

# FORWARD-LOOKING HIRING BEHAVIOR AND THE DYNAMICS OF THE AGGREGATE LABOR MARKET

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## Abstract

What drives aggregate labor market dynamics and, in particular, what explains the high volatility of unemployment and worker flows? The paper shows that a key mechanism underlying these dynamics is a forward-looking, asset-type mechanism. Fluctuations in the present value of the job-worker match drive job vacancies creation and consequently all key variables. In particular two elements play a crucial role: (i) convex hiring costs, affecting the response of vacancy creation to the present value of the match; and (ii) the volatility and persistence of the dissolution rate of matches, affecting the discounting of future values. The paper uses the search and matching model to examine these issues empirically with U.S. data. It offers a workable, empirically-grounded version of the model for the analysis of aggregate U.S. labor market dynamics.

*Key words:* U.S. labor market, labor market flows, search, matching, vacancies, business cycles.

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# FORWARD-LOOKING HIRING BEHAVIOR AND THE DYNAMICS OF THE AGGREGATE LABOR MARKET <sup>1</sup>

## 1 Introduction

What drives aggregate labor market dynamics and, in particular, what explains the high volatility of unemployment and worker flows? This paper shows that a key mechanism underlying these dynamics is a forward-looking, asset-type mechanism. The paper uses the search and matching model<sup>2</sup> to examine these issues empirically with U.S. data. It offers a workable, empirically-grounded version of the model for the analysis of aggregate U.S. labor market dynamics.

Recent papers have questioned this model's empirical performance in U.S. data. This literature poses doubts with respect to the model's ability to account for the observed behavior of key variables – unemployment, vacancies, worker flows, wages, and the duration of unemployment. In this context, two questions are addressed here: (i) Does the model fit the data, and, if so, on what dimensions? (ii) Does the data “fit” the model, i.e. which data are relevant in the context of the model? This examination shows whether the model is indeed useful for the analysis of U.S. labor market dynamics. The idea is to make precise the dimensions on which the model does well and those on which it does not do well or even fails.

When coming to undertake such analysis a number of fundamental problems arise:

(i) How to model the driving shocks? It is known from the business cycle literature that fluctuations often mimic the pattern of the shocks and that the model itself may lack propagation mechanisms. A similar problem may be the case here too.

(ii) How to avoid the “contamination” of labor market analysis by mis-specification of other parts of the macroeconomy? General equilibrium models have well-known problems that are yet to

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<sup>2</sup>Developed by Diamond, Mortensen and Pissarides [see Mortensen and Pissarides (1999 a,b) for surveys and Pissarides (2000) for a detailed exposition].

be resolved. Studying the empirical performance of the search and matching model in conjunction with other parts of the macroeconomy could thus lead to rejection of the model due to the aforementioned “contamination.”

(iii) How to model the frictions so as to determine what role they play in explaining the data? Many studies in the literature are highly stylized, use short cuts, or employ reduced-form modelling that is useful for theoretical exposition but is problematic when attempting an empirical examination of the model. It is then difficult to see how the degree of frictions matters in accounting for the data. For a model that puts frictions at center stage this is highly problematic. Thus, in cases where the model is shown not to fit the data, it is not always clear whether a particular specification of labor market frictions is responsible for the difficulty or whether this reflects a more essential problem.

(iv) How to determine the equilibrium effects of shocks? There is a need for a structural model that can trace out the effects of shocks on equilibrium dynamics and determine what is the role of frictions in this process.

(v) What are the proper data to use for the key elements of the model – the pool of searching workers and the stock of searching jobs (vacancies)? Questions regarding these data have been raised a number of times by different authors but have not been systematically treated in the current context. Hence, it is unclear to which extent the model-data fit is subject to problems with the data used rather than to problems with the model.

The paper takes the following approach to address these problems: to deal with (i) and (ii), it uses a partial equilibrium model and a reduced-form VAR of the actual data to specify the driving shocks. This ‘agnostic’ approach precludes the possibility that labor market dynamics will be affected by misspecifications in other parts of a more general macroeconomic model. Thus it does not take a particular stand on the sources of the driving shock processes nor does it formulate an explicit structure for the rest of the macroeconomy. To deal with (iii) and (iv) it uses a structural approach: it specifies agents’ objectives and constraints, their optimal behavior, and the dynamic paths of key variables in equilibrium. The structural parameters quantify the degree of frictions. The fifth issue is treated by looking at alternative formulations of the pool of searching workers, taking into account also non-employed workers outside the official unemployment pool. The paper

also uses newly available gross worker flows data that are compatible with the model’s formulations, rather than vacancy data, which are shown to be inconsistent with the concepts of the model.

The model is a stochastic version of the search and matching model. It characterizes firms’ optimal search behavior, deriving a relationship that equates marginal hiring costs with the present value of a hire – the “asset value” of the job-worker match. The firm’s decision on vacancy creation feeds into a matching function, which generates a flow of hires given the stocks of job vacancies and unemployment. The wage is determined by the Nash bargaining solution, which splits the surplus from the job-worker match. The resulting model is a partial equilibrium model, with employment (and unemployment), vacancies, and the wage defined by the equilibrium solution.

The non-linear equilibrium solution of the model is log-linearized using a first-order Taylor approximation to obtain a system of linear difference equations. The system formalizes the dynamics of the key variables in terms of exogenous, stochastic variables and in terms of structural parameters that quantify the frictions. The model is then calibrated. As mentioned, a reduced-form VAR determines the relevant parameters for the exogenous shock processes. These include labor productivity shocks, shocks to the match separation rate, and shocks to the discount rate. Other calibrated values are set using historical data averages or econometric estimates. The model’s implied second moments are compared to U.S. data in terms of persistence, co-movement, and volatility.

We find that for the most part the model fits U.S. labor market data relatively well. This includes capturing the high persistence and high volatility of most of the key variables, as well as the negative co-variation of unemployment and vacancies (the ‘Beveridge curve’). The main reason for the fit is a forward-looking, asset-type mechanism: fluctuations in the present value of the job-worker match drive vacancy creation and consequently all key variables. In particular two elements play a crucial role: (i) convex hiring costs and (ii) the second moments of the dissolution rate of job-worker matches. The main failure is an inability to simultaneously account for some dimensions of the stochastic behavior of the labor share in income, though the model captures some of those for other specifications of the data.

The paper makes several contributions: first, it offers an empirically-grounded model of aggregate U.S. labor market dynamics as implied by the search and matching model. Doing so it

provides macroeconomists some guidance concerning the relevant “building block” for modelling the labor market. Second, by calibrating the model and evaluating it against alternative formulations of the data, it is able to show on what dimensions the model fits the data and which data series are the relevant ones. Third, particular specifications of the model are able to replicate key empirical regularities in U.S. data which other models have been unable to capture; the reasons for this improved performance are discussed. In terms of the calibration-simulation methodology, there is an innovation in the “agnostic” approach taken with respect to the formulation of shocks.

The paper proceeds as follows: Section 2 briefly discusses the literature on the empirical performance of the model. Section 3 presents the search and matching model. Section 4 derives the equilibrium dynamics and steady state, highlighting the role of frictions. Section 5 discusses the data, presents the properties of the data that are to be matched, and proposes alternative formulations of the data to be used. Section 6 calibrates the model and evaluates the model-data fit. Section 7 examines the mechanism underlying the results, deriving the modelling and data lessons for using the model to study the U.S. labor market. Section 8 concludes.

## 2 Some Recent Questioning

The search and matching model has gained much attention in macroeconomic analysis. It provides a rich framework for the analysis of fluctuations in employment. Hall (2003) notes that “the idea that trading frictions play an important role in shaping aggregate labor market outcomes has become increasingly standard over the past years. The early work of Peter A. Diamond, Dale Mortensen, and Christopher Pissarides has spawned a class of models that have become the standard in formalizing these trading frictions.” Rogerson (1997) describes it as a modern theory of unemployment that caters well for dynamics in the labor market and that has proved useful in accounting for the time series behavior of aggregate unemployment. Merz (1995) and Andolfatto (1996) have shown that when incorporated in the RBC framework, the result outperforms the standard RBC model. Yashiv (2000a) has structurally estimated the model’s main ingredients in an aggregate data-set that fits well with the concepts inherent in the model. The main findings indicate that the model is corroborated by the data. In recent work, the model has been applied to study key macroeconomic issues such as growth, business cycles, inequality, and labor market

policy [see the references in Hall (1999, 2003), Mortensen and Pissarides (1999a,b) and Pissarides (2000)].

Recently, however, some authors have questioned the empirical performance of the standard model. Cole and Rogerson (1999) found that the model can account for business cycle facts only if the average duration of unemployment is relatively high (9 months or longer) and substantially longer than average duration in BLS data on the unemployed. The intuition for this critique is the following: the data point to a negative correlation between job destruction and job creation. Job destruction leads to higher unemployment thereby raising the incentives for job creation. In order for job creation to be contemporaneously negatively correlated with job destruction there has to be a mitigating factor and that role is played by the relatively low worker hazard rate, which is the inverse of unemployment duration.

Shimer (2004) showed that in the standard matching model productivity shocks of empirically plausible magnitude cannot generate the observed, large cyclical fluctuations in unemployment and vacancies. The key reason for this result is that the standard model assumes that wages are determined by Nash bargaining, which in turn implies that wages are “too flexible.” Thus, for example, following a positive productivity shock wages increase, absorbing the shock, thereby dampening the incentives of firms to create new jobs. In particular, while the model predicts roughly the same volatility of the vacancy to unemployment ratio and of productivity, the data indicate this ratio is actually 18 times more volatile. Hall (2004 a,b) has built on these insights to construct a formulation of wage setting that is consistent with (i) no unrealized bilateral gains to trade and (ii) wages that are relatively unresponsive to shocks to the value of the match. As a result, he provides an internally consistent model of labor market fluctuations, that can replicate the main stylized facts. His model departs from the standard formulation which uses Nash-bargained wages.

Veracierto (2002) has shown that the model fails to simultaneously account for the observed behavior of employment, unemployment and out of the labor force worker pools. Analyzing the RBC model with search and matching that makes an explicit distinction between these states, he finds that the model has serious difficulties in reproducing the labor market dynamics observed in U.S. data. In particular, employment fluctuates as much as the labor force while in the data it is three times more variable, unemployment fluctuates as much as output while in the data it is six

times more variable, and unemployment is acyclical while in the data it is strongly countercyclical. An underlying reason is that search decisions respond too little to aggregate productivity shocks.

Fujita (2004) conducted empirical tests showing that vacancies are much more persistent in the data than the low persistence implied by the model.

Taken together these studies cast doubt with respect to the model fit on two key dimensions: the volatility and duration of unemployment, and the volatility and persistence of vacancies. They also point to serious shortcomings in accounting for aggregate wage behavior.

The current paper uses a relatively general framework that enables the study of all of these issues. The idea is to postulate the shocks using the actual data, to allow for simultaneous shocks (for example, both productivity and separation shocks), and to examine alternative formulations of the data series in order to see which are compatible with the formulations of the model. Hence the analysis is able to determine whether a given problem is with the model not being able to explain the data under any plausible configuration or just in a particular case, and whether the data are the relevant ones to be explained by the model. Note that following most of these papers, the current paper uses a representative agent framework, which is consistent with the aim of providing the relevant “building block” for macroeconomic models (that employ such a framework too).

### 3 The Search and Matching Model

In this section we briefly present a stochastic, discrete-time version of the prototypical search and matching model.<sup>3</sup> An important addition to the standard analysis is a convex formulation for the hiring costs function, which has the standard linear formulation as a special case. This in turn necessitates taking into account the effect of employment on wages in the firm’s optimality condition and in the wage solution.

#### 3.1 The Basic Set-Up

There are two types of agents: unemployed workers ( $U$ ) searching for jobs and firms recruiting workers through vacancy creation ( $V$ ). Firms maximize their intertemporal profit functions with

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<sup>3</sup>A detailed exposition may be found in Pissarides (2000). The stochastic, discrete time formulation presented here follows Yashiv (2004).

the choice variable being the number of vacancies to open. Each firm produces a flow of output ( $F$ ), paying workers wages ( $W$ ) and incurring hiring costs ( $\Gamma$ ). Workers and firms are faced with different frictions such as different locations leading to regional mismatch or lags and asymmetries in the transmission of information. These frictions are embedded in the concept of a matching function which produces hires ( $M$ ) out of vacancies and unemployment, leaving certain jobs unfilled and certain workers unemployed. Workers are assumed to be separated from jobs at a stochastic, exogenous rate, to be denoted by  $\delta$ . The labor force ( $L$ ) is growing with new workers flowing into the unemployment pool. The set-up, whereby search is costly and matching is time-consuming, essentially describes the market as one with trade frictions. Supply and demand are not equilibrated instantaneously, so at each date  $t$  there are stocks of unemployed workers and vacant jobs.

The model assumes a market populated by many identical workers and firms. Hence we shall continue the discussion in terms of “representative agents.” Each agent is small enough so that the behavior of other agents is taken as given. As is well known this creates various externalities. In particular, more search activity creates a positive trading externality for the trading partner and a congestion externality for similar agents [see the discussion in Pissarides (2000, chapter 7)].

### 3.2 Matching

A matching function captures the frictions in the matching process; it satisfies the following properties

$$\begin{aligned} M_{t,t+1} &= \widetilde{M}(U_t, V_t) \\ \frac{\partial \widetilde{M}}{\partial U} &> 0, \quad \frac{\partial \widetilde{M}}{\partial V} > 0 \end{aligned} \tag{1}$$

Empirical work [see the survey by Petrongolo and Pissarides (2001)] has shown that a Cobb-Douglas function is useful for parameterizing it:

$$M_{t,t+1} = \mu U_t^\sigma V_t^{1-\sigma} \tag{2}$$

where  $\mu$  stands for matching technology. From this function the hazard rates –  $P$ , the worker probability of finding a job and  $Q$ , the firm’s probability of filling the vacancy – are derived:



$$P_{t,t+1} = \frac{M_{t,t+1}}{U_t} = \mu \left( \frac{V_t}{U_t} \right)^{1-\sigma} \quad (3)$$

$$Q_{t,t+1} = \frac{M_{t,t+1}}{V_t} = \mu \left( \frac{V_t}{U_t} \right)^{-\sigma} \quad (4)$$

The parameter  $\sigma$  reflects the relative contribution of unemployment to the matching process and determines the elasticity of the hazard rates with respect to market tightness.

### 3.3 Firms

Firms maximize the expected, present value of profits (where all other factors of production have been “maximized out”):

$$\max_{\{V\}} E_0 \sum_{t=0}^{\infty} \left( \prod_{j=0}^t \beta_j \right) [F_t - W_t N_t - \Gamma_t] \quad (5)$$

where  $\beta_j = \frac{1}{1+r_{t-1,t}}$ .

This maximization is done subject to the employment dynamics equation given by:

$$N_{t+1} = (1 - \delta_{t,t+1})N_t + Q_{t,t+1}V_t \quad (6)$$

The Lagrangean of this problem is (where  $\Lambda$  is the discounted Lagrange multiplier):

$$\mathfrak{L} = E_0 \sum_{t=0}^{\infty} \left( \prod_{j=0}^t \beta_j \right) [F_t - W_t N_t - \Gamma_t + \Lambda_t \{(1 - \delta_{t,t+1})N_t + Q_{t,t+1}V_t - N_{t+1}\}] \quad (7)$$

The F.O.C are:

$$\frac{\partial \Gamma_t}{\partial V_t} = Q_{t,t+1} E_t \Lambda_t \quad (8)$$

$$\Lambda_t = E_t \beta_{t+1} \left[ \frac{\partial F_{t+1}}{\partial N_{t+1}} - \frac{\partial \Gamma_{t+1}}{\partial N_{t+1}} - W_{t+1} - N_{t+1} \frac{\partial W_{t+1}}{\partial N_{t+1}} \right] + E_t (1 - \delta_{t+1,t+2}) \beta_{t+1} \Lambda_{t+1} \quad (9)$$

$$N_{t+1} = (1 - \delta_{t,t+1})N_t + Q_{t,t+1}V_t \quad (10)$$

and the transversality condition:

$$\lim_{T \rightarrow \infty} E_t \left[ \left( \prod_{j=0}^{T-1} \beta_j \right) \left\{ \frac{\partial F_T}{\partial N_T} - W_T - \frac{\frac{\partial \Gamma_{T-1}}{\partial N_{T-1}}}{\beta_{T-1}} - N_{T-1} \frac{\partial W_{T-1}}{\partial N_{T-1}} \right\} N_T \right] = 0 \quad (11)$$

The first, intratemporal condition (equation 8) sets the marginal cost of hiring  $\frac{\partial \Gamma_t}{\partial V_t}$  equal to the expected value of the multiplier times the probability of filling the vacancy. The second, intertemporal condition (equation 9) sets the multiplier equal to the sum of the expected, discounted marginal profit in the next period  $E_t \beta_{t+1} \left[ \frac{\partial F_{t+1}}{\partial N_{t+1}} - W_{t+1} - \frac{\partial \Gamma_{t+1}}{\partial N_{t+1}} - N_{t+1} \frac{\partial W_{t+1}}{\partial N_{t+1}} \right]$  and the expected, discounted (using also  $\delta$ ) value of the multiplier in the next period  $E_t (1 - \delta_{t+1,t+2}) \beta_{t+1} \Lambda_{t+1}$ . Note that because we postulate that  $\Gamma$  depends on  $N$  (see below), the net marginal product for the firm depends on  $N$ . This marginal product is part of the match surplus bargained over, and therefore part of the wage solution discussed below. Hence the term  $\frac{\partial W_{t+1}}{\partial N_{t+1}}$ , usually absent, is not zero in this formulation.

For production we assume a standard Cobb-Douglas function:

$$F_t = A_t K_t^\alpha N_t^{1-\alpha} \quad (12)$$

where  $A$  is technology and  $K$  is capital.

Hiring costs refer to the costs incurred in all stages of recruiting: the cost of advertising and screening – pertaining to all vacancies ( $V$ ), and the cost of training and disrupting production – pertaining to actual hires ( $QV$ ). For the functional form we use a general power function formulation which encompasses the widely-used linear and quadratic functions as special cases. This functional form emerged as the preferred one – for example as performing better than polynomials of various degrees – in structural estimation of this model reported in Yashiv (2000a,b) and in Merz and Yashiv (2004). The former study used an Israeli data-set that is uniquely suited for such estimation with a directly measured vacancy series that fits well the model’s definitions. The latter study used U.S. data. Formally this function is given by:

$$\Gamma_t = \frac{\Theta}{1 + \gamma} \left( \frac{\phi V_t + (1 - \phi) Q_t V_t}{N_t} \right)^{\gamma+1} F_t \quad (13)$$

Hiring costs are function of the weighted average of the number of vacancies and the number of

hires. They are internal to production and hence are proportional to output. The function is linearly homogenous in  $V, N$  and  $F$ . It encompasses the cases of a fixed cost per vacancy (i.e. linear costs,  $\gamma = 0$ ) and increasing costs ( $\gamma > 0$ ). When  $\gamma = 1$  we get the quadratic formulation  $(\frac{\Theta}{2}(\frac{\phi V_t + (1-\phi)Q_t V_t}{N_t})^2 F_t)$  which is analogous to the standard formulation in ‘‘Tobin’s q’’ models of investment where costs are quadratic in  $\frac{I}{K}$ . Note that  $\Theta$  is a scale parameter,  $\phi$  is the weight given to vacancies as distinct from actual hires, and  $\gamma$  expresses the degree of convexity.<sup>4</sup>

### 3.4 Wages

In this model, the matching of a worker and a vacancy against the backdrop of search costs, creates a joint surplus relative to the alternatives of continued search. Following Diamond (1982) and Mortensen (1982), the prototypical search and matching model derives the wage ( $W$ ) as the Nash solution of the bargaining problem of dividing this surplus between the firm and the worker [see the discussion in Pissarides (2000, Chapters 1 and 3)].

Formally this wage is:

$$W_t = \arg \max (J_t^N - J_t^U)^\xi (J_t^F - J_t^V)^{1-\xi} \quad (14)$$

where  $J^N$  and  $J^U$  are the present value for the worker of employment and unemployment respectively;  $J^F$  and  $J^V$  are the firm’s present value of profits from a filled job and from a vacancy respectively; and  $0 < \xi < 1$  reflects the degree of asymmetry in bargaining.<sup>5</sup>

Using the Stole and Zweibel (1996) approach as implemented by Cahuc, Marque and Wasmer (2004) to solve (14) taking into account the fact that  $\frac{\partial W_{t+1}}{\partial N_{t+1}} \neq 0$ , the wage is given by:<sup>6</sup>

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<sup>4</sup>Note that its derivatives, used below, are given by:

$$\begin{aligned} \frac{\partial \Gamma_t}{\partial V_t} &= \Theta (\phi + (1-\phi)Q_t) \left( \frac{\phi V_t + (1-\phi)Q_t V_t}{N_t} \right)^\gamma \frac{F_t}{N_t} \\ \frac{\partial \Gamma_t}{\partial N_t} &= \Theta \left( \frac{\phi V_t + (1-\phi)Q_t V_t}{N_t} \right)^{\gamma+1} \frac{F_t}{N_t} \left[ \frac{1-\alpha}{1+\gamma} - 1 \right] \end{aligned}$$

<sup>5</sup>Under certain conditions, this type of solution may be justified by application of strategic bargaining theory (see Binmore, Rubinstein and Wolinsky (1986)). In terms of this theory, asymmetry of the solution ( $\xi \neq \frac{1}{2}$ ) may reflect asymmetries in procedure, in the parties’ beliefs or in their time preferences.

<sup>6</sup>The solution entails postulating the asset values of a filled job and of a vacant job for the firm and

$$W_t = \xi \left( (1 - \alpha)A \left(\frac{K_t}{N_t}\right)^\alpha \left[ \frac{1}{1 - \alpha\xi} + \Theta\left(\frac{\phi V_t + (1 - \phi)Q_t V_t}{N_t}\right)^{\gamma+1} \frac{\alpha + \gamma}{(1 + \gamma)(1 - \alpha)(1 - \xi(1 + \alpha + \gamma))} \right] \right. \\ \left. + P_{t,t+1}\Lambda_t \right) + (1 - \xi)b_t \quad (15)$$

where  $b$  is the income of the unemployed, such as unemployment benefits.

We assume that  $b_t$  is proportional to  $W_t$  i.e.  $b_t = \tau W_t$ , so  $\tau$  may be labelled the ‘‘replacement ratio.’’ Hence:

$$W_t = \frac{\xi}{1 - (1 - \xi)\tau} \left( (1 - \alpha)A \left(\frac{K_t}{N_t}\right)^\alpha \left[ \frac{1}{1 - \alpha\xi} + \Theta\left(\frac{\phi V_t + (1 - \phi)Q_t V_t}{N_t}\right)^{\gamma+1} \frac{\alpha + \gamma}{(1 + \gamma)(1 - \alpha)(1 - \xi(1 + \alpha + \gamma))} \right] \right. \\ \left. + P_{t,t+1}\Lambda_t \right)$$

Denoting  $\frac{\xi}{1 - (1 - \xi)\tau}$  by  $\eta$  and formulating the wage in terms of the labor share in income (by dividing this wage by the average product) we get:

$$W_t = s_t \frac{F_t}{N_t} \\ s_t = \eta \left( (1 - \alpha) \left[ \frac{1}{1 - \alpha\xi} + \Theta\left(\frac{\phi V_t + (1 - \phi)Q_t V_t}{N_t}\right)^{\gamma+1} \frac{\alpha + \gamma}{(1 + \gamma)(1 - \alpha)(1 - \xi(1 + \alpha + \gamma))} \right] \right. \\ \left. + P_{t,t+1}\lambda_t \right) \quad (16)$$

This wage solution has the following properties:

- a. Wages are proportional to productivity  $\frac{F_t}{N_t}$  but the proportionality coefficient  $s_t$  is time-varying.
- b. Wages and the wage share increase with worker bargaining power ( $\xi$ ) or replacement ratio ( $\tau$ ) as expressed by  $\eta$ .
- c. Wages and the wage share are a positive function of the future value given by  $P_{t,t+1}\lambda_t$ .

Thus wages are positively related to the asset value of the match.

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the asset values of employment and unemployment for the worker in (14). See the technical appendix at <http://www.tau.ac.il/~yashiv/performance.html> for details of the derivation.

### 3.5 Equilibrium

The stocks of unemployment and employment and the flow of hiring emerge as equilibrium solutions. Solving the firms' maximization problem yields a dynamic path for vacancies; these and the stock of unemployment serve as inputs to the matching function; matches together with separation rates and labor force growth change the stocks of employment and unemployment. Formally:

$$\begin{aligned}
V_t &= \arg \max E_0 \sum_{t=0}^{\infty} \left( \prod_{j=0}^t \beta_j \right) [F_t - W_t N_t - \Gamma_t + \Lambda_t \{(1 - \delta_{t,t+1})N_t + Q_{t,t+1}V_t - N_{t+1}\}] \\
U_t &= L_t - N_t \\
M_{t,t+1} &= \widetilde{M}(U_t, V_t) \\
N_{t+1} &= (1 - \delta_{t,t+1})N_t + M_{t,t+1} \\
W_t &= \arg \max (J_t^N - J_t^U)^\xi (J_t^F - J_t^V)^{1-\xi}
\end{aligned} \tag{17}$$

This dynamic system may be solved for the five endogenous variables  $V, U, M, N$  and  $W$  given initial values  $U_0, N_0$  and given the path of the exogenous variables. The latter include  $\beta, \delta$  and  $L$ , variables included in the firms' profit function and variables included in the workers unemployment value function  $J^U$ .

As noted above, this is a partial equilibrium model. The exogenous variables include the worker's marginal product, the discount factor and the separation rate. If the production function is CRS and if the capital market is perfect – as we shall assume – the capital-labor ratio will be determined in equilibrium at the point where the marginal product of capital equals the interest rate plus the rate of depreciation. This in turn will determine production and the marginal product of labor. This set-up is consistent with several different macroeconomic models. For example Merz (1995) and Andolfatto (1996) have shown that a special case of this model may be combined with traditional elements of RBC models to yield a dynamic general equilibrium model. In these models the interest rate equals the marginal rate of intertemporal substitution in consumption.

In the following section we solve explicitly for a stochastic, dynamic equilibrium using a stochastic structure for the exogenous variables.

## 4 Steady State and the Stochastic Dynamic Equilibrium

We use a log-linear approach, transforming the non-linear problem into a first-order, linear, difference equations system through approximation and then solving the system using standard methods. More specifically we undertake the following steps:

(i) We postulate a stochastic set-up for the exogenous variables. As noted in the introduction, we use a reduced-form VAR procedure to characterize the shocks rather than imposing a structural formulation.

(ii) Using the F.O.C, we characterize the non-stochastic steady state.

(iii) The deterministic version of the F.O.C of the firm's problem, including the flow equation for employment, is linearly approximated in the neighborhood of this steady state, using a first-order Taylor approximation.

(iv) This yields a first-order, linear, difference equations system, which solution gives the dynamic path of the control and the endogenous state variables as functions of sequences of the exogenous variables.

(v) Working from a certainty equivalence perspective, the deterministic sequences for the exogenous variables are then replaced with the conditional expectations at time  $t$  for the afore-cited stochastic processes.

### 4.1 Dynamic Solution

Basically we are interested in exploring the dynamics of the labor market relative to its growth trend. Growth comes from two sources: productivity growth and population growth. To abstract from population growth, in what follows we cast all labor market variables in terms of rates out of the labor force  $L_t$ , denoting them by lower case letters. Productivity growth is captured by the evolution of  $A$ , which enters the model through the dynamics of  $\frac{F_t}{N_t}$  so we divide all variables by the latter. This leaves a system that is stationary and is affected by shocks to labor force *growth*, to productivity *growth*, as well as to the interest rate and to the job separation rate, to be formalized below.

We now further develop the F.O.C. in order to get a stationary representation with fewer variables. We begin with (8). Transforming the equation into a stationary one through the afore-

mentioned divisions the equation becomes:

$$\Theta(\phi + (1 - \phi)Q_t) \left( \frac{\phi V_t + (1 - \phi)Q_t V_t}{N_t} \right)^\gamma = Q_{t,t+1} E_t \lambda_t \quad (18)$$

where we define:

$$\lambda_t \equiv \frac{\Lambda_t}{\frac{F_t}{N_t}} \quad (19)$$

Inserting into (9) and dividing throughout by  $\frac{F_{t+1}}{N_{t+1}}$ :

$$\frac{\lambda_t}{G_{t+1}^X} = E_t \beta_{t+1} \left[ (1 - \eta) \left( (1 - \alpha) \left[ + \Theta \left( \frac{\phi V_{t+1} + (1 - \phi) Q_{t+1} V_{t+1}}{N_{t+1}} \right)^{\gamma+1} \frac{1}{1 - \alpha \xi} \frac{\alpha + \gamma}{(1 + \gamma)(1 - \alpha)(1 - \xi(1 + \alpha + \gamma))} \right] \right) \right] + E_t (1 - \delta_{t+1,t+2}) \beta_{t+1} \lambda_{t+1} - \eta P_{t+1,t+2} \lambda_{t+1} \quad (20)$$

where the (gross) rate of productivity growth is given by:

$$G_{t+1}^X \equiv \frac{\frac{F_{t+1}}{N_{t+1}}}{\frac{F_t}{N_t}} \quad (21)$$

Dividing (6) by  $L_t$  throughout we get:

$$n_{t+1} G_{t+1}^L = (1 - \delta_{t,t+1}) n_t + Q_{t,t+1} v_t \quad (22)$$

where we define 1+the growth rate of the labor force as:

$$G_{t+1}^L \equiv \frac{L_{t+1}}{L_t} \quad (23)$$

## 4.2 The Stochastic Set-Up

The model has four exogenous variables. These are productivity growth ( $G^X - 1$ , see equation 21), labor force growth ( $G^L - 1$ , see equation 23), the discount factor ( $\beta$ ) and the separation rate ( $\delta$ ). Empirical testing reveals that  $G^L$  can be modelled as white noise around a constant value. When we tried to add it as a stochastic variable to the framework below the results were not affected.

We thus treat it as a constant. It is the other three variables that inject shocks into this system. These variables are basically the variables that make up the firm's future marginal profits, with higher  $G^X$  increasing these profits and higher  $\beta$  or  $\delta$  decreasing them. As mentioned, we do not formulate the underlying shocks structurally. Instead, we postulate that they follow a first-order VAR (in terms of log deviations from their non-stochastic steady state values):

$$\begin{bmatrix} \widehat{G}_{t+1}^X \\ \widehat{\beta}_{t+1} \\ \widehat{\delta}_{t+1} \end{bmatrix} = \Pi \begin{bmatrix} \widehat{G}_t^X \\ \widehat{\beta}_t \\ \widehat{\delta}_t \end{bmatrix} + \Sigma \quad (24)$$

In the empirical section below we use reduced-form VAR estimates of the data to quantify the coefficient matrix  $\Pi$  and the variance-covariance matrix of the disturbances  $\Sigma$ . Thus the current model is consistent with both RBC-style models that emphasize technology shocks as well as with models that emphasize other shocks.

### 4.3 The Non-Stochastic Steady State

In the non-stochastic steady state the rate of vacancy creation is given by:

$$\Theta (\phi + (1 - \phi)Q) \left( \frac{\phi V + (1 - \phi)QV}{N} \right)^\gamma = Q \frac{G^X \beta}{[1 - (1 - \delta)G^X \beta]} \pi \quad (25)$$

The LHS are marginal costs; the RHS is the match asset value. It is probability of filling a vacancy ( $Q$ ) times the marginal profits accrued in the steady state. The latter are the product of per-period marginal profits  $\pi$  and a discount factor  $\frac{G^X \beta}{1 - G^X \beta (1 - \delta)}$  that takes into account the real rate of interest, the rate of separation and productivity growth.

Labor market flows are given by:

$$(\delta + G^L - 1) = \frac{m}{n} = \frac{Qv}{n} \quad (26)$$

This expression equates the rate of increase in employment through matching with the sum of the rates of separation and increase in the labor force. From this equation the rate of unemployment in equilibrium is given by:



$$u = \frac{\delta + (G^L - 1)}{\delta + (G^L - 1) + P} \quad (27)$$

#### 4.4 Log-Linear Approximation

We log-linearly approximate the deterministic version of the F.O.C in the neighborhood of the steady state. For each variable  $Y$ , we use the notation  $\widehat{Y}_t = \frac{Y_t - Y}{Y} \approx \ln Y_t - \ln Y$  where  $Y$  is the steady state value, so all variables are log deviations from steady state.

Approximating the intratemporal condition (18), the intertemporal condition (20), and the dynamic equation for employment (22) and combining all of them we get (the technical appendix available at <http://www.tau.ac.il/~yashiv/performance.html> spells out the complete derivation):

$$\begin{bmatrix} \widehat{n}_{t+1} \\ \widehat{\lambda}_{t+1} \end{bmatrix} = W \begin{bmatrix} \widehat{n}_t \\ \widehat{\lambda}_t \end{bmatrix} + R \begin{bmatrix} \widehat{G}_{t+1}^X \\ \widehat{\beta}_{t+1} \\ \widehat{\delta}_{t+1,t+2} \end{bmatrix} + Q \begin{bmatrix} \widehat{G}_t^X \\ \widehat{\beta}_t \\ \widehat{\delta}_{t,t+1} \end{bmatrix} \quad (28)$$

where  $W, R, Q$  are matrices which are functions of the parameters and steady state values.

This system is a first-order, linear, difference equation system in the state variable  $\widehat{n}$  and the co-state  $\widehat{\lambda}$  with three exogenous variables,  $\widehat{G}^X$ ,  $\widehat{\beta}$  and  $\widehat{\delta}$ . The matrices of coefficients are defined by the parameters  $\alpha, \Theta, \gamma, \phi, \mu, \sigma$  and  $\eta$  and by the steady state values of various variables. The solution of this system enables us to solve for the control variable – vacancies, and for other variables of interest, such as unemployment, hires, the matching rate, and the labor share of income.

## 5 U.S. Labor Market Data

In coming to relate the model to U.S. data, a number of important issues arise. The following discussion shows that the different variables have multiple representations in the data and some are not consistent with the concepts of the model. The idea is to select those series that do match these concepts and to examine alternative representations wherever relevant.

We proceed as follows: in 5.1 we discuss the relevant pool of searching workers, who are defined as  $u$ , unemployed, in the model; in 5.2 we conduct a similar discussion for job vacancies  $v$ ; in 5.3 we consider the flow of hires  $m$  and in 5.4 the relevant wage data  $s$ ; in 5.5 we briefly

discuss other data series; finally, in 5.6 we present summary statistics, including second moments, and discuss the properties of the data in terms of persistence, volatility, and co-movement.

## 5.1 The Relevant Pool of Unemployment

In order to see how the model relates to the data, a key issue that needs to be resolved is the size of the relevant pool of searching workers. The question is whether this pool is just the official unemployment pool or a bigger one. The model speaks of two states – employment and unemployment; in the model matches are flows from unemployment to employment and separations are flows from employment to unemployment.<sup>7</sup> In the actual data – taken from the CPS – several important issues arise:

(i) Flows between the pool out of the labor force and the labor force, including flows directly to and from employment, are sizeable. Blanchard and Diamond (1990) report that in worker flows data – adjusted according to the methodology of Abowd and Zellner (1985) and covering the period 1968-1986 – unemployment to employment flows are only slightly bigger – at 1.6 million workers per month – than out of the labor force to employment flows, at 1.4 million per month. More recent data, computed using the methodology of Bleakley et al (1999) for the period 1976-2003, indicate that unemployment to employment flows are on average 1.9 million workers per month, while out of the labor force to employment flows are 1.5 million workers per month on average.

(ii) Clark and Summers (1979) have argued that there is substantial misclassification of unemployment status and that “many of those not in the labor force are in situation effectively equivalent to the unemployed” (p.29), providing several measures to substantiate this claim. One implication that emerges from their study is that including such misclassified workers in the unemployment pool generates a longer average duration for unemployment and hence a smaller matching hazard ( $P$  in the model) relative to the official unemployment pool. Data collectors were aware of this issue: following the recommendations of the Gordon committee, which recognized that there could be some form of “hidden unemployment,” beginning in January 1967 the CPS included questions on out of the labor force people who could potentially be defined as unemployed. This generated a quarterly series on people that responded affirmatively to the question if they “wanted

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<sup>7</sup> Additionally, labor force growth (with new participants joining the unemployment pool) is an exogenous variable.

a job now” (we report this series below). Castillo (1998) [who offers a detailed discussion and additional references concerning the CPS procedure] reports, using 1994 data, that about 30% of this pool actually searched for work in the 12 months prior to the survey, and 47% were in the prime age category (25-54).

(iii) Working on the re-designed CPS data in the period 1994-1998, Jones and Riddell (2000) further demonstrate the importance of these distinctions. Key results include estimates of the hazard rates for three worker groups: the unemployed, the marginally attached, and the unattached, the last two being officially classified as out of the labor force. Their monthly hazard rates are 0.20-0.35 for the unemployed, 0.10-0.20 for the marginally attached, and below 0.05 for the unattached. Various tests indicate that these are indeed three distinct states.

(iv) The out of the labor force flows exhibit markedly different cyclical properties relative to flows between employment and unemployment: the unemployment to employment flows are countercyclical or a-cyclical while the out of the labor force to employment flows are pro-cyclical.<sup>8</sup>

Given this evidence, it appears natural to consider pools of workers outside the labor force when coming to study labor market dynamics and worker flows. The question is how to add the unobserved ‘unemployment’ pool from out of the labor force to the ‘official’ pool of unemployment. Blanchard and Diamond (1989, 1990) suggest that the relevant additional pool is made up of the people described in point (ii) above. Another method to estimate this pool is to compute the number of people within the out of the labor force group that would generate matching rates ( $\frac{M}{U}$ ) identical to those that are observed for unemployment to employment flows.<sup>9</sup> Figure 1 shows the

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<sup>8</sup>There are two major data sets on worker flows: Blanchard and Diamond (1990, see in particular Figures 1,10 and 11) and Bleakley et al (1999).

The unemployment to employment flows are negatively correlated with the rate of employment (  $-0.91$  for the 1968-1986 Blanchard and Diamond data and  $-0.50$  for the 1976-2003 Bleakley et al data) and negatively correlated ot uncorrelated with GDP per capita ( $-0.61$  for the 1968-1986 data and  $0.12$  for the 1976-2003 data).

The out of the labor force to employment flows are positively correlated with the rate of employment ( $0.56$  for the 1968-1986 Blanchard and Diamond data and  $0.70$  for the 1976-2003 Bleakley et al data) and with GDP per capita ( $0.56$  for the 1968-1986 data and  $0.76$  for the 1976-2003 data).

<sup>9</sup>This estimate – based on the relationship  $P = \frac{M}{U}$  – is computed as follows: first note that

$$P^{UE} = \frac{M^{UE}}{U}$$

where  $M^{UE}$  is the unemployment to employment flow and  $U$  is the official pool. Then, assuming  $P^{UE}$  holds true,

resulting two new series, as well as the official unemployment rate, for the period in which the series exist.

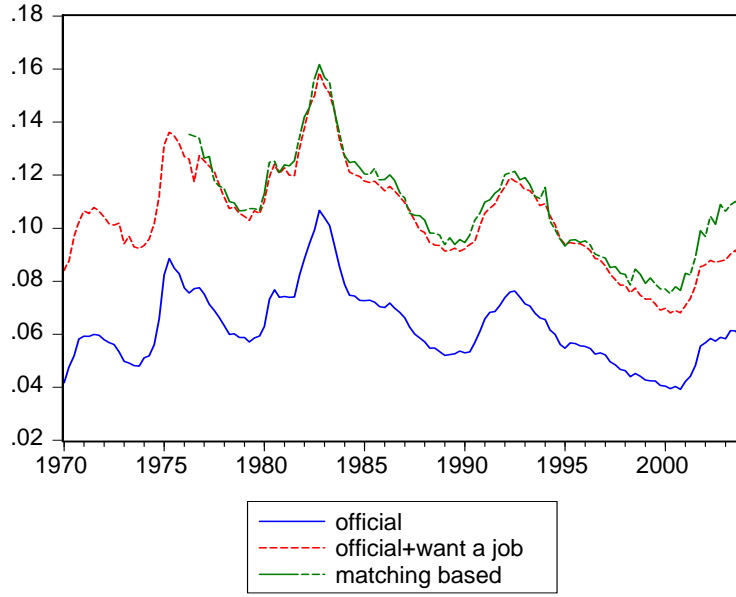


Figure 1: Alternative unemployment series (rates)

The official rate in this period has a mean of 6.3%. The series that adds workers who said they want a job has a mean of 10.4% and the second series has a mean of 10.9% of the relevant 

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and using data on out of the labor force to employment  $M^{NE}$  flow, we compute:

$$U^* = \frac{M^{UE} + M^{NE}}{P^{UE}} = U \left( \frac{M^{UE} + M^{NE}}{M^{UE}} \right) = U \left( 1 + \frac{M^{NE}}{M^{UE}} \right)$$

The official rate is given by  $\frac{U}{N+U}$  while the new rate is given by  $\frac{U^*}{N+U^*}$ . The relation between the two series in rates is thus:

$$\frac{\frac{U^*}{N+U^*}}{\frac{U}{N+U}} = \frac{U^*}{U} \frac{N+U}{N+U^*} = \left( 1 + \frac{M^{NE}}{M^{UE}} \right) \frac{1 + \frac{U}{N}}{1 + \frac{U^*}{N}}$$

labor force. As can be seen in the figure, the series are highly correlated: the new series have a 0.97 correlation between them and high correlations with the official rate (0.98).

These series, however, are likely to be *lower bounds* on the relevant unemployment pool: the first series was constructed in reference to indicating a desire for a job *now*; this precludes other out of the labor force groups who wish to work at a future date, for example people in training or in school. In terms of our two-state model these people could be subsequently flowing directly to the employment pool and thus have to be classified as unemployed. As to the second series, the data support the proposition that the matching rate ( $P = \frac{M}{U}$ ) used to generate it is an *upper* bound on the relevant rate, as persons out of the labor force are likely to experience higher unemployment durations. Hence the resulting unemployment pool is a lower bound on the relevant pool. For example, analysis of 1994 data in Castillo (1998) shows that while matching rates for the officially unemployed were 0.53, they were only 0.31 for the “wanted a job” pool. The issue, then, is how many non-employed workers does one add from out of the labor force to the already expanded pools shown in Figure 1.

The strategy we use in the empirical work is as follows. We look at the three “natural” candidate series: (i) official unemployment, (ii) official unemployment and the “want a job” category (depicted above) and (iii) the entire working age population. As the relevant pool may lie between the second and third cases and as no measured pool is available, we look at two additional specifications that try to approximate that pool. Thus, starting from case (ii) we gradually add parts of the remaining workers from the out of the labor force pool. Table 1 provides sample statistics of these five series.

**Table 1**

Two features stand out: while the mean of the series evidently rises with the expansion of the pool, the volatility hardly changes going from the official +want a job pool to the larger pools; the series are highly correlated (though the correlation slightly declines as the pool expands).

In what follows, we look at the properties of the data under these different specifications. In Section 6 below, we compare the performance of the calibrated model against these alternative

specifications and decide on a benchmark specification for the subsequent analysis.<sup>10</sup>

## 5.2 Job Vacancies

The relevant concept of vacancies in the model is the one relating to those vacancies that are to be filled with workers from outside the employment pool (we shall denote it by  $V^{UN}$ ). But the available and widely-used data series pertains to another concept, which also includes vacancies that are subsequently filled with workers moving from job to job (to be denoted by  $V_t^{NN}$ ).<sup>11</sup> Simply this can be expressed as follows:

$$V_t^{tot} = V_t^{UN} + V_t^{NN}$$

The  $V_t^{tot}$  series in the U.S. economy has two representations: one is the index of Help Wanted advertising in newspapers published by the Conference Board; this series was analyzed and discussed in Abraham (1987). A newer series is the job openings series available from the BLS since December 2000 using the Job Openings and Labor Turnover Survey (JOLTS).<sup>12</sup> The two series have a correlation of 0.88 over 37 monthly observations. The following figure plots these two vacancies series as well as a third series: the gross flows of workers from outside employment (unemployment and out of the labor force) to employment, which can be taken to represent  $Q \times V^{UN}$ . The latter was recently compiled at the Boston Fed based on CPS data [see the Data appendix below and Bleakley et al (1999)]. The figure shows all three series normalized:

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<sup>10</sup>Previous papers have taken on a variety of approaches to this issue, which are special cases of the current examination: Blanchard and Diamond (1989) have essentially considered the relevant pool to be the first series reported in Figure 1, Merz (1995) took the official pool of unemployment, and Andolfatto (1996) considered the entire working age population as unemployed in the model.

<sup>11</sup>This should not be taken to mean that firms post two types of vacancies. The idea is just to say that some vacancies are ex-post filled by previously unemployed workers and the rest by previously employed workers moving directly from job to job.

<sup>12</sup>A job is “open” only if it meets all three of the following conditions: (i) a specific position exists and there is work available for that position; the position can be full-time or part-time, and it can be permanent, short-term, or seasonal; (ii) the job could start within 30 days, whether or not the establishment finds a suitable candidate during that time; and (iii) there is active recruiting for workers from outside the establishment location that has the opening.

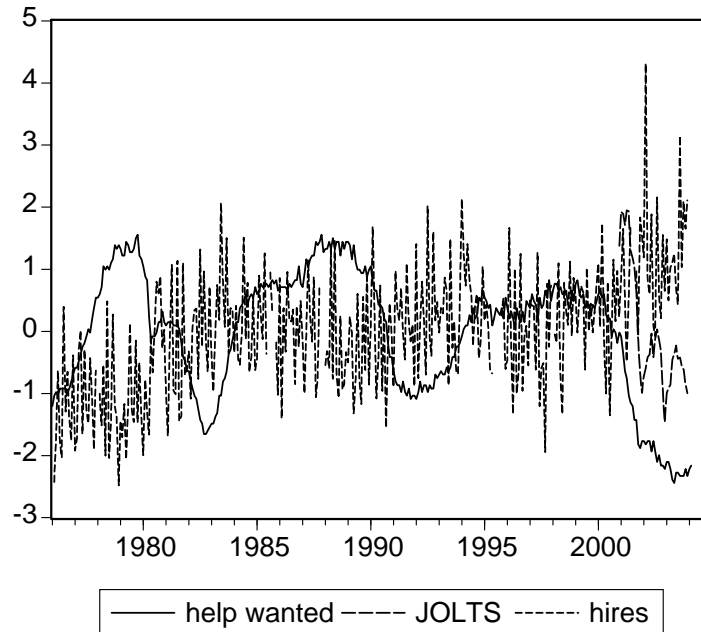


Figure 2: Hiring vs. the two vacancy measures (normalized)

The hiring flows series is negatively correlated with the two vacancy series:  $-0.27$  with the JOLTS series and  $-0.36$  with the Help Wanted ads series. The flows series also appears to be much less persistent than the vacancies series.

Table 2 presents the coefficient of variation (or standard deviation) and autocorrelation for the two vacancy series and for the hiring series. It does so first for their respective individual samples and then for the common sample period. It reports the variables in levels, in logged HP-filtered form, and subsequently in terms of their ratios to unemployment, which can be taken as a measure of market tightness.

### Table 2

Compared to the volatility of the Help Wanted Index, hiring volatility is about a seventh in levels, a fifth in HP filtered terms, and about a third in ratios to unemployment terms. It is

similarly less volatile than the JOLTS data series. In terms of persistence, hiring is substantially less persistent in levels and even more so in HP-filtered terms; it has similar persistence in market tightness terms.

This comparison – between  $V^{tot}$  and  $Q \times V^{UN}$  – suggests that  $V^{tot}$  may either be very different from  $V^{UN}$  or that the behavior of  $Q$  generates these discrepancies. Without direct measures of  $V^{UN}$  it is not possible to determine which explanation holds true but there is indirect evidence. Using the data on  $Q \times V^{UN}$  shown above one can run a regression of the matching function as follows:

$$\ln \left( \frac{Q_{t,t+1} V_t^{UN}}{N_t} \right) = \ln \hat{\mu} + \hat{\sigma} \ln \left( \frac{U_t}{N_t} \right) + (1 - \hat{\sigma}) \ln \left( \frac{V_t}{N_t} \right)$$

Previous estimates of the matching function – as surveyed by Petrongolo and Pissarides (2001) and as exemplified by the Blanchard and Diamond (1989) analysis – indicate that  $\hat{\sigma}$  is around 0.4 to 0.5. This means that the regression should yield around 0.4 to 0.5 for the coefficient of  $\ln \left( \frac{U_t}{N_t} \right)$  and around 0.6 to 0.5 for the coefficient of  $\ln \left( \frac{V_t}{N_t} \right)$ . Using the Help Wanted index for  $V$  one gets an estimate of around 0.3 for the coefficient of  $\ln \left( \frac{U_t}{N_t} \right)$  but only 0.1 for the coefficient of  $\ln \left( \frac{V_t}{N_t} \right)$ , irrespective of the estimation technique (OLS, TSLS or GMM) or the instruments used. The extremely low value for the latter coefficient implies that hiring rates (from outside employment, i.e.  $\frac{Q_{t,t+1} V_t^{UN}}{N_t}$ ) do not relate well to the vacancy rate when the latter is measured by the Help Wanted Index. This is consistent with the interpretation that the behavior of the Help Wanted Index, which relates to the broader  $V^{tot}$ , is different from the relevant series here  $V^{UN}$ .

Another exercise is to compute a “reconciling  $Q$ ” to be denoted  $Q_{t,t+1}^{sim}$  i.e.

$$Q_{t,t+1}^{sim} = \frac{M_{t,t+1}^{UN}}{V_t^{tot}}$$

which makes the series  $(V^{tot}, V^{UN})$  compatible and then see whether it reasonably fits with the model. The model predicts the following relationship:

$$Q_{t,t+1} = \mu \left( \frac{V_t}{U_t} \right)^{-\sigma}$$

The simulated  $Q$  can be written as:



$$Q_{t,t+1}^{sim} = \frac{M_{t,t+1}^{UN}}{V_t^{tot}} = Q_{t,t+1}^{UN} \frac{V_t^{UN}}{V_t^{tot}}$$

Note that as  $\frac{V_t^{UN}}{V_t^{tot}} \rightarrow 1$ , then  $Q_{t,t+1}^{sim} \rightarrow Q_{t,t+1}^{UN}$ . Running a regression based on the model's formulation i.e.:

$$\ln Q = \ln k - a \ln \left( \frac{V_t}{U_t} \right)$$

using  $Q^{sim}$  and the Help Wanted Index as  $V$ , we get an estimate of  $a$  of about 0.7, higher than the 0.4 to 0.5 range cited above for  $\sigma$ . In other words the simulated  $Q$  is more responsive to variations in market tightness than usual estimates of  $\sigma$  indicate. This is likely to occur the more different are  $V_t^{UN}$  and  $V_t^{tot}$ .<sup>13</sup>

Another body of evidence is to be found in recent work on gross worker flows. Fallick and Fleischman (2004), using CPS data in the period 1994-2003, find that employment to employment flows are large; in fact 2/5 of new jobs represent employer changes. They also show that the cyclical properties of employment to employment flows are very different from those of the flows into and out of employment. Nagypal (2004), using micro data sets, provides additional, supporting evidence on the prevalence of job to job worker flows. Pissarides (1994) offers one possible explanation for the higher volatility of vacancies that relate to both job to job movements ( $V^{NN}$  in the above

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<sup>13</sup>The regression postulates

$$Q_{t,t+1}^{sim} = k \left( \frac{V_t^{tot}}{U} \right)^{-a}$$

The model posits:

$$Q_{t,t+1}^{sim} = \mu \left( \frac{V_t^{UN}}{U} \right)^{-\sigma} \frac{V_t^{UN}}{V_t^{tot}}$$

Hence:

$$k \left( \frac{V_t^{tot}}{U} \right)^{-a} = \mu \left( \frac{V_t^{UN}}{U} \right)^{-\sigma} \frac{V_t^{UN}}{V_t^{tot}}$$

Taking logs and re-arranging we get:

$$a = \sigma - \frac{\ln \mu - \ln k}{\ln \left( \frac{V_t^{tot}}{U} \right)} - \frac{(1 - \sigma) \ln \left( \frac{V_t^{UN}}{V_t^{tot}} \right)}{\ln \left( \frac{V_t^{tot}}{U} \right)}$$

Only when  $\frac{V_t^{UN}}{V_t^{tot}} = 1$  and when  $\mu = k$  will the estimate  $a$  equal  $\sigma$ .

notation) and to movements from out of employment ( $V^{UN}$ ): these “broader” vacancies (i.e.  $V^{tot}$ ) are more responsive to productivity changes and hence are more volatile. The reason for the increased responsiveness is that employed workers, as well as unemployed workers, change their search activities following productivity changes, thereby affecting the incentives of firms in opening job vacancies.

The conclusion from this discussion is the following: the present model, being an aggregate, representative firm-type of model, does not deal with job to job movements (as is also the case for the cited literature), and thus the Help Wanted Index is not the relevant series to use as it probably does not behave like the relevant series ( $V^{UN}$ ). Given that the latter is unobserved, we resort to focusing on the observed worker flow series (i.e. on  $Q \times V^{UN}$ ) whenever comparing the model to the data (though the model will generate predictions also with respect to  $V$ ). The above discussion also implies that care must be taken when using or discussing vacancy data in the U.S. economy.

### 5.3 The Flow of Hires and the Worker Hazard Rate

The discussion on the relevant pool of unemployment makes it clear that it is important to analyze the flow of matches or hires using out of the labor force to employment flows as well as unemployment to employment. This implies that in addition to different unemployment pools  $U$ , there will be different matching flows  $M$  and consequently different worker hazard rates  $P = \frac{M}{U}$ . Figure 4 presents these different rates.

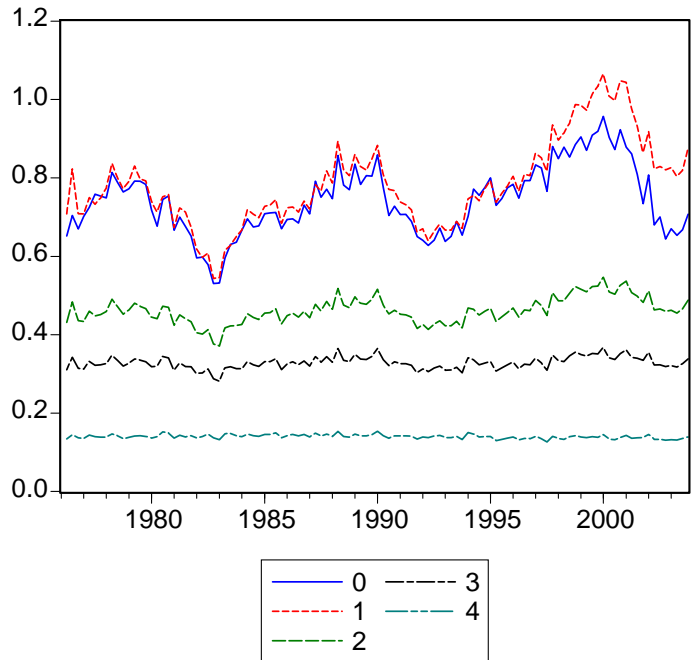


Figure 4: Alternative specifications of the hazard rate  $P$

The 0 and 1 series represent the ones derived from the official rate and from the official+want a job series respectively, both depicted in Figure 2 above. They imply unemployment durations of 17.4 weeks and 16.5 weeks respectively. In comparison, official BLS data on duration indicate 15.1 weeks on average (with 2.5 weeks standard deviation) in the same period. The other series – specifications 2,3 and 4 – evidently imply higher durations: 28.0, 39.0 and 91.8 weeks respectively. All rates are highly correlated with the notable exception of specification 6, which is almost uncorrelated with the 0 and 1 specifications (correlations of 0.10 and -0.03 respectively) and weakly to moderately correlated with the 2 and 3 specifications (0.23 and 0.53 respectively).

The following table compares these estimates, based on aggregate worker flow and non-employment stock data, to the estimates of the afore-cited Jones and Riddell (2000) micro-based, CPS 1994-1998 study:

### Table 3

The official pool has a slightly higher hazard than the upper bound of the Jones and Riddell estimate (i.e. 0.80 compared to 0.73). From the three intermediate pools, one can think of specification 2 with a hazard rate (0.47) that is consistent with a mixture of the unemployed and the marginally attached. The largest pool that includes all non-employed workers (specification 4 here) has a hazard rate (0.14) that seems to be consistent with a mixture of the unattached with the other two groups.

## 5.4 Wages

The existence of diverse data series for wages with different cyclical properties was noted by several papers [see in particular Abraham and Haltiwanger (1995), Abraham, Spletzer and Stewart (1999), and Krueger (1999)]. The discussion in these papers does not lead to any definite conclusion as to which series is the most appropriate. Figure 5 illustrates one aspect of this issue by plotting BEA series of the labor share  $s = \frac{W}{F/N}$ , once using total compensation<sup>14</sup> and once using wages.

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<sup>14</sup>Defined as total compensation of employees relative to GDP; this includes, beyond wages and salaries, supplements such as employer contribution for employee pension and insurance funds and employer contribution for government social insurance.

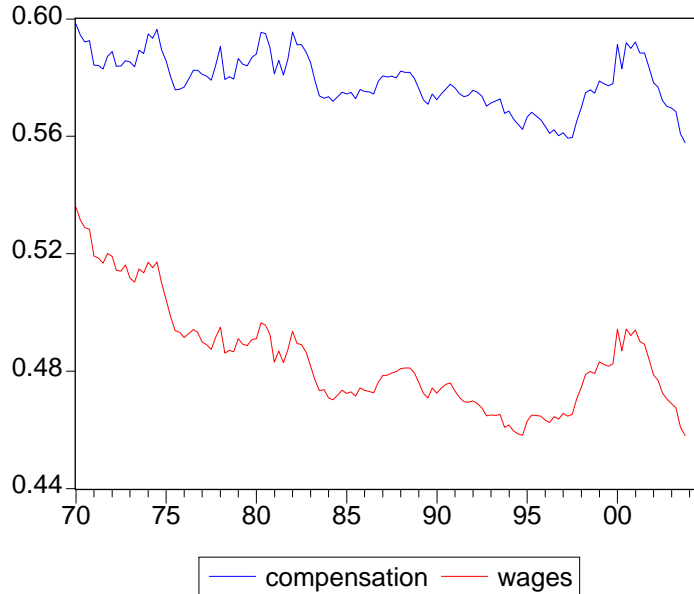


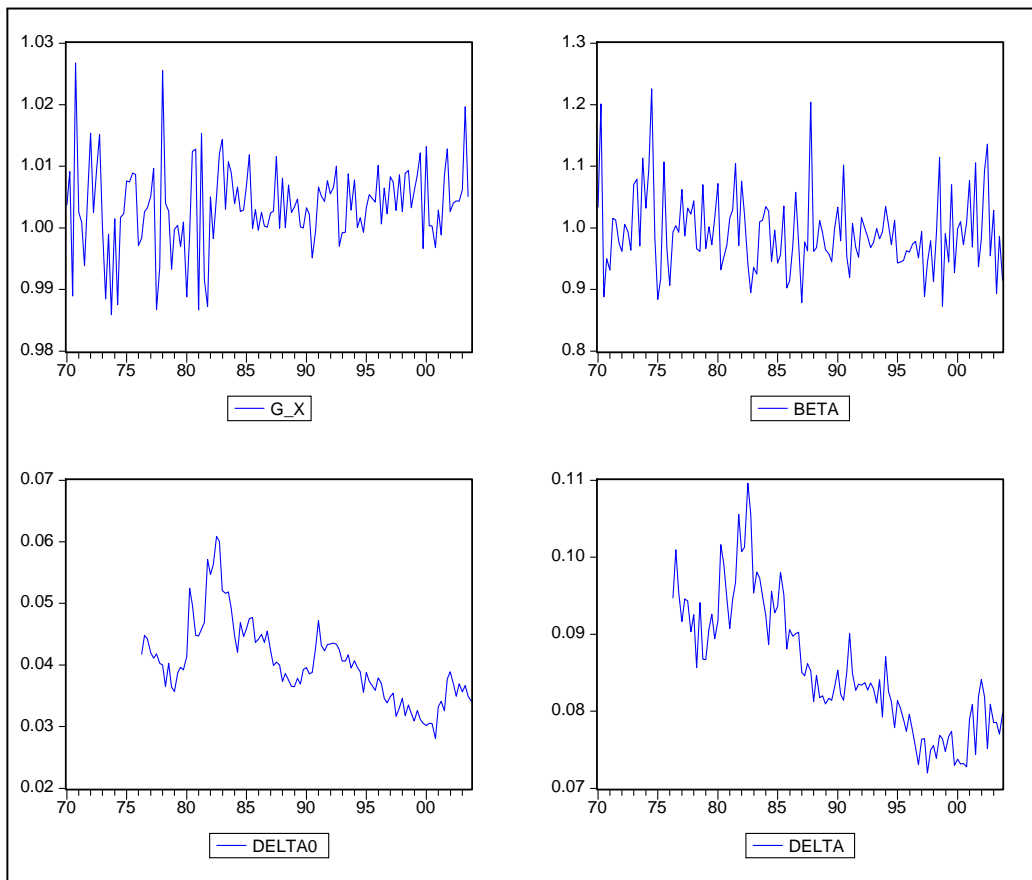
Figure 5: the labor share  $s$

The series are correlated 0.83 but have a number of important differences: the wage series declines more over time, is lower by ten percentage points on average, and displays much more variation (coefficient of variation of 0.037 relative to 0.016 for the other series). It should also be noted that both series have very weak correlation with the cycle: the compensation series has -0.05 correlation with the employment rate and the wage series has a 0.12 correlation.

In what follows we use the compensation series as it takes all firm's wage-related costs, which is the relevant concept in the model.

## 5.5 Other Data Series

Figure 6 shows the other data series to be used in the empirical work below: productivity growth  $G^X$  (gross rate), the discount factor  $\beta$ , and the separation rate  $\delta$  (separately for the official unemployment specification  $\delta_0$  and for the others  $\delta$ ).



**Figure 6:**  $G^X, \beta, \delta_0, \delta$

For productivity growth we take the rate of change of GDP per worker; for the discount factor we take  $\beta = \frac{1}{1+r}$  where  $r$  is the cost of firm finance (weighted average of equity finance and debt finance); for the separation rate we take the flow from employment divided by the stock of employment.

## 5.6 Data Properties

Table 4 reports the first two moments of all relevant data series, including measures of persistence and co-movement. When looking at these moments it is important recall that the business cycle is most clearly manifested in the labor market – there is high correlation between employment and output and their volatility is similar. The table describes the key moments across the different

specifications of the unemployment pool (and the labor force). Column 0 is the benchmark specification – it uses the official unemployment pool. Column 1 is the ‘lower bound’ specification with unemployment including the official pool and those workers out of the labor force indicating that they “want a job.” Columns 2 and 3 add to column 1 a fraction – 15% and 30% respectively – of the remaining persons out of the labor force. Column 4 considers the entire working age population as the labor force.

#### Table 4

The following major properties can be said to characterize the data:

*Persistence.* All the main labor market variables are persistent: the rate of unemployment (and thus employment), hiring, separation, and the wage share all exhibit high persistence. There is some variation across the different cases: unemployment persistence increases slightly as the unemployment pool is expanded and the reverse is true for hiring and separation. At the same time productivity growth and the discount factor are not persistent at all. Note that one of the three driving shocks – the separation rate – is persistent.

*Volatility.*

(i) The table indicates differential volatility across variables and across specifications of the pool of searching workers. With only the officially unemployed considered searching (column 0), the volatility of the unemployment rate is the highest at 0.22 in terms of log levels; less volatile are the hiring and separation rates, which have comparable volatility at about 0.13-0.16; the discount factor has a volatility of 0.06 in the same terms; the volatility of the rate of employment ( $n = \frac{N}{L}$ ) is of the same order of magnitude as that of the wage share at around 0.015 (in the same log terms); productivity growth at 0.007 has even lower volatility. Note that this is somewhat akin to investment behavior: the capital stock (here the employment stock) is much less volatile than investment (here the hiring flow). Note too the relatively high volatility of the rate of separation and of the discount factor, which are driving factors.

(ii) When moving across columns to a broader specification of the pool of searching workers some patterns change: in relative terms, hiring and separation become more volatile (at 0.07-0.10 in column 4) than unemployment (at 0.07). In absolute terms the employment rate becomes more volatile while unemployment, hiring and separation (in rates) all become less volatile.

*Co-movement.*

(i) Hiring rates( $m$ ) co-vary positively with the unemployment rate, while the worker hazard rate usually co-varies negatively with the same variable. This means that hiring flows (in rates) are counter-cyclical (see the discussion of the various unemployment pools above)<sup>15</sup> but job-finding rates  $P = \frac{m}{u}$  are pro-cyclical. Note that in recessions  $U$  rises and  $P$  falls; hence  $\frac{M}{L} = \frac{P \cdot U}{N+U}$  rises as the effect of a rising  $U$  overcomes the fall in  $P$  and any change in  $L$ .

(ii) Separation rates ( $\delta$ ) are counter-cyclical. Note that hiring and separation rates move together, unlike the widely known negative correlation between job creation and job destruction in the manufacturing sector. This issue is discussed in Nagypal (2004).

(iii) The labor share in income ( $s$ ) varies between a-cyclical and counter-cyclical (with respect to employment) according to the specification of the labor force. Throughout it covaries positively with the hiring rate. This means that when  $\frac{F}{N}$  rises in booms, the wage ( $W$ ) rises by less and thereby the labor share ( $s$ ) declines.

(iv) There is low correlation between employment and productivity growth, a fact which has received considerable attention in the RBC literature. Note that the data here are in terms of the log of the employment rate and the first differences of the log of productivity but that they display essentially the same co-movement pattern that is discussed in the business cycle literature in terms of HP-filtered and logged productivity and employment.

(v) Employment has low co-variation with the discount factor.

Note some non-intuitive aspects of these data moments: in booms hiring and separation rates fall and the labor share either does not change or falls. Hiring is strong at the same time as the share of wages is high.

We turn now to the calibration of the model and evaluation of its performance against these data.

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<sup>15</sup>This is also true for logged and HP-filtered variables.



## 6 Calibration and Model-Data Fit

In this section we calibrate the model and examine its performance, taking into account the alternative formulations of the pool of searching workers discussed above. We begin by discussing calibration values in sub-section 6.1. We then (6.2) examine the performance of the model.

### 6.1 Calibration

Calibration of the model requires the assignment of values to the parameters of the production, hiring costs, and matching functions as well as to the wage bargain, to the steady state values of the exogenous variables (labor force growth, productivity growth, the discount factor and the separation rate) and to the steady state values of the endogenous variables. To do so we use, wherever possible, results from econometric studies and average values of the longest sample period available. We then solve for all other values using the steady state relations (equations 25 and 26).

There are three structural parameters that are at the focal point of the model and that reflect the operation of frictions. These are the matching function parameter  $\sigma$  (elasticity of unemployment), the wage parameter  $\eta$ , and the hiring function convexity parameter  $\gamma$ .

For  $\sigma$  we use Blanchard and Diamond's (1989) estimate of 0.4. Structural estimation of the model using U.S. corporate sector data in Merz and Yashiv (2004) indicates a value of  $\gamma$ , the convexity parameter of the hiring cost function, around 2, i.e. a cubic function ( $\gamma+1 = 3$ ) for hiring costs. Later on we also present the results of using a counter-factual value for  $\gamma$ . These costs fall on vacancies and on actual hires, with  $\phi$  being the weight on the former. We follow the estimates in Yashiv (2000a) and set it at 0.3.

The wage parameter  $\eta$  depends on the asymmetry of the bargaining solution ( $\xi$ ) and on the replacement ratio ( $\tau$ ). This is obviously a difficult case for calibration as  $\xi$  is not directly observed and  $\tau$  depends not only on the value of benefits relative to wages but also on actual take-up rates. Following Anderson and Meyer (1997) we postulate a value of 0.25 for the latter; to test for robustness we also report a far higher value, finding that this change has a very small effect on the resulting moments. For  $\xi$  rather than imposing it, we solve it out of the steady state relations.

For the production function parameter  $\alpha$  we use a fairly traditional value of 0.68, which is also the structural estimate of this parameter in Merz and Yashiv (2004).

For the values of the exogenous variables we use sample average values as follows: in quarterly terms the sample average of the rate of productivity growth ( $G^X - 1$ ) is 0.35%; the sample average rate of labor force growth ( $G^L - 1$ ) varies according to the definition of the unemployment pool between 0.43% and 0.36%; the stochastic discount factor  $\beta$  – defined as the weighted price of capital (equity and debt financed) – has a sample average of 0.993, which is close to the value used in many business cycle studies; the sample average for the separation rate  $\delta$  is 8.5%<sup>16</sup>.

For the steady state values of the endogenous variables, given the discussion above on the relevant unemployment pool, we modify  $n$  and  $u$  according to the specifications used above. Calibration of  $Q$ , the matching rate for vacancies, is problematic as there are no wide or accurate measures of vacancy durations for the U.S. economy. Using a 1982 survey, Burdett and Cunningham (1998) estimated hazard functions for vacancies both parametrically and semi-parametrically finding that the general form of the hazard function within the quarter is non-monotonic; based on their estimates the quarterly hazard rate should be in the range of 0.8 – 1. We thus take  $Q = 0.9$  which is also the value used by Merz (1995) and Andolfatto (1996). This implies a certain value for the vacancy rate ( $v$ ). We use the average of the labor share in income  $s$  which is 0.58.

With the above values, we solve the steady state relations (25-26) for the steady state vacancy rate  $v$ , the hiring cost scale parameter  $\Theta$ , the matching function scale parameter  $\mu$ , and the wage parameter  $\xi$ . We can then solve for the steady state values of market tightness  $\frac{v}{u}$ , the worker hazard rate  $P$ , per period profits  $\pi$  and the match asset value  $\lambda$ .

The following table summarizes the calibrated values for the different specifications of the unemployment pool.

**Table 5**

Note two features of the implied results:

- (i) The implied wage parameter  $\eta$ , encompassing the worker bargaining strength and the replacement ratio, varies between 0.4 and 0.6 across specifications.

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<sup>16</sup>For the case of the official unemployment pool we use  $\delta = 4\%$  reflecting the rate of separation from employment to this pool only.

(ii) Across specifications 1-4, per period profits ( $\pi$ ) are around 0.07-0.13 (in average output terms) and the asset value of the match ( $\lambda$ ) is around 0.7-1.5. Given that  $s = 0.58$  this means that asset values are about 1.2 – 2.6 the labor share in income. In other words the match is worth around 1 to 2.5 quarters of wages in present value terms. This quantification of asset values is based on the values of panel (a). The latter include three values which have not been much investigated in the literature: the vacancy match rate  $Q$  and the parameters  $\phi$  and  $\gamma$  of the hiring cost function. When varying the latter values within reasonable ranges, there was no significant change in the results. Hence the steady state calibration gives a sense of the magnitude of hiring costs or match asset values.

As to the stochastic shocks, in order to get numerical values for the coefficient matrix  $\Pi$  and for the variance-co-variance matrix of  $\Sigma$  we estimate a first-order VAR in labor productivity growth, the discount factor and the rate of match separation. We estimate this VAR with the relevant data discussed above (see the Appendix for definitions and sources).

## 6.2 Model-Data Fit

We now turn to examine the performance of the model.<sup>17</sup> Table 6 shows the moments implied by the model and those of the data (repeating the moments reported in Table 4).

**Table 6**

The following conclusions can be drawn:

*Persistence.* The model captures the fact that across all specifications  $u$ ,  $m$  and  $s$  are highly persistent. The model tends to somewhat overstate this persistence.

*Volatility.* Column 1 captures very well the volatility of employment and unemployment. Hiring volatility is understated by the model; best performing is column 0 which captures three quarters of this volatility. As to the labor share, the model overstates its volatility with column (4) being the closest to the data. No single specification produces a high model-data fit for all variables.

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<sup>17</sup>We use a modified version of a program by Craig Burnside in Gauss to solve the model [see Burnside (1997)].

*Co-Movement.* Under most specifications of the model, the counter-cyclical behavior of hiring and the pro-cyclical behavior of the worker hazard rate are well captured. The behavior of the labor share is not captured: while in the data it is a-cyclical to counter-cyclical (across specifications of the unemployment pool) and co-varies moderately with the hiring rate, it is strongly pro-cyclical in the model and has a strong negative relationship with hiring, except for column 4 where it is positive but overstated.

*Overall fit.* The model captures the persistence, volatility and some of the co-movement in the data. The major problem concerns the labor share in income which is not well captured. Column 1 of the model fits the data in terms of employment and unemployment behavior and has reasonable but limited success in fitting hiring flows (fits persistence and cyclicity, understates volatility). Column (4) seems to be providing the better fit for the labor share, doing relatively well on volatility, reasonably well on persistence, moderately well on co-movement with hiring and doing badly in terms of cyclical behavior.

The model also generates predictions with respect to the behavior of  $v$  and  $q$  which are unobserved in U.S. data as explained in sub-section 5.2 above. Strictly speaking the moments involving these variables cannot be compared to the data. But some predictions look reasonable based on the theory and on experience in other economies: the negative correlation of  $u$  and  $v$  (the ‘Beveridge curve’) and persistent vacancy rates.

No single specification matches the data on all dimensions. Focusing on the behavior of unemployment, employment and hiring, it looks as though specification 1 (the official unemployment pool and the ‘want a job’ category) is the most fitting, though specification 0 (official unemployment pool) cannot be ruled out. We thus pick specification 1 as the benchmark specification for what follows. The performance of the model with respect to the labor share is further discussed below.

We turn now to look at the mechanisms driving this model-data fit.

## **7 The Underlying Mechanism**

The discussion up till now has shown which data specification is best explained by the model. The natural question now is what underlines the fit of the model.

## 7.1 A Key Relationship

In order to understand the essential mechanism in operation, it is best to consider the following steady-state equation which elaborates on the formulation in (25) and combines it with (26):

$$\frac{\Theta}{\mu} \tilde{Q}^{\gamma+1} \frac{\left(\frac{v}{1-u}\right)^\gamma}{\left(\frac{v}{u}\right)^{-\sigma}} = \underbrace{\left[ \frac{G^X \beta}{1 - G^X \beta (1 - \delta)} \right]}_{\Phi} \underbrace{\left[ \frac{(1 - \eta) \left( (1 - \alpha) \left[ \frac{1}{1 - \alpha \xi} + \Theta \left( \frac{\phi V + (1 - \phi) Q V}{N} \right)^{\gamma+1} \frac{\alpha + \gamma}{(1 + \gamma)(1 - \alpha)(1 - \xi(1 + \alpha + \gamma))} \right) \right]}{1 + \eta P \left[ \frac{G^X \beta}{1 - G^X \beta (1 - \delta)} \right]} \right]}_{\pi} \quad (29)$$

where  $\tilde{Q} = \phi + (1 - \phi)Q$

The LHS are marginal vacancy creation costs. They take into account  $Q$ , the probability of filling the vacancy. It is clear that the responsiveness of vacancies ( $v$ ) depends on the two elasticity parameters  $\gamma$  (of the hiring cost function) and  $\sigma$  (of the matching function). The higher is each of these, the less responsive is vacancy creation. Many studies have assumed this is a linear function ( $\gamma = 0$ ), thereby imposing a particular shape on this marginal cost function.

The RHS is the asset value of the match. This value can vary because per period profits  $\pi$  vary or because the discount factor  $\Phi$  varies. The former may be due to changes in the surplus itself or changes in the sharing of the surplus, with a key parameter being  $\eta$ . Any policy change in the replacement ratio  $\tau$ , for example, will change  $\eta$  and consequently the sharing of the match surplus. Changes in the discount factor  $\Phi$  can happen because of changes in productivity growth ( $G^X$ ), changes in the discount factor ( $\beta$ ), or changes in match dissolution rate ( $\delta$ ).

Hence the essential mechanism is this: changes in the long run (non-stochastic steady state) and in the stochastic dynamics are generated by changes in the surplus, in surplus sharing, in productivity growth, in the discount rate and in the match dissolution rate, that change the match asset values (profitability in present value terms).

If the model holds true, the following ingredients are therefore essential:

(i) The *shape of the hiring costs function* determining the LHS of equation (29) i.e. the marginal cost function. In this context  $\gamma$  and  $\sigma$  are key parameters.

(ii) The formulation of the *match surplus* – this depends both on the data used and on all key parameters of the model.

(iii) the *surplus sharing rule*, where  $\eta$  is the key parameter.

(iv) the *discounting* of the match surplus – here the data used and their stochastic properties are key.

In order to see the role played by each of these elements, we undertake an analysis of counterfactual parameter values.

## 7.2 Counterfactual Analysis

Taking specification 1 of the data, we change the value of one parameter at a time. Table 7 reports the moments implied by the model when changing various parameter values. In each case we re-work the non-stochastic steady-state and re-compute the moments.

**Table 7**

(i) *Linear hiring costs.*

Much of the literature uses a linear formulation for hiring costs. In terms of the current analysis this means setting  $\gamma = 0$ . When solving for the steady state under this formulation, the implied wage bargaining parameter  $\eta$  is solved out to be 0.82, much higher than the benchmark of 0.48. This results in a lower share of the surplus for firms and hence lower profits and match asset values. This implies that formulating linear hiring costs and setting the bargaining weight to be 0.50 are incompatible with U.S. data at the steady state.

The results with respect to the moments implied by the model, as reported in Table 7, show that the fit worsens on almost all dimensions: persistence declines, the volatility of  $n$  and  $u$  decline while those of  $v$  and  $s$  rise, and the co-movement of  $u$  and  $P$  and  $m$  and  $\delta$  decline. Moreover the persistence of  $v$  is reduced to zero and the co-movement of  $v$  with  $u$  becomes positive. The one change that does improve the fit is the higher volatility of  $m$ .

The basic intuition for these results is that linear, rather than convex, costs mean less sluggishness in vacancy creation. Hence persistence declines and vacancy rate volatility rises. Looking more in depth, the reason for these outcomes is best understood by examining a re-writing of (29):

$$\frac{\Theta}{\mu} \tilde{Q}^{\gamma+1} \left(\frac{v}{1-u}\right)^{\gamma} \left(\frac{v}{u}\right)^{\sigma} = \Phi\pi \quad (30)$$

According to the equation,  $v$  responds to  $\Phi, \pi$  and  $u$  and the relevant elasticity is  $\gamma + \sigma$ . When  $\gamma = 0$ , and given that  $\sigma$  is relatively small (0.4 at the benchmark value), then  $v$  responds a lot to  $\Phi$  and  $\pi$ ; for given values of the latter variables, it responds positively to  $u$ . When  $\gamma > 0$ , the responsiveness of vacancy creation to  $\Phi$  and  $\pi$  declines with  $\gamma$  and the negative relation with  $u$  (via the  $1 - u$ ) term rises. Hence we get the following:

When  $\gamma = 0$  and there is high responsiveness of  $v$ , it has no persistence and has relatively high volatility. Hiring ( $m = qv$ ) volatility is relatively high as a consequence. Vacancy creation moves with unemployment and hence the counterfactual positive correlation between  $u$  and  $v$ .

As  $\gamma$  rises, the decline in responsiveness to the asset value  $\Phi\pi$  generates sluggishness in hiring and hence increased persistence. This is due to the adjustment cost aspect of the analysis. Consequently the other variables become more persistent too, and their volatility increases as this happens.<sup>18</sup> With a positive  $\gamma$  the vacancy rate co-varies negatively with unemployment.

Note the difference between the  $\gamma = 0$  and  $\gamma > 0$  with respect to wage behavior ( $s$ ): in the linear case it is less persistent and more volatile counter-factually, and it co-varies negatively with vacancies rather than positively. The unchanged features are that it is still highly pro-cyclical and negatively correlated with hiring, counter-factually.

As many papers have adopted a linear specification, i.e. assumed  $\gamma = 0$ , it is not surprising that they get the following counter-factual results: a positive, rather than negative, unemployment-vacancy relationship, insufficient persistence of all variables, insufficient volatility of  $n$  and  $u$ , and too much volatility of wages.

*(ii) Matching elasticity.*

We set a value of  $\sigma = 0.8$  rather than the benchmark of 0.4. This results in slightly lower persistence statistics, significantly lower volatility of  $n, u$  and  $s$ , higher volatility of  $m, v$  and  $Q$ , and mildly stronger co-movement statistics. A higher  $\sigma$  makes the worker hazard rate  $P$  respond less to market tightness (as  $P = \mu\left(\frac{v}{u}\right)^{1-\sigma}$ ). This leads the wage, which depends on  $P$ , to fluctuate

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<sup>18</sup>The intuition for this result is easily seen in a univariate setting: if  $x_t = \rho x_{t-1} + \varepsilon_t$  then  $var(x_t) = \frac{1}{1-\rho^2} var(\varepsilon_t)$  and as  $\rho$  increases the variance of  $x$  increases.

less. Hence the volatility of  $s$  declines. With more rigid wages, profitability fluctuates more and so vacancy creation fluctuates more, i.e. the standard deviation of  $v$  rises. Now the firm hazard rate  $Q$  responds more to market tightness (as  $Q = \mu(\frac{v}{u})^{-\sigma}$ ) so the standard deviation of  $Q$  rises and so does the standard deviation of hiring ( $m = qv$ ). The standard deviation of  $u$  declines as the higher  $\sigma$  more than offsets the higher hiring volatility.

*(iii) Surplus sharing.*

We explore a different sharing of the surplus by increasing the replacement ratio from the benchmark of  $\tau = 0.25$  to an extremely high  $\tau = 0.90$ . This results in a new steady state, with the value of the wage bargaining parameter solved out to be  $\eta = 0.82$  rather than the 0.48 of the benchmark formulation. This change brings down steady state profits ( $\pi$ ) and asset values ( $\lambda$ ) as the firm gets a lower share of the surplus but the moments exhibit very little change.

*(iv) Persistence and volatility of the separation rate.*

The actual data exhibit high persistence of the separation rate  $\delta$ . This is estimated to be 0.92 in the  $\Pi$  matrix of the VAR (24). We set it counter-factually to be 0.10. The change in the moments induced by this counter-factual is substantial: the persistence of all variables falls from 0.98 to 0.55-0.76 and the volatility of all variables is reduced so that standard deviations are 2%-5% of their benchmark, actual values. The co-movement statistics weaken: most of the co-movement relations weaken moderately, and the negative co-movement relations of the labor share with the hiring rate and of unemployment with vacancies weaken substantially.

We reset the persistence to its actual value and reduce the variance to a tenth of its actual value. This has hardly any effect on the persistence and co-movement statistics but does have an equal sized effect on the volatility of all variables, i.e. it is reduced to about a tenth.

Hence the second moments of the separation rate play an important role. Why is the separation rate so important? This is so because it is a major determinant of match asset values. Note that this value (the RHS of (30)) is given by  $\Phi\pi$ . At the benchmark steady state  $\pi = 0.07$  and  $\Phi = 10.4$  so that the asset value of the match is  $\lambda = \Phi\pi = 0.73$ . Thus the discount factor – given by  $\Phi = \frac{G^X\beta}{[1-(1-\delta)G^X\beta]}$  – has a key role. To see the role of  $\delta$  in the latter, set  $G^X = \beta = 1$  and so  $\Phi = \frac{1}{\delta} = 11.710$ . We can therefore deduce that  $\delta$  plays the key role in  $\Phi$  and consequently in the match asset value  $\lambda$ .



*(v) Interest rate and productivity growth volatility*

Two other relevant counter-factuals are changing the volatility of the other driving variables:  $\beta$ , the discount factor based on the relevant interest rate, which we reduce to a tenth of its variance, and the rate of productivity growth,  $G^X$  which variance we increase by a factor of 10. These changes have no effect on the moments implied by the model.

### 7.3 Understanding Model Performance

Summing up the above discussion we see the following: equation (29) shows that the vacancy creation decision as an optimality condition equating the marginal costs of hiring with the asset value of the match. For the former the convexity of costs is key, hence the important role of  $\gamma$ . For asset values a key role is played by the separation rate  $\delta$ , which is the main determinant of the discount factor. Thus the performance of the model hinges to a large extent on the formulation of  $\gamma$  and on the stochastic properties of  $\delta$ .

Why do the other ingredients of the model have a much smaller role? The wage parameter  $\eta$  basically has a scale effect on per period profits and hence on the scale of asset values. It therefore affects the value of the variables at the steady state but does not affect the dynamics, as it does not affect the response of vacancy creation to asset values. The matching function parameter  $\sigma$ , that does have this ‘elasticity’ type of affects, has a possible range of variation that is much smaller than the variation in values of  $\gamma$ . For example, a reasonable change in  $\sigma$  would be 0.1 or 0.2 relative to the benchmark value, but a move from linear ( $\gamma = 0$ ) to cubic ( $\gamma = 2$ ) costs is a change of 2 in the value of  $\gamma$ . The interest rate and the rate of productivity growth in their turn play a much smaller role than the separation rate in discounting future values. While  $\delta$  has a sample mean of 8.6% and standard deviation of 0.8%, the rate of productivity growth ( $G^X - 1$ ) has a sample mean of 0.4% and standard deviation of 0.6% and the rate of interest ( $\frac{1}{\beta} - 1$ ) has a sample mean of 1.4% and standard deviation of 5.5%.

## 8 Conclusions

The paper has formulated a model of the U.S. aggregate labor market using log-linear approximation of the search and matching model. It has looked at alternative formulations of the data that would be consistent with the concepts of the model. Using a VAR of the actual data it injected driving shocks into the model. Comparing the resulting moments implied by the model to the moments in U.S. data, it has shown that it can account for much of observed labor market fluctuations. In particular, the model fits the data on persistence and volatility of most variables, on the negative relationship between vacancies and unemployment, and on the pro-cyclicality of the work hazard. For the same formulation of the data, it fails to capture the a-cyclicality of the labor share and its moderate positive co-variation with hiring, and it understates the latter's volatility; it is, however, able to come closer to capturing these features given a different configuration of the data.

The analysis has produced an empirically-grounded version of the search and matching model – complete with parameter values and data series – that can be used to study the U.S. labor market, including policy questions. It has shown that employing some widely-used parameter values and data series hinder the performance of the model.

The paper raises a number of issues for further research. A key one is the need to further explain the mechanisms in operation. In particular, one may ask what is the role of search and matching frictions in the dynamics; for example, how would different degrees of frictions lead to different outcomes This issue is the subject of current research. Other issues are: do the results carry over to other economies and, if so, what are the cross-country differences in parameter values and in the dynamics? For example it would be of interest to see whether such differences can explain the different U.S.– European unemployment experiences.

## Data: Sources and Definitions

*Data Sample.* All data are quarterly for the period 1970:I - 2003:IV, except for hires and separations which begin in 1976:I and end in 2003:III.

*Fundamental variables.*

variable	symbol	Source
Unemployment – official pool	$U^0$	CPS, BLS series id: LNS13000000
Workers out of l.f. who “want a job”	$WAJ$	CPS, BLS series id: LNU05026639
Unemployment – additional pool (see below)	$\tilde{U}$	CPS, BLS
Employment (total), household survey	$N$	CPS, BLS series id: LNS12000000
Vacancies – Index of Help Wanted ads	$V$	Conference Board <sup>1</sup>
Hires	$QV$	CPS-based data, Boston Fed computations <sup>2</sup>
Separations	$\delta N$	CPS-based data, Boston Fed computations <sup>2</sup>
Working age population <sup>4</sup>	$POP$	CPS, BLS series id: LNU00000000
Labor share <sup>5</sup>	$s = \frac{WN}{F}$	Table 1.12. NIPA, BEA <sup>3</sup>
Productivity	$\frac{F}{N}$	BLS
Cost of finance (equity and debt <sup>6</sup> )	$r$	Tables 1.1.5; 1.1.6 NIPA, BEA

### Notes:

BLS series are taken from <http://www.bls.gov/cps/home.htm>

1. Data were downloaded from Federal Reserve Bank of St. Louis:

<http://research.stlouisfed.org/fred2/series/HELPWANT/downloaddata?? ??>

2. See Bleakley et al (1999) for construction methodology. We thank Jeffrey Fuhrer and Elizabeth Walat for their work on this series.

3. <http://www.bea.gov/bea/dn/nipaweb/SelectTable.asp?Selected=N??>

4. Total civilian noninstitutional population 16 years and older.

5. Total compensation of employees divided by GDP.

6. This is a weighted average of the returns to debt,  $r_t^b$ , and equity,  $r_t^e$  constructed as follows:

$$r_t = \omega_t r_t^b + (1 - \omega_t) r_t^e,$$

with

$$\begin{aligned} r_t^b &= (1 - \tau_t) r_t^{CP} - \theta_t \\ r_t^e &= \frac{\widetilde{cf}_t}{\widetilde{s}_t} + \widetilde{s}_t - \theta_t \end{aligned}$$

where:

(i)  $\omega_t$  is the share of debt finance as reported in Fama and French (1999).

(ii) The definition of  $r_t^b$  reflects the fact that nominal interest payments on debt are tax deductible.  $r_t^{CP}$  is Moody's seasoned Aaa commercial paper rate. The commercial paper rate for the first month of each quarter represents the entire quarter. The tax rate is  $\tau$  as discussed above.

(iii)  $\theta$  denotes inflation and is measured by the GDP-deflator of NFCB GDP.

(iv) For equity return we use the CRSP Value Weighted NYSE, Nasdaq and Amex nominal returns deflated by the inflation rate  $\theta$ .

The additional pool of unemployment  $\tilde{U}$  has 5 permutations as follows:

permutation number	$\tilde{U}$
0	0
1	want a job
2	want a job $+0.15(POP - N - U^0)$
3	want a job $+0.3(POP - N - U^0)$
4	$POP - N$

*Transformations of variables.*

variable	symbol
Unemployment – total pool	$U = U^0 + \tilde{U}$
Labor force	$L = N + U$
Employment rate	$n = \frac{N}{N+U^0+\tilde{U}}$
Unemployment rate	$u = 1 - n$
Hiring rate	$m = \frac{QV}{N}$
Gross rate of labor force growth	$G^L = \frac{U_{t+1}+N_{t+1}}{U_t+N_t}$
Discount factor	$\beta = \frac{1}{1+r}$

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**Table 1**  
**Stochastic Properties of Alternative Measures**  
**of the Pool of Searching Workers**  
Quarterly Data

pool of workers	mean	std.	correlations				
			<i>0</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>
<i>0=official pool <math>U^o</math></i>	0.06	0.014	1				
<i>1=official+want a job</i>	0.10	0.020	0.976	1			
<i>2=official+want a job+0.15(<math>POP - N - U^o</math>)</i>	0.16	0.021	0.959	0.992	1		
<i>3=official+want a job+0.3(<math>POP - N - U^o</math>)</i>	0.21	0.021	0.937	0.977	0.996	1	
<i>4=<math>POP - N</math></i>	0.39	0.022	0.856	0.908	0.953	0.977	1

**Notes:**

1. *POP* is working age population; *N* is civilian employment.
2. See data appendix for sources.



**Table 2**  
**Stochastic Properties of Vacancies and Hires**  
Quarterly Data

		<i>individual samples</i>			<i>common sample</i>	
	<i>levels</i>	<i>sample coverage</i>	$\frac{std.}{mean}$	<i>auto-correlation</i>	$\frac{std.}{mean}$	<i>auto-correlation</i>
<i>V</i>	help wanted index	1951:I–2003:IV	0.34	0.98	0.27	0.56
<i>V</i>	JOLTS job openings	2001:I–2003:IV	0.24	0.61	0.24	0.61
<i>QV</i>	hires	1976:II–2003:IV	0.05	0.61	0.03	0.17
	<i>logs, HP-filtered</i>		<i>std.</i>	<i>auto-correlation</i>		
<i>V</i>	help wanted index	1951:I–2003:IV	0.14	0.89		
<i>V</i>	JOLTS job openings	2001:I–2003:IV	0.28	0.60		
<i>QV</i>	hires	1976:II–2003:IV	0.03	0.05		
	<i>ratio to unemployment</i>		$\frac{std.}{mean}$	<i>auto-correlation</i>	<i>std.</i>	<i>auto-correlation</i>
$\frac{V}{U}$	help wanted index	1951:I–2003:IV	0.44	0.95	0.47	0.62
$\frac{V}{U}$	JOLTS job openings	2001:I–2003:IV	0.35	0.66	0.35	0.66
$\frac{QV}{U}$	hires	1976:II–2003:IV	0.16	0.94	0.15	0.64

**Note:**  
See data appendix for sources.

**Table 3**  
**Worker Hazard Rates  $P$**   
**Quarterly 1994-1998**

specification	average		specification	Jones and Riddell (2000) range
0	0.80		unemployed	0.49-0.73
1	0.82			
2	0.47		marginally attached	0.27-0.49
3	0.39			
4	0.14		unattached	around 0.10, always below 0.14

**Notes:**

1. The five specifications – columns (0) to (4) correspond to the definitions in the notes to Table 1.

**Table 4**  
**Data Properties**

**a. Sample Means**

	(0)	(1)	(2)	(3)	(4)
unemployment rate $u$	0.06	0.10	0.16	0.22	0.39
hiring rate rate $m = qv$	0.046	0.079	0.074	0.070	0.054
labor force growth $G^L - 1$	0.0043	0.0042	0.0041	0.0040	0.0036
separation rate $\delta$	0.040	0.086			
productivity growth $G^X - 1$	0.0036				
discount factor $\beta$	0.993				
labor share $s$	0.58				

**b. Persistence (auto-correlation)**

	(0)	(1)	(2)	(3)	(4)
$\rho(\hat{u}_t, \hat{u}_{t-1})$	0.96	0.97	0.98	0.985	0.989
$\rho(\hat{m}_t, \hat{m}_{t-1})$	0.91	0.85	0.84	0.84	0.81
$\rho(\hat{s}_t, \hat{s}_{t-1})$	0.88				
$\rho(\hat{G}_t^X, \hat{G}_{t-1}^X)$	-0.014				
$\rho(\hat{\delta}_t, \hat{\delta}_{t-1})$	0.92	0.89			
$\rho(\hat{\beta}_t, \hat{\beta}_{t-1})$	0.02				

**c. Volatility (standard deviation)**

	(0)	(1)	(2)	(3)	(4)
$\hat{n}$	0.015	0.022	0.024	0.028	0.042
$\hat{u}$	0.22	0.19	0.12	0.10	0.07
$\hat{m}$	0.13	0.09	0.08	0.08	0.07
$\frac{\hat{m}}{u}$	0.12	0.14	0.07	0.05	0.04
$\hat{s}$	0.016				
$\hat{G}^X$	0.007				
$\hat{\delta}$	0.16	0.10			
$\hat{\beta}$	0.06				

**d. Co-movement (correlation)**

	(0)	(1)	(2)	(3)	(4)
$\rho(\widehat{u}_t, \widehat{m}_t)$	0.92	0.81	0.86	0.88	0.87
$\rho(\widehat{u}_t, \widehat{P}_t)$	-0.91	-0.93	-0.80	-0.59	0.24
$\rho(\widehat{n}_t, \widehat{s}_t)$	-0.06	-0.16	-0.30	-0.39	-0.52
$\rho(\widehat{m}_t, \widehat{s}_t)$	0.27	0.45	0.45	0.44	0.41
$\rho(\widehat{n}_t, \widehat{G}_t^X)$	-0.10	-0.03	0.001	0.02	0.06
$\rho(\widehat{n}_t, \widehat{\beta}_t)$	0.15	0.10	0.04	0.00	-0.08
$\rho(\widehat{n}_t, \widehat{\delta}_t)$	-0.93	-0.88	-0.91	-0.92	-0.91

**Notes:**

1. All data are quarterly for the period 1970:I - 2003:IV, except for hires and separations which begin in 1976:I and end in 2003:III.

2. The different columns differ with respect to the specification of the unemployment pool. We define  $U = U^0 + \widetilde{U}$  and  $L = N + U$  where  $U^0$  is the official pool and  $\widetilde{U}$  are the following additions: Column 1 adds workers out of the labor force indicating that they “want a job.” Columns 2 and 3 add to the latter column 15% and 30% of the remaining persons out of the labor force. Column 4 considers the entire working age population as the labor force. Column 0 is the official unemployment pool.

**Table 5**  
**Baseline Calibration Values**

**a. Parameters, Exogenous Variables and Steady State Values**

Parameter/Variable	symbol	(0)	(1)	(2)	(3)	(4)
Production	$1 - \alpha$	0.68				
Matching	$\sigma$	0.4				
Hiring	$\gamma$	2				
Hiring II	$\phi$	0.3				
productivity growth	$G^X - 1$	0.003536				
labor force growth	$G^L - 1$	0.004296	0.004199	0.004078	0.003974	0.003631
discount factor	$\beta$	0.9929				
separation rate	$\delta$	0.0404	0.0854			
Unemployment	$u$	0.063	0.104	0.164	0.217	0.395
Labor share	$s$	0.579				
Vacancy matching rate	$Q$	0.9				

**b. Implied Values**

		(0)	(1)	(2)	(3)	(4)
Matching	$\mu$	0.80	0.85	0.69	0.60	0.42
Hiring	$\Theta$	465	82	109	127	171
Wage bargaining parameter	$\xi$	0.37	0.41	0.45	0.49	0.56
Wage parameter	$\eta$	0.44	0.48	0.53	0.56	0.63
Vacancy rate	$v$	0.047	0.089	0.083	0.078	0.060
Market tightness	$\frac{v}{u}$	0.74	0.86	0.51	0.36	0.15
Workers' hazard	$P$	0.67	0.77	0.46	0.32	0.14
Profits	$\pi$	0.05	0.07	0.09	0.10	0.13
Asset Value	$\lambda$	1.02	0.73	0.96	1.12	1.49

**Notes:**

1. The implied values of  $v$ ,  $\mu$ ,  $\Theta$  and  $\eta$  are solved for using the steady state relationships as follows:

**Matching hazard:**

$$Q = \mu \left( \frac{v}{u} \right)^{-\sigma}$$

**Beveridge curve:**

$$(\delta + G^L - 1) = \mu \left( \frac{v}{u} \right)^{-\sigma} \left( \frac{v}{1 - u} \right)$$

**Vacancy creation:**

$$\Theta(\phi + (1 - \phi)Q) \left( \frac{\phi V + (1 - \phi)QV}{N} \right)^\gamma = Q \frac{G^X \beta}{[1 - (1 - \delta)G^X \beta]} \pi \quad (31)$$

**Wage solution**

$$s = \eta \left( (1 - \alpha) \left[ \frac{1}{1 - \alpha \xi} + \Theta \left( \frac{\phi V + (1 - \phi)QV}{N} \right)^{\gamma+1} \frac{\alpha + \gamma}{(1 + \gamma)(1 - \alpha)(1 - \xi(1 + \alpha + \gamma))} \right] + P\lambda \right)$$

The other variables are solved for using:

$$P = \frac{Qv}{u}$$

$$\pi = \frac{(1 - \eta) \left( (1 - \alpha) \left[ + \Theta \left( \frac{\phi V + (1 - \phi)QV}{N} \right)^{\gamma+1} \frac{1}{1 - \alpha \xi} \frac{\alpha + \gamma}{(1 + \gamma)(1 - \alpha)(1 - \xi(1 + \alpha + \gamma))} \right] \right)}{1 + \eta P \frac{G^X \beta}{[1 - (1 - \delta)G^X \beta]}}$$

$$\lambda = \frac{G^X \beta}{[1 - (1 - \delta)G^X \beta]} \pi$$

2. The five specifications – columns (0) to (4) correspond to the definitions in the notes to Table 4.

**Table 6**  
**Model Evaluation: Alternative Specifications**

**a. Model vs. Data**

		(0)	(1)	(2)	(3)	(4)
$\rho(\hat{u}_t, \hat{u}_{t-1})$	Data	0.962	0.971	0.980	0.985	0.989
	Model	0.989	0.983	0.990	0.993	0.996
$\rho(\hat{m}_t, \hat{m}_{t-1})$	Data	0.907	0.853	0.844	0.836	0.805
	Model	0.991	0.986	0.991	0.993	0.992
$\rho(\hat{s}_t, \hat{s}_{t-1})$	Data	0.884	0.884	0.884	0.884	0.884
	Model	0.982	0.976	0.988	0.992	0.996
$std(\hat{n}_t)$	Data	0.015	0.022	0.024	0.028	0.042
	Model	0.020	0.021	0.032	0.042	0.072
$std(\hat{u}_t)$	Data	0.218	0.188	0.124	0.100	0.069
	Model	0.298	0.182	0.165	0.151	0.110
$std(\hat{m}_t)$	Data	0.125	0.085	0.083	0.080	0.073
	Model	0.097	0.051	0.035	0.022	0.018
$std(\hat{s}_t)$	Data	0.016	0.016	0.016	0.016	0.016
	Model	0.089	0.056	0.052	0.049	0.038
$\rho(\hat{u}_t, \hat{m}_t)$	Data	0.920	0.810	0.860	0.880	0.870
	Model	0.997	0.997	0.998	0.999	-0.982
$\rho(\hat{u}_t, \hat{P}_t)$	Data	-0.909	-0.933	-0.802	-0.591	0.242
	Model	-0.999	-1.000	-1.000	-1.000	-1.000
$\rho(\hat{n}_t, \hat{s}_t)$	Data	-0.060	-0.160	-0.300	-0.390	-0.520
	Model	0.995	0.997	0.999	1.000	1.000
$\rho(\hat{m}_t, \hat{s}_t)$	Data	0.272	0.451	0.445	0.440	0.414
	Model	-0.985	-0.989	-0.996	-0.999	0.984

**b. Model predictions**

	(0)	(1)	(2)	(3)	(4)
$\rho(\hat{v}_t, \hat{v}_{t-1})$	0.952	0.961	0.986	0.992	0.996
$\rho(\hat{q}_t, \hat{q}_{t-1})$	0.988	0.981	0.989	0.992	0.996
$std(\hat{v}_t)$	0.040	0.037	0.052	0.064	0.103
$std(\hat{q}_t)$	0.135	0.088	0.087	0.086	0.085
$\rho(\hat{u}_t, \hat{v}_t)$	-0.956	-0.985	-0.998	-1.000	-0.999
$\rho(\hat{v}_t, \hat{q}_t)$	-0.966	-0.990	-0.999	-1.000	-1.000
$\rho(\hat{v}_t, \hat{s}_t)$	0.981	0.995	0.999	1.000	0.999

**Notes:**

The different specifications correspond to the definitions in Table 4.

**Table 7**  
**Model Evaluation: Counterfactuals**

**a. Model vs. Data**

	<b>Data</b>	<b>b. mark</b>	$\gamma = 0$	$\tau = 0.9$	$\sigma = 0.8$	$\delta \text{ AR}=0.1^1$	$0.1\sigma_\delta$	$0.1\sigma_\beta$	$10\sigma_{G^x}$
$\rho(\hat{u}_t, \hat{u}_{t-1})$	0.971	0.983	0.881	0.982	0.960	0.763	0.983	0.983	0.983
$\rho(\hat{m}_t, \hat{m}_{t-1})$	0.853	0.986	0.467	0.985	0.959	0.696	0.986	0.986	0.986
$\rho(\hat{s}_t, \hat{s}_{t-1})$	0.884	0.976	0.592	0.984	0.962	0.553	0.973	0.976	0.976
$std(\hat{n}_t)$	0.022	0.021	0.015	0.021	0.013	0.001	0.007	0.021	0.021
$std(\hat{u}_t)$	0.188	0.183	0.126	0.180	0.107	0.004	0.058	0.183	0.183
$std(\hat{m}_t)$	0.085	0.052	0.090	0.052	0.067	0.001	0.016	0.052	0.052
$std(\hat{s}_t)$	0.016	0.056	0.068	0.058	0.032	0.002	0.018	0.056	0.056
$\rho(\hat{u}_t, \hat{m}_t)$	0.810	0.997	0.888	0.998	1.00	0.886	0.996	0.997	0.997
$\rho(\hat{u}_t, \hat{P}_t)$	-0.933	-1.00	-0.743	-1.00	-1.00	-0.984	-0.999	-1.00	-1.00
$\rho(\hat{n}_t, \hat{s}_t)$	-0.160	0.997	0.936	0.999	1.00	0.917	0.996	0.997	0.997
$\rho(\hat{m}_t, \hat{s}_t)$	0.451	-0.989	-0.993	-1.00	-1.00	-0.627	-0.985	-0.989	-0.989
$\rho(\hat{m}_t, \hat{\delta}_t)$	0.908	0.862	0.599	0.858	0.881	0.304	0.865	0.862	0.862

**b. Model predictions**

	<b>benchmark</b>	$\gamma = 0$	$\tau = 0.9$	$\sigma = 0.8$	$\delta \text{ AR}=0.1^1$	$0.1\sigma_\delta$	$0.1\sigma_\beta$	$10\sigma_{G^x}$
$\rho(\hat{v}_t, \hat{v}_{t-1})$	0.961	0.001	0.963	0.961	0.287	0.950	0.961	0.961
$\rho(\hat{q}_t, \hat{q}_{t-1})$	0.981	0.707	0.981	0.961	0.697	0.980	0.981	0.981
$std(\hat{v}_t)$	0.037	0.085	0.033	0.096	0.002	0.012	0.037	0.037
$std(\hat{q}_t)$	0.088	0.041	0.085	0.163	0.002	0.028	0.088	0.088
$\rho(\hat{u}_t, \hat{v}_t)$	-0.985	0.580	-0.988	-1.00	-0.733	-0.979	-0.985	-0.985
$\rho(\hat{v}_t, \hat{q}_t)$	-0.990	-0.114	-0.992	-1.00	-0.842	-0.986	-0.990	-0.989
$\rho(\hat{v}_t, \hat{s}_t)$	0.995	-0.830	0.982	1.00	0.943	0.993	0.995	0.995

**Notes:**

1. The auto correlation parameter of  $\delta$  contained in  $\Pi$  is reduced to 0.1
2. The change in the volatility statistics is a multiplication of  $\sigma_\delta$  and  $\sigma_\beta$  by 0.1 and of  $\sigma_{G^x}$  by 10.