneur cannot start a new business. Therefore, the repudiation value becomes the reservation utility $R(\mathbf{s}_\tau)$. In the balanced growth path the repudiation value can be written as $R \cdot H_t$.

The optimization problem can be written as in (8). However, the repudiation values $D(\mathbf{s}_\tau, h_\tau)$ and $\widehat{D}(\mathbf{s}_\tau, h_{\tau+1})$ are now equal to the reservation utility $R(\mathbf{s}_\tau)$. The enforcement constraints become:

$$\sum_{j=\tau}^{\infty} \bar{\beta}^{j-\tau} \Big[ d_j - \varphi(H_j; h_j, h_{j+1}) \Big] \geq R(\mathbf{s}_\tau) \tag{32}$$

$$d_\tau + \sum_{j=\tau+1}^{\infty} \bar{\beta}^{j-\tau} \Big[ d_j - \varphi(H_j; h_j, h_{j+1}) \Big] \geq R(\mathbf{s}_\tau) \tag{33}$$

which must hold for any $\tau \geq t$.

As in the competitive economy, the second constraint is always satisfied once we impose the first. Therefore, in characterizing the solution we can ignore constraint (33).

The optimization problem can still be written as in (18) and (19), after imposing $D(\tilde{h}) = R$, and the solution is characterized by the first order conditions (20)-(27). Because of the different repudiation value, the equilibrium growth rate in the balanced growth path may be different compared to the growth rate in the competitive economy. In particular, because $R < V(\tilde{h}^N)$, the constraints to the optimization problem for a new firm started by a newborn entrepreneur are less tight. This implies that the initial $\mu_0$ may differ when compared to the case of a competitive economy. We first state the following lemma:

**Lemma 3** *In the non-competitive economy* $\mu_0 = 1$.

**Proof 3** *Suppose that* $\mu_0 = \mu' < 1$. *This implies that the entrepreneur receives zero payments in the first period. Because* $D(\tilde{h}) = R$*, condition (20) can be written as* $R \leq -\varphi(\tilde{h}_0, g\tilde{h}') + g\bar{\beta}R$*, which is clearly violated. Q.E.D.*

Therefore, assuming that $R$ is sufficiently small so that a solution exists, the investment policy is not constrained. This is not always the case in the competitive economy, where the initial $\mu_0$ could be smaller than 1. More specifically, when there is competition, the repudiation value of the entrepreneur is higher. As a result, a larger share of the surplus will be appropriated by the entrepreneur and the investor may not break even. This would be the case if the set-up cost $\kappa$ is large. To reduce the share of the surplus going to the entrepreneur, the initial weight $\mu_0$ assigned to the entrepreneur must be smaller than 1.

The initial value of $\mu_0$ affects the investment policy of new firms. In particular, lower values of $\mu_0$ induce greater initial investment in knowledge. This can be shown using the first order condition for a new entrant firm, that is, condition (22). After substituting the envelope condition, we have:

$$\mu'\varphi_2(\tilde{h}^N, g\tilde{h}') + \mu''\bar{\beta}\varphi_1(\tilde{h}', g\tilde{h}'') = \bar{\beta}\Big[\pi_1(\tilde{h}', \tilde{k}', g\tilde{h}'') - (\mu'' - \mu')D_1(\tilde{h}')\Big]$$

where $\tilde{h}^N$ is the knowledge capital of a new firm.

The left-hand-side is the cost of accumulating knowledge for the entrepreneur. The right-hand-side is the benefit for the investor. These costs and benefits are weighted by the variables $\mu'$ and $\mu''$. Lower values of $\mu_0$ imply lower values of $\mu'$ and $\mu''$, which tend to reduce the left-hand-side term. This stimulates more investment in knowledge. On the other hand, the term $(\mu'' - \mu')D_1(\tilde{h}')$ decreases the benefit of investing in knowledge. Although this cannot be seen directly from the above condition, the first effect dominates the second and lower values of $\mu_0$ generate greater initial investment in knowledge. We will show this result numerically in Section 7.

Once we have shown that in the competitive economy new entrant firms choose a higher initial investment—assuming that the condition $\mu_0 < 1$ is satisfied—it is easy to see how this generates higher aggregate growth. Because new firms invest more, the next period aggregate level of knowledge is higher. Thanks to the spillover, this reduces the cost of accumulating knowledge in the next period, which in turn increases the investment of all firms. As a result, the economy experiences faster grow. This finding is formalized in the following proposition.

**Proposition 3** *Suppose that in the competitive economy $\mu_0 < 1$. Then the balanced growth rate in the competitive economy is higher than in the non-competitive economy. If $\mu_0 = 1$, the two economies grow at the same rate.*

In Section 7 we will show this result numerically for a parameterized version of the model.

### 6.3 Non-competitive economy without investor's commitment

The last environment to consider is the economy without competition and without investor's commitment. The optimization problem is as in the economy in which the investor commits but constraint (33) must be satisfied with equality. Substituting this constraint in (32) and re-arranging we get:

$$-\varphi(\mathbf{s}_\tau; h_\tau, h_{\tau+1}) + R(\mathbf{s}_\tau) \geq R(\mathbf{s}_\tau)$$

which is satisfied only if $h_{\tau+1} = h_\tau$. Therefore,

**Proposition 4** *In the non-competitive economy without investor's commitment, there is no investment in knowledge and the economy stagnates.*

**Proof 3** *It trivially follows from the above condition.* *Q.E.D.*

Also this result has a simple intuition. Without commitment from the investor, the entrepreneur is unable to get rewarded for the effort in accumulating knowledge. With competition, the entrepreneur is still willing to invest because the accumulated knowledge has a value outside the firm. But in absence of competition, the entrepreneur's knowledge does not have any value outside the firm. Therefore, we reach the conclusion that, in an economy without competition and without investor's commitment, there will be no investment in knowledge and the economy stagnates.

We would like to emphasize that this result relies on both the limited commitment of the investor and the entrepreneur. If the entrepreneur could commit, the problem could be solved by making payments to the entrepreneurs before the investment in knowledge. Without commitment, however, the entrepreneur would quit after receiving the payments.

### 6.4 Summary results

The equilibrium properties of the infinite horizon model are summarized in Table 3. We denote with $g^* = h_1/h_0 - 1 > 0$ the steady state growth rate in the competitive economy with one-side commitment. As before, we use this economy as a reference of comparison. The key finding is that limited enforcement of contracts is not a cause of stagnation as long as there is competition. On the contrary, limited enforcement may even enhance growth if there is competition. At the same time, the lack of competition is detrimental to growth independently of whether there is commitment. This last finding differentiates the infinite horizon model from the two-period model studied in Section 4. In the analysis of the two-period model we have seen that the lack of competition affects growth only in absence of commitment. In the general model, instead, the lack of competition is harmful for growth even if there is commitment from the investors.

Table 3: Summary results: The infinite horizon model.

| | *Competitive Economy* | *Non-competitive Economy* |
|---|---|---|
| *Commitment* | Growth $= g^*$ | Growth $\leq g^*$ |
| *No commitment* | Growth $> g^*$ | Growth $= 0$ |

## 7 Numerical example

In this section we present a numerical example to show some additional features of the model we could not prove analytically. The goal of this section is not to provide a rigorous calibration exercise but simply to validate with a numerical example the properties of the model emphasized in the previous section. We start with the assignment of the parameter values and the specification of functional forms.

The discount factor is $\beta = 0.96$. The survival probability of entrepreneurs is $p = 0.96$. The initial knowledge of entrepreneurs is half the average knowledge in the economy, that is, $\tilde{h}^N = 0.5$.

The set-up cost of a new firm is $\kappa = 0.35$. The production function takes the form $Ah^{1-\alpha}k^{\alpha}$, with $A = 0.115$ and $\alpha = 0.9$. The depreciation of capital is specified as:

$$\delta_t = \psi\left(\frac{z_{t+1}}{z_t}\right) = \left(\frac{z_{t+1} - z_t}{z_t}\right)^{\phi}$$

where $\phi = 1.1$.

The effort cost function is derived from the accumulation equation for the stock of knowledge. More specifically, let's assume that the individual knowledge evolves according to:

$$h_{t+1} = h_t + H_t^{\rho} e_t^{1-\rho}$$

where $H_t$ is the average knowledge in the economy and $e_t$ is the effort from the entrepreneur. The parameter $\rho$ captures the importance of leakage or spillover effects, to which we assign the value of 0.3. By inverting we get the effort cost function:

$$e_t \;=\; \varphi(H_t; h_t, h_{t+1}) \;=\; \left(\frac{h_{t+1} - h_t}{H_t^{\rho}}\right)^{\frac{1}{1-\rho}}$$

After the assignment of the parameter values, we can solve for the balanced growth equilibrium. The steady state growth rates for the four different environments are reported in Table 4. Let's consider first the case in which the investor commits to the long-term contract, that is, the first row of the table. Our computation shows that the competitive economy grows at a steady state rate of 3.01% while the steady state growth rate in the non-competitive economy is only 2.55%. Therefore, the lack of competition

Table 4: Steady state growth rates in the parameterized economy.

| | *Competitive Economy* | *Non-competitive Economy* |
|---|---|---|
| *Commitment* | 3.01% | 2.55% |
| *No commitment* | 7.28% | 0.00% |

impacts negatively on the long-run growth of the economy even if the investor commits.

To understand why the lack of competition is detrimental for growth, we have to compare the dynamics experienced by newly created firms in the economies with and without competition. This is shown in Figure 5. This figure plots the stock of knowledge accumulated overtime by new entrepreneurs in the competitive as well as non-competitive economies. In both economies, the initial level of knowledge is half the average knowledge (where the average knowledge is 1). The key difference is that in the competitive economy the entrepreneur accumulates higher knowledge initially and it reaches the long-term level faster.

The finding that in the competitive economy new entrepreneurs accumulate higher knowledge initially can be explained as follows. The possibility of starting a new business (competition) increases the repudiation value of entrepreneurs. This makes it harder for the investor to break even if the entrepreneur's payments cannot be negative. As a result, the unconstrained solution for the accumulation of knowledge is not feasible. Technically, $\mu_0$ must be smaller than 1. For the break even condition to be satisfied, the entrepreneur has to accumulate higher initial knowledge. By increasing the initial knowledge, the subsequent growth will be smaller. This implies smaller depreciations of physical capital, which is a cost for the investor. By reducing this cost, the investor is able to extract a higher share of the surplus, which guarantees his participation in the contract.[6]

[6]The higher investment in knowledge reduces the total surplus. However, the reduction in the surplus is absorbed by the entrepreneur, not the investor. It is as if the entrepreneur

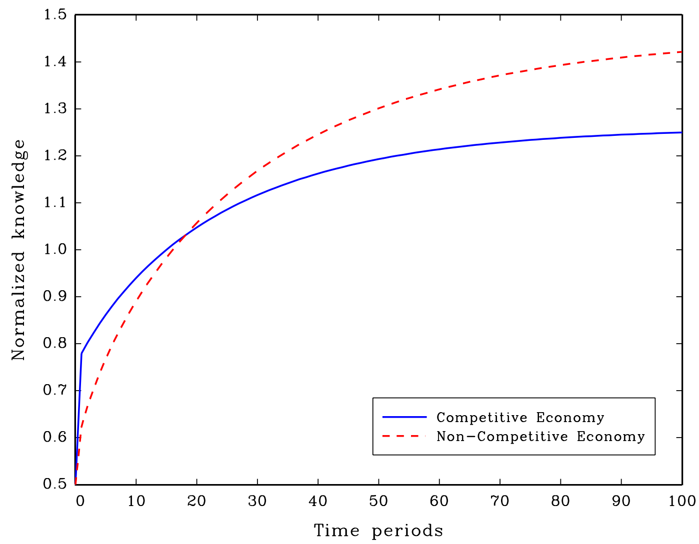


Figure 5: Knowledge capital dynamics with and without competition.

We can now see the aggregate implications of the different accumulation of knowledge in the economy with and without competition. If the two economies start with the same aggregate knowledge, the next period level will be higher in the competitive economy. Because of the spillover, this implies that the next period cost of accumulating knowledge is lower, which encourages more accumulation. As a result, the competitive economy will experience faster growth.

Let's consider now the case in which the investor does not commit to the long-term contract (last row of Table 4). Thanks to competition, the lack of commitment from the investor increases the equilibrium growth rate from 3.01% to 7.28%. However, if there is not competition, the growth rate will fall to zero and the economy stagnates. These findings confirm the qualitative results summarized in Table 3.

makes initial side payments to the investor. The payments, however, are not in cash but in the form of higher efforts.

## 8 Conclusion

Modern technologies are highly complementary to skilled labor. This implies that the adoption of these technologies requires the accumulation of innovation skills or knowledge from workers/managers. In absence of a commitment devise or enforcement for entrepreneurs and investors, under-accumulation of skills may result. However, we have shown that limited enforcement alone is not sufficient to impair the accumulation of knowledge capital and the long-term growth. It is the simultaneous lack of enforcement and competition—that is, the ability of workers/managers to use their skills to start new businesses—which is detrimental to growth. At the same time, the lack of competition or barriers to business entry can be detrimental to growth even if investors commit to the long-term contract.

Our paper provides a theoretical foundation for the empirical finding that the cost of starting a business and the cost of contract enforcement are negatively associated to the level of development and growth of a country.

## A Proof of Lemma 1

Conditions (9) and (10) can be rewritten as:

$$\begin{aligned}\sum_{j=\tau}^{\infty}\bar{\beta}^{j-\tau}\Big[d_j-\varphi(\mathbf{s}_j;h_j,h_{j+1})\Big] &\geq D(\mathbf{s}_\tau;k_\tau)\\ \sum_{j=\tau}^{\infty}\bar{\beta}^{j-\tau}\Big[d_j-\varphi(\mathbf{s}_j;h_j,h_{j+1})\Big] &\geq -\varphi(\mathbf{s}_\tau;h_\tau,h_{\tau+1})+\widehat{D}(\mathbf{s}_\tau;h_{\tau+1})\end{aligned}$$

Therefore, to show that the second constraint is satisfied when the first constraint is satisfied, it is enough to show that $D(\mathbf{s}_\tau;h_\tau)\geq-\varphi(\mathbf{s}_\tau;h_\tau,h_{\tau+1})+\widehat{D}(\mathbf{s}_\tau;h_{\tau+1})$ for any value of $h_{\tau+1}$. From the definition of the repudiation values—equations (13) and (14)—we have that $D(\mathbf{s}_\tau;h_\tau)=\max_h\{-\varphi(\mathbf{s}_\tau;h_\tau,h)+\widehat{D}(\mathbf{s}_\tau;h)\}$. This is at least as big as $-\varphi(\mathbf{s}_\tau;h_\tau,h_{\tau+1})+\widehat{D}(\mathbf{s}_\tau;h_{\tau+1})$ for any $h_{\tau+1}$. *Q.E.D.*

## B Proof of Lemma 2

We use a guess and verify procedure. Suppose that $\widehat{D}(H_t;h_{t+1})=\widehat{D}(g_t\tilde{h}_{t+1})\cdot H_t$. Obviously $D(H_t;h_t)$ takes a similar form:

$$\begin{aligned}D(H_t;h_t) &= \max_{g_t}\Big\{\varphi(\tilde{h}_t,g_t\tilde{h}_{t+1})\cdot H_t+\widehat{D}(g_t\tilde{h}_{t+1})\cdot H_t\Big\}\\ &= \max_{g_t}\Big\{\varphi(\tilde{h}_t,g_t\tilde{h}_{t+1})+\widehat{D}(g_t\tilde{h}_{t+1})\Big\}\cdot H_t\\ &= D(\tilde{h}_t)\cdot H_t\end{aligned}$$

We want to show next that $V(\mathbf{s}_t;h_t)=V(\tilde{\mathbf{s}}_t;\tilde{h}_t)\cdot H_t$. This can be easily proved by normalizing all variables by $H_t$ in problem (8). After normalizing, the optimization is over $\{\tilde{d}_\tau,\tilde{k}_{\tau+1},\tilde{h}_{\tau+1}\}_{\tau=t}^{\infty}$. Simple inspection of the normalized problem proves our claim. *Q.E.D.*

## C Saddle-point formulation

Consider problem (15). Given $\gamma_\tau$ the Lagrange multiplier associated with the enforcement constraint (16) and $\lambda_t$ the Lagrange multiplier associated with the enforcement constraint (17), the Lagrangian can be written as:

$$\begin{aligned}\mathcal{L} &= \sum_{\tau=t}^{\infty}\bar{\beta}_{t,\tau}\Big[\tilde{d}_\tau-\varphi(\tilde{h}_j,g_j\tilde{h}_{j+1})\Big]\\ &+ \sum_{\tau=t}^{\infty}\bar{\beta}_{t,\tau}\gamma_\tau\left\{\sum_{j=\tau}^{\infty}\bar{\beta}_{\tau,j}\Big[\tilde{d}_j-\varphi(\tilde{h}_j,g_j\tilde{h}_{j+1})\Big]-D(\tilde{h}_\tau)\right\}\end{aligned}$$

$$+ \quad \lambda_t \left\{ -\tilde{d}_t - \bar{\kappa} - g_t \tilde{k}_{t+1} + \sum_{\tau=t+1}^{\infty} \bar{\beta}_{t,\tau} \Big[ \pi(\tilde{h}_\tau, \tilde{k}_\tau, g_\tau \tilde{h}_{\tau+1}) - \tilde{d}_\tau - g_\tau \tilde{k}_{\tau+1} \Big] \right\}$$

Define $\tilde{\mu}_\tau$ recursively as follows: $\tilde{\mu}_{\tau+1} = \tilde{\mu}_\tau + \gamma_\tau$, with $\tilde{\mu}_t = 0$. Using this variable and rearranging terms, the Lagrangian can be written as:

$$\begin{aligned} \mathcal{L} \quad &= \quad \sum_{\tau=t}^{\infty} \bar{\beta}_{t,\tau} \Big\{ (1 + \tilde{\mu}_{\tau+1}) \Big[ \tilde{d}_\tau - \varphi(\tilde{h}_\tau, g_\tau \tilde{h}_{\tau+1}) \Big] - (\tilde{\mu}_{\tau+1} - \tilde{\mu}_\tau) D(\tilde{h}_\tau) \Big\} \\ &+ \quad \lambda_t \left\{ -\tilde{d}_t - \bar{\kappa} - g_t \tilde{k}_{t+1} + \sum_{\tau=t+1}^{\infty} \bar{\beta}_{t,\tau} \Big[ \pi(\tilde{h}_\tau, \tilde{k}_\tau, g_\tau \tilde{h}_{\tau+1}) - \tilde{d}_\tau - g_\tau \tilde{k}_{\tau+1} \Big] \right\} \end{aligned}$$

Define $\mu_\tau = (1 + \tilde{\mu}_\tau)/\lambda_t$ for $\tau \geq t+1$ with $\mu_t = 1/\lambda_t$. Substituting we get:

$$\begin{aligned} \mathcal{L} \quad = \quad & \lambda_t \Big\{ -\tilde{d}_t - \bar{\kappa} - g_t \tilde{k}_{t+1} + \mu_{t+1} \Big[ \tilde{d}_t - \varphi(\tilde{h}_t, g_t \tilde{h}_{t+1}) \Big] - (\mu_{t+1} - \mu_t) D(\tilde{h}_t) \\ + & \sum_{\tau=t+1}^{\infty} \bar{\beta}_{t,\tau} \Big[ \pi(\tilde{h}_\tau, \tilde{k}_\tau, g_\tau \tilde{h}_{\tau+1}) - \tilde{d}_\tau - g_\tau \tilde{k}_{\tau+1} + \mu_{\tau+1} \Big[ \tilde{d}_\tau - \varphi(\tilde{h}_\tau, g_\tau \tilde{h}_{\tau+1}) \Big] \\ & \qquad - (\mu_{\tau+1} - \mu_\tau) D(\tilde{h}_\tau) \Big] \Big\} \end{aligned}$$

Dividing by $\lambda_t$ and looking at the special case of a balanced growth path in which the stock of aggregate knowledge grows at the constant rate $g$, the problem can be rewritten as:

$$\begin{aligned} \min_{\mu_{t+1} \geq \mu_t} \; \max_{\substack{\tilde{d}_t \geq 0, \\ \tilde{h}_{t+1}, \tilde{k}_{t+1}}} \; & \Big\{ -\tilde{d}_t - \bar{\kappa} - g \tilde{k}_{t+1} + \mu_{t+1} \Big[ \tilde{d}_t - \varphi(\tilde{h}_t, g \tilde{h}_{t+1}) \Big] \\ & - (\mu_{t+1} - \mu_t) D(\tilde{h}_t) + g \bar{\beta} W(\mu_{t+1}, \tilde{h}_{t+1}, \tilde{k}_{t+1}) \Big\} \end{aligned}$$

for given $\mu_t$ and with the function $W$ defined recursively as follows:

$$\begin{aligned} W(\mu, \tilde{h}, \tilde{k}) \quad = \quad \min_{\mu' \geq \mu} \max_{\substack{\tilde{d} \geq 0, \\ \tilde{h}', \tilde{k}'}} & \Big\{ \pi(\tilde{h}, \tilde{k}, g\tilde{h}') - \tilde{d} - g\tilde{k}' + \mu' \Big[ \tilde{d} - \varphi(\tilde{h}, g\tilde{h}') \Big] \\ & - (\mu' - \mu) D(\tilde{h}) + g \bar{\beta} W(\mu', \tilde{h}', \tilde{k}') \Big\} \end{aligned}$$

The initial state $\mu_t$ is determined such that the participation constraint for the investor is satisfied.

## D Proof of Proposition 2

When $p = 1$, all firms are alike in the balanced growth path. Therefore, $\tilde{h} = 1$. If $\mu_0 = 1$, the first order condition for the accumulation of knowledge, equation (31), can be written as:

$$\varphi_2(1, g) = \beta\Big[\pi_1(1, \tilde{k}, g) - \varphi_1(1, g)\Big] + g\beta f_1(1)\pi_3(1, \tilde{k}, g)$$

With commitment, the first order condition for the accumulation of knowledge is given by equation (26), which in the steady state becomes:

$$\varphi_2(1, g) = \beta\Big[\pi_1(1, \tilde{k}, g) - \varphi_1(1, g)\Big] + \pi_3(1, \tilde{k}, g)$$

Because $\pi_3(1, \tilde{k}', g) < 0$ and $g\beta f_1(1) < 1$, the marginal cost from innovating $\varphi_2(1, g)$ in the first condition must be greater than in the second condition. This implies that in the economy without commitment the growth rate $g$ is higher than in the economy with commitment.

Notice that, without obsolescence in physical capital, $\pi_3(1, \tilde{k}, g) = 0$. Therefore, the two conditions above are indistinguishable, which implies that the commitment of the investor does not affect the long-term growth. *Q.E.D.*

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