Risks For The Long Run And The Real Exchange Rate*

Riccardo Colacito†
Mariano Massimiliano Croce‡

Abstract

Brandt, Cochrane, and Santa-Clara (2004) point out that the implicit stochastic discount factors computed using prices on the one hand and consumption growth on the other hand have very different implications for their cross country correlation. They leave this as an unresolved puzzle. We explain it by combining Epstein and Zin (1989) preferences with a model of predictable returns and by positing a very correlated long run component. We also assume that the intertemporal elasticity of substitution is larger than one. This setup brings the stochastic discount factors computed using prices and quantities close together, by keeping the volatility of the depreciation rate in the order of 14% and the cross country correlation of consumption growth around 30%.

KEYWORDS: International Finance, Asset Pricing, Recursive Utility
FIRST DRAFT: October 31, 2004
THIS DRAFT: January 31, 2005

*We thank Dave Backus, Fabrizio Perri, Sidney Ludvigson and Tom Sargent for useful discussions and encouragement. Seminar participants of the NYU macro reading group provided useful comments.

†New York University, Department of Economics. Email: rc652@nyu.edu.
‡New York University, Department of Economics. Email: massimiliano.croce@nyu.edu.

1 Introduction

There is a growing literature in economics that goes under the name of risks for the long run. Two fundamental forces are at work in this kind of models. Individuals care about the timing of the resolution of uncertainty and consumption is modeled as having a slowly moving predictable component. These models have been very successful in explaining some long standing puzzles of financial economics, such as the high excess return of equities over the risk free rate. Examples of these economies can be found in Bansal and Yaron (2004), Bansal, Gallant, and Tauchen (2002) and Hansen, Heaton, and Li (2004). This paper extends this framework to a two country model. Financial markets seem to indicate a high degree of co-movement in the stochastic discount factors, while the poor correlation of consumption growth across countries points in the opposite direction. Brandt, Cochrane, and Santa-Clara (2004) construct an index, that they regard as an indicator of risk sharing, that highlights this finding: when financial data are used to derive the stochastic discount factors the index is very close to one, while if we assume identical CRRA utility functions in the two countries and construct the stochastic discount factors accordingly, this index drops to 0.3. Our attack to the problem relies on the assumption that consumption growth is determined by a small, but highly persistent and cross-country correlated component. When combined with the type of preferences described before, the puzzle seems to disappear. The economic intuition is that the concept of risk sharing entails both short run and long run risk. If the latter source of uncertainty is common to the two countries and people care about the timing of the resolution of uncertainty, it is possible to reconcile a low correlation of consumption with a high degree of risk sharing.

We prefer to reformulate this puzzle without invoking the idea of international risk sharing. Following Backus, Foresi, and Telmer (1996) for any stochastic process for the depreciation rate and returns on domestic and foreign currency denominated assets, there exist pricing kernels, whose ratio is equal to the depreciation rate, provided that there are no arbitrage opportunities. This "accounting" relationship allows to retrieve the correlation of the stochastic discount factors from their volatilities and the standard deviation of the rate of growth of the exchange rate. Using the Hansen and Jagannathan (1991) bound the variance of the pricing kernels is at least 25%, while Brandt, Cochrane, and Santa-Clara (2004) estimate the volatility of the depreciation rate between the US and three other countries to be around 14%. The implied correlation of the stochastic discount factors is not less than 0.96, when computed in this way. However data on consumption display a very low cross country correlation and the assumption of agents with identical CRRA preferences in the two countries would result in a correlation of the stochastic discount factors around 0.3. As in the equity premium puzzle literature it is impossible to reconcile the high volatility of the pricing kernel with the volatility of consumption growth unless an implausibly high coefficient of risk aversion is assumed, in an international context, CRRA preferences fail to reconcile the high cross country correlation of stochastic discount factors with the relative smoothness of the depreciation rate.

We model a two country economy, each of them populated by a representative consumer with Epstein and Zin (1989) recursive preferences. We specify both consumption and dividend growths as the sum of an i.i.d. shock and a predictable component, that we refer to as the long run term, whose magnitude is small when compared to the volatility of the consumption specific shock. Nevertheless, we impose a highly persistent autoregressive process for this component. The countries share this setup. We allow shocks to be cross country correlated. Bansal and Yaron (2004) show that in a closed economy, this setup is able to explain the realized equity premium, when the intertemporal elasticity of substitution is larger than one. In a two country economy, we show that adding highly correlated long run components can reconcile the degree of co-movement in the stochastic discount factors as measured from prices and from quantities.

The paper is organized as follows. In section 2 we describe the determinants of the puzzle that we want to explain. In section 3, we write down a simple model that we can solve and calibrate. This provides a useful instrument to show the internal transmission mechanism of the model. We then calibrate this model and show how we can match a large number of statistics in the data. In section 5, we extend the model to include assets that pay dividends and we explore what our model has to say about the correlation of the stock markets. Section 6 concludes the paper, summarizing the main findings.

2 The "international equity premium puzzle"

We analyze two economies that we denote as home (h) and foreign (f). Following Backus, Foresi, and Telmer (1996) and Brandt, Cochrane, and Santa-Clara (2004), no arbitrage conditions[1] imply the following relationship between the log-stochastic discount factors of two economies, m_t^h and m_t^f, and the log-depreciation rate, π_{t+1}, defined as the growth of the exchange rate:

$$\pi_{t+1} = m_{t+1}^f - m_{t+1}^h$$

By taking the variance operator on both sides and by denoting $\sigma_{m^i}, \forall i \in \{h, f\}$ as the standard deviation of the stochastic discount factor in the two countries, ρ_{m^h,m^f} as the correlation of the stochastic discount factors and σ_π as the volatility of the depreciation rate, we obtain:

$$\rho_{m^h,m^f} = \frac{\sigma_{m^h}^2 + \sigma_{m^f}^2 - \sigma_\pi^2}{2\sigma_{m^h}\sigma_{m^f}} \quad (1)$$

It is useful to restate the puzzle we are after in terms of equation (1). The Hansen and Jagannathan (1991) bound on the volatility of the stochastic discount factor is in the order of 50% in the US, United Kingdom and Germany[2] and the standard deviation of the depreciation rate between the same countries is in the order of 14%. These numbers and equation (1) imply a correlation of the stochastic discount factors of approximatively 96%. Using identical CRRA preferences, the pricing kernels are $m_t^i = -\gamma \Delta c_t^i$, $\forall i \in \{h, f\}$ and the correlation ρ_{m^h,m^f} is simply the correlation of consumption growth. This number is far below the 0.96 calculated from financial data and it ranges from 0.24 to 0.42 depending on the countries and on the frequency of the data, as in Brandt, Cochrane, and Santa-Clara (2004).

We can also see this as a restatement of Mehra and Prescott (1985) equity premium puzzle. It is well known that when CRRA preferences are used, a high coefficient of risk aversion, γ, is needed to reconcile the low volatility of consumption

[1]See Backus, Foresi, and Telmer (1996) for a detailed explanation of how to derive this relationship.

[2]See for example Brandt, Cochrane, and Santa-Clara (2004).

growth with the degree of volatility of the stochastic discount factor indicated by financial data. A 2% annual volatility of consumption growth would require a γ of 25 to obtain at least as much volatility as the Hansen and Jagannathan (1991) bound. If we specialize equation (1) to this case:

$$\rho_{\Delta c^h, \Delta c^f} = \frac{\gamma^2 \sigma^2_{\Delta c^h} + \gamma^2 \sigma^2_{\Delta c^f} - \sigma^2_\pi}{2\gamma^2 \sigma_{\Delta c^h} \sigma_{\Delta c^f}}$$

we would need the volatility of the depreciation rate to be at least 53%, that is almost 4 times what is observed in the data. In a one country model, the stochastic discount factor should be extremely volatile to explain the equity premium. In a two country model, it is also required that the stochastic discount factors are very correlated across countries to explain the smoothness of the depreciation rate.

This opens up to the rules of the game we want to play. We want to be able to reconcile the implications that both financial and consumption data have for equation (1), by controlling at the same time for risk aversion, cross country correlation of consumption growth and volatility of the depreciation rate.

3 Setup of the economy

3.1 Structure of the markets

We study a two country endowment economy that we shall denote as home (h) and foreign (f). We assume that there are only two goods in the whole economy and that these goods are country specific. To further simplify the setup, we impose that preferences are such that there is complete home bias, meaning that each country is willing to consume only the good that it is endowed with. Markets are complete, implying that returns are equalized across countries after accounting for the exchange rate. An equilibrium of this economy exists, in which each country behaves as in autarky both for consumption and asset holdings.

The need for this extreme structure of the markets is dictated by the fact that we want to use preferences of the Epstein and Zin (1989) type. There exists a literature that examines the dynamics of allocations when trades in the goods market are allowed and agents have recursive, but non time separable preferences. Anderson (2005) and Kan (1995) provide analytical and theoretical tools to study these economies. In our case, the need to also introduce a slowly moving predictable component in the endowment process would make these frameworks difficult to extend. For the sake of explanation, we prefer to study the no-trade limiting case and leave a more realistic structure of the goods market for future extensions of this paper.

3.2 Preferences and long run risks

We model the two economies as each having a representative consumer with Epstein and Zin (1989) preferences:

$$U_t^i = \left\{(1-\delta)(C_t^i)^{\frac{1-\gamma}{\theta}} + \delta \left[E_t\left[(U_{t+1}^i)^{1-\gamma}\right]\right]^{\frac{1}{\theta}}\right\}^{\frac{\theta}{1-\gamma}}, \forall i \in \{h, f\}$$

where γ is the coefficient of risk aversion and $\theta = \frac{1-\gamma}{1-1/\Psi}$ implicitly defines the intertemporal elasticity of substitution Ψ. The two economies are assumed to be symmetric, having the same preference and transition laws parameters. The implied pricing equation for the j^{th} asset is

$$E_t\left[M_{t+1}^i R_{j,t+1}^i\right] = 1, \forall i \in \{h, f\}$$

where the pricing kernel M_{t+1}^i is a stochastic process that depends on consumption growth, $\frac{C_{t+1}^i}{C_t^i}$, on the return on the asset that pays the consumption bundle, $R_{c,t+1}^i$ and on the preference parameters:

$$\log M_{t+1}^i = \theta \log \delta - \frac{\theta}{\Psi} \log\left(\frac{C_{t+1}^i}{C_t^i}\right) + (\theta - 1) \log R_{c,t+1}^i, \forall i \in \{h, f\} \quad (2)$$

In what follows, we will adopt the convention of denoting log-variables in small letters (hence $m_{t+1}^i = \log M_{t+1}^i$). The price of an assets that entitles to a stream of country i consumption bundle costs $P_{c,t}^i$ and has the following gross return:

$$R_{c,t+1}^i = \frac{v_{c,t+1}^i + 1}{v_{c,t}^i} \exp \Delta c_{t+1} \quad (3)$$

with $v^i_{c,t+1}$ being the price-consumption ratio in each country $i = \{h, f\}$. We complete the system by specifying exogenous laws of motion for consumption as:

$$\Delta c^i_t = x^i_{t-1} + \varepsilon^i_{c,t} \quad (4)$$

$$x^i_t = \rho_x x^i_{t-1} + \varepsilon^i_{x,t} \quad (5)$$

$\forall i = \{h, f\}$. All shocks are *iid* normally distributed within each country, but they are allowed to be cross-country correlated according to the following scheme:

$$\begin{bmatrix} \varepsilon^h_{c,t} & \varepsilon^f_{c,t} & \varepsilon^h_{x,t} & \varepsilon^f_{x,t} \end{bmatrix}' \sim N(0, \Sigma)$$

$$\Sigma = \begin{bmatrix} \sigma^2_c H_c & 0 \\ 0 & \sigma^2_x H_x \end{bmatrix}$$

where

$$H_c = \begin{bmatrix} 1 & \rho^{hf}_c \\ \rho^{hf}_c & 1 \end{bmatrix} \quad H_x = \begin{bmatrix} 1 & \rho^{hf}_x \\ \rho^{hf}_x & 1 \end{bmatrix}$$

In Appendix A we show that a first order Taylor expansion of the price-consumption schedule around its steady state $\overline{v}^i_c$ is:

$$v^i_{c,t} = \overline{v}^i_c \left(1 + \frac{(\Psi - 1)}{\Psi(1 - \rho_x \delta)} x^i_t\right), \forall i \in \{h, f\} \quad (6)$$

Notice that when the intertemporal elasticity of substitution Ψ is larger than one, the coefficient on x_t is positive. Bansal and Yaron (2004) identify this characteristic of the model as the one that allows high equity premia to be justified by a reasonable coefficient of risk aversion. Intertemporal substitution dominates the wealth effect with the consequence of a higher volatility of the return on the consumption asset.

In the Appendix we derive analytical expressions for the international correlation of the stochastic discount factors and the volatility of the depreciation rate. The following two propositions can be stated.

Proposition 1. *For a given choice of parameters and provided that $\rho^{h,f}_x \geq \rho^{h,f}_c$, the lowest cross country correlation of the stochastic discount factors is achieved for*

$$\Psi = \frac{1}{\gamma}\widetilde{\delta} \quad , \quad \rho_x = 0 \quad , \quad \rho^{hf}_x = \rho^{hf}_c$$

where $\widetilde{\delta} = \frac{1-2\rho_x\delta+\delta^2}{\delta^2(1-\rho_x^2)}$. *Furthermore, if* $\rho_x^{h,f} > \rho_c^{h,f}$*, then* $(\Psi, \rho_x) = \left(\frac{1}{\gamma}\widetilde{\delta}, 0\right)$ *is the unique minimizer.*

Proof. See Appendix B. □

Proposition 2. *For a given choice of parameters, the lowest volatility of the depreciation rate is achieved for* $\rho_x^{hf} = 1$.

Proof. See Appendix B. □

Proposition 1 argues that a highly persistent x_t component is needed in both countries in order to raise the correlation of the stochastic discount factors. Recent studies by Bansal, Gallant, and Tauchen (2002) and Bansal and Yaron (2004) provide estimates and calibrations of this number to a value very close to unity. Furthermore propositions 1 and 2 combined require a high cross-country correlation of the x_t components as a necessary condition to increase the correlation of the discount factors and to keep the volatility of the depreciation rate at a low level. By staring at equations (4) and (5) one might be tempted to say that the quasi unit root process of x_t together with the high correlation of x_t across countries is guiding the result, by increasing the cross country correlation of consumption. However an explanation of our result based exclusively on the magnitude effect of x_t on the cross-country correlation of consumption would completely overlook the key feature of the model and we would regard this as a failure of our analysis being the correlation of the consumption processes in the order of 30% in the data. As a consequence we will set the standard deviation of ε_x to a small number to offset the impact of $\rho_x \approx 1$.

A third ingredient must to be added to the picture. Proposition 1 calls for the need of Epstein-Zin preferences to break the link between risk aversion and intertemporal elasticity of substitution. As δ and ρ_x approach 1, the timing of the resolution of uncertainty becomes part of the problem and a considerable higher ρ_{m^h,m^f} can be achieved. As shown in equation (2), Epstein-Zin preferences introduce an extra term in the pricing kernel that involves the return on the consumption asset and that is not present with CRRA preferences. As x_t is a predictable and persistent component of consumption growth, it is going to affect crucially the stream of dividends paid by the consumption asset. Therefore it is going to be key in determining expected value and volatility of this return. Furthermore, the fact that this long run component is highly correlated across countries implies an extremely high correlation of the new term introduced in the pricing kernel by the Epstein and Zin preferences.

In the next section we show how it is only the combination of all the three ingredients put forward by Propositions 1 and 2 that delivers our result.

4 Results from a calibrated economy

In this section we report the results of a calibrated economy of the type discussed earlier. We assume that the countries share the same parametrization. Table 1 reports our baseline calibration. As the structure of our two parallel economies mimics those discussed by Bansal and Yaron (2004) and Bansal, Gallant, and Tauchen (2002) most of the coefficients used in our analysis are either estimated or calibrated in those papers. Notice that the coefficient of risk aversion γ is relatively low compared with what is commonly found in the equity premium puzzle literature and with the number proposed by Brandt, Cochrane, and Santa-Clara (2004) in its extension to an international context. The intertemporal elasticity of substitution, Ψ is equal to the one estimated by Bansal, Gallant, and Tauchen (2002) and combined with a high persistence of the predictable component of consumption growth, ρ_x, is what allows to explain the degree of equity premium observed in the data[3]. We set $\rho_x = .987$, that is the value estimated by Bansal, Gallant, and Tauchen (2002) and that is slightly higher than the 0.979 calibrated by Bansal and Yaron (2004). The standard error of ε_x is extremely small compared to the one of ε_c, allowing the latter to be the main determinant of the volatility of consumption growth. The standard deviation of consumption growth implied by our choice of parameters is 2.2%, that is the average growth of per capita consumption of nondurables and services[4] from 1975 to 1998 of the countries studied by Brandt, Cochrane, and Santa-Clara (2004).

[3]We will discuss results from an economy that includes an asset that pays dividends in a later section.

[4]Source International Monetary Fund's IFS database.

Table 1

Baseline Calibration.

Ψ	Intertemporal elasticity of substitution	2
γ	Risk aversion	5.5
δ	Subjective discount factor	0.998
ρ	Autoregressive coefficient of the long run component x_t	0.987
σ_x	Standard error of the long run shock (in %)	0.1
σ_c	Standard error of the short run shock to consumption (in %)	2.2
ρ_x^{hf}	Cross country correlation of the long run shock	1.0
ρ_c^{hf}	Cross country correlation of the short run shock to consumption	0.3

Notes - All figures are annualized. The two countries share the same calibration.

The monthly individual discount factor is 0.998, that accounts for an annual risk free rate of 2.5%. The choice of the correlations coefficients is driven by the need of matching key features of the data. We set ρ_x^{hf} to 1 as suggested by Proposition 2 to keep the volatility of the depreciation rate to about 14%. The cross country correlation of the shocks to consumption is chosen so to obtain a correlation of consumption growth in the order of 30%.

In the previous section we have discussed how it is the combination of long run risks, Epstein-Zin preferences and cross country highly correlated x_t to drive the results. Table 2 shows what happens if we shut down one or more of these components at a time as compared to using all of them together. The first line of this table can be regarded as the case discussed by Brandt, Cochrane, and Santa-Clara (2004). Despite a reasonable volatility of the exchange rate, the correlation of the stochastic discount factors is extremely low, originating the puzzle that is the main attention of this paper. Clearly no combination of up to two ingredients is able to explain the puzzle and the next lines of the table seem to reinforce this idea. As we combine the three of them, as shown in the last line of Table 3, we manage to increase the correlation of the pricing kernel to 0.91 without having to increase the variance of the depreciation rate. Also notice how the volatility of consumption is kept to about 2% and the correlation of the consumption process is almost unchanged compared to what reported by Brandt, Cochrane, and Santa-Clara (2004). We regard the findings of Table 2 as the main result of the paper.

Table 2

Persistency, Correlation and Intertemporal Elasticity.

Parametrization			Results			
ρ_x	ρ_x^{hf}	Ψ	ρ_{m^h,m^f}	σ_π	$\sigma_{\Delta c}$	$\rho_{\Delta c^h,\Delta c^f}$
0	0	$1/\gamma$	0.29	14.34%	2.20%	0.29
0.987	0	$1/\gamma$	0.27	15.11%	2.29%	0.27
0	1	$1/\gamma$	0.30	14.32%	2.20%	0.30
0.987	1	$1/\gamma$	0.27	15.11%	2.29%	0.27
0	0	2	0.29	14.33%	2.20%	0.29
0	1	2	0.30	14.32%	2.20%	0.30
0.987	0	2	0.03	14.32%	2.20%	0.30
0.987	**1**	**2**	**0.91**	**14.33%**	**2.20%**	**0.29**

Notes - All figures are annualized. All coefficients are set to the numbers reported in Table 1, except for the parameters reported in the first three columns.

Table 3 reports the behavior of other relevant variables as we increase the persistence of the long run component. As already observed, the correlation of the stochastic discount factors is increasing and even more so the closer we get to $\rho_x = 1$. The standard deviation of consumption growth does not change too much, except for the case in which $\rho_x = 0.999$. This is the result of our choice of setting the volatility of the long run term to a tiny number compared to the volatility of ε_c. Indeed the ratio $\sigma_{\varepsilon_c}/\sigma_{\Delta c}$ indicates that the contribution of long run risks to the volatility of consumption growth is always small unless $\rho_x \approx 1$. The two covariance terms that seem to be driving the results are the correlation of the consumption assets and the correlation of consumption growth and consumption asset returns across countries. As the persistence of the x_t component rises, the returns on the consumption asset will mainly reflect the long run perspectives of consumption growth, implying a low correlation with Δc_t in any country, that isdriven for a large part by the short run shock ε_c, as we have already discussed before. By the same token, returns are going to be increasingly correlated across country as a result of our choice of setting ρ_x^{hf} to one. Since these returns enter the stochastic discount factors of the two countries, a considerably higher correlation of the two can be achieved under our benchmark calibration. Last notice how our choice of setting the correlation of the long run components to one, as suggested by Proposition 2, manages to keep the volatility of the depreciation rate to a constant 14%.

Table 3
The Role Of Returns.

ρ_x	ρ_{m^h,m^f}	σ_π	$\sigma_{\Delta c}$	$\sigma_{\varepsilon_c}/\sigma_{\Delta c}$	σ_{r_c}	$\rho_{r_c^h,r_c^f}$	$\rho_{\Delta c^h,\Delta c^f}$	$\rho_{\Delta c^f,r_c^h}$
0	0.30	14.32	2.20	0.99	2.20	0.30	0.30	0.30
0.7	0.31	14.32	2.20	0.99	2.21	0.30	0.30	0.30
0.9	0.39	14.32	2.21	0.99	2.26	0.33	0.30	0.29
0.987	0.91	14.32	2.29	0.96	4.01	0.78	0.35	0.18
0.999	0.99	14.32	3.14	0.70	16.83	0.98	0.65	0.07

Notes - All figures are annualized. All coefficients are set to the numbers reported in Table 1, except for ρ_x that takes the values reported in the first column.

5 Other implications of the model

We introduce dividends in the model as following a process that is extremely related to the one that we have used for consumption growth:

$$\Delta d_t^i = \lambda x_{t-1} + \varepsilon_{d,t}^i, \forall i \in \{h, f\} \tag{7}$$

with $\varepsilon_{d,t}^i$ *i.i.d.* normal with mean zero and variance σ_d, $\forall i \in \{h, f\}$. The coefficient λ is referred to as the leverage and is usually set to a number larger than 1. The six shocks of the economy follow a multivariate normal process with covariance matrix $\widetilde{\Sigma}$:

$$\begin{bmatrix} \varepsilon_{c,t}^h & \varepsilon_{c,t}^f & \varepsilon_{x,t}^h & \varepsilon_{x,t}^f & \varepsilon_{d,t}^h & \varepsilon_{d,t}^f \end{bmatrix}' \sim N(0, \widetilde{\Sigma})$$

$$\widetilde{\Sigma} = \begin{bmatrix} \sigma_c^2 H_c & 0 & 0 \\ 0 & \sigma_x^2 H_x & 0 \\ 0 & 0 & \sigma_d^2 H_d \end{bmatrix}$$

where

$$H_c = \begin{bmatrix} 1 & \rho_c^{hf} \\ \rho_c^{hf} & 1 \end{bmatrix} \quad H_x = \begin{bmatrix} 1 & \rho_x^{hf} \\ \rho_x^{hf} & 1 \end{bmatrix} \quad H_d = \begin{bmatrix} 1 & \rho_d^{hf} \\ \rho_d^{hf} & 1 \end{bmatrix}$$

Define as $v_{d,t}$ the price-dividend ratio at time t. Appendix A shows that a log-linear approximation of $v_{d,t}$ around its steady state $\overline{v}_d$ is:

$$v_{d,t}^i = \overline{v}_d^i \left(1 + \frac{(\lambda\Psi - 1)}{\Psi(1-\rho_x\delta)} x_t^i \right), \forall i \in \{h, f\} \quad (8)$$

The pricing kernel is not affected by the introduction of this asset and therefore it keeps the form of equation (2). What we want to study is the consequence that the parametrization of the previous section has for the volatility and cross country correlation of the returns $r_{d,t}^i$, $\forall i \in \{h, f\}$. We know from the data that the annual volatility of stock markets' excess returns is in the order of 14% and that the international correlation of financial markets can be regarded as a number ranging from 35% to 60%[5].

Table 4

Calibration Of Additional Parameters.

λ	Leverage effect	1.8
σ_d	Standard error of the short run shock to dividends	12.0
ρ_d^{hf}	Cross country correlation of the short run shock to dividends	0.3

Notes - All figures are annualized.

Table 4 reports the baseline calibration of λ, σ_d^i and ρ_d^{hf}. The choice of specific numbers for the additional parameters introduced in this section follows the same logic of the economy with the consumption asset only. The volatility of the short run shock to dividend growth is the same of Bansal and Yaron (2004). Once again notice how this number is noticeably higher than σ_x^i, $\forall i \in \{h, f\}$. The coefficient λ is set in such a way that the ratio $\sigma_{\Delta d}/\sigma_{\Delta c}$ in the range $(4, 8)$ estimated by Ludvigson,

[5]Brandt, Cochrane, and Santa-Clara (2004) calculate the correlation of the US stock market with the British, German and Japanese ones to be 57%, 45% and 34%, respectively.

Lettau, and Wachter (2004). In particular, $\sigma_{\Delta d}/\sigma_{\Delta c}$ should be closer to the upper bound if we use postwar data. In our baseline calibration[6], we set $\lambda = 1.8$ that implies $\sigma_{\Delta d}/\sigma_{\Delta c} = 5.27$. The cross country correlation of the short run shocks to dividends, ρ_d^{hf}, is set to a value that allows us to capture an overall correlation of stock markets is in the range suggested above.

TABLE 5

THE ROLE OF RETURNS.

	ρ_{m^h,m^f}	σ_π	σ_{r_d}	$\rho_{r_d^h,r_d^f}$
Baseline Calibration	0.91	14.32	14.80	0.54

Notes - All figures are annualized. All coefficients are set to the numbers reported in Table 1 and Table 4.

Table 5 shows that our baseline calibration is able to match key features of the stock markets. The stochastic discount factors that have the property of being highly correlated produce an annualized volatility of excess returns of the assets that pay the dividend process in the order of 14%. Furthermore the cross-country correlation of the return processes is in the range we pointed out before.

6 Conclusions

We have shown that allowing for an intertemporal elasticity of substitution larger than one and for a persistent and highly cross-correlated forecastable component of consumption growth in the economy described by Brandt, Cochrane, and Santa-Clara (2004) it is possible to reconcile the measure of cross country correlation of the stochastic discount factors obtained from data on consumption and from data on prices. This result is achieved in combination with a lowly volatile depreciation rate and without requiring a high correlation of the consumption processes and coefficient of risk aversion. We have also shown how key features of the data can be described by the same parametrization that allows us to meet our primary goal, extending in

[6]The sensitivity of the results to this choice of λ is available upon request to the authors.

this way the set of properties of the models that take into account long run risks beyond what pointed out by Bansal and Yaron (2004).

Future extensions of this work should focus on providing empirical evidence for the high cross-country correlation of the long run component and on more realistic setups of the economy, in which agents in one country are willing to consume goods produced in the other country. We discussed how the presence of Epstein and Zin (1989) preferences and the introduction of the long run predictable component contribute to make this a very hard task. We remain optimistic about the main features of our setup to hold also in a more complicated setup.

References

Anderson, E. (2005). The dynamics of risk-sensitive allocations. *Forthcoming in Journal of Economic Theory*.

Backus, D., S. Foresi, and C. Telmer (1996, June). Affine models of currency pricing. NBER Working Papers 5623, National Bureau of Economic Research, Inc. available at http://ideas.repec.org/p/nbr/nberwo/5623.html.

Backus, D. K. and S. E. Zin (1994, March). Reverse engineering the yield curve. NBER Working Papers 4676, National Bureau of Economic Research, Inc. available at http://ideas.repec.org/p/nbr/nberwo/4676.html.

Bansal, R., A. R. Gallant, and G. Tauchen (2002). Rational pessimism and exuberance. *Working Paper, Department of Economics, University of North Carolina, Chapel Hill NC 27599-3305 USA*.

Bansal, R. and A. Yaron (2004). Risks for the long run: A potential resolution of asset pricing puzzles. *Forthcoming Journal of Finance*.

Brandt, W., J. Cochrane, and P. Santa-Clara (2004). International risk sharing is better than you think (or exchange rates are much too smooth). *Forthcoming Journal of Monetary Economics*.

Campbell, J. (2003). Consumption-based asset pricing. *forthcoming in Handbook of the Economics of Finance*. George Constantinides, Milton Harris, and Rene Stulz eds., North-Holland, Amsterdam.

Epstein, L. G. and S. E. Zin (1989, July). Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica 57*(4), 937–69. available at http://ideas.repec.org/a/ecm/emetrp/v57y1989i4p937-69.html.

Epstein, L. G. and S. E. Zin (1991, April). Substitution, risk aversion, and the temporal behavior of consumption and asset returns: An empirical analysis. *Journal of Political Economy 99*(2), 263–86. available at http://ideas.repec.org/a/ucp/jpolec/v99y1991i2p263-86.html.

Hansen, L., J. Heaton, and N. Li (2004). Consumption strikes back? *Working Paper*.

Hansen, L. P. and R. Jagannathan (1991). Implications of security market data for models of dynamic economies. *Journal of Political Economy 99*, 225262.

Kan, R. (1995). Structure of pareto optima when agents have stochastic recursive preferences. *Journal of Economic Theory 66*(2), 626–31.

Ludvigson, S., M. Lettau, and J. A. Wachter (2004). The declining equity premium: What role does macroeconomic risk play? *NYU mimeo*.

Mehra, R. and E. Prescott (1985). The equity premium: A puzzle. *Journal of Monetary Economics 15*, 145–161.

Appendix A. Derivation of moments

Let's first of all recall the list of our deeper coefficients:

$$\Upsilon = \left\{\theta, \psi, \delta, \rho_x, \lambda, \sigma_c, \sigma_x, \sigma_d, \rho_c^{hf}, \rho_x^{hf}, \rho_d^{hf}\right\}$$

We will express our relevant moments as a function of these parameters to be able to disentangle their specific effects. We will start from the formulas for the consumption growth, then we will focus on the formulas for the relevant financial variables. Remember that we assume:

$$\Delta c_{t+1}^i = x_t^i + \epsilon_{x,t+1}^i \; i = h, f$$

Then

$$Var(\Delta c_{t+1}^i) = \sigma_c^2 + \frac{\sigma_x^2}{1-\rho_x^2}$$

$$Cov(\Delta c_{t+1}^h, \Delta c_{t+1}^f) = \rho_c^{hf}\sigma_c^2 + \frac{\rho_x^{hf}\sigma_x^2}{1-\rho_x^2}$$

Now we can focus on $v_{c,t}^i$, just by applying the general NA condition formula to the asset that pays the entire consumption bundle. Remember that in this case the return from the asset is

$$R_{c,t+1}^i = \frac{v_{c,t+1}^i + 1}{v_{c,t}^i} \exp \Delta c_{t+1}^i \; i = h, f$$

We get the following formula:

$$(v_{c,t}^i)^\theta = E_t\left[\delta e^{\Delta c_{t+1}^i(1-\gamma)}(1+v_{c,t+1}^i)^\theta\right] \tag{A.1}$$

By imposing the steady state condition:

$$\begin{aligned} v_c^i &= \delta(1+v_c^i) \\ v_c^{i,ss} &= \frac{\delta}{1-\delta} \end{aligned}$$

Now we can proceed by linearizing around the steady state. Define:

$$\begin{aligned} F(\Delta c_{t+1}^i, v_{c,t+1}^i) &= \delta e^{\Delta c_{t+1}^i\left(1-\frac{1}{\psi}\right)}(1+v_{c,t+1}^i) \\ \frac{\partial F}{\partial \Delta c_{t+1}^i} \mid ss &= \delta\left(1-\frac{1}{\psi}\right)(1+\frac{\delta}{1-\delta}) \\ \frac{\partial F}{\partial v_{c,t+1}^i} \mid ss &= \delta \end{aligned}$$

Then we have:

$$v_{c,t}^i = E_t\left[\delta + \frac{\delta}{1-\delta}\left(1-\frac{1}{\psi}\right)\Delta c_{t+1}^i + \delta v_{c,t+1}^i\right]$$

Solve forward taking conditional expected value:

$$v^i_{c,t} = \alpha_c + \beta_c x^i_t \tag{A.2}$$

$$\alpha_c = \frac{\delta}{1-\delta} \tag{A.3}$$

$$\beta_c = \frac{\frac{\delta}{1-\delta}\left(1-\frac{1}{\psi}\right)}{1-\rho_x\delta} \tag{A.4}$$

In a completely similar way:

$$v^i_{d,t} = \alpha_d + \beta_d x^i_t \tag{A.5}$$

$$\alpha_d = \frac{\delta}{1-\delta} \tag{A.6}$$

$$\beta_d = \frac{\frac{\delta}{1-\delta}\left(\lambda-\frac{1}{\psi}\right)}{1-\rho_x\delta} \tag{A.7}$$

We are ready to look at the returns, we will log-linearize the following expressions:

$$R^i_{c,t+1} = \frac{v^i_{c,t+1}+1}{v^i_{c,t}}\exp\Delta c^i_{t+1}$$

$$R^i_{d,t+1} = \frac{v^i_{d,t+1}+1}{v^i_{d,t}}\exp\Delta d^i_{t+1}$$

In general the following holds:

$$r^i_{j,t+1} = \Delta j^i_{t+1} + log(1+v^{ss}_j) + \frac{1}{1+v^{ss}_j}(v^i_{j,t+1}-v^{ss}_j) - log(v^{ss}_j) - \frac{1}{v^{ss}_j}(v^i_{j,t}-v^{ss}_j) \tag{A.8}$$

$$j = c, d$$

By using (A.8) and (A.2) we get:

$$r^i_{c,t+1} = -log(\delta) + \left[\frac{1}{\psi}\right]x^i_t + \left[\delta\left(1-\frac{1}{\psi}\right)\frac{1}{1-\rho_x\delta}\right]\epsilon^i_{x,t+1} + \epsilon^i_{c,t+1} \tag{A.9}$$

At this point it is possible to find the reduced form of the discount factor by plugging (A.9) and the law for the consumption growth back into (2). The log-s.d.f. assumes the following form:

$$m^i_{t+1} = log\delta - \frac{1}{\Psi}x^i_t - \gamma\epsilon^i_{c,t+1} + \frac{\delta\,(1-\gamma\Psi)}{\Psi(1-\rho_x\delta)}\epsilon^i_{x,t+1} \tag{A.10}$$

It follows that:

$$Var(\pi) = 2\left[\left(\frac{1}{\Psi}\right)^2 \frac{1}{1-\rho_x^2} + \left[\frac{\delta\left(1-\gamma\Psi\right)}{\Psi\left(1-\rho_x\delta\right)}\right]^2\right](1-\rho_x^{hf})\sigma_x^2 + 2\gamma^2(1-\rho_c^{hf})\sigma_c^2 \quad (A.11)$$

and

$$corr(m_t^h, m_t^f) = \frac{\left(\frac{1}{\Psi}\right)^2 \frac{1}{1-\rho_x^2}\rho_x^{hf}\sigma_x^2 + \gamma^2\rho_c^{hf}\sigma_c^2 + \left[\frac{\delta(1-\gamma\Psi)}{\Psi(1-\rho_x\delta)}\right]^2 \rho_x^{hf}\sigma_x^2}{\left(\frac{1}{\Psi}\right)^2 \frac{1}{1-\rho_x^2}\sigma_x^2 + \gamma^2\sigma_c^2 + \left[\frac{\delta(1-\gamma\Psi)}{\Psi(1-\rho_x\delta)}\right]^2 \sigma_x^2} \quad (A.12)$$

Before starting to analyze the returns from the stocks let's focus on several important moments regarding $r^i_{c,t+1}$ and $\Delta c_{i,t+1}$:

$$Var(r^i_{c,t+1}) = \left[\left(\frac{1}{\Psi}\right)^2 \frac{1}{1-\rho_x^2} + \left[\delta\left(1-\frac{1}{\Psi}\right)\frac{1}{1-\rho_x\delta}\right]^2\right]\sigma_x^2 + \sigma_c^2 \quad (A.13)$$

$$Cov(r^h_{c,t+1}, r^f_{c,t+1}) = \left[\left(\frac{1}{\Psi}\right)^2 \frac{1}{1-\rho_x^2} + \left[\delta\left(1-\frac{1}{\Psi}\right)\frac{1}{1-\rho_x\delta}\right]^2\right]\rho_x^{hf}\sigma_x^2 + \rho_c^{hf}\sigma_c^2 \quad (A.14)$$

$$Cov(\Delta c^i_{t+1}, r^i_{c,t+1}) = \frac{1}{\Psi}\frac{\sigma_x^2}{1-\rho_x^2} + \sigma_c^2 \quad (A.15)$$

$$Cov(\Delta c^h_{t+1}, r^f_{c,t+1}) = \frac{1}{\Psi}\frac{\rho_x^{hf}\sigma_x^2}{1-\rho_x^2} + \rho_c^{hf}\sigma_c^2 \quad (A.16)$$

Assume the following process for dividends:

$$\Delta d^i_{t+1} = \lambda x^i_t + \epsilon^i_{d,t+1} \quad (A.17)$$

Now, by merging (A.5), (A.8), (A.17) we get:

$$r^i_{d,t+1} = -log(\delta) + \left[\frac{1}{\psi}\right]x^i_t + \left[\delta\left(\lambda - \frac{1}{\psi}\right)\frac{1}{1-\rho_x\delta}\right]\epsilon^i_{x,t+1} + \epsilon^i_{d,t+1} \quad (A.18)$$

We want to look also at the excess returns, so let us compute first the risk free rate in this economy. By using our NA condition:

$$E_t\left[\delta^\theta exp^{-\frac{\theta}{\psi}\Delta c^i_{t+1} + (\theta-1)r^i_{c,t+1}}\right] = \frac{1}{1+r^i_{free,t}} \quad (A.19)$$

By using (A.19) and (A.9) we are in front of the following (approximated) condition:

$$\delta^\theta E\left[e^{Normal\left((1-\theta)log(\delta) - \left[\frac{1}{\psi}\right]x^i_t,\ [\gamma]^2\sigma_c^2 + \left[\delta\left(1-\frac{1}{\psi}\right)\frac{1}{1-\rho_x\delta}\right]^2\sigma_x^2\right)}\right] = \frac{1}{1+r^i_{free,t}}$$

Take logs:

$$\ln(1+r^i_{free,t}) = -\log(\delta) + \frac{1}{2}\left[[\gamma]^2\sigma_c^2 + \left[\delta\left(1-\frac{1}{\psi}\right)\frac{1}{1-\rho_x\delta}\right]^2\sigma_x^2\right] + \frac{1}{\psi}x_t^i \quad \text{(A.20)}$$

Notice that our risk free rate has the following variance:

$$V(\ln(1+r^i_{free})) = \left(\frac{1}{\psi}\right)^2 V(x^i) \quad \text{(A.21)}$$

Now we can easily derive the expression for the excess returns:

$$er^i_{t+1} \equiv \ln(1+r^i_{d,t+1}) - \ln(1+r^i_{free,t}) = -\frac{1}{2}\left[[\gamma]^2\sigma_c^2 + \left[\delta\left(1-\frac{1}{\psi}\right)\frac{1}{1-\rho_x\delta}\right]^2\sigma_x^2\right] + \left[\delta\left(\lambda-\frac{1}{\psi}\right)\frac{1}{1-\rho_x\delta}\right]\epsilon^i_{x,t+1} + \epsilon^i_{d,t+1} \quad \text{(A.22)}$$

This implies that we have the following correlation across countries:

$$corr(er_t^h, er_t^f) = \frac{\left[\delta\left(\lambda-\frac{1}{\psi}\right)\frac{1}{1-\rho_x\delta}\right]^2\rho_x^{hf}\sigma_x^2 + \rho_\epsilon^{hf}\sigma_\epsilon^2}{\left[\delta\left(\lambda-\frac{1}{\psi}\right)\frac{1}{1-\rho_x\delta}\right]^2\sigma_x^2 + \sigma_\epsilon^2} \quad \text{(A.23)}$$

Appendix B. Proof of Propositions

Proof of Proposition 1.
The cross-country correlation of the stochastic discount factors is:

$$corr(m_t^h, m_t^f) = \frac{\left(\frac{1}{\Psi}\right)^2\frac{1}{1-\rho_x^2}\rho_x^{hf}\sigma_x^2 + \gamma^2\rho_c^{hf}\sigma_c^2 + \left[\frac{\delta(1-\gamma\Psi)}{\Psi(1-\rho_x\delta)}\right]^2\rho_x^{hf}\sigma_x^2}{\left(\frac{1}{\Psi}\right)^2\frac{1}{1-\rho_x^2}\sigma_x^2 + \gamma^2\sigma_c^2 + \left[\frac{\delta(1-\gamma\Psi)}{\Psi(1-\rho_x\delta)}\right]^2\sigma_x^2}$$

For any choice of the parameters that satisfies the following two inequalities, $\rho_x \neq 1$ and $\rho_x\delta \neq 1$, the following three partial derivatives $\frac{\partial corr(m_t^h,m_t^f)}{\partial\rho_x^{hf}}$, $\frac{\partial corr(m_t^h,m_t^f)}{\partial\Psi}$, $\frac{\partial corr(m_t^h,m_t^f)}{\partial\rho_x}$ exist and are well defined. Let's compute them and study their sign.

$$\frac{\partial corr(m_t^h, m_t^f)}{\partial\rho_x^{hf}} = \frac{\left[\left(\frac{1}{\Psi}\right)^2\frac{1}{1-\rho_x^2} + \left[\frac{\delta(1-\gamma\Psi)}{\Psi(1-\rho_x\delta)}\right]^2\right]\sigma_x^2}{\left(\frac{1}{\Psi}\right)^2\frac{1}{1-\rho_x^2}\sigma_x^2 + \gamma^2\sigma_c^2 + \left[\frac{\delta(1-\gamma\Psi)}{\Psi(1-\rho_x\delta)}\right]^2\sigma_x^2} > 0 \quad \text{(B.1)}$$

This derivative is positive for all the values of the parameters that respect the two conditions: $\rho_x \neq 1$ and $\rho_x\delta \neq 1$. This trivially implies that the correlation of the two stochastic discount factors is strictly increasing with respect to ρ_x^{hf}.

$$\frac{\partial corr(m_t^h, m_t^f)}{\partial \Psi} = \frac{-\frac{2\sigma_x^2}{\Psi}\left[\left(\frac{1}{\Psi}\right)^2 \frac{1}{1-\rho_x^2} + \frac{\delta^2\left(\frac{1}{\Psi}-\gamma\right)}{(1-\rho_x\delta)^2}\right]\gamma^2(\rho_x^{hf} - \rho_c^{hf})\sigma_c^2}{\left[\left(\frac{1}{\Psi}\right)^2 \frac{1}{1-\rho_x^2}\sigma_x^2 + \gamma^2\sigma_c^2 + \left[\frac{\delta(1-\gamma\Psi)}{\Psi(1-\rho_x\delta)}\right]^2\sigma_x^2\right]^2} \tag{B.2}$$

When $\rho_x^{hf} = \rho_c^{hf}$ this derivative is always zero, meaning that changes in Ψ, γ or δ do not affect the correlation of the two stochastic discount factors.
If $\rho_x^{hf} \neq \rho_c^{hf}$, this derivative is zero only if

$$\Psi = \frac{1}{\gamma}\widetilde{\delta} \tag{B.3}$$

where $\widetilde{\delta} = \frac{1-2\rho_x\delta+\delta^2}{\delta^2(1-\rho_x^2)}$. In particular, when $\rho_x^{hf} > \rho_c^{hf}$ the sign of this derivative is positive for $\Psi > \frac{1}{\gamma}\widetilde{\delta}$ and negative for $\Psi < \frac{1}{\gamma}\widetilde{\delta}$.
Notice, finally, that

$$\lim_{\delta \to 1^-} \frac{1}{\gamma}\widetilde{\delta} \geq \frac{1}{\gamma} \tag{B.4}$$

and that

$$\lim_{\rho_x \to 1^-} \lim_{\delta \to 1^-} \frac{1}{\gamma}\widetilde{\delta} = \frac{1}{\gamma} \tag{B.5}$$

So, for a high persistence of the long run component and an individual discount factor close to one, the minimum of the cross correlation of the two discount factors is achieved for $\Psi = \frac{1}{\gamma}$, that is when the Epstein-Zin preferences collapse to the standard CES utility function.

$$\frac{\partial corr(m_t^h, m_t^f)}{\partial \rho_x} = \frac{\frac{2\sigma_x^2}{\Psi^2}\left[\frac{\rho_x}{(1-\rho_x^2)^2} + \frac{\delta^3(1-\gamma\Psi)^2}{(1-\rho_x\delta)^3}\right]\gamma^2(\rho_x^{hf} - \rho_c^{hf})\sigma_c^2}{\left[\left(\frac{1}{\Psi}\right)^2 \frac{1}{1-\rho_x^2}\sigma_x^2 + \gamma^2\sigma_c^2 + \left[\frac{\delta(1-\gamma\Psi)}{\Psi(1-\rho_x\delta)}\right]^2\sigma_x^2\right]^2} \tag{B.6}$$

When it exists, this derivative is always positive when $\rho_x^{hf} > \rho_c^{hf}$, negative when $\rho_x^{hf} < \rho_c^{hf}$ and null when $\rho_x^{hf} = \rho_c^{hf}$. In the case in which $\rho_x^{hf} > \rho_c^{hf}$ the cross correlation of the stochastic discount factors is strictly increasing with ρ_x, so the minimum is achieved for ρ_x=0.
When $\rho_x^{hf} > \rho_c^{hf}$, the properties of these three derivatives jointly imply the existence of one unique minimizer in correspondence of $\rho_x = 0$, $\rho_x^{hf} = 0$, $\Psi = \frac{1}{\gamma}\widetilde{\delta}$.

Proof of Proposition 2.
The variance of the depreciation rate is:

$$\begin{aligned} Var(\pi) &= 2\left[\left(\frac{1}{\Psi}\right)^2 \frac{1}{1-\rho_x^2} + \left[\frac{\delta(1-\gamma\Psi)}{\Psi(1-\rho_x\delta)}\right]^2\right](1-\rho_x^{hf})\sigma_x^2 + \\ & 2\gamma^2(1-\rho_c^{hf})\sigma_c^2 \end{aligned} \tag{B.7}$$

The partial derivative of this expression with respect to ρ_x^{hf} exists and is well defined for any value of the other parameters provided that $\rho_x \neq 1$ and $\rho_x \delta \neq 1$:

$$\frac{\partial Var(\pi)}{\partial \rho_x^{hf}} = -2\left[\left(\frac{1}{\Psi}\right)^2 \frac{1}{1-\rho_x^2} + \left[\frac{\delta\,(1-\gamma\Psi)}{\Psi\,(1-\rho_x\delta)}\right]^2\right]\sigma_x^2 \leq 0 \tag{B.8}$$

In particular, when it exists, such a derivative is always negative implying that the volatility of the log-depreciation rate achieves its minimum when $\rho_x^{hf} = 1$.