A QUANTITATIVE MODEL OF COMPETITIVE ASSET PRICING UNDER ASYMMETRIC INFORMATION

Very preliminary and very incomplete

Please do not quote

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Abstract

In the context of a simple two-period model, this paper extends the standard Lucas/Mehra-Prescott model of competitive asset pricing to the case where individuals rationally hold different beliefs about stock returns. Thus, a large fraction of the trade in these markets is "speculative": it arises from individuals taking opposite positions due to their opposing beliefs. Speculation, therefore, is also a key determinant of asset prices. Not all of the available aggregate information relevant for predicting stock returns is revealed by market prices because aggregate trades also react to other factors. Each agent receives a private signals about the stock returns and in addition is subject to idiosyncratic income shocks which influence the desired level of intertemporal trade and risk-taking. Because both the signals and the income shocks have aggregate components and because neither is directly publicly observable, the signal extraction from prices is less than perfect. The model is too complex to solve analytically, and we proceed with numerical analysis. We find that asset prices overreact to aggregate income shocks, and we characterize how beliefs about assets differ in the population. All agents in the model are described from first principles—there are no "noise traders"—and are assumed to have CRRA preferences. We solve both for the price of equity and for the real risk-free interest rate.

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1 Introduction

The high trading activity observed in financial markets suggests that a significant fraction of trading behavior is driven by "speculation", i.e., by differences in opinions regarding the return distribution for assets. The workhorse asset pricing model used in macroeconomic applications, where both prices and quantities are analyzed and where there is an emphasis on general equilibrium, is the model analyzed by Lucas (1978) and used by among others Mehra and Prescott (1985). That model, however, relies on there being no differences in opinions among traders: asset prices reflect "objective" probability assessments of the future payoffs, and there is no room for speculation. Moreover, asset trades, to the extent there are trades (for a framework with trades see, e.g., the extension to heterogeneous agents in Heaton and Lucas (1996), are motivated purely by smoothing needs across dates and states of nature. In this paper, we begin the development of an asset pricing model that can allow us to interpret observed asset price movements, and the associated trades, from a perspective that is a little bit richer and, we think, more realistic than that used currently. One of the important questions one would like our framework to address is to what extent asset price fluctuations can be larger under asymmetric information: can similar fundamental shocks lead to very different price responses in this model than they do in the standard model?

The present paper stops short of answering the question just stated, but it at least contains some partial insights. As will be made clear below, the development of an appropriate model faces several challenges, and this paper only takes a partial step toward the ultimate goal. The most important limitations of the current work is that it is based on numerical model solution and that the model has two periods only. We do believe that the analysis cannot move forward without giving up on analytical tractability—so we settle for numerical approximations—and we intend to extend the model to a fully dynamic one in future work. Moreover, the numerical methods necessary for solving the present model are not entirely off-the-shelf, and extensions to multiple periods will be even more demanding.

The typical explanation for speculative trade is that not all relevant information required for forecasting an asset's performance is publicly available. Information is costly to acquire and agents have a clear incentive not to disclose it before they use it. However, this explanation faces a limitation from a theoretical point of view. Milgrom and Stokey (1982) and Tirole (1982) show that, in equilibrium, the presence of private information does not induce trade per se. They prove that rational agents do not have incentives to trade if the only source of heterogeneity relies on the information received, or if the initial allocation is ex ante efficient.[2] The previous results are known in the literature as no-trade theorems.

Two alternatives have been pursued in order to allow for equilibrium trading behavior in frameworks with imperfect information. The first alternative introduces additional motives for trade, e.g., different hedging needs. Most of the papers in this literature are based on the workhorse model developed by Grossman and Stiglitz (1980), Hellwig (1980), and Admati (1985). In that framework, agents enjoy gains from trade due to the presence of "noise traders". Noise traders are agents whose net demand for assets is exogenous and random and not publicly observed. With noise traders, one can show the existence of rational expectations equilibrium with partially revealing prices and heterogeneous beliefs. In other words, agents can learn about information they do not have directly by looking at prices but since prices also reflect the noise trading, they cannot learn perfectly. More recent papers have extended the basic model, allowing for the existence of partially revealing prices without resorting to the assumption of noise traders (see Bhattacharya and Matthew (1991), Rahi (1996), Marín and Rahi (2000) Ausubel (1990a), and Ausubel (1990b)). With the exception of Ausubel's papers, these papers assume CARA utility functions and Gaussian returns. Such a structure yields a tractable solution but has several limitations (see below).

The second alternative is to assume that agents are not rational and display behavioral biases. See Barberis and Thaler (2003) and Hirshleifer (2001) for surveys on this literature. A more recent example is Sheinkman and Xiong (2003).

The present project belongs to the first area. More precisely, the objective here is to extend the Lucas/Mehra-Prescott asset pricing model to an environment with asymmetric information and partially revealing prices. We maintain the assumption that agents are fully rational, and that they learn, in a standard Bayesian fashion, from the price observed in the market as well as from any private signals they receive. The latter differ across agents, leading to heterogeneous beliefs.

We consider a standard Arrow-Debreu general equilibrium framework. Agents live for two periods, which allows for intertemporal decisions. There is a "tree"—equity—which gives either high or low dividends in the second period. In addition, agents also have stochastic non-traded endowments, and the aggregated endowments are stochastic as well. Thus, there is aggregate uncertainty both from the tree's payoff and from endowments. Agents can transfer resources across periods and across future

[2] An example of the latter is when the initial allocation results from a prior round of trading. In this case, the arrival of new private information is fully impounded in the equilibrium prices, and leaves no room for the presence of heterogeneous beliefs.

states by trading between them, but there is no intertemporal or interstate production technology. The point of departure compared to the benchmark model is that the probability distribution over future states is not common knowledge. Instead, agents receive informative signals and learn from the market prices. Their individual signals include their own endowment, which is informative about the aggregate endowment, and a signal about the tree. Using this information, the prices in the market, and knowledge about the economy works, agents thus form the best possible posterior belief about the tree and trade accordingly. Thus, the trading behavior generated by the model is driven by three forces: risk-sharing across states of nature, consumption smoothing over time, and trading motivated by heterogeneous beliefs about the payoff distribution for the tree.

Our preliminary findings are that shocks to income lead asset prices to overreact compare to the case with full information. Conversely, aggregate signals about stock returns are not fully revealed and lead to price underreactions. Moreover, we find interesting belief dispersion: agents who receive positive income shocks, which by assumption are uncorrelated with the signal about stock returns, are pessimistic about stock returns compared to other agents, holding constant the signal they receive about the stock. These findings are true both in a simple framework based only on risksharing and in a framework with intertemporal trade.

One main difference between the present work and that in most of the existing literature is that we do not use the Gaussian-CARA setup. Although the latter is a useful tool for understanding some aspects of informational asymmetries—it allows tractable analysis even in some frameworks without noise traders but where information is only partially revealed—it has several limitations in other dimensions. There are some significant shortcomings of the Gaussian-CARA model as an asset pricing model. First, it is silent on the equilibrium prices of many classes of assets; for instance, it cannot be used to price a risk free bond or options. Second, it implies individual behavior that is strongly at odds to what is observed in the data: it features an absence of wealth effects. This feature, which is due to the CARA utility specification, means that the absolute demand for risky assets is unrelated to the agent's wealth; in contrast, the data certainly supports a higher absolute demand for risky assets by wealthier agents and, according to most empirical studies, also a higher portfolio share of risky assets. Thus, quantitative work on this topic necessitates a departure from the CARA assumption.

This paper also relates with Calvet et al. (1999). They analyze asset pricing models with heterogeneous beliefs. However, for tractability purposes they consider the case where beliefs are exogenously given. Gollier and Schlee (2003) study how the arrival of information affects the risk free rate and risk premia. But they do not allow for heterogeneity in beliefs.

The analysis starts in Section 2 with a simple model where there is no intertemporal trade: there is only consumption in two states of nature, prior to which there is trade based on risk-sharing needs and speculation. Section **??** then looks at a model with consumption in two consecutive periods.

2 An economy without intertemporal trade

We begin with a simple economy without intertemporal trade. Here, the only trade that will take place is due to risk-sharing and speculation. We thus consider a pure exchange economy with asymmetric information and heterogeneous agents. There is a single risky asset in the economy: a tree. The tree pays high dividends with probability ν and low dividends with probability $1-\nu$. The tree pays off only once and then dies. There is a measure 1 of agents in the economy and everybody is initially entitled to a share of the tree. Agents also receive a riskless endowment, though some of the agents are luckier than others: a fraction ϕ of the population receives a high endowment, while a fraction $1-\phi$ receives a low endowment.

The parameters ν and ϕ are drawn from a joint probability distribution $F(\nu, \phi)$ which is common knowledge. The random variable ν takes values on the unit interval $I \equiv [0, 1]$. We will, in particular, consider a uniform distribution and assume that ν and ϕ are drawn independently. The random variable ϕ is discrete and takes values on $\Phi = \{\phi_l, \phi_h\}$ with probabilities $1-\pi$ and π, respectively.

Agents are not able to observe the realizations of ν and ϕ but privately receive informative signals about the tree. Each signal, s, can be either good or bad: $s = 1$ denotes a good signal and $s = 0$ denotes a bad signal. Every agent receives one signal.[3] The realization of the signal an agent is receiving is drawn from a binomial distribution with parameter ν. Moreover, we impose a law of large numbers that guarantees that the fraction of agents who receive a good signal equals ν.

All the action takes place in a single period. Markets open in the morning, the tree pays off in the afternoon, and agents consume at the end of the day. In the absence of trade, agents consume their endowments and dividends paid by the tree. Actually, this is the equilibrium allocation if there is no heterogeneity across agents. This is not the case in the present framework; here, poor agents have a stronger preference for consumption smoothing than wealthy individuals, so there are gains from trade. This result follows if the utility function is concave and shows a decreasing coefficient of absolute risk aversion. The latter is defined as $\frac{-u''(c)}{u'(c)}$. The utility function assumed in the present paper (logarithmic)

[3] **?** considers the case of more than one signal and whether economies with more signals deliver higher welfare.

satisfies both these properties.

Agents can transfer resources freely across the two states of nature that can be realized, i.e., whether the tree pays high or low dividends. This means that consumers can trade in two Arrow-Debreu securities. One of them pays 1 unit of the consumption good if the high dividend state is realized. Otherwise, it pays zero. The other security only pays (1 unit) in the low dividend state. There is only one price to be determined: the relative price between these two securities. We normalize so that we can use p to denote the price of consumption good in the good dividend state and $1-p$ to denote that in the bad dividend state.

The equilibrium relative price of contingent claims depends on ν and ϕ. Intuitively, a higher value of ν means that the high dividend state is more likely to occur, which makes the contingent claim paying in that state more valuable. A higher value of ϕ implies that a small fraction of agents need insurance, which reduces the demand for contingent claims paying in the low state.

The critical assumption made in the paper is that agents are fully rational and use the information pooled by the equilibrium price to update their beliefs. Agents not only learn from their private signals, but they also understand how the price is determined in equilibrium. This allows them to make inferences about the realizations of ν and ϕ once they have observed the market price. In addition, the endowment realization also conveys valuable information, as will be described below. Finally, the paper assumes agents do not behave strategically. They take the price and everyone else's behavior as given. This is justified on the grounds that there is a large number of agents, so that each individual does not exert any influence on aggregate variables.

2.1 Definition of equilibrium

Agents maximize their expected utility of consumption taking asset prices as given. This leads to asset demands, and market clearing for assets pin down the equilibrium relative price of the assets. The key equilibrium object is the pricing function P: it maps the state, which consists of ν and ϕ, into a price realization $p \geq 0$; we use $p = P(\nu, \phi)$ for this function. Since ν and ϕ are random and drawn according to F, the price p is random as well, and its distribution can be derived based on F and the shape of the function P, which is determined in equilibrium.

In the equilibrium we consider, P plays a dual role: (i) it plays its usual role in agents's budget constraints but (ii) it also influences agents' beliefs about ν. In particular, one could imagine that an agent knew, say, ϕ; then, this agent could find out exactly what ν is from seeing the price and using the knowledge of the function P, provided that P is strictly monotone in its first argument. Throughout, including in the definition of equilibrium, we will presume that this monotonicity is satisfied. This presumption is based, first, on the intuition that a higher ν should lead to a higher demand for consumption in state g, and thus to an increase in p. Second, a higher ϕ, and thus an increase in the number of rich, should also lead to increased demand for consumption in state g, given our assumption on u: rich agents are less concerned with risk. Though intuitive, however, it is not a foregone conclusion that P is strictly increasing in both its arguments, so it is important to verify when the equilibrium is computed that the presumption is actually borne out.

Formally, let $\mathcal{I}_i$ denote the private information set of agent i. Thus, i can be of four kinds: the signal about the dividend can be either good or bad and the endowment could be either high or low. Expected utility of a given agent i is therefore based on $\mathcal{I}_i$ along with an observed price: it can be written in abstract as $E(u(c)|\mathcal{I}_i, p)$. Because we assume that the expected utility hypothesis is met, we use probability compounding to reduce this expectation to "beliefs" about ν:

$$E(u(c)|\mathcal{I}_i, p) = \tilde{\nu}^i(p)u\left(c_h\right) + (1 - \tilde{\nu}^i(p))u\left(c_l\right),$$

where c_j denotes planned consumption in state $j \in \{h, l\}$ and where $\tilde{\nu}^i(p)$ satisfies

$$\tilde{\nu}^i(p) = E(\nu|\mathcal{I}_i, p).$$

Given P, these expectations are straightforward applications of Bayes' rule: the agent knows that ν and ϕ are drawn according to a joint distribution F (in particular, recall that ν and ϕ are independent and that ν is uniform and ϕ is low with probability π) and that p is random and given by a function P of ν and ϕ, which is all the information necessary in order to perform the signal extraction. The next subsection describes in more detail how agents compute their beliefs in the class of economies we analyze. Notice, of course, that these beliefs are endogenous here: they depend on the pricing function P.

A type i consumer solves the following optimization problem:

$$\begin{aligned}
&\max_{c_h, c_l} \left\{\tilde{\nu}^i\left(p\right) u\left(c_h\right) + \left(1 - \tilde{\nu}^i\left(p\right)\right) u\left(c_l\right)\right\} \qquad (1) \\
&\text{subject to} \quad (1-p)c_l + pc_h = W \equiv a^i + (1-p)d_l + pd_h \\
&c_l, c_h \geq 0,
\end{aligned}$$

where a^i denotes the riskless endowment of a type i agent, d_j denotes the dividends paid by the tree in state j, and we define W as the individual's total wealth. As mentioned above, the sum of the prices of the Arrow-Debreu securities is normalized to 1. This maximization delivers demand functions $c_j^i(p)$ for $j = l$ and $j = h$.

Notice that once ν and ϕ are realized, the population will have four groups. The sizes of these groups are denoted $\mu^i(\nu, \phi)$. For example, group 1 consists of agents with a good signal and a high endowment, and its size is given by $\mu^1(\nu, \phi) = \nu\phi$; we will discuss these groups in detail in the section below, where will also introduce some additional notation to distinguish the groups.

Let $Y_j(\phi)$ denote the aggregate resources in state j and let $Z_i(p, \nu, \phi)$ denote the aggregate demand for consumption goods in state j. The latter is thus computed as follows:

$$Z_j(p, \nu, \phi) = \sum_{i=1}^{4} c_j^i(p)\, \mu^i(\nu, \phi).$$

We are now ready to define a competitive equilibrium for this class of economies.

Definition 1 *A rational expectations equilibrium, REE, consists of a price function $P : \mathrm{I} \times \Phi \to [0, 1]$ and individual demand functions $\left\{c_l^i(p), c_h^i(p)\right\}_{i=1}^{4}$ such that:*

(1) *consumers maximize utility, i.e., $\left\{c_l^i(p), c_h^i(p)\right\}$ solves consumer i's problem for all $i = 1, \ldots, 4$, given that consumers use P for determining the distribution for p; and*

(2) *markets clear, i.e., $Z_j(P(\nu, \phi), \nu, \phi) = Y_j(\phi)$ for all $j = l, h$ and for all $\nu \in \mathrm{I}$ and $\phi \in \Phi$.*

Radner (1979) provides a more general definition of the equilibrium concept defined above.[4] An important assumption implicit in Definition 2 is that the individuals' perceived price function coincides with the actual equilibrium function. Agents fully understand how prices are determined and take this information into account when forming their beliefs. Notice that, in general, finding an equilibrium requires solving for a fixed point of a functional equation: the price function perceived by the agents, P, must coincide with the price function generated by their behavior and market clearing, P.

2.2 Finding the equilibrium in a special case

We choose a logarithmic utility function because it has the advantage that individual demands are linear in wealth. The optimal consumption rules are straightforward to derive in this case; they are specified in equation (2) below.

$$c_h^i(p) = \tilde{\nu}^i(p)\,\frac{W^i}{p} \qquad c_l^i(p) = \left(1 - \tilde{\nu}^i(p)\right)\frac{W^i}{1-p} \tag{2}$$

[4]**?** criticize the REE approach because it assumes implicitly that prices pool individuals' private information before they trade. Nonetheless, the approach has been extensively used in the literature, showing that, despite its limitations, it constitutes a useful tool for analyzing problems with asymmetric information.

This equation says that under complete markets and with logarithmic utility, the resources spent on consumption in a state of nature has to equal a constant fraction of total wealth spent on consumption, where the fraction simply equals the probability of that state occurring. Under more general utility specifications, e.g., with $u(c)$ displaying constant relative risk aversion, the fraction of wealth spent on consumption in a given state also depends directly on the price of consumption in that state. Under logarithmic utility in the present specification, there is no such direct price effect, but on the other hand the price matters in a new way: the probability of the state here is given by a "belief", which is an endogenous object and which depends on the price observed. Moreover, different agents have different reactions to the price, which leads to different propensities to consume in a given state. Thus, there is no aggregation theorem here: wealth redistribution across agents of different types will influence prices.

The four groups of agents are listed below with their corresponding sizes and some new notation.

- $\nu\phi$ agents with high endowment and a good signal, a type denoted $\bar{1}$,
- $(1-\nu)\phi$ agents with high endowment and a bad signal, a type denoted $\bar{0}$,
- $\nu(1-\phi)$ agents with low endowment and a good signal, a type denoted $\underline{1}$, and
- $(1-\nu)(1-\phi)$ agents with low endowment and a bad signal, a type denoted $\underline{0}$.

In equilibrium, aggregate planned consumption for the high state must equal aggregate resources in that state. If that equality holds, by Walras' law, the other market is also in equilibrium. The market clearing condition is formally stated in equation (3):

$$\phi\left[\nu c_h^{\bar{1}}(p)+(1-\nu)\,c_h^{\bar{0}}(p)\right]+(1-\phi)\left[\nu c_h^{\underline{1}}(p)+(1-\nu)\,c_h^{\underline{0}}(p)\right]=\phi\bar{a}+(1-\phi)\,\underline{a}+d_h, \tag{3}$$

where $\bar{a}$ denotes the high value of the riskless endowment and $\underline{a}$ the low value.

The equilibrium price is obtained after replacing individual demands into the market clearing condition:

$$p=\frac{\phi\,(\bar{a}+d_l)\left[\nu\tilde{\nu}^{\bar{1}}(p)+(1-\nu)\tilde{\nu}^{\bar{0}}(p)\right]+(1-\phi)\,(\underline{a}+d_l)\left[\nu\tilde{\nu}^{\underline{1}}(p)+(1-\nu)\tilde{\nu}^{\underline{0}}(p)\right]}{\phi\bar{a}+(1-\phi)\,\underline{a}+d_h+\left\{\phi\left[\nu\tilde{\nu}^{\bar{1}}(p)+(1-\nu)\tilde{\nu}^{\bar{0}}(p)\right]+(1-\phi)\left[\nu\tilde{\nu}^{\underline{1}}(p)+(1-\nu)\,\tilde{\nu}^{\underline{0}}(p)\right]\right\}(d_l-d_h)}. \tag{4}$$

This equation, thus, finds the equilibrium p for a given (ν,ϕ) pair. If it were not for the dependence of the $\tilde{\nu}$s on the p, we would thus already have a closed-form expression for how the price depends on (ν,ϕ): we would have $P(\nu,\phi)$. Now, instead, the dependence of p on (ν,ϕ) is influenced by how the $\tilde{\nu}$s depend on p. We will discuss how this works below, but recall that this latter relation is not only potentially nonlinear, but *it is determined by the shape of $P(\nu, \phi)$ itself*. This is one way of illustrating how finding P is a nontrivial fixed-point problem in the present economy.

It is easy to show that this model does not have a fully revealing equilibrium. The reasoning is as follows. The agents' private signals and endowments do not convey enough information to reveal the realization of (ν, ϕ). Thus, the only way agents can infer the values of those variables is if in equilibrium there is a one-to-one mapping between (ν, ϕ) and the equilibrium price. In other words, for prices to be fully revealing, there must be only one possible realization of ν consistent with a given price and value of ϕ. The equilibrium relationship between ν and the last two variables in the fully revealing case is described in equation (5). This equation is obtained after replacing the individual beliefs $\tilde{\nu}^i$ in equation (4) by the actual realization of ν and simplifying to obtain

$$P^{FR}(\nu, \phi) = \frac{\nu \left[\phi \bar{a} + (1-\phi)\,\underline{a} + d_l\right]}{\phi \bar{a} + (1-\phi)\,\underline{a} + d_h - \nu\,(d_h - d_l)}. \tag{5}$$

It is apparent that there is more than one combination of ν and ϕ consistent with a given price. This contradicts the hypothesis that prices are fully revealing. Furthermore, it suggests that the equilibrium is "pairwise" revealing: the market price reveals that the probability of high dividends can take one of two possible values. Thus, individual beliefs consist of a weighted some of those values. The weights are determined by the signal and endowment received. Another important property is that is indicated here is that, when ν is realized to be either 1 or 0, the equilibrium price does not depend on ϕ: it must be 1 and 0, respectively. That is, prices are fully revealing of ν in the corners (though ϕ is not revealed). That is, a "guess" that $P = P^{FR}$ locally around these points works: agents need to know ν in their formation of beliefs, and P^{FR} allows them to figure it out at $(1, \phi)$ and at $(0, \phi)$, but not at any other values for (ν, ϕ).

In order to understand how the price depends on ν and ϕ, it is helpful to remain for a moment in the case where both variables are actually common knowledge, say, by being part of each individual's information set. The function P^{FR} shows how the price increases with ν: as the high state becomes more likely, agents demand more contingent claims paying in that state. It can easily be shown that the equilibrium price also increases with ϕ. We have already mentioned that "poor" agents (with a low riskless endowment) are de-facto more risk-averse than "rich" agents in this economy, so the former buy insurance from the latter. That is, agents with a low endowment transfer consumption from the high-dividend state to the low-dividend state.[5] As the fraction of rich individuals (ϕ) increases, less agents demand contingent claims that pay off in the low state, so the relative price decreases, i.e., p

[5]Let θ_h denote the net demand for contingent claims that pay only if the high state is realized. The agent is endowed

increases.

In the economy with asymmetric information, where we solve for the equilibrium numerically, we found that the just reported properties are satisfied as well: $P(\nu, \phi)$ is also increasing in both arguments. This is crucial for how beliefs are determined. It is not hard to be convinced that there is an economy for which this must be true. Suppose that, in contrast to the assumption we maintain here, some fraction of all agents are fully informed of (ν, ϕ), so that the agents with private signals only are only a subset of the overall population. This, in other words, is a slight generalization of the setup we consider here. When the group of agents who are not fully informed is of zero measure, $P(\nu, \phi) = P^{FR}(\nu, \phi)$ must be the equilibrium function. When the same group has a small positive measure, if an equilibrium exists, it must be close to P^{FR}: though not apparent yet, because the missing equations—those describing the determination of the beliefs $\tilde{\nu}$—have not been displayed, there is sufficient continuity in these equations to ensure that the pricing function changes continuously with the primitive parameters.

2.2.1 Beliefs

Since ϕ takes on only two values, we are essentially looking for two pricing functions each of which has one argument: one of these maps ν into a price when $\phi = \phi_h$ and one similarly describes a map for the case $\phi = \phi_l$. It will turn out to be convenient, however, to not solve directly for the price function P but for another, closely related function: we will look for its inverse with respect to ν. An illustration of this function appears in Figure 1, which is based on an equilibrium that was computed for parameter values that are discussed below. The figure shows the price function P as two separate functions—one for each ϕ. Agents thus use it to extract information from the market price. When an agent observes a particular price, such as p_0 in the figure, he infers, by using the inverse of the P function with respect to ν, that only two values of ν could have been realized. We denote the associated functions $\mathcal{V}(p_0, \phi_l)$ and $\mathcal{V}(p_0, \phi_h)$, respectively; the first of these corresponds to the value of ν consistent with a price p_0 and a low fraction of highly endowed agents, and the second one corresponds to the value of ν consistent with a low fraction of poorly endowed agents. Since agents do not observe the actual distribution of riskless

with $a + d_h$ of this asset. It can be shown that

$$\frac{\partial \theta_h}{\partial a} > 0 \Longleftrightarrow \frac{-u''(c_h)}{u'(c_h)} < \frac{-u''(c_l)}{u'(c_l)}$$

where

$$c_i = a + d_i + \theta_i \quad i = l, h$$

From the individual first-order conditions and the aggregate resource constraint, it transpires that $c_h > c_l$ for every agent. Thus, a sufficient condition for the previous inequality to hold is that the coefficient of absolute risk aversion decreases with consumption. The utility function assumed in the present paper satisfies this property.

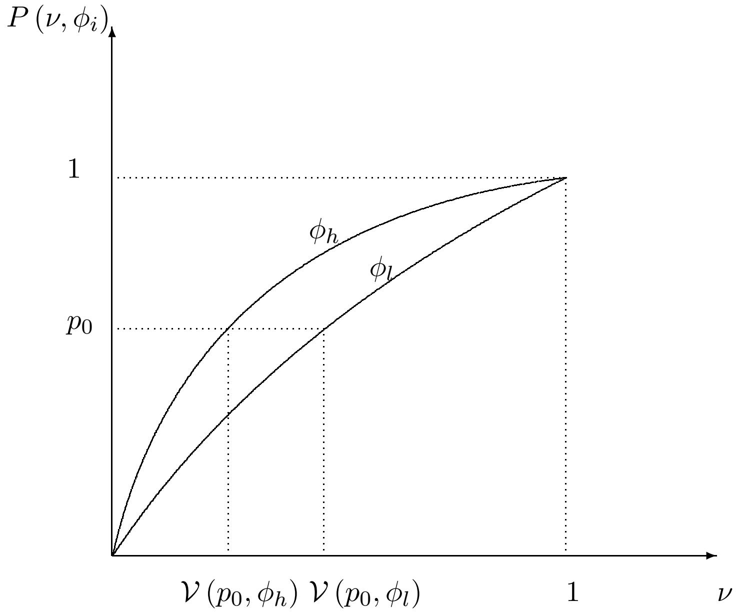


Figure 1: Information revealed by the price function

endowments, they cannot distinguish which of the values corresponds to the actual realization of ν.

In addition to learning from the market price, agents' private signals and endowments reveal information. Intuitively, an agent with a high endowment believes that it is more likely that the fraction of rich agents is ϕ_h rather than ϕ_l, so he assigns more weight to $\mathcal{V}(p_0, \phi_h)$. Similarly, an agent with a good signal about the tree believes that it is more likely that the highest ν was realized. Thus, four possible beliefs about ν emerge, and we should expect the most optimistic beliefs—in terms of a large ν—to come from agents who are poor and who have a good signal about the tree: the $\underline{1}$ type.

We now formalize the previous argument taking the case of an agent who has received a high riskless endowment and a good signal, i.e., an agent with $a = \bar{a}$ and $s = 1$. The updating of beliefs of the remaining agents follows the same logic. As stated above, each agent's belief regarding the probability that the tree pays high dividends consists of the expectation of ν conditional on his private information and the market price:

$$
\begin{aligned}
\tilde{\nu}^{\bar{1}}(p) &= E[\nu | s = 1, a = \bar{a}, p] \\
&= \mathcal{V}(p, \phi_h) Pr\{\phi_h | 1, \bar{a}, p\} + \mathcal{V}(p, \phi_l) Pr\{\phi_l | 1, \bar{a}, p\}.
\end{aligned}
$$

The second equality takes into account that once the agent has conditioned on the price, the probability ν

has a dichotomous distribution. Then we apply the law of conditional probabilities to the last expression, and use the fact that once we condition on ϕ, the following events are mutually independent: the tree pays high dividends, the agent receives a good signal, and the agent receives a high riskless endowment. The result is the following equation:

$$\tilde{\nu}^{\bar{1}}(p) = \frac{\mathcal{V}(p,\phi_h)\,Pr\{1|p,\phi_h\}\,Pr\{\bar{a}|p,\phi_h\}\,Pr\{p|\phi_h\}\,Pr\{\phi_h\} + \mathcal{V}(p,\phi_l)\,Pr\{1|p,\phi_l\}\,Pr\{\bar{a}|p,\phi_l\}\,Pr\{p\mid\phi_l\}\,Pr\{\phi_l\}}{Pr\{1|p,\phi_h\}\,Pr\{\bar{a}|p,\phi_h\}\,Pr\{p|\phi_h\}\,Pr\{\phi_h\} + Pr\{1|p,\phi_l\}\,Pr\{\bar{a}|p,\phi_l\}\,Pr\{p|\phi_l\}\,Pr\{\phi_l\}}.$$

Here, $Pr\{p|\phi_h\}$ and $Pr\{p|\phi_h\}$ refer to densities, to the extent they exist. Finally, equation (7) below is obtained after replacing the probabilities in the last expression by their actual values. Recall that the probability of receiving a good signal and a high riskless endowment coincides with the actual realizations of ν and ϕ, respectively.

$$\tilde{\nu}^{\bar{1}}(p) = \frac{[\mathcal{V}(p,\phi_h)]^2\,\phi_h g(p|\phi_h)\,\pi + [\mathcal{V}(p,\phi_l)]^2\,\phi_l g(p|\phi_l)\,(1-\pi)}{\mathcal{V}(p,\phi_h)\,\phi_h g(p|\phi_h)\,\pi + \mathcal{V}(p,\phi_l)\,\phi_l g(p|\phi_l)\,(1-\pi)} \tag{6}$$

The function $g(p|\phi_i)$ denotes the density of the price conditional on ϕ_i. It can be expressed in terms of the known density of ν, which is uniform in our example but in general can be denoted $f(\nu) \equiv F(d\nu, \phi)$ (here we use the fact that ν and ϕ are independent).[6] Thus it reads

$$g(p|\phi_i) = f(\mathcal{V}(p,\phi_i))\,\frac{\partial\mathcal{V}(p,\phi_i)}{\partial p}.$$

It is convenient to define the two functions $\mathcal{V}_h(p) \equiv \mathcal{V}(p,\phi_h)$ and $\mathcal{V}_l(p) \equiv \mathcal{V}(p,\phi_l)$. Thus we can restate the previous equation as

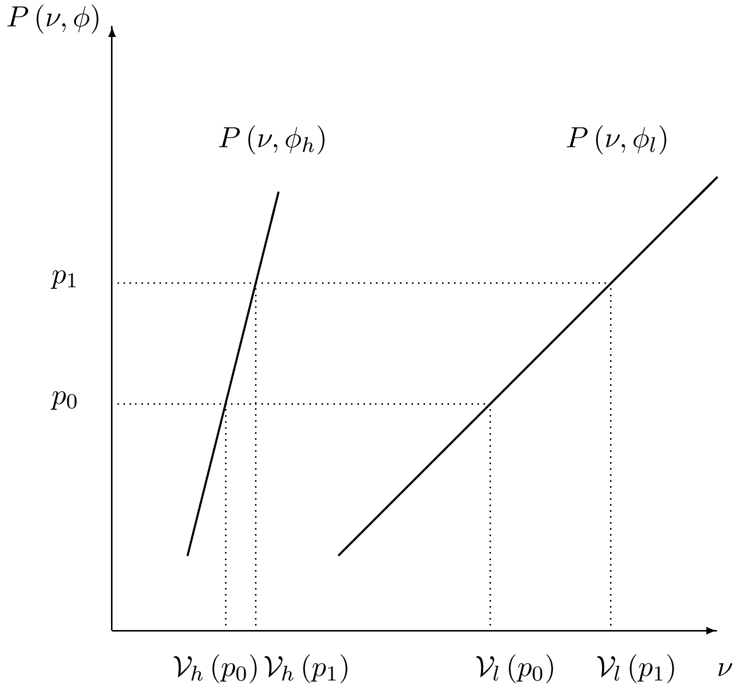
$$g(p|\phi_i) = f(\mathcal{V}_i(p))\,\mathcal{V}_i'(p).$$

The last equality just simplifies the notation. The subindex i denotes the fraction of rich agents in the economy, i.e., ϕ_i and $f(\cdot)$ denote the density function of ν.[7] The intuition for the formula of the conditional density is that a price p is likely to be observed when the value of ν consistent with that price is likely to be drawn, i.e., when $f(\mathcal{V}_i(p))$ is high, or when the price function $P(\nu,\phi_i)$ is not sensitive to ν at $\mathcal{V}_i(p)$. A heuristic description of the last argument is provided in the picture below. Consider a hypothetical case where it is known that the price belongs to the range $[p_0,\ p_1]$. Its actual value however, is not observed. In this case, agents infer that ν belongs to $[\mathcal{V}_h(p_0),\ \mathcal{V}_h(p_1)]$ if the fraction

[6] Recall that $\frac{d}{dx}\int_{a(x)}^{b(x)} c(x,z)dz = \int_{a(x)}^{b(x)} c_1(x,z)dz + c(x,b(x))b'(x) - c(x,a(x))a'(x)$. Thus, $g(p) \equiv \frac{d}{dp}\int_{\nu\le\mathcal{V}(p)} F(d\nu) = \frac{d}{dp}\int_0^{\mathcal{V}(p)} f(\nu)d\nu = f(\mathcal{V}(p))\mathcal{V}'(p)$.

[7] The paper assumes a uniform distribution over the interval $[0,\ 1]$, so the density is just the constant 1. However, it will assist the intuition to consider for the moment the more general case.

of rich agents is ϕ_h, and to $[\mathcal{V}_l(p_0), \mathcal{V}_l(p_1)]$ if the fraction is ϕ_l. In the case where ν is drawn from a uniform distribution, the probability of observing a price in $[p_0, p_1]$ consists of the length of the interval $[\mathcal{V}_i(p_0), \mathcal{V}_i(p_1)]$, which is clearly higher for the price function $P(\cdot, \phi_l)$. In the limit, as the length of the price range collapses to a single point, the likelihood of observing a particular price becomes inversely proportional to the derivative of the price function at that point, or directly proportional to $\mathcal{V}_i'(p)$.



Thus, we can describe the beliefs of all the four types as follows (where we now use the assumption that $f(\nu) = 1$ for all ν):

$$\tilde{\nu}^{\bar{1}}(p) = \frac{\pi \mathcal{V}_h'(p)\phi_h [\mathcal{V}_h(p)]^2 + (1-\pi)\mathcal{V}_l'(p)\phi_l [\mathcal{V}_l(p)]^2}{\pi \mathcal{V}_h'(p)\phi_h \mathcal{V}_h(p) + (1-\pi)\mathcal{V}_l'(p)\phi_l \mathcal{V}_l(p)} \tag{7}$$

$$\tilde{\nu}^{\bar{0}}(p) = \frac{\pi \mathcal{V}_h'(p)\phi_h (1-\mathcal{V}_h(p))\mathcal{V}_h(p) + (1-\pi)\mathcal{V}_l'(p)\phi_l (1-\mathcal{V}_l(p))\mathcal{V}_l(p)}{\pi \mathcal{V}_h'(p)\phi_h (1-\mathcal{V}_h(p)) + (1-\pi)\mathcal{V}_l'(p)\phi_l (1-\mathcal{V}_l(p))} \tag{8}$$

$$\tilde{\nu}^{\underline{1}}(p) = \frac{\pi \mathcal{V}_h'(p)(1-\phi_h)[\mathcal{V}_h(p)]^2 + (1-\pi)\mathcal{V}_l'(p)(1-\phi_l)[\mathcal{V}_l(p)]^2}{\pi \mathcal{V}_h'(p)(1-\phi_h)\mathcal{V}_h(p) + (1-\pi)\mathcal{V}_l'(p)(1-\phi_l)\mathcal{V}_l(p)} \tag{9}$$

$$\tilde{\nu}^{\underline{0}}(p) = \frac{\pi \mathcal{V}_h'(p)(1-\phi_h)(1-\mathcal{V}_h(p))\mathcal{V}_h(p) + (1-\pi)\mathcal{V}_l'(p)(1-\phi_l)(1-\mathcal{V}_l(p))\mathcal{V}_l(p)}{\pi \mathcal{V}_h'(p)(1-\phi_h)(1-\mathcal{V}_h(p)) + (1-\pi)\mathcal{V}_l'(p)(1-\phi_l)(1-\mathcal{V}_l(p))} \tag{10}$$

Notice that the equilibrium price affects the beliefs in two ways. First, for a given market price p, agents use the equilibrium price function to retrieve the possible realizations of ν: $\mathcal{V}_h(p)$ and $\mathcal{V}_l(p)$.

Second, they use the derivative of the price functions ($\mathcal{V}_h'(p)$ and $\mathcal{V}_l'(p)$) in order to assess how likely those points are.

We now restate the market clearing conditions for $\phi = \phi_h$ and for $\phi = \phi_l$ in terms of our new unknown functions $\mathcal{V}_h(p)$ and $\mathcal{V}_l(p)$:

$$p = \frac{\phi_h\left(\bar{a}+d_l\right)\left[\mathcal{V}_h(p)\tilde{\nu}^{\bar{1}}(p)+(1-\mathcal{V}_h(p))\tilde{\nu}^{\bar{0}}(p)\right]+(1-\phi_h)\left(\underline{a}+d_l\right)\left[\mathcal{V}_h(p)\tilde{\nu}^{\underline{1}}(p)+(1-\mathcal{V}_h(p))\tilde{\nu}^{\underline{0}}(p)\right]}{\phi_h\bar{a}+(1-\phi_h)\,\underline{a}+d_h-\left\{\phi_h\left[\mathcal{V}_h(p)\tilde{\nu}^{\bar{1}}(p)+(1-\mathcal{V}_h(p))\tilde{\nu}^{\bar{0}}(p)\right]+(1-\phi_h)\left[\mathcal{V}_h(p)\tilde{\nu}^{\underline{1}}(p)+(1-\mathcal{V}_h(p))\,\tilde{\nu}^{\underline{0}}(p)\right]\right\}\Delta}$$

and

$$p = \frac{\phi_l\left(\bar{a}+d_l\right)\left[\mathcal{V}_l(p)\tilde{\nu}^{\bar{1}}(p)+(1-\mathcal{V}_l(p))\tilde{\nu}^{\bar{0}}(p)\right]+(1-\phi_l)\left(\underline{a}+d_l\right)\left[\mathcal{V}_l(p)\tilde{\nu}^{\underline{1}}(p)+(1-\mathcal{V}_l(p))\tilde{\nu}^{\underline{0}}(p)\right]}{\phi_l\bar{a}+(1-\phi_l)\,\underline{a}+d_h-\left\{\phi_l\left[\mathcal{V}_l(p)\tilde{\nu}^{\bar{1}}(p)+(1-\mathcal{V}_l(p))\tilde{\nu}^{\bar{0}}(p)\right]+(1-\phi_l)\left[\mathcal{V}_l(p)\tilde{\nu}^{\underline{1}}(p)+(1-\mathcal{V}_l(p))\,\tilde{\nu}^{\underline{0}}(p)\right]\right\}\Delta},$$

where $\Delta \equiv d_h - d_l$.

The four equations expressing the beliefs as functions of the price, (7)-(10), can now be inserted into the market clearing equations. The system thus obtained consists of two functional equations—they each have to hold for all p—in the two unknown functions $\mathcal{V}_h(p)$ and $\mathcal{V}_l(p)$. The domain for the functions are given by the interval whose end points are the full revelation prices, i.e., by [0,1]. Notice that the two functional equations are differential equations and that they are interdependent, because the beliefs all involve both of the unknown functions.

The structure of the model is similar to Ausubel (1990a) and Ausubel (1990b), neither paper of which deals with asset pricing. He also analyzed an economy with partially revealing prices, where the state of the economy is characterized by two variables: a continuous one and a dichotomous one. In our framework, the first one is represented by ν and the second one by ϕ. In both of these papers, the framework is simplified by looking at "hierarchical" information structures: one type of agent is fully informed and the other type of agent has no private information and thus can only look at the price in order to make inference. Thus, uninformed agents are all alike. Here, in contrast, all agents are uninformed, and these uninformed agents are heterogeneous in their beliefs since they receive private signals. In particular, the uninformed agents in our setting cannot be ordered in terms of how much information they have. Moreover, the structure in Ausubel (1990a) allows closed-form solutions because of the shock structure. More precisely, there are two kinds of preference shocks, and although the uninformed can observe the demand of the informed, they cannot interpret it fully in terms of each of the shocks. Due to these assumptions, one can show that the belief functions corresponding to $\mathcal{V}_h(p)$ and

$\mathcal{V}_l(p)$ are proportional to each other, which allows a drastic simplification: the derivatives in the belief expressions can be cancelled out, and the final equilibrium condition is no longer a differential equation. We have not, unfortunately, found a way of introducing shocks so as to import this "trick" into an asset pricing environment. The structure in Ausubel (1990b) does not allow this simplification either, and there the characterization is carried out without closed-form solutions. Here, we find an approximate solution is arrived at using numerical techniques. The appendix provides a detailed description of these techniques.

2.2.2 Parameter selection

The model presented above builds on many restrictive assumptions. This allows us to find a (numerical) solution, but has the disadvantage that the resulting model is highly stylized and has limited ability to replicate real data. Thus, the parameters that characterize the dividend and endowment processes are not chosen following a standard calibration exercise, i.e., they are not based on actual data. There are other reasons that motivate this choice. The assumption of a risky asset that lives for only one period cannot mimic the returns to any real-world aggregate stock index.[8] Moreover, in order to calibrate the process of the riskless endowment it would be necessary to consider not only the labor income of stockholders, but also other sources of income, like the returns to private businesses, which are not easy to obtain.

The strategy is to choose baseline parameters that will illustrate the effect the paper tries to emphasize. To that end, the worst realization of the riskless endowment is allowed to take a relatively low value. This magnifies the different attitudes toward risk of rich and poor agents, which increases the sensitivity of the equilibrium to changes in the distribution of endowments (controlled by ϕ). Similarly, if the dividend dispersion was low, equilibrium state prices would lie close to the corresponding state probability, regardless of the realization of ϕ. In that case, agents' beliefs would tend to coincide with the actual realization of ν, and the economy would behave almost as if everyone were fully informed. A dispersed dividend realization is therefore a necessary ingredient. In summary, we restrict attention to the case where the lower realizations of the riskless endowment and dividend take small values compared

[8] If the risk free bond is taken as numéraire, the expected return of the tree for a given realization of ν and ϕ in an economy with full information is

$$R(\nu, \phi) = \frac{\nu d_h p}{d_l (1-p) + d_h p} + \frac{(1-\nu) d_l (1-p)}{d_l (1-p) + d_h p}, \quad where\ p = p(\nu, \phi).$$

The gross return is below 1 for almost all realizations of ν and ϕ. This implies that the model cannot generate positive net rates of returns of the risky asset, as it is observed in the data.

to their higher counterparts. The parameters chosen are specified in the table below.

$d_h = 1$	$d_l = 0.1$
$\bar{a} = 1$	$\underline{a} = 0.5$
$\phi_h = 0.8$	$\phi_l = 0.2$
$\pi = 0.5$	$\nu \sim U(0,1)$

2.2.3 Results

Figure 2 compares the equilibrium prices between an economy with full information and an economy with asymmetric information. The graph shows that the monotonicity property of the price function is preserved in the asymmetric information framework. It also illustrates that when the economy is hit with a good endowment shock ($\phi = \phi_h$), the relative price of the high dividend state is higher in the asymmetric information case than in the full information case. The result is reversed when the economy is hit with a bad shock. I.e., conditional on a value of ν, the price "overreacts" to the aggregate endowment shock compared to the case with full information. The explanation rests on the scheme designed to update beliefs. Consider again Figure 1 on page 12. We can interpret the picture as the price schedule in the case where all but a single agent are fully informed. The unlucky agent has to infer ν from the price observed in the market and from his private information. If the values $\mathcal{V}(p_0, \phi_l)$ and ϕ_l are realized, the agent's belief lies below the actual realization of ν. The equilibrium price is not affected by the behavior of this single individual, who has measure zero. However, if the fraction of agents who are imperfectly informed increases, the average belief in the economy decreases and the equilibrium price falls, as can be deduced from equation (4). Eventually, if no agent is fully informed, the average belief is below the actual realization of ν. This implies that the equilibrium price is below its level in the full information economy, as Figure 2 shows. The previous argument holds for any realization of ν. Similar logic can be used to explain why the equilibrium price is higher in an economy with asymmetric information and a high realization of ϕ.

Figures 3-4 graph the beliefs as a function of the price. It shows that the value of the riskless endowment conveys more information than the signal about the tree. Agents with low endowments are more optimistic than the rest, independently of the signal received. An agent hit with a low riskless endowment assigns more weight to the possibility that $\phi = \phi_l$ than a rich individual. This means that receiving a low endowment can be taken as a signal that the actual ν is closer to $\mathcal{V}_l(p)$ than to $\mathcal{V}_h(p)$. The first value is higher than the second one, explaining why poor agents tend to be more optimistic.

The heterogeneity in beliefs along with the difference in the endowments induce agents to trade. In Section 2 we stated that in an economy with full information, rich agents sell contingent claims that pay in the low state. This may not be true in the present case. Poor agents are more optimistic than wealthy individuals, so the former may now have an incentive to transfer resources to the high state.

3 The economy with intertemporal trade

We now consider another pure exchange economy with asymmetric information and heterogeneous agents but where there is also consumption in the first period. This economy will allow us to solve for endogenous real interest rates and asset returns. It will use a setup which is an extension of sorts of that considered above, and the methods used to find an equilibrium are parallel, though there are two prices to learn from in the model with intertemporal trade. Hence, to avoid full revelation of the private information, there need to be three dimensions of underlying uncertainty. On a general level, thus, we have one variable that captures endowment growth, one variable that captures some additional riskiness of the endowment income that accrues independently of the tree dividend in the second period, and one variable that captures the aggregate signal about the tree's payoff probabilities.

More specifically, as before, there is a single risky asset in the economy: a tree. The tree pays high dividends (d_h) with probability ν and low dividends (d_l) with probability $1 - \nu$. The tree pays only once and then dies. There is a measure 1 of agents in the economy. Agents live for two periods. A fraction λ of agents is endowed with A units of consumption good for the first period and receives no income in the second period. The remaining agents are endowed with B units of trees and some additional state-contingent endowment income. A fraction ϕ among the agents endowed with trees receive additional endowment income y_h when the tree pays high dividends and no additional such income when the tree pays low dividends. The remaining fraction $1 - \phi$ among the agents endowed with trees receive additional endowment income y_l when the tree pays low dividends and no additional such income otherwise. The distribution of resources across periods and states is described in the table below.

The variable λ represents the "aggregate growth variable". It specifies the fraction of total resources to be distributed in the first of the two periods. A low value of λ means high growth. The additional income in the second period allows for heterogeneous hedging needs. The demand for insurance varies across agents depending on how their income correlates with the returns of stocks. As ϕ increases, the net demand for insurance increases.

Fraction	Period 1	Period 2	
		High state	Low state
λ	A	0	0
$(1-\lambda)\,\phi$	0	$Bd_h + y_h$	Bd_l
$(1-\lambda)\,(1-\phi)$	0	Bd_h	$Bd_l + y_l$

Table 1: **Distribution of income across periods and states**

For the sake of simplicity, it is assumed that ν and λ are independent and drawn from uniform distributions with support $[0,1]$. The variable ϕ is drawn from a discrete distribution with support $\{\phi_l, \phi_h\}$. The probability that a fraction ϕ_h is realized equals π.

The probability distributions for ν, λ, and ϕ are common knowledge. However, agents do not observe the realizations of those variables. Instead, they receive informative signals about the expected performance of the tree. Each signal can be either good or bad. The probability of observing a good signal equals ν. Every agent receives only one signal.

Individuals live for two periods and maximize expected utility of present and future consumption flows, namely

$$u\left(c_0\right) + \beta E\left[u\left(c_1\right)|\mathcal{I}\right],$$

where $\mathcal{I}$ denotes the agent's information set.

There are two assets available, which amounts to complete markets in this setup: two Arrow-Debreu securities, or contingent claims. One of these pays 1 unit of the consumption good when the high dividends state is realized and otherwise it pays zero. The other security similarly pays 1 unit in the low dividends state and nothing otherwise. Thus, there are two prices to be determined: the relative price of consumption in the state where the tree has a high dividend—in terms of period-one consumption—and the corresponding relative price for when the tree pays a low dividend.

The consumer's optimization problem can then be expressed as follows

$$\underset{c_0, c_h, c_l}{Max}\left\{u\left(c_0\right) + \beta\left[E\left(\nu \mid \mathcal{I}\right) u\left(c_h\right) + \left(1 - E\left(\nu \mid \mathcal{I}\right)\right) u\left(c_l\right)\right]\right\}$$

subject to

$$c + p_h c_h + p_l c_l = W$$

The equilibrium prices depends on ν and λ and ϕ. This implies that market prices do not reveal all the information available in the economy. Agents are fully rational and use the information pooled by the equilibrium price when they update their beliefs. They learn from their private signals, but also understand how the price is determined in equilibrium. This allows them to make inferences about the realizations of ν, λ, and ϕ once they have observed the market prices. In addition, as in the simpler model, their endowment realization also conveys valuable information, as will be described below.

In this model, agents trade for three reasons. First, they want to smooth out consumption across periods. Agents endowed with initial consumption goods need to sell part of their endowment in order to buy future consumption. Similarly, the fraction endowed with future consumption wants to trade part of their endowments for current consumption. Second, agents display different hedging needs. Individuals with income highly correlated with the returns of trees are willing to sell contingent claims paying in the high state and buy consumption in the low state. Third, agents have different beliefs about the realization of ν. Optimistic agents are willing to buy contingent claims paying in the high dividend state.

3.1 Definition of equilibrium

As before, the belief about ν of an agent of type i is denoted buy $\tilde{\nu}^i$. For the agent to be able to unveil the information conveyed by the market prices, he must guess on the equilibrium relationship between prices, ν, λ, and ϕ. Thus, $\mathcal{P}_l^i$ and $\mathcal{P}_h^i$ denote the price functions perceived by agent i. They are used to extract information from the observed prices.

$$\tilde{\nu}^i\left(\mathcal{I}, p_l, p_h\right) = E\left[\nu | \mathcal{I}_i, p_l, p_h.\right]$$

There are six types of agents depending on whether the signal realization is good or bad, whether the agent has been endowed with consumption goods in the first or second period, and, in the last case, whether his income is positively correlated with dividend payments or not. The set of agents is denoted by Υ, where

$$\Upsilon = \left\{1E, 0E, 1Th, 0Th, 1Tl, 0Tl\right\}.$$

A value of 1 (0) denotes a good (bad) signal. E and T denote whether the individual is endowed with first-period income or with trees and second-period income, respectively. Finally, h and l denote whether the second-period income has high or low correlation with dividends. Denote by $\mu^i\left(\nu, \lambda, \phi\right)$ the measure of agents i in the population. This information is summarized in the table below.

Let $Y_{ts}\left(\lambda, \phi\right)$ denote the overall aggregate resources in period t and state s, $c_{ts}^i\left(p_l, p_h\right)$ denote the optimal consumption demand of agent i in period t and state s, and $Z_{ts}\left(p_l, p_h, \nu, \lambda, \phi\right)$ denote the

Type	Measure
$1E$	$\nu\lambda$
$0E$	$(1-\nu)\lambda$
$1Th$	$\nu(1-\lambda)\phi$
$0Th$	$(1-\nu)(1-\lambda)\phi$
$1Tl$	$\nu(1-\lambda)(1-\phi)$
$0Tl$	$(1-\nu)(1-\lambda)(1-\phi)$

aggregate demand for period t and state s consumption goods. The latter is computed as follows:

$$Z_{ts}(p_l, p_h, \nu, \lambda, \phi) = \sum_{i \in \Upsilon} c^i_{ts}(p_l, p_h)\, \mu^i(\nu, \lambda, \phi).$$

Denote the unit interval by I and the set of possible values of ϕ by Φ. Notice that ν and λ take values in I. We are now ready to define a competitive equilibrium for this class of economies.

Definition 2 *A rational expectations equilibrium, REE, consists of two measurable price functions* $P_l : \mathrm{I} \times \mathrm{I} \times \Phi \to [0, \infty]$, $P_h : \mathrm{I} \times \mathrm{I} \times \Phi \to [0, \infty]$ *and individual demands* $\left\{c^i_{ts}(p_l, p_h)\right\}_{i \in \Upsilon,\, ts=1,2h,2l}$ *such that:*

(1) $\left\{c^i_{ts}(p_l, p_h)\right\}_{ts=1,2h,2l}$ *solves consumer i's problem* $\forall\, i \in \Upsilon$ *and* $\forall\, \nu \in \mathrm{I}$, $\forall\, \lambda \in \mathrm{I}\, \forall \phi \in \Phi$, *given that consumers use* P_l *and* P_h *for determining the distributions for* p_l *and* p_h*; and*

(2) *markets clear, i.e.,* $Z_{ts}(P_l(\nu, \lambda, \phi), P_h(\nu, \lambda, \phi), \nu, \lambda, \phi) = Y_{ts}(\lambda, \phi)\ \forall\ ts = l, 2h, 2l$ *and* $\nu \in \mathrm{I}$, $\lambda \in \mathrm{I}$, $\phi \in \Phi$.

An important assumption implicit in Definition 2 is that individuals' perceived price functions coincide with the actual equilibrium functions. Agents fully understand how prices are determined and take this information into account when updating their beliefs. As in the simpler model, finding an equilibrium amounts to solving for a fixed point of a functional equation: the price functions perceived by the agents must coincide with the price functions generated by their behavior.

3.2 Solution

3.2.1 The fully revealing case

We begin by characterizing the equilibrium in the case where there is complete information. This will help to understand how the learning process works in the more general case, when agents extract information from prices. Equations (11) and (12) show the equilibrium prices when the realization of ν is common knowledge.

$$P_h(\nu, \lambda, \phi) = \frac{\beta\lambda A\nu}{(1-\lambda)\,B\,(d_h + \phi y_h)} \tag{11}$$

$$P_l(\nu, \lambda, \phi) = \frac{\beta\lambda A\,(1-\nu)}{(1-\lambda)\,B\,(d_l + (1-\phi)\,y_l)} \tag{12}$$

The value taken by λ does not affect the relative prices between the two contingent claims. It only affects the relative price between current and future consumption. For instance, an increase in λ uniformly increases the price of future consumption. The reason is simply that second period consumption goods become more scarce for larger realizations of λ.

On the other hand, the relative price $\frac{p_h}{p_l}$ clearly depends on the joint realizations of ν and ϕ. This price takes high values when the high dividend state is more likely (high ν), or the fraction with income correlated with dividend payments is small (low ϕ).

Suppose now that there is one agent in the economy who does not observe the joint realization of (ν, λ, ϕ), whereas everyone else does. The uninformed agent faces a signal extraction problem: he wants to learn the value of ν, but the only source of information he has are the prices observed. Trivially, in this case the equilibrium prices coincide with the ones specified above. They imply the following relationship:

$$\nu = \frac{(d_h + \phi y_h)\,\frac{p_h}{p_l}}{d_l + (1-\phi)\,y_l + (d_h + \phi y_h)\,\frac{p_h}{p_l}}.$$

If the uninformed agent were able to observe the actual realization of ϕ, the relative price $\frac{p_h}{p_l}$ would convey enough information to let him retrieve the actual value of ν. When this is not the case, the agent can only identify the possible ν realizations. His belief is computed therefore as a weighted sum of the possible values of ν consistent with the observed prices, namely,

$$\tilde{\nu} = \left(\frac{(d_h + \phi_h y_h)\,\frac{p_h}{p_l}}{d_l + (1-\phi_h)\,y_l + (d_h + \phi_h y_h)\,\frac{p_h}{p_l}}\right)\Pr\{\phi_h\} + \left(\frac{(d_h + \phi_l y_h)\,\frac{p_h}{p_l}}{d_l + (1-\phi_l)\,y_l + (d_h + \phi_l y_h)\,\frac{p_h}{p_l}}\right)\Pr\{\phi_l\}.$$

In the more general framework, the weights also depend on the agent's private information.

3.3 The model with partially revealing prices

Equations (15) and (16) below show the explicit form of the market clearing conditions leading to equilibrium prices for a given set of beliefs. In these equations, the beliefs depend on the price vector (p_l, p_h), although this dependence has been suppressed for readability. Each price, thus, is a function of three variables ν, λ, and ϕ.

$$p_h = \frac{\beta\lambda A}{(1-\lambda)B} \cdot$$
$$\cdot \frac{(d_l + (1-\phi)\,y_l)\left(\nu\tilde{\nu}^{1E} + (1-\nu)\,\tilde{\nu}^{0E}\right) + \beta\left[d_l\phi\left(\nu\tilde{\nu}^{1Th} + (1-\nu)\,\tilde{\nu}^{0Th}\right) + (d_l + y_l)(1-\phi)\left(\nu\tilde{\nu}^{1Tl} + (1-\nu)\,\tilde{\nu}^{0Tl}\right)\right]}{(1+\beta)(d_h + \phi y_h)(d_l + (1-\phi)\,y_l) + \beta\left(d_h + d_l\frac{y_h}{y_l} + y_h\right) y_l\phi(1-\phi)\left[\nu\left(\tilde{\nu}^{1Tl} - \tilde{\nu}^{1Th}\right) + (1-\nu)\left(\tilde{\nu}^{0Tl} - \tilde{\nu}^{0Th}\right)\right]} \quad (13)$$

$$p_l = \frac{\beta\lambda A}{(1-\lambda)B} \cdot$$
$$\cdot \frac{(1+\beta)(d_h + \phi y_h) - (d_h + \phi y_h)\left(\nu\tilde{\nu}^{1E} + (1-\nu)\,\tilde{\nu}^{0E}\right) - \beta\left[(d_h + y_h)\,\phi\left(\nu\tilde{\nu}^{1Th} + (1-\nu)\,\tilde{\nu}^{0Th}\right) + d_h(1-\phi)\left(\nu\tilde{\nu}^{1Tl} + (1-\nu)\,\tilde{\nu}^{0Tl}\right)\right]}{(1+\beta)(d_h + \phi y_h)(d_l + (1-\phi)\,y_l) + \beta\left(d_h + d_l\frac{y_h}{y_l} + y_h\right) y_l\phi(1-\phi)\left[\nu\left(\tilde{\nu}^{1Tl} - \tilde{\nu}^{1Th}\right) + (1-\nu)\left(\tilde{\nu}^{0Tl} - \tilde{\nu}^{0Th}\right)\right]} \quad (14)$$

In parallel with the procedure in the simpler model, we proceed to define inverses of the price functions, now with respect to both ν and λ: $\mathcal{V}_i(p_l, p_h)$ and $\Lambda_i(p_l, p_h)$ are defined jointly by

$$p_h = P_h\left(\mathcal{V}_i(p_l, p_h), \Lambda_i(p_l, p_h), \phi_i\right)$$

and

$$p_l = P_l\left(\mathcal{V}_i(p_l, p_h), \Lambda_i(p_l, p_h), \phi_i\right)$$

for $i \in \{l, h\}$ and all (p_l, p_h).

Let $G_i(p_l, p_h)$ denote the c.d.f. describing the joint distribution of (p_l, p_h) conditional on $\phi = \phi_i$. Given that both ν and λ are uniformly distributed over the interval $[0, 1]$, the density function for the price levels satisfies

$$dG_i(p_l, p_h) = \begin{vmatrix} \frac{\partial\mathcal{V}_i}{\partial p_l} & \frac{\partial\mathcal{V}_i}{\partial p_h} \\ \frac{\partial\Lambda_i}{\partial p_l} & \frac{\partial\Lambda_i}{\partial p_h} \end{vmatrix}.$$

We consider first how an agent that has received an endowment for the first period and a good signal about the tree. In the following, we use p to denote the vector (p_l, p_h). The belief of this agent thus consists of the following expectation:

$$\tilde{\nu}^{1E}(p) = \mathcal{V}_h(p)\,Pr\{\phi_h|1, E, p\} + \mathcal{V}_l(p)\,Pr\{\phi_l|1, E, p\}.$$

The second equality takes into account that once the agent has conditioned on the price, the probability ν has a dichotomous distribution. This expression becomes

$$\tilde{\nu}^{1E}(p) = \frac{\mathcal{V}_h(p)^2 \Lambda_h(p) \phi_h dG_h(p) \pi + \mathcal{V}_l(p)^2 \Lambda_l(p) \phi_l dG_l(p)(1-\pi)}{\mathcal{V}_h(p) \Lambda_h(p) \phi_h dG_h(p) \pi + \mathcal{V}_l(p) \Lambda_l(p) \phi_l dG_l(p)(1-\pi)}.$$

The beliefs of the remaining agents can be found using a similar logic:

$$\tilde{\nu}^{OE}(p) = \frac{\mathcal{V}_h(p)(1-\mathcal{V}_h(p)) \Lambda_h(p) \phi_h dG_h(p) \pi + \mathcal{V}_l(p)(1-\mathcal{V}_l(p)) \Lambda_l(p) \phi_l dG_l(p)(1-\pi)}{(1-\mathcal{V}_h(p)) \Lambda_h(p) \phi_h dG_h(p) \pi + (1-\mathcal{V}_l(p)) \Lambda_l(p) \phi_l dG_l(p)(1-\pi)}$$

$$\tilde{\nu}^{1Th}(p) = \frac{\mathcal{V}_h(p)^2 (1-\Lambda_h(p)) \phi_h dG_h(p) \pi + \mathcal{V}_l(p)^2 (1-\Lambda_l(p)) \phi_l dG_l(p)(1-\pi)}{\mathcal{V}_h(p)(1-\Lambda_h(p)) \phi_h dG_h(p) \pi + \mathcal{V}_l(p)(1-\Lambda_l(p)) \phi_l dG_l(p)(1-\pi)}$$

$$\tilde{\nu}^{OTh}(p) = \frac{\mathcal{V}_h(p)(1-\mathcal{V}_h(p))(1-\Lambda_h(p)) \phi_h dG_h(p) \pi + \mathcal{V}_l(p)(1-\mathcal{V}_l(p))(1-\Lambda_l(p)) \phi_l dG_l(p)(1-\pi)}{(1-\mathcal{V}_h(p))(1-\Lambda_h(p)) \phi_h dG_h(p) \pi + (1-\mathcal{V}_l(p))(1-\Lambda_l(p)) \phi_l dG_l(p)(1-\pi)}$$

$$\tilde{\nu}^{1Tl}(p) = \frac{\mathcal{V}_h(p)^2 (1-\Lambda_h(p))(1-\phi_h) dG_h(p) \pi + \mathcal{V}_l(p)^2 (1-\Lambda_l(p))(1-\phi_l) dG_l(p)(1-\pi)}{\mathcal{V}_h(p)(1-\Lambda_h(p))(1-\phi_h) dG_h(p) \pi + \mathcal{V}_l(p)(1-\Lambda_l(p))(1-\phi_l) dG_l(p)(1-\pi)}$$

$$\tilde{\nu}^{OTh}(p) = \frac{\mathcal{V}_h(p)(1-\mathcal{V}_h(p))(1-\Lambda_h(p))(1-\phi_h) dG_h(p) \pi + \mathcal{V}_l(p)(1-\mathcal{V}_l(p))(1-\Lambda_l(p))(1-\phi_l) dG_l(p)(1-\pi)}{(1-\mathcal{V}_h(p))(1-\Lambda_h(p))(1-\phi_h) dG_h(p) \pi + (1-\mathcal{V}_l(p))(1-\Lambda_l(p))(1-\phi_l) dG_l(p)(1-\pi)}.$$

It should be pointed out here that the model features some amount of information hierarchy between initial stockholders and initial non-stockholders. The former are more informed. The reason is that they receive an extra piece of information: how their income correlates with dividend payments.

In order to solve for an equilibrium, as in the simpler model we now need to substitute the belief functions into the market-clearing conditions (15) and (16). More precisely, equation (15) produces two equations, one for h, where one lets $\nu = \mathcal{V}_h(p_l, p_h)$ and $\lambda = \Lambda_h(p_l, p_h)$, and one for l, where one lets $\nu = \mathcal{V}_l(p_l, p_h)$ and $\lambda = \Lambda_l(p_l, p_h)$, with all the belief functions inserted in each case. Similarly, equation (16) produces two equations, one for h and one for l. In total this amounts to four functional (differential) equations that jointly determine $\mathcal{V}_i(p_l, p_h)$ and $\Lambda_i(p_l, p_h)$ for $i = h$ and $i = l$.

The procedure used to find a solution relies on the Projection Method described in Judd (1998). The functions $\mathcal{V}$ and Λ are parameterized as the weighted sum of Chebychev polynomials. This reduces the dimensionality of the problem. Instead of solving for infinite-dimensional objects, it is only necessary to solve for a finite set of parameters. Thus, a solution consists of a set of parameters values such that the behavior generated by these functions are consistent with equilibrium behavior.

3.4 A case with informed agents

In this section we consider the case where a fraction ρ of the population has full information, i.e., they know the actual realization of ν. The latter are referred as "fully informed" agents. For simplicity, it is assumed that the distribution of endowments across the fraction of fully informed individuals coincides with the distribution of endowments across individuals who only receive less than fully informative signals. That is, a fraction λ of the fully informed individuals receive an endowment of initial consumption goods, while a fraction $1-\lambda$ receive an endowment of period two consumption goods; every agent in the last group is entitled to B shares of the tree, and there is heterogeneity in the distribution of income: a fraction ϕ of the agents endowed with trees receive y_h consumption goods in the second period if the tree pays high dividend, and a fraction $1-\phi$ receive y_l consumption goods if the tree pays low dividends.

Thus, there are nine types of individuals depending on the quality of the information received (full information versus informative signals) and the endowment realization. The measure of agents is described in the table below.

Type	Measure
E	$\rho\lambda$
Th	$\rho\nu\phi$
Tl	$\rho\nu(1-\phi)$
$1E$	$(1-\rho)\,\nu\lambda$
$0E$	$(1-\rho)\,(1-\nu)\,\lambda$
$1Th$	$(1-\rho)\,\nu\,(1-\lambda)\,\phi$
$0Th$	$(1-\rho)\,(1-\nu)\,(1-\lambda)\,\phi$
$1Tl$	$(1-\rho)\,\nu\,(1-\lambda)\,(1-\phi)$
$0Tl$	$(1-\rho)\,(1-\nu)\,(1-\lambda)\,(1-\phi)$

This formulation allows us to encompass the fully revealing case with the partially revealing equilibrium described above. The first case corresponds to $\rho = 1$, while the second case corresponds to $\rho = 0$. The equilibrium pricing relationships are similar to the ones we found above, namely

$$p_h = \frac{\beta\lambda A \left\{ \begin{array}{l} (1-\rho)\left[\begin{array}{l} \left(\hat{d}_l + (1-\phi)\, y_l\right)\left(\nu\tilde{\nu}^{1E} + (1-\nu)\,\tilde{\nu}^{0E}\right) + \\ \beta\left[\hat{d}_l\phi\left(\nu\tilde{\nu}^{1Th} + (1-\nu)\,\tilde{\nu}^{0Th}\right) + \left(\hat{d}_l + y_l\right)(1-\phi)\left(\nu\tilde{\nu}^{1Tl} + (1-\nu)\,\tilde{\nu}^{0Tl}\right)\right] \end{array}\right] + \\ \rho\left(\hat{d}_l + (1-\phi)\, y_l\right)(1+\beta)\,\nu \end{array}\right\}}{(1-\lambda)\left\{\begin{array}{l} (1+\beta)\left(\hat{d}_h + \phi y_h\right)\left(\hat{d}_l + (1-\phi)\, y_l\right) + \\ (1-\rho)\,\beta\left(\hat{d}_h + \hat{d}_l\frac{y_h}{y_l} + y_h\right) y_l\phi\,(1-\phi)\left[\nu\left(\tilde{\nu}^{1Tl} - \tilde{\nu}^{1Th}\right) + (1-\nu)\left(\tilde{\nu}^{0Tl} - \tilde{\nu}^{0Th}\right)\right] \end{array}\right\}} \tag{15}$$

$$p_l = \frac{\beta\lambda A \left\{ \begin{array}{l} (1-\rho)\left[\begin{array}{l} (1+\beta)\left(\hat{d}_h + \phi y_h\right) - \left(\hat{d}_h + \phi y_h\right)\left(\nu\tilde{\nu}^{1E} + (1-\nu)\,\tilde{\nu}^{0E}\right) - \\ \beta\left[\left(\hat{d}_h + y_h\right)\phi\left(\nu\tilde{\nu}^{1Th} + (1-\nu)\,\tilde{\nu}^{0Th}\right) + \hat{d}_h(1-\phi)\left(\nu\tilde{\nu}^{1Tl} + (1-\nu)\,\tilde{\nu}^{0Tl}\right)\right] \end{array}\right] \\ +\rho\left(\hat{d}_h + y_h\phi\right)(1+\beta) \end{array}\right\}}{(1-\lambda)\left\{\begin{array}{l} (1+\beta)\left(\hat{d}_h + \phi y_h\right)\left(\hat{d}_l + (1-\phi)\, y_l\right) + \\ (1-\rho)\,\beta\left(\hat{d}_h + \hat{d}_l\frac{y_h}{y_l} + y_h\right) y_l\phi\,(1-\phi)\left[\nu\left(\tilde{\nu}^{1Tl} - \tilde{\nu}^{1Th}\right) + (1-\nu)\left(\tilde{\nu}^{0Tl} - \tilde{\nu}^{0Th}\right)\right] \end{array}\right\}}. \tag{16}$$

The difference with the version introduced in the previous section is that the present case features an additional channel through which the information about ν is impounded in the equilibrium prices: the trading behavior of fully informed agents. The other channel is the distribution of signals across partially informed agents.

The model was solved for the following parameter values

$d_h = 1$	$d_l = 0.2$
$A = 1$	$B = 1$
$\phi_h = 0.35$	$\phi_l = 0.25$
$y_h = 0.5$	$y_l = 0.5$
$\Pr(\phi_h) = 0.5$	$\beta = 0.96)$
$\rho = 0.2$	

3.4.1 Results

For convenience, we solve for the equilibrium functions $\mathcal{V}_l\left(p_l, \frac{p_h}{p_l}\right)$ and $\mathcal{V}_h\left(p_l, \frac{p_h}{p_l}\right)$. The choice of arguments is intended to facilitate the numerical approximation. In the fully revealing case, $\mathcal{V}_h$ and $\mathcal{V}_l$ display extremes degrees of curvature around $(p_l, p_h) = (0, 0)$. The reason is that both functions depend only on the relative price $\frac{p_h}{p_l}$, which is very sensitive to p_l and p_h when both are close to zero.[9]

The functions $\Lambda_i\left(p_l, \frac{p_h}{p_l}\right)$ can be found analytically from the market clearing condition for initial consumption goods, namely

$$\Lambda_i\left(p_l, \frac{p_h}{p_l}\right) = \frac{\frac{p_h}{p_l}\left(Bd_h + \phi y_h\right) + Bd_l + (1-\phi)\, y_l}{\frac{p_h}{p_l}\left(Bd_h + \phi y_h\right) + Bd_l + (1-\phi)\, y_l + A\beta}.$$

In order to help to visualize the results, the graphs below use a transformation of p_l and $\frac{p_h}{p_l}$ as input variables. In equilibrium, both prices take values in the interval $[0, \infty)$. The graphs are expressed in terms of $\hat{p}_l$ and $\hat{\frac{p_h}{p_l}}$, where

$$\hat{p}_l = \frac{p_l}{1+p_l}$$
$$\hat{\frac{p_h}{p_l}} = \frac{\frac{p_h}{p_l}}{1+\frac{p_h}{p_l}}.$$

Figure 5 describes the fully revealing equilibrium relationship between ν, $\hat{p}_l$ and $\hat{\frac{p_h}{p_l}}$ when the fraction of individuals with income negatively correlated with dividends is low, i.e., when $\phi = \phi_l$. Figure 6 shows the difference between the values of $\mathcal{V}_l$ and $\mathcal{V}_h$ for every possible price realization. It shows that the $\mathcal{V}_h$ is always above $\mathcal{V}_l$. The reason is that for a given relative price $\frac{p_h}{p_l}$, a higher value of ϕ implies a higher demand for insurance. That is, there is a higher fraction of individuals willing to sell contingent claims paying in the high state and demanding contingent claims that pay in the low state. Thus, in equilibrium a high value of ϕ must be associated with a high value of ν. A higher ν reduces the aggregate demand for contingent claims paying in the low state. In summary, the same relative price $\frac{p_h}{p_l}$ can be generated either by a high fraction of individuals with hedging needs and a low probability of the low consumption state scenario, or by a small fraction of individuals with hedging needs and a high probability of the low consumption state scenario.

The equilibrium relationship between λ, $\hat{p}_l$, $\hat{\frac{p_h}{p_l}}$, and ϕ is described below. Figure 9 shows the λ function when $\phi = \phi_l$. It is increasing in both arguments. Given the relative price $\frac{p_h}{p_l}$, higher values of p_l imply that the second-period consumption goods in both states become uniformly more valuable. In the model, this can only be explained by the fact that there are less consumption goods available

[9]In fact, neither $\mathcal{V}_h$ nor $\mathcal{V}_l$ are defined when $p_l = 0$ and $p_h = 0$.

in both future states, i.e., that the fraction of agents endowed with trees and second period income is low (λ is high). On the other hand, holding p_l fixed and increasing $\frac{p_h}{p_l}$ increases the wealth of agents endowed second period's consumption goods, but it does not affect the wealth of individuals endowed with period one consumption goods. This induces an increase in the net aggregate demand of period one consumption goods; hence, in equilibrium higher relative prices must be associated with a lower fraction of the population endowed with trees and second period income (higher λ).

From the functional form for λ, it is easy to observe that

$$\Lambda_l \left(\hat{p}_l, \frac{\hat{p_h}}{p_l} \right) > \Lambda_h \left(\hat{p}_l, \frac{\hat{p_h}}{p_l} \right) \quad if \quad \frac{p_h}{p_l} > 1$$

and

$$\Lambda_l \left(\hat{p}_l, \frac{\hat{p_h}}{p_l} \right) < \Lambda_h \left(\hat{p}_l, \frac{\hat{p_h}}{p_l} \right) \quad if \quad \frac{p_h}{p_l} < 1.$$

A higher fraction ϕ increases the number of individuals with income in the high dividend state and reduces the number of individuals with income in the low dividend state. When the relative price $\frac{p_l}{p_h}$ is low, the latter imply a reduction in the aggregate wealth of individual endowed with second-period consumption goods. The reason is that there are less individuals endowed with the most valuable second-period consumption good. In equilibrium, the latter must be associated with a lower fraction of agents endowed with period one consumption good so that the net supply of those goods is reduced. The opposite is true when the relative price $\frac{p_h}{p_l}$ takes a sufficiently high values.

Figure 11 illustrates the equilibrium function $\mathcal{V}_l \left(\hat{p}_l, \frac{\hat{p_h}}{p_l} \right)$ in a case where only 20% of the population has full information. The figure shows that the function has a similar shape compared to the function in the fully revealing case. The difference is that $\mathcal{V}_l$ and $\mathcal{V}_h$ now depend on $\hat{p}_l$, though they are not sensitive to this variable.

As expected, the function $\mathcal{V}_h$ takes higher values while $\mathcal{V}_l$ takes lower values compared to the fully revealing case. This is illustrated in Figures 12 and 13. The logic is similar to the one used in the single period framework. Figures 7 and 8 illustrate the result from another perspective. The graphs describe the relative price $\frac{p_h}{p_l}$ as a function of ν and λ when $\phi = \phi_l$. The graphs show that the equilibrium prices increase with ϕ in an economy with partial information.[10] In addition, equilibrium prices under partial information are higher than the values under full information. In other words, the lack of more imprecise information imply an overreaction of prices conditional on the values of ν and λ. We take away from this information that also in this version of our model, asset prices "overreact" to aggregate non-signal shocks: they respond more to shocks to ϕ and λ.

[10]We already know that this result holds in an economy with full information.

Figures 14 and 15 show that the equilibrium price $\frac{p_h}{p_l}$ react to λ in an economy with partial information. However, it does not depend on λ when agents are fully informed. The underlying reason is that λ plays a role in the beliefs. Since agents can retrieve valuable information from their endowment realizations, the distribution of endowments across time plays a new role in the determination of equilibrium prices.

Figures 16 and 17 show that the ranking of beliefs does not depend on prices. For the parameter values analyzed, the signal about the tree constitutes the most informative piece of information. Agents with good signals are always more optimistic than agents with bad signals. Since the endowment realization of agents that only receive period-2 consumption goods reveals valuable information, the latter class of agents display a wider dispersion of beliefs. The most optimistic agents are the ones receiving income contingent on the high dividend state and a good signal. The reason is that the endowment realization of these agents point towards a high realization of ϕ. It is more likely that they receive high state income if the fraction of individuals receiving high state income is high. Since a high value of ϕ is associated in equilibrium with a high realization of ν, this class of agents have two reasons to be optimistic.

Similarly, the most pessimistic agents are the ones receiving a bad signal and low dividend state contingent income. The beliefs of agents endowed with period one consumption goods lie in between.

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A Graphs

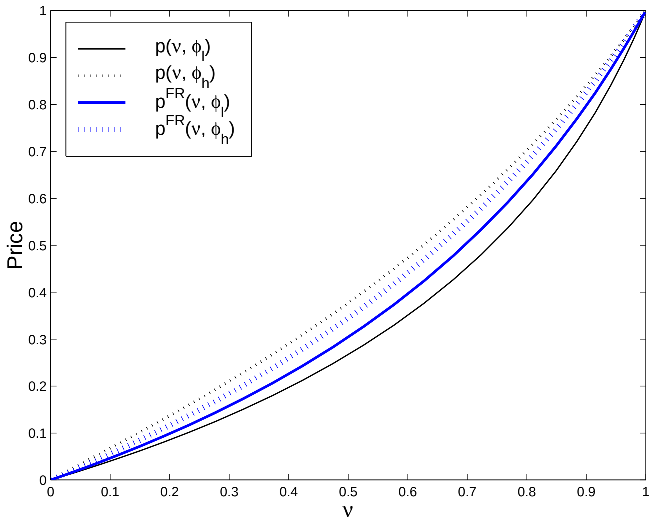

Figure 2: **Equilibrium price under full and partial information**

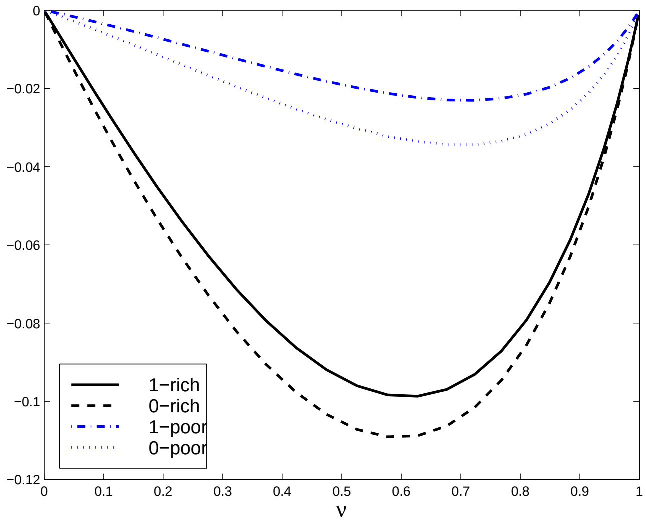

Figure 3: **Difference between individual beliefs and actual realizations of ν for the case** $\phi = \phi_l$

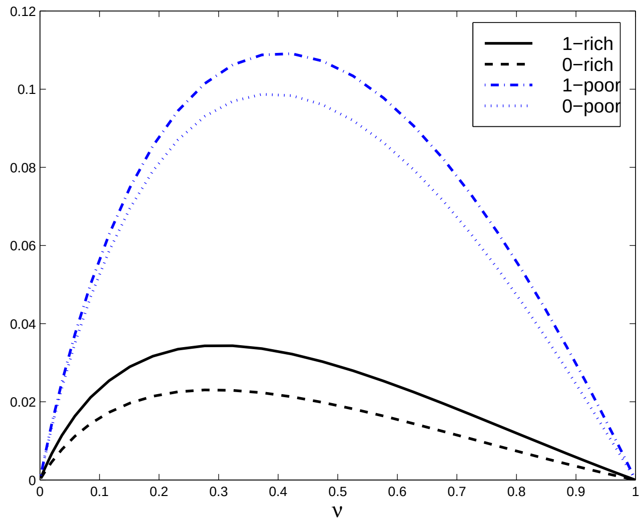

Figure 4: **Difference between individual beliefs and actual realizations of ν for the case** $\phi = \phi_h$

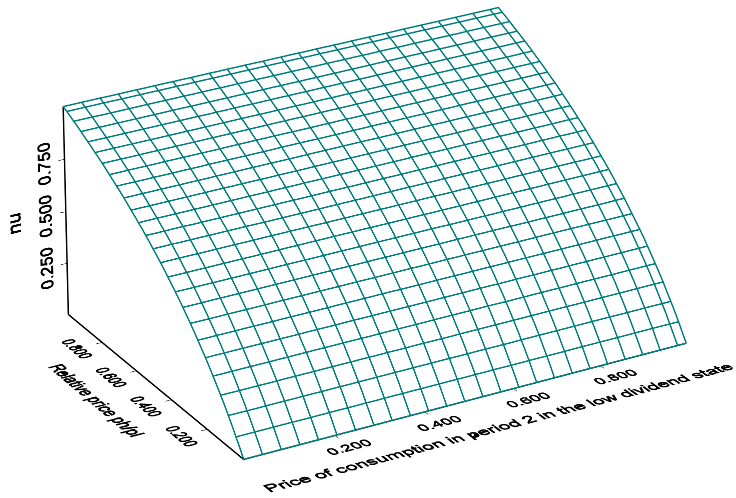

Figure 5: **Function $\mathcal{V}_l\left(\hat{p}_l, \frac{\hat{p_h}}{p_l}\right)$ when there is full information.**

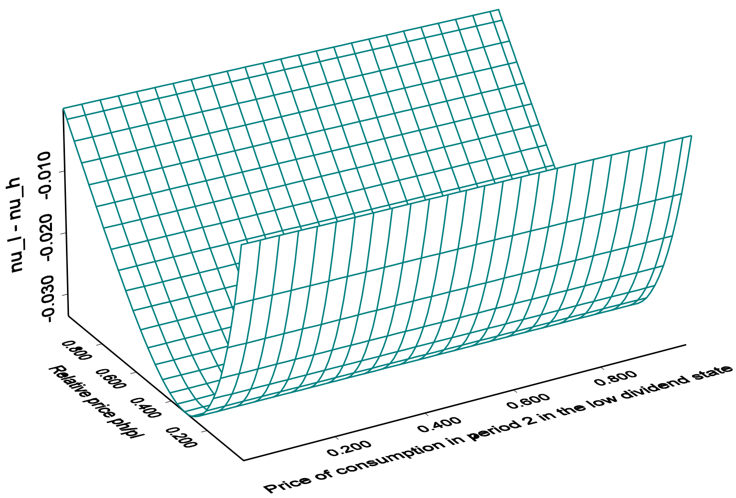

Figure 6: **Difference $\mathcal{V}_l\left(\hat{p}_l, \frac{\hat{p_h}}{p_l}\right) - \mathcal{V}_h\left(\hat{p}_l, \frac{\hat{p_h}}{p_l}\right)$ when there is full information.**

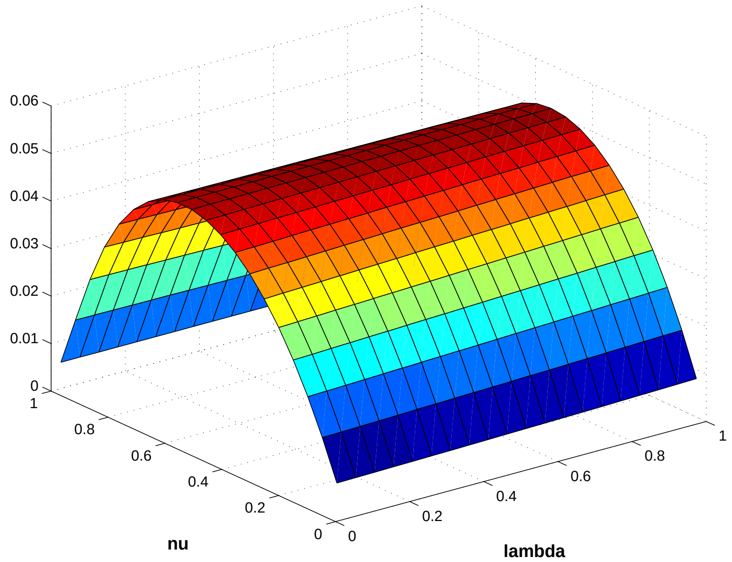

Figure 7: $P_l^{PI}(\nu,\lambda) - P_l^{FI}(\nu,\lambda)$

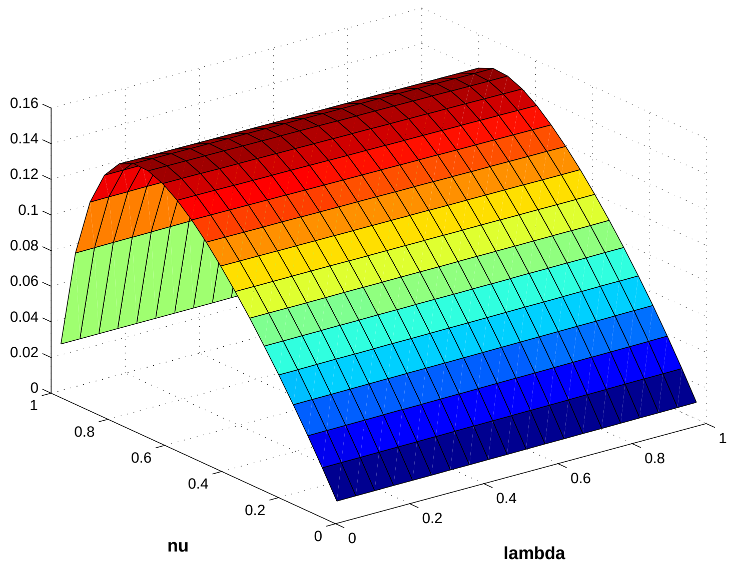

Figure 8: $P_l^{PI}(\nu,\lambda) - P_h^{PI}(\nu,\lambda)$

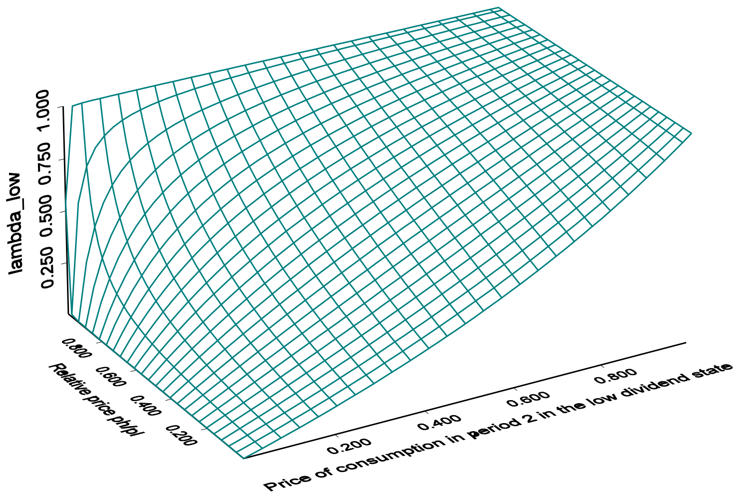

Figure 9: $\Lambda_l\left(\hat{p}_l, \frac{\hat{p_h}}{p_l}\right)$

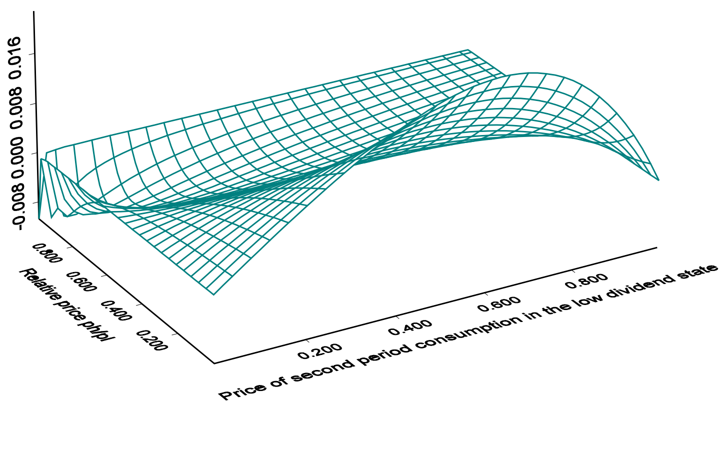

Figure 10: $\Lambda_l\left(\hat{p}_l, \frac{\hat{p_h}}{p_l}\right) - \Lambda_h\left(\hat{p}_l, \frac{\hat{p_h}}{p_l}\right)$

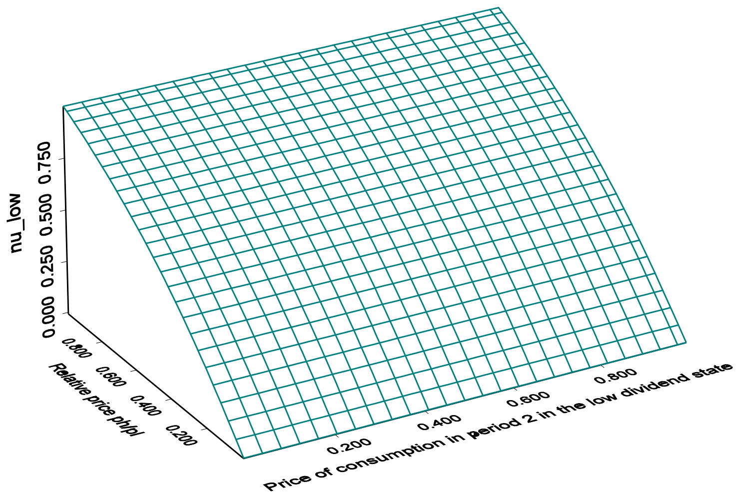

Figure 11: $\mathcal{V}_l^{PR}\left(\hat{p}_l, \frac{\hat{p_h}}{p_l}\right)$

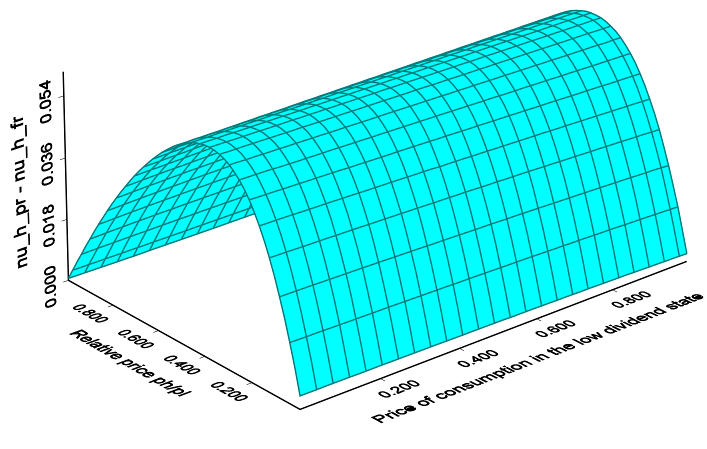

Figure 12: $\mathcal{V}_h^{PR}\left(\hat{p}_l, \frac{\hat{p_h}}{p_l}\right) - \mathcal{V}_h^{FR}\left(\hat{p}_l, \frac{\hat{p_h}}{p_l}\right)$

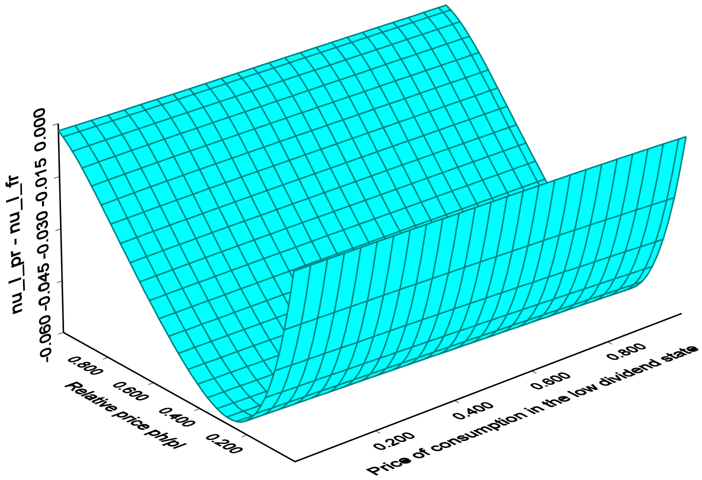

Figure 13: $\mathcal{V}_l^{PR}\left(\hat{p}_l, \frac{\hat{p_h}}{p_l}\right) - \mathcal{V}_l^{FR}\left(\hat{p}_l, \frac{\hat{p_h}}{p_l}\right)$

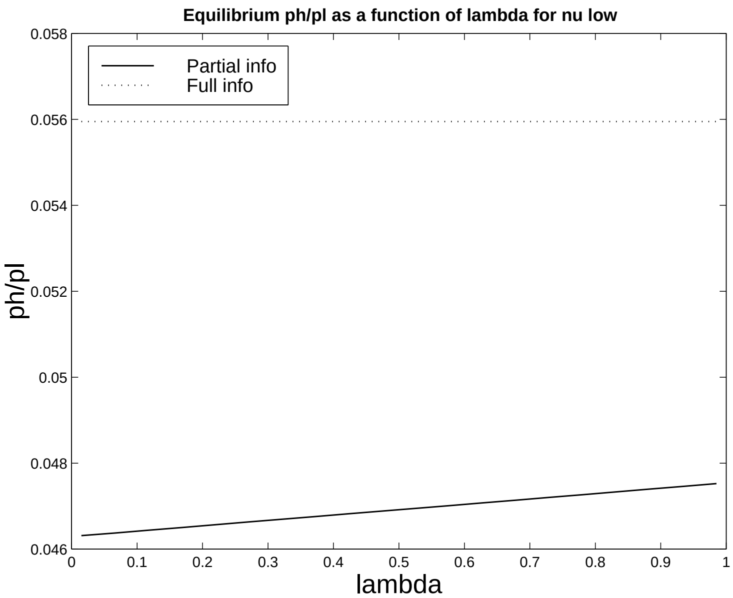

Figure 14:

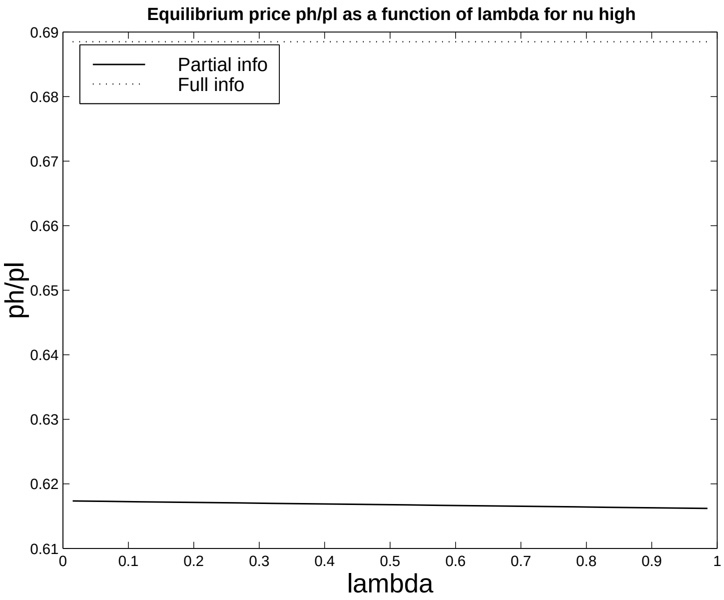

Figure 15:

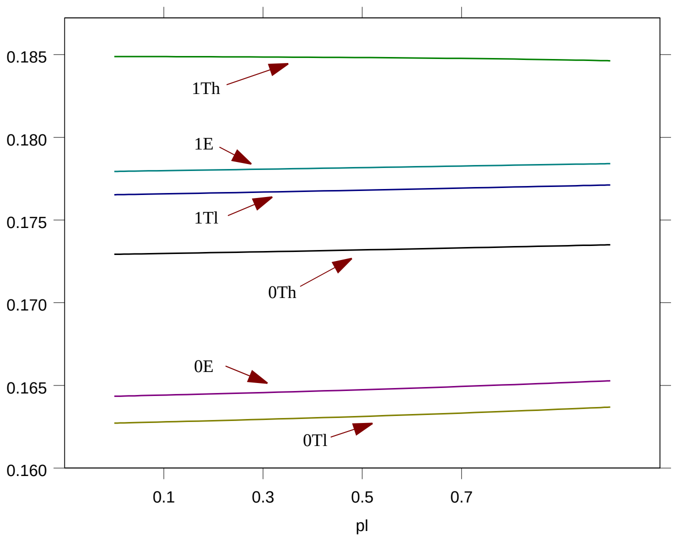

Figure 16: **Beliefs when** $\frac{\hat{p_h}}{p_l} = 0.0864$

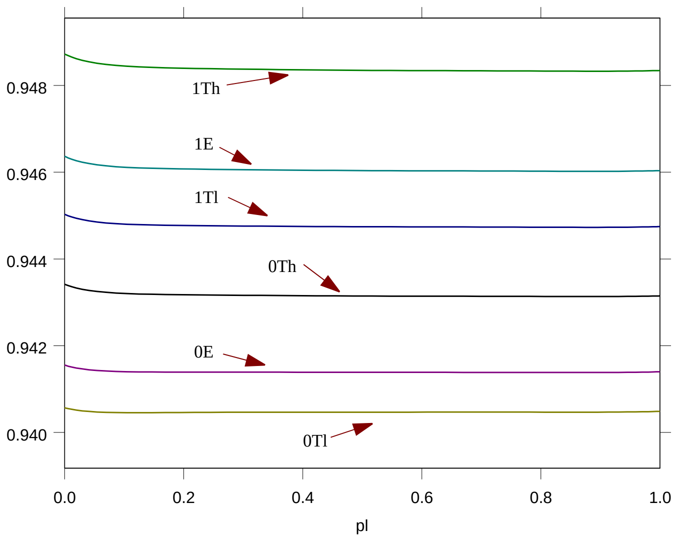

Figure 17: **Beliefs when** $\frac{\hat{p_h}}{p_l} = 0.895$