Housing and the Macroeconomy: The Role of Implicit Guarantees for Government Sponsored Enterprises*

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Abstract

This paper studies the macroeconomic effects of implicit government guarantees of Government-Sponsored Enterprises obligations. We construct a model with competitive housing and mortgage markets where the government provides banks with insurance against aggregate shocks to mortgage default risk. We use this model to evaluate aggregate and distributional impacts of this government subsidy to owner-occupied housing. Preliminary findings indicate that the subsidy leads to higher equilibrium housing investment, higher mortgage default rates and lower welfare. The welfare effects of this policy vary substantially across members of the population with different economic characteristics.

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1 Introduction

With close to 70% the United States displays one of the highest home ownership ratios in the world. Part of the attractiveness of owner-occupied housing stems from a variety of subsidies the government provides to homeowners. Apart from direct subsidies to low-income households via HUD programs, three important indirect subsidies exist. The first - and most well known - is the fact that mortgage interest payments (of mortgages up to $1 million) are tax-deductible. Second, the implicit income from housing investment (i.e. the imputed rental-equivalent) is not taxable, while other forms of capital income (e.g. interest, dividend and capital gains income) are being taxed. Gervais (2001) addresses the adverse effects of these two subsidies within a general equilibrium life-cycle model.

The third subsidy arises from the special structure of the US mortgage market. Essentially all home mortgages in the US are being sold from individual banks to so called Government Sponsored Enterprises (GSEs) who in turn refinance themselves via the bond market. The close link of GSEs to the federal government creates the impression that the government provides a guarantee to GSEs shielding them from aggregate risks, most notably aggregate credit risk which lowers their refinancing cost to below what private institutions would have to pay. Our paper is - to our knowledge - the first attempt to quantify the macroeconomic effects of this subsidy.

A formidable summary of the institutional details surrounding GSEs can be found in Frame and Wall (2002a) and (2002b). The three most important GSE are the two privately owned and publicly traded companies Fannie Mae (Federal National Mortgage Association) and Freddie Mac (Federal Home Loan Mortgage Association), and the FHLB (Federal Home Loan Bank system), a public and non-profit organization.

According to Frame and Wall, GSEs enjoy an array of government benefits for example being exempt from state and federal income taxes, a line of credit with the Treasury Department and very importantly a special status of GSE-issued debt. In particular, GSE securities can serve as substitutes to government bonds for transactions between public entities that normally require to be done in Treasuries. The Federal Reserve System also accepts GSE debt as a substitute for Treasuries in their portfolio of repurchase agreements. While no written federal guarantee for GSE debt exists, market participants view the special status of GSE debt as an indication of an implicit guarantee making them almost as safe as Treasury bills. The perception of a federal guarantee is further fueled by the sheer size of the GSE mortgage portfolio amounting to about 3 trillion dollars, 2.4 trillion dollars of which coming from the larger two GSEs, Fannie Mae and Freddie Mac. Insolvency of any one or both of these companies, say, due to an adverse shock in the real estate market that increases aggregate mortgage delinquency, will cause major disruptions in the financial system, which is why market participants consider housing GSEs to be too large to fail. Finally, two previous government bailouts of housing GSEs - Fannie Mae in the early 1980s and one of the smaller housing GSEs in the late 1980s - are further evidence of a bailout in case housing GSEs were to get into financial trouble.

The implicit federal guarantee is more than mere perception but most importantly it is reflected in interest rates GSEs pay when borrowing. GSEs can borrow at rates only marginally higher than the Treasury but about 40 basis points lower than private companies without a government guarantee according to the Congressional Budget Office CBO (2001). This is despite the fact that GSEs are highly leveraged entities with an equity cushion of only about 3% of their obligations, much lower than the 8.45% in the thrift industry (figures taken from Frame and Wall

(2002a)).

To the extend that part of the interest advantage of GSEs is passed through to homeowners, there exists a subsidy from the federal government to homeowners. The purpose of this paper is to set up a general equilibrium model with mortgage-financed housing to assess the macroeconomic effects of this subsidy on aggregate variables and the distributional effects. To this end we set up a heterogeneous agent model with aggregate uncertainty that drives the aggregate rate of mortgage delinquency. The aim is then to compare two economies, one in which the aggregate risk is priced into mortgages and one economy in which the government offers a tax-financed bailout in case of a bad aggregate shock, that is, the aggregate delinquency risk is not priced into mortgages. Due to the major computational burden of a model with aggregate uncertainty and heterogeneous agents we postpone the computation that particular model and instead start with a tax-financed direct subsidy on mortgage interest rates

In the numerical example in the current version of this paper aggregate uncertainty is not yet included. Due to the major computational burden of a model with aggregate uncertainty and heterogeneous agents we postpone that exercise to a later version of this paper and instead use a tax-financed direct subsidy on mortgage interest rates in an economy without aggregate uncertainty. The preliminary findings are that the subsidy produces over-investment in housing, reduces aggregate welfare, and creates adverse distributional effects. [1]

The remainder of the paper is organized as follows. Section 2 introduces the model and defines equilibrium in an economy with a housing and mortgage market. Section 3 characterizes equilibria. Section 4 describes the calibration of an econonomy without aggregate uncertainty and with a direct subsidy on mortgage interest rates. Section 5 details the numerical results comparing two steady states in economies with and without a mortgage interest subsidy. Section 6 concludes the paper.

2 The Model

The endowment economy is populated by a continuum of measure one of infinitely lived households, a continuum of competitive banks and a continuum of housing construction companies. Households face idiosyncratic endowment and housing depreciation shocks. In addition there may be aggregate shocks affecting endowments and housing depreciation. In what follows we will immediately proceed to describing the economy recursively, thereby skipping the (standard) sequential formulation of the economy.

2.1 Households

Households have endowment of the perishable consumption good given by yz. The aggregate part of endowments $z \in Z$ follows a finite state Markov chain with transition probabilities $\pi(z'|s)$ and unique invariant distribution $\Pi(z)$. The idiosyncratic part of endowments $y \in Y$ follows a finite state Markov chain with transition probabilities $\pi(y'|y, z', z)$ and unique invariant distribution $\Pi_z(y)$. That is, the distribution over idiosyncratic income shocks is allowed to depend on the aggregate state of the economy.

[1] Gruber and Martin (2003) also study the distributional effects of the inclusion of housing wealth in a general equilibrium model, but do not address the role of government housing subsidies for this question.

Households derive period utility $U(c,h)$ from consumption and housing services h, which can be purchased at a price p_l (relative to the numeraire consumption good). In addition to consumption and housing services the household can purchase two types of assets, one period bonds b' and houses g'. The price of bonds is denoted by P_b and the price of houses by P_h. Whereas households cannot short-sell bonds, they can borrow against their real estate property. Let by m' denote the size of their mortgage, and by P_m the receipt of resources (the consumption good) for each unit of mortgage issued and to be repaid tomorrow. These receipts will be determined in equilibrium by competition of banks, and will depend on the characteristics of households as well as the size of the mortgage m' as well as the size of the collateral g'. Houses depreciate stochastically; let $F_{\delta,z',y'}(\delta')$ denote the cumulative distribution function of the depreciation rate δ' tomorrow, which has support $D = [\underline{\delta}, \bar{\delta}]$ and may depend on the realized depreciation rate δ today as well as on the endowment realization of the household (z', y'). Households possess the option of defaulting on their mortgages, at the cost of losing their housing collateral. They will choose to do so whenever

$$m' > P_h(1-\delta')g'$$

If there is a government bailout guarantee, then the government levies taxes τ on endowments. It will use the receipts from these taxes to bail out part of the mortgages that private households have defaulted on. Finally let a denote cash at hand, that is, after tax endowment plus receipts from all assets brought into the period.

The individual state of a household consists of $s = (a, \delta, y)$, which reduces to $s = a$ in case idiosyncratic endowments and housing depreciation are *iid*. Let the cross-sectional distribution over individual states be given by μ; the aggregate state of the economy then consists of (z, μ). The dynamic programming problem of a household then reads as

$$v(s,z,\mu) = \max_{c,h,b',m',g' \geq 0} \left\{ U(c,h) + \beta \sum_{z'} \pi(z'|z) \sum_{y'} \pi(y'|y,z',z) \int_{\underline{\delta}}^{\bar{\delta}} v(s',z',\mu') dF_{\delta,z',y'}(\delta') \right\} \quad (1)$$

s.t.

$$c + b'P_b(z,\mu) + hP_l(z,\mu) + g'P_h(z,\mu) - m'P_m(s,g',m',z,\mu)) = a + g'P_l(z,\mu)$$

$$a'(\delta', y', m', g', z', \mu') = b' + \max\{0, P_h(z',\mu')(1-\delta')g' - m')\} + (1-\tau(z',\mu'))z'y'$$

with $\mu' = T(z, z', \mu)$. Note that the budget constraint implies the timing convention that newly purchased real estate g' can immediately be rented out in the same period. The function T describes the aggregate law of motion.

2.2 The Real Estate Construction Sector

Firms in the real estate construction sector act competitively and face the linear technology

$$I = A_h C_h$$

where I is the output of houses of a representative firm, C_h is the input of the consumption good and A_h is a technological constant, measuring the amount of consumption goods required to build one house. For now we assume that this technology is reversible, that is, real estate companies can turn houses back into consumption goods using the same technology. Thus the problem of a representative firm reads as

$$\begin{aligned} & \max_{I,C_h} P_h(z;\mu)I - C_h \\ & \text{s.t.} \\ I &= A_h C_h \end{aligned} \tag{2}$$

Thus the equilibrium house price necessarily satisfies

$$P_h(z;\mu) = \frac{1}{A_h}.$$

2.3 The Banking Sector

We assume (for now) that the risk free interest rate on one-period bonds r_b is exogenously given; one may interpret our economy as a small open economy. Thus $P_b = \frac{1}{1+r_b}$ is exogenously given as well. Mortgage receipts P_m for a mortgage of size m' against real estate of size g' are determined by perfect competition in the banking sector, which implies that banks make zero expected profits for *each mortgage* they issue (as in Chatterjee et al. (2002)), Banks take account of the fact that household may default on their mortgage, in which case the bank recovers the collateral value of the house, which we assume to be a fraction $\gamma \leq 1$ of the value of the real estate.

In order to define a typical banks' problem we first have to define the depreciation cut-off at which a household defaults on her mortgage. Define as $\kappa' = \frac{m'}{g'}$ the leverage (for $g' > 0$) of a mortgage m' backed by real estate g'. The default cutoff is defined by

$$\begin{aligned} m' &= (1-\delta^*(m',g',z',\mu'))P_h(z',\mu')g' \text{ or} \\ \delta^*(m',g',z',\mu') &= \begin{cases} \underline{\delta} & \text{if } 1-\frac{\kappa'}{P_h(z',\mu')} < \underline{\delta} \\ 1-\frac{m'}{g'P_h(z',\mu')} = 1-\frac{\kappa'}{P_h(z',\mu')} & \text{if } 1-\frac{\kappa'}{P_h(z',\mu')} \in [\underline{\delta},\bar{\delta}] \\ \bar{\delta} & \text{if } 1-\frac{\kappa'}{P_h(z',\mu')} > \bar{\delta} \end{cases} \end{aligned}$$

Evidently a household that obtains a mortgage $m' > 0$ without collateral, i.e. with $g' = 0$ defaults for sure. The receipt for this mortgage thus necessarily has to equal 0 as well, i.e. $P_m(s, g' = 0, m', z, \mu) = 0$. For other types of mortgages (m', g') with $m' > 0$ and $g' > 0$, the banks' problem is to choose the price $P_m(s, g', m', z, \mu)$ to maximize

$$\begin{aligned} & \max_{P_m(s,g',m',z,\mu)} \left[\begin{array}{c} -m'P_m(s,g',m',z,\mu) + P_b(z,\mu)\sum_{z'}\pi(z'|z)\sum_{y'}\pi(y'|y,z',z)* \\ \left\{ m'F_{\delta,z',y'}(\delta^*(m',g',z',\mu')) + \gamma P_h(z',\mu')g' \int_{\delta^*(m',g',z',\mu')}^{\bar{\delta}} (1-\delta')dF_{\delta,z',y'}(\delta') \right\} \end{array} \right] \\ = \; & m' \max_{P_m(s,g',m',z,\mu)} \left[\begin{array}{c} -P_m(s,g',m',z,\mu) + P_b(z,\mu)\sum_{z'}\pi(z'|z)\sum_{y'}\pi(y'|y,z',z)* \\ \left\{ F_{\delta,z',y'}(\delta^*(m',g',z',\mu')) + \frac{\gamma P_h(z',\mu')}{\kappa'} \int_{\delta^*(m',g',z',\mu')}^{\bar{\delta}} (1-\delta')dF_{\delta,z',y'}(\delta') \right\} \end{array} \right] \end{aligned} \tag{3}$$

In the presence of a government bailout, the government effectively subsidizes mortgages, in forms to be specified below.

2.4 The Government

As stated above the government levies endowment taxes $\tau(z,\mu)$ on households to subsidize mortgages. Subsidies take the form of interest rate subsidies (other forms of mortgage subsidies can be easily mapped in to these interest rate subsidies.

Define the interest rate on a mortgage with characteristics (m', g') as

$$r_m(s,g',m',z,\mu) = \frac{1}{P_m(s,g',m',z,\mu)} - 1$$

where $P_m(s,g',m',z,\mu)$ is the mortgage pricing function without subsidy. Define as $\hat{r}_m(s,g',m',z,\mu)$ and $\hat{P}_m(s,g',m',z,\mu)$ the corresponding entities with subsidy. Since the subsidy is a mortgage interest rate subsidy we model this as

$$\hat{r}_m(s,g',m',z,\mu) = r_m(s,g',m',z,\mu) - sub(s,g',m',z,\mu)$$

and thus

$$\hat{P}_m(s,g',m',z,\mu) = \frac{P_m(s,g',m',z,\mu)}{1 - sub(s,g',m',z,\mu) * P_m(s,g',m',z,\mu)} \geq P_m(s,g',m',z,\mu)$$

The total subsidy for a mortgage of characteristics (s,g',m',z,μ) is thus

$$\begin{aligned}
Sub(s,g',m',z,\mu) &= m'\left(\hat{P}_m(s,g',m',z,\mu) - P_m(s,g',m',z,\mu)\right) \\
&= m'P_m(s,g',m',z,\mu)\left(\frac{sub(s,g',m',z,\mu)P_m(s,g',m',z,\mu)}{1 - sub(s,g',m',z,\mu)P_m(s,g',m',z,\mu)}\right)
\end{aligned}$$

and the total economy-wide subsidy is

$$Aggsub(z,\mu) = \int Sub(s,g',m',z,\mu)d\mu$$

Thus taxes have to satisfy

$$\begin{aligned}
\tau(z,\mu)\int zyd\mu &= Aggsub(z,\mu) \\
\tau(z,\mu) &= \frac{Aggsub(z,\mu)}{z\bar{y}_z}
\end{aligned} \tag{4}$$

where $\bar{y}_z$ is average (aggregate) endowment if the aggregate state of the economy is z.

2.5 Equilibrium

We are now ready to define a Recursive Competitive Equilibrium. Let $S = R_+ \times D \times Y$ denote the individual state space and $\mathcal{M}$ the space of finite measures over the measurable space $(S, \mathcal{S})$, where $\mathcal{S} = \mathcal{B}(R_+) \times \mathcal{B}(D) \times \mathcal{P}(Y)$ and $\mathcal{B}$ is the Borel σ-algebra and $\mathcal{P}$ is the power set, so that $\mathcal{S}$ is a well-defined σ-algebra over S.

Definition 1 *Given a government subsidy policy* $sub : S \times R_+ \times R_+ \times Z \times \mathcal{M} \rightarrow R$*, a* ***Recursive Competitive Equilibrium*** *are value and policy functions for the households,* $v, c, h, b', m', g' : S \times Z \times \mathcal{M} \rightarrow R$*, policy functions for the real estate construction sector* $I, C_h : Z \times \mathcal{M} \rightarrow R$*, pricing functions* $P_l, P_h, P_b : Z \times \mathcal{M} \rightarrow R$*, mortgage pricing functions* $P_m, \hat{P}_m : S \times R_+ \times R_+ \times Z \times \mathcal{M} \rightarrow R$*, a government tax policy* $\tau : Z \times \mathcal{M} \rightarrow R$ *and an aggregate law of motion* $T : Z \times Z \times \mathcal{M} \rightarrow \mathcal{M}$ *such that*

1. *(Household Maximization) Given prices* $P_l, P_h, P_b, \hat{P}_m$ *and government policies the value function solves* (1) *and* c, h, b', m', g' *are the associated policy functions.*

2. *(Real Estate Construction Company Maximization) Given* P_h*, policies* I, C_h *solve* (2).

3. *(Bank Maximization) Given* P_h, P_b*, the function* P_m *solves* (3)

4. *(Small Open Economy Assumption) The function* P_b *is exogenously given by*

$$P_b(z, h) = \frac{1}{1 + r_b}$$

   where r_b *is the exogenously given fixed world risk free interest rate*

5. *(Government Budget Balance) The tax rate function* τ *satisfies* (4)*, given the functions* $m', P_m, \hat{P}_m, sub$*.*

6. *(Market Clearing in Rental Market) For all* (μ, z)

$$\int g'(s, z, \mu) d\mu = \int h(s, z, \mu) d\mu$$

7. *(Aggregate Law of Motion) The aggregate law of motion* T *is generated by the exogenous Markov processes* π *and the policy functions* m', g', b'

3 Theoretical Results

In this section we state theoretical properties of our model the use of which makes the computation of the model easier. These results consist of a characterization of the mortgage interest rate, a partial characterization of the solution to the household maximization problem and, finally, bounds on the equilibrium rental price $P_l(z, h)$.

3.1 Mortgage Interest Rates

From equation (3) and the fact that competition requires profits for all mortgages issued in equilibrium to be zero we immediately obtain a characterization of equilibrium mortgage payoffs as

$$\begin{aligned} P_m(s, g', m', z, \mu) &= P_b(z, \mu) \sum_{z'} \pi(z'|z) \sum_{y'} \pi(y'|y, z', z) * \\ & \left\{ F_{\delta, z', y'}(\delta^*(m', g', z', \mu')) + \frac{\gamma P_h(z', \mu')}{\kappa'} \int_{\delta^*(m', g', z', \mu')}^{\bar{\delta}} (1 - \delta') dF_{\delta, z', y'}(\delta') \right\} \\ &= P_m(s, \kappa', z, \mu) \end{aligned}$$

with implied interest rates

$$r_m(s,\kappa',z,\mu) = \frac{1}{P_m(s,\kappa',m',z,\mu)} - 1$$

We note the following facts:

1. Besides the aggregate state variables the only information determining mortgage interest rates are the individual states δ, y and the leverage of the mortgage $\kappa' = \frac{m'}{g'}$. If income and depreciation shocks are *iid*, then $P_m(s,\kappa',z,\mu) = P_m(\kappa',z,\mu)$ and mortgages are priced exclusively based on leverage and aggregate conditions.

2. $P_m(s,\kappa',z,\mu)$ is decreasing in κ', strictly so if the household defaults with positive probability. Thus mortgage interest rates are increasing in leverage κ'.

3. Households that repay their mortgage with probability one have $\delta^*(m',g',z',\mu') = \bar{\delta}$ and thus $P_m(s,g',m',z,\mu) = P_b$, i.e. can borrow at the risk free rate r_b.

4. Since for all $\delta' > \delta^*(m',g',z',\mu')$ we have $\gamma P_h(z',\mu')\kappa'(1-\delta') < 1$, households that do default with positive probability tomorrow receive $P_m(s,g',m',z,\mu) < P_b$ today, that is, they borrow with a risk premium $r_m(s,g',m',z,\mu) > r_b$.

3.2 Simplification of the Household Problem

In the household problem define as

$$\begin{aligned} u(c;P_l) &= \max_{\tilde{c},h\geq 0} U(\tilde{c},h) \\ & \quad s.t. \\ \tilde{c} + P_l(z,\mu)h &= c \end{aligned}$$

Then the above problem can be rewritten as

$$v(s,z,\mu) = \max_{c,b',m',g'\geq 0} \left\{ u(c;P_l(z,\mu)) + \beta \sum_{z'} \pi(z'|z) \sum_{y'} \pi(y'|y,z',z) \int_{\underline{\delta}}^{\bar{\delta}} v(s',z',\mu') dF_{\delta,z',y'}(\delta') \right\}$$

$$\begin{aligned} \text{s.t.} \qquad & c + b'P_b(z,\mu) + g'\left[P_h(z,\mu) - P_l(z,\mu)\right] - m'\hat{P}_m(s,g',m',z,\mu) = a \\ a'(\delta',h',m',g',z',\mu') &= b' + \max\{0, P_h(z',\mu')(1-\delta')g' - m'\} + (1-\tau(z',\mu'))z'y' \\ \mu' &= T(z,z',\mu) \end{aligned}$$

For future reference, in the absence of aggregate uncertainty and with individual shocks being *iid* the individual state variables collapse to just cash at hand a' and the problem becomes

$$v(a) = \max_{c,b',m',g'\geq 0} \left\{ u(c;P_l) + \beta \sum_{y'} \pi(y') \int_{\underline{\delta}}^{\bar{\delta}} v(a')dF(\delta') \right\}$$

$$\begin{aligned} \text{s.t.} \qquad & c + b'P_b + g'\left[P_h - P_l\right] - m'\hat{P}_m(\frac{m'}{g'}) = a \\ a'(\delta',y',m',g') &= b' + \max\{0, P_h(1-\delta')g' - m'\} + (1-\tau)y' \end{aligned}$$

3.3 Endogenous Borrowing Limit

We now want to show that it is never strictly beneficial for a household to obtain a mortgage with higher leverage than that level which will lead to default for sure. We will carry out the discussion in the next two subsections for the case without government bailout policy; the analysis goes through unchanged with government policy, mutatis mutandis. Remember that by construction $P_h(z', \mu') = P_h = \frac{1}{A_h}$. Define the leverage that leads to certain default by the smallest number $\bar{\kappa}$ such that

$$\begin{aligned} \delta^*(\bar{\kappa}, z', \mu') &= \underline{\delta} \\ \bar{\kappa} &= (1 - \bar{\delta})P_h = \frac{1 - \underline{\delta}}{A_h} \end{aligned}$$

Now we rewrite the budget constraint as

$$\begin{aligned} c + b'P_b(z, \mu) + g'\left[P_h(z, \mu) - P_l(z, \mu) - \frac{m'}{g'}P_m(s, \frac{m'}{g'}, z, \mu)\right] &= a \text{ or} \\ c + b'P_b(z, \mu) + g'\left[P_h(z, \mu) - P_l(z, \mu) - \kappa' P_m(s, \kappa', z, \mu)\right] &= a \text{ or} \\ c + b'P_b(z, \mu) + g'P(s, \kappa', z, \mu) &= a \end{aligned}$$

where

$$P(s, \kappa', z, \mu) = P_h(z, \mu) - P_l(z, \mu) - \kappa' P_m(s, \kappa', z, \mu)$$

is the is downpayment per unit of real estate purchased, net of rental income. With this definition the total downpayment is given by $g'P(s, \kappa', z, \mu)$

For all $\kappa' \geq \bar{\kappa}$ we have

$$\begin{aligned} \kappa' P_m(s, \kappa', z, \mu) &= P_b(z, \mu) \sum_{z'} \pi(z'|z) \sum_{y'} \pi(y'|y, z', z) * \\ &\quad \left\{ \kappa' F_{\delta, z', y'}(\underline{\delta}) + \gamma P_h(z', \mu') \int_{\underline{\delta}}^{\bar{\delta}} (1 - \delta') dF_{\delta, z', y'}(\delta') \right\} \\ &= P_b(z, \mu) \sum_{z'} \pi(z'|z) \sum_{y'} \pi(y'|y, z', z) \gamma P_h(z', \mu') \int_{\underline{\delta}}^{\bar{\delta}} (1 - \delta') dF_{\delta, z', y'}(\delta') \\ &= P_b(z, \mu) \sum_{z'} \pi(z'|z) \sum_{y'} \pi(y'|y, z', z) \gamma P_h(z', \mu')(1 - E_{\delta, z', y}(\delta')) \\ &= \bar{\kappa} P_m(s, \bar{\kappa}, z, \mu) \end{aligned}$$

and thus leveraging further does not bring extra revenues today and does not change resources obtained tomorrow (since the household defaults for sure and thus loses all real estate).[2] That is, the household faces an endogenous effective borrowing constraint of the form

$$\begin{aligned} \kappa' &\leq \bar{\kappa} \text{ or} \\ m' &\leq \left[\frac{1 - \underline{\delta}}{A_h}\right] g' \end{aligned}$$

One can interpret $1 - \bar{\kappa}$ as the minimum downpayment requirement in this economy.

[2]The household is obviously indifferent between choosing $\kappa' = \bar{\kappa}$ and $\kappa' > \bar{\kappa}$; from here on we resolve any indifference of the household by assuming that in this case he chooses $\kappa' = \bar{\kappa}$.

3.4 Bounds on the Rental Price of Housing

3.4.1 An Upper Bound

Evidently for all admissible choices of the household it has to be the case that $P(s, \kappa', z, \mu) \geq 0$, otherwise the household can obtain positive cash flow today by buying a house; the default option on the mortgage guarantees that the cash flow from the house is non-negative. Thus, the absence of this arbitrage in equilibrium requires $P(s, \kappa', z, \mu) \geq 0$. Therefore in particular

$$P(s, \kappa' = \bar{\kappa}, z, \mu) = P_h(z, \mu) - P_l(z, \mu) - \bar{\kappa} P_m(s, \kappa' = \bar{\kappa}, z, \mu) \geq 0$$

But

$$\begin{aligned} P(s, \bar{\kappa}, z, \mu) &= P_h(z, \mu) - P_l(z, \mu) - \bar{\kappa} P_m(s, \bar{\kappa}, z, \mu) \\ &= P_h(z, \mu) - P_l(z, \mu) - P_b(z, \mu) \sum_{z'} \pi(z'|z) \sum_{y'} \pi(y'|y, z', z) \gamma P_h(z', \mu')(1 - E_{\delta, z', y}(\delta')) \\ &\geq 0 \\ P_l(z, \mu) &\leq P_h(z, \mu) - P_b(z, \mu) \sum_{z'} \pi(z'|z) \sum_{y'} \pi(y'|y, z', z) \gamma P_h(z', \mu')(1 - E_{\delta, z', y}(\delta')) \end{aligned}$$

which places an upper bound on the equilibrium rental price.

Without aggregate uncertainty and *iid* income and depreciation shocks this inequality becomes

$$\begin{aligned} P_l &\leq P_h - \gamma P_b P_h (1 - E(\delta')) \\ &= P_h * \left[\frac{r_b + \gamma E(\delta) + 1 - \gamma}{1 + r_b} \right] \end{aligned}$$

If $\gamma = 1$, this condition simply states that the rental price P_l cannot be larger that the user cost of housing $\frac{r_b + E(\delta)}{1 + r_b}$.

3.4.2 A Lower Bound

Housing is an inherently risky asset. Since households are risk averse, for them to purchase the housing asset the expected return of housing at zero leverage has to be at least as high as the risk free interest rate. This implies

$$P_b(z, \mu) \sum_{z'} \pi(z'|z) \sum_{y'} \pi(y'|y, z', z) P_h(z', \mu') \int_{\underline{\delta}}^{\bar{\delta}} (1 - \delta') dF_{\delta, z', y'}(\delta') \geq P_h(z, \mu) - P_l(z, \mu)$$

Remembering that $P_h(z, \mu) = P_h(z', \mu') = P_h = \frac{1}{A_h}$ yields

$$\begin{aligned} P_b(z, \mu) P_h (1 - E_{\delta, z, y}(\delta')) &\geq P_h - P_l(z, \mu) \text{ or} \\ P_l(z, \mu) &\geq P_h \left[\frac{r_b + E_{\delta, z, y}(\delta')}{1 + r_b} \right] \end{aligned}$$

which states that the rental price of housing cannot be smaller than the (expected) user cost of housing in equilibrium (otherwise nobody would invest in housing, which cannot be an equilibrium given strictly positive demand for housing services by consumers).[3]

In summary, what these theoretical results buy us, besides being interesting in its own right, is a simplified household problem, a concise characterization of the high-dimensional equilibrium mortgage interest rate function and bounds for the equilibrium rental price, the only endogenous price to be determined in our analysis.

4 Calibration

4.1 Technology

Summary

Parameter	Interpretation	Value	Target
A_h	Technology Const. in Housing Constr.	1.0	none (normalized)
π	Transition Matrix for Income	see below	Tauchen $\rho = 0.98, \sigma_e = 0.30$
y	Income States	see below	Tauchen $\rho = 0.98, \sigma_e = 0.30$
μ_δ	Depreciation	0.0199	$E(\delta) = 0.0148$
σ_δ	Std. Dev. of Depreciation	0.10	OFHEO volatility
$\bar{\delta}$	Upper Bound on Depreciation	0.3429	$1 - \exp\{-\mu_\delta - 4\sigma_\delta\}$
$\underline{\delta}$	Lower Bound on Depreciation	-0.4624	$1 - \exp\{-\mu_\delta + 4\sigma_\delta\}$

Foreclosure technology The default technology parameter γ has been estimated by Pennington-Cross (2004) who looks at the sales revenue from foreclosed houses and compares it to a market price constructed via the OFHEO repeat sales index. He finds that on average the loss is 22%. The loss varies only slightly depending on the age of the loan, between 20% for loans 16-20 months old to 26% for loans up to 10 months old, so it is safe to assume that in the model $\gamma = 0.78$ for all loans.

The depreciation process The Office of Federal Housing Enterprise Oversight (OFHEO) models house prices as a diffusion process and estimates within-state and within-region annual house price volatility. The technical details can be found in the paper by Calhoun (1996). The ballpark figure for the eight census regions is $9 - 10\%$ volatility in the years 1998-2004. We use the upper bound $\sigma_\delta = 0.10$ to account for the fact that nationwide volatility is slightly higher than the within-region volaility. Assume that $(1 - \delta)$ is log-normally distributed, that is, $\log(1 - \delta) \sim N(-\mu_\delta, \sigma_\delta^2)$. The average depreciation for residential housing according to the Bureau of Economic Analysis was 1.48% between 1960 and 2002 (standard deviation 0.05%), computed as consumption of fixed capital in the housing sector (Table 7.4.5) divided by the

[3]Without aggregate uncertainty and $\gamma = 1$ we thus immediately obtain that the rental price of housing equals its user cost. In fact, what happens in this equilibrium is that households purchase houses, leverage such that they default for sure tomorrow and the houses end up in the hand of the banks. Since these are risk-neutral, default is fully priced into the mortgage and banks receive the full (depreciated) value of the house, banks rather than households (which are risk averse) should and will end up owning the real estate.

capital stock of residential housing. Since the mean of the log-Normal is $\exp\left\{-\mu_\delta + \frac{1}{2}\sigma_\delta^2\right\}$, we set $\mu_\delta = \frac{1}{2}\sigma_\delta^2 - \log 0.9852 \approx 1.99\%$ in order to match the average depreciation of 1.48%.

In the program we have to truncate the support for δ to $[\delta_l, \delta_h]$. We draw $\log(1-\delta)$ from a range of ± 4 standard deviations around μ_δ. This makes sure that the moments of the simulated truncated distribution are indistinguishable from theoretical moments and also the probabilities of drawing from the far right tail of the distribution - depreciation rates high enough to trigger default - are close enough to their theoretical values:

	Analytical	Truncated
$E(\delta)$	0.0148	0.0149
$St.dev.$	0.1000	0.0999
$P(\delta \geq 0.20)$	0.021055	0.021024
$P(\delta \geq 0.25)$	0.003705	0.003674
$P(\delta \geq 0.30)$	0.000379	0.000347

Housing Technology We normalize the housing construction constant at $A_h = 1.0$.

Income process For a continuous state $AR(1)$ process of the form

$$\begin{aligned} \log y' &= \rho \log y + (1-\rho^2)^{0.5}\varepsilon \\ E(\varepsilon) &= 0 \\ E\left(\varepsilon^2\right) &= \sigma_e^2 \end{aligned} \tag{5}$$

we can calculate the unconditional standard deviation to be σ_e and the one-period autocorrelation (persistence) to be ρ. Estimates for ρ range from $[0.53, 1]$ where the lower number is somewhat of an outlier. We choose $\rho = 0.98$. The estimates for the standard deviation range from 0.2 to 0.4, so we pick $\sigma_\varepsilon = 0.3$.

We approximate the continuous state AR(1) with a 5 state Markov chain using the procedure put forth by Tauchen and Hussey (1991). We get the five labor productivity shocks $y \in \{0.3586, 0.5626, 0.8449, 1.2689, 1.9909\}$ and the following transition matrix:

$$\Pi = \begin{bmatrix} 0.7629 & 0.2249 & 0.0121 & 0.0001 & 0.0000 \\ 0.2074 & 0.5566 & 0.2207 & 0.0152 & 0.0001 \\ 0.0113 & 0.2221 & 0.5333 & 0.2221 & 0.0113 \\ 0.0001 & 0.0152 & 0.2207 & 0.5566 & 0.2074 \\ 0.0000 & 0.0001 & 0.0121 & 0.2249 & 0.7629 \end{bmatrix}$$

which generates the stationary distribution $(0.190722, 0.206633, 0.205290, 0.206633, 0.190722)$ and and average labor productivity of one.

4.2 Preferences

For the utility function we start with a CES functional form:

$$u(c,h) = (1-\beta)\frac{\left(\theta c^\nu + (1-\theta)h^\nu\right)^{\frac{1-\sigma}{\nu}} - 1}{1-\sigma}$$

Notice that the first order conditions in the intratemporal optimization problem yield the condition

$$\frac{h}{c} = \left(P_l \frac{\theta}{1-\theta}\right)^{\frac{1}{\nu-1}}$$

which implies that in steady state θ and ν cannot be pinned down at the same time. We therefore assume for now that $\nu = 0$, and therefore:

$$u(c,h) = \frac{c^{\theta(1-\sigma)} h^{(1-\theta)(1-\sigma)} - 1}{1-\sigma}$$

which allows us to easily calibrate θ to the share of housing vs. non-housing consumption.

	Parameter	Interpretation	Value	Target
Summary	σ	Risk Aversion	TBA	Bond portfolio shares
	β	Time Discount Factor	TBA	Net Worth/Income
	θ	Share Parameter on Nondur. Cons.	0.86	Exp. Share in Data

Details The risk aversion and time discount parameters are calibrated to match targets in the data using the benchmark economy without aggregate uncertainty. We use data from the Survey of Consumer Finances and restrict our attention to only bonds and net real estate, that is real estate holdings net of mortgages, and compute the bond share and net worth to income ratio as a) the unrestricted mean over all households, b) the restricted mean of all households having a net worth smaller than 50 times median income[4] and c) the mean within the median net worth bin using 25 equally-sized bins along household net worth. The results are reported below:

	unrestricted mean	restricted mean	median bin
Bond Share	0.4473	0.3854	0.2832
Net worth / income	2.7733	2.2666	1.2137

One can see from this table that bond shares and net worth ratios are affected a lot by extremely high net worth households. Since we will have trouble matching the skewness of the wealth distribution we decided to match the moments at the median household. Using $\sigma = 1.74$ and $\beta = 0.95$ generates a bond share of around 40% and a net worth to income ratio of about 1.30 which is close enough for now.

The share of housing in total consumption θ is set to generate a realistic share of housing in total consumption which has been steady at 14% over the last 40 years with a standard deviation of only about 0.5%. Hence, we set $\theta = 0.86$.

4.3 Policy and Markets

	Parameter	Interpretation	Value	Target
Summary	sub	Implicit Interest Rate Subsidy	40 BP	Passmore
	r_b	Risk Free Interest Rate on Bonds	0.01	1 year real return on TIPS

[4] This would eliminate the top 0.93% of the wealth distribution.

Details On the interest rate subsidy we take the view that the pass-through is 100% to make the case for the GSEs as positive as possible. The subsidy is then chosen to match the estimated implicit interest rate differential of around 40 basis points.

The risk-free interest rate is set to the real return on risk-free government bonds with maturity equal to the length of model period, that is, probably a year. 1% is a resonable estimate.

5 Results

In this section we document preliminary results from the first thought experiment, that is, we compare steady states of economies with and without a mortgage interest rates subsidy of 40 basis points. Table I summarizes the main macroeconomic aggregates

Table 1: Consequences of Removing the Subsidy

Variable	Subsidy	No Subsidy	Difference
P_l	0.026875	0.029116	8.34%
H	5.407190	4.937504	-6.88%
M	3.752130	2.741853	-26.93%
Def. prob	0.0514%	0.0171%	-66.73%
median net worth	1.259915	1.250381	-0.76%
Wealth Gini	0.510921	0.503774	-0.007147
$Sub/\bar{y}$	0.014858	0.000000	-100%
τ	0.014867	0.000000	-100%
EV^{SS}	-1.416980	-1.416711	0.019%

We see that removing the subsidy decreases the equilibrium housing stock and rental demand H by almost 7% and increases the rental price by about 8%. Households use far fewer mortgages in the absense of the subsidy partially due to less housing consumption, but mainly due to lower leverage. Not surprisingly, the most significant impact of the subsidy is on mortgage default rates, which are substantially lower without the subsidy. The overall size of the subsidy and thus the tax rate to finance it is quite substantial at about 1.5%.

Removing the subsidy reduces median net worth by about three quarters of a percent and also reduces the wealth Gini coefficient by about 0.007. In terms of welfare, the subsidy policy reduces steady state welfare by a modest 2/100 of a percent: households consumption (of nondurables and housing services) in the steady state with the subsidy has to be increased by this amount to be indifferent between the steady state with and the one without subsidy policy. Which suggests that the subsidy tends to benefit mainly higher income and higher net worth households.

6 Conclusions

We constructed a model with competitive housing and mortgage markets where the government provides banks with insurance against aggregate shocks to mortgage default risk. We used this model to evaluate aggregate and distributional impacts of this implicit government subsidy to owner-occupied housing. Our main findings are that the subsidy policy leads to lower welfare, more mortgages issued and a higher housing stock as well as more mortgage delinquencies.

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