Optimal Stalling While Bargaining

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Abstract

Why do people stall while bargaining? We propose that the reason isn't fully explained by claiming people are engaged in a complicated signalling equilibrium with unguessable valuations. Rather we propose that staling is explained by the fact that agents are worried about a rival coming to bargain for the pie and creating a bidding war - this encourages speedy agreement. But when the beliefs as to how likely a rival is to come along differ, then agents may choose to stall, run the risk of a rival coming, but beliefs update and the possibility is seen as less likely the more time elapses. Thus, for example, house buyers trade off the risk of a bidding war against getting sellers to realise their house isn't so desirable buyers are falling over themselves for it.

Keywords Bargaining; Delay; Stalling; Learning

JEL C78; C73; D83

1 Introduction

In individual negotiations agents often consider stalling and not agreeing to a deal currently on the table. Why? Three main types of explanation have been put forward in the bargaining literature: uncertainty over the size of the pie leading to bargaining by signalling; extremal equilibria leading to punishment for early agreement and finally agents committing themselves to a stand that they find costly to reverse.

We propose that these do not however provide convincing explanations of the bargaining process in situations ranging from those as simple as house buying to others as involved as one off business to business agreements. We will show that stalling within bargaining can be explained by the parties having differing views, which they update during negotiations, on how likely a rival to one of the agents is to come along. Therefore stalling can serve two purposes: it allows time for a rival to come and a bidding war to ensue, while on the other hand no rival coming forces the other agent to readjust their beliefs and thus compromise more.

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We are therefore suggesting that a large part of the desire to reach speedy conclusions in bargaining situations such as house buying is because the buyer (for example) does not want a rival buyer to come along and start a bidding war. It is not the discount rate which drives the urge for agreement - the actual date of moving into the house will often be both uncertain and lie considerably further into the future. This is, as far as I am aware, a new explanation, with predictive power, for the stalling process.

We first consider why a new theory is needed.

Signalling and Secrets Though it is reasonable that agents may have private views on the value of a good, this alone leads to implausible predictions as far as bargaining and stalling are concerned. In the signalling theory of bargaining (Cramton (1984), Fudenberg and Tirole (1983), Watson (1998)) the agents make offer and counter offers to signal their types. The agent who is more patient or with a lower valuation is happy to postpone agreement, not withstanding the discount rate, as to agree early on would be too costly. The problem with these explanations are (i) many outcomes can be sustained depending on the postulated beliefs off some equilibrium signalling path - thus predictions are weak where possible at all; (ii) the equilibria assume that agents carefully formulate views on how agent $A's$ beliefs will be changed given $B's$ offer and given $B's$ response to $A's$ offer in response to $B's$ offer, and so on. This hyper-rationality is questionable; (iii) the issue of how a signalling equilibrium is arrived at in one shot interactions is ignored. For at least these three reasons, uncertainty alone does not seem to capture all there is to understand about bargaining impasses.[1]

Extremal Equilibria A second methodology for explaining bargaining delay has been depicted by Fernandez and Glazer (1991) in the context of union-firm negotiations. Here there is more than one possible agreement and thus a Nash equilibrium can be constructed with any given delay: if either party tries to agree earlier than the exogenously agreed upon time T, then the other party will play the strategy which gives them their maximal payoff. The issue here is how the equilibrium was constructed: in particular, how did the parties agree on how long the impasse would be. Repeated union-firm bargaining may provide an explanation, but this appears to apply less in house buying and one off business to business interactions.

Posturing Finally Crawford (1982) proposed that bargainers can find it in their interests to commit to positions in advance which are difficult to withdraw from. These ex ante positions prevent immediate ex post agreement. This appears a convincing rationale for explaining the bargaining process between representatives of a group - such as union firm interactions or the bargaining between Heads of State. However, in business to business interactions where bargains are secret and the principal (or decision maker) is the one conducting the bargaining, then this is less applicable.

[1] A much fuller account of bargaining with uncertainty is provided in Ausubel et al. (2002).

Thus the existing models of stalling within bargaining have little predictive power and, in parts, questionable requirements or implications. We offer a new model of bargaining, stalling and learning.

2 The Model

There are two parties bargaining over the division of a pie of size π. These players are labelled the buyer (B) and the seller (S). There exists a possibility that a rival for the buyer exists. It is assumed that there is no chance that a rival seller may exist. This is therefore a seller's market.

If competition for the buyer's spot does exist, then a competitor for the buyer arrives with a Poisson process at a rate of λ. Thus the probability of a rival buyer arriving in time dt is λdt and is independent of the time since bargaining commenced. If a rival buyer does arrive then the seller extracts all of the rents - the buyers compete against each other to make more and more generous offers.

Alternatively, we may be in a state of the world in which a rival buyer for this pie does not exist. The buyer and the seller disagree on how likely one state of the world is to the other. Thus, over time, the buyer's and the seller's beliefs as to the likelihood of a rival buyer coming (with Poisson process λ) change. We denote the buyer's belief that a rival could be coming as p_B and p_S is the corresponding seller's belief of this case. Thus the buyer believes that a rival will arrive with probability $p_B \lambda dt$ over the next dt of time, while the seller believes this scenario to occur with probability $p_S \lambda dt$.

The parties have a common discount rate of r. Thus an agreement reached at time t divides a total pie of size $e^{-rt}\pi$ in ex ante currency units.

2.1 Rate of Change of beliefs over time

We first consider how beliefs would be updated as time passes in the event that no rival buyer has arrived. Note that outside options come along with Poisson process $\lambda > 0$ or they don't come along at all. Suppose that at time t an agent viewed the possibility that a rival buyer exists as p_t. Thus, for this agent, the probability that a rival buyer comes along in the next time dt is $p_t \lambda dt$, and the probability that they don't is thus $1 - p_t \lambda dt$. Therefore, through Bayesian updating, we have:

$$\begin{aligned} \Pr\left(\text{Competition for } B \text{ exists} \,|\text{No one comes in } dt\right) &= \frac{\Pr\left(\text{Competition for } B \text{ exists and yet no one comes}\right)}{\Pr\left(\text{No one comes in } dt\right)} \\ p_t + dp_t &= \frac{\left(1-\lambda dt\right) p_t}{1 - p_t + \left(1-\lambda dt\right) p_t} \\ &\Rightarrow dp_t = -\lambda p_t \left(1-p_t\right) dt \qquad (1) \end{aligned}$$

Thus as time goes on, and no rival buyer has come forward, the players both become a little more pessimistic as to how likely a rival buyer is ever going to be found. That is, $p_t + dp_t \leq p_t$ with equality if the agent is certain of the state of the world $(p_t = 0 \text{ or } 1)$. Hence, as time goes on, B becomes more confident it can extract a larger slice of the pie and S less confident that it will be able to extract a larger slice of the pie.

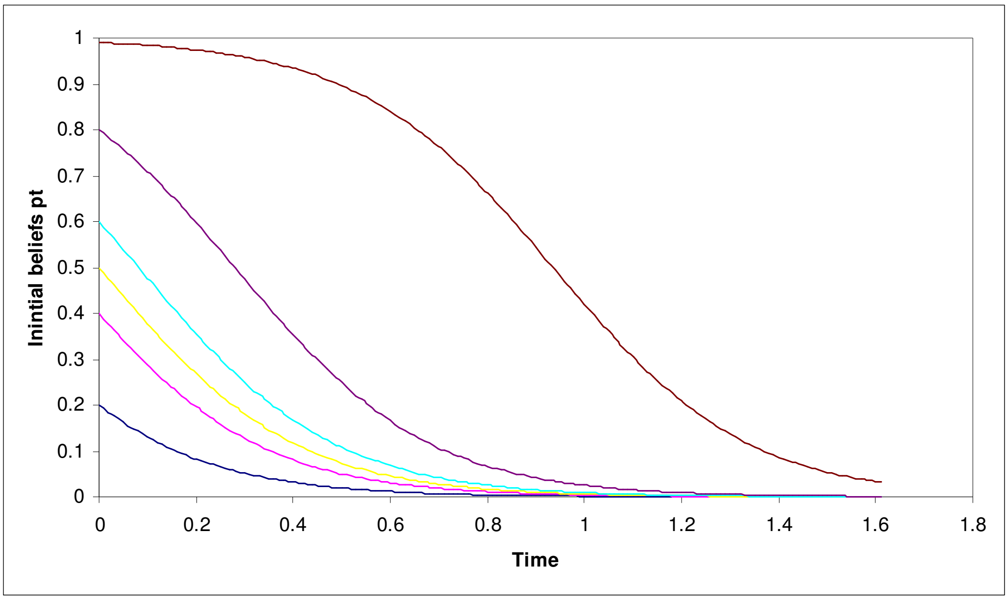

Figure 1: The change of beliefs as time (measured as a factor of λ) progresses

It is useful in developing intuitions to solve the differential equation given in (1). It can be checked that the general solution is given by

$$p\left(t\right)=\frac{1}{1+\left(\frac{1}{p(0)}-1\right)e^{\lambda t}}$$

Beliefs therefore take a downwards trajectory at which they fall, slowly at first if p_t is near 1, then rapidly and then tend asymptotically to 1. This is shown in Figure 1. The rate of change of beliefs thus depends on the initial level of beliefs. Specifically, for some intermediate ranges of beliefs, if one agent, say S, places a greater possibility of a rival buyer coming than the other (B), then $S's$ beliefs decline much more rapidly than $B's$ and so the two beliefs converge. It is this feature which will provide the motivation for the buyer to stall for some ranges of beliefs and not others. The same is true for the seller. Making this insight rigorous is the subject of this paper.

3 Method of Proof

This section describes the steps, which will be made rigorous, to determine the optimal stalling region in belief space. The parties can either agree immediately using a continuous form of Rubinstein bargaining or stall. If agreeing immediately, their payoffs would be given by $x_S\left(p_S, p_B\right)$ and $x_B\left(p_S, p_B\right)$ found in the standard way: the buyer must offer the seller at least what she can get by making a counter offer time dt later. Note that $x_S + x_B = \pi$.

Either the buyer or the seller can threaten to stall. Optimal such behaviour would guarantee payoffs S and B to the buyer and seller as a function of their beliefs. With any positive discount rate, stalling is not optimal at the origin as no learning takes place.

Stalling is not optimal if $p_S < p_B$ as seller is happy that buyer believes the chance of a rival is greater than she does - stalling would only erode this belief. In addition the buyer is happy that the seller is less sure of a rival than she is - stalling would only cause the seller to have a chance for the rival to come. We will thus see that stalling is only possible if $p_S \geq p_B$.

If $p_S = 1$ and $r > 0$ then stalling is not optimal for either the buyer or the seller as $x_S > S$ and $x_B > B$: stalling doesn't alter the seller's beliefs and so the effect of the discount rate dominates.

Stalling is optimal for some intermediate beliefs. Thus there must be at least two boundary points, along any belief line, between stalling and not.

The buyer and seller will seek to maximise their value along any path of beliefs from $(p_S(0), p_B(0))$ down towards the origin as time progresses. Subject to the restriction that there are at least two contact points - otherwise we would be implying stalling at the origin which is a contradiction.

The shape of x_S and S and similarly x_B and B imply that there is smooth pasting at the smaller (lower beliefs) contact point. This is because as p_S becomes small, the stalling value blows up and therefore can be increased until it is tangent here. This will suggest that either S or B is able to threaten to stall ($B > x_B$ or $S > x_S$) for longer.

The agent with the smaller stall zone will have to compromise. Hence B would have to smooth paste on to $\pi - S$ if the buyer must compromise, otherwise S will have to smooth paste on to $\pi - B$ if S must compromise first.

This smooth pasting requirement is because the compromiser is seeking to maximise their value subject to being forced to agree to an enlarged split of the pie for the rival.

Thus we have S and B. The actual stalling region is given implicitly by the curve $S + B = \pi$. If $S + B > \pi$ then agreement is impossible as the outside option of stalling is too large. Otherwise agreement would be reached by standard bargaining with outside options arguments (See Muthoo (1999, §5.2).

What follows is the formalisation of this methodology.

4 The Active Negotiation Phase

In this section we search for what the agreed split of the pie would be if the parties both preferred to reach an immediate agreement. The payoffs here would be given by the pie division functions $x_S(p_S, p_B)$ and $x_B(p_S, p_B)$. These functions are such that neither player would prefer to wait a moment and make a counteroffer. Thus the payoffs are essentially Rubinstein alternating offer bargaining functions in continuous time.[2]

The buyer will have to offer the seller as much as she thinks she can gain by waiting and making a

[2] For a full discussion of Rubsintein bargaining and its discrete time extensions, see Muthoo (1999).

counteroffer an instant later:

$$\pi - x_B\left(p_S, p_B\right) = p_S \lambda \pi dt + \left(1 - rdt\right)\left(1 - p_S \lambda dt\right) x_S\left(p_S + dp_S, p_B + dp_B\right)$$

Beliefs evolve according to Bayesian updating so that $dp = -\lambda p\left(1-p\right)dt$ and so, expanding $x_S\left(p_S + dp_S, p_B + dp_B\right)$. Using a Taylor series we have

$$\pi - x_B = p_S \lambda \pi dt + \left(1 - \left(r + p_S \lambda\right) dt\right)\left[x_S - \frac{\partial x_S}{\partial p_S}\lambda p_S\left(1 - p_S\right)dt - \frac{\partial x_S}{\partial p_B}\lambda p_B\left(1 - p_B\right)dt\right] + O\left(dt^2\right) \quad (2)$$

The seller will also have to offer the buyer as much as she thinks she can get by waiting. Hence

$$\pi - x_S = \left(1 - rdt\right)\left(1 - p_B \lambda dt\right)\left[x_B - \frac{\partial x_B}{\partial p_S}\lambda p_S\left(1 - p_S\right)dt - \frac{\partial x_B}{\partial p_B}\lambda p_B\left(1 - p_B\right)dt\right] \quad (3)$$

These equations need to be solved. From (3) we have

$$x_S = \pi - x_B + \left(r + p_B \lambda\right) x_B dt + \frac{\partial x_B}{\partial p_S}\lambda p_S\left(1 - p_S\right)dt + \frac{\partial x_B}{\partial p_B}\lambda p_B\left(1 - p_B\right)dt$$

And hence $\frac{\partial x_S}{\partial p_S} = -\frac{\partial x_B}{\partial p_S} + o\left(dt\right)$, $\frac{\partial x_S}{\partial p_B} = -\frac{\partial x_B}{\partial p_B} + o\left(dt\right)$. Thus in (2) we have

$$\begin{aligned} \pi - x_B &= \pi - x_B + \left(r + p_B \lambda\right) x_B dt + \frac{\partial x_B}{\partial p_S}\lambda p_S\left(1 - p_S\right)dt + \frac{\partial x_B}{\partial p_B}\lambda p_B\left(1 - p_B\right)dt \\ &\quad + dt\left\{p_S \lambda \pi - \left(r + p_S \lambda\right)\left(\pi - x_B\right) + \frac{\partial x_B}{\partial p_S}\lambda p_S\left(1 - p_S\right) + \frac{\partial x_B}{\partial p_B}\lambda p_B\left(1 - p_B\right)\right\} \end{aligned}$$

Dividing through by dt gives

$$0 = x_B\left[2r + \lambda\left(p_B + p_S\right)\right] - r\pi + 2\frac{\partial x_B}{\partial p_S}\lambda p_S\left(1 - p_S\right) + 2\frac{\partial x_B}{\partial p_B}\lambda p_B\left(1 - p_B\right)$$

Hence we have the differential equation

$$\frac{\partial x_B}{\partial p_S} p_S\left(1 - p_S\right) + \frac{\partial x_B}{\partial p_B} p_B\left(1 - p_B\right) + x_B\left[\frac{r}{\lambda} + \frac{1}{2}\left(p_B + p_S\right)\right] = \frac{r}{\lambda}\frac{\pi}{2} \quad (4)$$

The solution of the homogeneous equation is

$$C\left(\frac{r}{\lambda}, \pi\right)\sqrt{\left(1 - p_B\right)\left(1 - p_S\right)\Omega\left(p_B\right)^{\frac{r}{\lambda}}\Omega\left(p_S\right)^{\frac{r}{\lambda}}} \quad (5)$$

where $\Omega\left(p\right) = \frac{1-p}{p}$, the odds ratio and we have stressed that the constant of integration is independent of beliefs, but will depend on the discount rate, Poisson process and the size of the pie.

If $r = 0$, then (5) gives the full solution as (4) is a homogeneous differential equation. We note that if $p_S = p_B = 0$, then both players are certain that no rival buyer exists. In this case standard alternating offer Rubinstein bargaining requires that the parties agree on an equal split of the pie. Hence $C|_{r=0} = \frac{\pi}{2}$ and so we have the agreement functions

$$x_B\left(p_S, p_B\right) = \frac{\pi}{2}\sqrt{\left(1 - p_B\right)\left(1 - p_S\right)} \quad x_S = \pi - \frac{\pi}{2}\sqrt{\left(1 - p_B\right)\left(1 - p_S\right)} \quad (6)$$

However, if $r > 0$, then the requirement that the payoff to x_B must lie between 0 and $\frac{\pi}{2}$ forces $C = 0$. This is because (5) is unbounded as either belief becomes very small. The solution to (4) in this case of positive discounting lies in the particular integral. This partial differential equation can be solved in terms of power series. However, this is not particularly helpful here and so we delay this discussion. The partial differential equation (4) will prove invaluable in understanding the economics of bargaining whilst learning. To this end note that (3) implies that $x_S + x_B = \pi$ and so the equivalent differential equation for x_S, inserting this into (4) is

$$\frac{\partial x_S}{\partial p_S} p_S (1 - p_S) + \frac{\partial x_S}{\partial p_B} p_B (1 - p_B) + x_S \left[\frac{r}{\lambda} + \frac{1}{2} (p_B + p_S) \right] = \frac{\pi}{2} \left[\frac{r}{\lambda} + p_B + p_S \right] \tag{7}$$

5 Is Stalling Ever Optimal?

We have not as yet shown that it is ever optimal for either the buyer or the seller to stall. We consider the simpler case of no discounting $(r = 0)$. This section will show that stalling is optimal for the buyer and the seller whenever the seller is more optimistic than the buyer as to the possibility of a rival buyer existing. That is stalling is preferred if $p_S > p_B$.

5.1 Can B ever benefit from delay?

The above expression for x_B (equation (6)) comes from equating the best that B can make now as against waiting a small amount of time. Suppose we start with beliefs $p_S(0)$ and $p_B(0)$. We have already established that beliefs evolve according to $p(t) = \frac{1}{1 + \left(\frac{1}{p(0)} - 1\right) e^{\lambda t}}$. Thus seen at time $t = 0$, the probability of the buyer getting to time t is $e^{-p_B(0)\lambda t}$. Thus seen as a static problem, the buyer would choose $t \geq 0$ to maximise[3]

$$\frac{\pi}{2} e^{-\lambda t p_B(0)} \sqrt{(1 - p_S(t))(1 - p_B(t))} \propto e^{\lambda t (1 - p_B(0))} \frac{1}{\left[\left(1 + \left(\frac{1}{p_S(0)} - 1\right) e^{\lambda t}\right) \left(1 + \left(\frac{1}{p_B(0)} - 1\right) e^{\lambda t}\right) \right]^{\frac{1}{2}}}$$

Let $b = p_B(0)$ and $s = p_S(0)$ then differentiating this with respect to t and setting $t = 0$ we have

$$\frac{\frac{\lambda(1-b)}{\sqrt{sb}} - \frac{1}{2}\sqrt{sb} \cdot \left[\lambda \left(\frac{1}{s} - 1\right) \frac{1}{b} + \lambda \left(\frac{1}{b} - 1\right) \frac{1}{s}\right]}{\frac{1}{sb}} =_{\text{sign}} \sqrt{\frac{s}{b}} - \sqrt{\frac{b}{s}}$$

Thus we have delay from the buyer if $s > b$. That is, it is always worth the buyer stalling a little if the seller is more optimistic than the buyer as to the likelihood of others coming. Note that this implies that agreement can't be reached here - either the rival buyer comes and the seller gets all of the rents, or the rival buyer doesn't come and agreement can only be struck as a 50-50 split after an 'infinite' delay (or such time as the seller acknowledges there is no chance of an outside option).

[3] Note that this under-estimates the buyer's future payoff as the buyer becomes more confident of reaching time t without a rival as each instant of time passes. Therefore the value of stalling would be greater than is predicted here.

5.2 Can S ever benefit from delay?

If S actively negotiates at beliefs (p_S, p_B) she receives a payoff of $\pi - \frac{\pi}{2}\sqrt{(1-p_S)(1-p_B)}$. Evaluated at time 0, the ex ante expected payoff of waiting until time t is given by

$$e^{-\lambda t p_S(0)}\left\{\pi - \frac{\pi}{2}\sqrt{(1-p_S(t))(1-p_B(t))}\right\}+\left(1-e^{-\lambda t p_S(0)}\right)\pi = \pi - e^{-\lambda t p_S(0)}\frac{\pi}{2}\sqrt{(1-p_S(t))(1-p_B(t))}$$

Thus the rate of change of this expression with respect to t evaluated at $t = 0$ must have sign of $-\left[\sqrt{\frac{b}{s}} - \sqrt{\frac{s}{b}}\right]$ which is equal to $\sqrt{\frac{s}{b}} - \sqrt{\frac{b}{s}}$. Thus delay is preferred by the seller again when $s > b$.

By continuity of the value of stalling, there will be some beliefs and some non zero discount rates at which the agents will prefer to stall rather than agree on a deal immediately. We thus turn to the question of the optimal stalling period.

6 The Buyer's Greatest Stalling Value

Section 5 established that stalling can be optimal for some combinations of beliefs and discount rates. This section will explore exactly when stalling is preferred by the buyer to immediate agreement. During any periods when it is optimal for the buyer to stall, the buyer will be worth $B(p_S(t), p_B(t))$ at time t and would optimally wait and get $E_t\{B(p_B(t+dt), p_s(t+dt))\}$ where the expectation denotes the uncertainty around whether a rival buyer might come. Using Taylor we have

$$B(p_B(t+dt), p_s(t+dt)) = B(p_B(t), p_S(t)) - \lambda p_B(1-p_B)\frac{\partial B}{\partial p_B}dt - \lambda p_S(1-p_S)\frac{\partial B}{\partial p_S}dt + o\left(dt^2\right)$$

If stalling is optimal we thus have the Bellman equation:

$$\begin{aligned} B &= (1-p_B\lambda dt)(1-rdt)\left[B - \lambda p_S(1-p_S)\frac{\partial B}{\partial p_S}dt - \lambda p_B(1-p_B)\frac{\partial B}{\partial p_B}dt\right] \\ 0 &= B(r+\lambda p_B) + \lambda p_S(1-p_S)\frac{\partial B}{\partial p_S} + \lambda p_B(1-p_B)\frac{\partial B}{\partial p_B} \end{aligned} \tag{8}$$

The integrated function of this expression will have two constants of integration. Consider the general solution $Cp_S^a(1-p_S)^b p_B^c(1-p_B)^d$. Substituting this in and dividing by B gives

$$0 = \frac{r}{\lambda} + p_B + a(1-p_S) - bp_S + c(1-p_B) - dp_B$$

Equating the coefficients to zero implies that we have

$$B(p_S, p_B) = E(1-p_B)\,\Omega(p_B)^{\frac{r}{\lambda}}\left[\frac{p_S}{1-p_S}\frac{1-p_B}{p_B}\right]^a \quad \text{for some constants } a, E$$

We know that stalling is optimal for $p_S > p_B$ when $r = 0$. Thus for some small $r > 0$ as p_B tends to zero we must still have stalling optimal for some p_S by continuity of the payoff functions. Thus we require $a \leq -\frac{r}{\lambda}$ to avoid B vanishing. In addition the value of the buyer must be bounded between $\left[0, \frac{\pi}{2}\right]$ and not equal to zero when $p_B = 0$. Thus we require $a = -\frac{r}{\lambda}$. Therefore we have:

$$B(p_S, p_B) = E(1-p_B)\,\Omega(p_S)^{\frac{r}{\lambda}} \quad \text{for some constant } E \tag{9}$$

Note that the value of stalling must lie in the range $\left(0, \frac{\pi}{2}\right)$ for intermediate beliefs and so we must have $E > 0$. If $r = 0$ then we've seen that when $p_S \geq p_B$ the buyer will only agree to a 50-50 split along $p_B = 0$ and so we have $\left.E\right|_{r=0} = \frac{\pi}{2}$.

Lemma 1 *Stalling is not optimal for the buyer if* $p_S = 1$

Proof. Note that $B(1, p_B) = 0$ and yet the immediate payoff $x_B(1, p_B)$ must be larger than 0 if the buyer believes there is a possibility that no rival is coming.[4] ■

Stalling is clearly not optimal at the origin as there is no learning to take place. Thus, if stalling is to be optimal for the buyer then there will be at least two boundary points marking out some intermediate stalling region. Next note that as p_S tends to 0, $B(p_S, p_B)$ tends to infinity. The buyer seeks to maximise her value subject to having two stalling boundaries. Thus we require the lower of these boundary points to be smoothly pasted between x_B and B, otherwise E could be increased further.[5] Specifically, the agent must decide whether to wait dt before concluding her stalling or indeed whether she should have concluded her stall dt time before. That is, we must have $\frac{dB}{dt} = \frac{dx_B}{dt}$ (unless we are at a corner solution). However, the development of beliefs over time established in (1) above thus implies that we require

$$\frac{\partial x_B}{\partial p_S} p_S (1 - p_S) + \frac{\partial x_B}{\partial p_B} p_B (1 - p_B) = \frac{\partial B}{\partial p_S} p_S (1 - p_S) + \frac{\partial B}{\partial p_B} p_B (1 - p_B)$$

We thus seek the largest E such that $B(p_S, p_B)$ satisfies (4). This will then provide a curve $p_B(p_S)$ along which $x_B = B$ at some points. That is

$$\frac{\partial B}{\partial p_S} p_S (1 - p_S) + \frac{\partial B}{\partial p_B} p_B (1 - p_B) + B\left[\frac{r}{\lambda} + \frac{1}{2}(p_B + p_S)\right] = \frac{r}{\lambda}\frac{\pi}{2}$$

Basic algebra confirms that this expression collapses to

$$E\left(\frac{r}{\lambda}, \pi\right)(1 - p_B)(p_S - p_B)\,\Omega(p_S)^{\frac{r}{\lambda}} = \pi\frac{r}{\lambda} \tag{10}$$

where we have been clear that the constant of integration, E, depends on the parameter π, $\frac{r}{\lambda}$. Given E, the above gives the relationship, for small beliefs[6] between p_B and p_S which is the maximal range at which the buyer can threaten to stall.

6.1 The greatest value for E

Though the immediate agreement function $x_B(p_S, p_B)$ cannot be found generally. It can be found analytically along the line $p_B = 0$. We thus solve explicitly the case in which the buyer is certain there is no rival, but the seller believes there might be a rival buyer.

[4] One can show the same result by explicitly solving for $x_B(p_S, p_B)$ along the line $p_B = 0$ and then solving along the line $p_S = 1$.

[5] This is standard in the one dimensional context in the context of learning about Poisson processes - see Keller et al (2004, Proposition 3.1).

[6] Specificaly the smooth pasting rule applies for belief paths coming out of the stalling region.

In this case the value of stalling is given by (9) as

$$B(p_S,0) = E\Omega(p_S)^{\frac{r}{\lambda}} \qquad E>0,\ E|_{r=0} = \frac{\pi}{2} \tag{11}$$

From (4) we have that the solution $x_B(p_S,0)$ satisfies

$$\frac{\partial x_B}{\partial p_S}p_S(1-p_S) + x_B\left[\frac{r}{\lambda}+\frac{p_S}{2}\right] = \frac{r}{\lambda}\frac{\pi}{2} \tag{12}$$

Differentiating with respect to p_S gives the differential equation

$$\frac{\partial^2 x_B}{\partial p_S^2}p_S(1-p_S) + \frac{\partial x_B}{\partial p_S}\left[1+\frac{r}{\lambda}-\frac{3}{2}p_S\right] + \frac{1}{2}x_B = 0 \tag{13}$$

This is Gauss's hypergeometric differential equation and the solution has one base given by the hypergeometric function. Formally one solution to the differential equation $x(1-x)y'' + [c-(a+b+1)x]y' - aby = 0$ is given by $F(a,b,c;x)$ where a, b, and c are constants (see Bailey (1935) or Kreyszig (1999, §4.4). The hypergeometric function can be written out using the Pochhammer symbols (the rising factorial) where

$$\begin{aligned}(d)_n \quad &: = d(d+1)(d+2)\cdots(d+n-1)\\ \Rightarrow \quad &(1)_n = n!\end{aligned}$$

Thus we have

$$F(a,b,c;x) = \sum_{n=0}^{\infty}\frac{(a)_n(b)_n}{(c)_n}\frac{x^n}{n!}$$

The second basis function solving the general hypergeometric function is given by a second hypergeometric series multiplied by x^{1-c}. This basis will not have economic content here as the x^{1-c} will be unbounded at the origin.

The solution to (13) is thus given a multiple of the hypergeometric function $F(a,b,c;p_S)$ where $c = 1+\frac{r}{\lambda}$ and $ab = -\frac{1}{2}$, $a+b+1 = \frac{3}{2}$ so that $a = -\frac{1}{2}$, $b = 1$. The solution to (12) must satisfy $x_B(0,0) = \frac{\pi}{2}$ and so the general solution to (12) is

$$\begin{aligned} x_B(p_S,0) = \frac{\pi}{2}F\left(-\frac{1}{2},1,1+\frac{r}{\lambda};p_S\right) \quad &= \quad \frac{\pi}{2}\sum_{n=0}^{\infty}\frac{\left(-\frac{1}{2}\right)_n}{\left(1+\frac{r}{\lambda}\right)_n}p_S^n \\ &= \quad \frac{\pi}{2}\left\{1-\frac{1}{2\left(1+\frac{r}{\lambda}\right)}p_S+\frac{\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)}{\left(1+\frac{r}{\lambda}\right)\left(2+\frac{r}{\lambda}\right)}p_S^2+\cdots+\frac{\left(-\frac{1}{2}\right)_n}{\left(1+\frac{r}{\lambda}\right)_n}p_S^n+\right\}\end{aligned} \tag{14}$$

The exact stalling region can then be found by equating (14) with (11) and doing the same with the first derivative with respect to p_S. Hence, denoting the boundary between stalling and agreeing as σ, we require

$$\frac{\pi}{2}F\left(-\frac{1}{2},1,1+\frac{r}{\lambda};\sigma\right) = E\Omega(\sigma)^{\frac{r}{\lambda}} \quad \text{and} \quad -E\frac{r}{\lambda}\Omega(\sigma)^{\frac{r}{\lambda}}\frac{1}{\sigma(1-\sigma)} = \frac{\pi}{2}\frac{d}{d\sigma}F\left(-\frac{1}{2},1,1+\frac{r}{\lambda};\sigma\right)$$

Thus the boundaries of the stalling region on $p_B = 0$, denoted $\{\sigma_*, \sigma^*\}$, are given by the solution of

$$-\frac{r}{\lambda}F\left(-\frac{1}{2},1,1+\frac{r}{\lambda};\sigma\right) = \sigma(1-\sigma)\frac{d}{d\sigma}F\left(-\frac{1}{2},1,1+\frac{r}{\lambda};\sigma\right)$$

Expressing this in terms of the power series definitions gives:

$$\frac{r}{\lambda}\sum_{n=0}^{\infty}\frac{\left(-\frac{1}{2}\right)_n}{\left(1+\frac{r}{\lambda}\right)_n}\sigma^n+\sum_{n=1}^{\infty}n\cdot\frac{\left(-\frac{1}{2}\right)_n}{\left(1+\frac{r}{\lambda}\right)_n}\sigma^n-\sum_{n=2}^{\infty}(n-1)\cdot\frac{\left(-\frac{1}{2}\right)_{n-1}}{\left(1+\frac{r}{\lambda}\right)_{n-1}}\sigma^n=0 \tag{15}$$

However we have noted that $\{\sigma_*, \sigma^*\} \in (0,1)$. Therefore the larger powers in the above expression become dominated by the smaller powers at σ_* (which tends to 0 for small $\frac{r}{\lambda}$). We can therefore approximate (15) by the quadratic in σ and so have that σ_* is approximated by the smaller root of:

$$\begin{array}{rcl}\left\{\begin{array}{llll}\frac{r}{\lambda} & +\frac{r}{\lambda}\frac{\left(-\frac{1}{2}\right)}{\left(1+\frac{r}{\lambda}\right)}\sigma & +\frac{r}{\lambda}\frac{\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)}{\left(1+\frac{r}{\lambda}\right)\left(2+\frac{r}{\lambda}\right)}\sigma^2 & +O\left(\sigma^3\right)\\ & +\frac{\left(-\frac{1}{2}\right)}{\left(1+\frac{r}{\lambda}\right)}\sigma & +2\frac{\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)}{\left(1+\frac{r}{\lambda}\right)\left(2+\frac{r}{\lambda}\right)}\sigma^2 & +O\left(\sigma^3\right)\\ & & -\frac{\left(-\frac{1}{2}\right)}{\left(1+\frac{r}{\lambda}\right)}\sigma^2 & +O\left(\sigma^3\right)\end{array}\right\}_{\sigma_*} & \approx & 0\\ \frac{r}{\lambda}-\frac{1}{2}\sigma_*+\frac{1}{4\left(1+\frac{r}{\lambda}\right)}\sigma_*^2 & \approx & 0\end{array} \tag{16}$$

The above quadratic is not a good approximation for σ^* as this tends to 1 for small $\frac{r}{\lambda}$ and so more terms would be needed. Thus solving we have

$$\sigma_* \approx \left(1+\frac{r}{\lambda}\right)-\sqrt{\left(1+\frac{r}{\lambda}\right)\left(1-4\frac{r}{\lambda}\right)} \tag{17}$$

We note immediately that this has no solution if $\frac{r}{\lambda} > \frac{1}{4}$ and thus this places an upper bound on when stalling is optimal.

Now using the fact that $\frac{\pi}{2}F\left(-\frac{1}{2},1,1+\frac{r}{\lambda};\sigma_*\right)=E\Omega\left(\sigma_*\right)^{\frac{r}{\lambda}}$ we can solve for E using the approximation for the hypergeometric function at σ_*. Thus

$$\begin{aligned} E &= \frac{\pi}{2}\left[\frac{\sigma_*}{1-\sigma_*}\right]^{\frac{r}{\lambda}}F\left(-\frac{1}{2},1,1+\frac{r}{\lambda};\sigma_*\right) \\ &\approx \frac{\pi}{2}\left[\frac{\sigma_*}{1-\sigma_*}\right]^{\frac{r}{\lambda}}\left\{1-\frac{1}{2\left(1+\frac{r}{\lambda}\right)}\sigma_*+\frac{\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)}{\left(1+\frac{r}{\lambda}\right)\left(2+\frac{r}{\lambda}\right)}\sigma_*^2\right\}\end{aligned} \tag{18}$$

We have thus found the greatest extent to which the buyer can threaten to stall: down to beliefs σ_*. The question is, can the seller threaten to stall for longer coming out of a stalling region. If so then the buyer will need B to smooth paste on to $\pi - S$, rather than x_B.

7 The Seller's greatest stalling value

Thus far we have yet to establish whether the seller would seek to stall in a different region of belief space.

We proceed exactly as above to determine the equation satisfied by the seller's stalling region. Thus, when stalling the seller has a value $S(p_S, p_B)$ which satisfies the Bellman equation:

$$\begin{aligned} S &= p_S\lambda dt\pi+(1-p_S\lambda dt)(1-rdt)\left[S-\lambda p_S(1-p_S)\frac{\partial S}{\partial p_S}dt-\lambda p_B(1-p_B)\frac{\partial S}{\partial p_B}dt\right] \\ p_S\lambda\pi &= S(r+\lambda p_S)+\lambda p_S(1-p_S)\frac{\partial S}{\partial p_S}+\lambda p_B(1-p_B)\frac{\partial S}{\partial p_B}\end{aligned}$$

Note that a particular integral of this equation is given by $\frac{\pi}{\left(1+\frac{r}{\lambda}\right)}p_S$. The homogeneous version of this equation matches that for B given in (8) above. Therefore we again consider the general solution $Gp_S^a\left(1-p_S\right)^b p_B^c\left(1-p_B\right)^d$. Substituting this in to the homogeneous equation and dividing by S gives

$$0=\frac{r}{\lambda}+p_S+a\left(1-p_S\right)-bp_S+c\left(1-p_B\right)-dp_B$$

Equating the coefficients to zero implies that we have

$$S\left(p_S,p_B\right)=\frac{\pi}{\left(1+\frac{r}{\lambda}\right)}p_S+G\left(1-p_S\right)\Omega\left(p_B\right)^{\frac{r}{\lambda}+a}\Omega\left(p_S\right)^{-a}\qquad\text{for some constants } a,G$$

We first consider the case when $r=0$. Then $S\left(p_S,p_B\right)=\pi p_S+G\left(1-p_S\right)\left[\frac{\Omega(p_B)}{\Omega(p_S)}\right]^a$. If $p_S=p_B=0$ then stalling and instantaneous agreement have the same payoff of $\frac{\pi}{2}$. Thus $G|_{r=0}=\frac{\pi}{2}$. We have also noted that the seller would be willing to stall when $r=0$ if $p_S>p_B$. Thus, by continuity, for some small $r>0$ as p_B tends to zero we must still have stalling optimal for some p_S. In addition the value of the seller must be bounded between $\left[\frac{\pi}{2},\pi\right]$. Thus we require $a\leq-\frac{r}{\lambda}$.

Note that the first term is the discounted value of getting the whole pie at current beliefs p_S.[7] Thus this captures the value of waiting indefinitely. The second term gives the option value of being able to agree earlier. This value should not vanish as $p_B\to0$ as there is still scope for agreement at some ranges of belief. Thus we require $a=-\frac{r}{\lambda}$.[8] And so the stalling value is given by

$$S\left(p_S,p_B\right)=\frac{\pi}{\left(1+\frac{r}{\lambda}\right)}p_S+G\left(1-p_S\right)\Omega\left(p_S\right)^{\frac{r}{\lambda}}\tag{19}$$

Lemma 2 *Stalling is not optimal for the seller if $p_S=1$*

Proof. We note immediately that $S\left(1,p_B\right)=\frac{\pi}{\left(1+\frac{r}{\lambda}\right)}$. Noting that $x_S+x_B=\pi$, we need to find the value of $x_B\left(1,p_B\right)$. The lemma thus amounts to showing that $\pi-x_B\left(1,p_B\right)>\frac{\pi}{\left(1+\frac{r}{\lambda}\right)}$. The buyer's payoff will increase when a rival is less likely to exist. Hence we need only show the inequality at beliefs $(1,0)$. We have found the solution to $x_B\left(p_S,0\right)$ in (14) above. In particular we note that $x_B\left(1,0\right)=\frac{\pi}{2}F\left(-\frac{1}{2},1,1+\frac{r}{\lambda};1\right)$. Hence we only need to show that

$$1-\frac{1}{2}F\left(-\frac{1}{2},1,1+\frac{r}{\lambda};1\right)>\frac{1}{\left(1+\frac{r}{\lambda}\right)}$$

But this is true and can be shown graphically.[9] ■

As with the buyer, stalling is not optimal at the origin as there is no learning to take place. Therefore, if stalling is to be optimal for the seller then there will be at least two boundary points marking out some intermediate stalling region. Next note that as p_S tends to 0, $S\left(p_S,p_B\right)$ tends to infinity. The seller

[7]
$$E\left(\text{payoff}\right)=\int_{t=0}^{\infty}\pi\lambda e^{-\lambda t}e^{-rt}dt=\frac{\pi}{1+\frac{r}{\lambda}}$$

[8] A second way to show this is to note that S and x_S cannot be tangent along $p_B=0$ unless $a=-\frac{r}{\lambda}$.

[9] Clearly as $\frac{r}{\lambda}$ becomes very large the inequality holds with the left hand side tending to $\frac{1}{2}$ and the right hand side tending to 0.

seeks to maximise her value subject to having two stalling boundaries. Thus we require the lower of these boundary points to be smoothly pasted between x_S and S, otherwise G could be increased further. Therefore, along this lower boundary of the stalling region, value matching and smooth pasting between S and x_S is required implying that:

$$\frac{\partial S}{\partial p_S} p_S (1 - p_S) + \frac{\partial S}{\partial p_B} p_B (1 - p_B) + S \left[\frac{r}{\lambda} + \frac{1}{2} (p_B + p_S) \right] = \frac{\pi}{2} \left[\frac{r}{\lambda} + p_B + p_S \right]$$

Substituting in we have

$$(p_S - p_B) \left\{ \pi - \frac{\pi}{1 + \frac{r}{\lambda}} p_S - G (1 - p_S) \Omega (p_S)^{\frac{r}{\lambda}} \right\} = \frac{\pi r}{\lambda} \tag{20}$$

Thus given G, we have a relationship between p_B and p_S which gives the maximal stalling boundary for the seller.

7.1 The Seller's greatest value of G

The value of G and the stalling region boundary can be found by applying value matching and smooth pasting[10] along the line $p_B = 0$. Along here we have the value of immediate agreement given by

$$x_S (p_S, 0) = \pi - x_B (p_S, 0) = \pi - \frac{\pi}{2} F \left(-\frac{1}{2}, 1, 1 + \frac{r}{\lambda}; p_S \right)$$

Value matching and smooth pasting thus imply that at a boundary between stalling and not

$$\begin{aligned} \pi - \frac{\pi}{2} F \left(-\frac{1}{2}, 1, 1 + \frac{r}{\lambda}; \sigma \right) &= \frac{\pi}{\left(1 + \frac{r}{\lambda}\right)} \sigma + G (1 - \sigma) \Omega (\sigma)^{\frac{r}{\lambda}} \\ -\frac{\pi}{2} \frac{d}{d\sigma} F \left(-\frac{1}{2}, 1, 1 + \frac{r}{\lambda}; \sigma \right) &= \frac{\pi}{\left(1 + \frac{r}{\lambda}\right)} - G \Omega (\sigma)^{\frac{r}{\lambda}} \left(1 + \frac{r}{\lambda} \frac{1}{\sigma} \right) \end{aligned}$$

Thus removing G gives us the condition that

$$\left(\sigma + \frac{r}{\lambda} \right) F \left(-\frac{1}{2}, 1, 1 + \frac{r}{\lambda}; \sigma \right) + \sigma (1 - \sigma) \frac{d}{d\sigma} F \left(-\frac{1}{2}, 1, 1 + \frac{r}{\lambda}; \sigma \right) = 2 \frac{r}{\lambda} \tag{21}$$

Using the fact that $F\left(-\frac{1}{2}, 1, 1 + \frac{r}{\lambda}; \sigma\right) = \sum_{n=0}^{\infty} \frac{\left(-\frac{1}{2}\right)_n}{\left(1+\frac{r}{\lambda}\right)_n} \sigma^n$ allows us to express (21) as a power series relation:

$$\frac{r}{\lambda} \sum_{n=0}^{\infty} \frac{\left(-\frac{1}{2}\right)_n}{\left(1 + \frac{r}{\lambda}\right)_n} \sigma^n + \sum_{n=1}^{\infty} \left\{ \frac{\left(-\frac{1}{2}\right)_{n-1}}{\left(1 + \frac{r}{\lambda}\right)_{n-1}} + n \cdot \frac{\left(-\frac{1}{2}\right)_n}{\left(1 + \frac{r}{\lambda}\right)_n} \right\} \sigma^n - \sum_{n=2}^{\infty} (n - 1) \cdot \frac{\left(-\frac{1}{2}\right)_{n-1}}{\left(1 + \frac{r}{\lambda}\right)_{n-1}} \sigma^n = 2 \frac{r}{\lambda} \tag{22}$$

We seek the smaller of these two solutions, $\underline{\sigma}$, which tends towards zero and so we can approximate (22)

[10] In actual fact, along $p_B = 0$ we require

$$\frac{\partial S}{\partial p_S} p_S (1 - p_S) = \frac{\partial x_S}{\partial p_S} p_S (1 - p_S) \quad \forall p_S \in (0, 1)$$

which implies that $\frac{\partial S}{\partial p_S} = \frac{\partial x_S}{\partial p_S}$ as required.

by a quadratic in σ to give

$$\left\{\begin{array}{llll} \frac{r}{\lambda} & +\frac{r}{\lambda}\frac{\left(-\frac{1}{2}\right)}{\left(1+\frac{r}{\lambda}\right)}\sigma & +\frac{r}{\lambda}\frac{\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)}{\left(1+\frac{r}{\lambda}\right)\left(2+\frac{r}{\lambda}\right)}\sigma^2 & +O\left(\sigma^3\right) \\ & +\left\{1+\frac{\left(-\frac{1}{2}\right)}{\left(1+\frac{r}{\lambda}\right)}\right\}\sigma & +\left\{\frac{\left(-\frac{1}{2}\right)}{\left(1+\frac{r}{\lambda}\right)}+2\frac{\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)}{\left(1+\frac{r}{\lambda}\right)\left(2+\frac{r}{\lambda}\right)}\right\}\sigma^2 & +O\left(\sigma^3\right) \\ & & -\frac{\left(-\frac{1}{2}\right)}{\left(1+\frac{r}{\lambda}\right)}\sigma^2 & +O\left(\sigma^3\right) \end{array}\right\}_{\underline{\sigma}} \approx 2\frac{r}{\lambda}$$

$$-\frac{r}{\lambda}+\frac{1}{2}\underline{\sigma}-\frac{1}{4\left(1+\frac{r}{\lambda}\right)}\underline{\sigma}^2 \approx 0$$

But comparing this to (16) we have $\underline{\sigma}=\sigma_*$ with the actual value given in (17). Using the value matching condition, we then have

$$G=\frac{1}{1-\sigma_*}\left[\frac{\sigma_*}{1-\sigma_*}\right]^{\frac{r}{\lambda}}\left\{\pi\left[1-\frac{\sigma_*}{1+\frac{r}{\lambda}}\right]-\frac{\pi}{2}F\left(-\frac{1}{2},1,1+\frac{r}{\lambda};\sigma_*\right)\right\} \tag{23}$$

We have thus found the seller's greatest value of G. Thus along $p_B=0$, the buyer and the seller can stall down to the same small beliefs, $(\sigma_*,0)$.

8 The champion staller

The method of proof (Section 3) now required us to ascertain whether the buyer or seller could threaten to stall for longer. The agent who could stall less would have to smooth paste on to the rival's stalling value. However, we first note that $S+B$ increases as p_B decreases, and the stalling region will be give by the curve $S+B>\pi$ as agreement is not possible here. Thus if stalling is optimal anywhere, it is optimal on $p_B=0$. Next note that at $(\sigma_*,0)$, if we keep E unchanged then B meets x_B smoothly. But $x_B=\pi-x_S$, and at the same point S meets x_S smoothly by construction. Thus B meets $\pi-S$ smoothly. The same applies if G is left unchanged. Thus we have no slack to alter E or G and the stalling functions have been found.

Nevertheless, for completeness we establish that the seller can stall for longer than the buyer: it is the buyer who, at small beliefs p_B, must compromise when coming out of the stalling region. In particular, starting from beliefs at which both buyer and seller would seek to stall, the boundary between stalling and not for the buyer is given by (10) with the constant E given in (18). The boundary for the seller is given by (20) with the boundary G given by (23).

Proposition 3 *For small beliefs p_B, the seller can threaten to stall for longer (at less pessimistic beliefs) than the buyer*

Proof. We need to show that given a p_S close to σ_*, $p_B(p_S)$ is larger for the seller boundary than it is for the buyer boundary. We have already noted that along the line $p_B=0$, the maximal stalling boundaries both start at the same point: $p_S=\sigma_*$. Thus we seek to establish whether the buyer's or the seller's stalling boundary moves off from σ_* at the greatest rate. This is done by taking approximations for σ_* in terms of $\frac{r}{\lambda}$. The proof is completed in the appendix. ■

We have therefore shown that the seller will be able to stall for longer. Therefore the buyer cannot terminate a stalling phase on to x_B, rather the buyer will have to compromise and terminate on to $\pi - S(p_S, p_B)$ for small beliefs p_B. We therefore have the full stalling valuations given by (19) with the constant G given by (23) and (9) with the constant E given by (18). We have also shown that the condition (10) is not applicable. Rather the stalling valuations are defined upstream (in terms of beliefs) of the boundary given by (20).

9 The Stalling Region

We are therefore in a position to define the stalling region. To recap, note that the immediate agreement function was established using a continuous form of Rubinstein's alternating bargaining model allowing for the outside option that a rival buyer might come with some probability (disagreed on) in the next dt of time. However, agents also have the option of stalling for longer and can achieve values of $S(p_S, p_B)$ and $B(p_S, p_B)$ if they do so. If only one player wishes to stall, the other player must decide whether to compromise and offer the rival their stalling value or whether to wait themselves. The full stalling region is thus given by the curve implicitly defined by the equation stalling value equals total pie available

$$S(p_S, p_B) + B(p_S, p_B) = x_S(p_S, p_B) + x_B(p_S, p_B) = \pi$$

where the last equality follows from Section 4. We have already established $B(p_S, p_B)$ and $S(p_S, p_B)$. Thus the full stalling region can be written as:

$$\begin{aligned} \pi &= \frac{\pi}{\left(1+\frac{r}{\lambda}\right)} p_S + \Omega(p_S)^{\frac{r}{\lambda}} \left[G(1-p_S) + E(1-p_B)\right] \\ &\Rightarrow \ p_B = 1 - \frac{1}{E} \cdot \left[\pi\left(1 - \frac{p_S}{1+\frac{r}{\lambda}}\right)\Omega(p_S)^{-\frac{r}{\lambda}} - G(1-p_S)\right] \end{aligned} \tag{24}$$

This can then be plotted for general values of $\frac{r}{\lambda}$: the result is given in Figure 2. We note immediately a number of features which are now straight forwardly proved:

Lemma 4 *The stalling region is bounded by $p_S = p_B$ when $r = 0$*

Proof. Set $r = 0$ in (24) and recalling that $E|_{r=0} = G|_{r=0} = \frac{\pi}{2}$ gives us

$$\pi = \pi p_S + \pi - \frac{\pi}{2}(p_S + p_B) \Rightarrow p_S = p_B$$

■

Lemma 5 *If (p_S, p_B) lies in the stalling region, then so do (p_S, b) for all beliefs $b \in [0, p_B]$*

Proof. If (p_S, p_B) lies in the stalling region then agreement cannot be reached. Thus $[S+B]_{(p_S,p_B)} \geq \pi$. But S is constant with respect to p_B and B increases as p_B decreases so that $[S+B]_{(p_S,b)} > \pi \forall b \in [0, p_B)$ which gives the result. ■

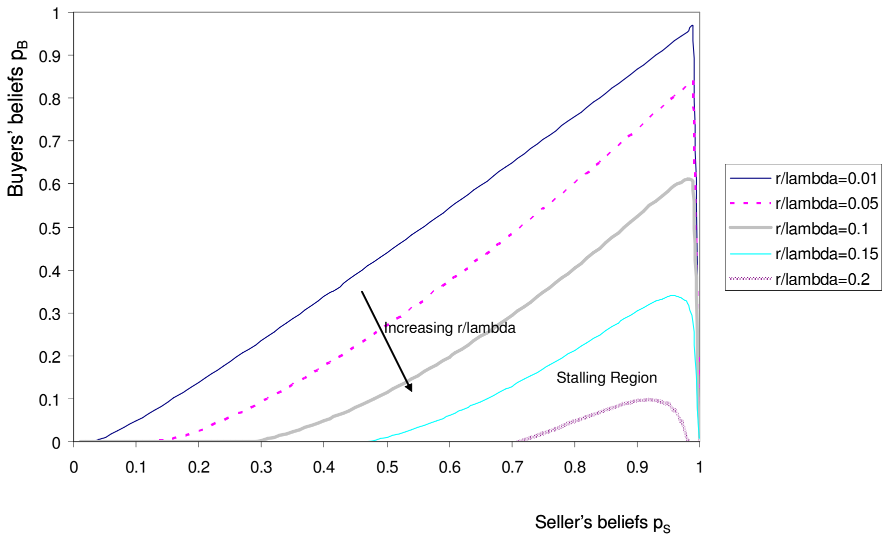

Figure 2: Overall stalling region

We have already noted that there is no solution to S meeting x_S along $p_B = 0$ twice if $\frac{r}{\lambda} > \frac{1}{4}$. Thus there can be no stalling region for $\frac{r}{\lambda} > \frac{1}{4}$, otherwise the seller would stall when beliefs were $(0, 0)$ which is a contradiction.

So we have the main result of this paper: stalling is optimal and to be expected when agents disagree as to the likelihood of a rival buyer coming. We can thus answer the question of when buyers and sellers will find it in their interests to stall:

- There is no stalling if $p_S \geq p_B$

If the discount rate is positive

- There is no stalling if $p_S = 1$: the buyer is certain a rival is coming and so sufficient concessions will not arrive to warrant stalling.[11]
- Stalling is optimal if the buyer and seller disagree strongly so that the buyer is confident no rival is coming while the seller takes the opposite view.
- Stalling is for a finite period of time: eventually either a rival buyer does come, or the seller's confidence in a rival buyer falls sufficiently to allow the buyer to compromise.

10 When the Buyer is Certain There is No Competition

We analyse in detail the example of the buyer being certain there is no rival $(p_B = 0)$. For expositional purposes we initially set $\frac{r}{\lambda} = 0.1$. The buyer's stalling value and agreement function are plotted in Figure

[11] See Lemmas 1 and 2.

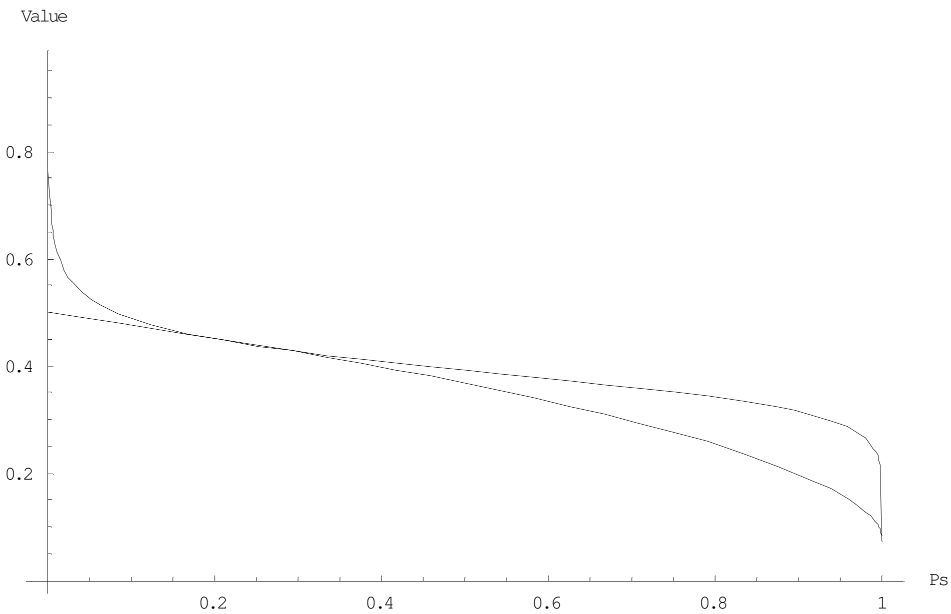

Figure 3: Buyer's stalling value $B(p_S, 0)$ - the upper curve, and $x_B(p_S, 0)$

3. The stalling value function is tangent to x_B at σ_*, the lower of the two contact points. For values of $p_S < \sigma_*$, the stalling function is not defined and so the graph drawn has no economic content - stalling cannot be optimal when $p_S = p_B = 0$. We can see that stalling begins very close to $p_S = 1$.

A similar graph is available for the seller and is given as Figure 4. The higher of the two graphs is the stalling value - a function which has no economic meaning at $p_S < \sigma_*$.

Next we consider the stalling values S and B along a belief path starting at $(0.999, 0.5)$ with $\frac{r}{\lambda}$ set to 0.05. We have selected such a small value for $\frac{r}{\lambda}$ as the approximations used in finding the constants E and G work best in this range. The origin in Figure 5 depicts the value of the stalling functions at this belief point. The upper curve is B, the lower one $\pi - S$.[12] As time progresses forward the value of B increases as does the value of $\pi - S$ (where π has been normalised to 1). Note that we have the smooth pasting condition of B on to $\pi - S$ as previously discussed. In addition note that stalling is already optimal at beliefs $(p_S = 0.999, p_B = 0.5)$ even though p_S is so close to 1.

11 Conclusion

We have shown that stalling is a key feature of an intuitive bargaining process in which the buyer and the seller disagree as to the chances of a rival buyer coming. The stalling value functions S and B bound

[12] Explicitly e have plotted

$$B\left[p_S(t), p_B(t)\right] = B\left[\frac{1}{1+\left(\frac{1}{p_S(0)}-1\right)e^{\lambda t}}, \frac{1}{1+\left(\frac{1}{p_B(0)}-1\right)e^{\lambda t}}\right]\Bigg|_{\substack{p_S(0)=0.999\\ p_B(0)=0.5}}$$

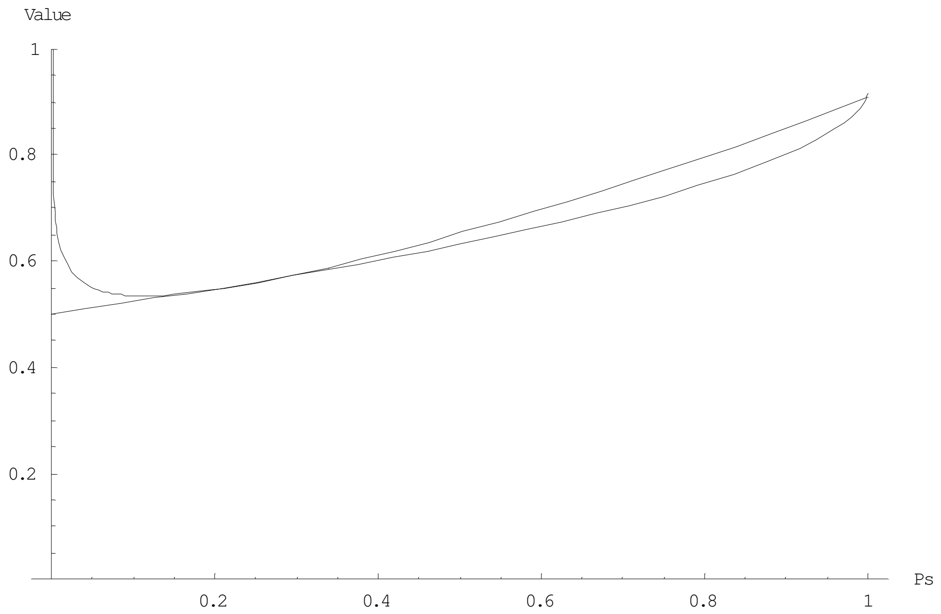

Figure 4: Seller's stalling value $S(p_S, 0)$ - the upper curve, and $x_S(p_S, 0)$

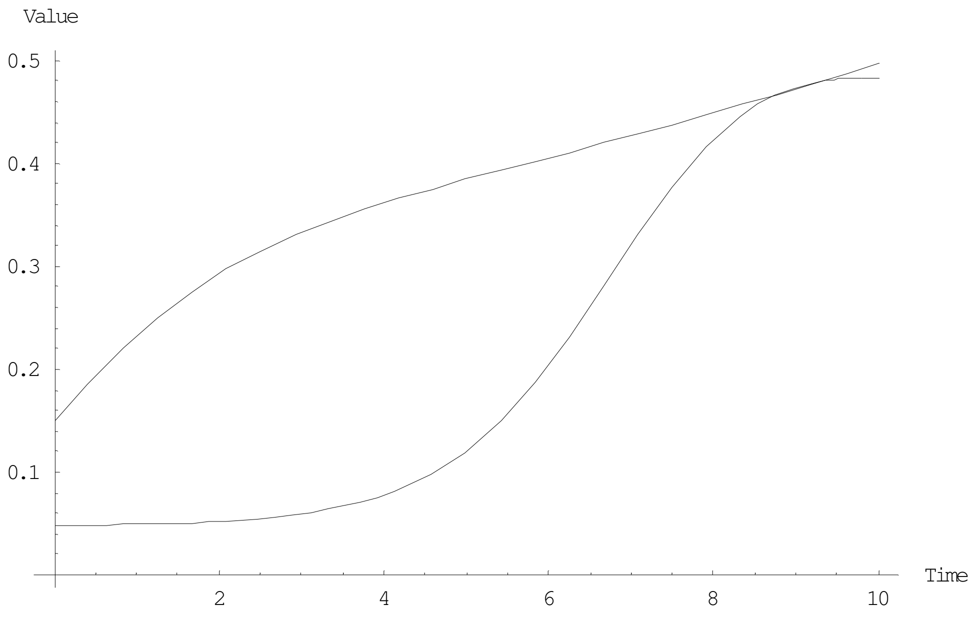

Figure 5: The stalling functions B (the upper curve) and $\pi - S$ as a function of time, measured as multilples of λ, starting at beliefs $(0.999, 0.5)$

the true stalling region which is ultimately shown in Figure 2 above.

The stalling areas shrink as the discount rate increases. If beliefs lie in the stalling areas, then agreement will not be reached until the beliefs have declined (through Bayesian updating as the result of no rival arriving) out of the stalling region or until a rival buyer has come. Thus agreement is always reached in finite time. The model provides a new explanation for stalling with concrete predictions possible as to whether stalling will occur, and for how long, given the initial disagreement between the parties.

A Other Proofs

Proof of Proposition 3. The buyer's stalling boundary is given by $E(1-p_B)(p_S-p_B)\Omega(p_S)^{\frac{r}{\lambda}} = \frac{\pi r}{\lambda}$. If we write $p_S = \sigma_* + s$ then noting that

$$\Omega(p_S)^{\frac{r}{\lambda}} = \Omega(\sigma_*)^{\frac{r}{\lambda}}\left(1-\frac{s}{1-\sigma_*}\right)^{\frac{r}{\lambda}}\left(1+\frac{s}{\sigma_*}\right)^{-\frac{r}{\lambda}} = \Omega(\sigma_*)^{\frac{r}{\lambda}}\left[1-\frac{r}{\lambda}\frac{s}{\sigma_*(1-\sigma_*)}\right]+O\left(s^2\right)$$

this boundary collapses to

$$E\left[\frac{1-\sigma_*}{\sigma_*}\right]^{\frac{r}{\lambda}}\left[\sigma_*+s\left(1-\frac{\frac{r}{\lambda}}{1-\sigma_*}\right)-p_B(1+\sigma_*)\right] = \frac{\pi r}{\lambda}+O\left(s^2, sp_B, p_B^2\right)$$

and thus the buyer's stalling boundary has gradient $\frac{1}{1+\sigma_*}\left(1-\frac{\frac{r}{\lambda}}{1-\sigma_*}\right)$.

The seller's stalling boundary is given by $(p_S-p_B)\left\{\pi-\frac{\pi}{1+\frac{r}{\lambda}}p_S - G(1-p_S)\Omega(p_S)^{\frac{r}{\lambda}}\right\} = \frac{\pi r}{\lambda}$. Replacing p_S by σ_*+s gives us that

$$\begin{aligned}
(\sigma_*+s-p_B)\left\{1-\frac{\sigma_*}{1+\frac{r}{\lambda}}-\frac{s}{1+\frac{r}{\lambda}}-\frac{G}{\pi}\Omega(\sigma_*)^{\frac{r}{\lambda}}\left[1-\sigma_*-s\left(1+\frac{r}{\lambda}\frac{1}{\sigma_*}\right)\right]\right\} &\approx \frac{r}{\lambda}\\
(\sigma_*+s-p_B)\left\{1-\frac{\sigma_*}{1+\frac{r}{\lambda}}-\frac{G}{\pi}\Omega(\sigma_*)^{\frac{r}{\lambda}}(1-\sigma_*)+s\left[\frac{G}{\pi}\Omega(\sigma_*)^{\frac{r}{\lambda}}\left(1+\frac{r}{\lambda}\frac{1}{\sigma_*}\right)-\frac{1}{1+\frac{r}{\lambda}}\right]\right\} &\approx \frac{r}{\lambda}\\
(\sigma_*+s-p_B)\left\{\frac{1}{2}F\left(-\frac{1}{2},1,1+\frac{r}{\lambda},\sigma_*\right)+s\left[\frac{G}{\pi}\Omega(\sigma_*)^{\frac{r}{\lambda}}\left(1+\frac{r}{\lambda}\frac{1}{\sigma_*}\right)-\frac{1}{1+\frac{r}{\lambda}}\right]\right\} &\approx \frac{r}{\lambda}\\
\begin{matrix} s\left[\frac{1}{2}F\left(-\frac{1}{2},1,1+\frac{r}{\lambda},\sigma_*\right)+\frac{G}{\pi}\Omega(\sigma_*)^{\frac{r}{\lambda}}\left(\sigma_*+\frac{r}{\lambda}\right)-\frac{\sigma_*}{1+\frac{r}{\lambda}}\right] \\ -p_B\frac{1}{2}F\left(-\frac{1}{2},1,1+\frac{r}{\lambda},\sigma_*\right)\end{matrix} + O\left(s^2, sp_B, p_B^2\right) &= \text{constant}
\end{aligned}$$

The gradient of the stalling boundary for the seller is thus given by

$$1+\frac{\frac{G}{\pi}\Omega(\sigma_*)^{\frac{r}{\lambda}}\left(\sigma_*+\frac{r}{\lambda}\right)-\frac{\sigma_*}{1+\frac{r}{\lambda}}}{\frac{1}{2}F\left(-\frac{1}{2},1,1+\frac{r}{\lambda},\sigma_*\right)}$$

The seller can stall for longer if

$$\begin{aligned}
\frac{\frac{G}{\pi}\Omega(\sigma_*)^{\frac{r}{\lambda}}\left(\sigma_*+\frac{r}{\lambda}\right)-\frac{\sigma_*}{1+\frac{r}{\lambda}}}{\frac{1}{2}F\left(-\frac{1}{2},1,1+\frac{r}{\lambda},\sigma_*\right)} &> \frac{1}{1-\sigma_*^2}\left(-\sigma_*-\frac{r}{\lambda}+\sigma_*^2\right)\\
\frac{G}{\pi}\Omega(\sigma_*)^{\frac{r}{\lambda}}\left(\sigma_*+\frac{r}{\lambda}\right) &> \frac{\sigma_*}{1+\frac{r}{\lambda}}-\frac{1}{1-\sigma_*^2}\left(\sigma_*+\frac{r}{\lambda}-\sigma_*^2\right)\frac{1}{2}F\left(-\frac{1}{2},1,1+\frac{r}{\lambda},\sigma_*\right)\\
\frac{\left(\sigma_*+\frac{r}{\lambda}\right)}{1-\sigma_*}\left\{\begin{matrix}\left[1-\frac{\sigma_*}{1+\frac{r}{\lambda}}\right]\\ -\frac{1}{2}F\left(-\frac{1}{2},1,1+\frac{r}{\lambda};\sigma_*\right)\end{matrix}\right\} &> \frac{\sigma_*}{1+\frac{r}{\lambda}}-\frac{1}{1-\sigma_*^2}\left(\sigma_*+\frac{r}{\lambda}-\sigma_*^2\right)\frac{1}{2}F\left(-\frac{1}{2},1,1+\frac{r}{\lambda},\sigma_*\right)
\end{aligned}$$

We require $\frac{r}{\lambda} < \frac{1}{4}$ for σ_* to have a solution. Thus we expand around small $\frac{r}{\lambda}$ up to first order. In this case $\sigma_* \approx \frac{5}{2}\frac{r}{\lambda}$, $F\left(-\frac{1}{2}, 1, 1 + \frac{r}{\lambda}, \sigma_*\right) \approx 1 - \frac{5}{4}\frac{r}{\lambda}$. Thus we require

$$\begin{aligned} \left(1 + \frac{5}{2}\frac{r}{\lambda}\right)\left(1 - \frac{5}{2}\frac{r}{\lambda} - \frac{1}{2} + \frac{5}{8}\frac{r}{\lambda}\right)\frac{7}{2}\frac{r}{\lambda} &> \frac{5}{2}\frac{r}{\lambda} - \frac{7}{4}\frac{r}{\lambda} \\ \frac{7}{4} &> \frac{5}{2} - \frac{7}{4} \end{aligned}$$

which is indeed true giving the result. ■

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