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WAGE-TENURE CONTRACTS, EXPERIENCE AND EMPLOYMENT STATUS

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Abstract

The objective of this paper is to study equilibrium in a labour market with search frictions à la Burdett and Mortensen (1998). Identical firms post wage-contracts and ex-ante identical workers search for a job while unemployed and for a better one while employed. Although this situation has been analysed before, Stevens (2004) and Burdett and Coles (2003), the main novelty of this paper is to allow firms to offer contracts according to the worker's initial experience and employment status. We construct an equilibrium in which firms compete in "promotion" contracts and offer unemployed workers longer "probation" periods than to employed workers. An interesting feature of this equilibrium is that outside offers become more generous with experience. This generates workers cohort effects within a firm that depend on the level of experience at which they where hired. The distribution of earnings within the firm is then such that workers who have acquired more "outside" firm experience and more tenure are higher in the earnings scale.

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1 Introduction

Why do firms offer different employment contracts to workers with different labour market status and experience? Many economists would argue that since experience is positively correlated with a worker's skills and unemployment depreciates human capital, workers are just being paid their marginal product at every point in time. This interpretation is so deeply rooted in labour economics that it is used as the standard explanation of why we observe positive returns to experience in wage equations. Moreover, it is also widely used to explain the adverse effects that unemployment inflicts on future earnings. The present paper argues that search frictions provides an alternative answer. We construct an equilibrium model in which firms offer wage-tenure contracts conditional on employment status and initial experience to discriminate between otherwise identical workers. We show that firms offer relatively higher paying jobs to employed workers and that outside offers become more generous with experience.

Recent theoretical analyses of non-stationary firm wage policies in the equilibrium search literature à la Burdett and Mortensen (1998), have enriched existing theories of wage dispersion and workers' labour market histories. In these models, wage dispersion is not only a market phenomenon, but can also be observed within each firm. In turn, changes in a worker's wage over time are determined by both job mobility and positive returns to tenure. These results are obtained under two types of wage policies, which differ in the firm's ability to wage discriminate otherwise similar workers. The degree of discrimination is reflected in assumptions on the information available to the firm when recruiting a worker and on the firm's policy when confronted with outside competition for its employees.

Postel-Vinay and Robin (2002 a) analyse the case of complete information and counter-offering, in which firms perfectly discriminate workers by their reservation wages. The firm posts a single wage that is re-negotiated every time the worker finds an alternative offer. Positive tenure effects are driven by the ability of a worker to engage his employer with other potential employers into Bertrand competition. Burdett and Coles (2003) and Stevens (2004) on the other hand, assume firms have no information about the worker and do not counter-offer, or precommit not to do so. The wage policy is similar to incentive/agency theory. In this case, firms post wage-tenure contracts with an increasing wage profile, to reduce the quit probability of their employees. The firm is able to minimise the expected loss caused by inefficient quits, by discriminating workers by their tenure.

This framework provides powerful theoretical tools for the empirical analysis of traditional issues in labour economics (see Manning, 2003 and Mortensen, 2003), promoting in particular the understanding of wage variation. However, in many cases labour markets do not appear to resemble the two extremes under which the theory is modelled. Firms do discriminate between potential employees by observable characteristics and rarely engage in offer matching. Therefore a more "realistic" setting is needed.

The present paper contributes to this literature by extending the analysis to an environment in which firms condition their job offers upon workers' observable characteristics that change over time, precommitting not to counter-offer any outside offers. In particular, we analyse the implications of this policy on the earnings distribution observed within each firm.[1]

[1] In an earlier paper, Carrillo-Tudela (2004), we use a simple version of the Burdett and Mortensen model to analyse the case in which firms discriminate between potential employees only by employment status. The present

Following Stevens (2004), we assume workers are risk neutral and liquidity contrained. The optimal wage-tenure contract for each level of initial experience and employment status is then described by a step-contract. Effectively, for each type of employment status, we segment Stevens homogenous case model into a continuum of markets (one for each experience) between which workers transition and engage in on-the-job search.[2] Given that firms always face outside competition for their employees, a step-contract is the only solution for the firm's problem.

We construct an equilibrium in which outside offers are degenerate and firms compete in "promotion" contracts. As in Carrillo-Tudela (2004) a "dual" labour market emerges within the firm (see Doeringer and Piore, 1971). All firms offer two type of contracts: "bad" jobs with longer probation periods to unemployed workers and "good" jobs with shorter ones to employed workers. Worker turnover occurs in the direction of bad towards good jobs. Since employed workers hired from unemployment earn less than their marginal revenue product until they get promoted, search frictions imply firms will have incentives to recruit them at positive experience markets. If firms offer contracts with short enough "probation" periods at the initial stages of a worker's career, outside offers become more generous with experience. In this case, firms find profitable to offer more than the workers' reservation value because they trade off a longer period during which they make positive profit with a higher chance the worker will quit in the future.

An interesting feature of this equilibrium is that there exists worker cohort effects within a firm

paper extends this work to a more general environment.

[2] This idea is similar to the one used in Van den Berg and Ridder (1998). They segment the homogenous Burdett and Mortensen model into a large (but finite) number of markets, to analyse the impact of between-market population heterogeneity in improving the fit of the earnings distribution. As in our case, in each particular market workers and firms are identical and markets differ in observable characteristics such as worker's age and educational level. However, they do not consider the possibility of workers moving between these markets as their characteristics change.

that depend on the level of experience at which they where hired. In turn, this implies that the distribution of earnings within the firm is such that when controlling for experience there exits a positive relation between tenure and earnings. Moreover, workers with the same tenure but with more “outside firm” experience are higher in the earnings ladder. Empirical evidence of these results can be found in Baker, Gibbs and Holmstrom (1994) and Medoff and Abraham (1980). We perform several comparative statics exercises by numerically analysing the model and obtain further insights on the interaction between the internal and external labour markets.

The next sections describe the general framework and discuss the workers’ and firms’ decision problems. Section 5 generalises Stevens results and show that the optimal wage-tenure contract is a step-contract for each level of initial experience and employment status. Sections 6 and 7, define and construct the market equilibrium. Using simulations we present some comparative statics exercises. Section 8 further discusses the results and concludes.

2 Basic Framework

Consider a labour market in steady state in which time is continuous and there is a fixed number of workers and firms each of measure one. Workers can either be employed (e) or unemployed (u) with experience, x, defined as total time spent in all previous employments. Firms post job offers at a zero cost on a take it or leave it basis. Both unemployed and employed workers of any experience search. Let $0 < \lambda < \infty$ denote the common Poisson arrival rate of these offers. Assume there is no recall should a worker quit or reject a job offer.[3]

[3] Although the no recall assumption does not bind in equilibrium it is important as it much simplifies the analysis that follows.

Although we will argue that firms offer different contracts to different experienced workers for reasons other than productivity, we start by given credit to the human capital explanation and analyse the case in which experience and productivity are positively correlated. We then show that this recruitment policy persists even when productivity is held constant. Hence, assume that when entering the labour market each worker is endowed with the same initial level of general human capital. As a worker gains more experience his human capital stock increases. Any worker with experience x would then have accumulated the same units of human capital over his employments spells. Firms use this human capital as a single input. All firms generate the same revenue, $p(x),$ for each worker of experience x they employ per unit of time. Assume $p : \Re_+ \rightarrow P$ is continuously differentiable, strictly increasing and concave, where for simplicity $P \subset \Re_{++}$ is a bounded set described by the interval $[\underline{p}, \overline{p}]$.

A job offer is described by a wage contract. Upon a meeting firms are able to observe the worker's experience and labour market status and condition their offers upon these characteristics. An important assumption is that firms pre-commit not to counter-offer any outside offer the worker might receive in the future. Contracts are then contingent on the worker's tenure, $t,$ defined as time spent working on the firm. A job offer is fully described by a wage-tenure contract conditional on the worker's initial experience and employment status.

An important simplification is that workers are liquidity constrained and cannot borrow against future earnings. As in Stevens (2004) the lack of capital markets constrain the set of feasible contracts available to the firm. In particular, they rule out contracts that require entry fees or quitting payments from the worker. Formally, a wage contract is described by a right-continuous function $w_i^x : \Re_+ \to W$ defined for all tenures t given employment status $i = u, e$ and starting experience x such that $W \subset \Re_+$ is bounded from below by $\underline{w} \geq 0$.

Both agents have a zero rate of time preference. Firms are risk neutral and infinitely lived. The objective of each firm is to maximize total steady state flow profits. Workers, on the other hand, are also risk neutral but their lives are of uncertain duration. Any worker's life is described by an exponential random variable with parameter $0 < \delta < \infty$. The inflow rate of new unemployed workers of zero experience into the market is δ.[4] The objective of any worker is to maximize total expected lifetime utility. Finally, let b denote the opportunity cost of employment per unit of time and assume $\underline{p} > b > \underline{w}$.

3 Worker's Payoffs and Job Search Strategies

Given contact with a firm, a worker of employment status i and experience x observes the posted contract w_i^x. Let V_i^x denote his expected lifetime utility conditional on accepting it and using an optimal quit strategy in the future. Further, let $F_i(V_i^x \mid x)$ denote the distribution of starting payoffs offered by firms to workers with employment status i and experience x. Random matching implies, given contact with a firm, $F_i(V_i^x \mid x)$ describes the probability that the outside offer has a value no greater than V_i^x. Although $F_i(. \mid x)$ will be endogenously determine in equilibrium, at this stage assume it is continuous in x and has a bounded support. Let $\underline{V}_i^x$ and $\overline{V}_i^x$ denote the infimum and supremum of the support for each i, x.

First consider the case of an unemployed worker. Let $U(x)$ denote the expected lifetime payoff

[4] Although agents do not discount the future, note that the worker's "death rate", δ, plays the role of a discount rate.

of this worker when he has experience x and follows an optimal search strategy. Conditional on receiving a job offer, the definition of $U(x)$ and the no recall assumption imply that his optimal policy is described by: accept a job offer if and only if $V_u^x \geq U(x)$ and reject a job offer otherwise. Since workers do not accumulate experience while unemployed, $U(x)$ then solves the following stationary Bellman equation

$$\delta U(x) = b + \lambda \int_{U(x)}^{\overline{V}_u^x} [V_u^x - U(x)] dF_u(V_u^x \mid x). \tag{1}$$

Now consider an employed worker who has been hired from state i with starting experience x on a wage contract w_x^i. Define $V_i^x(t; w_i^x)$ as this worker's expected lifetime payoff at tenure t when using an optimal quit strategy. Given any contract w_i^x and experience $x + t$ where $V_i^x(t; w_i^x) > U(x + t)$, the definition of $V_i^x(.; w_i^x)$ and the no recall assumption implies the worker's optimal strategy is to quit if and only if he receives a job offer which has starting value $V_e^{x+t} > V_i^x(t; w_i^x)$ and continue employment at the firm if and only if $V_e^{x+t} \leq V_i^x(t; w_i^x)$. Since the worker gains experience while employed, $V_i^x(.; w_i^x)$ then satisfies the following non-stationary Bellman equation

$$\delta V_i^x(t; w_i^x) = w_i^x(t) + \frac{dV_i^x(t; w_i^x)}{dt} + \lambda \int_{V_i^x(t; w_i^x)}^{\overline{V}_e^{x+t}} [V_e^{x+t} - V_i^x(t; w_i^x)] dF_e(V_e^{x+t} \mid x + t), \tag{2}$$

for $i = u, e$, and note that $V_i^x(.; w_i^x)$ is right-differentiable with respect to t at the point in which w_i^x is discontinuous.[5]

However, if $V_i^x(t; w_i^x) < U(x+t)$ for some accumulated experience $x+t$, the worker's optimal

[5] See Van den Berg (1990)- Theorem 1 for a formal derivation of this equation and its properties.

strategy is to quit into unemployment. To allow for this possibility define the set Υ_i^x by

$$\Upsilon_i^x = \{t \in \Re_+ : V_i^x(t; w_i^x) < U(x+t)\} \quad \text{for } i = u, e$$

and let $\underline{t}_i^x = \inf \Upsilon_i^x$. Hence, $\underline{t}_i^x$ denotes the tenure at which an employed worker hired from state i with initial experience x optimally quits into unemployment. If $V_i^x(t; w_i^x) \geq U(x+t)$ for all t, then define $\underline{t}_i^x = \infty$ and the worker never quits into unemployment.

Note that given a wage contract w_i^x and tenure $t < \underline{t}_i^x$, a worker's hazard rate is $\delta + \lambda[1 - F_e(V_i^x(t; w_i^x) \mid x+t)]$ for $i = u, e$. Hence, for tenures $t < \underline{t}_i^x$, the survival probability

$$\psi_i^x(t; w_i^x) = e^{-\int_0^t [\delta + \lambda(1 - F_e(V_i^x(s; w_i^x) | x+s)] ds} \quad \text{for } i = u, e \tag{3}$$

describes the probability a newly employed worker hired from state i with starting experience x does not leave the firm before tenure t. If $\underline{t}_i^x < \infty$, then $\psi_i^x = 0$ for all $t \geq \underline{t}_i^x$ and $i = u, e$.

4 Firm Payoffs and Optimal Strategies

As firms can perfectly discriminate by employment status and previous experience, to simplify the exposition consider for each employment status $i = u, e$ the market of experience x. In what follows assume all firms make acceptable offers to unemployed workers such that $\underline{V}_e^x \geq \underline{V}_u^x \geq U(x)$ for all x.[6] We will show later that this assumption is satisfied in equilibrium.

First we analyse the market of unemployed workers with experience x. Let $M(x)$ denote the steady state number of unemployed workers with experience no greater than x and $\mu(x)$ denote the steady state proportion of unemployed workers with experience x. Consider a firm which posts

[6] Note that at $x = 0$ the only offer distribution that is defined is $F_u(. \mid 0)$. As employed workers gain $x = 0^+$ experience firms have the possibility of hiring them from a competing firm and hence $F_e(. \mid x)$ is defined for all $x > 0$.

a contract w_u^x and let V_u^x denote a worker's expected lifetime payoff by accepting it. The firm's steady state flow profit per new hire in this market is then given by

$$\Omega_u^x(V_u^x, w_u^x) = \lambda\mu(x)\left[\int_0^\infty \psi_u^x(t; w_u^x)[p(x+t) - w_u^x(t)]dt\right], \tag{4}$$

where the first term is the probability of hiring an unemployed worker of experience x and the second term is the firm's expected profit per new hire. The firm's total steady state flow profit in the market of unemployed workers is then obtained by integrating (4) across all experience markets

$$\Omega_u(W_u) = \int_0^\infty \lambda\mu(x)\left[\int_0^\infty \psi_u^x(t; w_u^x)[p(x+t) - w_u^x(t)]dt\right]dM(x),$$

where W_u denotes the set of tenure contracts, w_u^x, the firm offers to unemployed workers for each experience $x \geq 0$.

Next, consider the market of employed workers with experience x. Let $N(x)$ denote the steady state number of employed workers that have experience no greater than x and $1 - G(V \mid x)$ denote the steady state proportion of employed workers that currently have experience x and a lifetime expected payoff of at least V. These two steady state measures include workers that where hired from unemployment and from a competing firm. Consider a firm which posts a contract w_e^x and let V_e^x denote a worker's expected lifetime payoff by accepting it. Note $G(V_e^x \mid x)$ describes the probability that an employed worker of experience x earning $V < V_e^x$ will accept the firms offer. The firm's steady state flow profit per new hire in the market of employed workers of experience x is then given by

$$\Omega_e^x(V_e^x, w_e^x) = \lambda G(V_e^x \mid x)\left[\int_0^\infty \psi_e^x(t; w_e^x)[p(x+t) - w_e^x(t)]dt\right]. \tag{5}$$

The firm's total steady state flow profit in the market of employed workers is obtained by integrating (5) across experience markets

$$\Omega_e(W_e) = \int_0^\infty \lambda G(V_e^x \mid x) \left[\int_0^\infty \psi_e^x(t; w_e^x)[p(x+t) - w_e^x(t)]dt \right] dN(x),$$

where W_e denotes the set of tenure contracts, w_e^x, the firm offers to employed workers for each experience $x > 0$.

Hence a firm's total steady state profit flow is given by

$$\Omega(W_u, W_e) = \Omega_u(W_u) + \Omega_e(W_e).$$

The objective of each firm is to choose two sets of wage contracts $\{W_u, W_e\}$, one for each employment status, to maximise $\Omega(W_u, W_e)$ given $F_u(. \mid x)$, $F_e(. \mid x)$, $U(x)$ for each market x and the turnover strategies of workers described in the previous section.[7] However, the no recall assumption implies a firm can maximise $\Omega(W_u, W_e)$ by choosing W_i independently to maximise $\Omega_i(W_i)$ for each $i = u, e$. Furthermore, no recall also implies that for a given i the firm can choose w_i^x to maximise Ω_i^x at each market x. This structure much simplifies the analysis as it allows us to focus on the firm's optimisation problem for each pair i, x.[8]

First, conditional on offering a new hire a starting payoff V_i^x an optimal contract in market x solves the programming problem

$$\max_{w_i^x(.) \geq \underline{w}} \int_0^\infty \psi_i^x(t; w_i^x)[p(x+t) - w_i^x(t)]dt$$

[7] Note that $\mu(x)$ and $G(. \mid x)$ are functions of $F_u(. \mid x)$ and $F_e(. \mid x)$.

[8] If we did not impose the no recall assumption, intertemporal inconsistency may arise in the firm's problem. An employed worker hired with initial experience x' might want to quit and be rehired by the same firm in a future date because of a more attractive contract. This would imply a different profit maximising strategy for the firm which much complicates the analysis. Although in a different context, see McAfee (1995) for a version of this extension.

subject to

$$V_i^x(0; w_i^x) = V_i^x.$$

Let $w_i^{x*}(.; V_i^x)$ denote this optimal contract and define $\Pi_i^{x*}(0; V_i^x)$ as the expected profit per new hire associated with it. The optimized steady state flow profit per new hire in the market of unemployed workers of experience x is then given by

$$\Omega_u^{x*}(V_u^x, w_u^{x*}) = \lambda\mu(x)\Pi_u^{x*}(0; V_u^x)$$

and the corresponding steady state flow profit per new hire in the market of employed workers of experience x is given by

$$\Omega_e^{x*}(V_e^x, w_e^{x*}) = \lambda G(V_e^x \mid x)\Pi_e^{x*}(0; V_e^x).$$

The firm then chooses V_i^x to maximise Ω_i^{x*}. Let $\overline{\Omega}_i^x$ denote the maximised value of Ω_i^{x*}.

5 Optimal Wage Contracts

5.1 The Contracting Problem

Given a match is formed at any market, search frictions provide the firm with a dynamic monopoly power that enables it to extract quasi rents from the worker. The latter is able to recover those rents (or part of them) over time through job shopping. A moral hazard problem then arises since the firm's profit depends on the worker's search strategies.[9] Quits are jointly inefficient. As firms cannot eliminate potential quits and workers are liquidity constraint, the optimal contract must then minimises the worker's quit rate by offering him an increasing share of the match rents.

[9] However, as argued by Postel-Vinay and Robin (2002 b), given workers cannot vary their search intensity, the moral hazard problem is not fully addressed by the firm.

In particular, when designing an optimal contract for a worker of employment status i and initial experience x, each firm takes as given the distribution of outside offers for each i and experience x, the expected lifetime utility of unemployed workers, and the optimal quit strategy of an employed worker given the contract offered. Formally, for each $i = u, e$ the firm's optimal contracting problem is defined as

$$\max_{w_i^x(.)} \int_0^{\infty} \psi_i^x(t)[p(x+t) - w_i^x(t)]dt \tag{6}$$

subject to

$$\frac{dV_i^x(t)}{dt} = \delta V_i^x(t) - w_i^x(t) - \lambda \int_{V_i^x(t)}^{\overline{V}_e^{x+t}} \left[V_e^{x+t} - V_i^x(t)\right] dF_e(V_e^{x+t} \mid x+t) \tag{7}$$

$$\frac{d\psi_i^x(t)}{dt} = -\left[\delta + \lambda(1 - F_e(V_i^x(t) \mid x+t))\right] \psi_i^x(t) \tag{8}$$

and the initial conditions

$$V_i^x(0) = V_i^x \text{ and } \psi_i^x(0) = 1 \tag{9}$$

and

$$w_i^x(.) \geq \underline{w}; \tag{10}$$

where (8) is obtained by differentiating (3) with respect to tenure (given an initial experience x) and $p(x+t)$ is determined exogenously.

Note that this optimisation problem describes a non-stationary programming problem. To help simplify the analysis we use the following assumption which must be consistent with the equilibrium described in the next section.

$A0$ **: The option of quitting into unemployment never binds on an optimal contract; i.e.** $V_i^x(t; w_i^{x*}) > U(x+t)$ **for all** $x+t > 0$ **and** $i = u, e$.

As outside offers are conditioned on experience for each employment status, the value of being unemployed could increase sufficiently quickly with experience such that firms have to distort the optimal contract to ensure $V_i^x(t; w_i^x) \geq U(x+t)$ for all $t > 0$. We avoid this complication and solve for the optimal contract by restricting attention only to the set of retaining contracts, where w_i^x is a retaining contract if $V_i^x(t; w_i^x) \geq U(x+t)$ and hence $\underline{t}_i^x = \infty$ for all x, t and $i = u, e$.

Let $J^x(t)$ denote the maximum expected value of a match between a firm and an employed worker of initial experience x at tenure t. Since by assumption firms do not engage in search, $J^x(t)$ also describes the expected lifetime utility of a worker of tenure t and starting experience x that is paid $w_i^x(t) = p(x+t)$ for all t and follows an optimal quit strategy. Note that such a contract is the only one that guarantees that the worker's privately optimal quit strategy is also jointly efficient.[10] Since this contract is optimal, it solves (6) conditional on $V_i^x = J^x(0)$, then $J^x(t) > U(x+t)$ for all $x+t > 0$ and so $J^x(t)$ solves the following Bellman equation[11]

$$\delta J^x(t) = p(x+t) + \frac{dJ^x(t)}{dt} + \lambda \int_{J^x(t)}^{\overline{V}_e^{x+t}} [V_e^{x+t} - J^x(t)] dF_e(V_e^{x+t} \mid x+t). \tag{11}$$

Given $J^x(t) > U(x+t)$ for all $x+t > 0$, there exists positive match rents in each market. Hence in the market of workers with employment status i and experience x, the firm will offer a wage contract with starting payoff $V_i^x \in [U(x), J^x(0))$.

The following claim establishes useful boundary conditions for the expected value of employment, $V_i^x(.; w_i^x)$ under an optimal contract.

[10] As the firm is receving zero payoff, this contract is jointly efficient because the firm is indifferent to an eventual separation.

[11] Note that (11) can then be rewritten as $\dot{J}_x(t) - \delta J_x(t) = p(x+t)$ and since $p(.)$ is a bounded function of experience and δ acts as a discount rate, the former describes a bounded first order differential equation that is solved by $J_x(t) = \int_t^\infty e^{-\delta(s-t)} p(x+s) ds$.

CLAIM 1: *Given* $A0$, *for an employment status* $i = u, e$, *any initial experience* x *and conditional on a* $V_i^x \in [U(x), J^x(0))$ *and a profile* $F_e(. \mid \varkappa)$ *for all* $\varkappa \geq x$, *an optimal contract implies* $U(x+t) < V_i^x(t; w_i^{x*}) \leq J^x(t)$ *for all* $t > 0$.

Proof: For a given $i = u, e$, fix an x, an initial starting payoff $V_i^x \in [U(x), J^x(0))$ and a profile $F_e(. \mid \varkappa)$ for all $\varkappa \geq x$.

(i) Note that $U(x+t) < V_i^x(t; w_i^{x*})$ for all $t > 0$ follows directly from assumption $A0$.

(ii) Next, suppose there exists a tenure $t' > 0$ such that $V_i^x(t'; w_i^{x*}) > J^x(t')$. Since $V_i^x(0; w_i^{x*}) < J^x(0)$ and $V_i^x(.; w_i^{x*})$ is continuous over t there exists an $s \in (0, t')$ such that $V_i^x(s; w_i^{x*}) = J^x(s)$. However, at that tenure the optimal contract implies $w_i^x(t) = p(x+t)$ for all $t \geq s$ and therefore $V_i^x(.; w_i^{x*}) = J^x(.)$ for all $t \geq s$ contradicting the optimality of w_i^{x*}. $\|$

5.2 Step-Contracts

Since $F_i(. \mid x)$ might have mass points at any x, standard dynamic optimisation techniques cannot be applied to obtain necessary or sufficient conditions that could help characterise the optimal contract. However, $A0$ imply that we can use similar arguments as in Stevens (2004). Given the worker is risk neutral and hence there is no gain in smoothing income, Proposition 1 shows that in equilibrium for an employment status i and experience x the optimal contract is described by a *step-contract*. In particular, the optimal contract is fully characterised by a promotion tenure z and wages paid satisfy:

$$
\begin{aligned}
w_i^x(t) &= \underline{w} \qquad \text{for } t < z, \\
w_i^x(t) &= p(x+t) \qquad \text{for } t \geq z;
\end{aligned} \tag{12}
$$

and the promotion tenure z is chosen so that the value of accepting the contract is V_i^x.

PROPOSITION 1: *Given $A0$, for any employment status $i = u, e$, any initial experience x and conditional on $V_i^x \in [U(x), J_x(0))$ and a profile $F_e(. \mid \varkappa)$ for all $\varkappa \geq x$, the optimal contract is a step-contract.*

Proof: See Appendix.

Note that as long as $V_i^x(t; w_i^x) < J^x(t)$ for some t, any contract w_i^x will generate inefficient quit behaviour, where a quit is jointly inefficient if the outside offer has value $V_e^{x+t} < J^x(t)$. The proof of Proposition 1 relies on showing that an optimal contract maximises the expected profit per new hire by simply maximising the growth rate of $V_i^x(t; w_i^x)$ and hence minimising the deadweight loss caused by inefficient quit behaviour. This is acheived by a contract such as (12).

The step-contract property is useful as a firm's optimal contract for each i, x is now fully described by a singleton, z. The worker quits if an outside offer that promises an earlier promotion date is received. To simplify the analysis, we use the following renormalisation. Consider an employed worker hired from state i and with initial experience x. Define $T_i^x = x + z$ as the accumulated experience when promotion arrives. Note that the step-contract offer z is equivalent to promotion when the worker's accumulated experience $x + t$ reaches T_i^x. This renormalisation is convenient since outside offers are conditioned on experience. A worker then quits if and only if he receives an outside offer at experience $x' < T_e^x$, where the corresponding promotion offer $T_e^{x'} = x' + z'$ satisfies $T_e^{x'} < T_i^x$.

6 Market Equilibrium

Given step-contracts as described in the previous section, we define the following notation. Let $V_i(x, T_i^x)$ denoted the expected value of an employed worker hired from state $i = u, e$ with experience x on a step-contract T_i^x; i.e. the worker will be promoted after $z = T_i^x - x$ further units of time (if the worker does not quit). Let $\Pi_i(x, T_i^x)$ denote the corresponding firm's expected profit. For workers of employment status i and experience x, the distribution of offers is described by $F_i(T_i^x \mid x)$, where $F_i(. \mid x)$ describes the probability that an outside offer implies promotion at accumulated experience no greater than T_i^x. Let $\underline{T}_i^x$ and $\overline{T}_i^x$ be the infimum and supremum of the support for each i, x. Note that offers always satisfy $T_i^x \geq x$. Also, conditional on experience $x \geq 0$, let $1 - G(T_i^x \mid x)$ denote the proportion of employed workers on a step-contract of at least T_i^x.

In the market for unemployed workers of experience x, a firm then offers T_u^x to maximise expected steady state flow profit per new hire

$$\Omega_u^x(T_u^x, x) = \lambda\mu(x)\Pi_u(x, T_u^x).$$

Similarly, in the market of employed workers with experience x, a firm offers T_e^x to maximise expected steady state flow profit per new hire

$$\Omega_e^x(T_e^x, x) = \lambda[1 - G(T_e^x \mid x)]\Pi_e(x, T_e^x).$$

DEFINITION: *A Market Equilibrium in step-contracts requires:*

(a) an employed worker (x, T_e^x) *quits if an outside offer* $T_e^{x'} < T_e^x$ *is received;*

(b) optimal job search by unemployed workers of experience x, where

$$\delta U(x) = b + \lambda \int_{\underline{T}_u^x}^{\overline{T}_u^x} \max[V_u(x, T_u^x) - U(x), 0] dF_u(T_u^x \mid x),$$

and an unemployed worker with experience x accepts offer T_u^x if and only if $V_u(x, T_u^x) \geq U(x)$.

(c) assumption $A0$ is satisfied.

(d) $\mu(x)$ and $G(. \mid x)$ are consistent with the distribution of contract offers $F_i(. \mid x)$ and the optimal quit turnover strategies for each i, x;

(e) steady state profit per new hire satisfies

$$\begin{aligned} \Omega_i^x(T_i^x, x) &= \overline{\Omega}_i^x \text{ for all } T_i^x \text{ in the support of } F_i(. \mid x), \\ &\leq \overline{\Omega}_i^x \text{ otherwise, for } i = u, e. \end{aligned}$$

We construct an equilibrium where all outside offers are deterministic.[12] Note that $A0$ requires that once an unemployed worker with no previous experience is hired, he will not quit to unemployment at any positive experience. This implies then that in the constructed equilibrium $\mu(x) = 0$ for all $x > 0$ and $\mu(0) = M(0)$ determines the total number of unemployed. Given this condition, let T_u^{0*} denote the optimal contract offered to unemployed workers with no experience and $T_e^*(x)$ denote the optimal contract offered to an employed worker with experience $x > 0$. Optimality implies $T_e^* < T_u^{0*}$ for all experiences $x < T_u^{0*}$ (otherwise the offer is rejected by workers hired from unemployment and the firm makes zero profit). Moreover, let T_e^* have the following

[12] Although an equilibrium with non-degenerate outside offers may exist, it is not trivial to construct one. Given $A0$, it can be shown that if $F_u(. \mid 0)$ is assumed continuous with connected support $\left[\underline{T}_u^0, \overline{T}_u^0\right]$, the profile $F_e(. \mid x)$ around $x = 0$ must be degenerate. By constructing a candidate equilibrium in which all $F_e(. \mid x)$ defined for $x < \overline{T}_u^0$ are degenerate it can be shown using a contradiction argument that the only $F_u(. \mid 0)$ consistent with the profile $F_e(. \mid x)$ is degenerate at $\overline{T}_u^0$.

properties:

$A1$**: For** $x \in (0, T_u^{0*})$, T_e^* **is continuously differentiable with**

(a) $T_e^*(0) = \underline{T} < T_u^{0*}$,

(b) it is strictly increasing with $T_e^*(x) > x$**, and**

(c) lim$_{x \to T_u^{0*}} T_e^*(x) = T_u^{0*}$.

That is, outside offers are such that firms offer two type of jobs conditioning on employment status. A "bad" job -T_u^{0*} contracts- to unemployed workers with no experience and "good" jobs -T_e^* contracts- to employed workers. Only those workers hired under T_u^{0*} contracts quit to firms that offer T_e^* contracts. Once a worker is hired under a T_e^* contract he stays in that firm until retirement.

However, note it must be shown that this outside offer structure is consistent with assumption $A0$ and no employed worker of positive experience is willing to quit into unemployment. To do so we analyse the out-of-equilibrium-path strategies of firms. We show that, given $A1,$ the optimal T_u^x firms will offer to any potential unemployed worker of positive experience is such that $A0$ is satisfied.

7 Identifying a Market Equilibrium

7.1 Firms' Contract Offers

Given $A1$ and that at market $x = 0$ it is optimal to offer T_u^{0*} to unemployed workers, we characterise the optimal contract offers to employed workers for all $x \in (0, T_u^{0*})$ given they behave optimally. To do this, we first describe the firm's expected profit given an employed worker with experience x is employed on contract T_e^x and then consider G. Equilibrium outside offers T_e^* are then characterise. Given the latter, we show that T_u^{0*} is indeed optimal. Without any loss of generality let $\underline{w} = 0$.

Step 1: Consider any market $x \in (0, T_u^{0*})$ such that $T_e^x \geq T_u^{0*}$, which will be the least generous contract offer in the market. As long as it is never optimal for the worker to quit into unemployment, the firm makes expected profit

$$\Pi_e(x, T_e^x) = \int_x^{T_e^x} e^{-(\lambda+\delta)(s-x)} p(s) ds \qquad \text{if } T_e^x \geq T_u^{0*}, \tag{13}$$

where the worker quits if an outside offer is received before T_e^x, and the firm pays marginal revenue product after $T_e^x - x$ further units of time. Note that Π_e is strictly increasing in T_e^x. Hence a necessary condition for a market equilibrium, so that firms do not offer $T_e^x > T_u^{0*}$, is that a worker prefers to remain unemployed rather than accept a contract $T_u^0 > T_u^{0*}$. We shall return to this condition later (see Claim 6 below). However, note this implies $\mu(0) = \delta/(\delta + \lambda)$ describes the unemployment rate.

Next consider a market $x \in (0, T_u^{0*})$ such that $T_e^x \leq \underline{T}$, which is the most generous contract. As $T_e^* \geq \underline{T}$ for all $x > 0$, a worker will never quit and the firm makes expected profit

$$\Pi_e(x, T_e^x) = \int_x^{T_e^x} e^{-\delta s} p(s) ds \qquad \text{if } T_e^x \leq \underline{T}.$$

As Π_e is strictly increasing in T_e^x, it follows that offering $T_e^x < \underline{T}$ is never optimal.

Consider now a market $x \in (0, T_u^{0*})$ in which a firm which makes contract offer $T_e^x \in (\underline{T}, T_u^{0*})$. Define κ where $T_e^x = T_e^*(\kappa)$. There are two cases depending on whether x exceeds κ. First suppose $x < \kappa$. Conditional on hiring the worker, $A1$ implies the firm's expected profit is:

$$\Pi_e(x, T_e^x) = \int_x^{\kappa} e^{-(\lambda+\delta)(s-x)} p(s) ds + e^{-(\lambda+\delta)(\kappa-x)} \int_{\kappa}^{T_e^x} e^{-\delta(s-\kappa)} p(s) ds \;\; \text{if } x < \kappa,$$

as the worker quits for experiences $x + s < \kappa$, does not quit thereafter, and the firm makes zero profit for experiences $x + s \geq T_e^x$.

Now suppose $x > \kappa$. The firm's expected profit (conditional on a hire and $A1$) is

$$\Pi_e(x, T_e^x) = \int_x^{T_e^x} e^{-\delta(s-x)} p(s) ds \qquad \text{if } x > \kappa,$$

as the worker never quits to an outside offer. Differentiating with respect to T_e^x and some rearranging establishes the following useful result.

CLAIM 2. *At any experience* $x \in (0, T_u^{0*})$ *and for* $T_e^x \in (\underline{T}, T_u^{0*})$:

$$\frac{\partial \Pi_e}{\partial T_e^x} = e^{-(\lambda+\delta)(\kappa - x)} \left[-\frac{\lambda \int_\kappa^{T_e^x} e^{-\delta(s-\kappa)} p(s) ds}{dT_e^*(\kappa)/dx} + p(T_e^x) e^{-\delta(T_e^x - \kappa)} \right] \text{ for } x < \kappa,$$

$$\frac{\partial \Pi_e}{\partial T_e^x} = p(T_e^x) e^{-\delta(T_e^x - x)} \text{ for } x > \kappa,$$

where κ *is defined by* $T_e^*(\kappa) = T_e^x$.

A marginal increase in T_e^x increases expected profit by the marginal revenue product of the worker at experience T_e^x, $p(T_e^x)$, multiplied by the probability that he remains employed at the firm until promotion date T_e^x. The loss, however, is that the worker is more likely to quit. In this case the firm (conditional on the worker receiving an outside offer) looses the profits that otherwise would have obtain from delaying the worker's promotion date.

To characterise the optimal contract offer, we also need to describe G, the equilibrium distribution of worker's reservation values.

CLAIM 3. *At any experience* $x \in (0, T_u^{0*})$:

$$1 - G(T_u^{0*} \mid x) = e^{-\lambda x}$$

and for $T_e^x \in (\underline{T}, T_u^{0*})$:

$$\partial G(T_e^x \mid x)/\partial T_e^x = 0, \qquad [1 - G(T_e^x \mid x)] = e^{-\lambda x} \quad \text{for } x < \kappa,$$

$$\partial G(T_e^x \mid x)/\partial T_e^x = \lambda e^{-\lambda\kappa}/[dT_e^*(\kappa)/dx], \quad [1 - G(T_e^x \mid x)] = e^{-\lambda\kappa} \quad \text{for } x > \kappa,$$

,where κ *is defined by* $T_e^*(\kappa) = T_e^x$.

Proof: See Appendix.

Notice that neither marginal profit $\partial\Pi_e/\partial T_e^x$ nor the density function $\partial G/\partial T_e^x$ are continuous at $T_e^x = T_e^*(x)$. In what follows consider left and right differentiation.

Step 2: Recall that $T_e^*(.)$ describes the equilibrium offer strategies of firms hiring employed workers of positive experience. Equilibrium also requires that $T_e^*(x)$ describes the firms' optimal contract offer for all $x \in (0, T_u^{0*})$. To show this is the case, fix a $x \in (0, T_u^{0*})$ and recall $A0$ implies the expected profit by offering contract $T_e^x \in (\underline{T}, T_u^{0*})$ is

$$\Omega_e(T_e^x, x) = \lambda[1 - G(T_e^x \mid x)]\Pi_e(x, T_e^x).$$

(a) Right differentiation: consider $T_e^x \in (T_e^*(x), T_u^{0*})$. Claims 2 and 3 imply

$$\begin{aligned} \frac{\partial\Omega_e}{\partial T_e^x} &= -\frac{\partial G}{\partial T_e^x}\Pi_e(x, T_e^x) + [1 - G(T_e^x \mid x)]\frac{\partial\Pi_e}{\partial T_e^x} \\ &= e^{-\lambda x}e^{-(\lambda+\delta)(\kappa - x)}\left[-\frac{\lambda\int_\kappa^{T_e^x} e^{-\delta(s-\kappa)}p(s)ds}{dT_e^*(\kappa)/dx} + p(T_e^x)e^{-\delta(T_e^x - \kappa)}\right]. \end{aligned}$$

Hence a necessary condition for optimality is that $\lim_{\varepsilon\to 0^+}[\partial\Omega_e(T_e^* + \varepsilon, x)/\partial T_e^x] \leq 0$, otherwise offering a $T_e^x > T_e^*(x)$ is optimal. This implies

$$-\frac{\lambda}{dT^*(x)/dx}\int_x^{T_e^*(x)} e^{-\delta(s-x)}p(s)ds + p(T_e^*(x))e^{-\delta(T_e^*(x)-x)} \leq 0.$$

(b) Left differentiation: consider $T_e^x \in (\underline{T}, T_e^{x*}(x))$. Claims 2 and 3 imply

$$\frac{\partial\Omega_e}{\partial T_e^x} = -\frac{\partial G}{\partial T_e^x}\Pi_e(x, T_e^x) + [1 - G(T_e^x \mid x)]\frac{\partial\Pi_e}{\partial T_e^x}$$

$$= \; -\frac{\lambda e^{-\lambda\kappa}}{dT_e^*(\kappa)/dx}\int_x^{T_e^x} e^{-\delta(s-x)}p(s)ds + e^{-\lambda\kappa}[p(T_e^x)e^{-\delta(T_e^x-x)}].$$

and a necessary condition for optimality is that $\lim_{\varepsilon\to 0^+}[\partial\Omega_e(T_e^*-\varepsilon,x)/\partial T_e^x] \geq 0$, otherwise offering a $T_e^x < T_e^*(x)$ is optimal. This implies

$$-\frac{\lambda}{dT_e^*(x)/dx}\int_x^{T_e^*(x)} e^{-\delta(s-x)}p(s)ds + p(T_e^*(x))e^{-\delta(T_e^*(x)-x)} \geq 0.$$

Hence the necessary conditions for the optimality of $T_e^x = T_e^*(x)$ are satisfied if and only if

$$p(T_e^x)e^{-\delta(T_e^x-x)} = \frac{\lambda\int_x^{T_e^x} e^{-\delta(s-x)}p(s)ds}{dT_e^*(x)/dx}$$

Rearranging this expression implies T_e^* must satisfy

$$\frac{dT_e^*(x)}{dx} = \frac{\lambda\int_x^{T_e^*(x)} e^{-\delta(s-x)}p(s)ds}{p(T_e^*(x))e^{-\delta(T_e^*(x)-x)}} \tag{14}$$

for any given $x \in (0, T_u^{0*})$. Hence equilibrium requires that T_e^* is described by the non-autonomous differential equation (14) for all $x \in (0, T_u^{0*})$, subject to the initial condition $T_e^*(0) = \underline{T}$. Given the properties of p, the fundamental theorem of differential equations imply T_e^* exists, is continuously differentiable in x and strictly increasing if $x < T_e^*(x)$. $A1$ also requires $\lim_{x\to T_u^{0*}} T_e^*(x) = T_u^{0*}$. As in any initial value problem, the stability of T_e^* and hence the existences of a fixed point T_u^{0*} will depend on the value of the initial condition, $T_e^*(0) = \underline{T}$. In the appendix we show, using simulations, that there exists an upper bound, $\widehat{\underline{T}}_1$, defined by

$$e^{-\delta\widehat{\underline{T}}_1}p(\widehat{\underline{T}}_1) = \lambda\int_0^{\widehat{\underline{T}}_1} e^{-\delta s}p(s)ds,$$

such that for any value of $\underline{T} < \widehat{\underline{T}}_1$, T_e^* converges to the fixed point T_u^{0*}.

PROPOSITION 2. *A necessary condition for a market equilibrium with T_e^* satisfying $A1$ requires:*

(a) $\underline{T} \in (0, \widehat{\underline{T}}_1)$,

(b) conditional on such a $\underline{T}$, T_e^* *is the solution to the differential equation (14) with initial value* $T_e^*(0) = \underline{T}$,*and*

(c) T_u^{0*} *is determined where* $dT_e^*(x)/dx = 0$.

Proof: See Appendix.

Note (14) describes how outside offers for employed workers must vary with experience in a market equilibrium. Under the conditions stated in Proposition 2 outside offers become more generous with experience. The offered promotion tenure, $z^* = T_e^*(x) - x$, decreases with previous experience x (see Figure 1). This guarantees that accumulating experience is valuable. More importantly, in the appendix we show that this property is maintained even if $p(x) = \underline{p}$ for all x. Hence, when a worker's productivity is not correlated with experience firms might still post outside offers that become more generous with experience. Also note that since $z^* > 0$ for all $x \in (0, T_u^{0*})$, $T_e^*(x) > x$ for such T_e. (14) then implies T_e^* is strictly increasing in x for $x < T_u^{0*}$. Hence a solution to the conditions of Proposition 2 yields a T_e^* which satisfies $A1$.

The next claim shows that Proposition 2 also implies firms offering promotions T_e^* at any market $x \in (0, T_u^{0*})$ are indifferent to increase (marginally) their promotion date $T_e^*(x)$. Thus, when posting contract $T_e^*(x)$ at any market $x > 0$ firms trade off a longer period during which they make positive profit with a higher chance the worker will quit in the future.

CLAIM 4. *Given a* T_e^* *satisfying Proposition 2, then*

$$\begin{aligned} \frac{\partial \Omega_e^x}{\partial T_e^x} &= 0 \text{ for all } T_e^x > T_e^*(x); \\ \frac{\partial \Omega_e^x}{\partial T_e^x} &> 0 \text{ for all } T_e^x < T_e^*(x) \end{aligned}$$

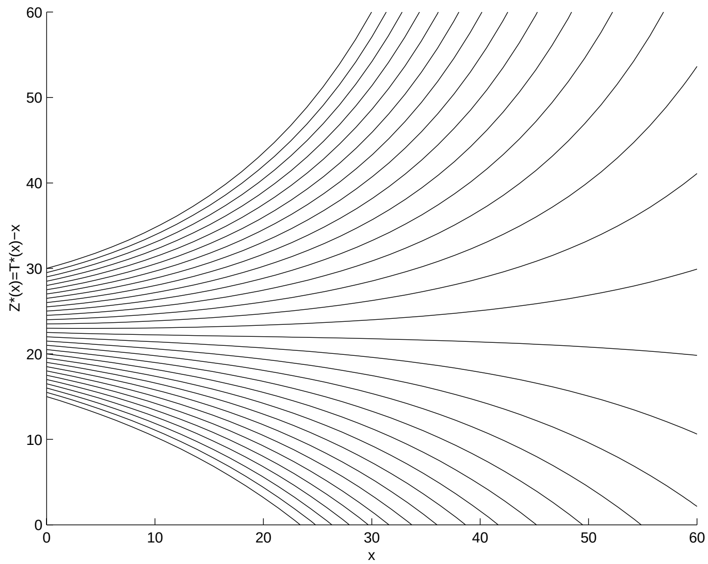

Figure 1: Stability of Outside Offers, z^*, for Initial Values of $z^* = [15, \ldots, 30]$

i.e. Proposition 2 is sufficient as well as necessary.

Proof: See Appendix.

Step 3: Given that at market $x = 0^+$ is optimal to set $\underline{T}$, then at market $x = 0$ firms can always deviate from T_u^{0*} by posting a step-contract $T_u^0 = \underline{T}$ and retain all the workers. Hence, optimality of T_u^{0*} requires that $\Pi_u(0, T_u^{0*}) \geq \Pi_u(0, \underline{T})$. However, at $x = 0^+$ firms then can still deviate by posting a step-contract $T_e^{0^+} = T_u^{0*} - \varepsilon$, where $\varepsilon > 0$ is arbitrarily small. Note that this contract attracts the worker with probability one. Hence, optimality of $\underline{T}$ at $x = 0^+$ requires $\Pi_u(0, \underline{T}) \geq \Pi_u(0, T_u^{0*} - \varepsilon)$ for any $\varepsilon > 0$. Continuity then implies that $\Pi_u(0, T_u^{0*}) = \Pi_u(0, \underline{T})$ must hold in equilibrium.

CLAIM 5. *Given a T_e^* satisfying Proposition 2, then*

$$\Pi_u(0, T_u^0) = \overline{\Pi}_u \text{ for all } T_u^0 \in [\underline{T}, T_u^{0*}].$$

Proof: See Appendix.

We now turn to analyse the behaviour of workers given the optimal strategies of firms.

7.2 Workers' Quit Strategies

Given that at market $x = 0$ it is optimal for firms to offer T_u^{0*} and that at markets $x > 0$ firms' optimal contract offers are described by a T_e^* satisfying Proposition 2, the next step is to compute the worker's expected payoffs. First, fix a step-contract $T_e^*(\kappa) \in (\underline{T}, T_u^{0*})$ and note only workers that where hired from unemployment who quit with experience κ are employed on this contract. Now consider such a worker and with no loss of generality consider experience $x \geq \kappa$. Assuming it is never optimal for the worker to quit into unemployment, and noting that $T_e^*(x) > T_e^*(\kappa)$ for all $x > \kappa$ implies the worker never quits, the worker's expected lifetime payoff is:

$$V(x, T_e^*(\kappa)) = e^{-\delta(T_e^*(\kappa)-x)} \int_{T_e^*(\kappa)}^{\infty} e^{-\delta(s-T_e^*(\kappa))} p(s) ds, \tag{15}$$

as the worker receives a zero wage until promoted, and earns marginal revenue product thereafter. Note V is increasing and convex in x for $x < T_e^*(\kappa)$.

Now consider the expected payoff of a worker employed under contract T_u^{0*}, the least generous contract. Assuming the worker never quits into unemployment, equilibrium implies the expected lifetime payoff at experience $x < T_u^{0*}$ is

$$V(x, T_u^{0*}) = \lambda \int_x^{T_u^{0*}} e^{-(\lambda+\delta)(s-x)} V(s, T_e^*(s)) ds + e^{-(\lambda+\delta)(T_u^{0*}-x)} \int_{T_u^{0*}}^{\infty} e^{-\delta(s-T_u^{0*})} p(s) ds, \tag{16}$$

where

$$V(s, T_e^*(s)) = e^{-\delta(T_e^*(s)-s)} \int_{T_e^*(s)}^{\infty} e^{-\delta(\tau-T_e^*(s))} p(\tau) d\tau, \tag{17}$$

is the starting payoff offered by a contract T_e^* at market $s \in (x, T_u^{0*})$. Under this contract the worker also gets paid a zero wage until promotion and marginal revenue product thereafter, but before promotion arrives (which happens after $T_u^{0*} - x$ units of time) he might quit to a contract $T_e^*(s)$ and receive $V(s, T_e^*(s))$. Note that in this case $V(x, T_u^{0*})$ is also strictly increasing and convex in $x \in (0, T_u^{0*})$ as outside offers get more generous with experience.[13]

Finally, consider an unemployed worker of experience x. In general, the expected value of unemployment is given by

$$\delta U(x) = b + \lambda \max \left[V(x, T_u^*(x)) - U(x), 0\right] \qquad \text{for all } x \geq 0,$$

where $T_u^*(x)$ are the optimal contract firms offer to unemployed workers. Conditional on $A0$, note that when constructing the firms' outside offers we have only define $T_u^*(0) = T_u^{0*}$. Equilibrium requires that an unemployed worker must be just indifferent to accept the least generous offer, T_u^{0*}. Otherwise, equation (13) implies that firms could increase profits by offering $T_u^{0*} + \varepsilon$, where $\varepsilon > 0$ is arbitrarily small. The next claim follows.

CLAIM 6. *A necessary condition for a market equilibrium is that*

$$U(0) = V_u(0, T_u^{0*}) = b/\delta \tag{18}$$

and unemployed workers get no surplus.

However, to show that $A0$ is satisfied in equilibrium and no employed worker will quit to unemployment at positive experience, we have to define the optimal offers firms would offer unemployed workers of experience $x > 0$, $T_u^*(x)$. Given optimal outside offers for employed workers

[13] See Van den Berg (1990)-Theorem 2 for a similar argument when the distribution of outside offers increase in the sense of first order stochastic dominance.

at positive experience are degenerate and described by T_e^* satisfying Proposition 2, consider an unemployed worker of experience $x > 0$. Since firms can observe this worker's employment status, it follows that the optimal T_u^x will imply $V_u(0, T_u^x) = U(x)$ for any $x > 0$. Firms have no incentive to improve this offer since they have a constant hiring rate, $\lambda\mu(x)$, and the worker will accept any contract that gives him at least $U(x)$ and quit as soon as a T_e^* contract arrives. Hence this implies $U(x) = b/\delta$ for all x. However, these strategies are never materialised and only act as a credible threat to the worker.

Since the expected value of unemployment can be regarded as constant over time and $V(x, T_u^{0*})$ is strictly increasing in $x \in (0, T_u^{0*})$, Claim 6 implies the option of quitting into unemployment never binds at contracts T_u^{0*}. Moreover, Claim 7 below gives a sufficient condition such that this is also true for contract $T_e^*(x)$ at any $x \in (0, T_u^{0*})$.

CLAIM 7. *Given a T_e^* satisfying Proposition 2, workers employed on a $T_e^*(x)$ contract will never quit to unemployment if and only if $T_e^*(0) = \underline{T} < \widehat{\underline{T}}_2$, where $\widehat{\underline{T}}_2$ is defined by*

$$\int_{\widehat{\underline{T}}_2}^{\infty} e^{-\delta s} p(s) ds = b/\delta.$$

Proof: Consider a T_e^* satisfying Proposition 2. Note $V(x, T_e^*(x))$ is strictly increasing in x and $U(x)$ is given by b/δ for all x. Hence, workers hired under T_e^* contracts at any market $x \in (0, T_u^{0*})$ will never quit to unemployment if and only if $V(x, T_e^*(x)) > b/\delta$ at experience $x = 0^+$. Evaluating (17) at $T_e^*(0^+)$ this implies that $\underline{T}$ must also satisfy

$$\int_{\underline{T}}^{\infty} e^{-\delta s} p(s) ds > b/\delta. \tag{19}$$

By defining $\widehat{\underline{T}}_2$ as the value of $\underline{T}$ that makes $\int_{\underline{T}}^{\infty} e^{-\delta s} p(s) ds = b/\delta$ we obtain the condition stated in the claim.$\|$

Given a T_e^* that satisfies Proposition 2, the conditions in Claim 6 and 7 imply unemployed workers with no previous experience hired under contract T_u^{0*} and employed workers hired under contract T_e^* never quit to unemployment. Hence, existence of equilibrium is guaranteed if and only if there exists a T_e^* such that Proposition 2, Claim 6 and Claim 7 are simultaneously satisfied. In particular, Proposition 2 and Claim 7 imply that any equilibrium T_e^* must be a solution to (14) subject to the initial condition $T_e^*(0) < \underline{\widehat{T}}_1$ and $T_e^*(0) < \underline{\widehat{T}}_2$. Claim 6 and (16) then require that these solutions also satisfy

$$\lambda \int_0^{T_u^{0*}} e^{-(\lambda+\delta)s} V(s, T_e^*(s)) ds + e^{-(\lambda+\delta)T_u^{0*}} \int_{T_u^{0*}}^{\infty} e^{-\delta(s-T_u^{0*})} p(s) ds = \frac{b}{\delta},$$

where $V(s, T_e^*(s))$ is given by (17). However, since (14) cannot be solved explicitly for T_e^* we do not provide an existence proof but instead analyse the model numerically to show existence.

7.3 Numerical Analysis

Without any loss of generality assume the function p takes the following exponential form

$$p(x) = \overline{p} - p_0 e^{-\alpha x} \quad \text{for all } x \geq 0, \tag{20}$$

where $\overline{p} > p_0 > 0$ and $\alpha > 0$. Note α describes the worker's "learning" rate (a high α implies workers acquire human capital faster) and $\overline{p}$ determines the maximum human capital stock any worker could achieve.

Next we show that if $U = b/\delta$ is sufficiently close to $J^0(0) = \int_0^\infty e^{-\delta s} p(s) ds$ and λ is small enough, there exists a unique solution, T_e^*, to (14) such that the conditions in Proposition 2, Claim 6 and 7 are simultaneously satisfied. We then consider comparative statics.

7.3.1 Existence

Substituting out for $p(x)$ in (14) outside offers are now described by

$$\frac{dT_e^*(x)}{dx} = \frac{\overline{p}\frac{\lambda}{\delta}\left(1 - e^{-\delta(T_e^*(x)-x)}\right) + p_0\frac{\lambda}{\alpha+\delta}e^{\delta x}\left(e^{-(\alpha+\delta)T_e^*(x)} - e^{-(\alpha+\delta)x}\right)}{e^{-\delta(T_e^*(x)-x)}\left(\overline{p} - p_0 e^{-\alpha T_e^*(x)}\right)}, \tag{21}$$

subject to the initial condition $T_e^*(0) = \underline{T}$ such that $\underline{T} < \widehat{\underline{T}}_1$ and $\underline{T} < \widehat{\underline{T}}_2$, where $\widehat{\underline{T}}_1$ is defined by

$$\overline{p}e^{-\delta\widehat{\underline{T}}_1}\left[1 + \frac{\lambda}{\delta}\right] - p_0 e^{-(\alpha+\delta)\widehat{\underline{T}}_1}\left[1 + \frac{\lambda}{\alpha+\delta}\right] = \lambda\left[\frac{\overline{p}}{\delta} - \frac{p_0}{\alpha+\delta}\right],$$

and $\widehat{\underline{T}}_2$ solves

$$\frac{\overline{p}}{\delta}e^{-\delta\widehat{\underline{T}}_2} - \frac{p_0}{\alpha+\delta}e^{-(\alpha+\delta)\widehat{\underline{T}}_2} = \frac{b}{\delta}.$$

Claim 6 then requires any equilibrium T_e^* to also satisfy

$$\lambda\int_0^{T_u^{0*}} e^{-\lambda s}\left[\frac{\overline{p}}{\delta}e^{-\delta T_e^*(s)} - \frac{p_0}{\alpha+\delta}e^{-(\alpha+\delta)T_e^*(s)}\right]ds + e^{-(\lambda+\delta)T_u^{0*}}\left[\frac{\overline{p}}{\delta} - \frac{p_0}{\alpha+\delta}e^{-\alpha T_u^{0*}}\right] = \frac{b}{\delta}. \tag{22}$$

Assuming $\overline{p} = 20$, $p_0 = 3$, $b = 14.5$, $\delta = 0.01$, $\lambda = 0.04$ and $\alpha = 0.1$, Figures 2 and 3 show the unique solutions for T_e^* and z^*. Figure 4 shows the corresponding solutions for $V(x, T_u^{0*})$ and $V(x, T_e^*(x))$ given U and $J^0(x)$. Note that with these parameter values $\widehat{\underline{T}}_1 = 23.3$, $\widehat{\underline{T}}_2 = 32.1$, $\underline{T} = 19.5$, $T_u^{0*} = 41.7$.[14]

The intuition behind these results is straightforward. First, note that as long as $0 < \underline{T} < \widehat{\underline{T}}_1$ and hence a T_u^{0*} exists, (21) implies $T_e^*(x)$ increases continuosly with $\underline{T}$ for all $x \in [0, T_u^{0*}]$. In

[14] To show existence numerically, we apply the following algorithm. Given any $\underline{T}$ satisfying Proposition 2(a) and $\underline{T} < \widehat{\underline{T}}_2$, solve (14) given the initial condition $T_e^*(0) = \underline{T}$ such that its solution, T_e^*, implies a function $V(x, T_u^{0*})$ that solves

$$\frac{dV(x, T_u^{0*})}{dx} = (\lambda+\delta)V(x, T_u^{0*}) - \lambda V(x, T_e^*(x)) \text{ for all } x \in [0, T_u^{0*}).$$

given the boundary conditions $V(0, T_u^{0*}) = b/\delta$ and $V(T_u^{0*}, T_u^{0*}) = \int_{T_u^{0*}}^{\infty} e^{-\delta(s-T_u^{0*})}p(s)ds$ and also satisfies (22).

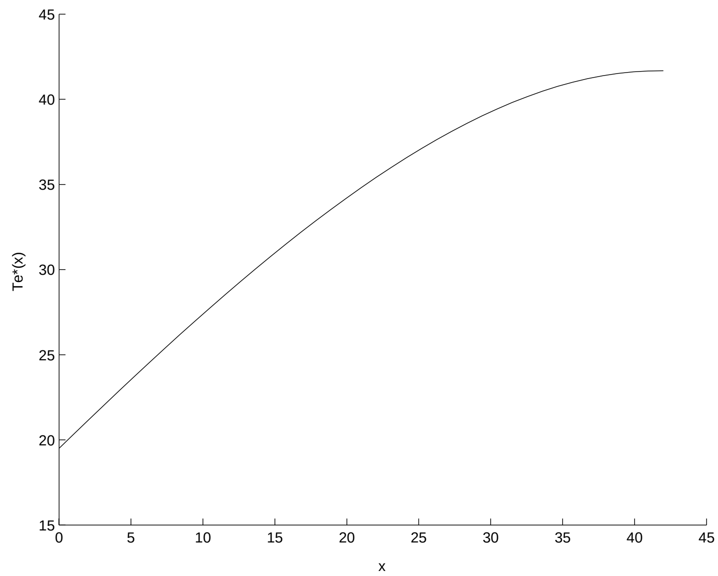

Figure 2: Equilibrium Outside Offers at Positive Experience Markets, $T_e^*(.)$

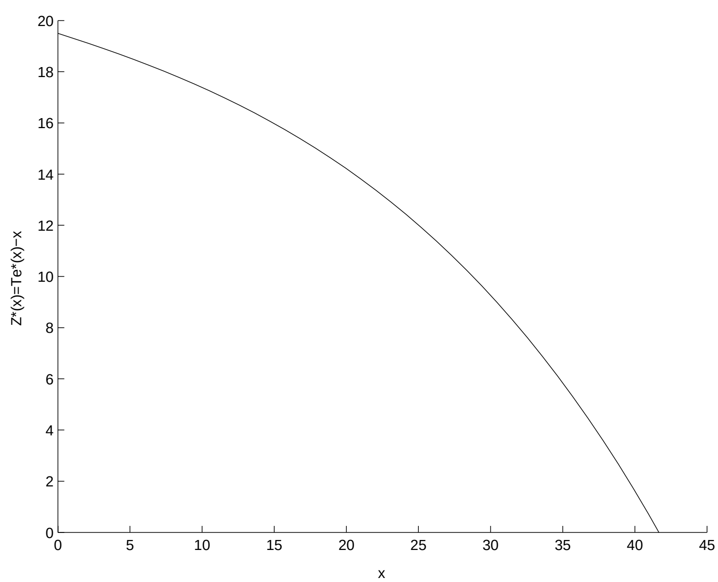

Figure 3: Equilibrium Outside Offers at Positive Experience Markets, $z^*(.)$.

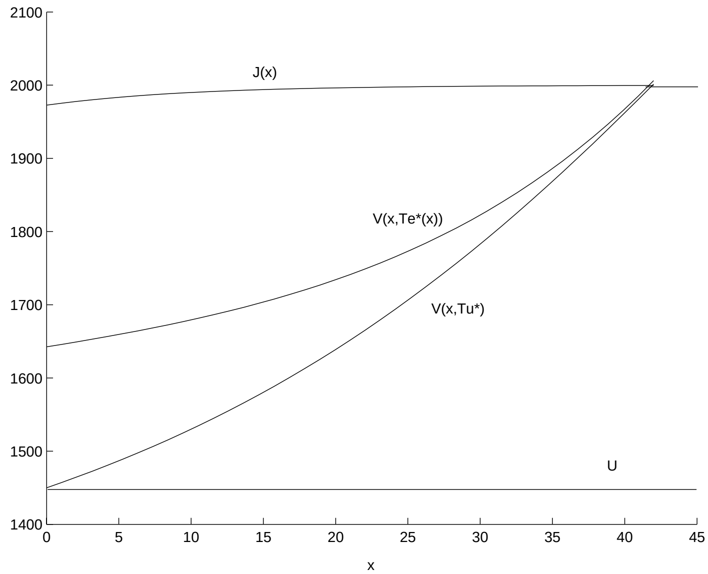

Figure 4: Equilibrium Expected Value of Employment, $J^0(x)$, $V(x, T_e^*(x))$, $V(x, T_u^{0*})$, U.

terms of Figure 2, the function T_e^* shifts upwards. It then follows that, $V(0, T_u^{0*})$ -the LHS of (22)- must be continuous and strictly decreasing in $\underline{T}$. On the other hand, equation (17) shows that $V(0, T_e^*(0))$ -the LHS of (19)- is also continuous and strictly decreasing in $\underline{T}$. Figure 5 shows this relationship. The locuses MG and LG describe $V(0, T_e^*(0))$ and $V(0, T_u^{0*})$, respectively for the parameter values given above. Let $\widehat{V}$ denote the limit of $V(0, T_u^{0*})$ as $\underline{T} \to \widehat{\underline{T}}_1$ and note (21) implies that as $\underline{T} \to 0$, then $T_u^{0*} \to 0$ and, hence, $V(0, T_u^{0*}) \to J^0(0)$. Equation (17) shows that $V(0, T_e^*(0))$ also converges to $J^0(0)$ as $\underline{T} \to 0$. Finally, note that $T_u^{0*} > \underline{T} > 0$ implies $V(0, T_e^*(0))$ is strictly greater than $V(0, T_u^{0*})$ for any given $\underline{T} \in (0, \widehat{\underline{T}}_1)$.

It follows from Figure 5 that for any value of $U = b/\delta \in (\widehat{V}, J^0(0))$ there exists a unique $\underline{T}'$ satisfying $\underline{T}' < \widehat{\underline{T}}_1$ and $\underline{T}' < \widehat{\underline{T}}_2$ such that the corresponding solution, $T_e^{*\prime}$, to (21) solves (22). However, as shown below, $\widehat{V}$ increases with λ and hence for values of λ high enough there is no

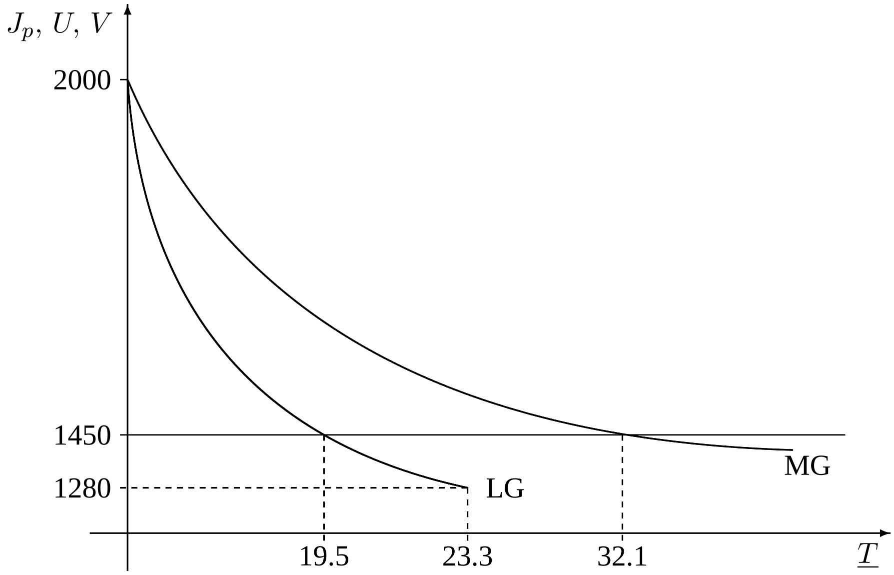

Figure 5: Existence of Equilibrium. Given the parameter values mentioned in the text, on the horizontal-axis, we find that $T_e^*(0) = \underline{T}' = 19.5$, $\widehat{\underline{T}}_1 = 23.3$ and $\widehat{\underline{T}}_2 = 32.1$. On the vertical-axis, $\widehat{V} = 1280$, $U = 1450$ and $J^0(0) = 1972.7$.

$b < p(0)$ such that equilibrium outside offers T_e^* exists.

7.3.2 Comparative Statics

Note that Figure 5 implies for any $b \in (\delta\widehat{V}, p(0))$, b and T_e^* are inversely related. As b decreases (and U decreases), firms at $x = 0$ market are able to attract unemployed workers by offering them a contract with a longer probation period. Since poaching firms at any market $x > 0$ will also increase their promotion dates, a decrease (increase) in b shifts outwards (inwards) T_e^* for all $x \in (0, T_u^{0*})$. Table 1 shows this relationship for the same parameter values as above.

Similarly, Table 2 shows the relationship between λ and T_e^*. When search frictions increase (λ gets smaller), equation (21) implies that dT_e^*/dx decreases for all $x \in (0, T_u^{0*})$ and promotion dates offered by good jobs at positive experience markets converge to the ones offered by bad jobs; i.e. $T^*(x) \to T_u^{0*}$ for all $x \in (0, T_u^{0*})$. In the limit, equilibrium converges to the pure monopoly case. On the other hand, as frictions disappear $\widehat{\underline{T}}_1$ decreases. This implies that $\underline{T}$ must also decrease. At the same time workers hired in bad jobs have more opportunities to get a better job before promotion, more markets open and hence T_u^{0*} increases. However, as mentioned earlier, an increase in λ also increases $\hat{V}$ and hence reduces the set of values of b for which equilibrium can exists. In this case, equilibrium fails to exist for $\lambda > 0.05$. For those cases $\widehat{V} > U$.

Finally, Table 3 shows the relationship between α and T_e^*. Note that for the initial values of α, an increases in the learning rate of workers increases both $\underline{T}$ and T_u^{0*}. As workers increase their productivity over a long time span, firms find profitable to increase their probation period and obtain higher match rents. However, as α reaches a value of 5% there is hardly any change in T_e^*. At this point most of the increase in productivity has been achieved and hence there is barely any impact on the firm's contract.

Table 1

b	$\underline{T}$	T_u^{0*}	$\widehat{\underline{T}}_2$
12.9	22.5	106.6	43.8
13.5	21.6	63.6	39.3
14.5	19.5	41.6	32.1
15.5	16.9	24.7	22.2
16.5	13.9	20.6	19.1
16.9	12.6	17.6	16.6

Table 2

λ	$\underline{T}$	T_u^{0*}	$\widehat{\underline{T}}_1$	$\widehat{V}$
0.005	29.4	32.0	110.5	223
0.01	27.3	32.3	70	502
0.02	24.1	33.9	41.8	887
0.03	21.6	36.8	29.9	1125
0.04	19.5	41.6	23.3	1280
0.05	17.7	52.4	19.1	1386

Table 3

α	$\underline{T}$	T_u^{0*}	$\widehat{\underline{T}}_1$	$\widehat{\underline{T}}_2$
0	12.1	16.8	22.3	15.9
0.01	17.2	30.7	22.7	26.2
0.05	19.3	40.6	23.2	31.7
0.1	19.4	41.7	23.3	32.1
0.5	19.5	42.0	23.4	32.2
1	19.5	42.1	23.5	32.2

8 Interpretation and Conclusions

We have constructed an equilibrium in which firms post wage-tenure contracts conditional on the worker's employment status and initial experience. As is Stevens (2004) the firm is able to wage discriminate its employees by their tenure by first setting a probation period in which the worker is paid the minimum acceptable wage, $\underline{w}$. After this period, the worker is promoted and paid marginal revenue product, $p(x)$. In equilibrium, firms hire workers with no previous experience from unemployment. In this market firms extract all the match rents from the workers; i.e. the Diamond outcome (see Diamond, 1971). Since these workers earn less than their marginal revenue product until they get promoted, firms have incentives to recruit them at positive experience markets $(0 < x < T_u^{0*})$. Proposition 2 then shows that if $\underline{T} < \widehat{\underline{T}}_1$, equilibrium outside offers become more generous with experience. This result is solely determined by tenure effects driven by search frictions and is independent of the worker's human capital accumulation. As the experience-earnings profile of workers hired under a T_u^{0*} contract increases steeply during the probation period, firms at markets $x \in (0, T_u^{0*})$ must offer contracts with increasing starting values if they are going to successfully recruit these workers before promotion arrives (see Figure 4). In this case, firms find profitable to offer employed workers with positive experience more than their reservation value. Firms trade off a longer period during which they make positive profit with a higher chance the worker will quit in the future (Claim 5). Hence, a "dual" labour market emerges within a firm in which worker turnover occurs in the direction of bad jobs with longer probation periods towards good jobs with shorter ones.

Note step-contracts are the unique optimal wage-tenure contract in this framework, since firms always face outside competition for their employees. Moreover, by using step-contracts in positive experience markets firms successfully create an internal labour market that shields their employees from outside competition as workers hired under T_e^* contracts never quit.

There is evidence that firms segment the labour market internally offering two types of jobs (see Doeringer and Piore (1971) and Saint-Paul (1996)). In the upper tier of their labour market firms create an internal labour market offering high wages, employment stability and promotions. The lower tier is characterised by low wages and a high degree of turnover. We argue that search frictions alone can give an alternative explanation of why this "dual" labour market might appear within a firm. In our model, profit maximising firms exploit their monopsony power and offer a low paying job to an unemployed worker with no previous experience and a high paying one to an employed with positive experience because it knows the former does not have an outside option while the latter does. This is in contrast is the typical explanations based on efficiency wages and the existence of monitoring cost.

Since more experienced workers are offered contracts with higher starting values, the model predicts cohort effects within a firm. Figure 4 implies that the expected value of employment of two workers hired at experiences x and x' follow a common pattern that is independent of outside offers. Hence, much of the variation between cohort's expected value of employment implied by the earnings distribution G (see Claim 3) comes from the difference in starting payoff described by $V(x, T_e^*(x))$ and persists until promotion arrives. Through simulations we have further derived the impact of changes in market conditions on equilibrium outside offers and hence on how these cohort effects behave when frictions, unemployment insurance payments and the learning rate of workers change.

Baker, Gibbs and Holmstrom (1994) find some empirical support for this prediction. Using personnel data of a mayor US corporation during the period 1969-1988 they find strong evidence of cohort effects that depend on the year in which workers where hired. In their study, employee's wages of different cohorts follow a common pattern (increasing and convex with tenure) and move in parallel. New entrant wages, however, follow a more idiosyncratic and erratic path which is described by external market conditions. Not surprisingly, they argue that these wage patterns are consistent with wage policies found in the incentive/agency theory.

Furthermore, the earnings distribution also implies that inside a firm, holding tenure constant, workers that where hired with more pre-company experience (experience gained outside the firm) are higher in the earnings scale and when controlling for experience workers with more tenure have higher earnings. These predictions are also consistent with the empirical findings of Medoff and Abraham (1980). Their analysis of personnel data of two US manufacturing companies shows that in both cases for managerial and professional employees there exists positive returns to outside firm (pre-company) experience and positive returns to tenure when controlling for tenure and experience, respectively. Interestingly, they find that these effects explain nearly 40% of wage differentials found within a job level.

The results presented in this paper then suggest that allowing for more complex firm wage policies in search equilibrium type of environments can proof useful to further understanding the interaction between search frictions and the worker's wage pattern inside the firm.

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Appendix

Proof of Proposition 1:

Assume workers do not quit into unemployment. For any employment status $i = u, e$ and initial experience x fix a $V_i^x \in [U(x), J_x(0))$ and profile $F_e(. \mid \varkappa)$ for all $\varkappa \geq x$.

Step 1: Define the "surplus function" for any market $x + t$ as

$$\varphi(V_i^x(t); x+t) \equiv \int_{V_i^x(t)}^{\overline{V}_e^{x+t}} [V_e^{x+t} - V_i^x(t)] dF_e(V_e^{x+t} \mid x+t). \tag{23}$$

Note that for a given $F_e(. \mid x+t)$, φ is continuous and non increasing in $V_i^x(t) \in [U(x+t), J^x(t)]$. (7) then implies $V_i^x(t)$ evolves according to the differential equation

$$\frac{dV_i^x(t)}{dt} = -w_i^x(t) + \delta V_i^x(t) - \lambda\varphi(V_i^x(t); x+t), \tag{24}$$

given a starting payoff $V_i^x(0) = V_i^x \in [U(x), J^x(0))$. Let $V_i^{xb}(t)$ denote the solution to (24) when the liquidity constraint (10) binds and $V_i^{xnb}(t)$ when it does not. By subtracting the corresponding differential equations we obtain the following expression

$$[\frac{dV_i^{xb}(t)}{dt} - \frac{dV_i^{xnb}(t)}{dt}] = \left[w_i^{xnb}(t) - \underline{w}\right] + \delta[V_i^{xb}(t) - V_i^{xnb}(t)] - \lambda[\varphi(V_i^{xb}(t); x+t) - \varphi(V_i^{xnb}(t); x+t)]. \tag{25}$$

Since $V_i^x = V_i^{xb}(0) = V_i^{xnb}(0)$ by assumption, it follows from (25) that $dV_i^{xb}(0)/dt > dV_i^{xnb}(0)/dt$. Continuity of $V_i^x(.)$ and the properties of (23) then imply $V_i^{xb}(t) > V_i^{xnb}(t)$ and $dV_i^{xb}(t)/dt > dV_i^{xnb}(t)/dt$ for all t.

Step 2: Note that at each tenure t the worker's hazard rate, $\delta + \lambda[1 - F_e(V_i^x(t) \mid x+t)]$, is a non increasing function of $V_i^x(.)$. This implies the survival probability, $\psi_i^x(.)$, is a non decreasing function of $V_i^x(.)$. Furthermore, note that the firm's profit flow per worker, $[p(x+t) - w_i^x(t)]$, is a decreasing function of $w_i^x(t)$. It then follows from Step 1 and Claim 1 that the firm maximises expected profit per new hire given V_i^x by setting $w_i^x(t) = \underline{w}$ for those t in which $V_i^x(t) < J^x(t)$. When $V_i^x(t) = J^x(t)$, say at $t = z$, the jointly efficient contract then pays marginal revenue product. Hence the optimal contract is a step contract as described by (12), where the promotion tenure, z, is determined by $V_i^x(0; w_i^x(.)) = V_i^x$.$\|$

<u>Proof of Claim 3</u>:

Consider first T_u^{0*}. As all starting offers imply T_u^{0*} then $1 - G(T_u^{0*} \mid 0) = 1$. Further, as $T_e^* < T_u^{0*}$ for $x > 0$, then conditional on remaining in the labour market, workers quit to $T_e^* < T_u^{0*}$ at rate λ. Hence $1 - G(T_u^{0*} \mid x) = e^{-\lambda x}$ for $0 < x < T_u^{0*}$.

Now fix a $x \in (0, T_u^{0*})$ and a $T_e^x \in (\underline{T}, T_u^{0*})$. Define κ where $T_e^*(\kappa) = T_e^x$. If $x < \kappa$, then all offers $T_e^*(s) < T_e^x$ for all experiences $s \in [0, x]$, and so $\partial G/\partial T_e^x = 0$. It then follows from the first part of the proof that $1 - G(T_e^x \mid x) = e^{-\lambda x}$.

Suppose instead $x > \kappa$. Steady state turnover implies for dx arbitrarily small

$$G(T_e^*(\kappa + dx) \mid x) - G(T_e^*(\kappa) \mid x) = \lambda dx[1 - G(T_e^*(\kappa + dx) \mid \kappa)] + O(dx^2),$$

where conditional on remaining in the labour market, the proportion of workers on contract $T_e^x \in [T_e^*(\kappa), T_e^*(\kappa + dx)]$ at experience x are those who at experience κ and on contract T_u^{0*} received an outside offer and so quit to a contract $T_e^\kappa \in [T_e^*(\kappa), T_e^*(\kappa + dx)]$. But at experience κ, $1 - G(T_e^*(\kappa + dx), \kappa) = e^{-\lambda\kappa}$. Dividing by dx and taking the limit $dx \to 0$ implies the condition stated in the Claim.

Finally, note that $A1$ implies that workers under contracts no greater than $T_e^*(\kappa)$ will never quit to an outside offer and that at experience κ, $G(T_e^*(\kappa), \kappa) = 1 - e^{-\lambda\kappa}$. Hence, conditional on those workers remaining in the labour market at experience x, $1 - G(T_e^*(\kappa), x) = e^{-\lambda\kappa}$. $\|$

Proof of Proposition 2:

Without any loss of generality assume p takes the following exponential form

$$p(x) = \overline{p} - p_0 e^{-\alpha x} \quad \text{for all } x \geq 0,$$

where $\overline{p} > p_0 > 0$ and $\alpha > 0$, where $\overline{p}$ determines the maximum human capital stock any worker could achieve and α describes the worker's "learning" rate. First consider the case of $\alpha = 0$. Workers do not accumulate human capital with experience and hence the marginal revenue product is given by a constant $p = p(0)$. In this case T_e^* is described by

$$\frac{dT_e^*(x)}{dx} = \frac{\lambda}{\delta}\left[e^{\delta(T_e^*(x)-x)} - 1\right]. \tag{26}$$

subject to the initial condition $T_e^*(0) = \underline{T}$, where $\underline{T} < \widehat{\underline{T}}_1$ and $\widehat{\underline{T}}_1 = \ln(1+\delta/\lambda)^{1/\delta}$ is the value of $T_e^*(0)$ given by (26) when $dT_e^*(0)/dx = 1$. Let $z^* = T_e^* - x$ denote the corresponding promotion tenure offered to a worker with experience x. In that case,

$$\frac{dz^*(x)}{dx} = \frac{\lambda}{\delta}\left[e^{\delta z^*} - 1\right] - 1, \tag{27}$$

which must be solved subject to the boundary condition $z^* = \underline{T}$ at $x = 0$. The corresponding phase diagram implies that if $\underline{T} < \widehat{\underline{T}}_1$, then z^* is strictly decreasing for all x. Further $T_e^* \to T_u^{0*}$ as $x \to T_u^{0*}$, which requires $z^* \to 0$, and then $T_u^{0*} > 0$ is determined where $z^*(T_u^{0*}) = 0$.

Assuming $\alpha > 0$ and following the same procedure, numerical solutions of (14) give similar diagrams for the corresponding z^*. In particular, letting $\overline{p} = 20$, $p_0 = 3$, $\lambda = 0.04$, $\delta = 0.01$ and $\alpha = 0.1$, Figure 1 (in the text) shows the behaviour of outside offers for several values of $z^*(0) = \underline{T}$. Note that in this case $\widehat{\underline{T}}_1 = 23.3$.

Proof of Claim 4:

First consider $T_e^x > T_e^*(x)$. Then

$$sign[\frac{\partial\Omega_e^*}{\partial T_e^x}] = sign[e^{-\lambda x}e^{-(\lambda+\delta)(\kappa-x)}\left[-\lambda\int_\kappa^{T_e^x} e^{-\delta(s-\kappa)}p(s)ds + [dT_e^*(\kappa)/dx]\,p(T_e^x)e^{-\delta(T_e^x-\kappa)}\right]].$$

Substituting out $dT_e^*(\kappa)/dx$ from (14) and using Proposition 2 implies the first part of the claim.

Now consider $T_e^x < T_e^*(x)$,and so

$$sign[\frac{\partial\Omega_e^*}{\partial T_e^x}] = sign[-\lambda\int_x^{T_e^x} e^{-\delta(s-x)}p(s)ds + [dT_e^*(\kappa)/dx]\,p(T_e^x)e^{-\delta(T_e^x-x)}].$$

Substituting out $dT_e^*(\kappa)/dx$ gives

$$sign[\frac{\partial\Omega_e^*}{\partial T_e^x}] = sign[-\lambda\int_x^{T_e^x} e^{-\delta(s-x)}p(s)ds + \lambda\int_\kappa^{T_e^x} e^{-\delta(s-\kappa)}p(s)ds].$$

$$sign[\frac{\partial\Omega_e^*}{\partial T_e^x}] = sign[\int_\kappa^{T_e^x} e^{-\delta(s-\kappa)}p(s)ds - \int_x^{T_e^x} e^{-\delta(s-x)}p(s)ds] > 0$$

as $\kappa < x$. $\|$

Proof of Claim 5:

Fix $x = 0$. Suppose a firm offers a contract $T_u^0 \in [\underline{T}, T_u^{0*}]$ to an unemployed worker with no experience. Define κ where $T_u^0 = T_e^*(\kappa)$ such that T_e^* satisfies Proposition 2. Given $A0$ and conditional on a hire, the firm makes expected profit

$$\Pi_u(0, T_u^0) = \int_0^\kappa e^{-(\lambda+\delta)s}p(s)ds + e^{-(\lambda+\delta)\kappa}\int_\kappa^{T_u^0} e^{-\delta(s-\kappa)}p(s)ds \tag{28}$$

as the worker quits for experience $s < \kappa$, does not quit thereafter, and the firm makes zero profits for experiences $s > T_u^0$. Differentiating the above equation with respect to T_u^0 yields

$$\frac{\partial\Pi_u(0,T_u^0)}{\partial T_u^0} = e^{-(\lambda+\delta)\kappa}\left[-\frac{\lambda\int_\kappa^{T_u^0} e^{-\delta(s-\kappa)}p(s)ds}{dT_e^*(\kappa)/dx} + p(T_u^0)e^{-\delta(T_u^0-\kappa)}\right].$$

Using (14) to substitute out $dT_e^*(\kappa)/dt$ such that T_e^* satisfies Proposition 2 leads to $\partial\Pi_u(0, T_u^0)/\partial T_u^0 = 0$ which implies the condition stated in the claim.$\|$