The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited*

Marcus Hagedorn†
University of Frankfurt

Iourii Manovskii‡
University of Pennsylvania

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Abstract

Recently, a number of authors have argued that the standard search model cannot generate the observed business-cycle-frequency fluctuations in unemployment and job vacancies, given shocks of a plausible magnitude. We use implications of the model to pin down its two key parameters - the value of non-market activity and the bargaining weights. Our calibration implies that the model is, in fact, consistent with the data.

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†Department of Economics, Johann Wolfgang Goethe University Frankfurt am Main, Mertonstr. 17, PF 81, 60054 Franfurt am Main, Germany. E-mail: hagedorn@wiwi.uni-frankfurt.de.

‡Department of Economics, University of Pennsylvania, 160 McNeil Building, 3718 Locust Walk, Philadelphia, PA, 19104-6297 USA. E-mail: manovski@econ.upenn.edu.

1 Introduction

The Mortensen-Pissarides (MP) search and matching model (Mortensen and Pissarides (1994), Pissarides (1985, 2000)) has become the standard theory of equilibrium unemployment. It was recently pointed out by Shimer (2005) that the standard calibration of the model fails to account for the cyclical properties of its two central variables - unemployment and vacancies. These variables are much more volatile in U.S. data than in the MP model. This result came as a call for a better understanding of the elements of the model. The ensuing literature has suggested that the wage-setting mechanism in the MP model has to be altered: the close link between wages and productivity has to be broken. Farmer (2004) and Shimer (2004) suggest that some rigidity in wage formation may be necessary. In Hall (2005) a social norm, that does not change over the business cycle, renders wages not responsive to productivity changes. Menzio (2004) adds asymmetric information about productivity to make wages rigid.[1]

We take a different route in this paper. We suggest that the problem lies not in the model itself, but in the way the model is typically calibrated. We propose a new calibration strategy for the two central parameters of the MP model - the worker's value of non-market activity and the worker's bargaining power. The usual strategy is to identify non-market activity with receiving unemployment benefits. The bargaining weight is then picked in a way that guarantees the efficiency of the model (i.e., to satisfy the Hosios (1990) condition). This choice of the bargaining power typically implies that wages in the model are close to productivity and follow productivity closely over the business cycle. The problem is then clear. An increase in productivity is mostly absorbed by an increase in wages and profits remain little changed. Therefore, firms' incentives to post vacancies do not increase much either.

The calibration strategy that we propose does not impose the assumption that the return

[1]Many other authors have explored whether a search model is consistent with business cycle facts, including Lilien (1982), Abraham and Katz (1986), Blanchard and Diamond (1989), Merz (1995), Andolfatto (1996), Ramey and Watson (1997), Cole and Rogerson (1999), Den Haan, Ramey, and Watson (2000), Gomes, Grenwood, and Rebelo (2001), Fujita (2004) and Pries (2004). See Shimer (2005) for a discussion.

to non-market activity is identical to receiving unemployment benefits. In a model without search frictions, for example a standard real business cycle model, market and non-market productivity are equalized on the margin: workers are indifferent between working one more hour at home or in the market (e.g., Benhabib, Rogerson, and Wright (1991), Greenwood and Hercowitz (1991)). Thus, it seems arbitrary to assume a large gap between the value of market and non-market activities. We, instead, pin down the value of non-market activity and worker's bargaining power through matching two targets in the data: small accounting profits and the mild procyclicality of wages. Our estimates imply that the value of the non-market activity has to be considerably higher than the typical replacement ratio of unemployment insurance, and the bargaining power of the workers has to be relatively small. We find that the model calibrated in this way is, in fact, consistent with the business cycle facts. The explanation is straightforward. As in the "sticky wage" models of Hall (2005), Farmer (2004), Menzio (2004) and Shimer (2004), an increase in productivity is not absorbed by wages and leads to an increase in profits. Since profits are small, the percentage change in profits is large, and firm's incentives to post vacancies increase considerably.[2]

The paper is organized as follows. A discrete time stochastic version of the Pissarides (1985, 2000) search and matching model is laid out in Section 2. In Section 3 we discuss our calibration strategy and perform a quantitative analysis. We show that the model accounts very well for the volatility of unemployment and vacancies when we pin down the value of non-market activity and worker's bargaining power by matching the data on profits and on the cyclicality of wages. We analyse our calibration strategy theoretically and show that the model can replicate any elasticity of vacancies and unemployment if the researcher had complete freedom to pick the value of non-market activity and worker's bargaining weight. The model pins down the present value of wages only. Therefore, in Section 4 we explore the robustness of our finding to using the cyclical volatility of the present value of wages in

[2]Shimer (2005) and Costain and Reiter (2003) have noted that an increase in the value of the non-market activity improves the performance of the model in matching the business cycle facts. Both paper, however, dismiss a relatively high value of the non-market activity as implausible for several reasons. We address their concerns below.

the data as an alternative calibration target. In Section 5 we discuss various implications of our calibration approach. Among other issues, we suggest that our calibration of the model implies that it is much closer to the efficient benchmark than what is implied by the standard calibration. Section 6 concludes.

2 The Model

We consider a (aggregate) stochastic discrete time version of the Pissarides (1985, 2000) search and matching model. The exposition of the model follows Shimer (2005). The only difference is that from the outset we write down the discrete time model that we will use in quantitative exercise.

2.1 Workers and Firms

There is a continuum of infinitely lived workers and a continuum of infinitely lived firms, each of measure one. Workers maximize their expected life time utility:

$$\mathbb{E}\sum_{t=0}^{\infty}\delta^t y_t, \tag{1}$$

where y_t represents income in period t and $\delta \in (0,1)$ is workers' and firms' common discount factor. Firms have a constant returns to scale production technology that uses labor as the only input. Output of each unit of labor is denoted by p_t. Labor productivity p follows a first order Markov process in discrete time, according to some distribution $G(p', p) = Pr(p_{t+1} \leq p' \mid p_t = p)$.

There is free entry of firms. Firms attract unemployed workers by posting a vacancy at the flow cost c. Once matched, workers and firms separate exogenously with probability s. Employed workers are paid a wage w_p, contingent on the aggregate productivity level p. Firms make profit of $p - w_p$ per worker each period in which they operate. Unemployed workers get flow utility z from leisure/non-market activity. We assume that p_t always exceeds z.[3] Workers and firms split the surplus from a match according to the generalized Nash bargaining solution. The bargaining power of workers is $\beta \in (0, 1)$.

2.2 Matching

Let u_t denote the unemployment rate (or the number of unemployed people) and $n_t = 1 - u_t$ the employment rate (or the number of matched workers). Let v_t be the number of vacancies posted in period t. We refer to $\theta_t = v_t/u_t$ as the market tightness at time t.

The number of new matches (starting to produce output at $t+1$) is given by a constant returns to scale matching function $m(u_t, v_t)$. Employment evolves according to the following law of motion:

$$n_{t+1} = (1 - s)n_t + m(u_t, v_t). \tag{2}$$

The probability for an unemployed worker to be matched with a vacancy next period equals $f(\theta_t) = m(u_t, v_t)/u_t = m(1, \theta_t)$. The probability for a vacancy to be filled next period equals $q(\theta_t) = m(u_t, v_t)/v_t = m(1/\theta_t, 1) = f(\theta_t)/\theta_t$. We restrict $m(u_t, v_t) \leq \min(u_t, v_t)$.

2.3 Equilibrium

Denote the firm's value of a job (a filled vacancy) by J, the firm's value of an unfilled vacancy by V, the worker's value of having a job by W, and the worker's value of being unemployed by U. Let $E_p X_{p'}$ denote next periods' expected value of an arbitrary variable X, conditional on the current state p. With this notation, the following Bellman equations

[3] This is an equilibrium result in a real business cycle model with search and with a constant-returns-to-scale technology in capital and labor (as in Merz (1995) and Andolfatto (1996)). Without search, p equals z. To see this, consider a family of measure one. The family decides what fraction of its members, L, should work in the market, given that each worker can produce z at home, to $max_L\{Lp + (1 - L)z\}$, where $p = F_L(L, K)$ denotes the marginal product of labor. The optimal choice of L implies $p = z$.

describe the model:[4]

$$J_p = p - w_p + \delta(1-s)E_p J_{p'} \tag{3}$$

$$V_p = -c + \delta q(\theta_p) E_p J_{p'} \tag{4}$$

$$U_p = z + \delta\{f(\theta_p)E_p W_{p'} + (1 - f(\theta_p))E_p U_{p'}\} \tag{5}$$

$$W_p = w_p + \delta\{(1-s)E_p W_{p'} + sE_p U_{p'}\}. \tag{6}$$

The interpretation is straightforward. Operating firms earn profits $p - w_p$ and the matches are exogenously destroyed with probability s. A vacancy costs c and is matched with a worker (next period) with probability q. An unemployed worker derives utility z and finds a job next period with probability f. An employed worker earns wage w_p but may loose her job next period with probability s and become unemployed.

Nash bargaining implies that a worker and a firm split the surplus $S_p = J_p + W_p - U_p$ such that

$$J_p = (1-\beta)S_p, \tag{7}$$

$$W_p - U_p = \beta S_p. \tag{8}$$

Free entry implies that the value of posting a vacancy is zero: $V_p = 0$ for all p and, therefore,

$$\begin{aligned} c &= \delta q(\theta_p) E_p J_{p'} \\ &= \delta q(\theta_p)(1-\beta)E_p S_{p'}. \end{aligned} \tag{9}$$

The Bellman equation for the surplus is:

$$\begin{aligned} S_p &= p - z + \delta(1-s)E_p(W_{p'} + J_{p'}) + \delta E_p(sU_{p'} - f(\theta_p)W_{p'} - (1-f(\theta_p))U_{p'}) \\ &= p - z + \delta(1-s)E_p(W_{p'} + J_{p'} - U_{p'}) - \delta f(\theta_p)E_p(W_{p'} - U_{p'}) \\ &= p - z - \delta f(\theta_p)\beta E_p S_{p'} + \delta(1-s)E_p(W_{p'} + J_{p'} - U_{p'}) \\ &= p - (z + \delta f(\theta_p)\beta E_p S_{p'}) + \delta(1-s)E_p S_{p'}. \end{aligned} \tag{10}$$

[4] As Shimer (2005), we implicitly assume that the value functions only depend on p and not on the unemployment rate. Existence of such an equilibrium is straightforward and follows from the existence of a solution to equation 18 derived below.

An existing match generates p units of output every period. It is destroyed next period with probability s. In this case, the value of the firm drops to zero, the value of a vacancy. The worker, on the other hand, becomes unemployed and gets utility z every period until he becomes employed again with probability $f(\theta_p)$ per period. An employed worker keeps a share β of the match surplus. With probability $1-s$, the match exists next period and generates surplus depending on the realization of p'.

We now derive the expressions for equilibrium wages and profits. Using $J_p = (1-\beta)S_p$, it follows from the free-entry condition and the flow equation for J that:

$$(1-\beta)S_p = p - w_p + (1-s)c/q(\theta_p). \tag{11}$$

Free entry and (10) imply that

$$S_p = p - z + ((1-s) - f(\theta_p)\beta)\frac{c}{q(\theta_p)(1-\beta)}. \tag{12}$$

Thus, we have that

$$(1-\beta)S_p = (1-\beta)(p-z) + c\frac{1-s-f(\theta_p)\beta}{q(\theta_p)}. \tag{13}$$

This implies that wages are given by

$$\begin{aligned} w_p &= p - (1-\beta)S_p + (1-s)c/q(\theta_p) && (14)\\ &= \beta p + (1-\beta)z + c\beta\theta_p, && (15) \end{aligned}$$

and profits are given by

$$\Pi_p = p - w_p = (1-\beta)(p-z) - c\beta\theta_p. \tag{16}$$

3 Cyclical Behavior of Unemployment and Vacancies

3.1 Calibration

In this section we calibrate the model to match U.S. facts documented in Shimer (2005), and our two additional targets, profits and the cyclicality of wages. The following parameters have to be determined: average productivity $\overline{p}$, the value of non-market activity z, the discount factor δ, the separation rate s, the bargaining power β, the vacancy cost c, and the two matching function parameters - introduced below - μ and α.

Basics. We choose the model period to be one month, consistent with the frequency of the employment data we use to calibrate the model. The data used to compute some of the targets have quarterly or annual frequency and we aggregate the model appropriately when matching those targets. We set $\delta = 0.9966$. Shimer (2005) estimates the average monthly job finding rate from 1951 to 2003 to be 0.45 and the separation rate to be 0.034. Thus, we use $f = 0.45$ as the target for the mean job finding rate, and set $s = 0.034$.

Productivity. The stochastic process for labor productivity is chosen as follows. We approximate through a 35-state Markov chain the continuous-valued AR(1) process

$$logp_{t+1} = \rho \cdot logp_t + \epsilon_{t+1}, \tag{17}$$

where $\rho \in (0,1)$ and $\epsilon \sim N(0, \sigma_\epsilon^2)$. To calibrate ρ and σ_ϵ^2, we consider quarterly averages of monthly productivity and HP-filter (Prescott (1986)) this process (as in Shimer (2005), with a smoothing parameter of 10^5). For this process Shimer (2005) reports an autocorrelation of 0.878 and an unconditional variance of 0.020. This requires setting $\rho = 0.97$ and σ_ϵ, the standard deviation of ϵ, to 0.007. The mean of p is normalized to one.

Matching Function. We choose the functional form of the matching function to be $m(u,v) = min\{v, u, \mu u^\alpha v^{1-\alpha}\}$. We choose $\alpha = 0.5$, the midpoint of the range of values typically used in the literature (e.g., Shimer (2005), Hall (2005), Farmer (2004), Andolfatto (1996), Merz (1995)). We study the quantitative importance of this parameter in Section 3.4.3. The average v/u ratio (θ) is intrinsically meaningless since any change in θ can be offset by changes in μ and c. As in Shimer (2005), we normalize the average θ. $\mu = 0.345$ and $c = 0.212$ are found to match the targets for (average) $f = 0.45$ and $\theta = 1.8$.[5]

[5] The normalization of the average $\theta = 1.8$ does not affect our findings on the volatility of unemployment and vacancies, but is convenient because it guarantees $m(u,v) < min(u,v)$ in the model, given the other calibrated parameters.

Bargaining Weights and Value of Non-market Activity. We choose the remaining two parameters z and β to match data on average profits and on the elasticity of wages with respect to productivity.

Basu and Fernald (1997) estimate that the typical industry in the U.S. has an average profit rate not higher than 3 percent.[6] Thus, in the benchmark calibration we target a value of $\Pi = 0.03$. The corresponding statistic in the model was derived in equation (16). In Section 3.3 below we discuss this target in more detail.

We estimate the cyclicality of wages from the PSID data. We find that a 1 percentage point increase in productivity is associated with a 0.47 percentage points increase in real wage growth.[7] We use this as a calibration target. Since PSID has annual frequency, we first aggregate the monthly data from the simulated model, and then estimate a regression identical to the one estimated on the PSID data. Our calibration implies $z = 0.943$ and $\beta = 0.061$.

In Sections 3.4.1 and 3.4.2, we will derive that the lower are the profits and the lower is the volatility of wages over the business cycle, the higher is the cyclical volatility of the market tightness, θ. Our benchmark calibration assumes that profits are in the upper range of the estimates, and that wages are quite flexible over the business cycle. Thus, our benchmark calibration provides a lower bound on the volatility of θ.

Computation. First, we use the free entry condition (9) and flow equation for the surplus

[6]They use confidential data from Jorgensen to compute the user cost of capital for 50 types of capital, plus land and inventories. The user cost depends on the required rate of return on capital, assumed to be equal to the dividend yield on the S&P 500, the depreciation rate, the investment tax credit, the present value of depreciation allowances and the corporate tax rate.

[7]We use the measures of wages computed by Solon, Barsky, and Parker (1994) (available at http://www.princeton.edu/~jparker/research/rwbc.html) that avoid the cyclical selection bias. As measures of productivity, we use seasonally adjusted real average output per person in the non-farm business sector constructed by the BLS from the National Income and Product Accounts and the Current Employment Statistics. We regress the change in log wages between each two consecutive years on the corresponding changes in log productivity and a time trend. The estimates vary, depending on the exact sample and specification used, around the value of 0.47.

(10) to derive the following difference equation in θ:

$$\begin{aligned}
\frac{c}{rq(\theta_p)} &= (1-\beta)E_p S_{p'} \\
&= (1-\beta)E_p\{p' - (z + \delta f(\theta_{p'})\beta E_{p'} S_{p''}) + \delta(1-s)E_{p'}S_{p''}\} \\
&= E_p\{(1-\beta)(p'-z) - \frac{f(\theta_{p'})\beta c}{q(\theta_{p'})} + \frac{(1-s)c}{q(\theta_{p'})}\} \\
&= E_p\{(1-\beta)(p'-z) - c\beta\theta_{p'} + \frac{(1-s)c}{q(\theta_{p'})}\}. \qquad (18)
\end{aligned}$$

We solve this difference equation to find θ as a function of productivity p. Next, given θ_p for every productivity level, we simulate the model to generate artificial time series for productivity, unemployment, vacancies, wages and profits. To do so, we start with an initial value for unemployment and productivity and draw a new productivity shock according to the Markov chain derived above. We then know θ and, thus, the job finding rate and the new unemployment rate. Iterating this procedure generates the time series of interest.

The performance of the model in matching calibration targets is described in Table 1. We are able to match the targets exactly. Values of calibrated parameters are summarized in Table 2.

3.2 Main Result

The statistics of interest computed from U.S. data in Shimer (2005), are presented in Table 3. Our goal in this paper is to evaluate whether a reasonably calibrated MP model can replicate these statistics.

We use the calibrated parameter values to simulate the model to create artificial time series. We then compute various relevant moments from these time series. Table 4 provides the results from the calibrated model. A comparison with the corresponding statistics in the data reveals that the model matches the key business cycle facts quite well. In particular, the volatility of unemployment, vacancies, and market tightness are close to those in the data.[8]

[8] Table 4 reveals that the stochastic properties of vacancies and labor market tightness are closely linked to those of productivity. In particular, the correlation of labor market tightness and productivity is too

3.3 A Comment on Using Profits as a Calibration Target

In the calibration above we assumed that all costs of posting vacancies are unmeasured, in the sense that they do not show up in wages nor in capital costs. We implicitly assumed that there is a perfect second-hand market for capital. This assumption is admittedly extreme since it implies that firms can buy capital only (and immediately) after they hired a worker. In this subsection we show that our results are robust even if we make the opposite extreme assumption: firms posting a vacancy must rent the same capital stock as if they already found a worker. The only Bellman equation that changes is the that for V, the value of a vacancy:

$$V_p \quad = \quad -c - (r+d)k + \delta q(\theta_p) E_p J_{p'}, \tag{19}$$

where r is the interest rate and d is the depreciation rate. The cost of posting a vacancy is $\tilde{c} = c + (r+d)k$. With c replaced by $\tilde{c}$, the rest of the model remains unchanged. But the assumption about the timing of capital purchases has consequences for our calibration strategy. So far, to compute per period accounting profits, we subtracted all wages and all capital costs from firms' revenues. Given our new timing assumption, part of the capital stock is held by firms who posted a vacancy but do not produce yet. To quantify this we need to be serious about the average value of θ. van Ours and Ridder (1992) find that the average vacancy lasts for 45 days in the Dutch economy, that is 1.5 months. This corresponds to a $\theta = 0.675$. This number appears reasonable, if not too high, as Hall (2005) reports, using Job Openings and Labor Turnover Survey (JOLTS), the value of 0.539. And the lower the value for θ, the smaller is the understatement of profits of operating firms.

How much of aggregate capital cost are borne by vacancy posting firms? With an unemployment level of 5%, the share of capital held by vacant firms is given by $\frac{v}{v+(1-u)} =$

high compared to the data. The problem is well known and seems to be that we allow for only one shock, to productivity, in our model. Since this paper is motivated by the facts on volatility of unemployment and vacancies, we do not address this concern here. We conjecture, however, that adding a shock to the matching function correlated with the level of productivity may help solve the problem. The correlation is not unreasonable if one accepts the idea that sectoral reallocation accounts for some of the cyclical volatility.

0.0343. Aggregate capital costs account for 1/3 of output, so that the capital costs borne by vacancies equal $0.0343/3 = 0.0114$. Thus, when we compute profits, we subtract 1.14% too much.

In addition, some wages may be paid to workers who are engaged in hiring. Given the model, the wages of these workers should be added to the profits of operating firms. The PSID shows that at most 0.5% of the employed workers can be considered as hiring personnel (Occupation code 056 in the 1970 US Census Occupational Classification: "Personnel and labor relation workers"). These workers can receive some secretarial, etc., support. So we can generously allow for approximately 1% of all wages paid to be paid for the servises of workers engaged in hiring, directly or indirectly.

Thus we get an upper bound of approximately 5% per period accounting profits of operating firm-worker pairs (= 3% + (1.14% + 1%) · 0.95). To assess the performance of the model, we now have to use a different matching function, that makes sure that the probability of finding a job and of filling a vacancy lies between 0 and 1 (since the precise value of θ is now important, we can no longer conveniently normalize it as we did above). We follow Den Haan, Ramey, and Watson (2000)(HRW) and choose

$$m(u, v) = \frac{u \cdot v}{(u^l + v^l)^{1/l}}.$$

Thus, our calibration targets for z, β, c, and l are a profit rate of 0.05, a productivity elasticity of wages of 0.47, an average value of θ of 0.675 and an average job finding rate of 0.45. Table 5 contains the results and the parameter values that generate the best (virtually exact) match between these targets and the corresponding statistics in the model.

As can be seen in Table 5, the model matches the key business cycle facts even if we make the extreme assumption that firms rent capital at the moment of posting a vacancy, and allow for a fraction of the wage bill in the data to represent hiring costs. Once again we find that the model generates the volatility of unemployment, vacancies, and market tightness that are close to those in the data.[9]

[9] We should note that the results are not driven by the difference in the matching function. If one were to recalibrate the model by following the common practice in the literature and picking $z = 0.4$ and setting

3.4 Analysis

3.4.1 Productivity Elasticity of θ and the Value of z

In this subsection we consider the model without aggregate uncertainty ($p = p'$) and derive the elasticity of θ with respect to aggregate productivity, $\epsilon_{\theta,p}$. We show that this elasticity is increasing in the value of the non-market activity of the workers, z.
In case of no aggregate uncertainty we can solve for the surplus:

$$S_p = \frac{p - z}{1 - \delta(1-s) + \delta f(\theta)\beta}. \tag{20}$$

Plugging this into the free entry condition yields:

$$\frac{p - z}{1 - \delta(1-s) + \delta f(\theta)\beta} = \frac{c}{\delta q(\theta)(1-\beta)}, \tag{21}$$

and, equivalently,

$$\frac{1 - \delta(1-s)}{\delta q(\theta)} + \beta\theta = \frac{p-z}{c}(1-\beta). \tag{22}$$

The elasticity of market tightness θ with respect to productivity p, $\epsilon_{\theta,p}$, equals $\frac{\partial\theta}{\partial p}\frac{p}{\theta}$. Implicit differentiation yields:

$$\begin{aligned}
\frac{\partial\theta}{\partial p} &= \frac{(1-\beta)/c}{\beta - \frac{\frac{\partial q(\theta)}{\partial\theta}}{q(\theta)^2}\frac{1-\delta(1-s)}{\delta}} && (23)\\
&= \frac{1}{p-z}\frac{\beta\theta + \frac{1-\delta(1-s)}{\delta q(\theta)}}{\beta - \frac{\frac{\partial q(\theta)}{\partial\theta}}{q(\theta)^2}\frac{1-r(1-s)}{r}} && (24)\\
&= \frac{\theta}{p-z}\frac{\beta\theta q(\theta) + \frac{1-\delta(1-s)}{\delta}}{\beta\theta q(\theta) - \frac{\frac{\partial q(\theta)}{\partial\theta}\theta}{q(\theta)}\frac{1-\delta(1-s)}{\delta}} && (25)\\
&= \frac{\theta}{p-z}\underbrace{\frac{\beta f(\theta) + (1-\delta(1-s))/\delta}{\beta f(\theta) + (1-\eta)(1-\delta(1-s))/\delta}}_{=:k1}, && (26)
\end{aligned}$$

where η is the elasticity of $f(\theta)$ with respect to θ.

β according to Hosios' condition, the model will essentially replicate the result in Shimer (2005), i.e. the finding that the volatility of vacancies and unemployment is smaller in the model than in the data by a factor of ten.

Thus,

$$\epsilon_{\theta,p} = \frac{p}{p-z} k_1. \tag{27}$$

Since it turns out that, given the estimated parameter values, $k_1 \approx 1.4$, the elasticity is as large as in the data only if $p - z$ is small. In a competitive model without search frictions $p - z$ equals zero. In a search model this can not be the case as it would result in zero operating profits and, consequently, in the absence of vacancy posting by the firms. (Firms have to make positive profits when operating to be willing to post vacancies.) Since we do not want to choose z arbitrarily, we have used the data on (positive) profits and wages to pin down $p - z$.

3.4.2 The Role of β and z

In this subsection we relate the elasticity of θ with respect to aggregate productivity, $\epsilon_{\theta,p}$, to the level of profits and the cyclical properties of wages. We show that the fact that profits are small and wages are only mildly procyclical in the data implies that the outside option of the workers, z, has to be close to their productivity level, p, and workers' bargaining weight has to be relatively small.

Without aggregate uncertainty it holds that

$$J_p = p - w + \delta(1-s)J_p \tag{28}$$

and, thus,

$$w = p - J_p(1-\delta(1-s)) = p - (1-\beta)S_p(1-\delta(1-s)) \tag{29}$$

$$w = p - (1-\beta)(1-\delta(1-s))\frac{p-z}{1-\delta(1-s)+\delta f(\theta)\beta}. \tag{30}$$

Profits are equal to $p - w$:

$$\Pi = \frac{(1-\beta)(1-\delta(1-s))}{1-\delta(1-s)+\delta f(\theta)\beta}(p-z). \tag{31}$$

Finally, consider the derivative of wages with respect to productivity:

$$\frac{\partial w}{\partial p} = 1-\frac{(1-\beta)(1-\delta(1-s))}{1-\delta(1-s)+\delta f(\theta)\beta} \tag{32}$$

$$+ \delta\beta(1-\beta)(1-\delta(1-s))\frac{p-z}{(1-\delta(1-s)+\delta f(\theta)\beta)^2}\frac{\partial f(\theta)}{\partial p}. \tag{33}$$

Since $\frac{\partial f(\theta)}{\partial p}$ is positive, $\frac{\partial w}{\partial p}$ is small if $\frac{(1-\beta)(1-\delta(1-s))}{1-\delta(1-s)+\delta f(\theta)\beta}$ is large. Profits, on the other hand, are small only if $(p-z)\frac{(1-\beta)(1-\delta(1-s))}{1-\delta(1-s)+\delta f(\theta)\beta}$ is small. This immediately leads to the conclusion that $p-z$ and β have to be small. The explanation is easy. Small profits means that $p-w$ is small and mildly procyclical wages mean that $w-z$ is small.

To illustrate the quantitative importance of properly calibrating z and β, we recalibrate the model by following the common practice in the literature (e.g. Shimer (2005)) and picking $z = 0.4$ and setting $\beta = \alpha$ according to Hosios condition. We use $\alpha = 0.72$, as in Shimer (2005) in this experiment. The results are summarized in Table 6. Not surprisingly, the standard calibration of the model suggests that the volatility of vacancies and unemployment is smaller in the model than in the data by a factor of ten.

3.4.3 The Role of α

The values of α used in the literature vary considerably. Shimer (2005) considers $\alpha = 0.72$, Andolfatto (1996) $\alpha = 0.6$, Farmer (2004) $\alpha = 0.5$, Merz (1995) $\alpha = 0.4$ and Hall (2005) $\alpha = 0.235$. Changing α can have an effect on the volatility of vacancies but it definitely has a large effect on the volatility of unemployment. Hall (2005) finds the volatility of unemployment to be high, much higher than for vacancies. In Farmer (2004) vacancies are slightly more volatile than unemployment, and in Shimer (2005) vacancies are much more volatile than unemployment. The following simple (deterministic) steady state comparative statics arguments show why. In a steady state the outflow from unemployment equals the inflow into unemployment:

$$s(1-u) = m(u,v). \tag{34}$$

Implicit differentiation of u with respect to v gives:

$$\frac{\partial u}{\partial v} = -\frac{(1-\alpha)m/v}{s+\alpha m/u}. \tag{35}$$

Thus, for the elasticity $\epsilon_{u,v}$ it holds that

$$|\frac{1}{\epsilon_{u,v}}| = \frac{s+\alpha m/u}{(1-\alpha)m/v}\frac{1}{\theta} \tag{36}$$

$$= \frac{\alpha}{(1-\alpha)} + \frac{s}{(1-\alpha)f}. \tag{37}$$

As s is small relative to f, the first term $\frac{\alpha}{(1-\alpha)}$ matters a lot. For $\alpha = 0.72$ it equals 2.57, for $\alpha = 0.5$, it equals 1 and for $\alpha = 0.235$ it equals 0.31. Quite a difference. And it affects the quantitative results. In Table 7 we redo our calibration exercise with only one change, α now equals 0.72. The volatility of unemployment changes a lot whereas the volatility of vacancies remains almost unchanged. For lower α, a percentage change in vacancies results in a much higher change in matches and thus unemployment. The table also shows that vacancies become less persistent. If vacancies increase today, next period's unemployment rate is low and the market is tight. Therefore next period's incentives to post vacancies are lower than this period's incentives.

Viewed from a different perspective, $\alpha = 0.5$ means that the variance of $logv/u$ is close to be evenly split between (log) vacancies and (log) unemployment. For the volatility of vacancies not to decrease when α decreases, labor market tightness has to become more volatile. In Section 3.4.1 we have shown, for a deterministic version of the model, that the elasticity of θ with respect to productivity equals $\frac{1}{p-z} \cdot \frac{\beta f + (1-\delta(1-s))/\delta}{\beta f + \alpha(1-\delta(1-s))/\delta}$. Since our estimate of β is small, this elasticity goes up substantially when α is decreased. As a result, a decrease in the volatility of vacancies is not necessarily an implication of a decrease in α. With our calibration strategy it remains almost unchanged when α is decreased. If we impose the Hosios' condition, the elasticity increases by a smaller amount but volatilities are smaller. Again, the volatility of vacancies remains largely unchanged.

4 Expected Wages and the Value of β

In the benchmark calibration we took seriously all the features of the MP model, including its assumption about the formation of spot wages. Different wage profiles with different statistical properties, such as contemporaneous correlation with productivity, may generate the same expected total wages during the employment relationship. The expected present value of wages is all that matters both for the employer and the employee. In this Section we use data on expected wages to pin down agents' bargaining power.

One caveat is in order, however. In the MP model, Nash bargaining not only determines the sharing of surplus in the first period but there is continuous Nash-renegotiation. If both parties prefer a different path of wages when they first meet, they need some form of commitment to stick to this path. Although in a more general model there may be incentives to do so ex-ante, e.g. risk-sharing, it is not clear how binding these agreements are. There are no restrictions on worker mobility. Whenever a worker gets an offer from another firm, his wage has to be adjusted (perhaps, to his actual productivity level). Since the probability of finding a job is as high as 45% per month reported in Shimer (2005), outside offers are likely to be the rule rather than the exception (see also Nagypal (2004b)). Furthermore, the finding in Shimer (2005) that the average job lasts for about 2.5 years also undermines the scope of binding long-term contracts. Overall, the data on job-finding and job-duration suggest that there are frequent wage-negotiations and, thus, changes in wages are closely related to changes in productivity. The empirical elasticity of spot wages then says by how much.

Notwithstanding these concerns, assume that there is some form of commitment. Agents then agree on a possibly different (from continuous renegotiation) path of wages but with the same expected present value. For example, wages can be constant as in Shimer (2004). A good robustness check, or a possible calibration strategy for the worker's bargaining weight β, then, is to compare the volatility of the present value of wages during employment relationships starting at different stages of the business cycle.

Computing this statistic in the data is complicated due to the selection bias discussed in Solon, Barsky, and Parker (1994). Because the employment shares of low-wage groups of workers tend to be procyclical, the aggregate wage statistics give greater weight to low-skill workers during expansions than during recessions. To alleviate this problem, we adopt the following strategy. We use the PSID data over the 1968-1997 period. We restrict the sample to workers who have an even number of employment spells with equal number of them starting when productivity is above and below the trend. We compute the total sum of real wages during the first (at most) seven years of each employment spell. Next, we separately compute the expected present value of wages of the spells starting above and below the trend, respectively. We find the ratio of the expected wage of the spells starting above the trend to the expected wage of the spells starting below the trend of about 6 percent.[10]

We then compute the same statistic in the model. An expression for expected wages is easy to derive. We know that the continuously renegotiated wage equals

$$w_t \quad = \quad \beta p_t + (1-\beta)z + c\beta\theta_t \tag{38}$$

and gives the worker exactly his expected wage. The expected present value of wages W_p, when entering the employment match at productivity level p, then equals the expected present value of w_t:

$$W_p \quad = \quad E_p(\sum_{t=0}^{\infty}(\delta(1-s))^t(\beta p_t + (1-\beta)z + c\beta\theta_t)). \tag{39}$$

This expression for W_p can be derived without making explicit use of w. The crucial argument is that the surplus S_p is independent from the exact sequence of spot wages.

The average expected present value of wages for someone getting a job when p is above or below its mean $\overline{p}$, W^{Boom} and W^{Bust}, respectively, equal

$$W^{Boom} \quad = \quad \sum_{p>\overline{p}} f(\theta_p)\cdot u\cdot W_p, \tag{40}$$

$$W^{Bust} \quad = \quad \sum_{p\leq\overline{p}} f(\theta_p)\cdot u\cdot W_p. \tag{41}$$

Since the unemployment rate u is a state variable, we compute this statistic from a simulation. Because the PSID is annual while our model period is a month, we replicate the

[10]This number is not very robust, varying from 4 to 10 percent, depending on the exact procedures and specification. We are working on obtaining a more robust estimate. Our biggest concern now is the way to deal with the top-coding issue. Solon, Barsky, and Parker (1994) overcame this problem in their analysis of the volatility of spot wages by actually obtaining the true values of top-coded observations from the original PSID interview forms.

sampling procedures of the PSID on the simulated data. As in the data, we restrict to the first seven years of the spells that last more then seven years. The ratio of W^{Boom} to W^{Bust} is equal to 7.4% given our benchmark calibration. This seems within the range of estimated parameters of the data, but we are reluctunt to take it too seriously for now.

5 Discussion

In this section we discuss several implications and possible concerns about our procedure and our results.

First, our calibration strategy implies a value of the bargaining power parameter that is considerably lower than the standard estimate of the elasticity of the matching function with respect to unemployment. This appears to suggests that the economy is far from the efficient benchmark. We show that this argument is incorrect. In particular, we prove that in the presence of taxes, welfare is maximized with the worker's bargaining weight lower than that implied by the Hosios condition.

Second, our calibration strategy implies that the returns to the market and non-market activities are fairly close to each other. We discuss why we think this finding is quite "reasonable".

Third, our calibration implies that there are strong effects of productivity shocks on unemployment and wages. This suggests that policy changes, such as taxation and unemployment insurance can have large effects as well. We explore these policy implications in section 5.3.

5.1 Efficiency with Taxation

Our quantitative analysis so far has ignored taxes. This section shows that this has two consequences. First, market activity provides more than 6 percent incremental value over non-market activity. Second, the Hosios condition ceases to imply efficiency.

The crucial step when adding taxes to the model, is to derive wages. They are determined through the generalized Nash bargaining solution, which selects, for every p, w to maximize

$$(J_p(w \cdot (1+\tau_f)))^{1-\beta}(W_p((1-\tau_w)\cdot w) - U_p)^{\beta}, \tag{42}$$

where, as before, $J_p(), W_p()$ are the values to the firm and the worker at productivity level p, τ_f is a wage tax to be paid by the firm and τ_w is the wage tax to be paid by the worker, respectively. Set $\tilde{w}_p = w_p(1-\tau_w)$ and $\hat{w}_p = w_p(1+\tau_f)$.

The optimum satisfies:

$$(1-\beta)(1+\tau_f(\theta_p))(W_p(\tilde{w}_p) - U_p) = \beta J_p(\hat{w}_p)(1-\tau_w). \tag{43}$$

This implies that

$$J_p(\hat{w}_p) = (1-\beta)(\frac{1+\tau_f}{1-\tau_w}(W_p(\tilde{w}) - U_p) + J_p(\hat{w}_p)), \tag{44}$$

and

$$W_p(\tilde{w}_p) - U_p = \beta\frac{1-\tau_w}{1+\tau_f}(\frac{1+\tau_f}{1-\tau_w}(W_p(\tilde{w}_p) - U_p) + J_p(\hat{w}_p)). \tag{45}$$

The flow equation for J_p is:

$$J_p(\hat{w}_p) = (\tilde{p} - \hat{w}_p) + \delta(1-s)E_pJ_{p'}(\hat{w}_{p'}), \tag{46}$$

where $\tilde{p}$ is the after sales tax revenue/productivity. We ignore profit taxes since they can be replicated through a sales tax and a tax on wages to be paid by firms.

The free entry equation is unchanged:

$$c = \delta q(\theta_p)E_pJ_{p'}(\hat{w}_{p'}). \tag{47}$$

Define the surplus

$$S_p = (\frac{1+\tau_f}{1-\tau_w}(W_p(\tilde{w}_p) - U_p) + J_p(\hat{w}_p)). \tag{48}$$

It holds that

$$\begin{aligned}
S_p &= \frac{1+\tau_f}{1-\tau_w}(\tilde{w} + \delta(1-s)E_p(W_{p'}(\tilde{w}_{p'})) + \delta sE_p(U_{p'}) \\
&- \frac{1+\tau_f}{1-\tau_w}(z + \delta f(\theta_p)E_pW_{p'}(\tilde{w}_{p'}) + \delta(1-f(\theta_p))E_pU_{p'}) \\
&+ (\tilde{p} - \hat{w}) + \delta(1-s)E_pJ_{p'}(\hat{w}_{p'}) \\
&= \tilde{p} - \frac{1+\tau_f}{1-\tau_w}z + \delta(1-s)E_pS_{p'} - \delta f(\theta_p)\beta E_pS_{p'}.
\end{aligned}$$

Thus,

$$J_p(\hat{w}) = (1-\beta)(\tilde{p} - \frac{1+\tau_f}{1-\tau_w}z) + \delta(1-s)E_p J_{p'}(\hat{w}) - c\beta\theta_p. \tag{49}$$

For wages we have that

$$(\tilde{p} - \hat{w}_p) = (1-\beta)(\tilde{p} - \frac{1+\tau_f}{1-\tau_w}z) - c\beta\theta_p, \tag{50}$$

and

$$\hat{w}_p = \beta\tilde{p} + (1-\beta)\frac{1+\tau_f}{1-\tau_w}z + c\beta\theta_p, \tag{51}$$

and

$$\tilde{w}_p = \beta\frac{1-\tau_w}{1+\tau_f}\tilde{p} + (1-\beta)z + c\beta\frac{1-\tau_w}{1+\tau_f}\theta_p. \tag{52}$$

Profits equal

$$\Pi = \tilde{p} - \hat{w}_p = (1-\beta)\tilde{p} - (1-\beta)\frac{1+\tau_f}{1-\tau_w}z - c\beta\theta_p. \tag{53}$$

Effective average tax rates, that are consistent with the concept of aggregate tax rates at the national level, are provided by Mendoza, Razin, and Tesar (1994). In 1987 the consumption tax rate equaled 5.1 percent and the labor tax rate equaled 29.1 percent.[11] Their results imply $\tau_f = 0$, $\tau_w = 0.291$ and $\tilde{p} = (1-0.051)p$. As said above, this has two implications. First, when we estimate z, we really estimate $\frac{1+\tau_f}{1-\tau_w}z$. Our estimate for z is 0.943 but the true value of z is 0.67. Instead of normalizing p to 1 we really normalize $\tilde{p}$ to be one. The implicit normalization on p is then $p = 1/0.949 = 1.054$. Thus instead of 0.06, $p - z$ equals 0.385.

Second, we can compute the bargaining power that maximizes social welfare. This number will be lower than α.

The efficient levels of θ's are the solution to the following optimization problem:

$$SW_p(u) = Max_\theta(zu + p(1-u) - cu\theta + \delta E_p SW_{p'}((1-s)u - f(\theta)u)). \tag{54}$$

[11]Lucas (1990) and Prescott (2004) use an effective tax rate of 0.4. The reason for their higher value is that in a competitive model the marginal tax rate matters whereas here the average tax is relevant.

Using the envelope theorem, one can show that the optimum for productivity level p, θ_p^*, satisfies:

$$\frac{c}{\delta q(\theta_p^*)} = E_p\{(1-\alpha)(p'-z) - c\alpha\theta_{p'}^* + \frac{(1-s)c}{q(\theta_{p'}^*)}\}. \tag{55}$$

Consider a deterministic version and let θ^* be the optimal market tightness. It solves

$$\frac{c}{\delta q(\theta^*)} = (1-\alpha)(p-z) - c\alpha\theta^* + \frac{(1-s)c}{q(\theta^*)}. \tag{56}$$

For $\delta = 0.997$, $s = 0.034$, $c = 0.212$, $p = 1.054$, $z = 0.67$ and $\alpha = 0.5$, θ^* equals 1.55. We can now solve for the bargaining power such that the efficient amount of vacancies is posted.

$$\frac{c}{\delta q(\theta^*)} - \frac{(1-s)c}{q(\theta^*)} = (1-\alpha)(p-z) - c\alpha\theta^* = (1-\beta)(\tilde{p} - \frac{1+\tau_f}{1-\tau_w}z) - c\beta\theta^*. \tag{57}$$

The result is $\beta = 0.073$.

5.2 The Value of Non-Market Activity

Our calibrated value for z, the value of non-market activity, implies that a typical unemployed worker is not substantially worse off than being employed. Our estimate appears reasonable since z is a sum of the value of leisure, unemployment benefits, home production, self-employment, dis-utility of work, etc. A value of $z \approx 0.4$, typically used in the literature, on the other hand, seems to be unreasonably low as non-market activity is identified with receiving unemployment benefits only. Unemployment benefits are not likely to constitute a substantial fraction of the total value of being unemployed as suggested by the finding in Anderson and Meyer (1997) that only between 24% and 54% of workers eligible to receive unemployment insurance even bother to apply for it.

Our finding that a typical unemployed worker does not suffer a substantial decline in utility has to be interpreted with caution, however. In the model, z does not depend on the length of the unemployment spell. This is a strong assumption. Long-term unemployed for sure are worse off as they face problems replacing their durable consumption goods (a broken TV, dishwasher, microwave, etc.). Furthermore, having a month off to enjoy leisure has a high value, but the enjoyment of a year of unemployment is questionable. In our calibration we (implicitly) estimate the average z of all unemployed. Since the job finding rate equals 45% per month on average, short-term unemployed are the bulk of observations. Thus, our estimate of z represents the value of unemployment for the marginal worker, who finds employment quickly. It is not informative about the value of long-term unemployment since this is a low probability event.

Taking into account that z decreases with the length of the unemployment spell makes z endogenous. When productivity goes down, the average duration of unemployment increases and thus the average z of the unemployment pool goes down as well. This is an interesting and, we believe, productive way to add curvature on the worker side to the model. It is also useful to add curvature through assuming decreasing returns to labor. In Appendix I we show that the standard linear model is an approximation to a more general model with curvature in utility and production. It is important to use the nonlinear version of the model if large changes in productivity or policy are considered, since a linear version is not a good approximation anymore.

To formalize the discussion above, we now develop a model with endogenous z. Suppose that z can take $N+1$ values $z_0 > z_1 > \ldots z_{N-1} > z_N$ and D describes the state of depreciation, that takes integer values between 0 and N. For unemployed, z depreciates with probability $p_d = Prob(D = d+1 \mid D = d)$ in state $D < N$. There is no further depreciation in state $D = N$. For employed, z accumulates with probability $a_d = Prob(D = d-1 \mid D = d)$ in state $D > 0$. There is no further accumulation in state $D = 0$. π_d is the number of unemployed with value of non market activity z_d and ϕ_d is the number of employed with value of non market activity z_d. The number of unemployed workers then equals $\sum_{d=0}^{N} \pi_d$, and the number of employed workers is $\sum_{d=0}^{N} \phi_d$.

Let W_d be the value of becoming employed if the value of non-market activity has depreciated d times. W_0 is the maximal value of having a job (value of non-market activity is at its maximum z_0). J_d is the value of a filling a vacancy with a worker whose z has depreciated d times. J_0 is the value of a filled vacancy if the worker' z is at its maximum. U^d is the value of being unemployed if the value of non-market activity has depreciated

d times. A newly unemployed worker has the same value of z he would have had in the case of no separation (whether accumulation took place does not depend on the exogenous separation.). The remaining features of the model remain unchanged.

The following Bellman equations now describe the model:

$$
\begin{aligned}
J_d &= p - w_d + \delta(1-s)E_p(a_d J'_{d-1} + (1-a_d)J'_d) \\
J_0 &= p - w_0 + \delta(1-s)E_p J'_0 \\
U_d &= z_d + \delta(f E_p(p_{d+1}W'_{d+1} + p_d W'_d) + (1-f)E_p(p_{d+1}U'_{d+1} + p_d U'_d)) \\
U_N &= z_N + \delta((1-f)U'_N + f W'_N) \\
W_d &= w_d + \delta((1-s)E_p(a_d W'_{d-1} + (1-a_d)W'_d) + sE_p(a_d U'_{d-1} + (1-a_d)U'_d)) \\
W_0 &= w_0 + \delta((1-s)E_p(W'_0) + sE_p U'_0) \\
V_p &= -c + \delta q(\theta_p)E\sum_{d=0}^{N}\frac{\pi_d}{u}J'_d.
\end{aligned}
$$

The law of motion for π_d is as follows:

$$
\begin{aligned}
\pi'_d &= (1-f(\theta))(\pi_d(1-p_{d+1}) + \pi_{d-1}p_d) + s(a_{d+1}\phi_{d+1} + (1-a_d)\phi_d) \quad \text{for } 1 \le d \le N-1. \\
\pi'_0 &= (1-f(\theta))\pi_0(1-p_0) + (\phi_0 + a_1\phi_1)s. \\
\pi'_N &= (1-f(\theta))(\pi_N + p_{N-1}\pi_{N-1}) + s\phi_N(1-a_N).
\end{aligned}
$$

The law of motion for ϕ_d is as follows:

$$
\begin{aligned}
\phi'_d &= (1-s)(a_{d+1}\phi_{d+1} + (1-a_d)\phi_d) + f(\theta)((1-p_d)\pi_d + p_{d-1}\pi_{d-1}) \quad \text{for } 1 \le d \le N-1. \\
\phi'_0 &= (1-s)(\phi_0 + a_1\phi_1) + f(\theta)(1-p_0)\pi_0. \\
\phi'_N &= (1-s)(1-a_N)\phi_N + f(\theta)(\pi_N + p_{N-1}\pi_{N-1}).
\end{aligned}
$$

It is straightforward to add curvature in labor to the model. For productivity it holds that

$$
p_t = A_t F_L(1-u_t, \overline{K}) = A_t(1-\eta)(1-u_t)^{-\eta}\overline{K}^{\eta},
$$

where $F(L, K)$ is the production function (assumed to be Cobb-Douglas for the second equality), A_t is the technology shock, and $\overline{K}$ is the fixed capital stock[12]. In equilibrium $V_p = 0$. Wages are formed through axiomatic Nash-bargaining. That is, W, J and U are related as follows:

$$(W_d - U_d)(1-\beta) = J_d\beta. \tag{58}$$

These equations determine wages in equilibrium.

The state variables are now: $A, \pi_0, \ldots, \pi_N,\ \phi_0, \ldots \phi_N$, and the values functions are: $U_d(\theta, A), W_d(\theta, A), J_d(\theta, A)$, where labor market tightness θ is a function of A and π_i: $\theta(\pi_0, \ldots, \pi_N, A)$.

The computational strategy is as follows: Guess on the value function U_d, W_d and J_d. Plug these values into the right hand side Bellman equations. Compute θ from the free-entry condition and plug in it. We get today's values for U, W and J as a function of wages w. We then use the axiomatic Nash bargaining solution to pin down the wage. Thus we have new candidates for our three value functions. We use a discretized $AR(1)$ process for technology A. The law of motions for technology, π and ϕ are used to compute expectations. We iterate until convergence is reached.

Quantitative analysis of this version of the model will be performed in the next draft. Modeling curvature, however, is unlikely to dampen the ability of the model to replicate business cycles facts. It creates some procyclicality in z but our calibration strategy implies that the bargaining power will likely have to be reduced further.[13] Since the effects of productivity shocks are relatively short lived, the average duration of unemployment, and

[12]We assume no capital accumulation for simplicity only. This assumption can be relaxed. With perfect rental market for capital, $r = 1/\delta + d$, where d is the depreciation rate. The capital stock is not fixed but solves $F_K = r$. This results in a fixed-point problem as the capital stock and employment have to be determined jointly. Labor productivity will depend on the amount of capital as well.

[13]The value of non-market activity can be cyclical for several other reasons, such as correlation between market and home technology. Unfortunately, the correlation between p and z and the bargaining power β are not jointly identified from observation of wages $w_p = \beta p + (1-\beta)z + c\beta\theta_p$. A related problem plagues the estimation of real business cycles models with home production (for example McGrattan, Rogerson, and Wright (1997)). They cannot simultaneously identify the elasticity of substitution between market and non-market activities and the correlation between market and non-market productivity.

thus the average z, is unlikely to change much over the business cycle. If anything, modeling depreciation of z may amplify the effects of productivity shocks.

5.3 Response of the Model to Changes in Policy

Costain and Reiter (2003) suggest that z cannot be too large relative to market productivity in the MP model because in this case changes in unemployment insurance would have strong effects on unemployment. They suggest that these effects are counterfactual. Unfortunately, the effects of changes in unemployment insurance are hard to measure in the data. There exist a number of microeconomic studies, surveyed in Meyer (1995), that suggest that these effects are small. A problem with this approach to evaluating the MP model lies in a fact that, in general, a change in policy affects two decisions. Those by unemployed to search and those by firms to post vacancies. Microeconomic studies address the first decision. In a typical microeconomic study, a small fraction of unemployed is given a bonus if they find a job fast. Their expected unemployment duration decreases by much less than what would be predicted by the MP model in the case of a permanent subsidy to all unemployed. If we replicate these studies in the model, the effect would be virtually zero. Because matching is random, firms expected profits do not change if a small fraction of unemployed has a higher z. In our view, these microeconomic studies explore the elasticity of search decisions to monetary incentives. We take the evidence, that these effects are small, as a justification for not endogenizing worker's search effort. It is the effect on vacancy posting that makes the response in MP large but it is not measured by microeconomic studies. We are also skeptical that the effect of UI can be isolated and that endogeneity problems can be overcome in cross-country regressions. In particular, we think that results from these regressions (summarized in Layard and Nickell (1999)) cannot be used to assess the success of a model.[14]

Moreover, any model where shocks to productivity are strongly amplified, is likely to

[14]More direct evidence on economy wide changes in policy is provided by studies on the effects of the earned income tax credit (EITC). The consensus in the literature (summarized in Hotz, Mullin, and Scholz (2001)) seems to imply very strong effects of changes in this policy on employment.

exhibit strong effects of policies as well. The argument is simple. Any sequence of productivity shocks can be replicated through a sequence of sales taxes. In a basic real business cycle model, productivity and tax changes have identical effects both on first-order conditions and households' budget constraint – the conditions that characterize the equilibrium. In a model with employment lotteries (e.g., Rogerson (1988)), increasing unemployment insurance acts like a wage tax, effects of which are close to that of a productivity shock. Consequently, our calibration of the MP model makes it useful for studying the effects of policies. Thus, the basic search and matching model joins the pool of models which can rationalize differences in output and unemployment through differences in policy.[15]

Our major concern with the policy analysis in the MP model lies in its linearity. As we show in Appendix I, the basic model is a linear approximation to a more general model with curvature in p. With a decreasing marginal product of labor p, a one percentage increase in p requires more than a one percent change in productivity. This is unproblematic for our results on the amplification of productivity since what matters and what we measure in the data is p.[16] But the effect of a one percentage point change in tax rates is damped through the curvature in labor.

Additional arguments apply to changes in unemployment insurance. Anderson and Meyer (1997) show that 50% of unemployed are eiligible and at most 50% of those eligible apply for unemployment benefits. Thus a change in unemployment insurance (given the same application behavior) affects the wages of at most 25% of the unemployed. As a result, expected profits decrease by much less than in a world where all unemployed receive unemployment insurance.

The size of changes in unemployment insurance is presumably dampened further if we allow for on-the-job-search as in Pissarides (1994), Burdett and Mortensen (1998) and Nagypal (2004a). Firms post vacancies to attract both employed and unemployed workers, but z has a strong effect on the wages of unemployed only. The overall effect of z on profits

[15]Prescott (2004), for example, finds that differences in marginal tax rates alone can explain why Americans work 50 percent more than do the Germans, French and Italians. And Ljungqvist and Sargent (1998) argue that the welfare state is responsible for the persistently high European unemployment rates.

[16]If we measure say a two percent increase in p, technology went up by more than two percent.

and vacancy posting depends on how much profits firms make out of hiring an employed or an unemployed. If firms prefer to hire employed, as in Nagypal (2004a), the effect of changes in z will be reduced.

Another feature of the labor market that diminishes the effect of changes in unemployment insurance is the presence of minimum wages and of the public sector with largely administered wages. For some workers a government job is the relevant outside option, for others the minimum wage is binding, i.e. it is higher than the outcome of negotiations w_p. Changes in z then affect wages only of the remaining workers, whose wages equal w_p and their outside option is unemployment (if the minimum wage is still binding for others). Both these institutional features do not affect the volatility of v or θ since the model has to be re-calibrated. What matters here is the elasticity of v and θ with respect to productivity p. And this elasticity depends on the size of profits and the average elasticity of wages, our two calibration targets. Of course, parameters values change, but the elasticity and, thus, the volatility, do not.

Considering these arguments, we doubt that there are very strong effects of unemployment insurance on unemployment and wages. Changes in taxation, on the other hand, are likely to have stronger effects.

6 Conclusion

The Mortensen-Pissarides search and matching model has become the standard theory of equilibrium unemployment. As Shimer (2005) puts it, "the model is attractive for a number of reasons: it offers an appealing description of how the labor market functions; it is analytically tractable; it has rich and generally intuitive comparative statics; and it can easily be adapted to study a number of labor market policy issues, such as unemployment insurance, firing restrictions, and mandatory advanced notification of layoffs." Moreover, in this paper we found that a reasonably calibrated model is consistent with the key business cycle facts. Given the recent controversy in the literature on this issue, our finding is comforting.

The main contribution of this paper lies in the way we calibrate two key parameters of the model - the value of non-market activity and the bargaining weight of the workers. We use data on profits and on the cyclicality of wages to pin down these two parameters.

Our calibration implies that the value of non-market activity is close to market productivity. This is the result one would expect in a frictionless competitive environment. It then seems reasonable that a search and matching model, that shares many features with an RBC model, exhibits a similar relationship. A typical practice in calibrating the MP model is to set the value of non-market activity to the replacement rate of unemployment insurance. We note, however, that it also includes the value of leisure and home production, and, given the structure of the model, the costs of earning income. We should reiterate that we only found a high value of non-market activity for a marginal worker, i.e., a worker with the average unemployment duration of about two months. Our finding does not imply a high value of non-market activity for the long-term unemployed. We simply do not observe many such individuals in the unemployment pool.

We also find a relatively low value for the workers' bargaining weight. In this sense, our calibration of the model implies that the wage setting mechanism is close to wage posting.

Our finding that policies may have strong effects on employment and wages makes the model a useful laboratory to conduct economic experiments. A key shortcoming of the standard model for the purpose of policy analysis, is its linearity. This is easily fixed, however.

Table 1: Matching the Calibration Targets.

	Target	Value	
		Data	Model
1.	Average Profit	0.030	0.030
2.	Elasticity of wages w.r.t. productivity, $\epsilon_{w,p}$,	0.470	0.470
3.	Average job finding rate, f,	0.450	0.450
4.	Average market tightness (normalization), θ,	1.800	1.800

Note - The table describes the performance of the model in matching the calibration targets.

Table 2: Calibrated Parameter Values.

Parameter	Definition	Value
z	value of non-mrkt activity	0.943
β	workers' bargaining power	0.061
μ	scale parameter of matching function	0.345
α	elasticity parameter of matching function	0.500
c	cost of vacancy	0.212
δ	discount factor	0.997
s	separation rate	0.034
ρ	persistence of productivity process	0.970
σ_ϵ^2	variance of innovations in productivity process	0.007

Table 3: Summary Statistics, quarterly US data, 1951 to 2003.

		u	v	v/u	f	p
Standard Deviation		0.190	0.202	0.382	0.118	0.020
Quarterly Autocorrelation		0.936	0.940	0.941	0.908	0.878
	u	1	-0.894	-0.971	-0.949	-0.408
	v	—	1	0.975	0.897	0.364
Correlation Matrix	v/u	—	—	1	0.948	0.369
	f	—	—	—	1	0.396
	p	—	—	—	—	1

Source - Shimer (2005).

Note - Seasonally adjusted unemployment u is constructed by the Bureau of Labor Statistics (BLS) from the Current Population Survey (CPS). The seasonally adjusted help-wanted advertising index v is constructed by the Conference Board. The job finding rate f is constructed from the seasonally adjusted employment, unemployment, and mean unemployment duration, all computed by the BLS from the CPS. u, v, and f are quarterly averages of monthly series. Average labor productivity p is seasonally adjusted real average output per person in the non-farm business sector, constructed by the BLS from the National Income and Product Accounts and the Current Employment Statistics. All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5.

Table 4: Results from the Calibrated Model.

		u	v	v/u	f	p
Standard Deviation		0.174	0.235	0.391	0.196	0.020
Quarterly Autocorrelation		0.916	0.760	0.874	0.874	0.878
	u	1	-0.828	-0.941	-0.941	-0.934
	v	—	1	0.969	0.968	0.942
Correlation Matrix	v/u	—	—	1	1	0.981
	f	—	—	—	1	0.981
	p	—	—	—	—	1

Note - All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5. Calibrated parameter values are described in Table 2.

Table 5: Results from the Calibration with 5% Profits and HRW Matching Function.

		u	v	v/u	f	p
Standard Deviation		0.243	0.209	0.406	0.299	0.020
Quarterly Autocorrelation		0.916	0.599	0.905	0.837	0.878
	u	1	-0.609	-0.913	-0.900	-0.857
	v	—	1	0.880	0.879	0.804
Correlation Matrix	v/u	—	—	1	0.992	0.927
	f	—	—	—	1	0.873
	p	—	—	—	—	1

Note - All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5. Calibrated parameter values are: $z = 0.920$, $\beta = 0.042$, $l = 1.309$, $c = 0.928$, $\delta = 0.997$,$s = 0.034$, $\rho = 0.970$, $\sigma_\epsilon^2 = 0.007$.

Table 6: Results form the Standard Parameterization of the Model.

		u	v	v/u	f	p
Standard Deviation		0.008	0.025	0.032	0.09	0.020
Quarterly Autocorrelation		0.912	0.840	0.878	0.878	0.878
	u	1	-0.914	-0.951	-0.951	-0.951
	v	—	1	0.995	0.995	0.994
Correlation Matrix	v/u	—	—	1	1	0.999
	f	—	—	—	1	0.999
	p	—	—	—	—	1

Note - This table replicates the experiment in Shimer (2005). The workers' value of non-market activity is set to $z = 0.4$ and workers' bargaining weight and the elasticity of matching function with respect to unemployment, α, is set to $\alpha = \beta = 0.72$. All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5.

Table 7: Results from the Model Calibrated with $\alpha = 0.72$.

		u	v	v/u	f	p
Standard Deviation		0.07	0.215	0.275	0.009	0.020
Quarterly Autocorrelation		0.913	0.835	0.878	0.878	0.878
	u	1	-0.91	-0.948	0.951	-0.939
	v	—	1	0.994	0.994	0.979
Correlation Matrix	v/u	—	—	1	1	0.986
	p	—	—	—	—	1

Note - All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5. The only difference from benchmark calibration is that the elasticity of matching function with respect to unemployment, α, is set to $\alpha = 0.72$. We still choose β and z to match the data on profits and the elasticity of wages with respect to productivity. All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5.

References

Abraham, K. G., and L. F. Katz (1986): "Cyclical Unemployment: Sectoral Shifts or Aggregate Disturbances?," *Journal of Political Economy*, 94(3), 507–522.

Anderson, P., and B. Meyer (1997): "Unemployment Insurance Takeup Rates and the After-Tax Value of Benefits," *Quarterly Journal of Economics*, 112(3), 913–937.

Andolfatto, D. (1996): "Business Cycles and Labor-Market Search," *American Economic Review*, 86(1), 112–132.

Basu, S., and J. G. Fernald (1997): "Returns to Scale in U.S. Production: Estimates and Implications," *Journal of Political Economy*, 105(2), 249–283.

Benhabib, J., R. Rogerson, and R. Wright (1991): "Homework in Macroeconomics: Household Production and Aggregate Fluctuations," *Journal of Political Economy*, 99(6), 1166–1187.

Blanchard, O. J., and P. Diamond (1989): "The Beveridge Curve," *Brookings Papers on Economic Activity*, 1, 1–60.

Burdett, K., and D. Mortensen (1998): "Wage Differentials, Employer Size, and Unemployment.," *International Economic Review*, 39(2), 257–273.

Cole, H. L., and R. Rogerson (1999): "Can the Mortensen-Pissarides Matching Model Match the Business-Cycle Facts?," *International Economic Review*, 40(4), 933–959.

Costain, J. S., and M. Reiter (2003): "Business Cycles, Unemployment Insurance, and the Calibration of Matching Models," CESifo Working Paper No. 1008.

Den Haan, W., G. Ramey, and J. Watson (2000): "Job Destruction and Propagation of Shocks," *American Economic Review*, 90(3), 482–498.

Farmer, R. E. A. (2004): "Shooting the Auctioneer," mimeo, UCLA.

Fujita, S. (2004): "Vacancy Persistence," Working Paper 04-23, Federal Reserve Bank of Philadelphia.

Gomes, J., J. Grenwood, and S. Rebelo (2001): "Equilibrium Unemployment," *Journal of Monetary Economics*, 48(1), 109–152.

Greenwood, J., and Z. Hercowitz (1991): "The Allocation of Capital and Time Over the Business Cycle," *Journal of Political Economy*, 99(6), 1188–1214.

Hall, R. (2005): "Employment Fluctuations with Equilibrium Wage Stickiness," *Forthcoming, American Economic Review*.

Hosios, A. (1990): "On the Efficiency of Matching and Related Models of Search and Unemployment," *Review of Economic Studies*, 57(2), 279–298.

Hotz, J. V., C. H. Mullin, and J. K. Scholz (2001): "The Earned Income Tax Credit and Labor Market Participation of Families on Welfare," in *The Incentives of Government Programs and the Well-Being of Families*, ed. by B. Meyer, and G. Duncan. Joint Center for Poverty Research.

Lilien, D. (1982): "Sectoral Shifts and Cyclical Unemployment," *Journal of Political Economy*, 90(4), 777–793.

Ljungqvist, L., and T. J. Sargent (1998): "The European Unemployment Dilemma," *Journal of Political Economy*, 106(3), 514–550.

Lucas, R. J. (1990): "Supply Side Economics: An Analytical Review," *Oxford Economic Papers*, 42(2), 293–316.

McGrattan, E., R. Rogerson, and R. Wright (1997): "An Equilibrium Model of the Business Cycle with Household Production and Fiscal Policy," *International Economic Review*, 38(2), 267–290.

Mendoza, E., A. Razin, and L. Tesar (1994): “Efective Tax Rates in Macroeconomics: Cross-country Estimates of Tax Rates on Factor Incomes and Consumption,” *Journal of Monetary Economics*, 34, 297–323.

Menzio, G. (2004): “High-Frequency Wage Rigidity,” mimeo, Northwestern University.

Merz, M. (1995): “Search in the Labor Market and the Real Business Cycle,” *Journal of Monetary Economics*, 36(2), 269–300.

Meyer, B. (1995): “Lessons from the U.S. Unemployment Insurance Experiments,” *Journal of Economic Literature*, 33(1), 91–131.

Mortensen, D., and C. Pissarides (1994): “Job Creation and Job Destruction in the Theory of Unemployment,” *Review of Economic Studies*, 61(3), 397–415.

Nagypal, E. (2004a): “Amplification of Productivity Shocks: Why Vacancies Don’t like to Hire the Unemployed,” mimeo, Northwestern University.

——— (2004b): “Worker Reallocation over the Business Cycle: The Importance of Job-to-Job Transitions,” mimeo, Northwestern University.

Pissarides, C. (1985): “Short-Run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages,” *American Economic Review*, 75(4), 676–690.

——— (1994): “Search Unemployment with On-the-Job Search.,” *Review of Economic Studies*, 61(3), 457–475.

——— (2000): *Equilibrium Unemployment Theory*. MIT Press, Cambridge, MA, second edn.

Prescott, E. (1986): “Theory Ahead of Business Cycle Measurement,” *Federal Reserve Bank of Minneapolis Quarterly Review*, 10, 9–22.

——— (2004): “Why Do Americans Work So Much More Than Europeans,” *Federal Reserve Bank of Minneapolis Quarterly Review*, 28(1), 2–13.

PRIES, M. (2004): "Persistence of Employment Fluctuations: A Model of Recurring Job Loss," *Review of Economic Studies*, 71(1), 193–215.

RAMEY, G., AND J. WATSON (1997): "Contractual Fragility, Job Destruction, and Business Cycles," *Quarterly Journal of Economics*, 112(3), 873–911.

ROGERSON, R. (1988): "Indivisible Labor, Lotteries and Equilibrium," *Journal of Monetary Economics*, 21(1), 3–16.

ROGERSON, R., R. SHIMER, AND R. WRIGHT (2005): "Search-Theoretic Models of the Labor Market: A Survey," Working Paper.

SHIMER, R. (2004): "The Consequences of Rigid Wages in Search Models," *Journal of the European Economic Association*, 2(2-3), 469–79.

——— (2005): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *Forthcoming, American Economic Review.*

SOLON, G., R. BARSKY, AND J. A. PARKER (1994): "Measuring the Cyclicality of Real Wages: How Important is Composition Bias," *Quarterly Journal of Economics*, 109(1), 1–25.

VAN OURS, J., AND G. RIDDER (1992): "Vacancies and the Recruitment of New Employees," *Journal of Labor Economics*, 10, 138–55.

APPENDIX

I Adding Curvature to the Model

In this section we show that the linear model in the main part of the text is characterized by, with one exception, the same equations as a model with curvature both in utility and production. The exception is that labor productivity is not exogenous any more but depends on the level of capital and employment.

In adding capital we follow Pissarides (2000). There is a CRS aggregate production function $F(K, AE)$, where A denotes an labor-augmenting productivity parameter, K is the capital stock and $E = 1 - u$ denotes employment. $k = K/(AE)$ is the capital stock per efficiency unit of labor and $f(k) = F(k, 1)$ is output per efficiency unit of output. The interest cost equals Ark and Adk is capital depreciation, where r is the interest rate and d denotes the depreciation rate. Revenue per worker, net of capital cost, then equals $p := A(f(k) - (r + d)k)$.

We also allow for curvature in utility. We assume complete consumption markets. This implies that the worker just maximizes income because he can smooth consumption afterwards (see Rogerson, Shimer, and Wright (2005)). Let $\kappa_{t,A,E}$ be the time 0 price for consumption at date t with technology level A and employment E.

The worker now maximizes

$$E \sum \kappa_{t,A,E} x_{t,A,E}, \tag{A1}$$

where $x_{t,A,E}$ denotes income at time t, technology level A, and employment level E.
The flow equations are now as follows (denoting next period values with a prime):

$$\begin{aligned}
J_{A,E} &= \kappa_{t,A,E}(p - w_{A,E}) + (1 - s)E_{A,E}(\kappa_{t+1,A',E'} J_{A',E'}) && \text{(A2)}\\
V_{A,E} &= -c\kappa_{t,A,E} + q(\theta_{A,E})E_{A,E}(\kappa_{t+1,A',E'} J_{A',E'}) && \text{(A3)}\\
U_{A,E} &= \kappa_{t,A,E} z + \{f(\theta_{A,E})E_p \kappa_{t+1,A',E'} W_{A',E'} + (1 - f(\theta_{A,E}))E_p \kappa_{t+1,A',E'} U_{A',E'}\} && \text{(A4)}\\
W_{A,E} &= \kappa_{t,A,E} w_{A,E} + \delta\{(1 - s)E_{A,E}\kappa_{t+1,A',E'} W_{A',E'} + E_{A,E}\kappa_{t+1,A',E'} U_{A',E'}\}. && \text{(A5)}
\end{aligned}$$

Nash bargaining implies that a worker and a firm split the surplus $S = J + W - U$ so that

$$J = (1 - \beta)S, \tag{A6}$$

$$W - U = \beta S. \tag{A7}$$

The Bellman equation for the surplus now reads as follows (omitting subscripts):

$$\begin{aligned} S &= \kappa(p - w + w - z) + E\kappa'\{(1-s)J' + (1-s)W' + sU' - fW' - (1-f)U'\} \\ &= \kappa(p - z) + E\kappa'\{(1-s)S' - f(W' - U')\} \\ &= \kappa(p - z) + E\kappa'\{(1-s)S' - f\beta S'\}. \end{aligned}$$

We can now derive wages, again through using two different expressions for J.

$$\begin{aligned} J &= \kappa(p - w) + (1-s)E\kappa' J_{p'} = \kappa(p - w_p) + c(1-s)\kappa/q \\ &= (1-\beta)\kappa(p - w + w - b) + (1 - s - f\beta)c\kappa/q. \end{aligned}$$

This implies that

$$(1-\beta)\kappa(p - z) - cf\theta\kappa = \kappa(p - w), \tag{A8}$$

and

$$w = \beta p + (1-\beta)z + cf\theta \tag{A9}$$

- exactly the same wage equation as for the linear case. The law of motion for unemployment remains unchanged. But p is not exogenous any longer. It is now a function of technology, of the employment level and of the capital stock. Since $f'(k) = r + d$, p equals the marginal product of an efficiency unit of labor: $p = z(f(k) - kf'(k)) = F_L(K, zL)$. Thus, treating p as an exogenous process (as in the standard model) is only an approximation that becomes invalid if large changes in the employment level are considered. But this does not affect our two calibration targets, per period accounting profits and the productivity elasticity of wages. For the first target the level of p matters. And for the second target we match how wages react to a percentage change in labor productivity. If labor productivity changes because of changes in technology or in capital or in employment is irrelevant.